

# Effective theories for accelerator neutrino cross sections

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28th Rencontres de Blois

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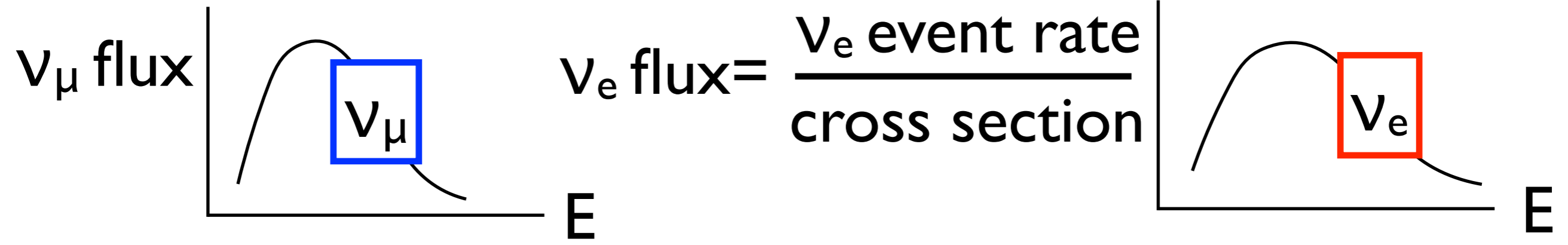
## Overview

- introduction: importance of accelerator neutrino cross sections ( $E_\nu \sim \text{GeV}$ )
- deuteron constraints on the elementary signal process  $\sigma(\nu_\ell n \rightarrow \ell^- p)$ 

*1603.03048, with A. Meyer, M. Betancourt and R. Gran and related work with B. Bhattacharya and G. Paz*
- new formalism for radiative corrections impacting  $\sigma(\nu_e n \rightarrow e^- p)/\sigma(\nu_\mu n \rightarrow \mu^- p)$ 

*1605.02613, and related work with J. Arrington, G. Lee*
- summary

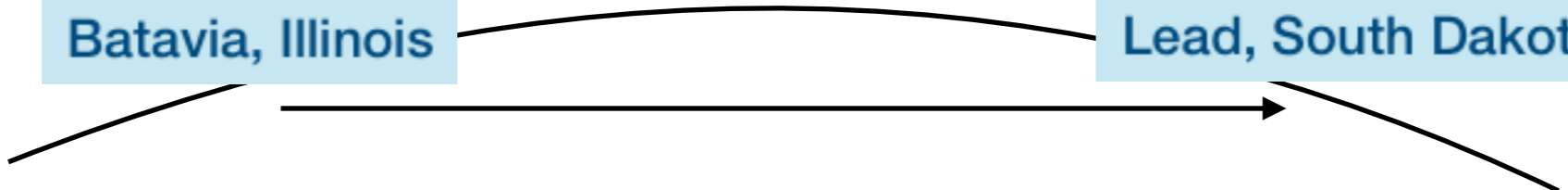
# probability of $\nu_\mu \rightarrow \nu_e \Rightarrow$ fundamental neutrino properties



E.g. DUNE

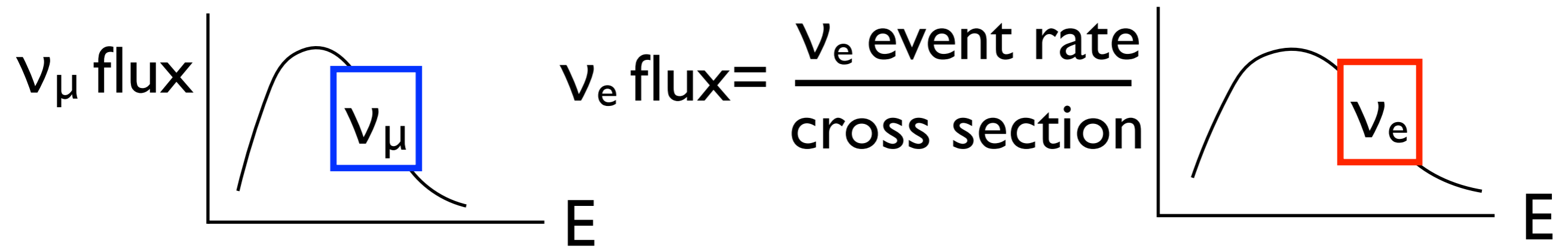
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Cross section translates observed event rate to  $\nu_e$  appearance prob.

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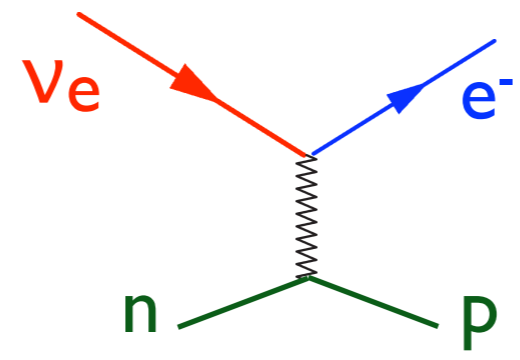
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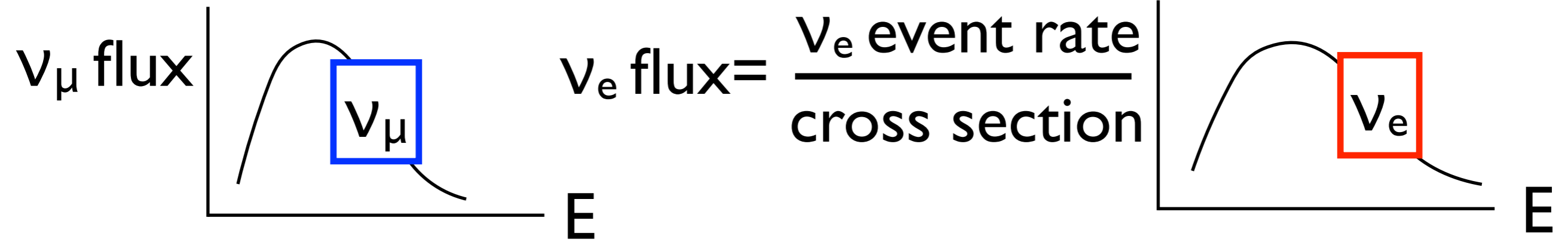
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*Basic signal process: charged current quasi elastic scattering  
(large event sample, "reconstructible" neutrino energy, theoretically "clean")*



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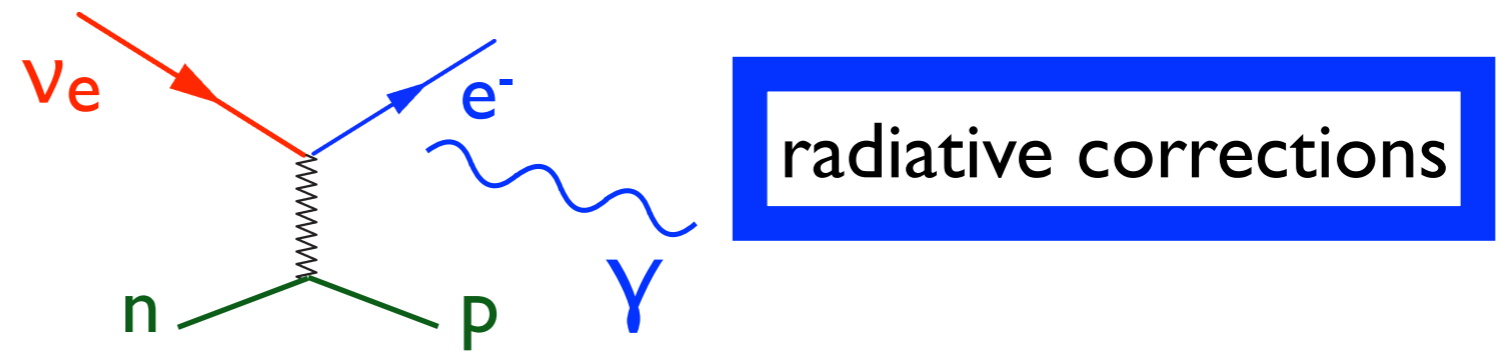
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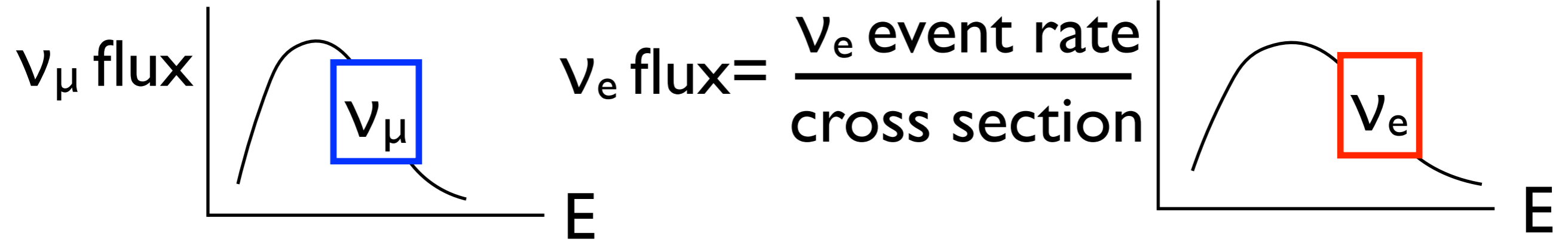
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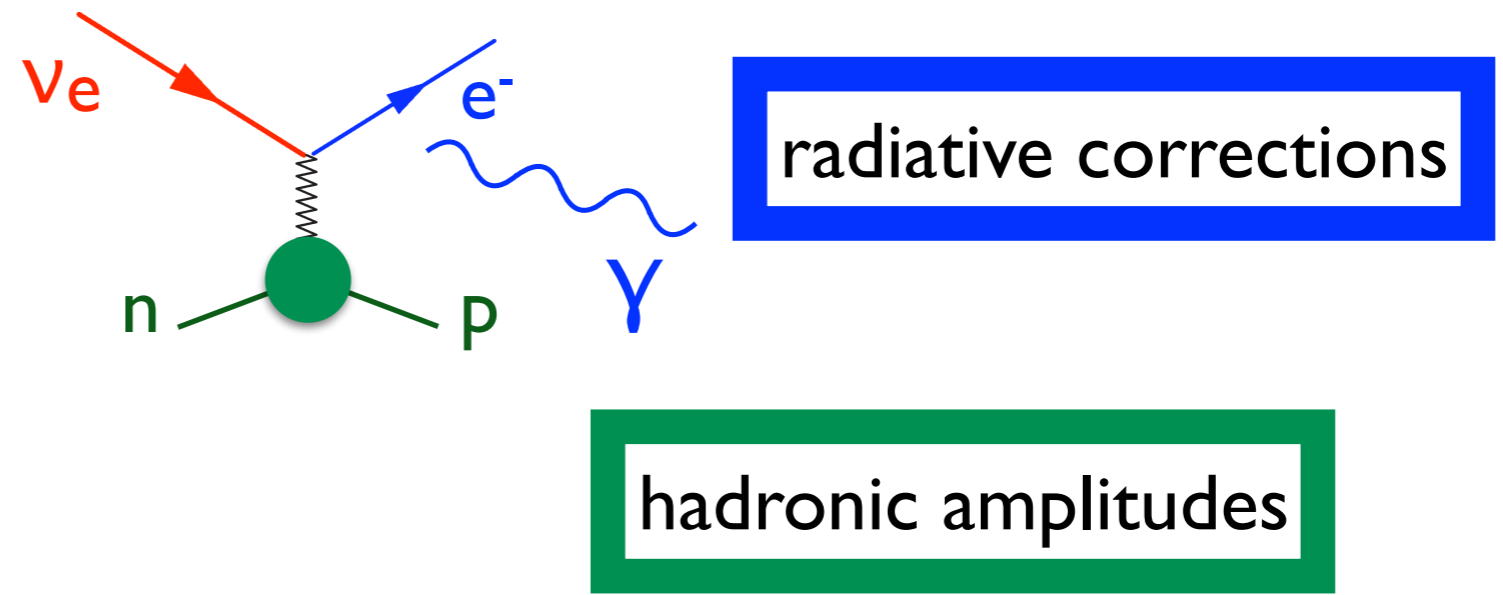
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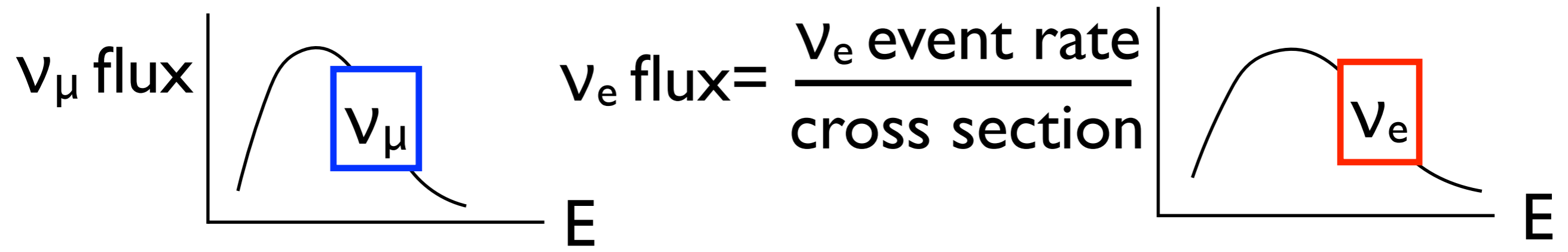
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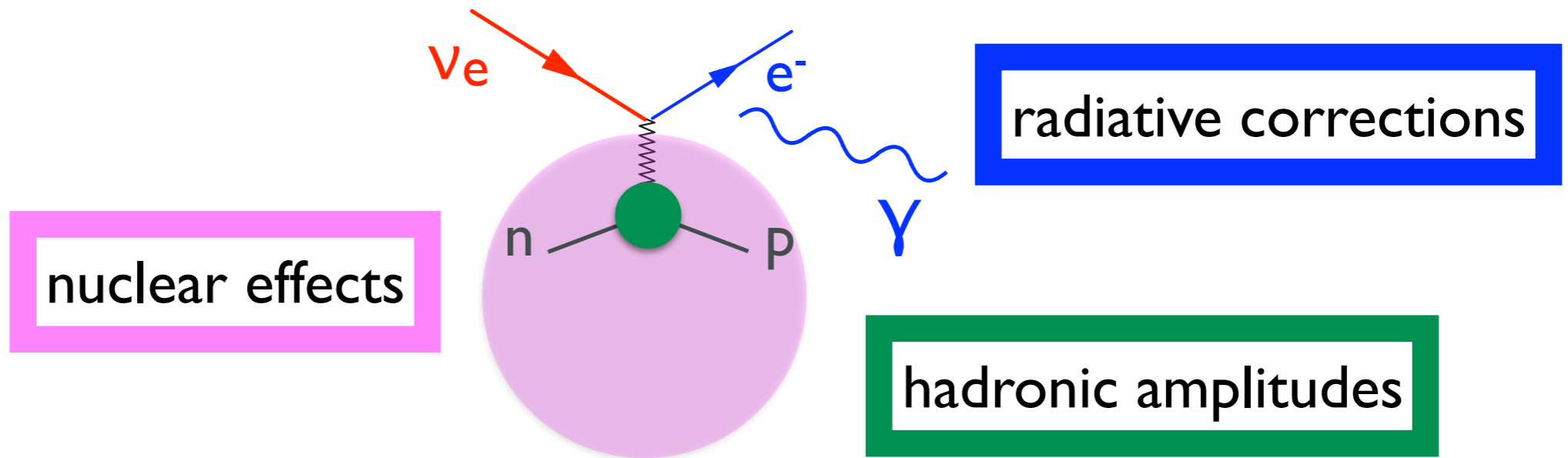
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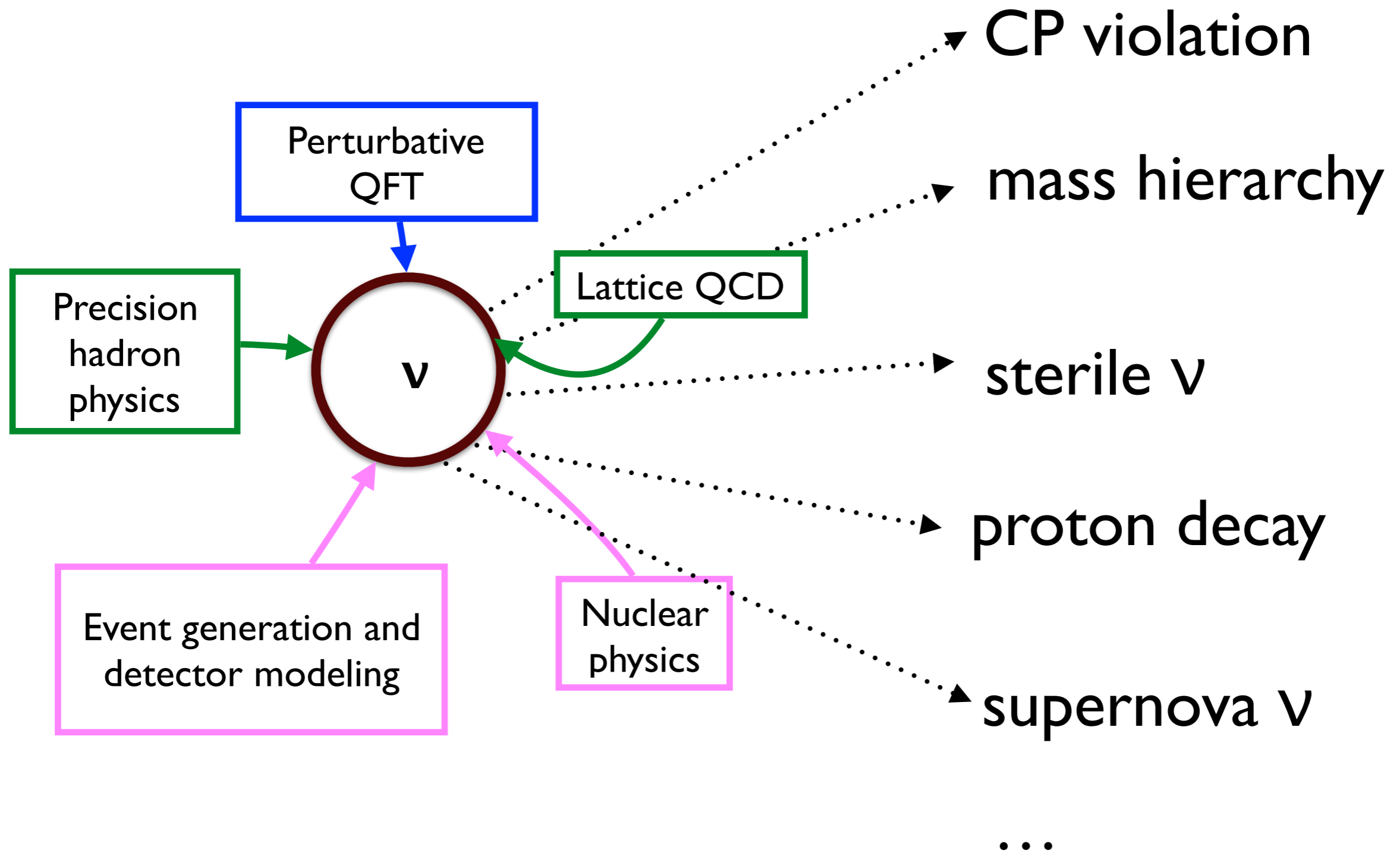
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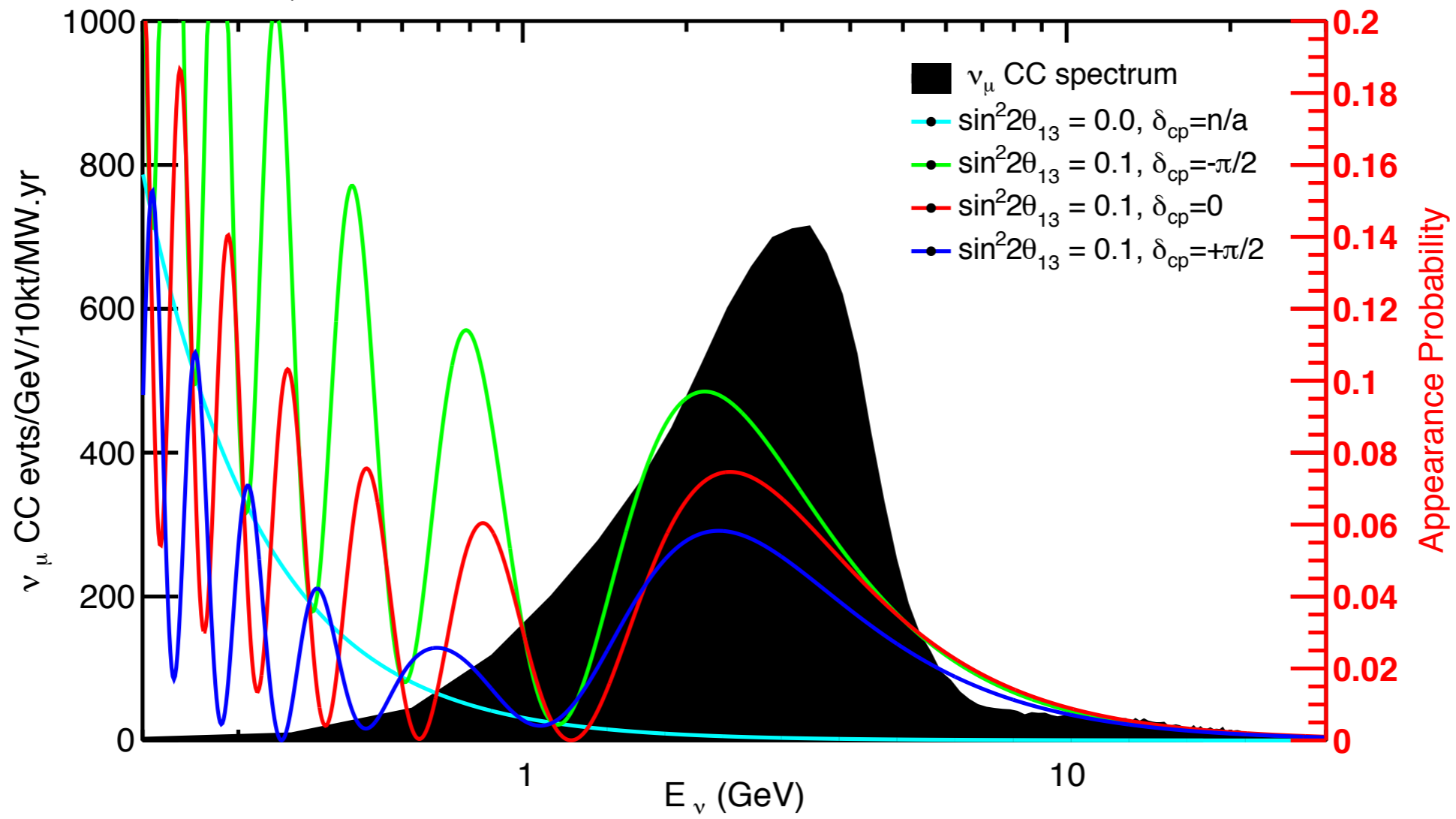


