

Leptonic CP Violation

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Plan of the Lecture

1. Introduction.
2. Massive Neutrinos, Neutrino Mixing and Oscillations: Overview.
3. Three Neutrino Mixing. Massive Majorana versus Massive Dirac Neutrinos. Dirac and Majorana CP Violation.
4. The Neutrino Mass Spectrum and the Absolute Scale of Neutrino Masses.
5. Open Questions and Fundamental Problems in Physics of Massive Neutrinos.
6. Neutrino Oscillations in Vacuum: Theory and Experimental Evidences.
7. Matter Effects in Neutrino Oscillations.
8. Dirac CP Violation in the Lepton Sector: CP Violation in Neutrino Oscillations. Current and Future LBL Neutrino Oscillation Experiments on $\text{sgn}(\Delta m_{31}^2)$ and CP Violation.
9. Understanding the Pattern of Neutrino Mixing.
10. Determining the Nature of Massive Neutrinos.
11. Outlook.

3 Families of Fundamental Particles

$$\begin{pmatrix} \nu_e & u \\ e & d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu & c \\ \mu & s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau & t \\ \tau & b \end{pmatrix} \quad + \text{ their antiparticles}$$

- 3 types (flavours) of active ν 's and $\tilde{\nu}$'s
- The notion of “type” (“flavour”) - dynamical;
 ν_e : $\nu_e + n \rightarrow e^- + p$; ν_μ : $\pi^+ \rightarrow \mu^+ + \nu_\mu$; etc.
- The flavour of a given neutrino is Lorentz invariant.
- $\nu_l \neq \nu_{l'}$, $\tilde{\nu}_l \neq \tilde{\nu}_{l'}$, $l \neq l' = e, \mu, \tau$; $\nu_l \neq \tilde{\nu}_{l'}$, $l, l' = e, \mu, \tau$.

The states must be orthogonal (within the precision of the corresponding data): $\langle \nu'_l | \nu_l \rangle = \delta_{l'l}$, $\langle \tilde{\nu}'_l | \tilde{\nu}_l \rangle = \delta_{l'l}$, $\langle \tilde{\nu}'_l | \nu_l \rangle = 0$.

- Data (relativistic ν 's): ν_l ($\tilde{\nu}_l$) - predominantly LH (RH).
Standard Theory: ν_l , $\tilde{\nu}_l$ - $\nu_{lL}(x)$;
 $\nu_{lL}(x)$ form $SU(2)_L$ doublets with $l_L(x)$, $l = e\mu, \tau$:

$$\begin{pmatrix} \nu_{lL}(x) \\ l_L(x) \end{pmatrix}, \quad l = e, \mu, \tau.$$

- No (compelling) evidence for existence of (relativistic) ν 's ($\tilde{\nu}$'s) which are predominantly RH (LH): ν_R ($\tilde{\nu}_L$).
If ν_R , $\tilde{\nu}_L$ exist, must have much weaker interaction than ν_l , $\tilde{\nu}_l$: ν_R , $\tilde{\nu}_L$ - “sterile”, “inert”.

In the formalism of the ST, ν_R and $\tilde{\nu}_L$ - RH ν fields $\nu_R(x)$; can be introduced in the ST as $SU(2)_L$ singlets. B. Pontecorvo, 1967

No experimental indications exist at present whether the SM should be minimally extended to include $\nu_R(x)$, and if it should, how many $\nu_R(x)$ should be introduced.

$\nu_R(x)$ appear in many extensions of the ST, notably in $SO(10)$ GUT's.

The RH ν 's can play crucial role

- i) in the generation of $m(\nu) \neq 0$,
- ii) in understanding why $m(\nu) \ll m_l, m_q$,
- iii) in the generation of the observed matter-antimatter asymmetry of the Universe (via leptogenesis).

The simplest hypothesis is that to each $\nu_{lL}(x)$ there corresponds a $\nu_{lR}(x)$, $l = e, \mu, \tau$.

ST + $m(\nu) = 0$: $L_l = \text{const.}$, $l = e, \mu, \tau$;
 $L \equiv L_e + L_\mu + L_\tau = \text{const.}$

There have been remarkable discoveries in neutrino physics in the last ~ 18 years.

Compellings Evidence for ν -Oscillations

$-\bar{\nu}_{\text{atm}}$: SK UP-DOWN ASYMMETRY

θ_Z -, L/E - dependences of μ -like events

Dominant $\nu_{\mu} \rightarrow \nu_{\tau}$ K2K, MINOS, T2K; CNGS (OPERA)

