Type Ia Supernovae meet Maximum Likelihood Estimators

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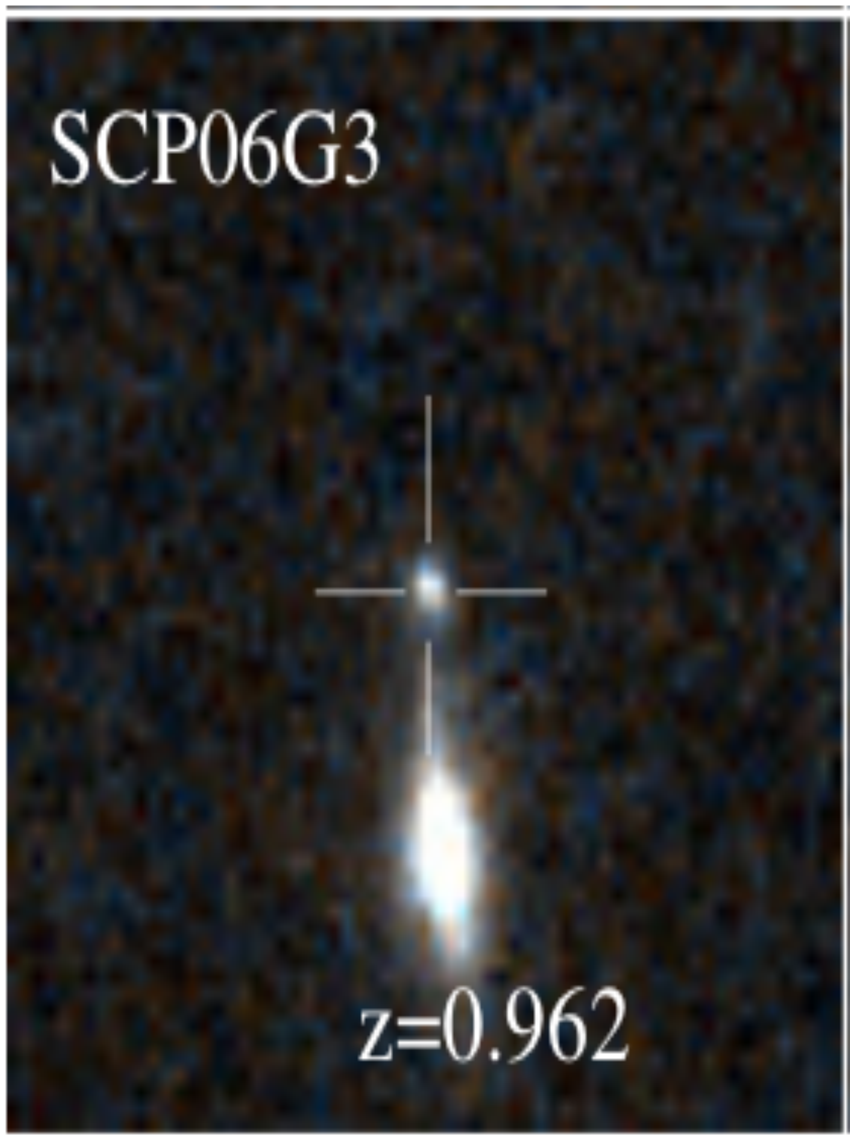
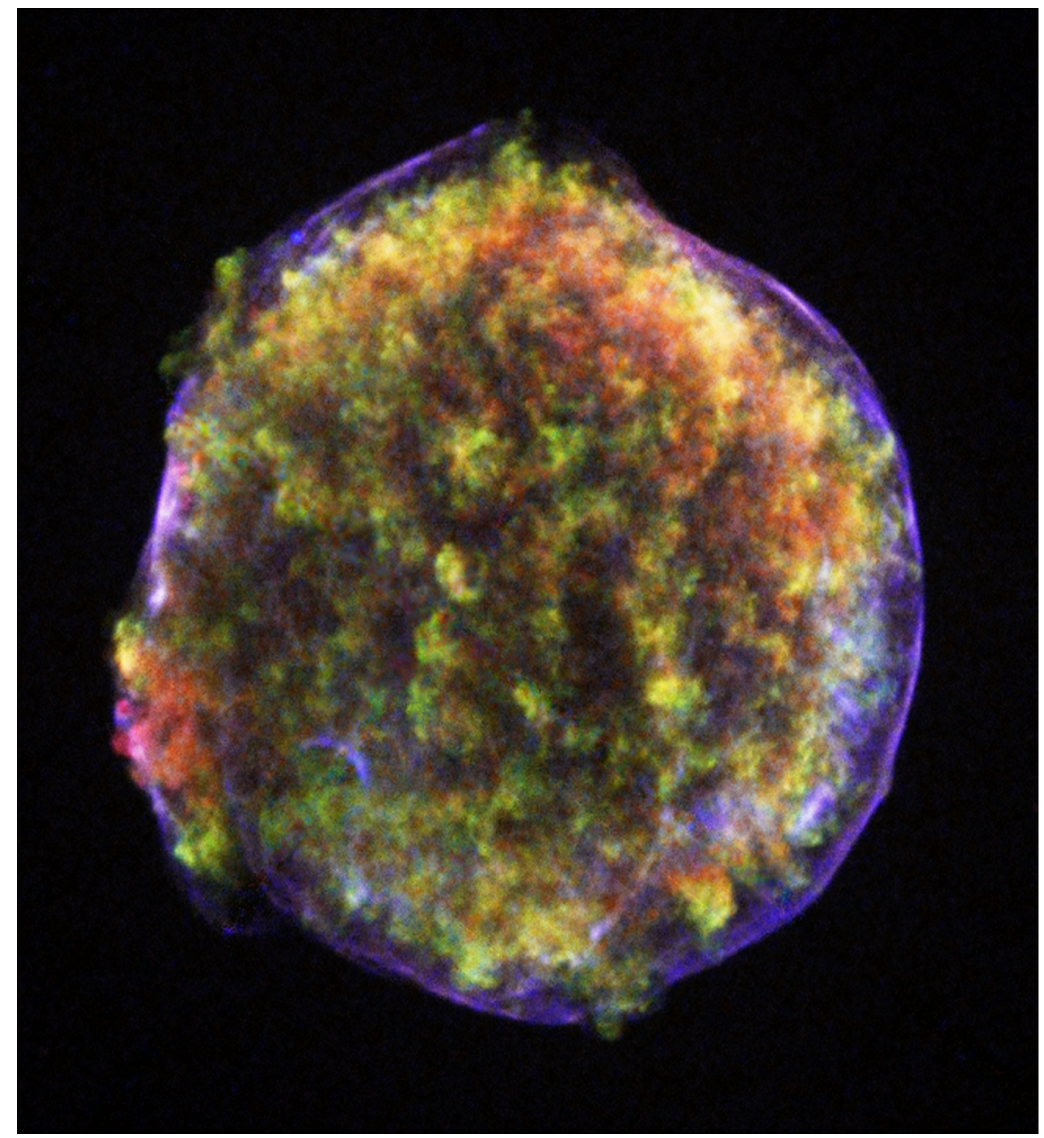
J. Trøst Nielsen, AG and S. Sarkar, arXiv:1506.01354
J. Trøst Nielsen, arXiv:1508.07850

XXVIIIth Rencontres de Blois - Particle Physics and Cosmology
Blois, 29 May - 3 June, 2016