QCD in many regimes critical to extracting fundamental physics in the neutrino sector

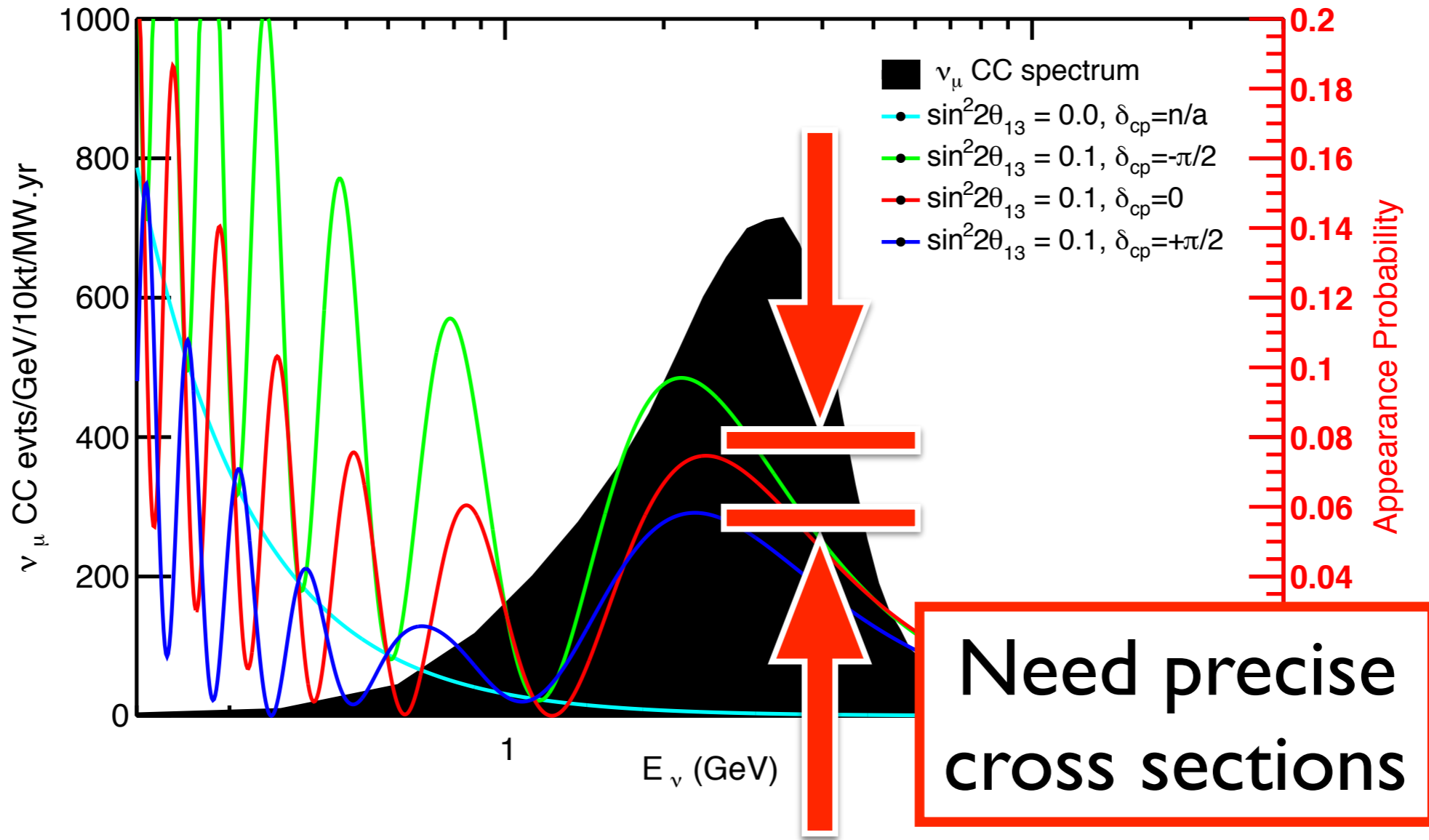




$\nu_\mu$  CC spectrum at 1300 km,  $\Delta m_{31}^2 = 2.4e-03 \text{ eV}^2$

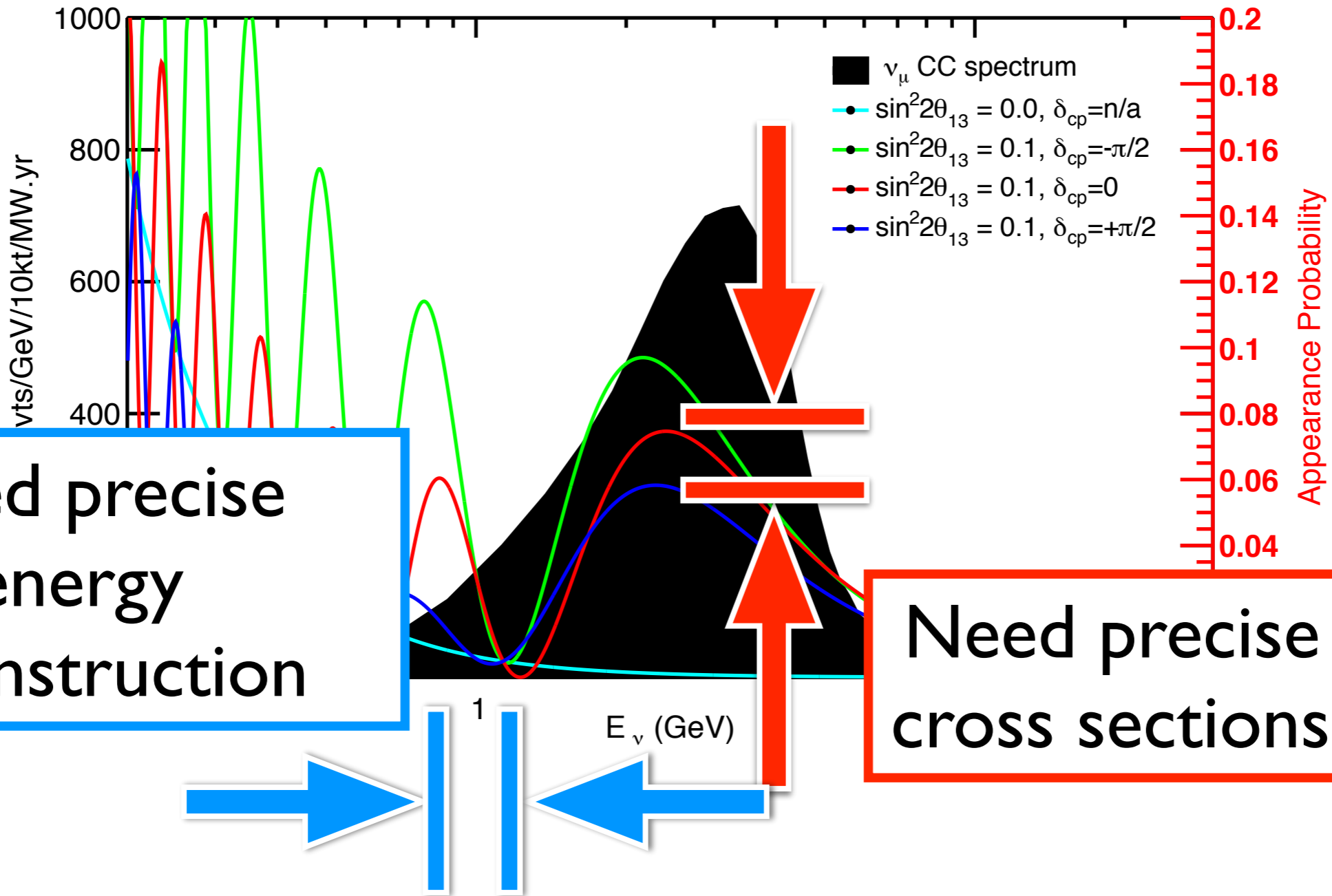


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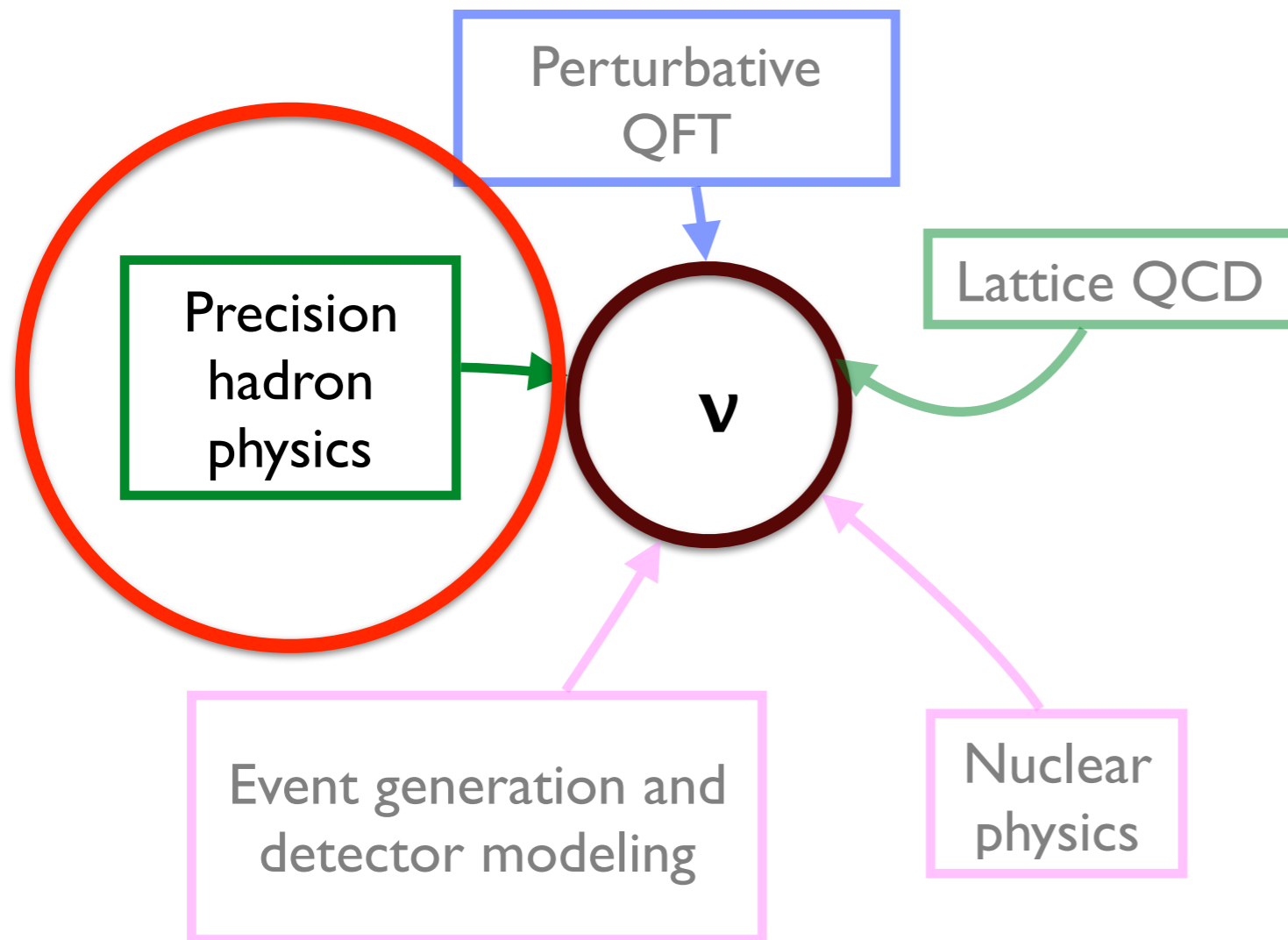


LBNE, 1307.7335

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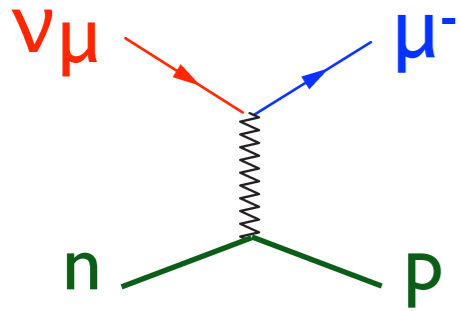


LBNE, 1307.7335



Every neutrino-nucleus cross section prediction relies on nucleon-level amplitudes constrained by deuterium experiments of the 1970's and 80's, fit to simple models. What is the actual uncertainty?

## Start with the basic process



$$\sigma(\nu n \rightarrow \mu p) = |\cdots \cdot F_A(q^2) \cdots|^2$$

**poorly known axial-vector form factor**

A common ansatz for  $F_A$  has been employed for the last  $\sim 40$  years:

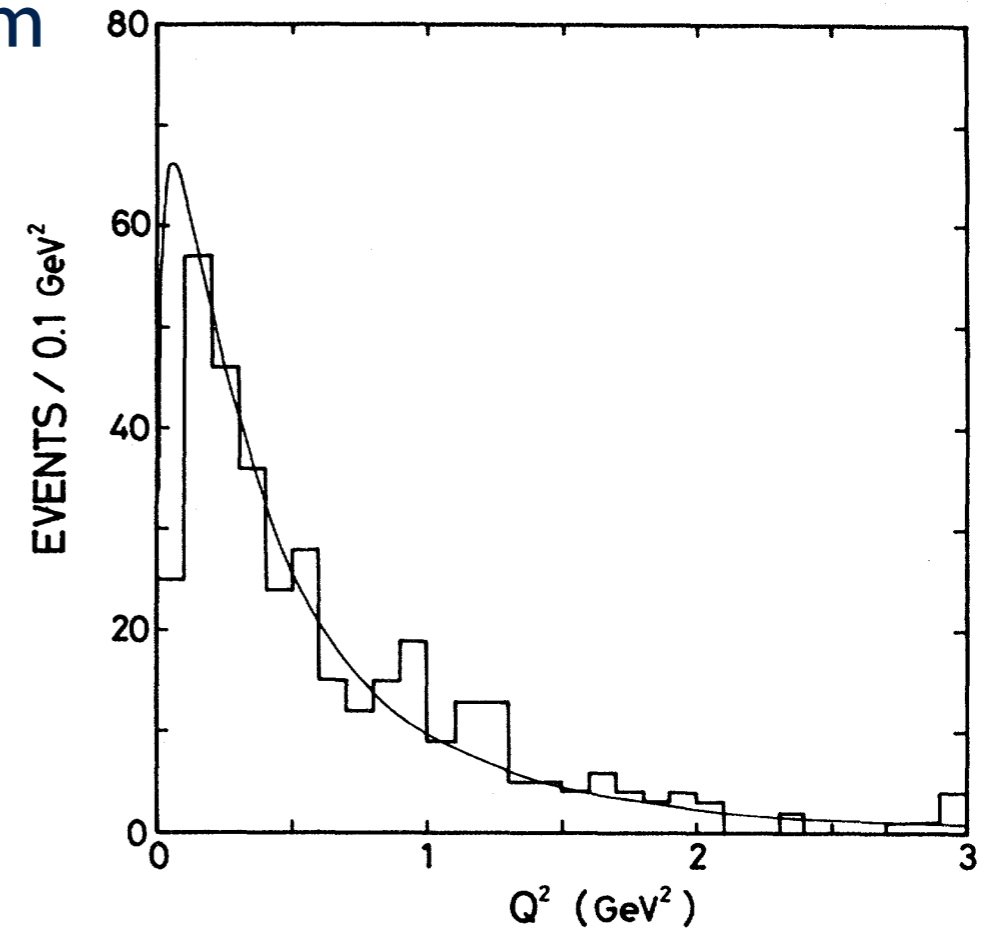
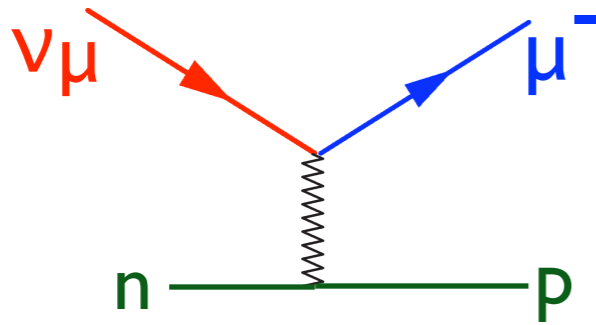
$$F_A^{\text{dipole}}(q^2) = F_A(0) \left(1 - \frac{q^2}{m_A^2}\right)^{-2}$$

Inconsistent with QCD.