$-\bar{\nu}_{\odot}$: Homestake, Kamiokande, SAGE, GALLEX/GNO

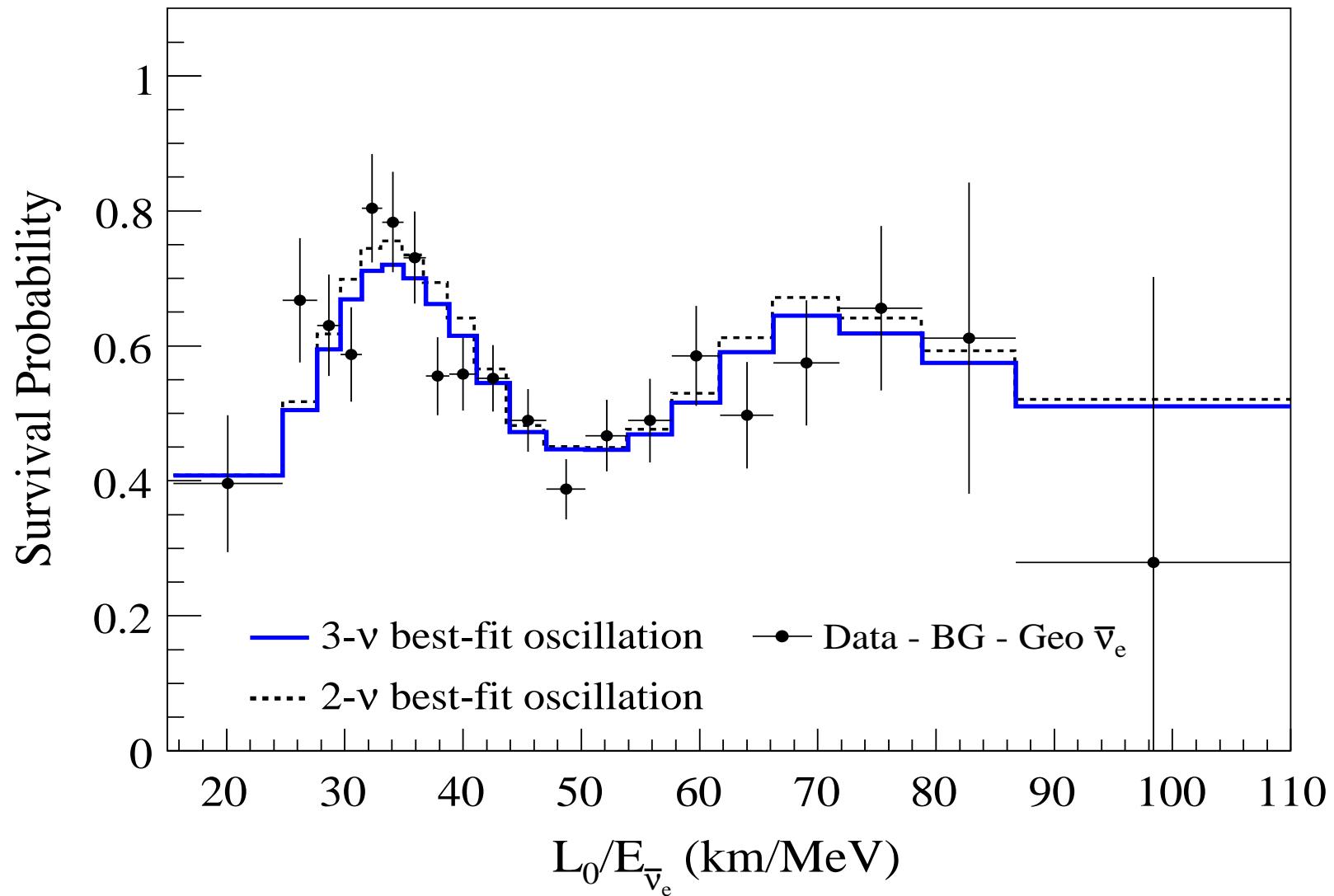
Super-Kamiokande, SNO, BOREXINO; KamLAND

Dominant $\nu_e \rightarrow \nu_{\mu,\tau}$ BOREXINO

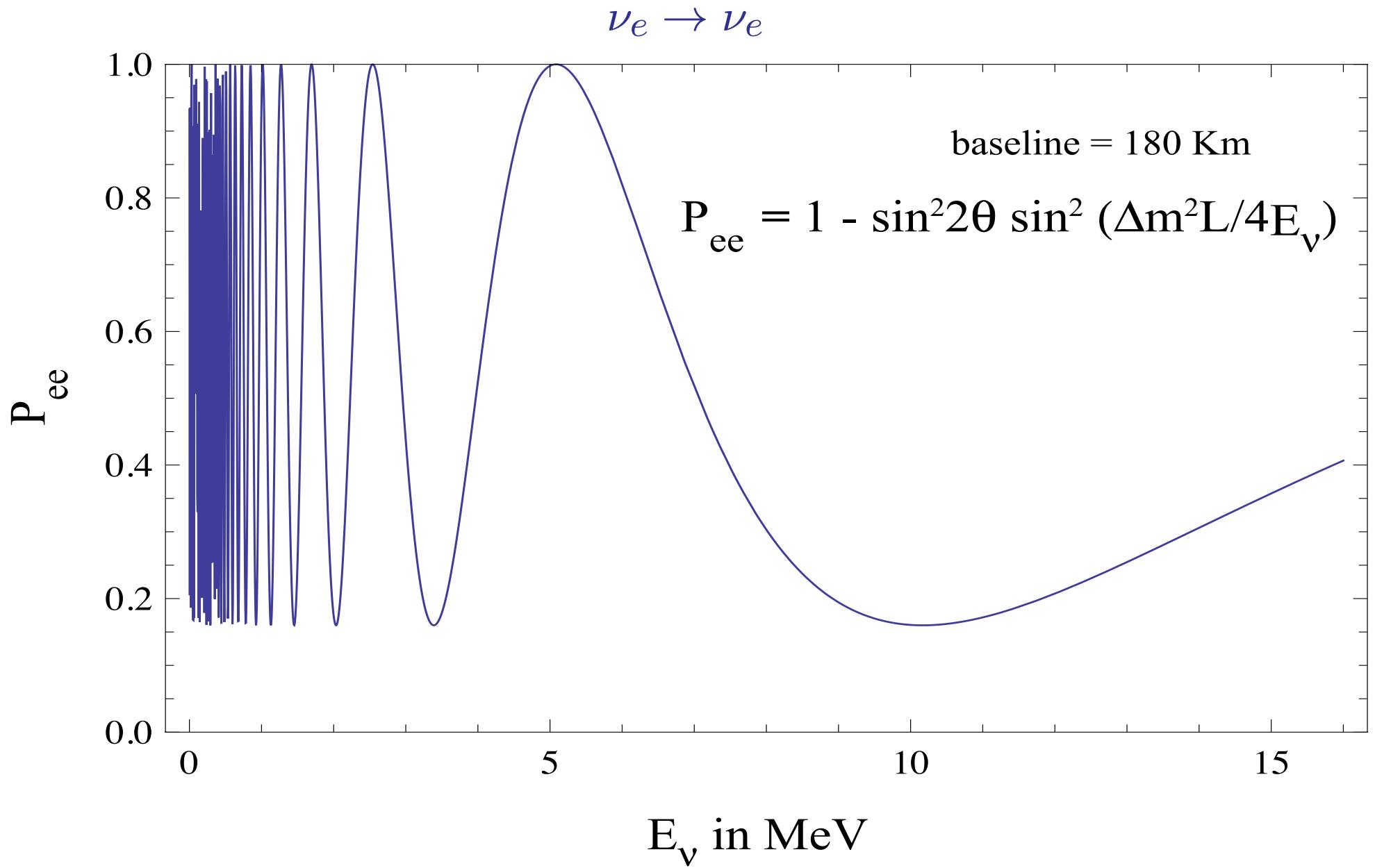
$-\bar{\nu}_e$ (from reactors): Daya Bay, RENO, Double Chooz