Type Ia Supernovae

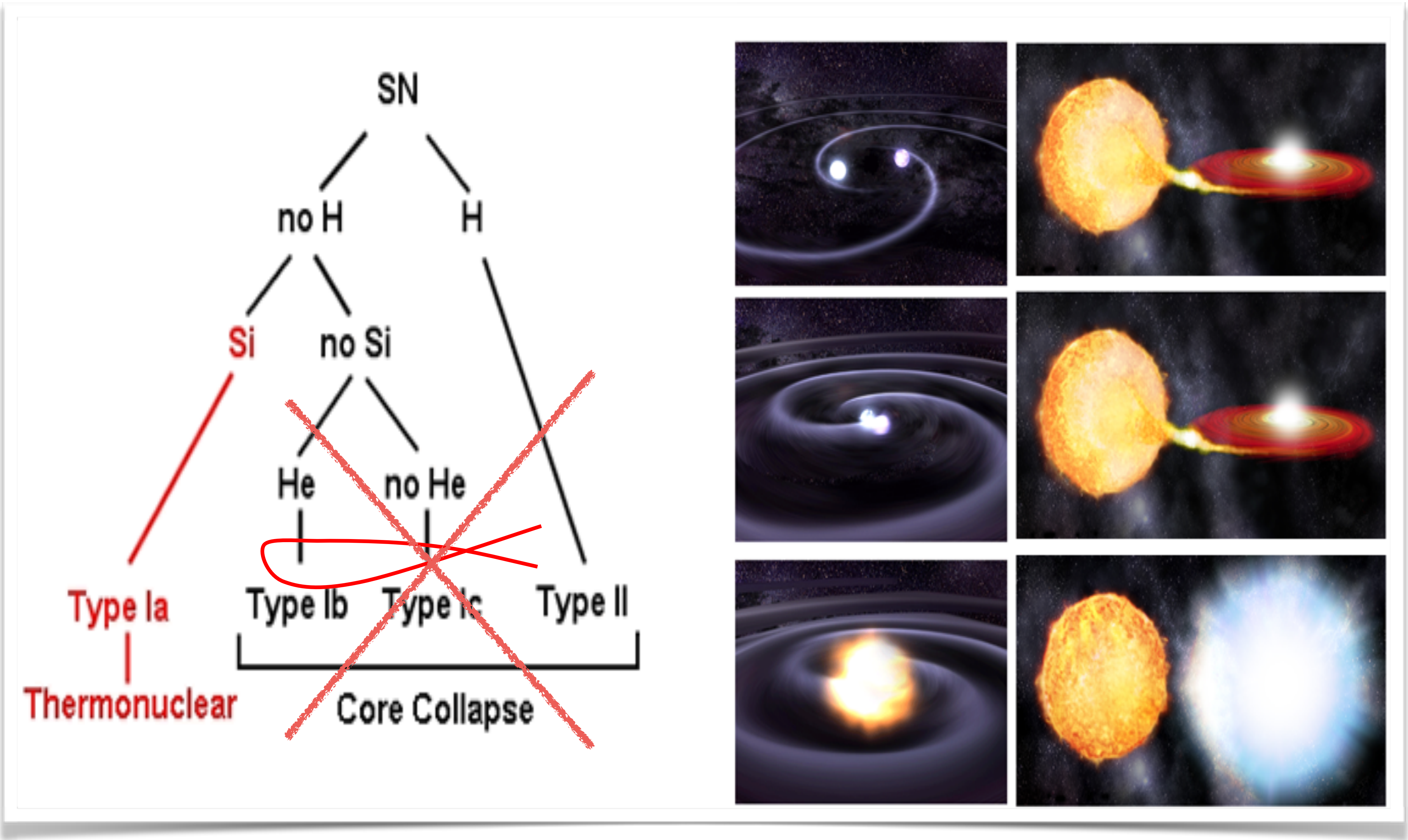


Suzuki *et al*, 1105.3470

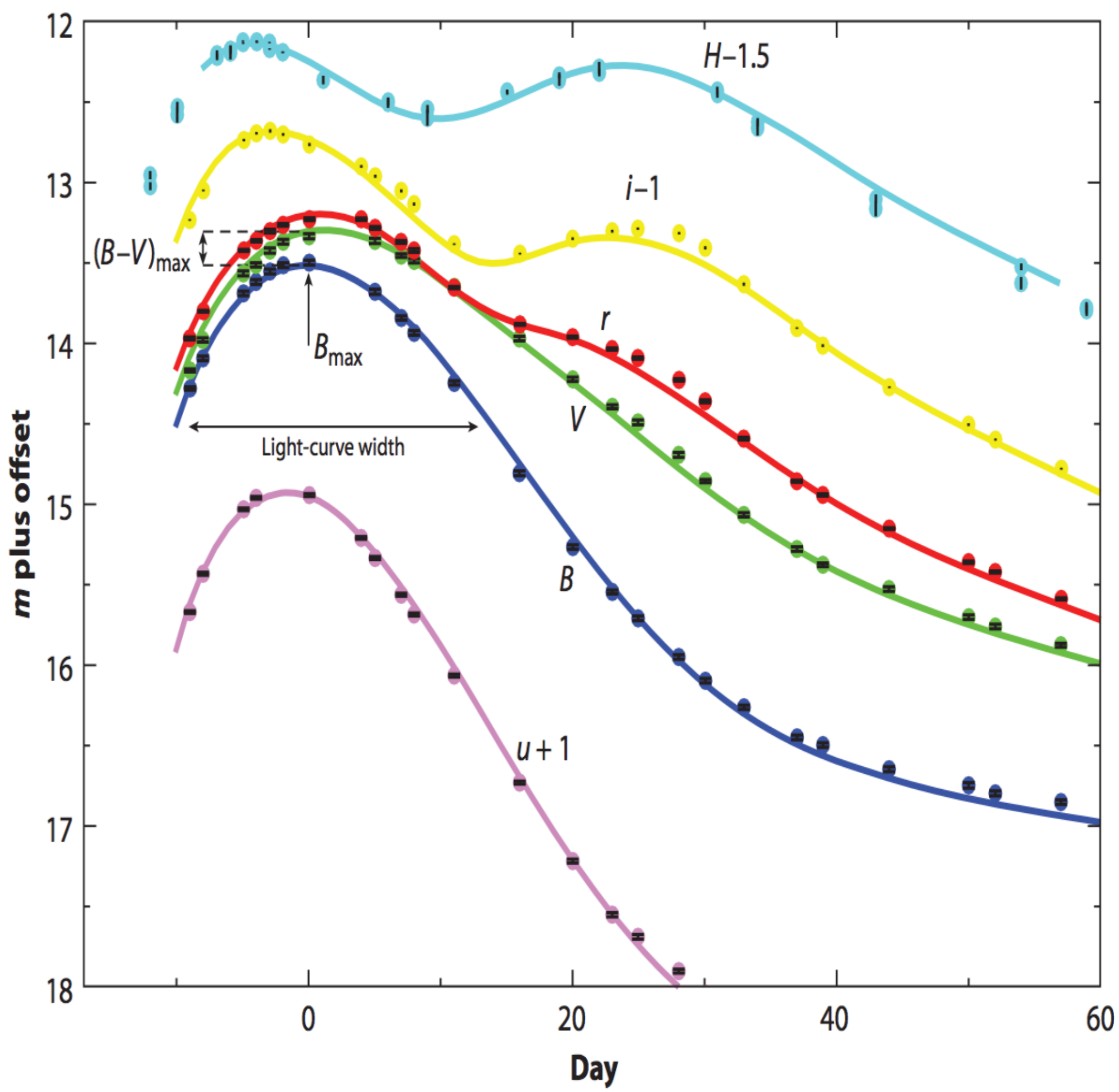


SN 1572 (Tycho)
NASA/CXC/Rutgers/Warren & Hughes

Type Ia Supernovae

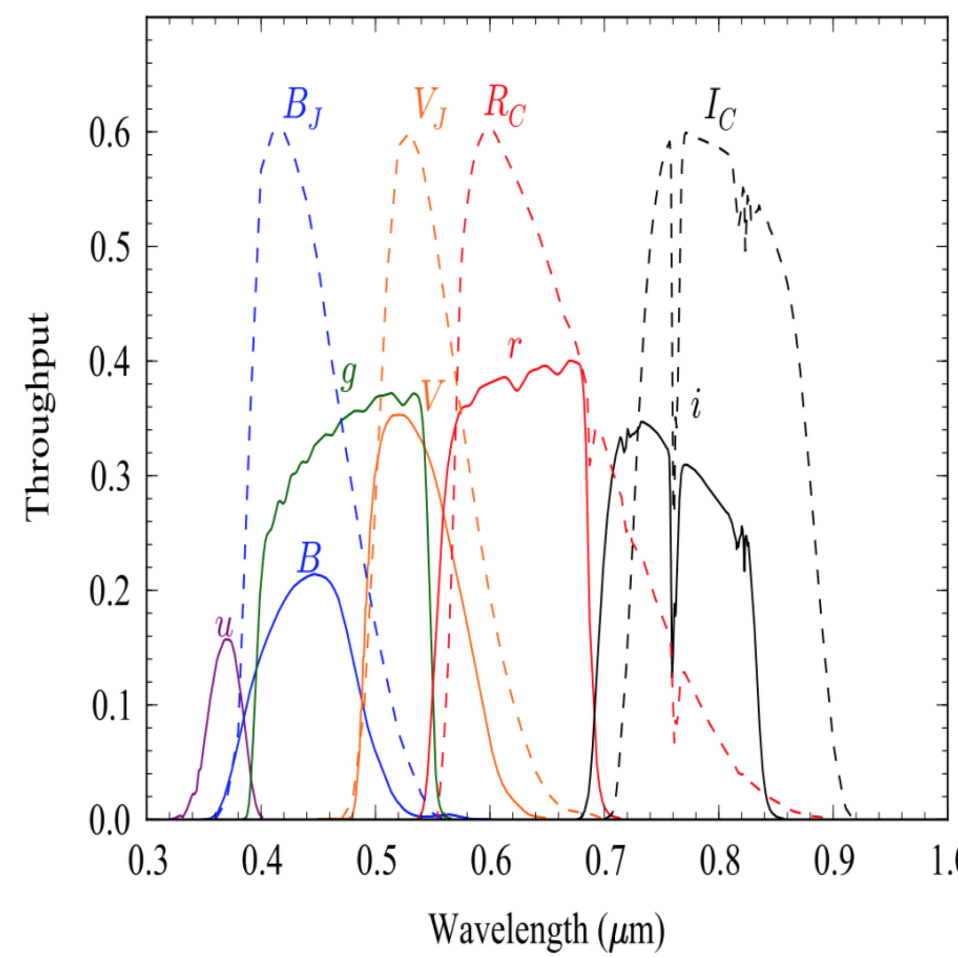


Type Ia Supernovae

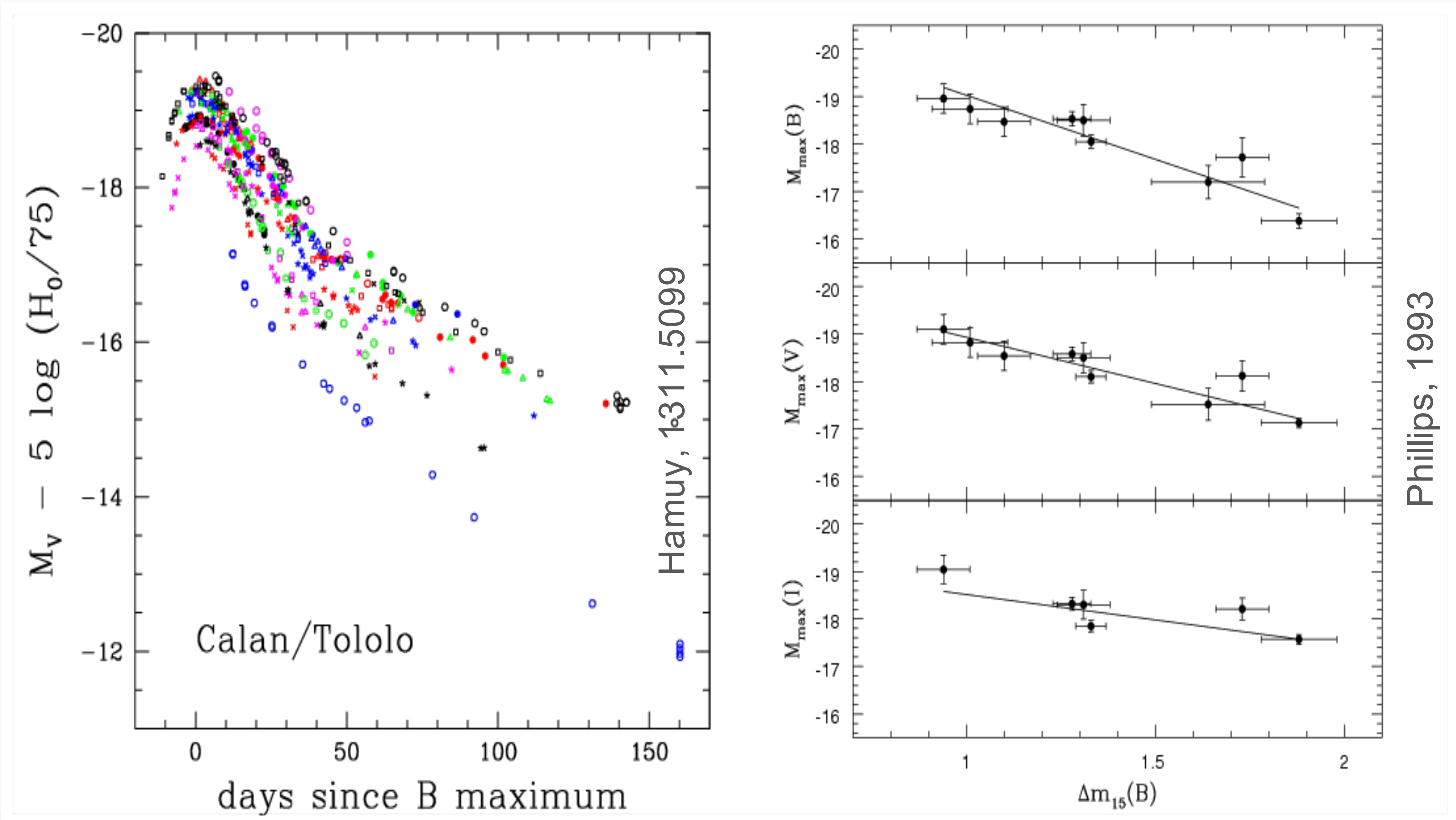


Goobar & Leibundgut, 1102.1431

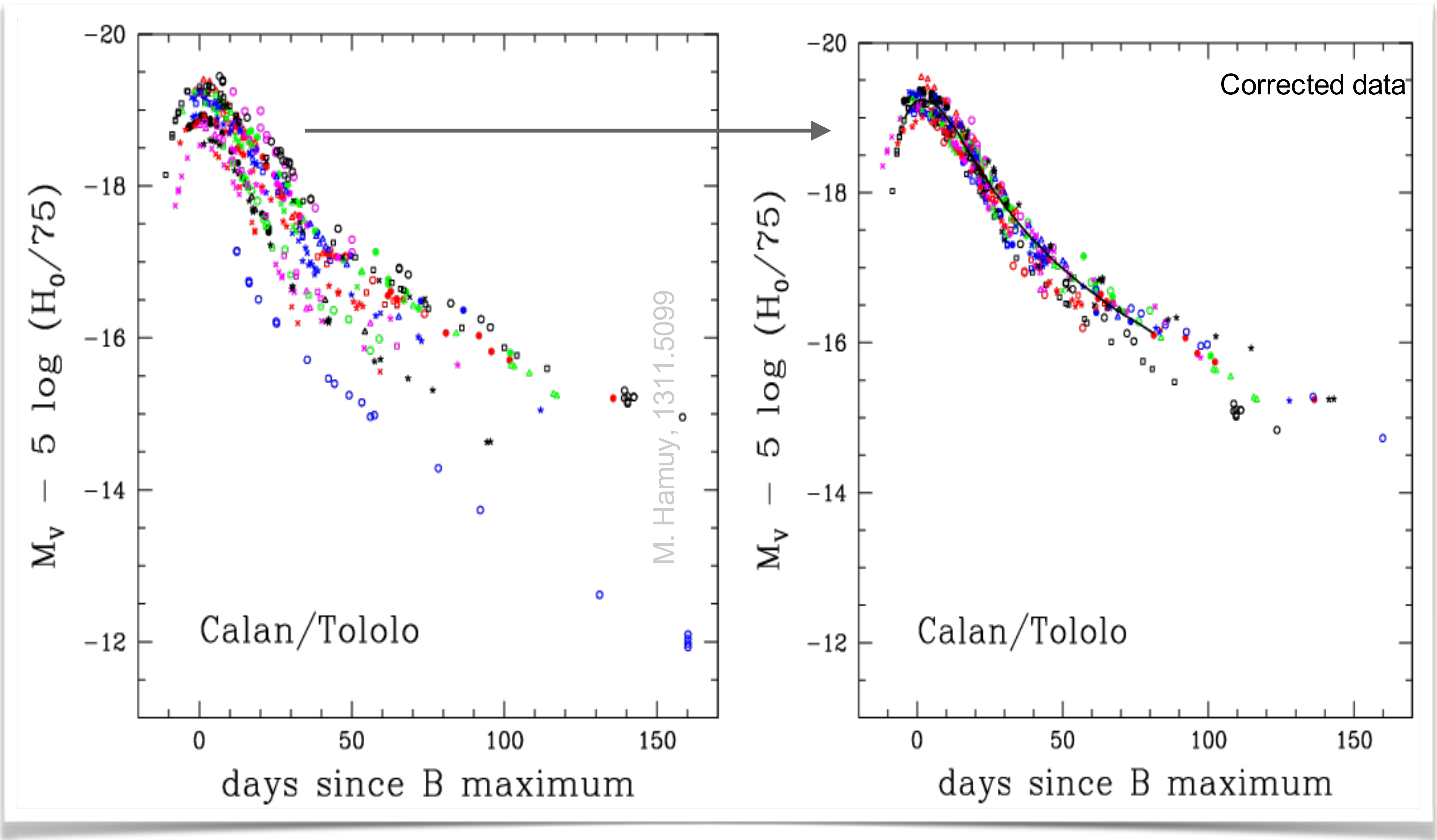
$$m = -2.5\log(F/F_{\mathrm{ref}})$$



Type Ia Supernovae - Standard Candles?



Type Ia Supernovae - Standardizable Candles



Type Ia Supernovae - Standardizable Candles

- We take as input to our analysis SN Ia standardises using the SALT2 LC fitting procedure

[Guy et al, 2007, 2010]

SALT 2 parameters

Betoule *et al.*, 1401.4064

Name	$z_{\rm cmb}$	$m_B^\star$	X_1	C	$M_{\rm stellar}$	?
03D1ar	0.002	23.941 ± 0.033	−0.945 ± 0.209	0.266 ± 0.035	10.1 ± 0.5	?
03D1au	0.503	23.002 ± 0.088	1.273 ± 0.150	−0.012 ± 0.030	9.5 ± 0.1	?
03D1aw	0.581	23.574 ± 0.090	0.974 ± 0.274	−0.025 ± 0.037	9.2 ± 0.1	?
03D1ax	0.495	22.960 ± 0.088	−0.729 ± 0.102	−0.100 ± 0.030	11.6 ± 0.1	?
03D1bp	0.346	22.398 ± 0.087	−1.155 ± 0.113	−0.041 ± 0.027	10.8 ± 0.1	?