Typically quoted uncertainties are (too) small (e.g. compared to proton charge form factor)

$$\frac{1}{F_A(0)} \left. \frac{dF_A}{dq^2} \right|_{q^2=0} \equiv \frac{1}{6} r_A^2 \quad r_A = 0.674(9) \text{ fm}$$

# Best source of almost-free neutrons: deuterium



*Fermilab 15-foot deuterium bubble chamber, PRD 28, 436 (1983)*

*also:*

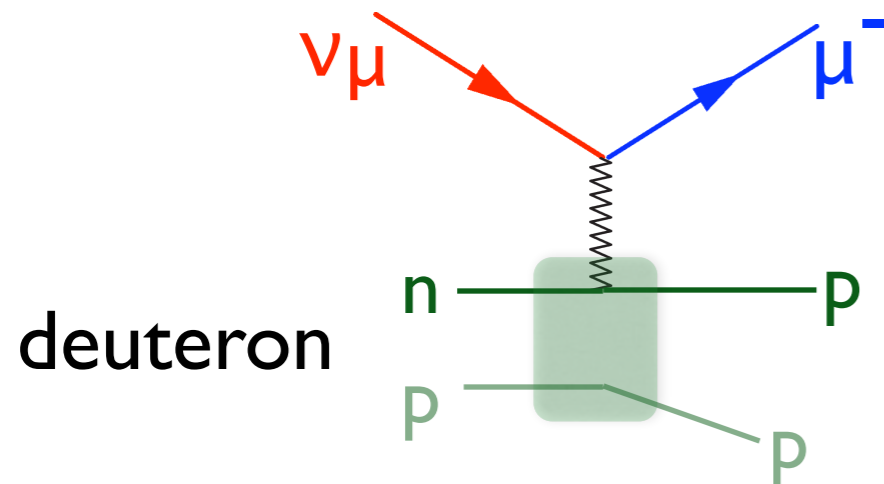
*ANL 12-foot deuterium bubble chamber, PRD 26, 537 (1982)*

*BNL 7-foot deuterium bubble chamber, PRD23, 2499 (1981)*

## Deuterium bubble chamber data

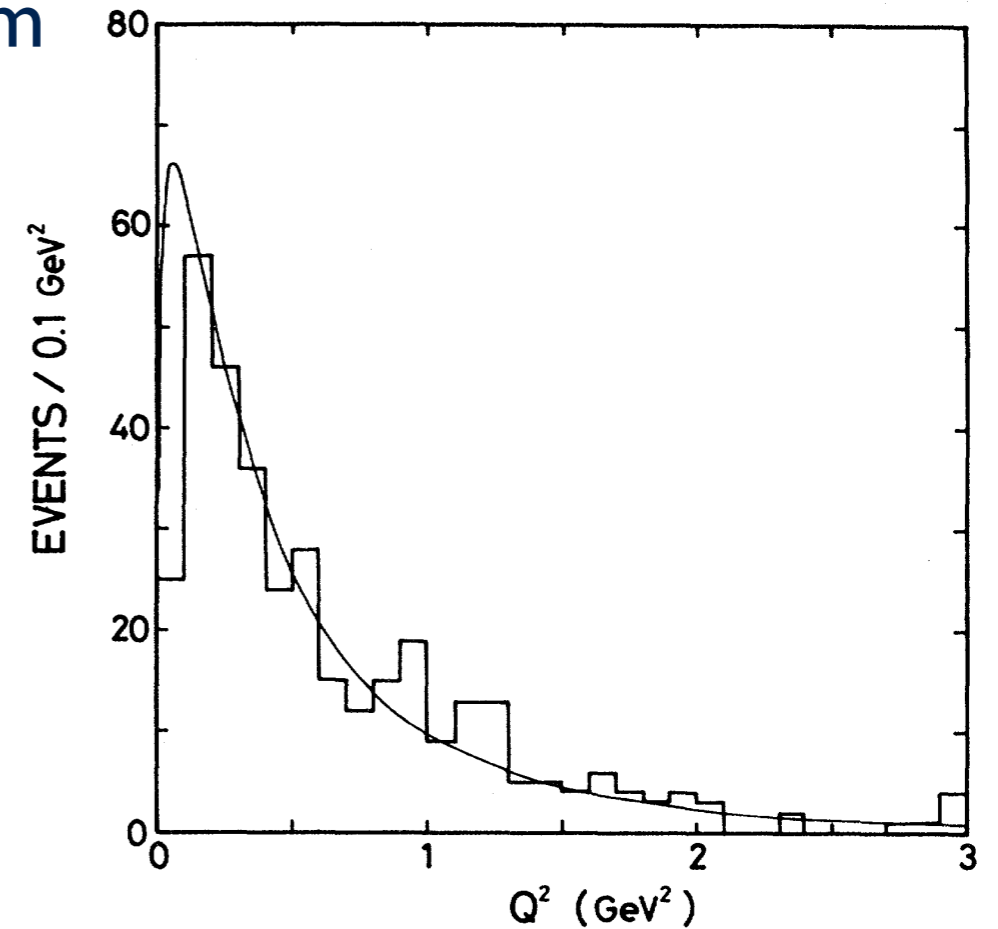
- small(-ish) nuclear effects
- small(-ish) experimental uncertainties
- small statistics, ~3000 events in world data

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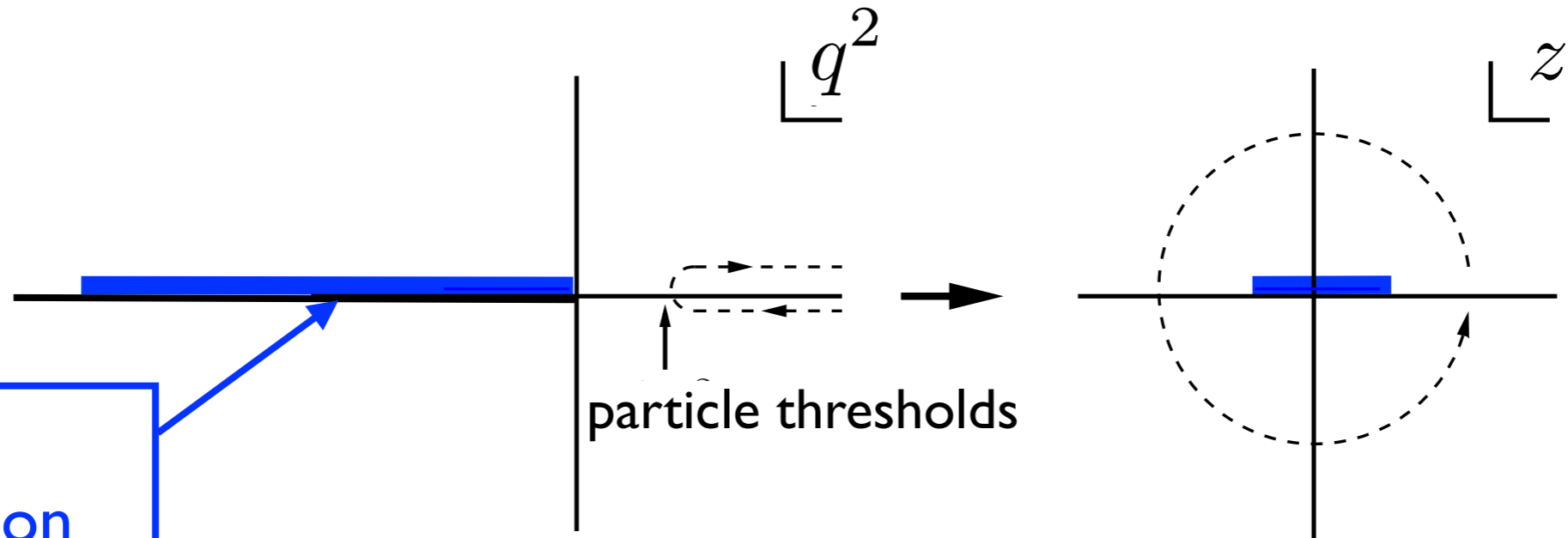
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# HEP toolbox is being applied to precision lepton-nucleon scattering

Underlying QCD tells us that Taylor expansion in appropriate variable is rapidly convergent



experimental  
kinematic region

$$F_A(q^2) = \sum_k a_k [z(q^2)]^k$$

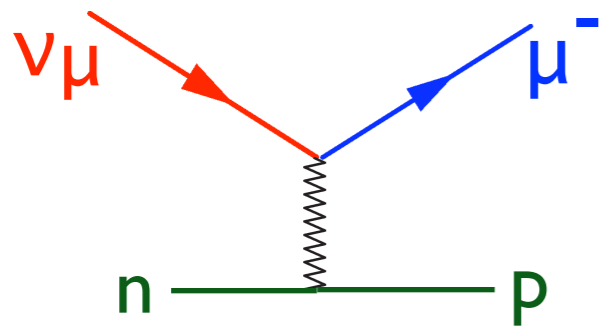
$$z(q^2, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - q^2} + \sqrt{t_{\text{cut}} - t_0}}$$

coefficients in rapidly  
convergent expansion encode  
nonperturbative QCD

Systematically improvable, quantifiable uncertainties



# Adapt these tools for neutrino - hadron scattering



$$\nu_{\mu} + n \rightarrow \mu^{-} + p,$$
$$0 < Q^2 < 3 \text{ GeV}^2$$

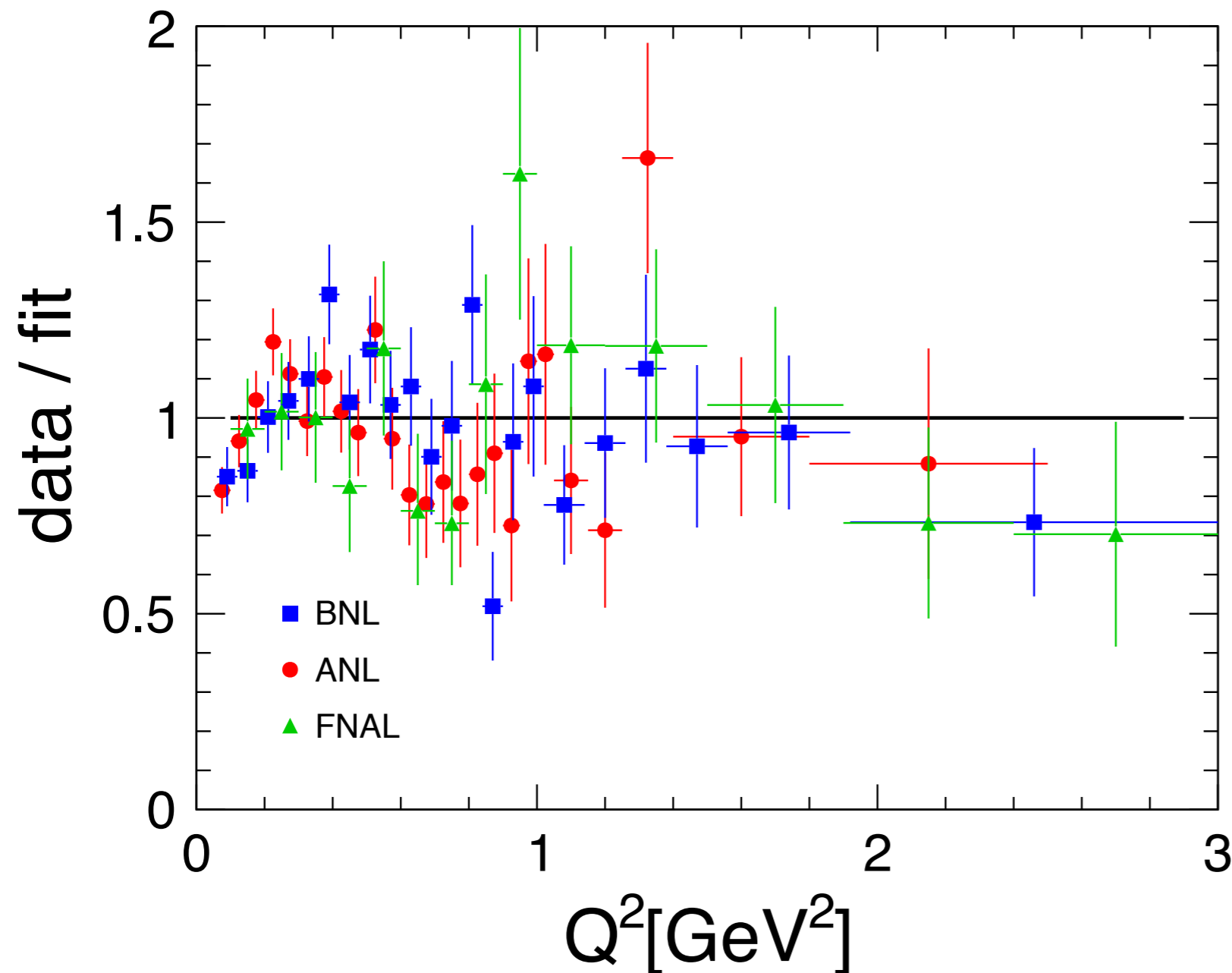
$$|z| < 0.35$$

- Event-level data from the deuterium experiments has been lost
- Ab initio flux estimates have poorly constrained systematics.
  - Use published distributions in neutrino energy to determine flux:

$$\Phi(E_{\nu})dE_{\nu} = \frac{1}{\sigma(E_{\nu}, F_A)} \frac{dN}{dE_{\nu}} dE_{\nu}$$

- Fit to published  $Q^2$  distributions to determine  $F_A$
- Reproduced results of original publications under same assumptions
- Replaced dipole  $F_A$  with model-independent  $z$  expansion

# Data are in tension with any $F_A$ described by QCD

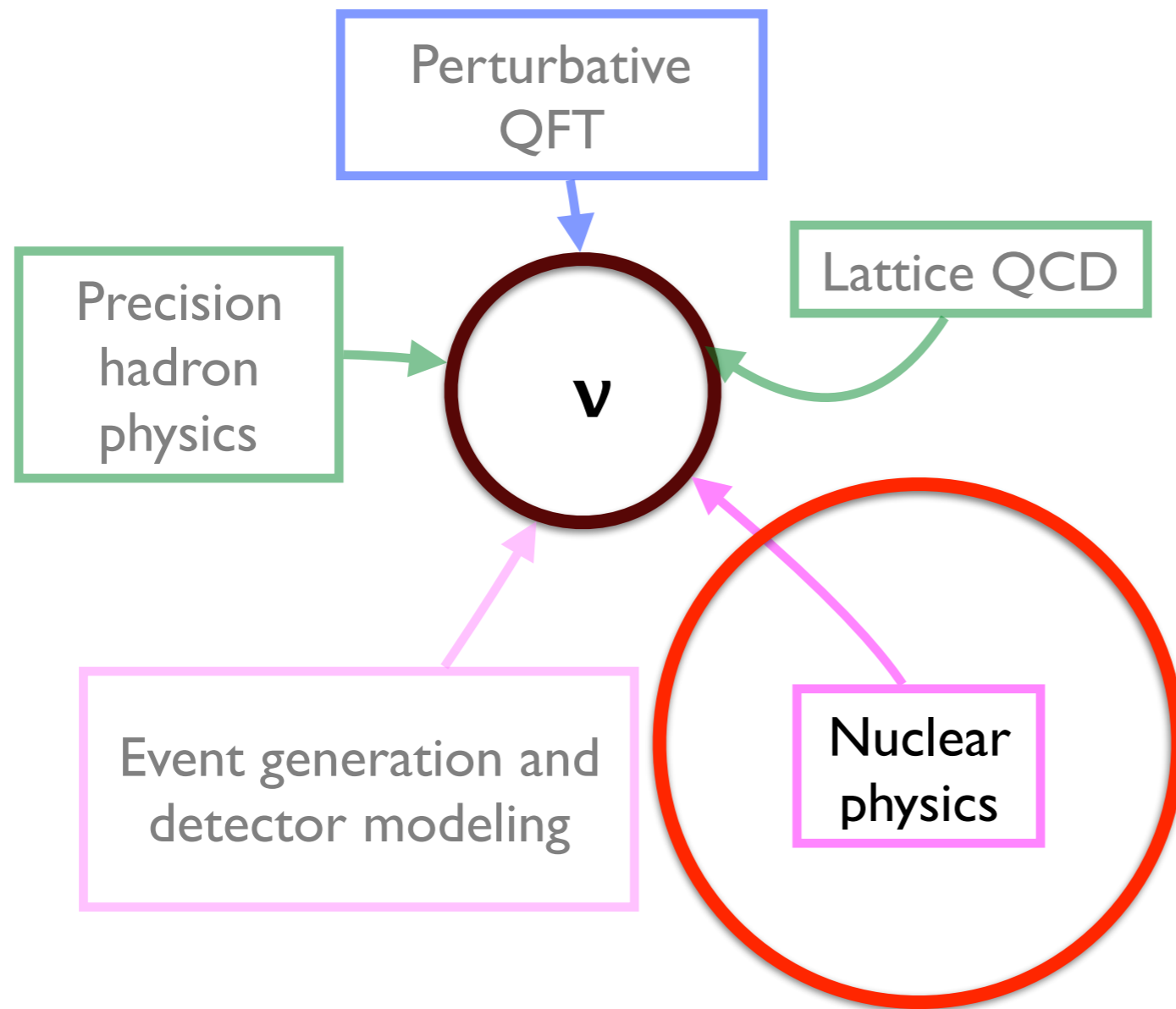


$\chi^2 = 30$  (16 points, BNL)

$\chi^2 = 32$  (19 points, ANL)

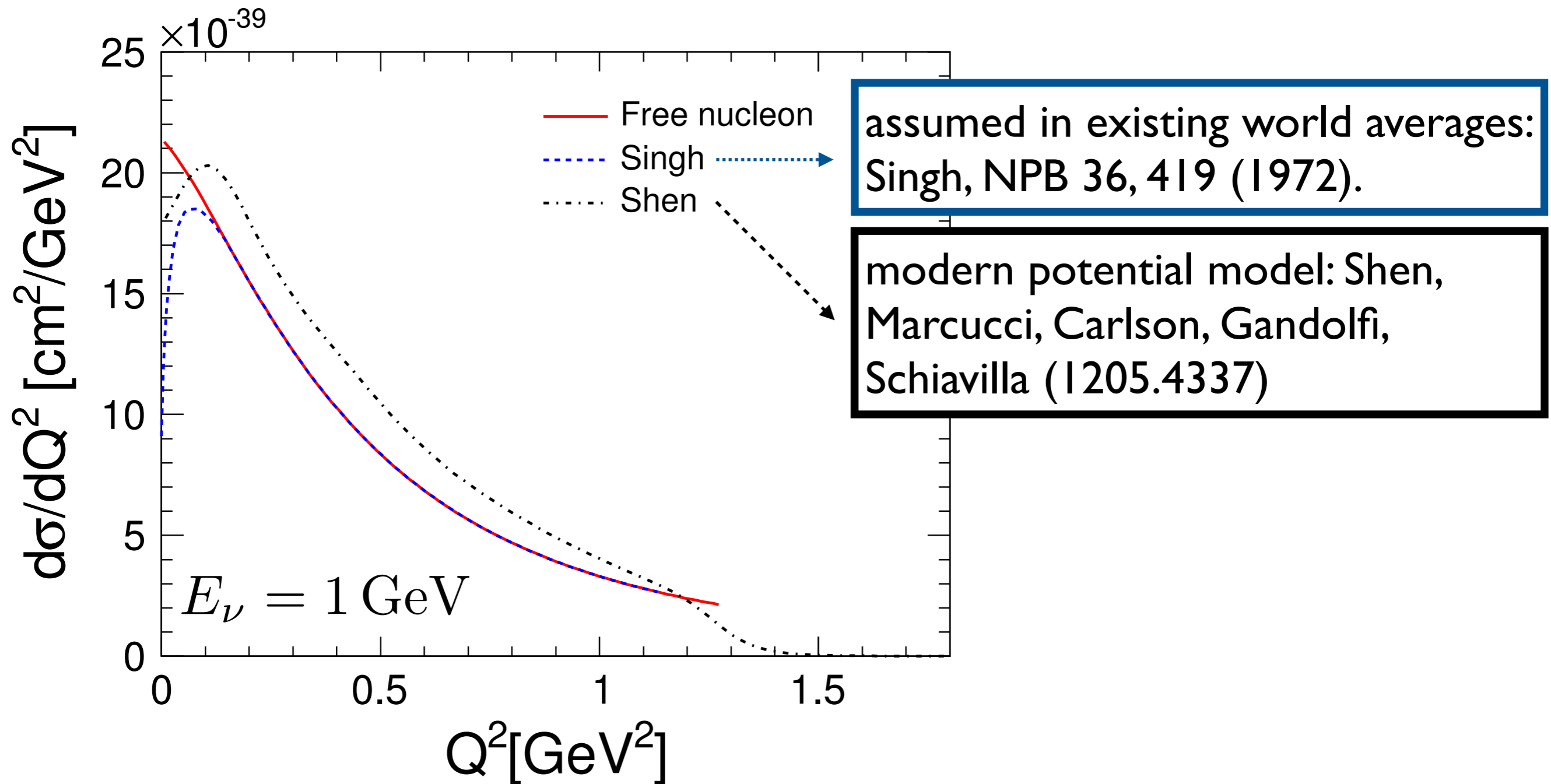
Possible correlated effect between datasets, including deficit at small  $Q^2$

Revisit systematics: - experimental acceptance/efficiency correction  
- theoretical deuteron correction



To validate nuclear models for  $^{12}\text{C}$ ,  $^{16}\text{O}$ ,  $^{40}\text{Ar}$ , ... , should first master  $^2\text{H}$  (work remains - new experimental data would be very valuable)

