3 hours with Axions

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1 Quick Intro

The QCD axion, or simply the axion, is a hypothetical 0^- particle arising in the Peccei-Quinn generic mechanism to ease the strong CP problem. Furthermore, axions are candidates for the cold dark matter of the Universe. Axions have been searched in a small number of experiments and constrained with astrophysical and cosmological arguments, but not yet found. Many different theoretical realisations have been proposed, which relate axion physics with other theories beyond the standard model. After some years of abandon and despair, the interest in axions is growing strong again, and many new experiments have been proposed and will be built in the next few years. In this lectures, we will introduce the axion in the context of the strong CP problem and study its phenomenological consequences in astrophysics and dark matter, as well as its experimental signatures.

These notes are quickly scribbled to provide a closer recollection of the first lecture I gave in the 2016 Invisibles school (SISSA Trieste, 5-9 July 2016). The 3 hours I had assigned were unpurposely stretched to the limit, and yet my talents did reach to cover all aspects of this exciting field. For that reason, and it is always advisable to complement any lecture with different approaches to the problem, I list here other pedagogical readings that I encourage to get acquainted with. I was very lucky to enjoy the lectures of the 1st Joint ILIAS-CAST-CERN Training back in 2005 [1], which produced excellent lecture notes. The very same Roberto Peccei taught on Axions and the strong CP problem [2], Pierre Sikivie on Axion Cosmology [3] and Georg Raffelt on Astrophysical Axion bounds [4]. As an easy and enjoyable read for the moments where everything seems uphill, I also recommend [5]. For a thorough review on Axions with a well fed collection of references, see the review of J. E. Kim and G. P. Carosi [6] (take a deep breath before). I am always available for requests on deeper readings on specific aspects and updated experimental proposals. Use me.

2 Strong CP ~~problem~~ hint

The strong CP problem is a conceptual issue with QCD being the theory of strong interactions in the standard model of particle physics. From a theoretical point of view we expect that such an SU(3) gauge theory coupled to massive quarks is "generically" CP violating, and yet there is no sign of CP violation in the strong interactions. Sure, the heart of the matter is on what we mean by generically.

When $SU(3)_c$ was proposed as a theory of the strong interactions, one of the designer's choice was CP conservation, which was already a clear constraint from the experimental point of view. The low energy theory of $SU(3)_c$ had, however, a mysterious problem: Weinberg's $U(1)_A$ "missing meson" problem. Its resolution by 't Hooft triggered the recongnition of the strong CP problem and thus is our starting point for these lectures.

2.1 $U(1)_A$ missing meson problem

Consider QCD with 2 quark flavours, u, d in a vector notation $q = (u, d)$

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + i\bar{q}\not{D} q - (\bar{q}_L m_q e^{i\theta_\lambda} q_R + \text{h.c.}) \tag{1}$$

where m_q is a diagonal mass matrix with the m_u, m_d masses and θ_λ a common phase. Note that in the SM, quark masses come from Yukawa couplings of the Higgs and Yukawa matrices are allowed to be completely general, so we expect θ_λ to be there in a general case.

In the $m_q \to 0$ limit, the quark phase transformations

$$q_R \to e^{i(\theta_0 + \theta.\sigma)} q_R \quad ; \quad q_L \to e^{-i(\theta_0 + \theta.\sigma)} q_L, \quad \text{or} \quad q \to e^{i\gamma_5(\theta_0 + \vec{\theta}_\pi \cdot \vec{\sigma})} q \tag{2}$$

are a four-parameter $(\theta_0, \vec{\theta}_\pi)$ symmetry, $U(2)_A = U(1)_A \otimes SU(2)_A$. The U(1) part shifts a common phase of u_R and d_R quarks at the same time and opposite for LH, while the SU(2) part shifts phases differently for u and d flavours. The symmetry is explicitly violated by the quark masses, but since they are much smaller than QCD energy scales we can think about them as a perturbation. Note that a $U(1)_A$ transformation can be used to reabsorb θ_λ in the quark fields, and thus it should have unobservable effects.