Dominant $\bar{\nu}_e \rightarrow \bar{\nu}_{\mu,\tau}$

T2K, MINOS (ν_{μ} from accelerators): $\nu_{\mu} \rightarrow \nu_e$



KamLAND: L/E -Dependence (reactor $\bar{\nu}_e$, $\bar{L} = 180$ km, $E = (1.8 - 10)$ MeV)



Compelling Evidences for ν -Oscillations: ν mixing

$$|\nu_l\rangle = \sum_{j=1}^n U_{lj}^* |\nu_j\rangle, \quad \nu_j : m_j \neq 0; \quad l = e, \mu, \tau; \quad n \geq 3;$$

$$\nu_{lL}(x) = \sum_{j=1}^n U_{lj} \nu_{jL}(x), \quad \nu_{jL}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$

B. Pontecorvo, 1957; 1958; 1967;
Z. Maki, M. Nakagawa, S. Sakata, 1962;

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix.

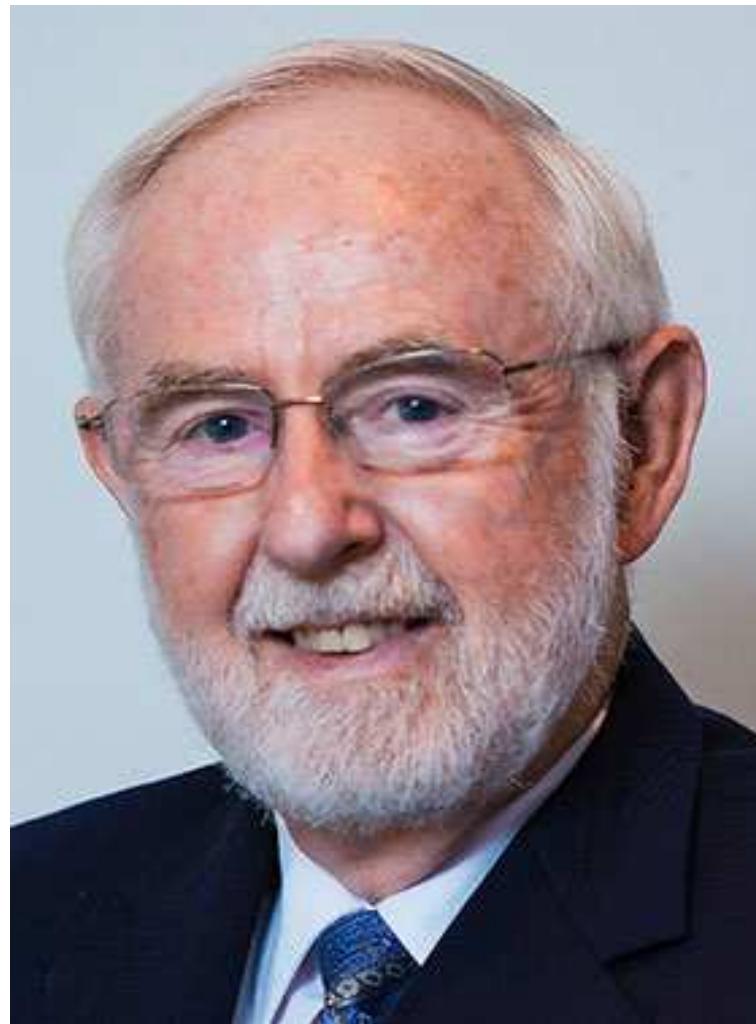
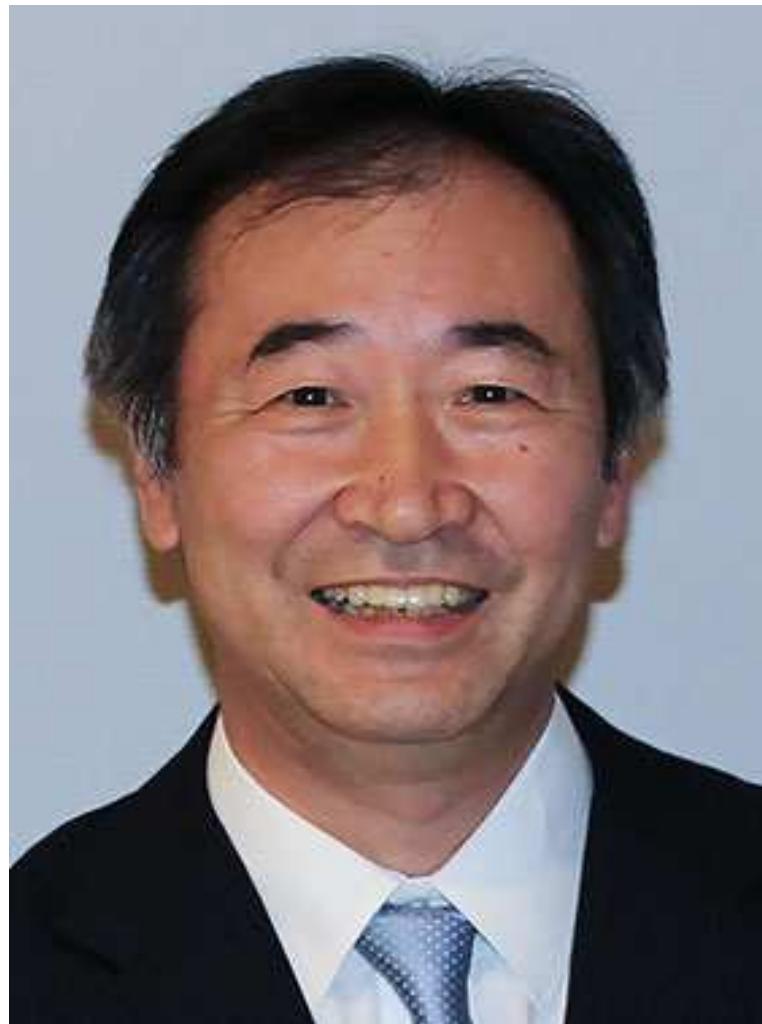
$\nu_j, m_j \neq 0$: Dirac or Majorana particles.