- theoretical systematic: deuteron correction



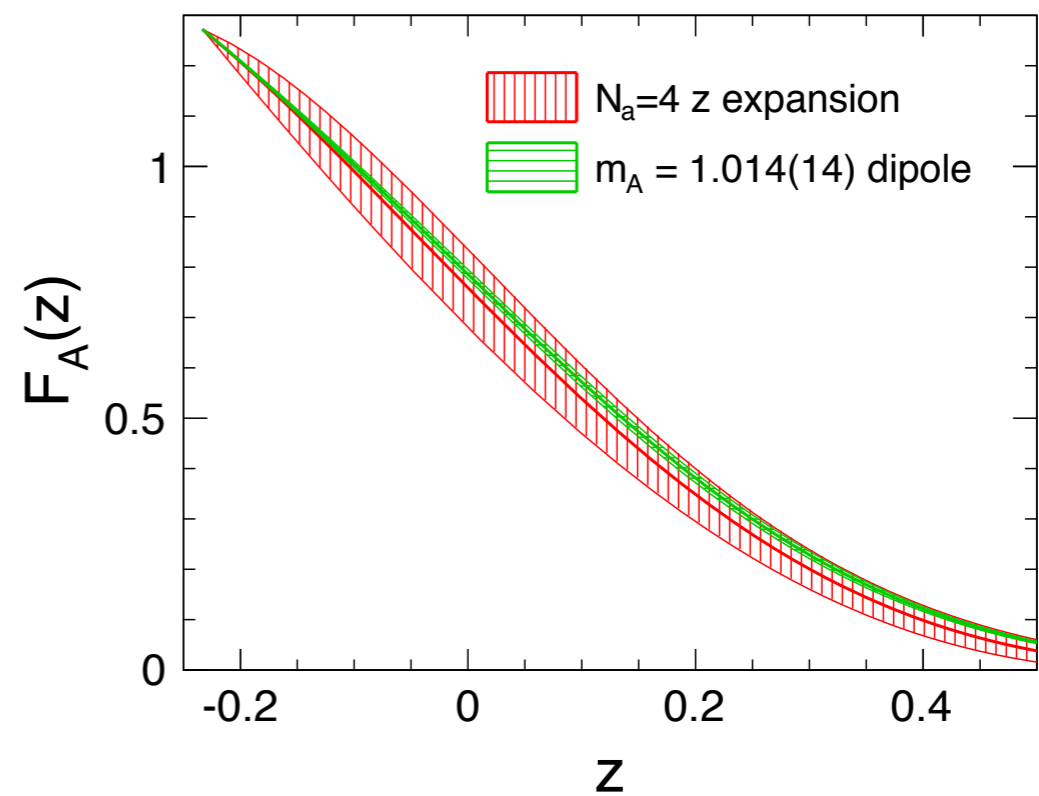
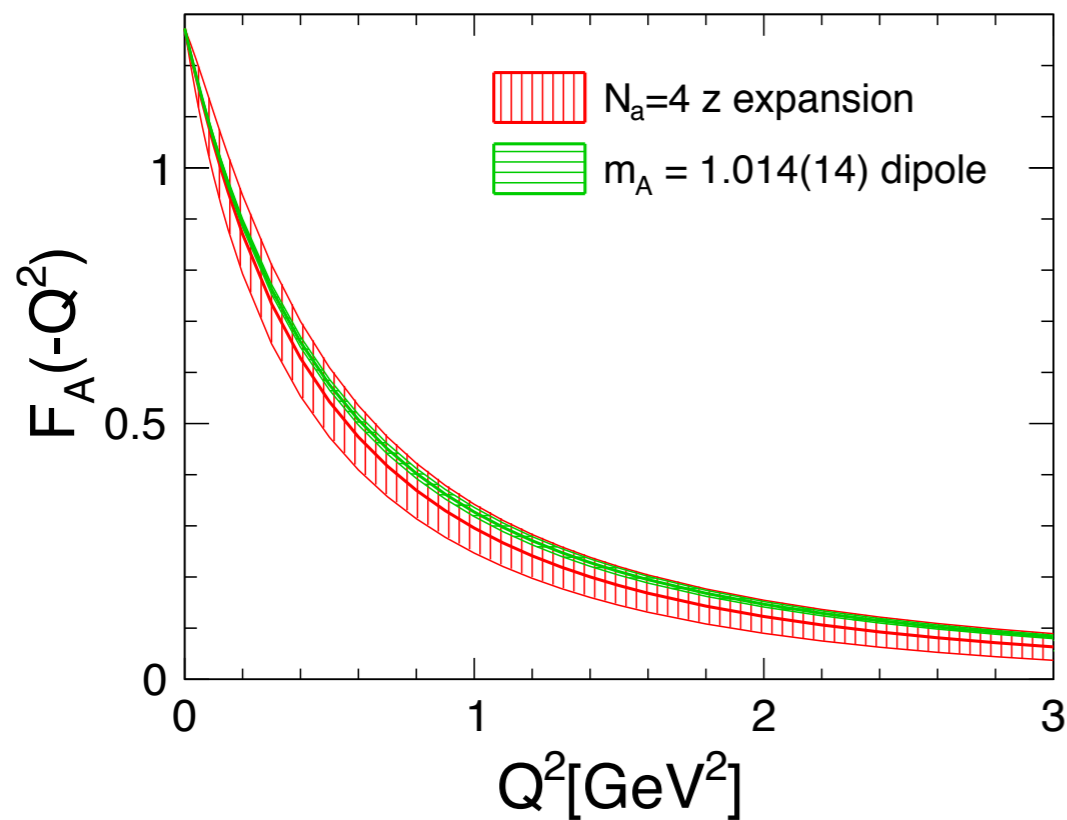
- An open problem to quantify uncertainty, especially at larger energy

- $F_A$  with complete error budget, correlations:

$$F_A(q^2) = \sum_k a_k [z(q^2)]^k$$

$$[a_1, a_2, a_3, a_4] = [2.30(13), -0.6(1.0), -3.8(2.5), 2.3(2.7)]$$

$$C_{ij} = \begin{pmatrix} 1 & 0.350 & -0.678 & 0.611 \\ 0.350 & 1 & -0.898 & 0.367 \\ -0.678 & -0.898 & 1 & -0.685 \\ 0.611 & 0.367 & -0.685 & 1 \end{pmatrix}$$



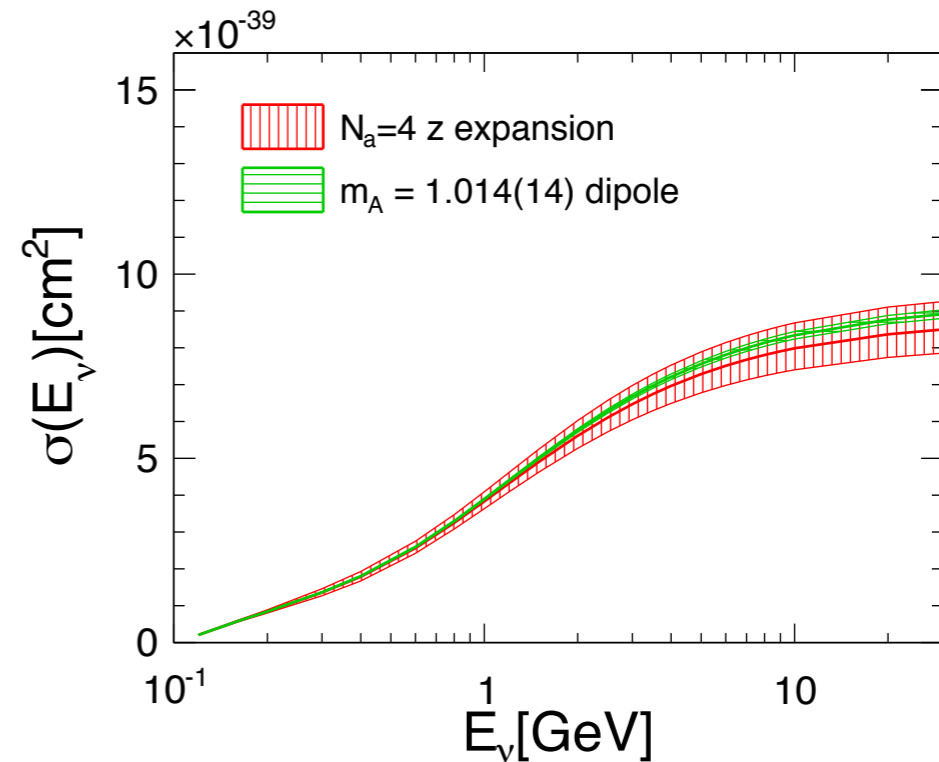
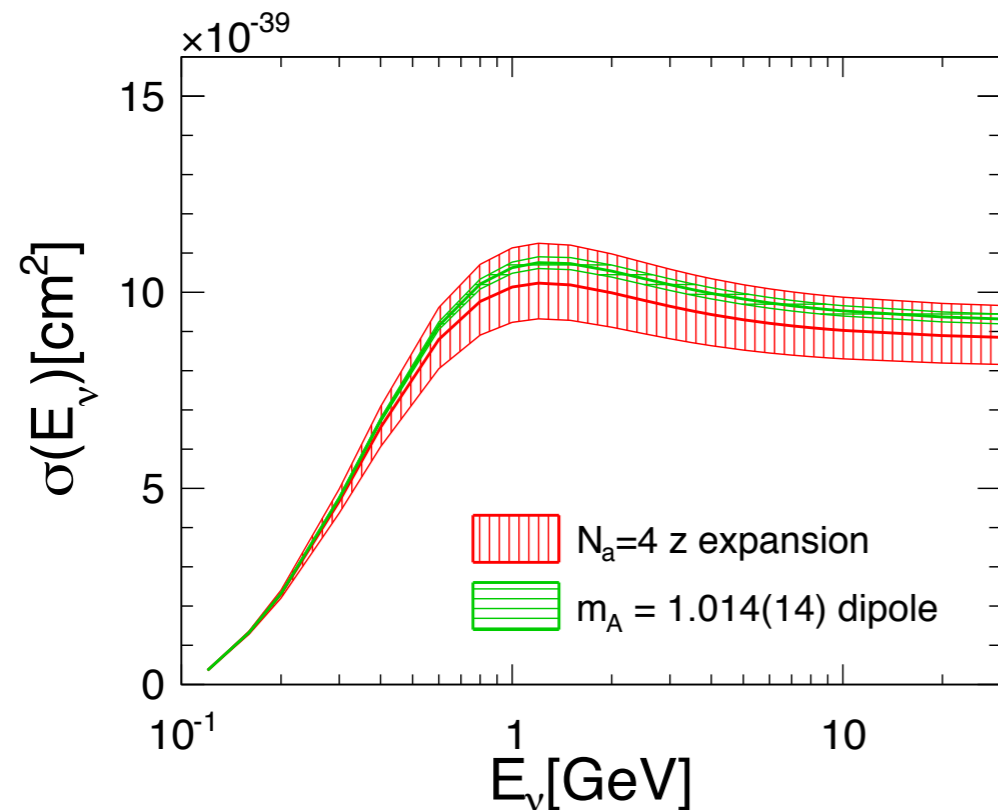
## Derived observables: 1) axial radius

$$\frac{1}{F_A(0)} \left. \frac{dF_A}{dq^2} \right|_{q^2=0} \equiv \frac{1}{6} r_A^2$$

$$r_A^2 = 0.46(22) \text{ fm}^2$$

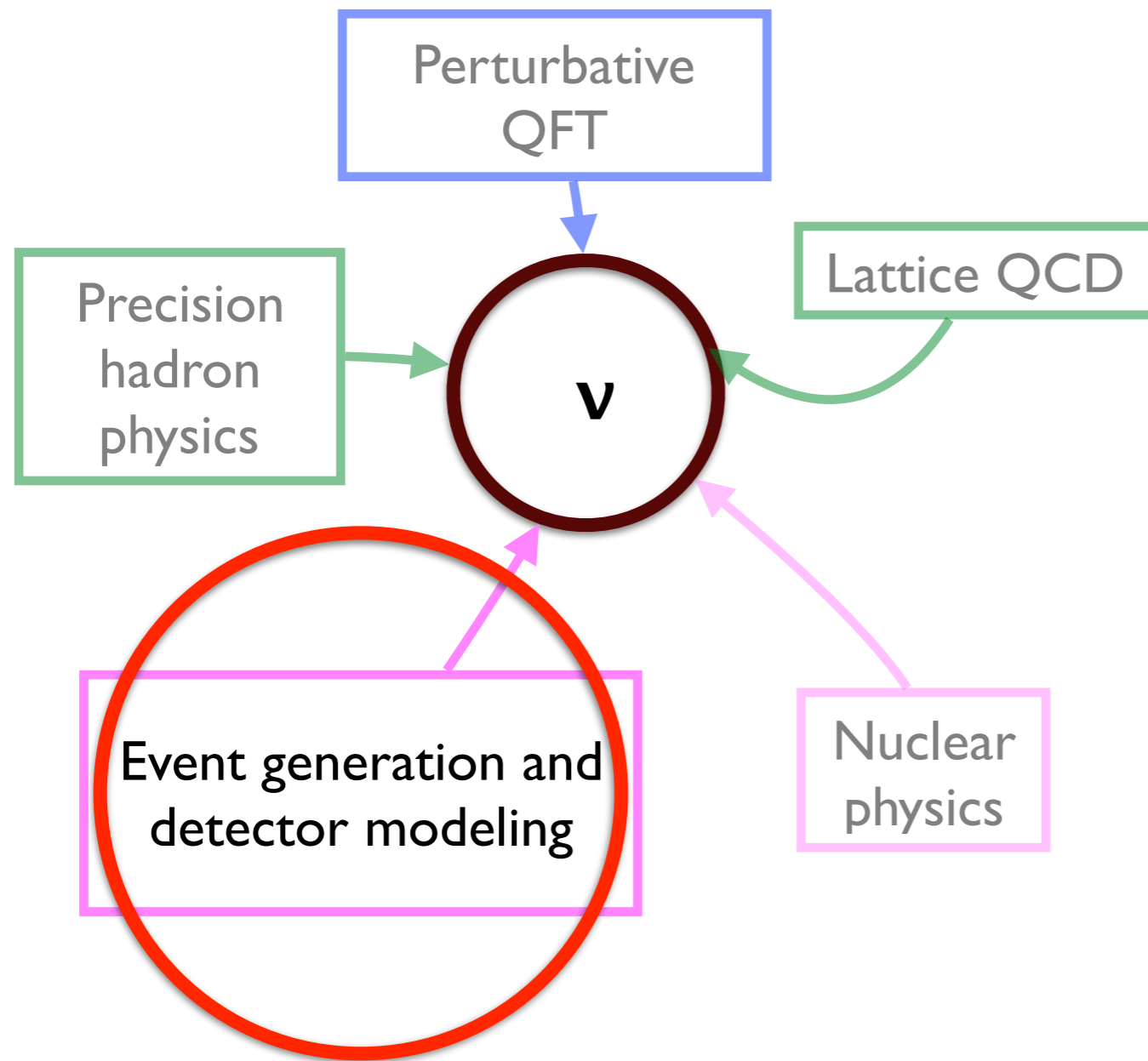
- a basic parameter of nucleon structure
- order of magnitude larger uncertainty compared to dipole fits
- impacts comparison to other data, e.g. pion electroproduction, muon capture

## Derived observables: 2) neutrino-nucleon quasi elastic cross sections



$$\sigma_{\nu n \rightarrow \mu p}(E_\nu = 1 \text{ GeV}) = 10.1(0.9) \times 10^{-39} \text{ cm}^2 \quad (\sim \text{T2K})$$

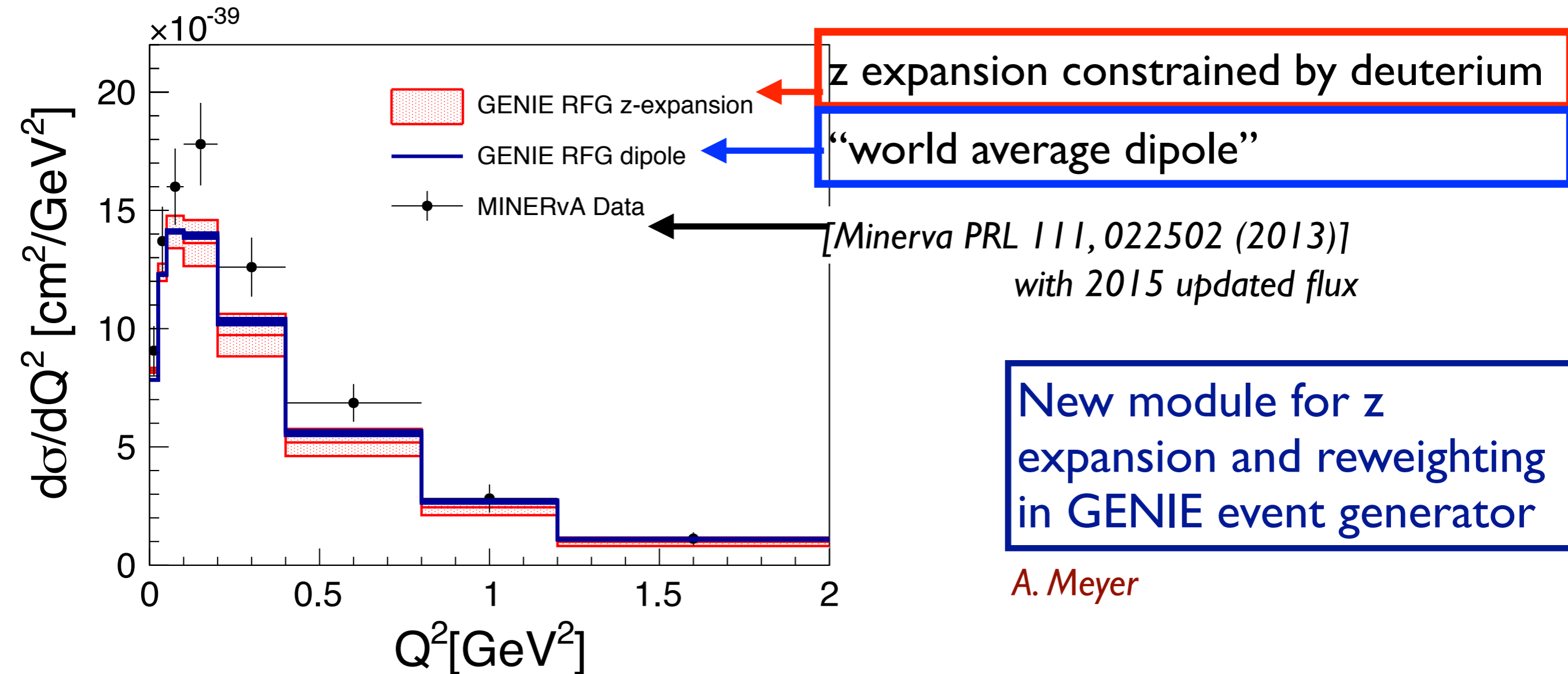
$$\sigma_{\nu n \rightarrow \mu p}(E_\nu = 3 \text{ GeV}) = 9.6(0.9) \times 10^{-39} \text{ cm}^2 \quad (\sim \text{DUNE})$$



The model independent  $z$  expansion has been implemented in **GENIE** event generator for input to nuclear models