When QCD grows strong at low energies, this symmetry becomes spontaneously broken by the quark condensate $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -v^3$. According to the Goldstone theorem, a global symmetry spontaneously broken implies the existence of Nambu-Golstone bosons (NGB), massless particles that appear in the low energy effective theory. The NGB's have quantum numbers of the symmetry generators which are spontaneously broken. A 4-parameter symmetry implies 4 Goldstone bosons, which are associated with the η^0 and the 3 pions π^0, π^+, π^-. Since the symmetry is not perfect, i.e. it is violated by the small mass terms, the NGBs become massive, and are usually called pseudo-NGB's. Let us compute the spectrum of mesons.

Meson masses can be computed by promoting our parameters θ_0, θ_π to NGB fields

$$\theta_0 = \frac{\eta^0(x^\mu)}{f_0} \quad \vec{\theta}_\pi = \frac{\vec{\pi}(x^\mu)}{f_\pi} \tag{3}$$

where f_0, f_π are energy scales related to $\Lambda_{\rm QCD}$. For simplicity in the exposition I take $f_0 = f_\pi = f$, which does not compromise the main points under discussion. We define Goldstone-less quarks $\tilde{q}$

$$q_R = e^{+i(\theta_0+\vec{\theta}_\pi\cdot\vec{\sigma})/2}\tilde{q}_R \quad ; \quad q_L = e^{-i(\theta_0+\vec{\theta}_\pi\cdot\vec{\sigma})/2}\tilde{q}_L \quad \text{or} \quad q = e^{i\gamma_5(\theta_0+\vec{\theta}_\pi\cdot\vec{\sigma})/2}\tilde{q} \tag{4}$$

Note that after this redefinition, $\mathrm{U}(2)_A$ transformations will appear as shifts of the $\eta^0, \vec{\pi}$ fields.

Under this redefinition, the quark mass term in the Lagrangian leads to a potential for the NGB's when subject to the quark condensate, In the charged sector, we get

$$\bar{q}_R m_q q_L + \text{h.c.} \to -(m_u+m_d)v^3\cos(\sqrt{\theta_-\theta_+}) = (m_u+m_d)v^3 + \frac{(m_u+m_d)v^3}{2f_\pi^2}\pi_-\pi_+, \tag{5}$$

which sets the charged pion mass $m_\pi^2 = (m_u+m_d)v^3/f^2$.

In the neutral sector η^0, π_3 we have ($\theta_3 = \pi_3/f$ because it appears with σ_3)

$$\bar{q}_L m_q q_R + \text{h.c.} \to -m_u v^3\cos(\theta_0+\theta_3) - m_d v^3\cos(\theta_0-\theta_3), \tag{6}$$

which gives mass to two linear combinations of η^0, π_3, one of which has to be the neutral pion π^0 and the other something related with η' (because of quantum numbers). However, the ratio of the neutral pion and eta masses

$$\frac{m_\eta}{m_\pi} \sim \frac{m_u}{m_d} \sim 0.5 \tag{7}$$

is very far away from the experimental values. The theory as it is predicts a pNGB with similar mass to the neutral pion, which was not observed in nature. The puzzle stays when including the strange quark, which forces us to consider $\mathrm{U}(3)_A$ and has three neutral mesons that have to be assotiated with π^0, η, η'.

2.2 $G\widetilde{G}$ and QCD instantons solve the issue, but ...

The divergence of the $\mathrm{U}(1)_A$ current $j_A^\mu = \bar{u}\gamma^\mu\gamma_5 u + \bar{d}\gamma^\mu\gamma_5 d$ gets a contribution from the triangle loop diagram

$$\partial_\mu j_A^\mu = -2m_u\bar{u}i\gamma_5 u - 2m_d\bar{d}i\gamma_5 d + 4\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}_a^{\mu\nu} \tag{8}$$

where $\widetilde{G}_a^{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}G_a^{\alpha\beta}/2$ is the gluon field strength. The current is not conserved even for non-zero masses $\partial_\mu j_A^\mu \neq 0$. In other words, $\mathrm{U}(1)_A$ is not a symmetry even when m_u, m_d

are zero. The factor of 4 comes from the 2 flavours and two chiralities that run in the loop. The first two terms, proportional to quark masses, simply state that quark masses violate the symmetry too, but this we already knew.

> Generically, an axial phase transformation of one quark (SU(3) fermionic triplet)
>
> $$u \to e^{i\alpha\gamma_5} u \tag{9}$$
>
> implies that the current associated has a triangle anomaly
>
> $$\partial_\mu(\bar{u}\gamma^\mu\gamma_5 u) = ... + 2\frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a. \tag{10}$$
>
> Transformations along the π_3 direction, i.e. σ_3 =diag{1,-1} are not colour anomalous because the u and d parts cancel out. But the $\mathrm{U}(1)_A$ part is proportional to the identity in flavour space and all the quarks contribute the same.