Data: at least 3 ν s are light: $\nu_{1,2,3}, m_{1,2,3} \lesssim 1$ eV.

The Charged Current Weak Interaction Lagrangian:

$$\mathcal{L}^{CC}(x) = -\frac{g}{2\sqrt{2}} \sum_{l=e,\mu,\tau} \bar{l}(x) \gamma_\alpha (1 - \gamma_5) \nu_{lL}(x) W^\alpha(x) + \text{h.c.},$$

$$\nu_{lL}(x) = \sum_{j=1}^n U_{lj} \nu_{jL}(x), \quad \nu_{jL}(x) : m_j \neq 0; \quad l = e, \mu, \tau.$$



Dr. T. Kajita, Prof. A. McDonald, Nobel Prize for Physics winners, 2015



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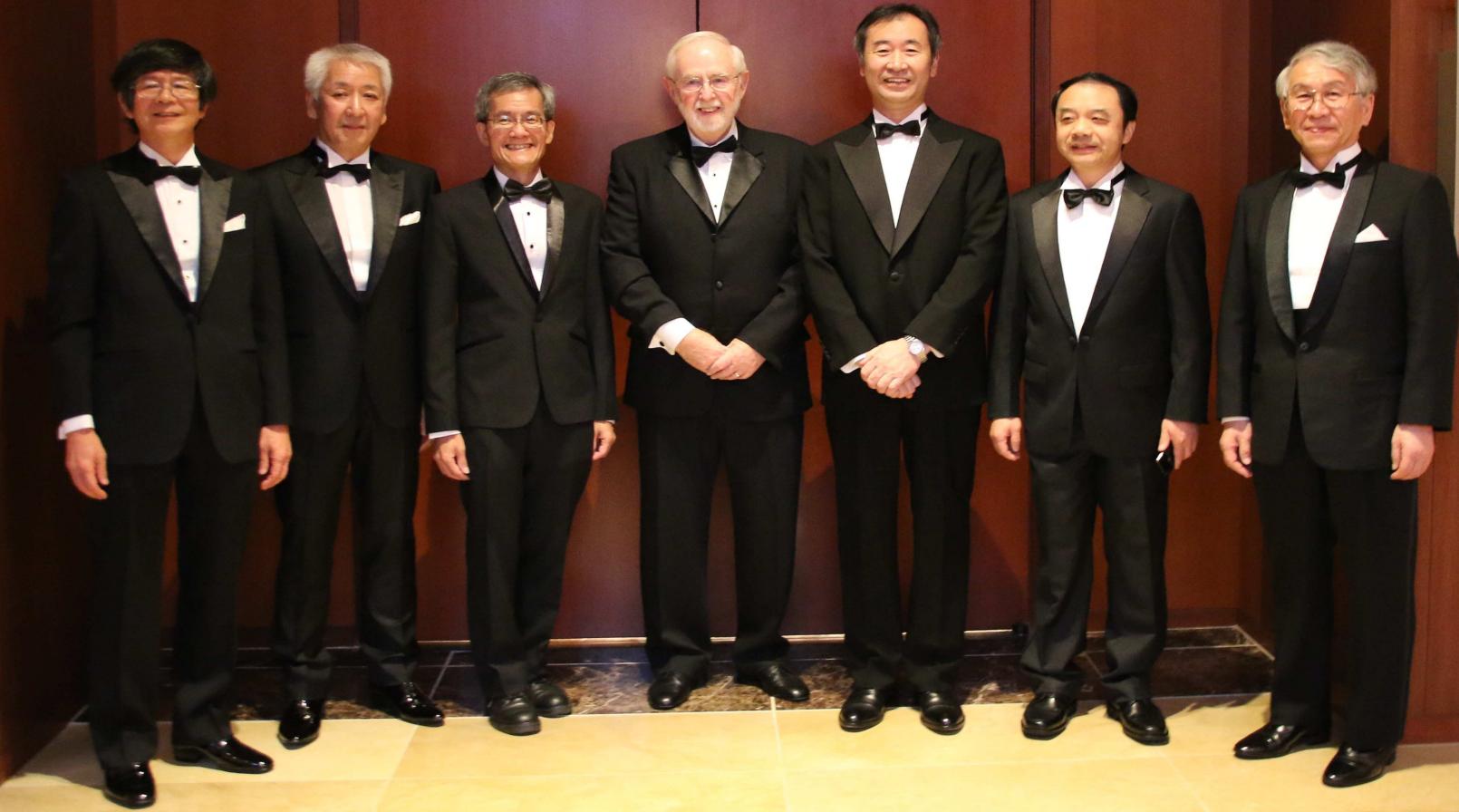
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FOUR SEASONS HOTEL
Mae Fah Luang