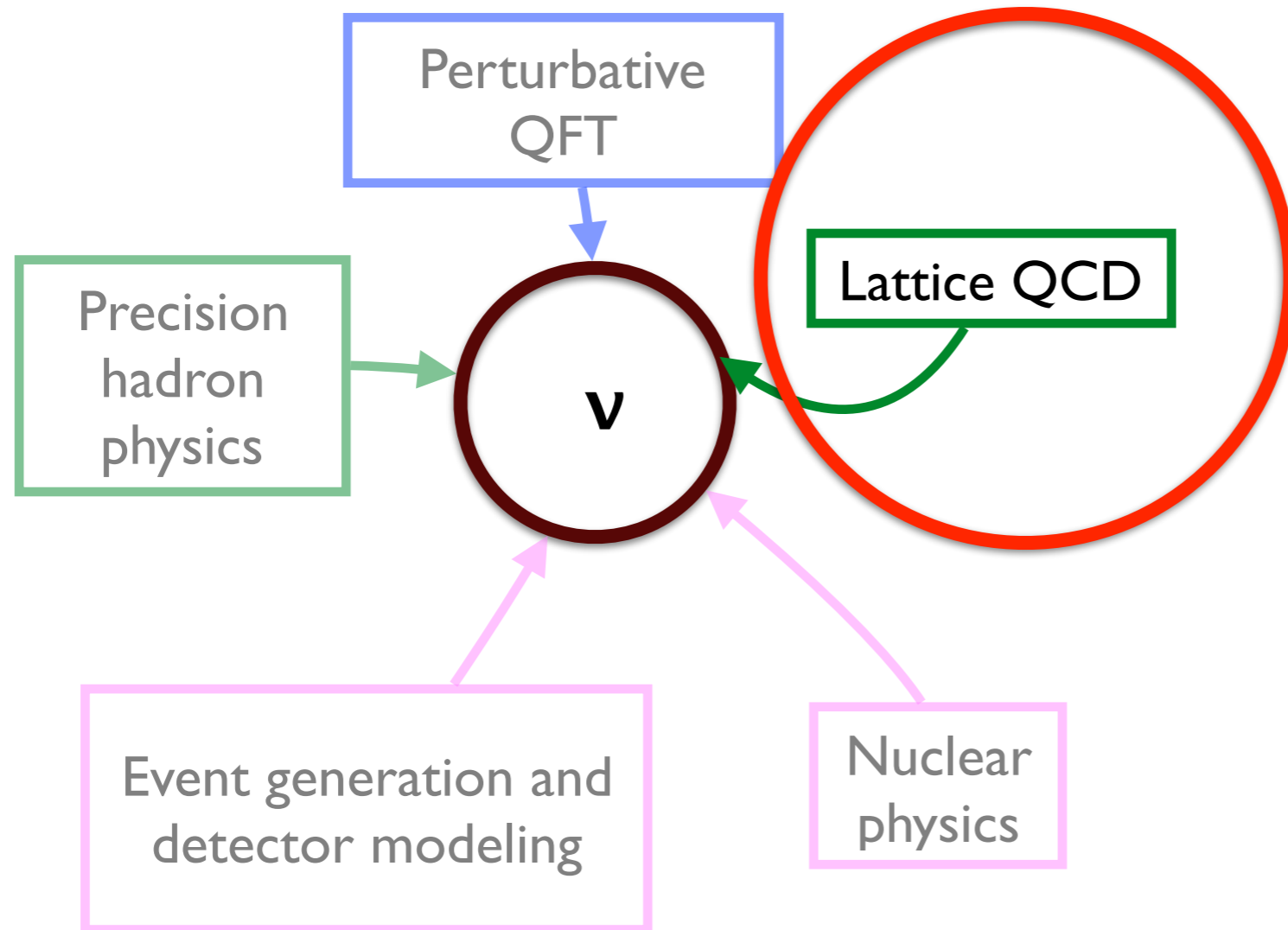


## Derived observables: 3) neutrino-nucleus quasi elastic cross sections



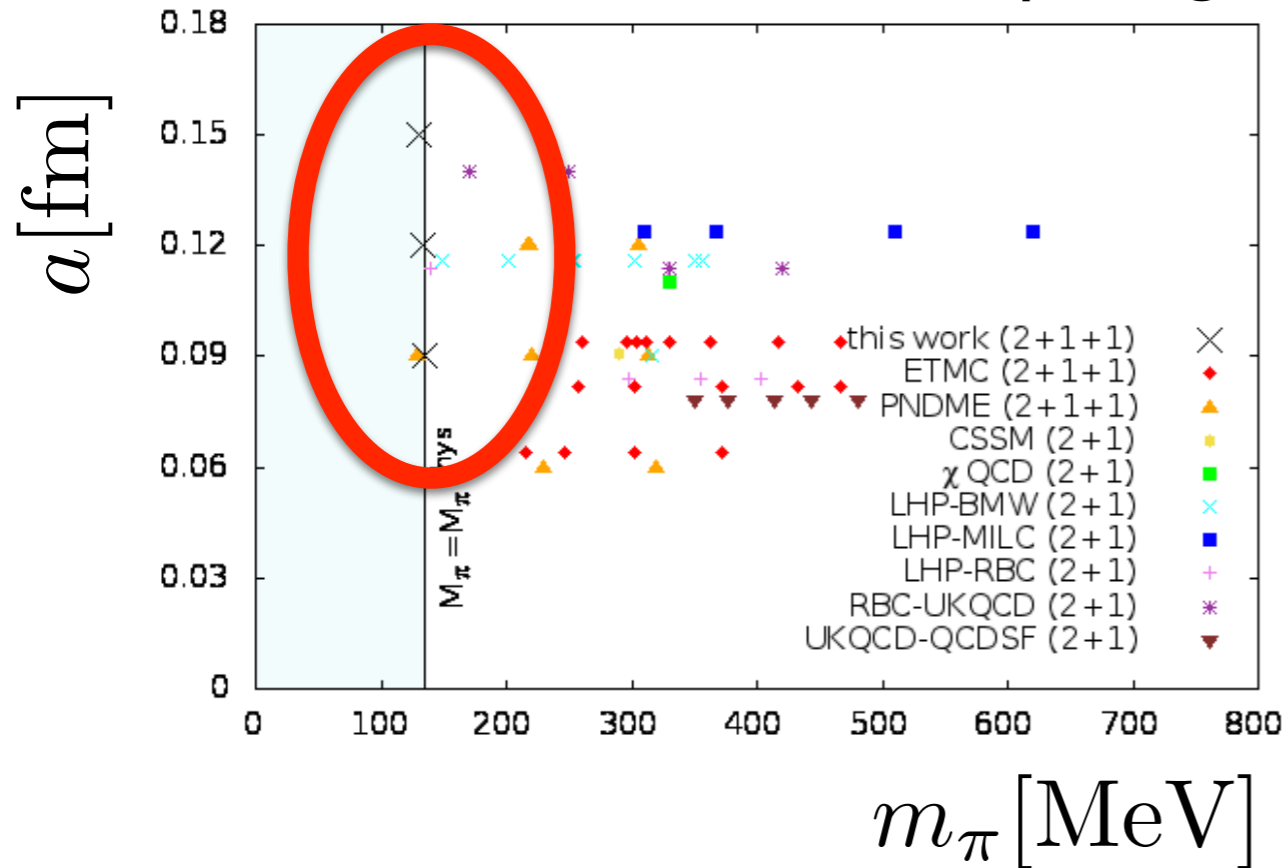
- errors have different kinematic dependence than dialing  $m_A$  in dipole ansatz

- z expansion (with correlations, reweighting) coded in GENIE, can be readily implemented with nuclear models

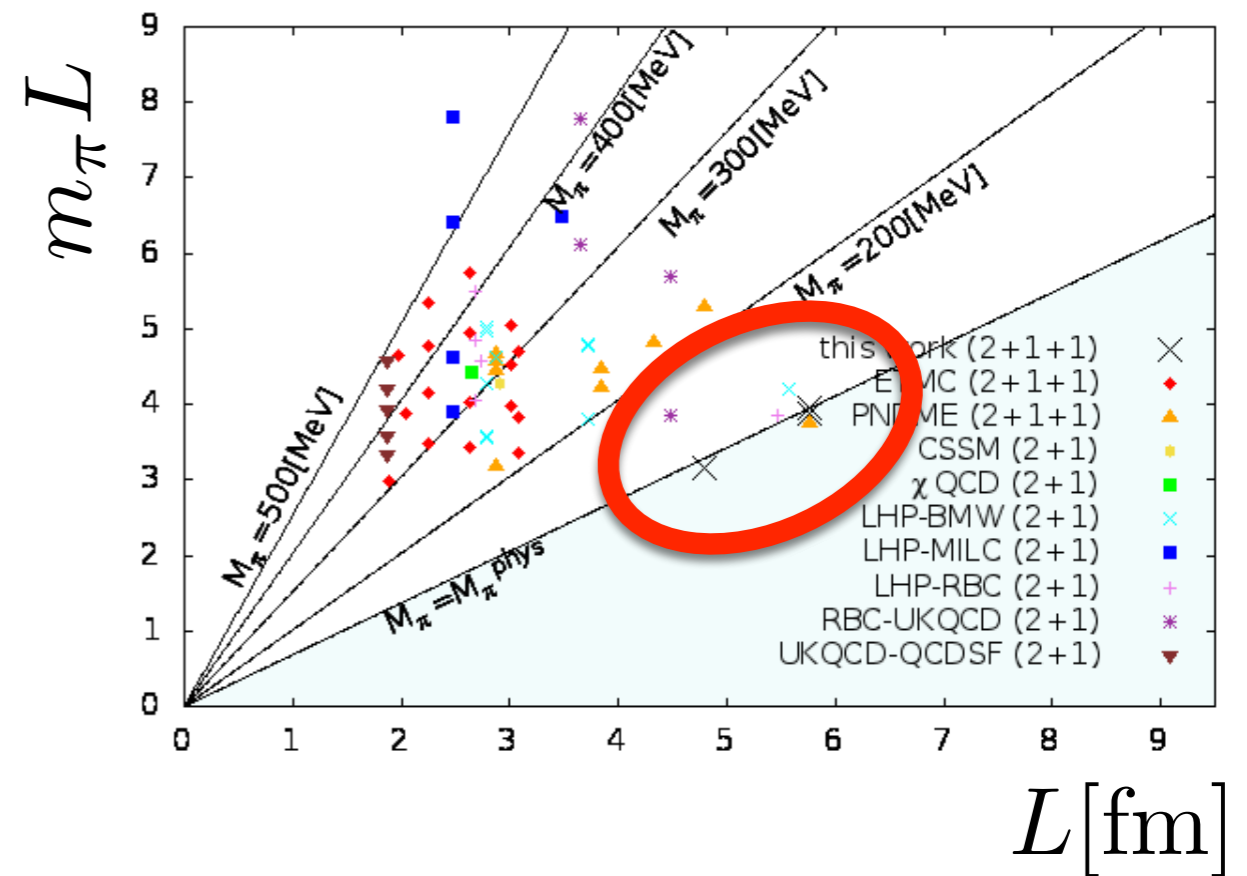


Practical obstacles to modern neutrino experiments with elementary (hydrogen or deuterium) target. Lattice QCD is poised to contribute.

Pion mass vs. lattice spacing



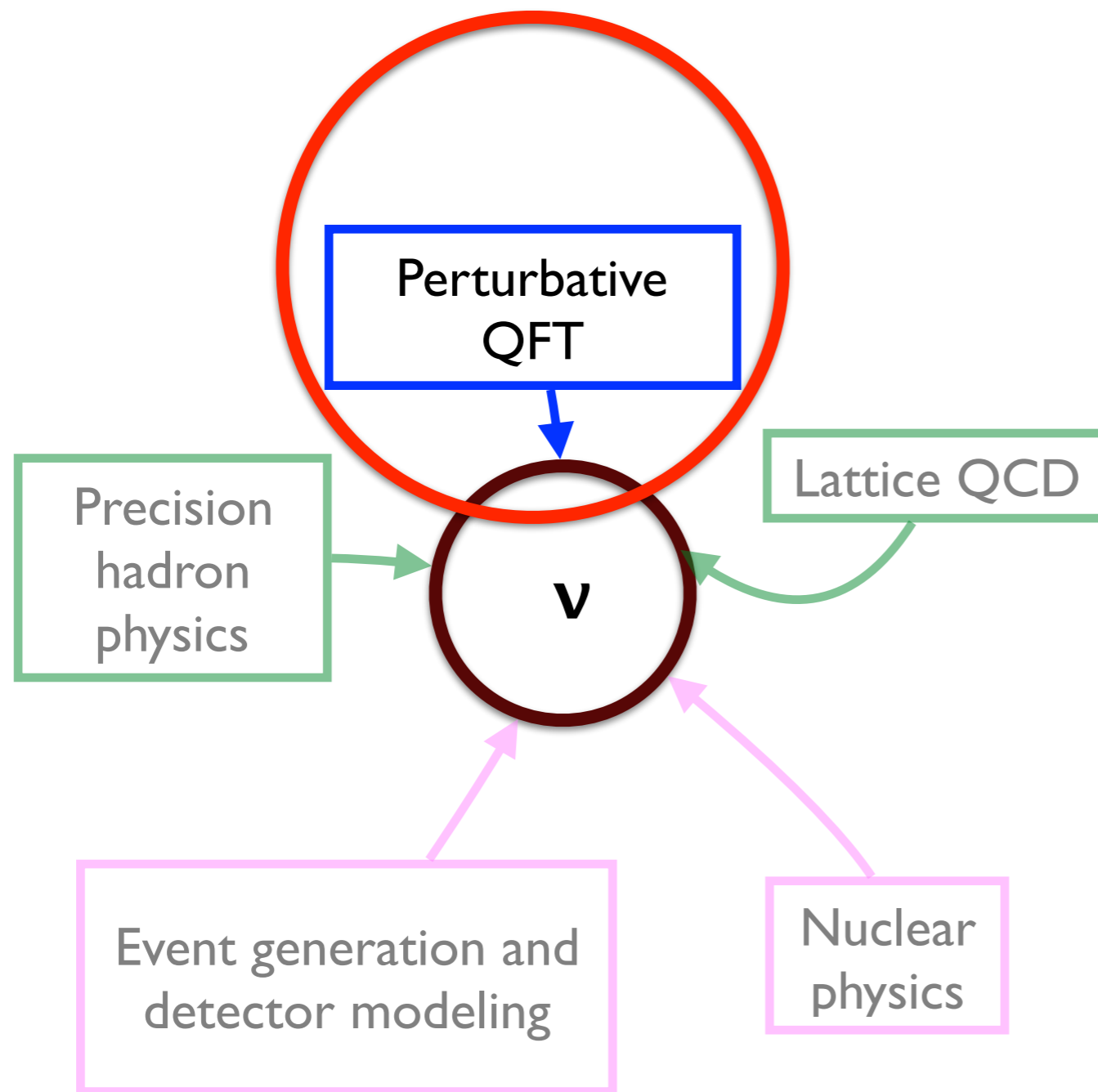
Lattice Extent vs. Pion Mass



Big lattices, multiple spacings, physical quark masses

Other targets: neutral currents; resonance couplings and form factors; pion final states

Advantages: independent of detector-dependent radiative corrections and nuclear effects (and for lattice QCD: no underground safety hazard)



QED radiative corrections impact, e.g.,  $\nu_e$  appearance signal. Validate with electron-proton scattering. (Actually, progress in radiative corrections required here also.)

# Some facts about the Rydberg constant puzzle (a.k.a. proton radius puzzle)

1) It has generated a lot of attention and controversy



The New York Times

2) The *most* mundane resolution necessitates:

- $5\sigma$  shift in fundamental Rydberg constant
- discarding or revising decades of results in e-p scattering and hydrogen spectroscopy

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This problem has broad ownership, e.g.:

3) Systematic effects in electron-proton scattering impact neutrino-nucleus scattering, *at a level large compared to long baseline precision requirements*



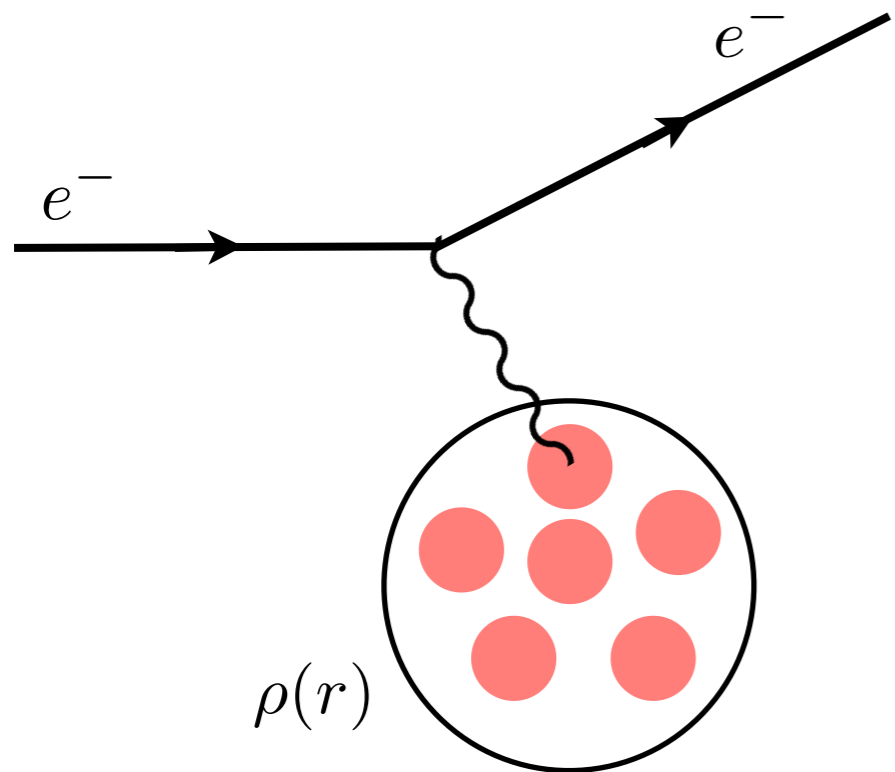
**“The good news is that it’s not my problem”**

# What is the proton charge radius?

recall scattering from extended classical charge distribution:

$$\frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{pointlike}} |F(q^2)|^2$$

$$\begin{aligned} F(q^2) &= \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \rho(\mathbf{r}) \\ &= \int d^3r \left[ 1 + i\mathbf{q}\cdot\mathbf{r} - \frac{1}{2}(\mathbf{q}\cdot\mathbf{r})^2 + \dots \right] \rho(\mathbf{r}) \\ &= 1 - \frac{1}{6}\langle r^2 \rangle \mathbf{q}^2 + \dots \end{aligned}$$



for the relativistic, QM, case, *define* radius as slope of form factor

$$\begin{aligned} \langle J^\mu \rangle &= \gamma^\mu F_1 + \frac{i}{2m_p} \sigma^{\mu\nu} q_\nu F_2 \\ G_E &= F_1 + \frac{q^2}{4m_p^2} F_2 \quad G_M = F_1 + F_2 \end{aligned}$$

$$r_E^2 \equiv 6 \frac{d}{dq^2} G_E(q^2) \Big|_{q^2=0}$$



Recall hydrogen spectrum:

$$E_n \sim \frac{R_\infty}{n^2} + \frac{r_E^2}{n^3}$$

$hcR_\infty = \frac{m_e c^2 \alpha^2}{2} \approx 13.6 \text{ eV}$       proton charge radius

Disentangle 2 unknowns,  $R_\infty$  and  $r_E$ , using well-measured 1S-2S hydrogen transition *and*

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- (3) a muonic hydrogen interval (2S-2P)

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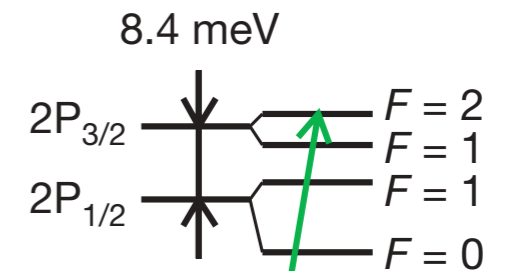
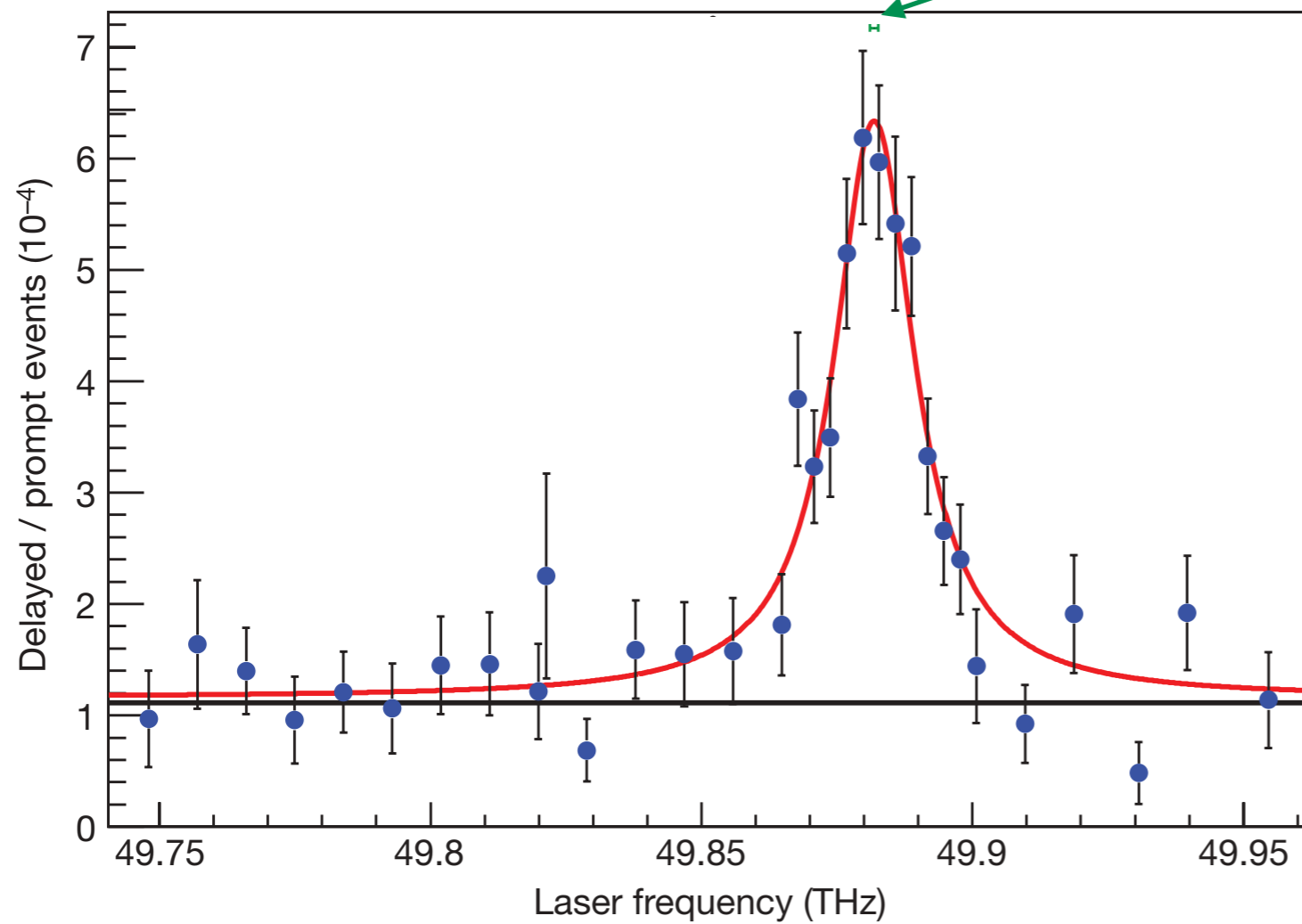
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$5\sigma$  discrepancy in Rydberg constant from (1+2) versus (3)