Physical effects of the $G\widetilde{G}$ term were neglected in early times because it turns out to be a total derivative,

$$G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a = \partial_\mu K^\mu \tag{11}$$

with

$$K^\mu = \epsilon^{\mu\nu\alpha\beta} A_{a\nu}\left(F_{a\alpha\beta} - \frac{g_s}{3} f_{abc} A_{b\alpha} A_{c\beta}\right) \tag{12}$$

(note that it is not gauge-invariant).

By partial integration all its effects are defined by field configurations at infinity, which shall not contribute to local processes. But Gerard 't Hooft realised that there are actually topologically non-trivial field configurations, called instantons, that contribute to this operator, and thus it cannot be neglected. For the remainder of these lectures we will not need the fine points of instantons so I will skip the discussion as much as I can. The important points I cannot avoid to list are the following:

- The term violates P and T, or equivalently, P and CP (see Lecture notes by Cohen)
- A $G\widetilde{G}$ term must be admitted in our Lagrangian (1), because it is compatible with all symmetries of the SM gauge group and instanton configurations contribute to it. Thus, we are led to consider

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + i\bar{q}\not{D}q - (\bar{q}_L m_q e^{i\theta_\lambda} q_R + \mathrm{h.c.}) - \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a \theta_{\mathrm{QCD}}. \tag{13}$$

where θ_{QCD} is a (coupling) constant to be determined[1].

[1] It is actually related to the QCD vacuum structure, which turned out to be non-trivial, but we do not need to enter into this now.

- $G\widetilde{G}$ violates the $\mathrm{U}(1)_A$ symmetry explicitly (even if only at the quantum level, i.e. after the triangle radiative correction is included) so it will generate a mass term for the η_0 field, just like m_u generated mass for the $\theta_0 + \theta_3$ combination.

When an infinitesimal $\mathrm{U}(1)_A$ transformation(redefinition) like (2) with parameter α is performed on quark terms, the Lagrangian gets a new piece

$$\delta\mathcal{L} = \alpha\partial_\mu j^\mu_A \tag{14}$$

that, of course, vanishes if $\mathrm{U}(1)_A$ would be a symmetry with its current conserved. This leads to

$$\delta\mathcal{L} = \alpha\partial_\mu j^\mu_A = -2m_u\bar{u}i\alpha\gamma_5 u - 2m_d\bar{d}\alpha\gamma_5 d + \frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a \times 4\alpha. \tag{15}$$

The first two terms are the phase shifts of quark masses, expected because $\mathrm{U}(1)_A$ transformations do not leave invariant mass terms, and the last redefines(shifts) θ_{QCD}. Therefore, note that when performing these transformations, we are effective shifting a phase from θ_λ to θ_{QCD}. For instance, we can rotate θ_λ away from the quark mass term. Then, the combination

$$\theta_{\mathrm{SM}} = \theta_{\mathrm{QCD}} - \theta_Y/4 \tag{16}$$

appears multiplying the $G\widetilde{G}$ term in the Lagrangian. Only this combination is thus physical and all the CP violation observables are going to depend on it. The $G\widetilde{G}$ term will solve the missing meson problem, but it brings with it CP violation that we thought was absent in QCD.

Note that $\theta_{\mathrm{SM}} = \theta_{\mathrm{QCD}} - 2\theta_Y$ it is a sum of two phases which in principle have a different origin: θ_Y is a common phase of the Yukawa couplings and θ_{QCD} comes from the strong interaction sector. Therefore, in principle we shall not expect any cancellation. Moreover, the only CP violating phase observed so far, which appears in the CKM has a similar origin than θ_Y and it is O(1) ($\gamma \sim 60$ degrees).