03D1co	0.678	24.078 ± 0.098	0.619 ± 0.404	−0.039 ± 0.067	8.6 ± 0.3	?
03D1dt	0.611	23.285 ± 0.093	−1.162 ± 1.641	−0.095 ± 0.050	9.7 ± 0.1	
03D1ew	0.866	24.354 ± 0.106	0.376 ± 0.348	−0.063 ± 0.068	8.5 ± 0.8	
03D1fc	0.331	21.861 ± 0.086	0.650 ± 0.119	−0.018 ± 0.024	10.4 ± 0.0	
03D1fq	0.799	24.510 ± 0.102	−1.057 ± 0.407	−0.056 ± 0.065	10.7 ± 0.1	
03D3aw	0.450	22.667 ± 0.092	0.810 ± 0.232	−0.086 ± 0.038	10.7 ± 0.0	
03D3ay	0.371	22.273 ± 0.091	0.570 ± 0.198	−0.054 ± 0.033	10.2 ± 0.1	
03D3ba	0.292	21.961 ± 0.093	0.761 ± 0.173	0.116 ± 0.035	10.2 ± 0.1	
03D3bl	0.356	22.927 ± 0.087	0.056 ± 0.193	0.205 ± 0.030	10.8 ± 0.1	

$$\boxed{\mu_B = m_B^* - M + \alpha X_1 - \beta C}$$

Distance moduli as probes of cosmology

$$F = \frac{L}{4\pi d_L^2}$$

$$d_L = (1+z)\frac{d_H}{\sqrt{\Omega_k}}\sinh\left(\sqrt{\Omega_k}\int_0^z \frac{H_0 dz'}{H(z')}\right)$$

$$\mu_C = m - M = -2.5\log\frac{F/F_{\mathrm{ref}}}{L/L_{\mathrm{ref}}} = 5\log\frac{d_L}{10\mathrm{pc}}$$

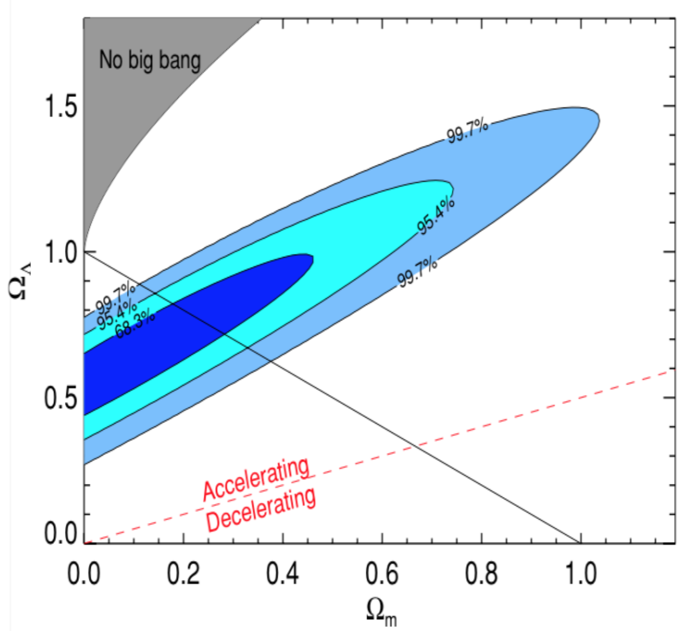
$$\chi^2 = \sum_{objects} \frac{(\mu_B - 5\log_{10}(d_L(\theta,z)/10pc))^2}{\sigma^2(\mu_B) + \sigma_{int}^2}$$