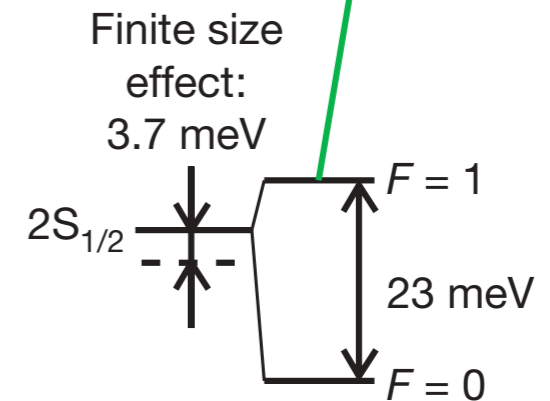
# muonic hydrogen Lamb shift measurement

*Pohl et al., Nature 466, 213 (2010)*

measured frequency



206 meV  
50 THz  
6  $\mu\text{m}$

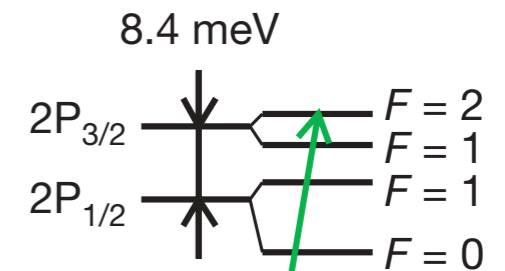
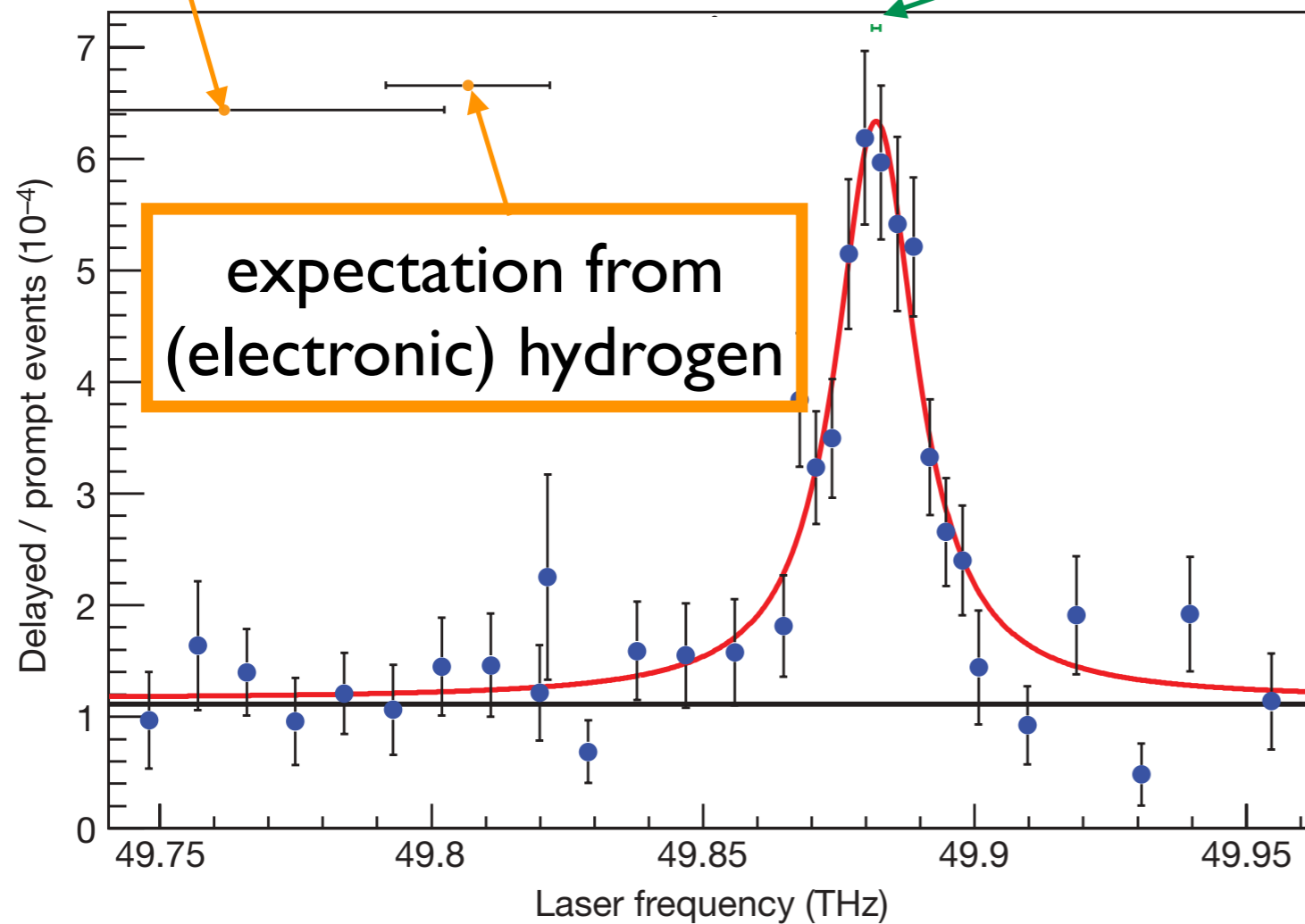


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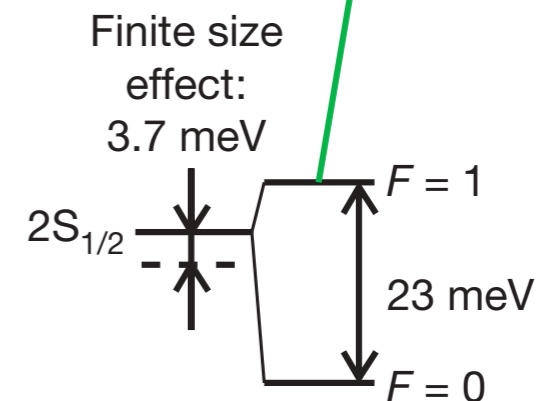
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expectation from  
e-p scattering

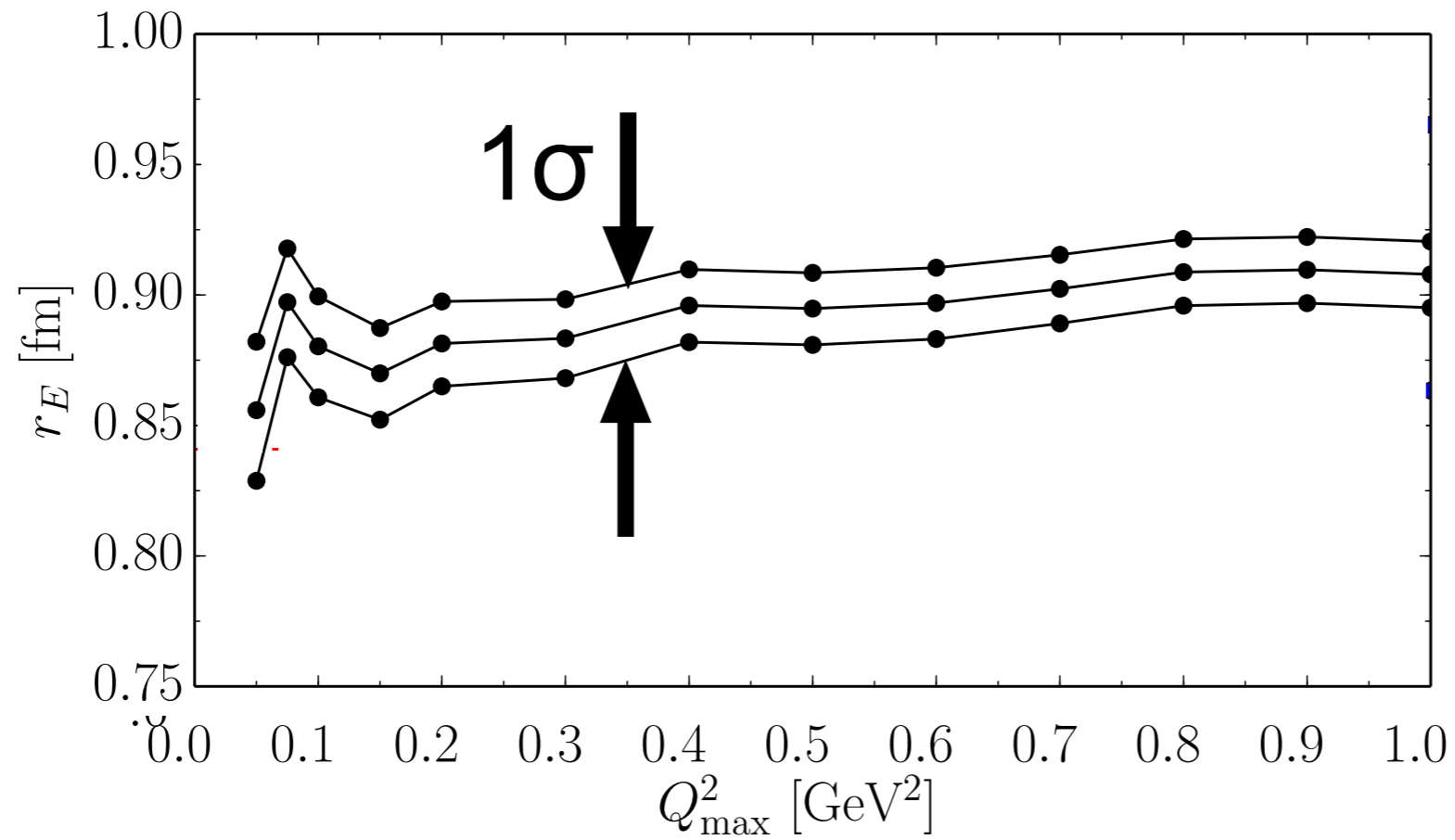
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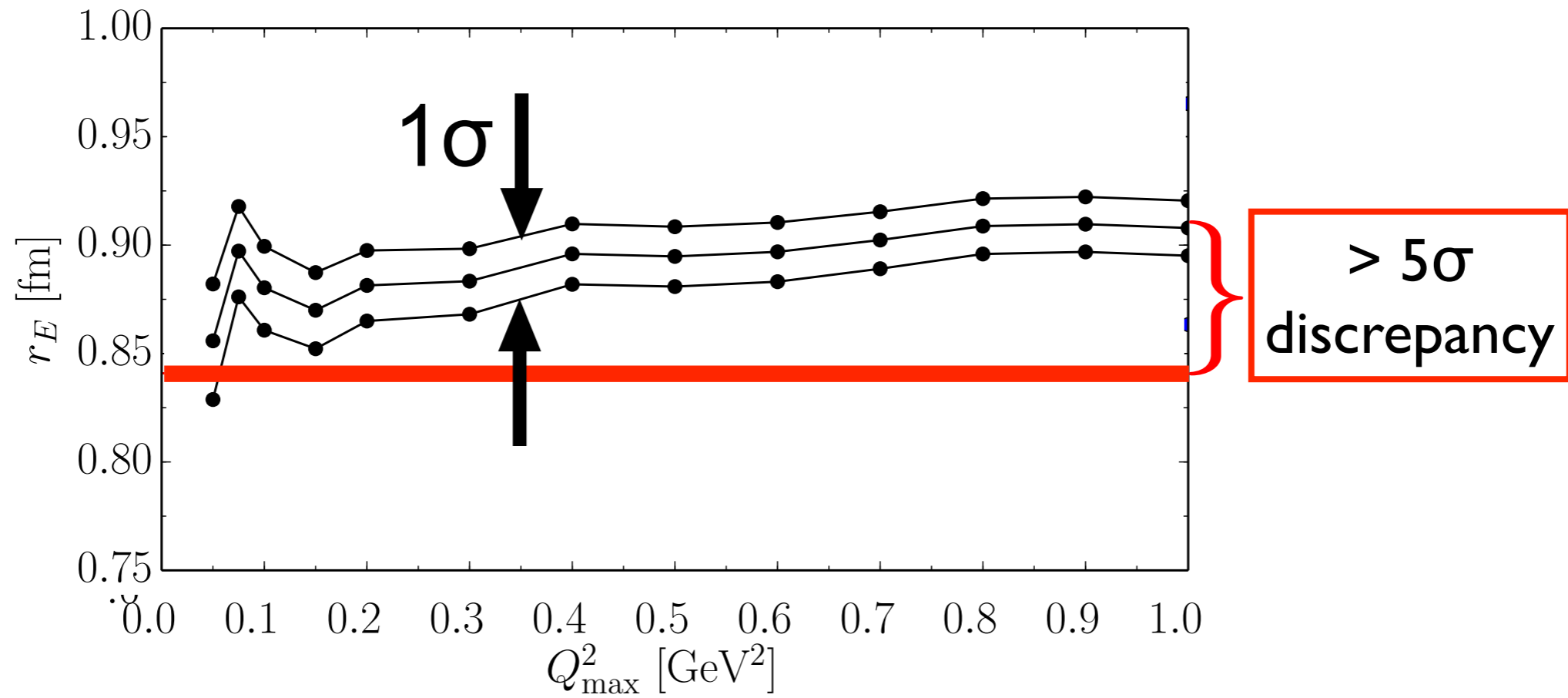


# Tension between radius extracted from different $Q^2$ ranges

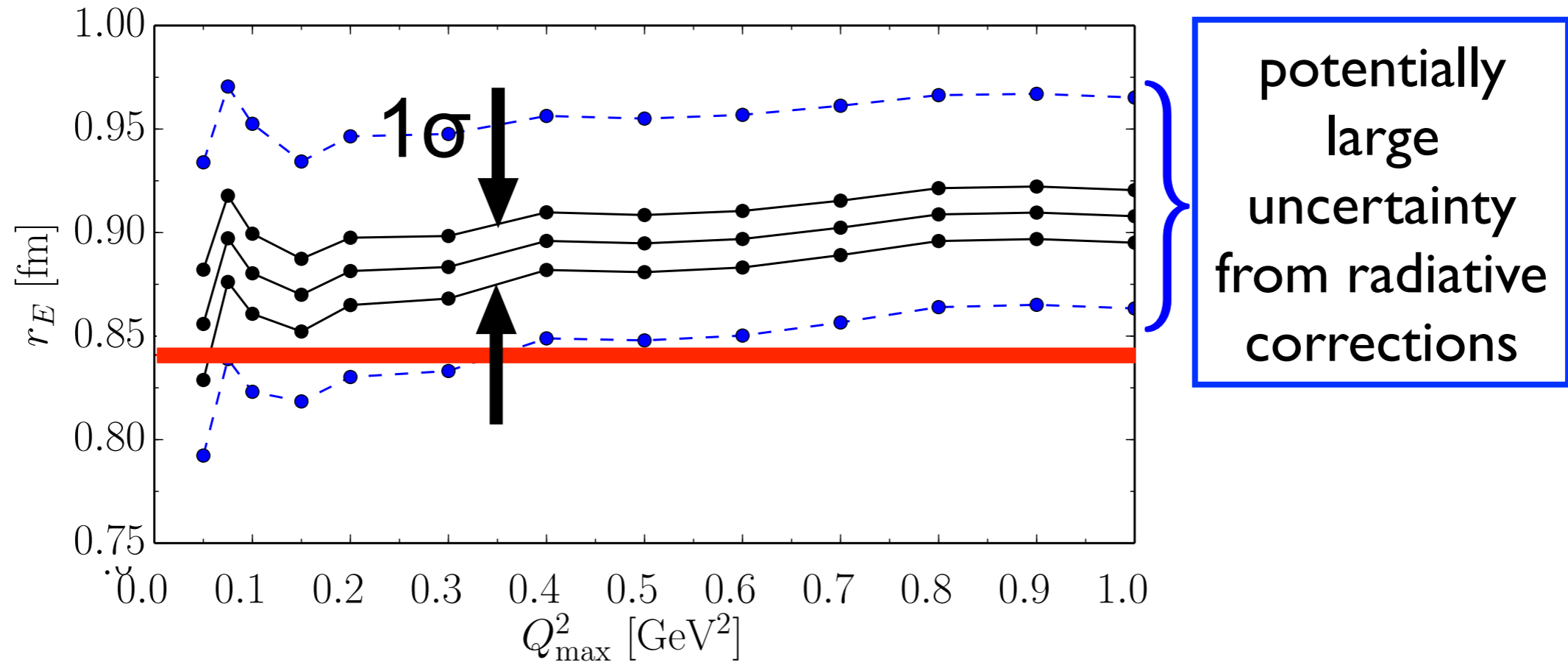




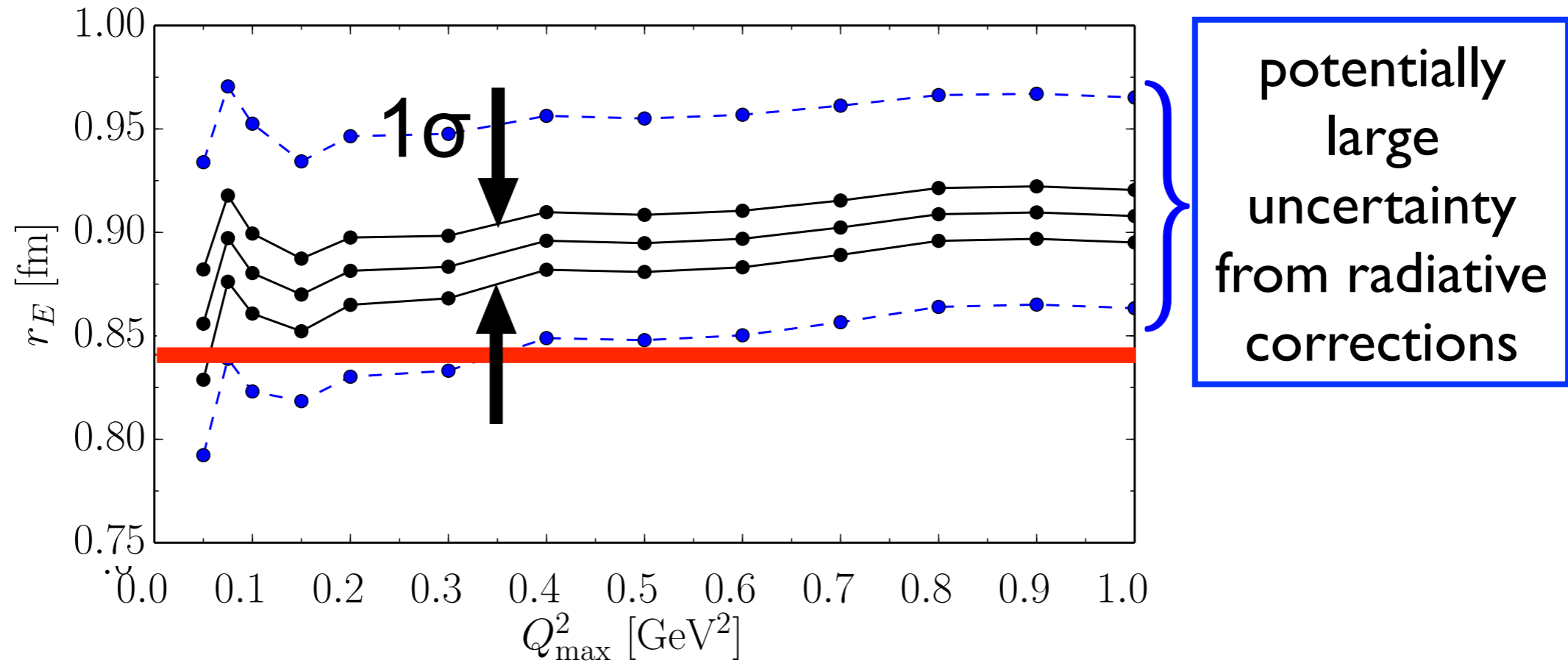
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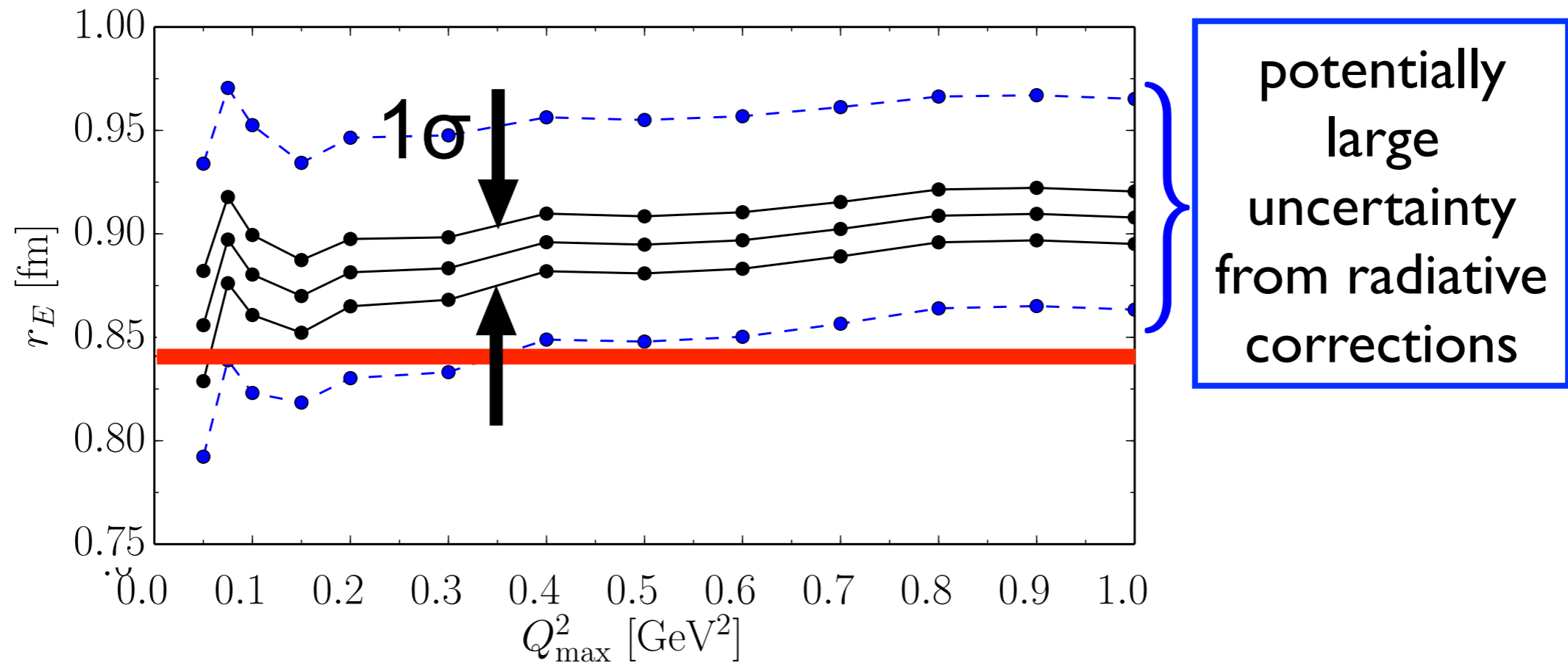