Let us now discuss some important details of how the eta' mass has to be generated. We aim at guessing the contribution to the meson potential due to the new $G\widetilde{G}$ term. There are three points to consider:

- The spacetime integral of $G\widetilde{G}$ is a very special object. It only cares about special field configurations, instantons, and only about their behaviour at infinity. Indeed, it turns out to be an integer, the Pontryagin index,

$$\int d^4x \frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a = n, \tag{17}$$

  related to the change of winding number of the gluon field configuration integrated. For more information about the topological properties of instantons and the QCD vacuum see [2] and references therein. Here we only want to highlight that n is an integer and thus any quantity that depends on $\theta_{\rm SM}$ must be 2π-periodic

$$\theta_{\rm SM} \equiv \theta_{\rm SM} + 2\pi. \tag{18}$$

In particular, the energy density (or effective potential) dependence on for $\theta_{\rm SM}$ (Euclidean path integral)

$$e^{-\int d^4x V[\theta]} = \int \mathcal{D}A_{a\mu} e^{-S_E[A_{a\mu}] - i\theta \int d^4x_E \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a} \tag{19}$$

will satisfy $V(\theta) = V(\theta + 2\pi)$.

- The effective potential has its absolute minimum[2] at $\theta_{\rm QCD} = 0$

$$\begin{aligned} e^{-\int d^4x V[\theta]} &= \left| \int \mathcal{D}A_{a\mu} e^{-S_E[A_{a\mu}] - i\theta \int d^4x_E \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a} \right| & (20) \\ &\leq \int \mathcal{D}A_{a\mu} e^{-S_E[A_{a\mu}]} \left| e^{-i\theta \int d^4x_E \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a} \right| & (21) \\ &\leq \int \mathcal{D}A_{a\mu} e^{-S_E[A_{a\mu}]} = e^{-\int d^4x V[0]} & (22) \end{aligned}$$

  so $V[0] \leq V[\theta]$.

- The VEV of the η^0 meson also contributes to CP violation. When we define Goldstone-less quarks in (4) we are effectively doing a position dependent $\mathrm{U}(1)_A$ transformation with parameter $\theta_0(x)/2$. This produces the term in the Lagrangian

$$\mathcal{L} = \frac{\theta_0}{2}\partial_\mu j^\mu_A \ni \frac{\alpha_s}{8\pi} G\widetilde{G} \times 2\theta_0. \tag{23}$$

  which adds the dynamical field $2\theta_0(x)$ to the theta-angle.

With these considerations in mind, we can write a new contribution to the meson potential, which reads now,

$$V \sim -m_u v^3 \cos(\theta_0 + \theta_3) - m_d v^3 \cos(\theta_0 - \theta_3) - \Lambda^4 \cos(2\theta_0 - \theta_{\rm SM}). \tag{24}$$

We have modelled the effect from QCD instantons as $-\Lambda^4 \cos(2\theta_0 - \theta_{\rm SM})$ with Λ an energy scale related with non-perturbative QCD to be determined from the eta' mass. The reasons are: 1) the induced term has to have its minimum at whatever value multiplies $\frac{\alpha_s}{8\pi} G\widetilde{G}$ in

[2] In this proof we have assumed that the Euclidean action at $\theta = 0$, $S_E[A^a_\mu]$ is real, which is the case when the $G\widetilde{G}$ term is the only source of CP violation. Since the EW sector of the SM has also a phase in the CKM matrix, this is not exactly true, but quantitatively irrelevant for these lectures. However, the CP-interested student will shall remember this.

the Lagrangian, i.e. $2\theta_0 - \theta_{\rm SM}$, 2) it has to give a large mass ∼GeV to η' so it must have a non-zero second derivative at the minimum and 3) it must be periodic in $2\theta_0 - \theta_{\rm SM}$. The cosine form is a simple choice which at this point I decided[3]. The potential can be computed analytically in the so-called dilute-instanton-gas-approximation (DIGA) to give precisely this form, but this calculation is only physically justified at high temperatures (when multi-instanton configurations do not interact). Introducing the cosine gave me the excuse to tell you about all this, but I will actually only use the position of the minimum and the fact that its second derivative is large (because eta' is much more massive than π^0) so forgive me for the liberty.

The meson mass matrix is now,

$$= \frac{v^3}{f^2}\begin{pmatrix} m_u + m_d & m_u - m_d \\ m_u - m_d & m_u + m_d \end{pmatrix} + 4\Lambda^4 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{25}$$

which allows to fit the large hierarchy between η' and π^0 masses (and have $m_{\pi^0} = m_{\pi^\pm}$ up to corrections). Essentially, eta' takes its mass from the new term and pions from chiral symmetry breaking by quark masses and the quark condensate,

$$m_\pi^2 = \frac{(m_u + m_d)v^3}{f^2} \quad ; \quad m_{\eta'}^2 \simeq 4\Lambda^4 + \mathcal{O}(m_q v^3). \tag{26}$$

2.3 ... create the strong CP problem

Let us turn into CP violation. The meson potential allows also to identify formally the VEVs of the η^0 and π^0, which behave as contributions to CP violating phases.