Astier *et al.*, 2006

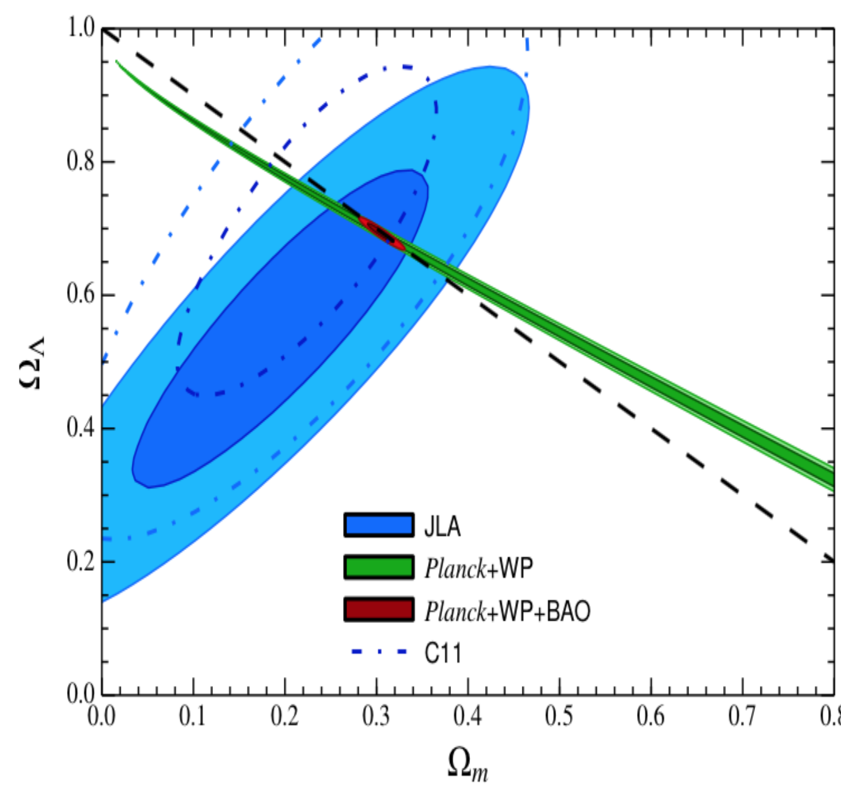
Supernova Cosmology: the conventional wisdom

"SN data alone require cosmic acceleration at >99.999% confidence, including systematic effects"

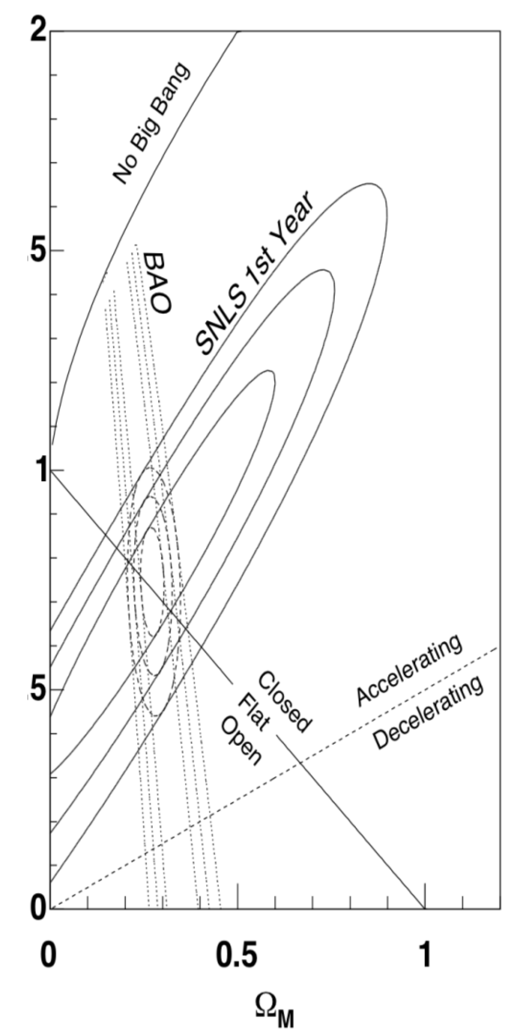
Conley *et al*, 2011



Betoule *et al,* 2014



Astier *et al*, 2006



A critical appraisal of the standard constrained χ^2

$$\mu_B = m_B^{\max} + \alpha \cdot x_1 - \beta \cdot c + \delta \cdot P(m_*^{\text{true}} < m_*^{\text{threshold}}) - M_B,$$

where M_B is the absolute B-band magnitude of a SN Ia with $x_1 = 0$, $c = 0$ and $P(m_\star^{\text{true}} < m_\star^{\text{threshold}}) = 0$. The parameters α, β, δ and M_B are nuisance parameters that are fitted simultaneously with the cosmological parameters. The SN Ia

4.4. *Fitting the Cosmology*

Following Amanullah et al. (2010), the best-fit cosmology is determined by minimizing

$$\chi^2_{\text{stat}} = \sum_{\text{SNe}} \frac{[\mu_B(\alpha, \beta, \delta, M_{\text{B}}) - \mu(z; \Omega_m, \Omega_w, w)]^2}{\sigma^2_{\text{lc}} + \sigma^2_{\text{ext}} + \sigma^2_{\text{sample}}}. \quad (4)$$

A detailed discussion of the terms in this equation can be found in Amanullah et al. (2010). We only comment on the final term in the denominator, σ^2_{sample}, which is computed by setting the reduced χ^2 of each sample to unity. This term was referred to as "$\sigma^2_{\text{systematic}}$" in Kowalski et al. (2008); Amanullah et al. (2010). We note that σ^2_{sample} includes intrinsic dispersion as well as sample-dependent effects. This term effectively further deweights samples with poorer-quality data that has sources of error which have not been accounted for. As noted in Amanullah et al. (2010), this may occasionally deweight an otherwise well-measured supernova.

4.5. *Systematic errors*

In this paper, we follow the systematics analysis we presented in Amanullah et al. (2010). Systematic errors that directly affect supernova distance measurements (calibration, and galactic extinction, for example) are treated as nuisance parameters to be fit simultaneously with the cosmology. Minimizing over these nuisance parameters gives additional terms to add to the distance modulus covariance matrix

$$U_{ij} = \sum_{\epsilon} \frac{d\mu_i(\alpha, \beta)}{d\epsilon} \frac{d\mu_j(\alpha, \beta)}{d\epsilon} \sigma^2_\epsilon\,, \quad (5)$$

where the sum is over each of these distance systematic errors in the analysis. (Although the distance modulus depends on δ as well as α and β, the derivatives with respect to the zero-points do not.) In this analysis, α and β have little interaction with cosmological parameters. When computing cosmologi-

Union2.1 - arXiv:1105.3470

A critical appraisal of the standard constrained χ^2

$$\mu_B = m_B^{\max} + \alpha \cdot x_1 - \beta \cdot c + \delta \cdot P(m_*^{\text{true}} < m_*^{\text{threshold}}) - M_B,$$

C. EMPIRICAL ADJUSTMENT OF UNCERTAINTIES

The propagated uncertainties are underestimates, as they do not account for the pixel-pixel covariance introduced by warping, sub-sampling, stacking, and convolution of the images. In order to empirically determine by how much the uncertainties are underestimated, we measure the flux f_r and its uncertainty σ_r at random positions in a given difference image in exactly the same way we measure the SN flux. We calculate the weighted mean $\bar{f}_r$ of these flux measurements. In order to guard against reduction and image artifacts, we apply a 3σ cut to the normalized flux distribution $(f_r - \bar{f}_r)/\sigma_r$, rather than cutting on the underestimated errors, σ_r, for the following reason: let's assume that all uncertainties are underestimated by the same factor s_r. If we nominally apply a N-sigma cut using these underestimated uncertainties, we effectively apply an N/s_r-cut, e.g. for a nominal 3-sigma cut and $s_r = 1.5$, the real cut-off is at 2-sigma. In order to avoid this, we determine the normalized flux distribution $(f_r - \bar{f}_r)/\sigma_r$, which has a standard deviation of s_r. The true 3-sigma outliers can then be identified and removed by doing an 3-sigma cut on the normalized flux distribution. Note that the standard deviation s_r is equivalent to the square-root of the chi-square distribution