S.T. Petcov, INVISIBLES School, SISSA, 09/09/2016

These data imply that

$$m_{\nu_j} \ll m_{e,\mu,\tau}, m_q, \quad q = u, c, t, d, s, b$$

For $m_{\nu_j} \lesssim 1$ eV: $m_{\nu_j}/m_{l,q} \lesssim 10^{-6}$

For a given family: $10^{-2} \lesssim m_{l,q}/m_{q'} \lesssim 10^2$

**These discoveries suggest the existence of
New Physics beyond that of the ST.**

The New Physics can manifest itself (can have a variety of different “flavours”):

- In the existence of more than 3 massive neutrinos: $n > 3$ ($n = 4$, or $n = 5$, or $n = 6, \dots$).
- In the observed pattern of neutrino mixing and in the values of the CPV phases in the PMNS matrix.
- In the Majorana nature of massive neutrinos.
- In the existence of LFV processes: $\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e$, $\mu - e$ conversion, etc., which proceed with rates close to the existing upper limits.
- In the existence of new particles, e.g., at the TeV scale: heavy Majorana Neutrinos N_j , doubly charged scalars, ...
- In the existence of new (FChNC, FCFNSNC) neutrino interactions.
- In the existence of “unknown unknowns” ...

We can have $n > 3$ ($n = 4$, or $n = 5$, or $n = 6, \dots$) if, e.g., **sterile** ν_R , $\tilde{\nu}_L$ exist and they mix with the active flavour neutrinos ν_l ($\tilde{\nu}_l$), $l = e, \mu, \tau$.

Two (extreme) possibilities:

i) $m_{4,5,\dots} \sim 1$ eV;

in this case $\nu_{e(\mu)} \rightarrow \nu_S$ oscillations are possible (hints from LSND and MiniBooNE experiments, re-analyses of short baseline (SBL) reactor neutrino oscillation data (“reactor neutrino anomaly”), data of radioactive source calibration of the solar neutrino SAGE and GALLEX experiments (“Gallium anomaly”); tests (STEREO, SOX (BOREXINO), BEST (SAGE), DANS, RICOCHET, PROSPECT, Fermilab (3 LAr TPC detectors), CeLAND (KamLAND),...) under way).

ii) $M_{4,5,\dots} \sim (10^2 - 10^3)$ GeV, TeV scale seesaw models;
 $M_{4,5,\dots} \sim (10^9 - 10^{13})$ GeV, “classical” seesaw models.

All compelling data compatible with 3- ν mixing:

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} \quad l = e, \mu, \tau.$$

The PMNS matrix U - 3×3 unitary to a good approximation (at least: $|U_{l,n}| \lesssim (<<)0.1$, $l = e, \mu, n = 4, 5, \dots$).

$\nu_j, m_j \neq 0$: Dirac or Majorana particles.

3- ν mixing: 3-flavour neutrino oscillations possible.

ν_μ, E ; at distance L : $P(\nu_\mu \rightarrow \nu_{\tau(e)}) \neq 0$, $P(\nu_\mu \rightarrow \nu_\mu) < 1$

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_l \rightarrow \nu_{l'}; E, L; U; m_2^2 - m_1^2, m_3^2 - m_1^2)$$

Three Neutrino Mixing

$$\nu_{lL} = \sum_{j=1}^3 U_{lj} \nu_{jL} .$$

U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix,

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

- U - $n \times n$ unitary:

n	2	3	4	
mixing angles:	$\frac{1}{2}n(n-1)$	1	3	6

CP-violating phases:

• ν_j – Dirac:	$\frac{1}{2}(n-1)(n-2)$	0	1	3
• ν_j – Majorana:	$\frac{1}{2}n(n-1)$	1	3	6

$n = 3$: 1 Dirac and

2 additional CP-violating phases, Majorana phases

S.M. Bilenky, J. Hosek, S.T.P., 1980

Majorana Neutrinos

Can be defined in QFT using fields or states.

Fields: $\chi_k(x)$ - 4 component (spin 1/2), complex, m_k

Majorana condition:

$$C (\bar{\chi}_k(x))^T = \xi_k \chi_k(x), \quad |\xi_k|^2 = 1, \quad C^{-1} \gamma_\mu C = - \gamma_\mu^T$$

- Invariant under proper Lorentz transformations.
- Reduces by 2 the number of components in $\chi_k(x)$.