Note also that the potential also reflects the fact that under $\mathrm{U}(1)_A$ transformations, which are now shifts of the η^0 field ($\eta^0 \to \eta_0 + \alpha f$) allow us to move the $\theta_{\rm SM}$ phase from the QCD instanton term to quark mass terms, but they do not allow to redefine it away. Also, we learn that, if any of the quark masses is zero, one could reabsorb $\theta_{\rm SM}$ inside θ_0 and θ_3 and thus CP violation will be absent.

Before minimising V, note that if $\theta_{\rm SM} = 0$, every of the three terms of the potential can be made minimum by $\theta_0 = \theta_3 = 0$. Since all the phases are zero, the theory is of course CP-conserving. Considering now a small value of $\theta_{\rm SM}$, and a perturbation the CP conserving solution (expand the cosines at second order). Minimisation of V leads to a linear system for θ_0, θ_3 which solves to

$$2\theta_0 - \theta_{\rm SM} \sim -\frac{m_u m_d v^3}{M^2 f^2 (m_u + m_d)}\theta_{\rm SM}, \tag{27}$$

$$\theta_0 + \theta_3 \sim \frac{m_d}{(m_u + m_d)}\theta_{\rm SM}, \tag{28}$$

$$\theta_0 - \theta_3 \sim \frac{m_u}{(m_u + m_d)}\theta_{\rm SM}. \tag{29}$$

[3] In principle, this term can be computed in lattice QCD but we still do not have the adequate algorithms to sample non positive definite path integrals. People is working on it, though.

Two things are again obvious here. First $\theta_{\rm SM} = 0$ implies CP conservation. Second, the effects of $\theta_{\rm SM}$ on CP violation also disappear in the case that any of the masses is zero, for instance $m_u \to 0$. CP violation appears multiplied by quark masses, if $m_u = 0$ it has to be proportional to m_d but the phase $\theta_d = \theta_0 - \theta_3 \to 0$ and $\theta_0 \to 0$ too.

We could have advanced this by noting that a phase redefinition of the u quark alone can shift $\theta_{\rm SM}$ to zero in the theta-term of the Lagrangian. If $m_u = 0$ this redefinition only shifts the theta-term, so the theta-term must have no physical consequences. In its absence, the η^0, π_3 VEVs can not either violate CP. For all we know, there is no massless quark in SM, but the fact that u and d have small masses, suppresses a bit CP violation observables.

A most discused CP violating observable arising from $\theta_{\rm SM}$ is the neutron electric dipole moment (NEDM). Its calculation is a bit cumbersome so instead I will quote the results

$$d_n = \frac{g_{\pi NN}\bar{g}_{\pi NN}}{4\pi^2 m_N} \log\left(\frac{m_N}{m_\pi}\right) \sim 4.5 \times 10^{-15} \text{ecm} \tag{30}$$

where CP violation enters in the CP violating pion-nucleon coupling $\bar{g}_{\pi NN} \sim -\theta_{\rm SM} m_u m_d/(m_u + m_d)$, as it comes from $2\theta_0 - \theta_{\rm SM}$.

The last attempt to measure the NEDM reported an upper limit

$$d_n < 3 \times 10^{-26} \text{ecm} \tag{31}$$

which implies the amazingly stringent constraint

$$\theta_{\rm SM} < 0.7 \times 10^{-11}. \tag{32}$$

The fact that $\theta_{\rm SM}$ is that small while on general grounds we could expect it to be O(loop correction times m_u suppression) is dubbed the strong CP problem. It is actually not a technical problem, because $\theta_{\rm SM}$ does not receive large radiative corrections in the SM. It could just be that nature chose small $\theta_Y, \theta_{\rm QCD}$ or a fine tuning among them.