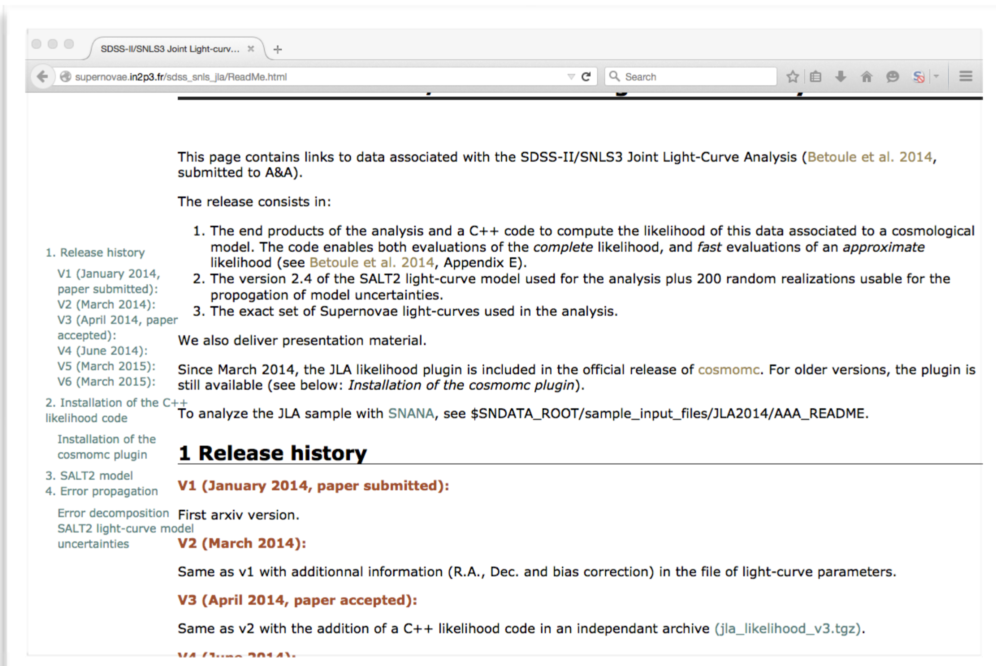
$$s_r = \sqrt{\chi_r^2} = \frac{1}{N-2}\sum^{N}\left(\frac{f_r}{-}\bar{f}_r\sigma_r\right)^2 \tag{C1}$$

We multiply all uncertainties by the factor s_r in order to empirically correct the uncertainties. We find that it is imperative to employ this robust way of determining s_r for the method to work correctly. The fact that the reduced chi-square of the baseline flux of the SN light curves peaks at 1.0 validates our method (see §5.3.3).

~~In addition, for a given difference image, $\bar{f}_r$ is an estimate of the bias in the flux measurements.~~ The values of $\bar{f}_r$ are in general very small, much smaller than the typical uncertainties. Nevertheless, we adjust all fluxes by this value.

Pan-STARRS1 - arXiv:1310.3828

Data - JLA catalogue



SDSS-II/SNLS3 Joint Light-curv...

supernovae.in2p3.fr/sdss_snls_jla/ReadMe.html

1. Release history
   - V1 (January 2014, paper submitted):
   - V2 (March 2014):
   - V3 (April 2014, paper accepted):
   - V4 (June 2014):
   - V5 (March 2015):
   - V6 (March 2015):
2. Installation of the C++ likelihood code
   - Installation of the cosmomc plugin
3. SALT2 model
4. Error propagation
   - Error decomposition
   - SALT2 light-curve model uncertainties

This page contains links to data associated with the SDSS-II/SNLS3 Joint Light-Curve Analysis (Betoule et al. 2014, submitted to A&A).

The release consists in:

1. The end products of the analysis and a C++ code to compute the likelihood of this data associated to a cosmological model. The code enables both evaluations of the *complete* likelihood, and *fast* evaluations of an *approximate* likelihood (see Betoule et al. 2014, Appendix E).
2. The version 2.4 of the SALT2 light-curve model used for the analysis plus 200 random realizations usable for the propogation of model uncertainties.
3. The exact set of Supernovae light-curves used in the analysis.

We also deliver presentation material.

Since March 2014, the JLA likelihood plugin is included in the official release of cosmomc. For older versions, the plugin is still available (see below: *Installation of the cosmomc plugin*).

To analyze the JLA sample with SNANA, see $SNDATA_ROOT/sample_input_files/JLA2014/AAA_README.

1 Release history

V1 (January 2014, paper submitted):

First arxiv version.

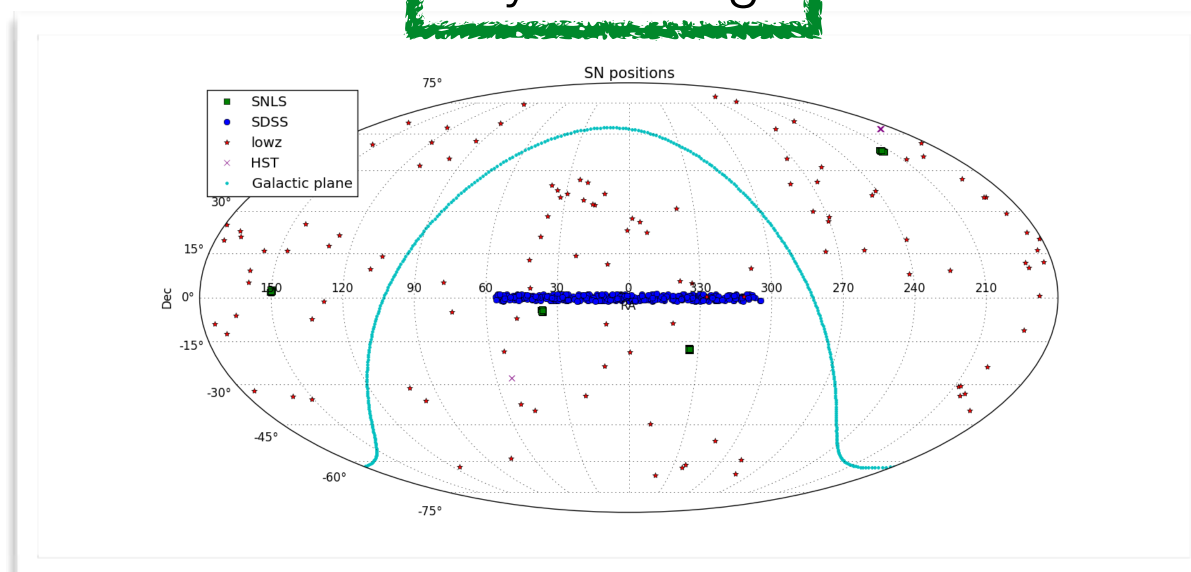
V2 (March 2014):

Same as v1 with additionnal information (R.A., Dec. and bias correction) in the file of light-curve parameters.

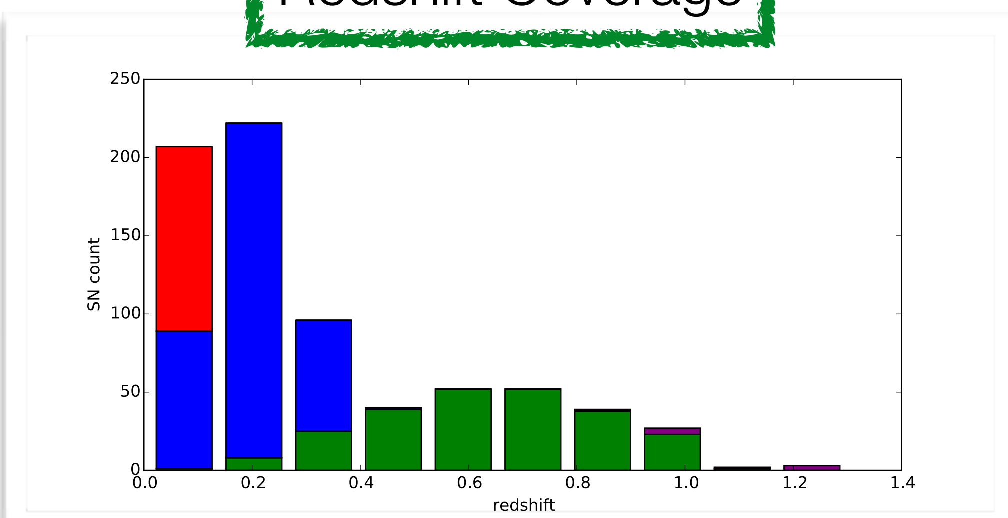
V3 (April 2014, paper accepted):

Same as v2 with the addition of a C++ likelihood code in an independant archive (jla_likelihood_v3.tgz).

Sky Coverage



Redshift Coverage



Our analysis is based on (and makes use of) the public information available through from the JLA website

Maximum Likelihood Estimator analysis

It all starts with a likelihood

$$\mathcal{L} = \text{“probability density”}(\text{data}|\text{model})$$

Disclaimer

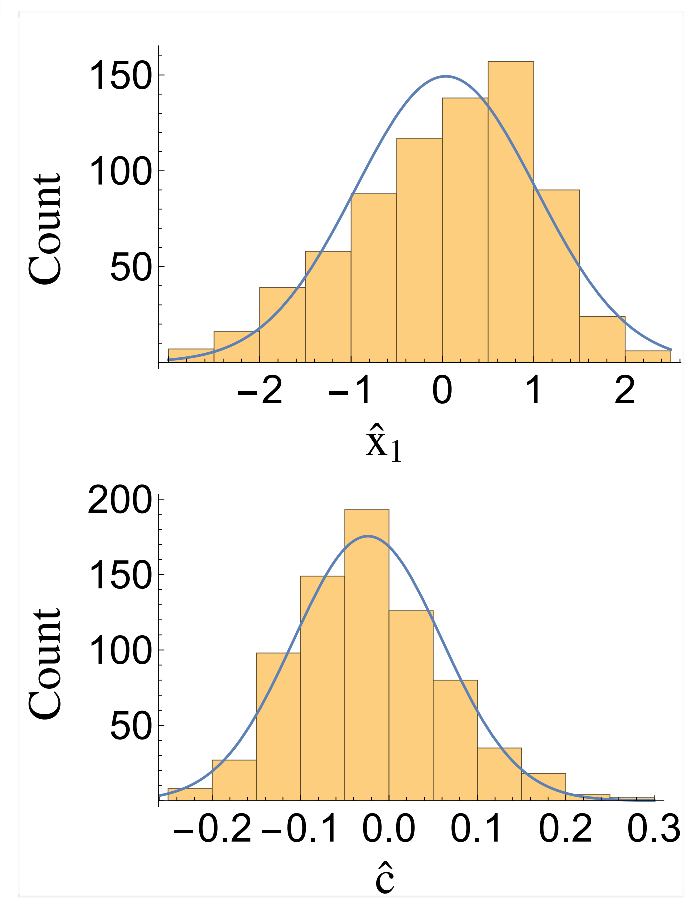
(for my statistics-savvy friends)