Implications:

$$U(1) : \chi_k(x) \rightarrow e^{i\alpha} \chi_k(x) - \text{impossible}$$

- $\chi_k(x)$ cannot absorb phases.
- $Q_{U(1)} = 0 : Q_{\text{el}} = 0, L_l = 0, L = 0, \dots$
- $\chi_k(x)$: 2 spin states of a spin 1/2 absolutely neutral particle
- $\chi_k \equiv \bar{\chi}_k$

Propagators: $\Psi(x)$ -Dirac, $\chi(x)$ -Majorana

$$\langle 0 | T(\Psi_\alpha(x) \bar{\Psi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y) ,$$

$$\langle 0 | T(\Psi_\alpha(x) \Psi_\beta(y)) | 0 \rangle = 0 , \quad \langle 0 | T(\bar{\Psi}_\alpha(x) \bar{\Psi}_\beta(y)) | 0 \rangle = 0 .$$

$$\langle 0 | T(\chi_\alpha(x) \bar{\chi}_\beta(y)) | 0 \rangle = S_{\alpha\beta}^F(x - y) ,$$

$$\langle 0 | T(\chi_\alpha(x) \chi_\beta(y)) | 0 \rangle = -\xi^* S_{\alpha\kappa}^F(x - y) C_{\kappa\beta} ,$$

$$\langle 0 | T(\bar{\chi}_\alpha(x) \bar{\chi}_\beta(y)) | 0 \rangle = \xi C_{\alpha\kappa}^{-1} S_{\kappa\beta}^F(x - y)$$

$$U_{CP} \chi(x) U_{CP}^{-1} = \eta_{CP} \gamma_0 \chi(x'), \quad \eta_{CP} = \pm i .$$

PMNS Matrix: Standard Parametrization

$$U = V P, \quad P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$, $\theta_{ij} = [0, \frac{\pi}{2}]$,
- δ - Dirac CPV phase, $\delta = [0, 2\pi]$; CP inv.: $\delta = 0, \pi, 2\pi$;
- α_{21}, α_{31} - Majorana CPV phases; CP inv.: $\alpha_{21(31)} = k(k')\pi$, $k(k') = 0, 1, 2, \dots$
S.M. Bilenky, J. Hosek, S.T.P., 1980
- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.37 \times 10^{-5} \text{ eV}^2 > 0$, $\sin^2 \theta_{12} \cong 0.297$, $\cos 2\theta_{12} \gtrsim 0.29$ (3σ)
- $|\Delta m_{31(32)}^2| \cong 2.53$ (2.43) $\times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_{23} \cong 0.437$ (0.569), NO (IO) ,
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0214$ (0.0218), Capozzi et al. NO (IO).
F. Capozzi et al. (Bari Group), arXiv:1601.07777v1.

- $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2 \cong 7.54 \times 10^{-5}$ eV $^2 > 0$, $\sin^2 \theta_{12} \cong 0.308$, $\cos 2\theta_{12} \gtrsim 0.28$ (3σ),
- $|\Delta m_{31(32)}^2| \cong 2.47$ (2.42) $\times 10^{-3}$ eV 2 , $\sin^2 \theta_{23} \cong 0.437$ (0.455), NO (IO),
- θ_{13} - the CHOOZ angle: $\sin^2 \theta_{13} = 0.0234$ (0.0240), NH (IH).
- $1\sigma(\Delta m_{21}^2) = 2.6\%$, $1\sigma(\sin^2 \theta_{12}) = 5.4\%$;
- $1\sigma(|\Delta m_{31(23)}^2|) = 2.6\%$, $1\sigma(\sin^2 \theta_{23}) = 9.6\%$;
- $1\sigma(\sin^2 \theta_{13}) = 8.5\%$;
- $3\sigma(\Delta m_{21}^2) : (6.99 - 8.18) \times 10^{-5}$ eV 2 ; $3\sigma(\sin^2 \theta_{12}) : (0.259 - 0.359)$;
 $(3\sigma(\Delta m_{21}^2) : (6.93 - 7.97) \times 10^{-5}$ eV 2 ; $3\sigma(\sin^2 \theta_{12}) : (0.250 - 0.354);)$
- $3\sigma(|\Delta m_{31(23)}^2|) : 2.27(2.23) - 2.65(2.60) \times 10^{-3}$ eV 2 ;
 $(2.40(2.30) - 2.66(2.57) \times 10^{-3}$ eV 2 ;
 $3\sigma(\sin^2 \theta_{23}) : 0.374(0.380) - 0.628(0.641)$;
 $(3\sigma(\sin^2 \theta_{23}) : 0.379(0.383) - 0.616(0.637))$
- $3\sigma(\sin^2 \theta_{13}) : 0.0176(0.0178) - 0.0296(0.0298)$
 $(3\sigma(\sin^2 \theta_{13}) : 0.0185(0.0186) - 0.0246(0.0248).)$

F. Capozzi et al. (Bari Group), arXiv:1312.2878v2 (May 5, 2014)
(F. Capozzi et al. (Bari Group), arXiv:1601.07777v1.)

- $\text{sgn}(\Delta m_{\text{atm}}^2) = \text{sgn}(\Delta m_{31(32)}^2)$ not determined

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 > 0$, normal mass ordering (NO)

$\Delta m_{\text{atm}}^2 \equiv \Delta m_{32}^2 < 0$, inverted mass ordering (IO)