However, small numbers like this could very well have a dynamical origin hinting at new dynamics and new physics. In this lectures we will discuss a very elegant mechanism to cancel (almost) completely the effect of $\theta_{\rm SM}$ the Peccei-Quinn mechanism based on the axion. The most appealing aspect in my opinion is that the dynamics required is already build in the SM, concretely in the strong interactions. As a timely side-effect, it turns out that the mechanism provides a very intriguing cold dark matter candidate, and a hint for a new (high) energy scale in nature.

2.4 A new degree of freedom: Axion solution

Each of the terms of the meson potential

$$V \sim -m_u v^3 \cos(\theta_0 + \theta_3) - m_d v^3 \cos(\theta_0 - \theta_3) - \Lambda^4 \cos(2\theta_0 - \theta_{\rm SM}) \quad (33)$$

has the tendency to minimise a given combination of CP violating VEVs and phases. Unfortunately, we have three terms in the potential and only two degrees of freedom so the minimisation of them all at a time has to find a compromise, which is generically CP violating. This suggests a possible explanation to the shocking absence of NEDM: could there be a new meson-like degree of freedom?

If we include a new meson-like *without introducing new terms in the potential* we will have three degrees of freedom to minimise three terms, each of which depends on three different linear combinations of our degrees of freedom. The system has now enough freedom to set all-three combinations to zero, i.e. to go to the CP conserving absolute minimum dynamically. Peccei and Quinn argued in a different way, but it all boils down to the above argument.

The simplest axion realisation involves a new meson-like field ϕ, that will be called axion. The important pieces of its Lagrangian are just 2: a kinetic term and an anomalous coupling to gluons, just like the theta-term

$$\mathcal{L}_\phi \in \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\widetilde{G}^{\mu\nu}_a \frac{\phi}{f_\phi}. \quad (34)$$

The energy scale f_ϕ is called axion decay constant and will play a very important role in phenomenology. We can define $\theta_\phi(x) = \phi(x)/f_\phi$. At low energies, below QCD confinement, the axion appears in the instanton contribution to the potential

$$V \sim -m_u v^3 \cos(\theta_0 + \theta_3) - m_d v^3 \cos(\theta_0 - \theta_3) - \Lambda^4 \cos(2\theta_0 + \theta_\phi - \theta_{\rm SM}) \quad (35)$$

The minimum of the potential is given by $\partial_{\theta_0} V = \partial_{\theta_3} V = \partial_{\theta_\phi} V = 0$, which is equivalent to

$$\begin{aligned} \theta_0 + \theta_3 &= 0 && (36)\\ \theta_0 - \theta_3 &= 0 && (37)\\ 2\theta_0 + \theta_\phi - \theta_{\rm SM} &= 0 && (38) \end{aligned}$$

i.e.

$$\theta_0 = \theta_3 = 0 \quad ; \quad \theta_\phi = \theta_{\rm SM} \quad (39)$$

the axion VEV is adjusted *by the QCD potential* to cancel any possible value of $\theta_{\rm SM}$, and thus any effect of CP violation.

Another way to look at the absence of CP violation in the presence of axions is that we can redefine $\theta_a \to \theta_\phi - \theta_{\rm SM}$ at the Lagrangian level. TYhis completely wipes out any dependence on $\theta_{\rm SM}$ and thus of CP violation. In other words, the presence of such axion field makes $\theta_{\rm SM}$ unphyiscal (this was closer to the thinking of Peccei and Quinn).

Yet another saying you will hear: the axion promotes $\theta_{\rm SM}$ to a dynamical variable, i.e. the role of $\theta_{\rm SM}$ in the SM is now played by the axion field $\theta_\phi(x)$. This dynamical variable can now respond to the QCD potential, adjusting its VEV to cancel $\theta_{\rm SM}$.

What Peccei and Quinn missed was to realise that θ_ϕ, as a dynamical field, has particle excitations: axions. This was realised very fast by Weinberg and Wilczek independently, which worked out their properties. The minimal version presented here is called the hadronic axion and turns out remarkably predictive.