I apologise for the abuse of language of defining the likelihood function as a probability density, since technically it is not normalised … life (and talks) are often too short to discuss all the details

Maximum Likelihood Estimator analysis

$$
\begin{aligned}
\mathcal{L} &= p[(\hat{m}_B^*, \hat{x}_1, \hat{c})|\theta] \\
&= \int p[(\hat{m}_B^*, \hat{x}_1, \hat{c})|(M, x_1, c), \theta]\; p[(M, x_1, c)|\theta]\mathrm{d}M\mathrm{d}x_1\mathrm{d}c,
\end{aligned}
$$

$$
\begin{aligned}
&p[(M, x_1, c)|\theta] = p(M|\theta)p(x_1|\theta)p(c|\theta), \quad \text{where:} \\
&p(M|\theta) = (2\pi\sigma_{M_0}^2)^{-1/2} \exp\left\{-\left[(M - M_0)/\sigma_{M_0}\right]^2/2\right\}, \\
&p(x_1|\theta) = (2\pi\sigma_{x_{1,0}}^2)^{-1/2} \exp\left\{-\left[(x_1 - x_{1,0})/\sigma_{x_{1,0}}\right]^2/2\right\}, \\
&p(c|\theta) = (2\pi\sigma_{c_0}^2)^{-1/2} \exp\left\{-\left[(c - c_0)/\sigma_{c_0}\right]^2/2\right\}.
\end{aligned}
$$



Maximum Likelihood Estimator analysis

$$p(Y|\theta) = \frac{1}{\sqrt{|2\pi\Sigma_l|}} \exp\left[-\frac{1}{2}(Y - Y_0)\Sigma_l^{-1}(Y - Y_0)^{\mathrm{T}}\right]$$

$$p(\hat{X}|X,\theta) = \frac{1}{\sqrt{|2\pi\Sigma_d|}} \exp\left[-\frac{1}{2}(\hat{X} - X)\Sigma_d^{-1}(\hat{X} - X)^{\mathrm{T}}\right]$$

intrinsic distributions

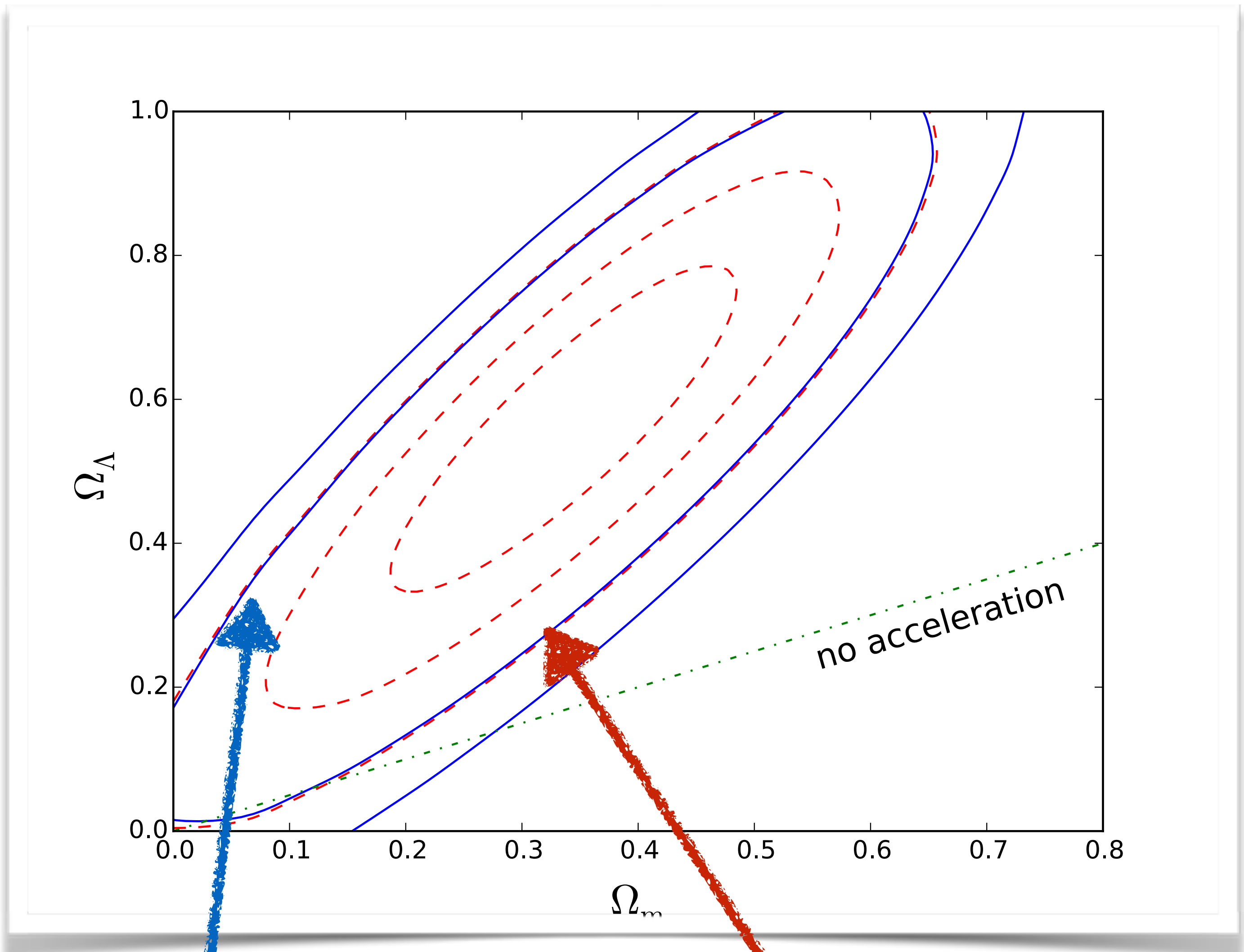
$$\mathcal{L} = \frac{1}{\sqrt{|2\pi(\Sigma_d + A^{\mathrm{T}}\Sigma_l A)|}} \times \exp\left(-\frac{1}{2}(\hat{Z} - Y_0 A)(\Sigma_d + A^{\mathrm{T}}\Sigma_l A)^{-1}(\hat{Z} - Y_0 A)^{\mathrm{T}}\right)$$

cosmology

SALT2

MLE fit result

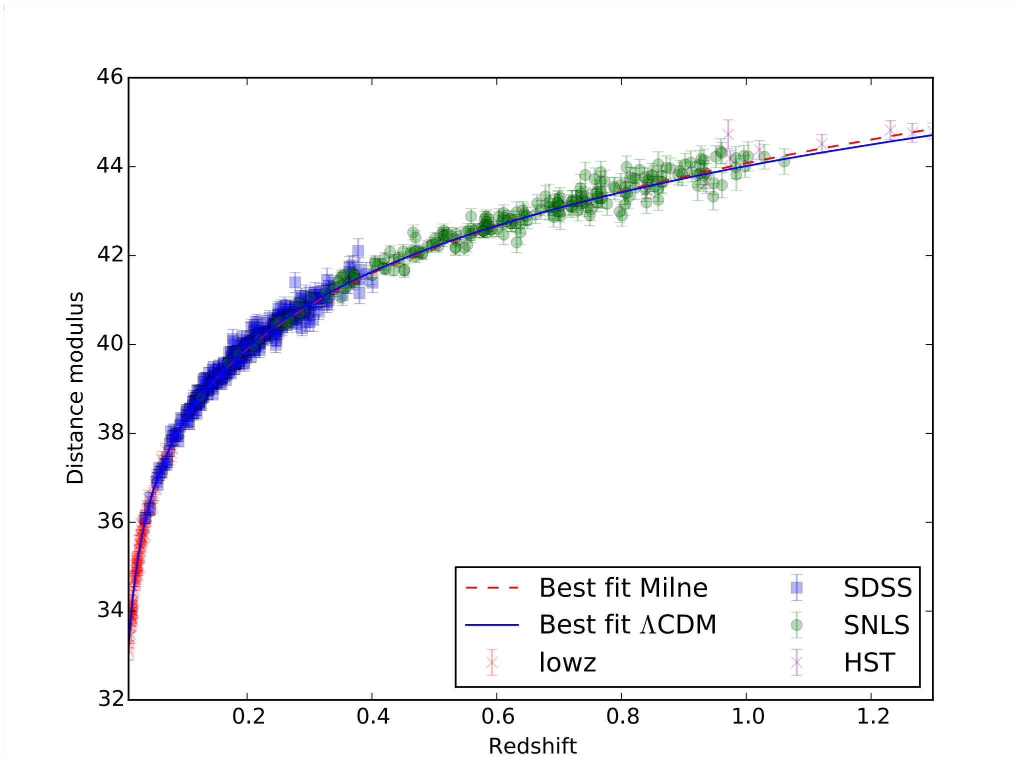
MLE, *best fit*	
Ω_M	0.341
Ω_Λ	0.569
α	0.134
x_0	0.038
σ^2_{x0}	0.931
β	3.058
c_0	-0.016
σ^2_{c0}	0.071
M_0	-19.05
σ^2_{M0}	0.108