Convention: $m_1 < m_2 < m_3$ - NO, $m_3 < m_1 < m_2$ - IO

$$\Delta m_{31}^2(\text{NO}) = -\Delta m_{32}^2(\text{IO})$$

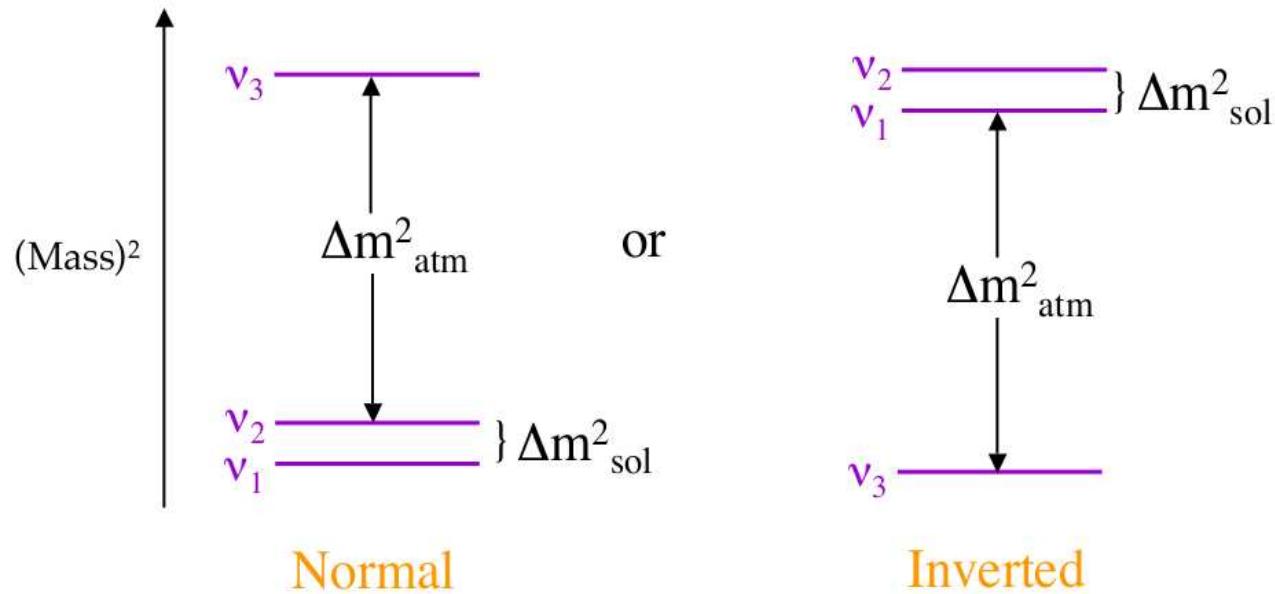
$$m_1 \ll m_2 < m_3, \quad \text{NH},$$

$$m_3 \ll m_1 < m_2, \quad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg |\Delta m_{31(32)}^2|, \quad \text{QD}; \quad m_j \gtrsim 0.10 \text{ eV}.$$

- $m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}$ - NO;
- $m_1 = \sqrt{m_3^2 + \Delta m_{23}^2 - \Delta m_{21}^2}, \quad m_2 = \sqrt{m_3^2 + \Delta m_{23}^2}$ - IO;