The meson mass matrix is now, using $\beta = f/2f_a$ in the (π_3, η^0, a) basis

$$= \begin{pmatrix} m_u + m_d & m_u - m_d & 0 \\ m_u - m_d & m_u + m_d & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{v^3}{f^2} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \beta \\ 0 & \beta & \beta^2 \end{pmatrix} \frac{4\Lambda^4}{f^2}. \tag{40}$$

Integrating out $\eta'(x) = \eta^0(x) + \beta\phi(x) = 0$ at the tree-level the system becomes 2x2 and one easily finds the mass eigenstates,

$$\pi^0 = \pi_3 + \varphi_{a\pi}\phi \quad ; \quad m_\pi^2 = \frac{(m_u + m_d)v^3}{f^2}, \tag{41}$$

$$a = \phi - \varphi_{a\pi}\pi_3 \quad ; \quad m_a^2 = \frac{m_u m_d v^3}{(m_u + m_d) f_a^2} = \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f^2}{f_a^2}, \tag{42}$$

where we have redefined $f_a = f_\phi$ and the pion-axion mixing angle is

$$\varphi_{a\pi} = \frac{m_d - m_u}{2(m_u + m_d)} \frac{f}{f_a}. \tag{43}$$

People uses the word axion for both ϕ and a. Here we will reserve the symbol a for the physical mass eigenstate. Note that, in a sense, a takes some features of Weinberg's missing meson, like becoming massless in the m_u limit and its mixing to the π, proportional to $m_u - m_d$. This mixing angle allows to compute axion couplings to nucleons and self-couplings by simply taking standard expressions for axion-less theories and substituing

$$\pi_3(x) \to \pi^0(x) - \varphi_{a\pi}\phi(x) \sim \pi^0(x) - \varphi_{a\pi} a(x). \tag{44}$$

Note that the larger the axion decay constant f_a is, the smaller the axion mass and the weaker its interactions with photons, hadrons, etc. Soon we will be forced to consider $f_a > 10^9$ GeV, so that axions will be indeed low mass and weakly interacting.

Axion Gymnastics 1

2.4.1 Axion mass and mixing

At low energies, below QCD confinement and absorbing $\theta_{\rm SM}$ in the axion field,

$$V \sim -m_u v^3 \cos(\theta_0 + \theta_3) - m_d v^3 \cos(\theta_0 - \theta_3) - \Lambda^4 \cos(2\theta_0 + \theta_\phi) \tag{45}$$

Integrating out $\eta'(x) = \eta^0(x) + \beta\phi(x) = 0$ ($\beta = f/2f_\phi$) the system becomes 2x2. Find the linear combinations of mass eigenstates that diagonalise the mass matrix in the $\beta \to 0$ limit, and their masses.

Can you estimate the 1st correction due to the finite η' mass?

2.4.2 Compute the axion to photon coupling for hadronic axions

Even if axions would not couple to photons directly, they would inherit a coupling from their mixing with η^0 and π_3. These anomalous couplings follow from the divergence of the $\mathrm{U}(1)_A$ and third generator of $\mathrm{SU}(2)_A$

$$\begin{aligned}\mathcal{L} &\ni [6\left(\frac{2}{3}\right)^2 + 6\left(\frac{1}{3}\right)^2]\frac{\eta^0}{f}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} + [6\left(\frac{2}{3}\right)^2 - 6\left(\frac{1}{3}\right)^2]\frac{\pi_3}{f}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} && (46)\\ &= \frac{10}{3}\frac{\eta^0}{f}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} + 2\frac{\pi_3}{f}\frac{\alpha}{8\pi}F_{\mu\nu}\widetilde{F}^{\mu\nu} && (47)\end{aligned}$$

2.4.3 Compute the axion/nucleon couplings for hadronic axions

Take the dust out of your ChPT notes and compute the axion coupling to protons and neutrons. Part of the problem is to find the right references ;-).

2.4.4 Extend the results to SU(3)$_A$

Including the strange quark is relatively straightforward and not very illuminating but makes a good sport. The axial quasi-symmetry group is now $\mathrm{U}(3)_A$=$\mathrm{U}(1)_A \otimes \mathrm{SU}(3)_A$ acting on $q = (u, d, s)$ and the phase transformations for the relevant neutral bosons are three Gell-Mann matrices

$$q_R \to \exp\left(i\mathrm{diag}\left\{\bar\theta_0 + \bar\theta_3 + \bar\theta_8, \bar\theta_0 - \bar\theta_3 + \bar\theta_8, \bar\theta_0 - \bar\theta_3 - 2\bar\theta_8\right\}\right) q_R \tag{48}$$

(opposite sign for q_L) where barred θ's canonically normalise to unbarred as $\bar\theta_0 = \theta_0\sqrt{\frac{2}{3}}$, $\bar\theta_3 = \theta_3$ and $\bar\theta_8 = \theta_8\frac{1}{\sqrt{3}}$.