Projected 10D confidence regions

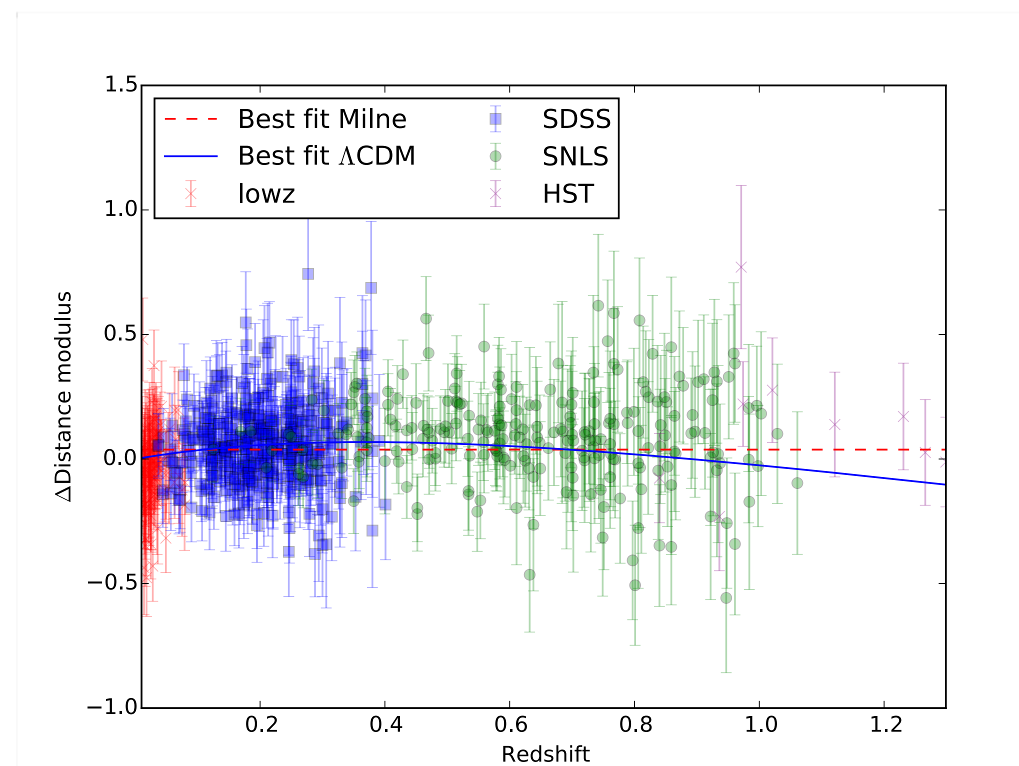
2D confidence regions

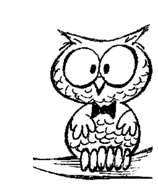
MLE fit result - does it look like a good fit?



Looks like a good fit when looking at the Hubble diagram

… but in all honesty … any (half-decent) fit would …

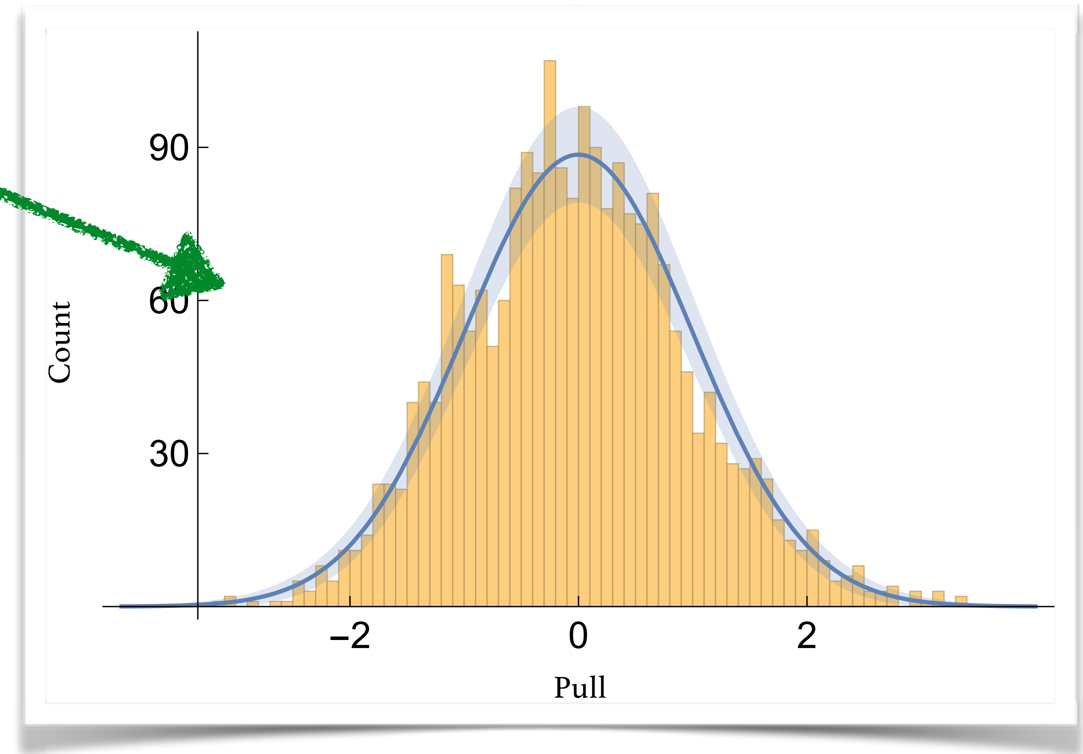


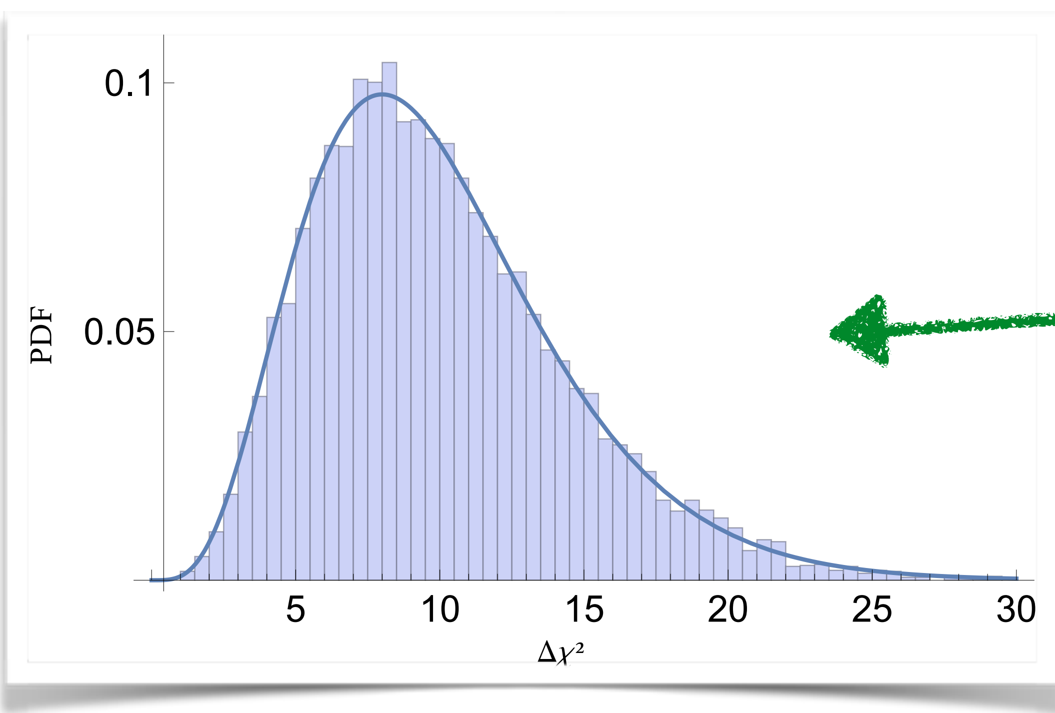


MLE fit result - Is it a good fit?

Pulls distribution

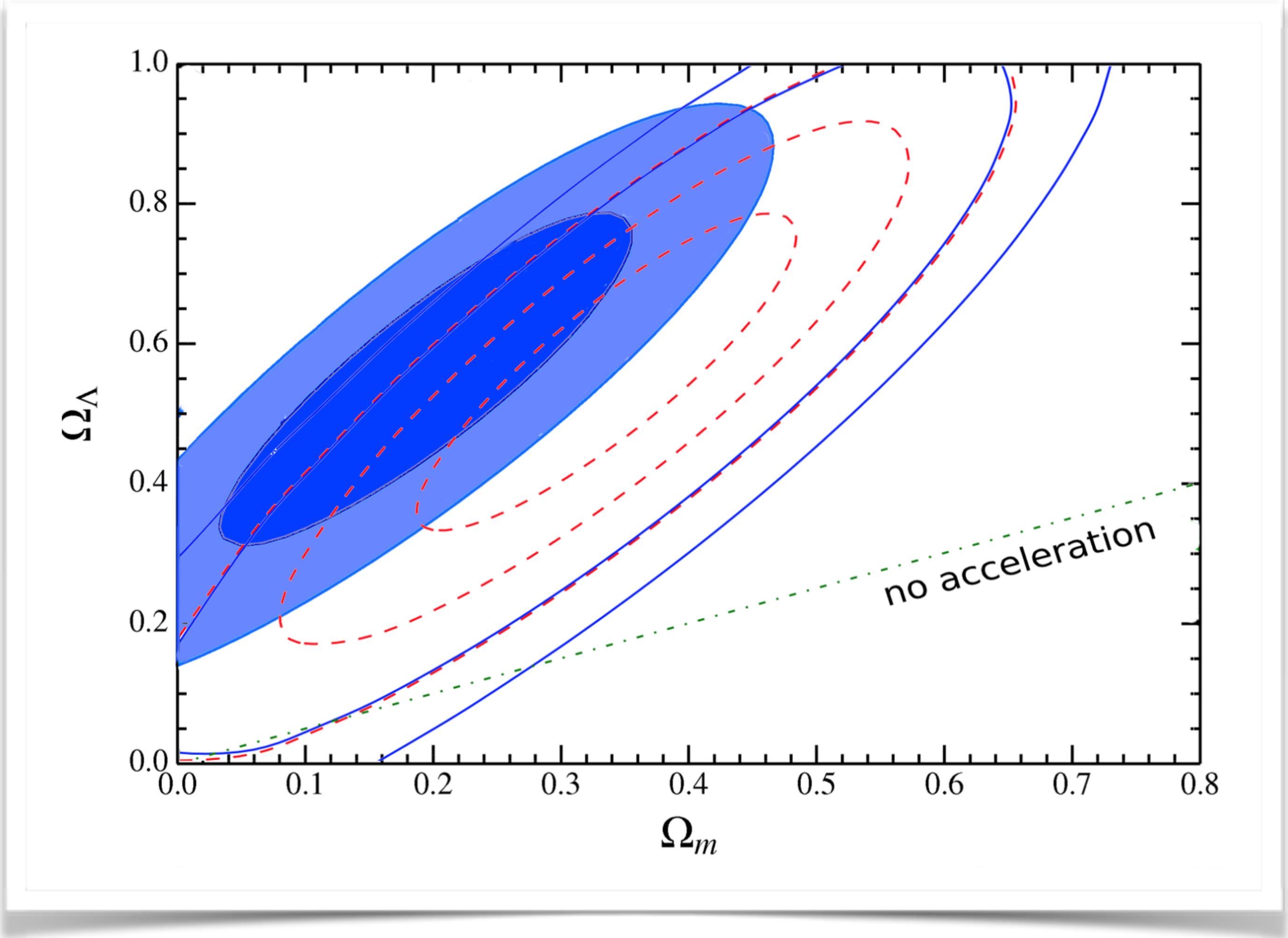
$$(\Sigma_d + A^{\mathrm{T}}\Sigma_l A)^{-1/2}(\hat{Z} - Y_0 A)$$





Do the best fit and its uncertainties satisfy Wilks' theorem?