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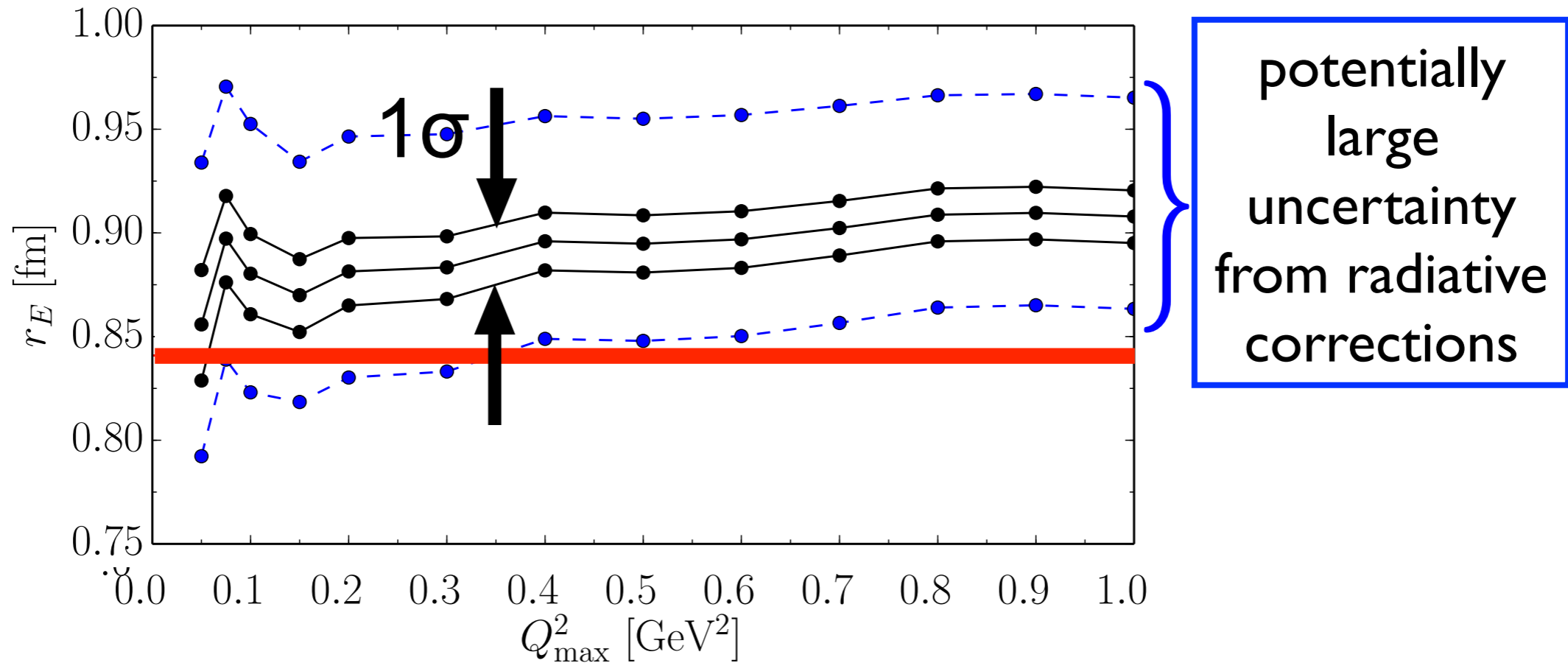
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- soft-collinear effective theory developed to systematically compute/resum large logarithms *details: 1605.02613*

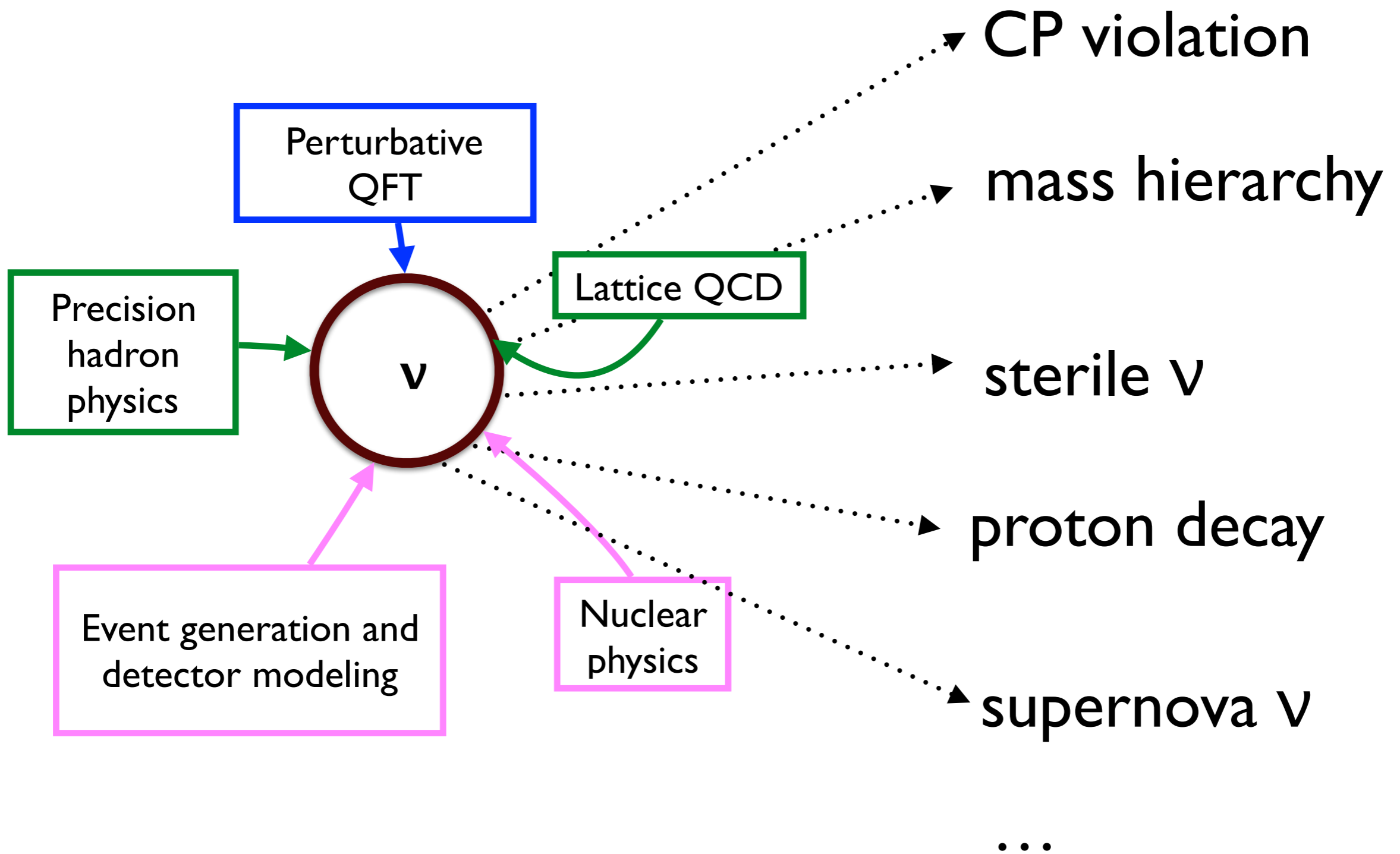
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- same formalism applies to neutrino processes, impacting  $\nu_e/\nu_\mu$  cross section ratios, critical for long baseline oscillation program

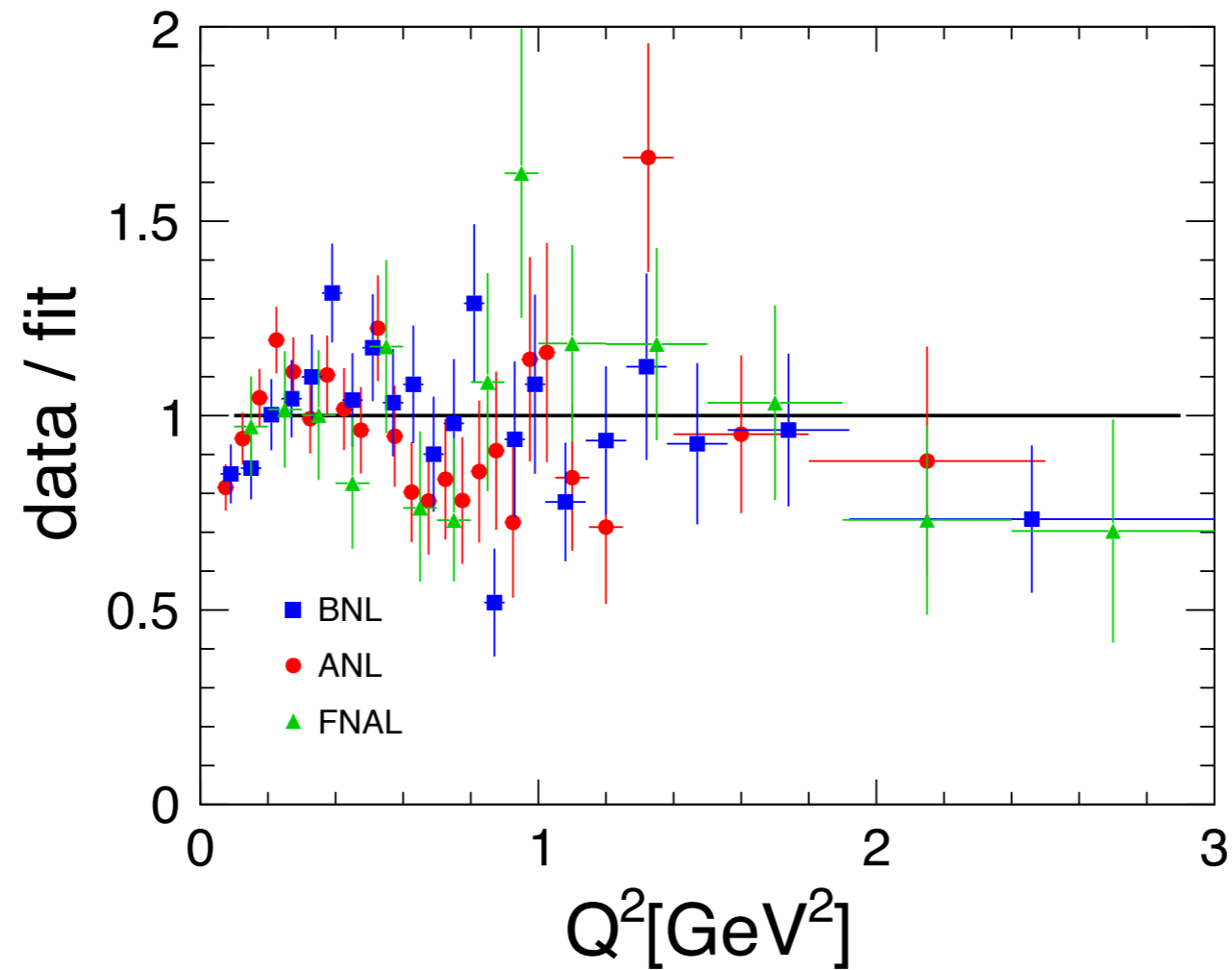
## Summary

- progress in neutrino cross sections critical for probing new physics of the neutrino sector
- z expansion + lattice QCD: *model independent analysis of elementary signal process  $\sigma(\nu_\ell n \rightarrow \ell^- p)$*
- soft-collinear effective theory: *radiative corrections to  $\sigma(\nu_e n \rightarrow e^- p)/\sigma(\nu_\mu n \rightarrow \mu^- p)$*
- active field, modern theory techniques: *soft-collinear effective theory, lattice gauge theory, ab initio nuclear responses*
- many unavoidable connections with other intensity/cosmic/energy frontier problems



backup





In final determination :

- joint fit to all data (ANL, BNL, FNAL)
- include correlated efficiency correction (for each dataset)
- include additional uncorrelated error to achieve  $\chi^2/\text{d.o.f.} = 1$  ( $\delta N/N \approx 10\%$ )

- experimental acceptance/efficiency correction

see: 1603.03048

allow for correlated variation:  $\eta=0 \pm 1$

$$\frac{dN}{e(Q^2)} \rightarrow \frac{dN}{e(Q^2) + de(Q^2)} = \frac{dN}{e(Q^2)} \left( 1 + \eta_e \frac{de(Q^2)}{e(Q^2)} \right)^{-1}$$

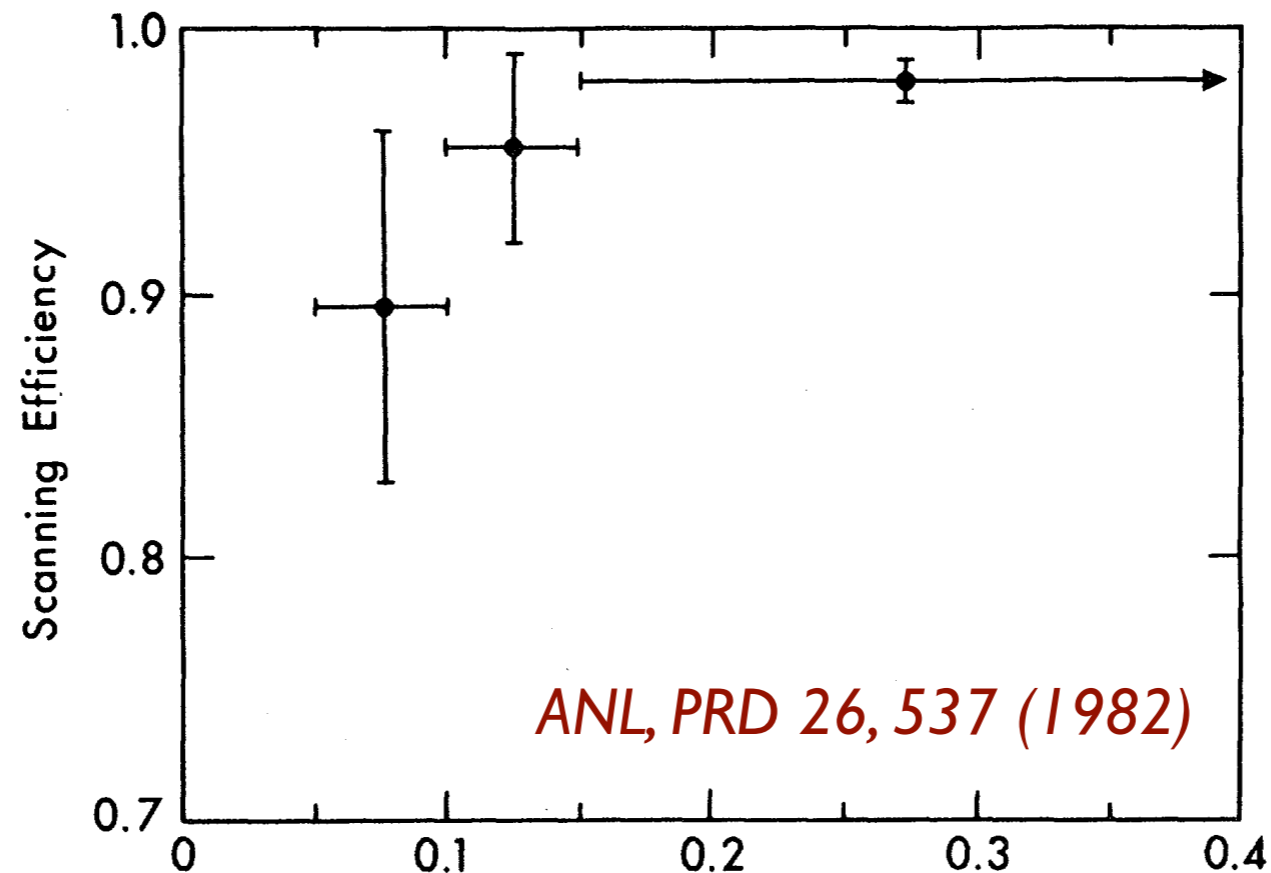
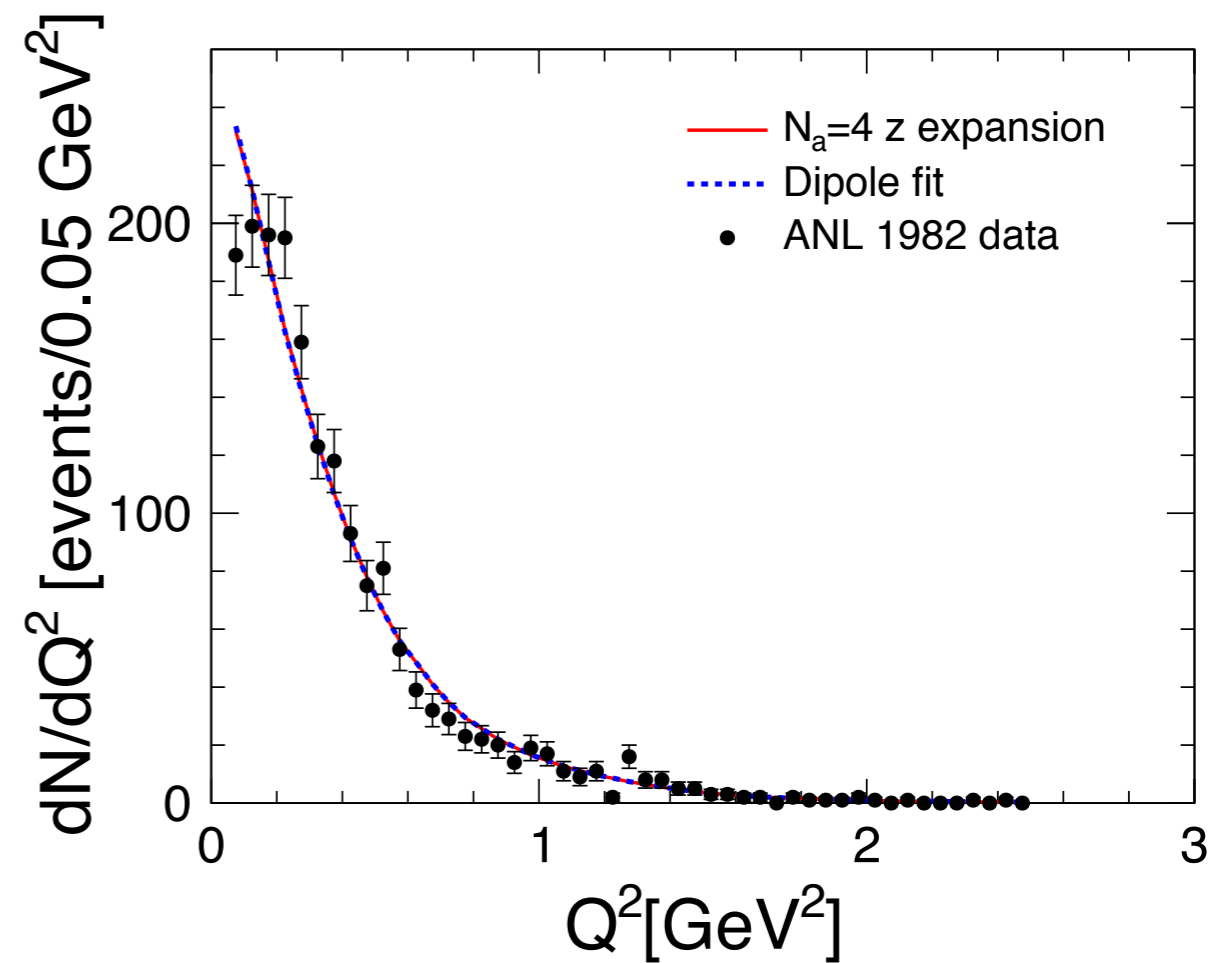
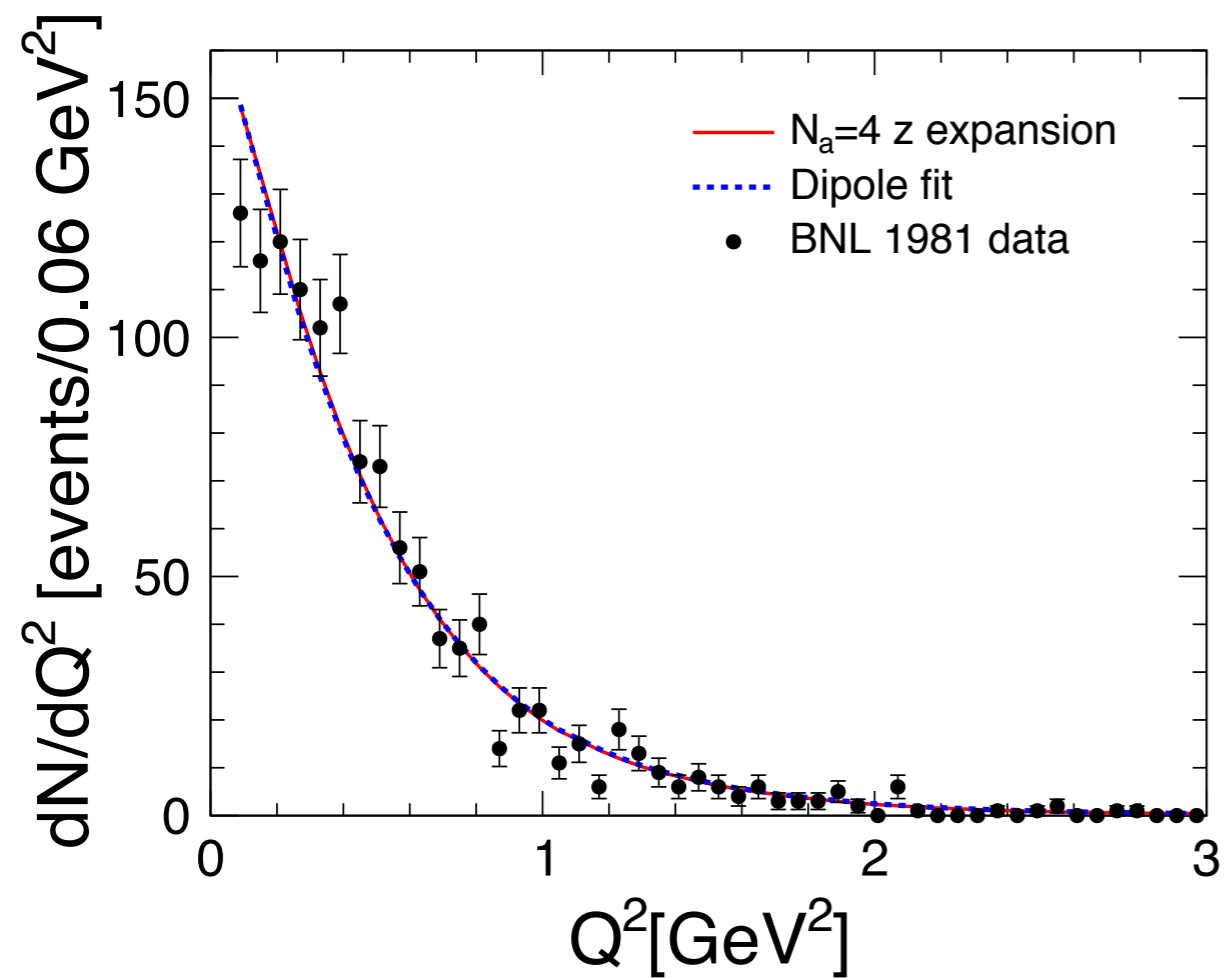