Neglecting $G\widetilde{G}$, build the meson potential and study the spectrum. Using values for m_u, m_d, m_c you shall find again the missing meson problem.

Include $G\widetilde{G}\theta_{\rm SM}$. What is the coefficient of the contribution of θ_0 to $\theta_{\rm SM}$?. Guess again the instanton contribution to the meson potential. How many degrees of freedom and how many terms do we have? Can you generalise the above to n-quarks?

Introduce the axion, integrate η' at tree level, find the mass eigenstates (at least approximately) and their mixing angles. Recompute the axion-photon coupling.

2.4.5 The KSVZ model

The simplest hadronic model was proposed soon after the original PQWW model was ruled out. It consists on adding to the SM a new quark singlet Q and a new scalar field Φ with a mexican-hat potential coupled with a Yukawa term,

$$\mathcal{L}_\Phi = i\bar{Q}\not{D}Q - (y\bar{Q}_L\Phi Q_R + \text{h.c.}) - \frac{\lambda}{4}\left(|\Phi|^2 - v_{\rm PQ}^2\right)^2. \tag{49}$$

This Lagrangian has a $\text{U}(1)_A$ symmetry, which becomes spontaneously broken by the Φ VEV. Identify the symmetry. As it turns out, the symmetry is anomalous, calculate the anomaly, i.e. $\partial_\mu j^\mu$ (by analogy with the η^0 case discussed). After spontaneous symmetry breaking, both the radial part of the Φ field and Q become massive and can be integrated out. There is of course a Goldstone boson, which appears as the radial degree-of-freedom of $\Phi = \rho e^{i\phi/v_{\rm PQ}}$. Show that at low energies, the effective Lagrangian is exactly what we need for a hadronic axion model, i.e. (34).

Consider an extension of the model where there are several Q's, which can have hypercharge (and thus electric charge). Compute the relation between f_a and $v_{\rm PQ}$ and the anomalous coupling to two photons.

2.4.6 Additional axion potential

Consider the KSVZ model and add a term of the short

$$\mathcal{L} \ni c\frac{\Phi^n}{M^{n+4}} + \text{h.c.} \tag{50}$$

with c a generally complex constant and M another energy scale $M > f_a$. After SSB compute the contribution to the axion potential.

Adding it to the meson-axion potential, find the minimum of the potential. Show that $2\theta_0 + \theta_\phi - \theta_{\rm SM}$ is in general non-zero.

Global symmetries are violated by gravity effects (black hole's no-hair theorem). Some authors thus suggest that the violation of the PQ symmetry (axion shift-symmetry) is violated by the above type of operators where M is the Planck scale $M_P = 1/\sqrt{G_N} = 1.22 \times 10^{19}$ GeV. Choose $c = |c|e^{i\delta}$ and discuss the NEDM constraint on $|c|$ as a function of δ and n (In this framework, you can use $\bar{g}_{\pi NN} \propto 2\theta_0 + \theta_a - \theta_{\rm SM}$).

3 Axion Dark Matter

The slides of this lecture can be found in the school's web page.

4 Axion Searches

The slides of this lecture can be found in the school's web page.

References

[1] `http://indico.cern.ch/event/426464/`

[2] R. D. Peccei, "The Strong CP problem and axions," Lect. Notes Phys. **741** (2008) 3 doi:10.1007/978-3-540-73518-2_1 [hep-ph/0607268].

[3] P. Sikivie, "Axion Cosmology," Lect. Notes Phys. **741** (2008) 19 doi:10.1007/978-3-540-73518-2_2 [astro-ph/0610440].

[4] G. G. Raffelt, "Astrophysical axion bounds," Lect. Notes Phys. **741** (2008) 51 doi:10.1007/978-3-540-73518-2_3 [hep-ph/0611350].

[5] P. Sikivie, "The Pool table analogy to axion physics," Phys. Today **49N12** (1996) 22 doi:10.1063/1.881573 [hep-ph/9506229].

[6] J. E. Kim and G. Carosi, "Axions and the Strong CP Problem," Rev. Mod. Phys. **82** (2010) 557 doi:10.1103/RevModPhys.82.557 [arXiv:0807.3125 [hep-ph]].