Comparison to JLA results



Size of "ellipses" comparable to JLA
Systematic shift towards larger values of Ω_m

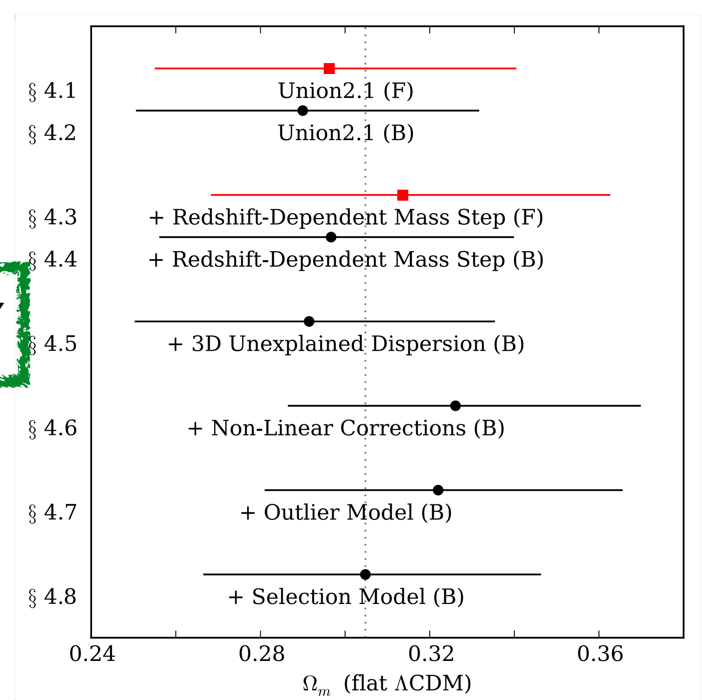
Recent developments - Unity & BAHAMAS

- More recently two extensive analyses based on Bayesian frameworks have appeared.

[UNITY, Rubin et al., arXiv:1507.01602]
[BAHAMAS, H. Shariff *et al.*, arXiv:1510.05954]

- The frameworks they develop allow for the systematic study of effects that have been, up to now, disregarded (or poorly treated) in SN cosmology studies, *i.e.* a modelling of the redshift dependence of the color and stretch corrections

UNITY



BAHAMAS

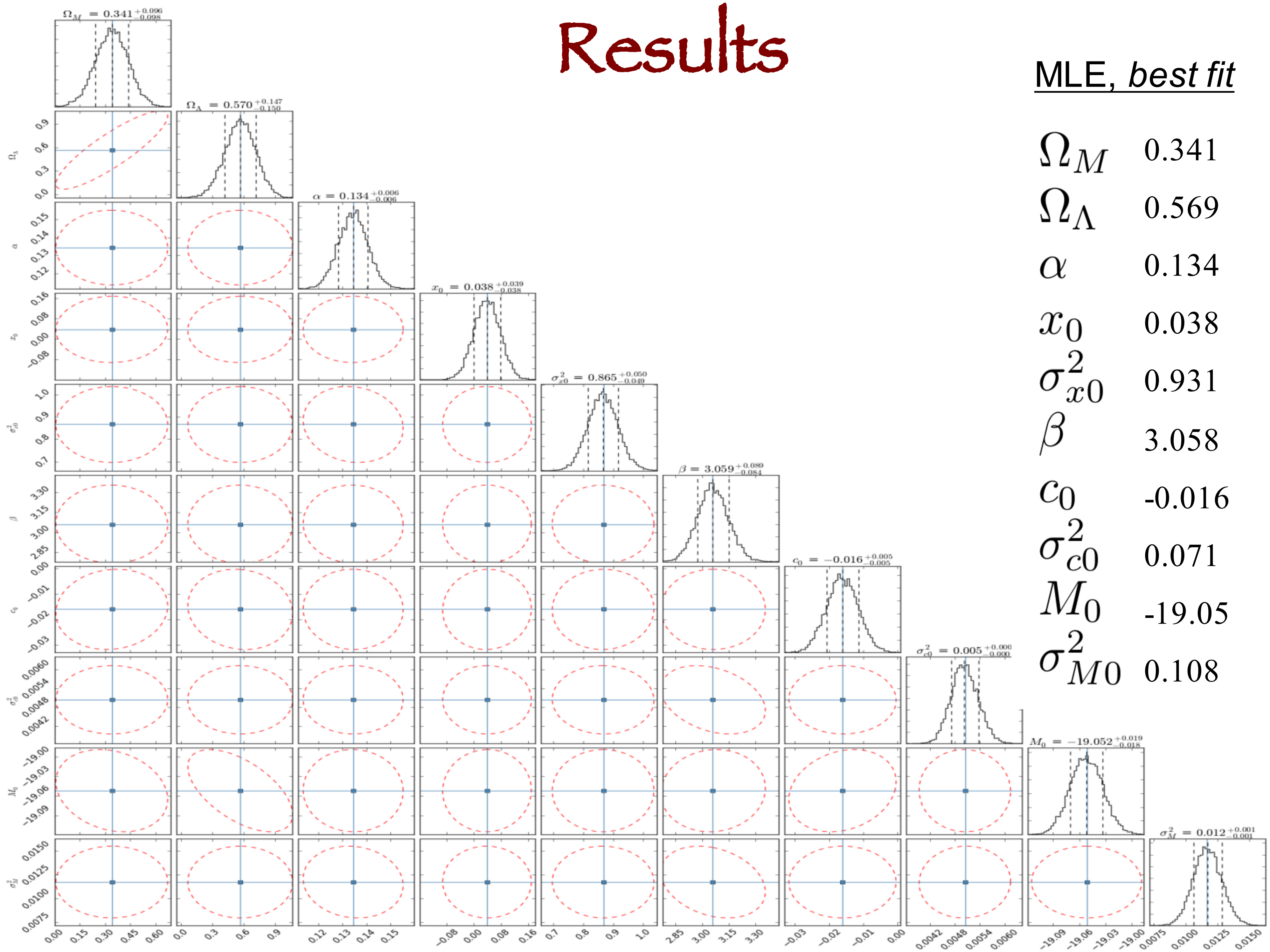
	JLA SNIa only			JLA SNIa + Planck 2015		
	Baseline	z-Linear color Corr	z-Jump color Corr	Baseline	z-Linear color Corr	z-Jump color Corr
			Baseline Model parameters			
$\Omega_{\rm m}$	0.340 ± 0.101	0.362 ± 0.094	0.429 ± 0.097	0.399 ± 0.027	0.420 ± 0.031	0.425 ± 0.025
Ω_Λ	0.542 ± 0.157	0.557 ± 0.145	0.632 ± 0.155	0.625 ± 0.020	0.609 ± 0.025	0.604 ± 0.019
Ω_κ	0.119 ± 0.249	0.081 ± 0.230	-0.061 ± 0.244	-0.024 ± 0.008	-0.028 ± 0.008	-0.029 ± 0.007
α	0.137 ± 0.006	0.136 ± 0.006	0.136 ± 0.006	0.137 ± 0.006	0.135 ± 0.007	0.136 ± 0.006
β	3.058 ± 0.085	n/a	n/a	3.068 ± 0.097	n/a	n/a
			Redshift evolution of color correction parameters			
β_0	n/a	3.211 ± 0.120	3.137 ± 0.092	n/a	3.219 ± 0.119	3.136 ± 0.096
β_1	n/a	-0.622 ± 0.342	n/a	n/a	-0.732 ± 0.360	n/a
$\Delta\beta$	n/a	n/a	-1.120 ± 0.240	n/a	n/a	-1.145 ± 0.243
z_t	n/a	n/a	0.662 ± 0.055	n/a	n/a	0.670 ± 0.056
			Intrinsic magnitude and residual dispersion parameters			
M_0	-19.140 ± 0.022	-19.140 ± 0.020	-19.144 ± 0.021	-19.140 ± 0.018	-19.138 ± 0.018	-19.140 ± 0.016
$\sigma_{\rm res}$	0.104 ± 0.005	0.104 ± 0.005	0.103 ± 0.005	0.105 ± 0.005	0.105 ± 0.004	0.103 ± 0.005

Conclusions & Outlook

- Type Ia Supernovae are a crucial probe of cosmic acceleration
- The precision of present and upcoming (*i.e.* DES …) data needs to be matched by a comparably sound and accurate statistical analysis
- Maximum Likelihood Estimators methods provide such a framework, forcing one to make explicit the assumption made in the modelling of the data
- When MLE techniques are applied to the data from the data in the JLA catalogue marginal evidence (~3 sigma) for cosmic acceleration is supported by SN data alone
- A better modelling of SN standardisation corrections is crucial to improve the quality of the analysis

Backup Slides

Best fit correlations



Results

<u>MLE, *best fit*</u>

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