FIG. 1. Scanning efficiency as a function of momentum transfer squared.

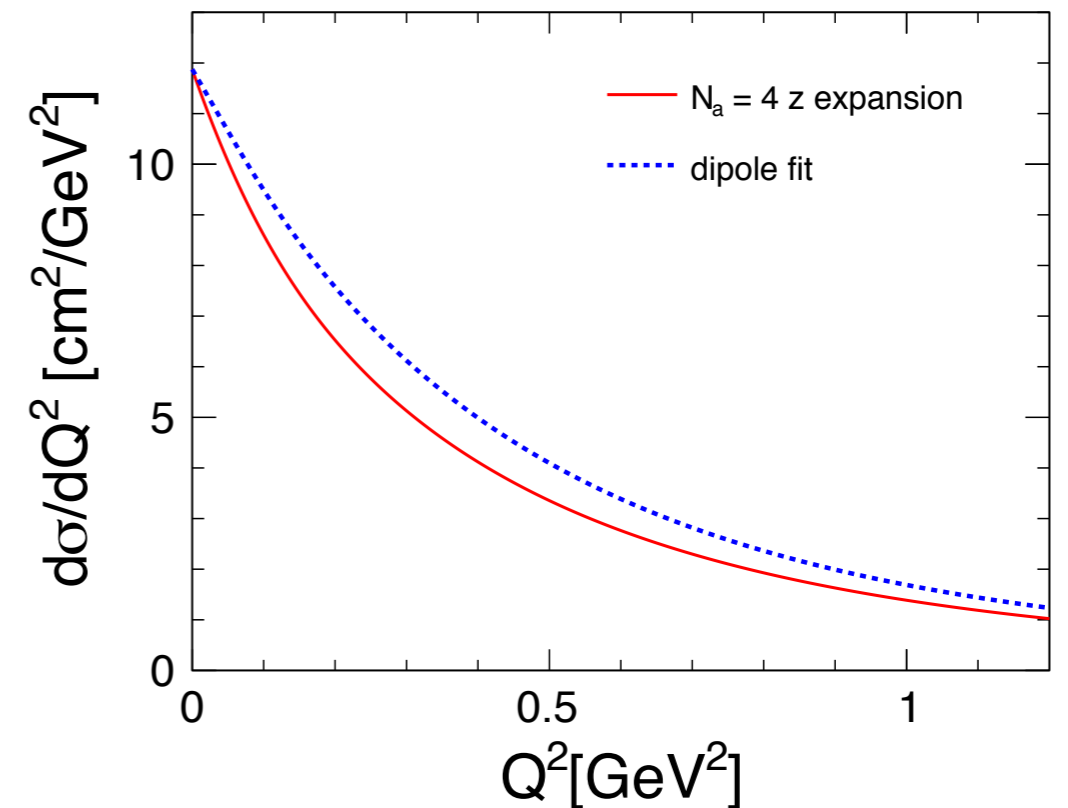
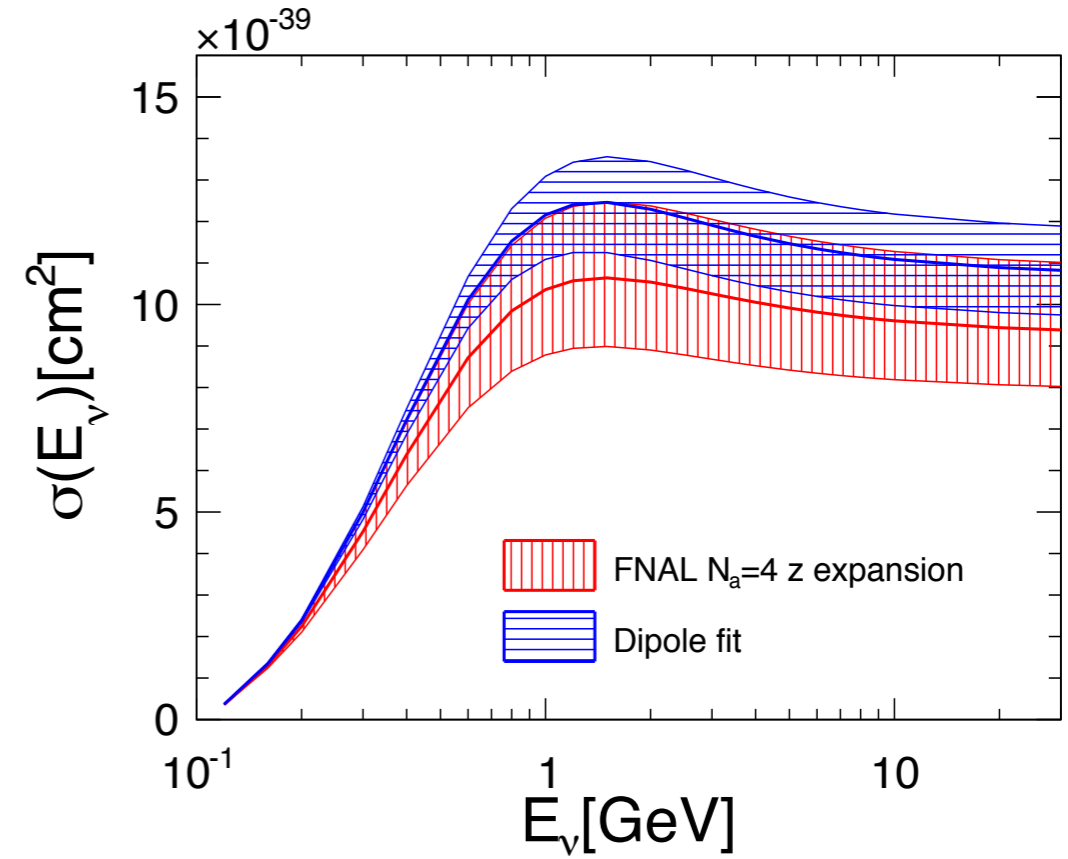
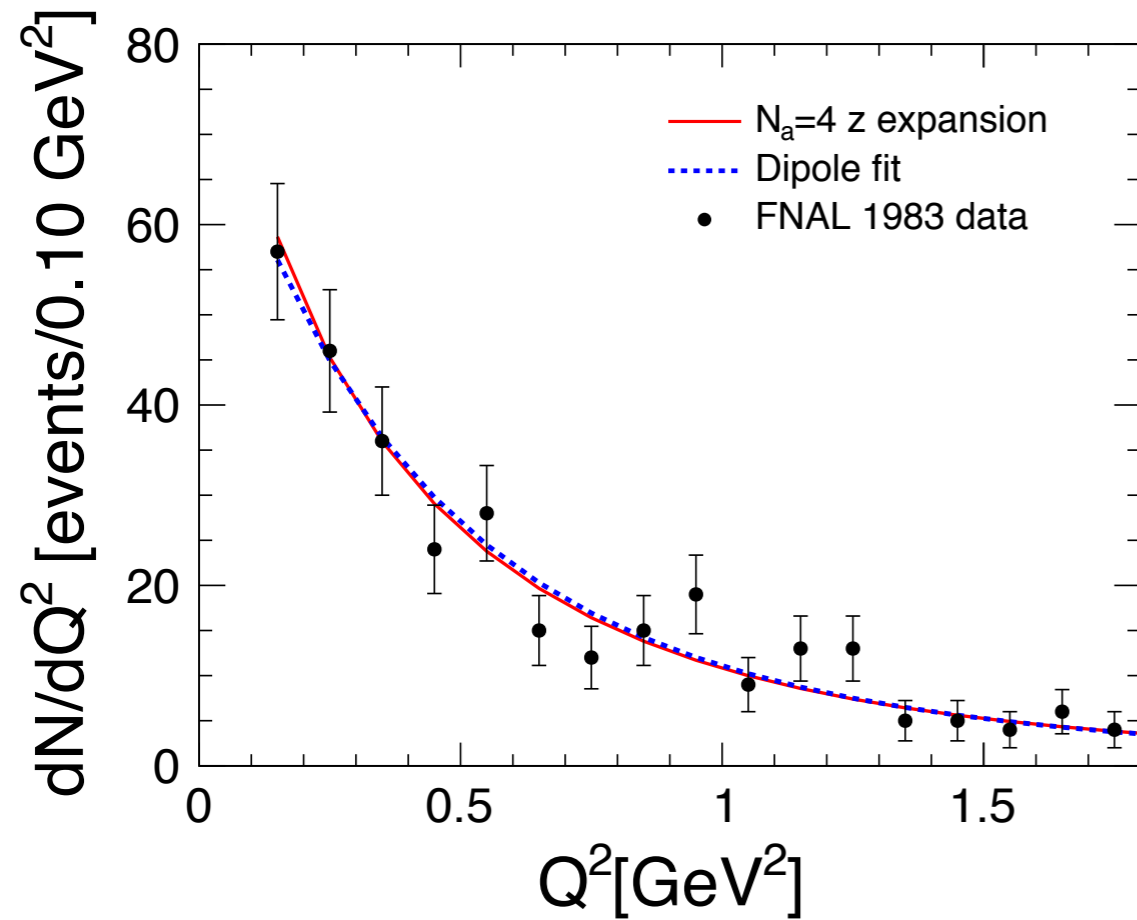
data prefer  $\eta \neq 0$  (ANL:  $\eta = -1.9$ , BNL:  $\eta = -1$ ), but no significant improvement in fit quality



- Poor fit quality, symptom of underestimated systematic errors

# Dipole and z expansion yield different $F_A$

see: 1603.03048



(recall floating normalization and self-consistent flux: different  $F_A$  can yield similar  $dN/dQ^2$  in fit range)

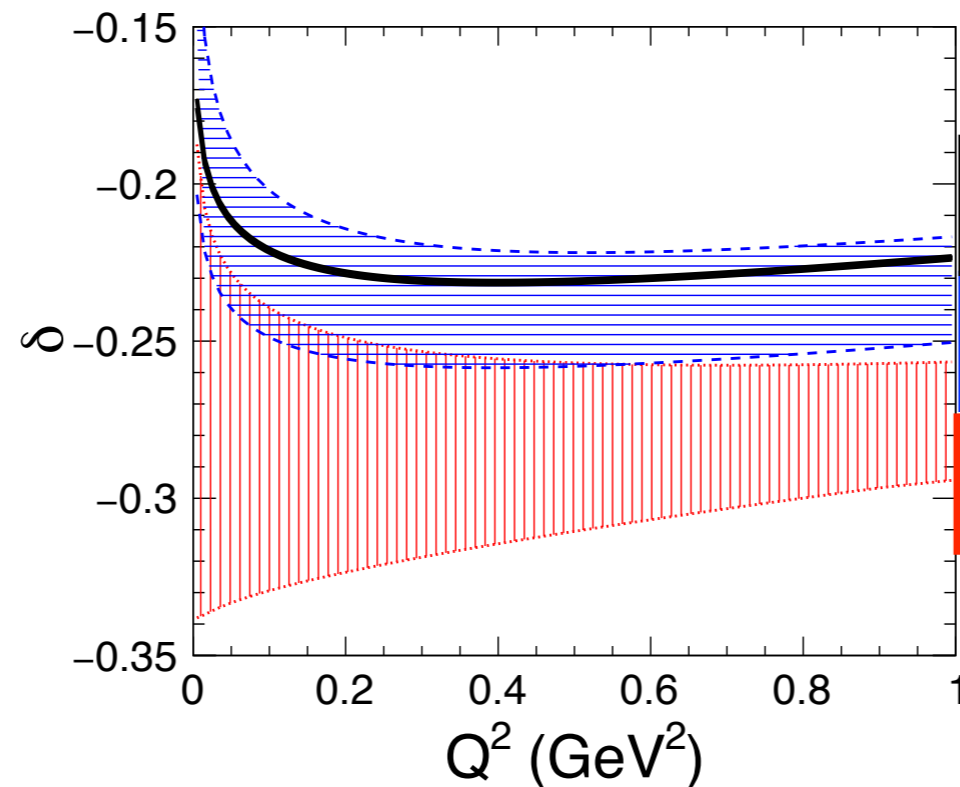
$$d\sigma = H(M) \times \underbrace{\frac{H(\mu)}{H(M)} \times J(\mu) \times S(\mu)}_{\text{total radiative correction}}$$

numerically:  $\alpha L^2 = \alpha \log^2 \frac{Q^2}{m^2} \sim 1 \quad \Rightarrow \quad \alpha L \sim \alpha^{\frac{1}{2}}, \text{ etc.}$

electron energy:  $E = 1 \text{ GeV}$

electron energy loss cut:  $\Delta E = 5 \text{ MeV}$

total radiative  
correction



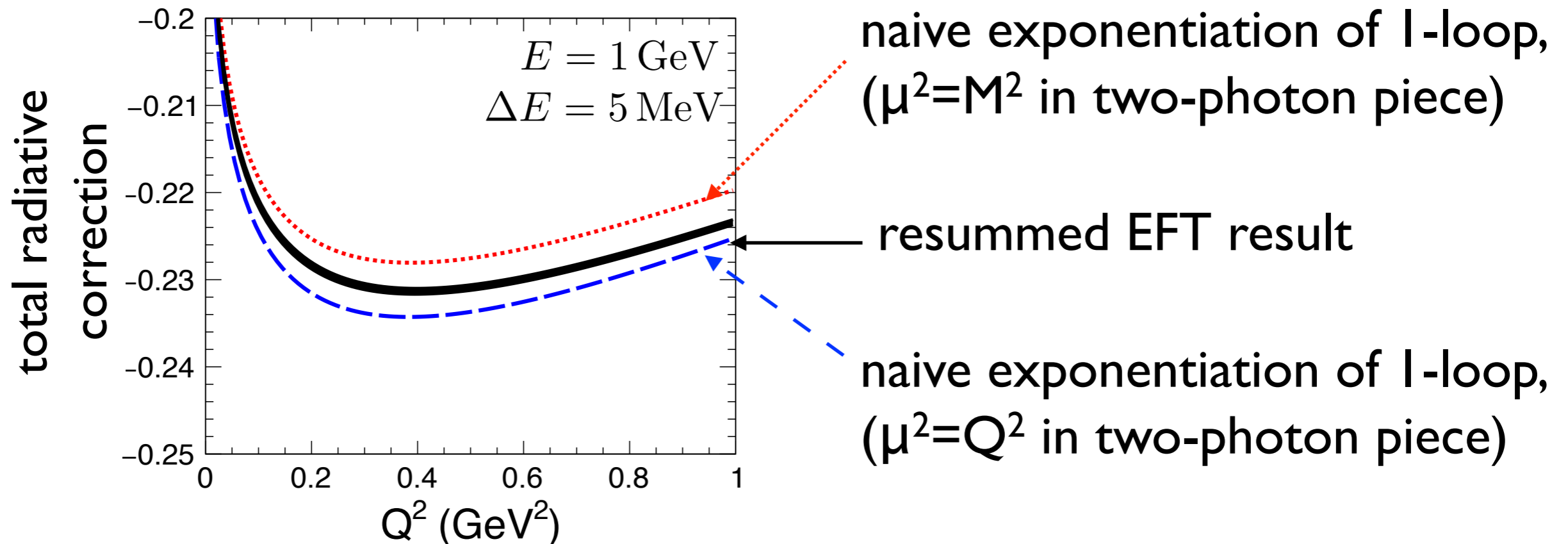
correct  
through:

$$\mathcal{O}(\alpha)$$

$$\mathcal{O}(\alpha^{\frac{1}{2}})$$

$$\mathcal{O}(1)$$

Comparison to previous implementations of radiative corrections, e.g. in A1 collaboration analysis of electron-proton scattering data



- discrepancies at 0.5-1% compared to currently applied radiative correction models (cf. 0.2-0.5% systematic error budget of A1)

- conflicting implicit scheme choices for 1PE and 2PE

- complete analysis: account for floating normalizations, correlated shape variations when fitting together with backgrounds

Broader context: Sudakov logs ubiquitous, appear whenever kinematic invariants large compared to particle masses. Poor convergence, or even breakdown of fixed order perturbation theory

- massive boson production at proton collider

$$\alpha_s \log^2 \frac{m_Z^2}{q_T^2} \quad q_T \sim \text{GeV}$$

- dark matter annihilation

$$\alpha_2 \log^2 \frac{M_{\text{DM}}^2}{m_W^2} \quad M_{\text{DM}} \sim \text{TeV}$$

- Lepton-nucleon scattering

$$\alpha \log^2 \frac{Q^2}{m_e^2} \quad Q \sim \text{GeV}$$

Effective theories differ in detail. For lepton-nucleon scattering: explicit lepton mass, bremsstrahlung energy cut, nuclear recoil and charge corrections