The (Mass)² Spectrum

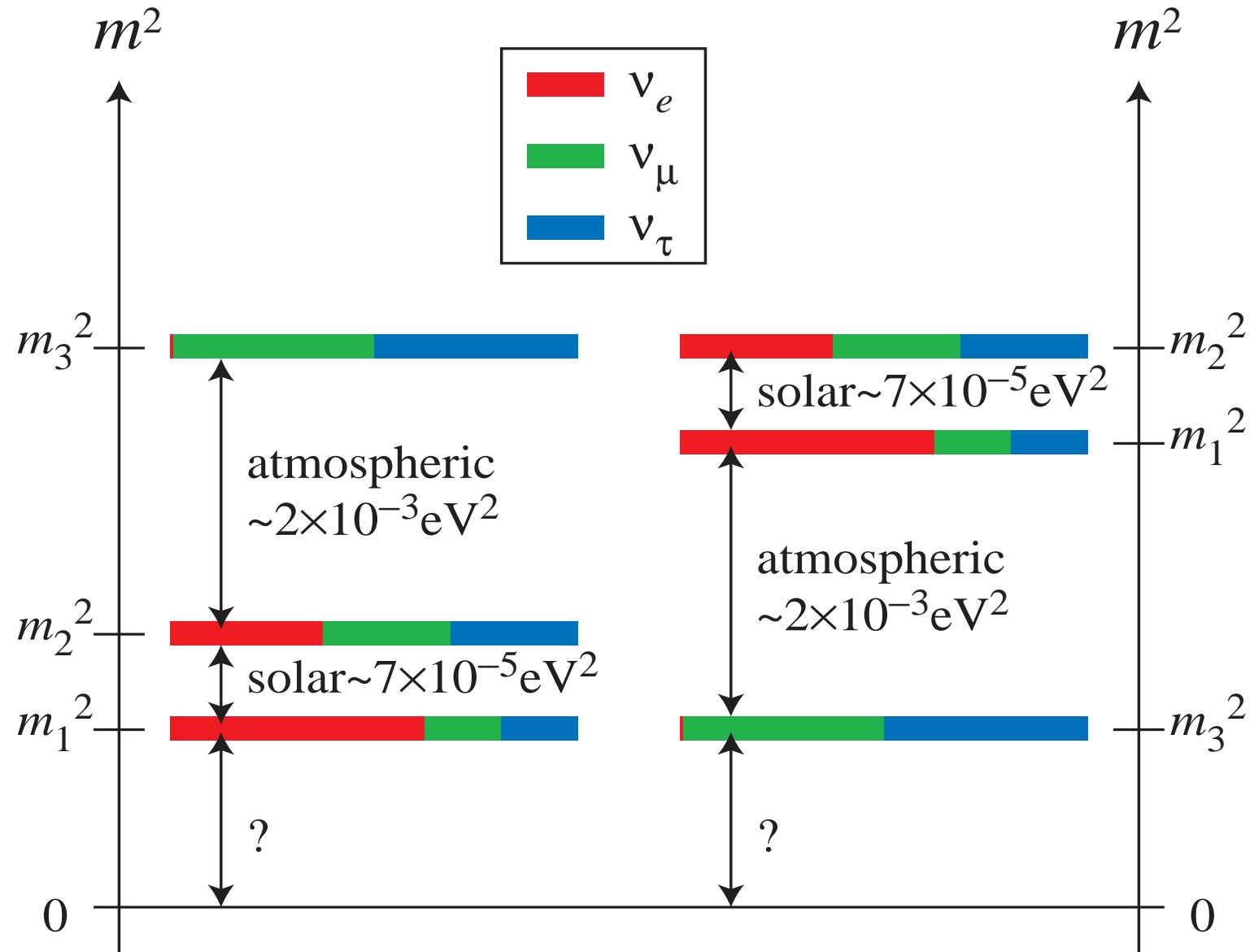


$$\Delta m^2_{\text{sol}} \cong 7.6 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{\text{atm}} \cong 2.4 \times 10^{-3} \text{ eV}^2$$

Are there *more* mass eigenstates, as LSND suggests,
and MiniBooNE recently hints?

3

Due to B. Kayser



S. King, Ch. Luhn, 2013

- Dirac phase δ : $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$, $l \neq l'$; $A_{CP}^{(l,l')} \propto J_{CP} \propto \sin \theta_{13} \sin \delta$:

P.I. Krastev, S.T.P., 1988

$$J_{CP} = \text{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

Current data: $|J_{CP}| \lesssim 0.035$ (can be relatively large!); b.f.v. with $\delta = 3\pi/2$: $J_{CP} \cong -0.035$.

- Majorana phases α_{21}, α_{31} :

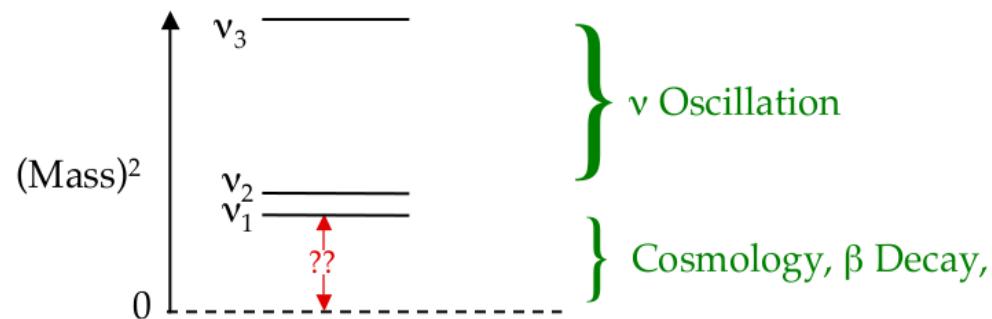
– $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}$ not sensitive;

S.M. Bilenky, J. Hosek, S.T.P., 1980;
P. Langacker, S.T.P., G. Steigman, S. Toshev, 1987

- $|\langle m \rangle|$ in $(\beta\beta)_{0\nu}$ -decay depends on α_{21}, α_{31} ;
- $\Gamma(\mu \rightarrow e + \gamma)$ etc. in SUSY theories depend on $\alpha_{21,31}$;
- BAU, leptogenesis scenario: $\delta, \alpha_{21,31}$!

Absolute Neutrino Mass Scale

The Absolute Scale of Neutrino Mass



How far above zero
is the whole pattern?

$$\text{Oscillation Data} \Rightarrow \sqrt{\Delta m_{\text{atm}}^2} < \text{Mass[Heaviest } v_i]$$

4

Due to B. Kayser

Absolute Neutrino Mass Measurements

Troitzk, Mainz experiments on ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$:
 $m_{\nu_e} < 2.2 \text{ eV}$ (95% C.L.)

We have $m_{\nu_e} \cong m_{1,2,3}$ in the case of QD spectrum. The upcoming **KATRIN** experiment is planned to reach sensitivity

KATRIN: $m_{\nu_e} \sim 0.2 \text{ eV}$

i.e., it will probe the region of the QD spectrum.

Improved β energy resolution requires a **BIG** β spectrometer.





9

Mass and Hierarchy from Cosmology

Cosmological and astrophysical data on $\sum_j m_j$: the Planck + WMAP (low $l \leq 25$) + ACT (large $l \geq 2500$) CMB data + Λ CDM (6 parameter) model + assuming 3 light massive neutrinos, implies

$$\sum_j m_j \equiv \Sigma < 0.66 \text{ eV} \quad (95\% \text{ C.L.})$$

Adding data on the baryon acoustic oscillations (BAO) leads to:

$$\sum_j m_j \equiv \Sigma < 0.23 \text{ eV} \quad (95\% \text{ C.L.})$$

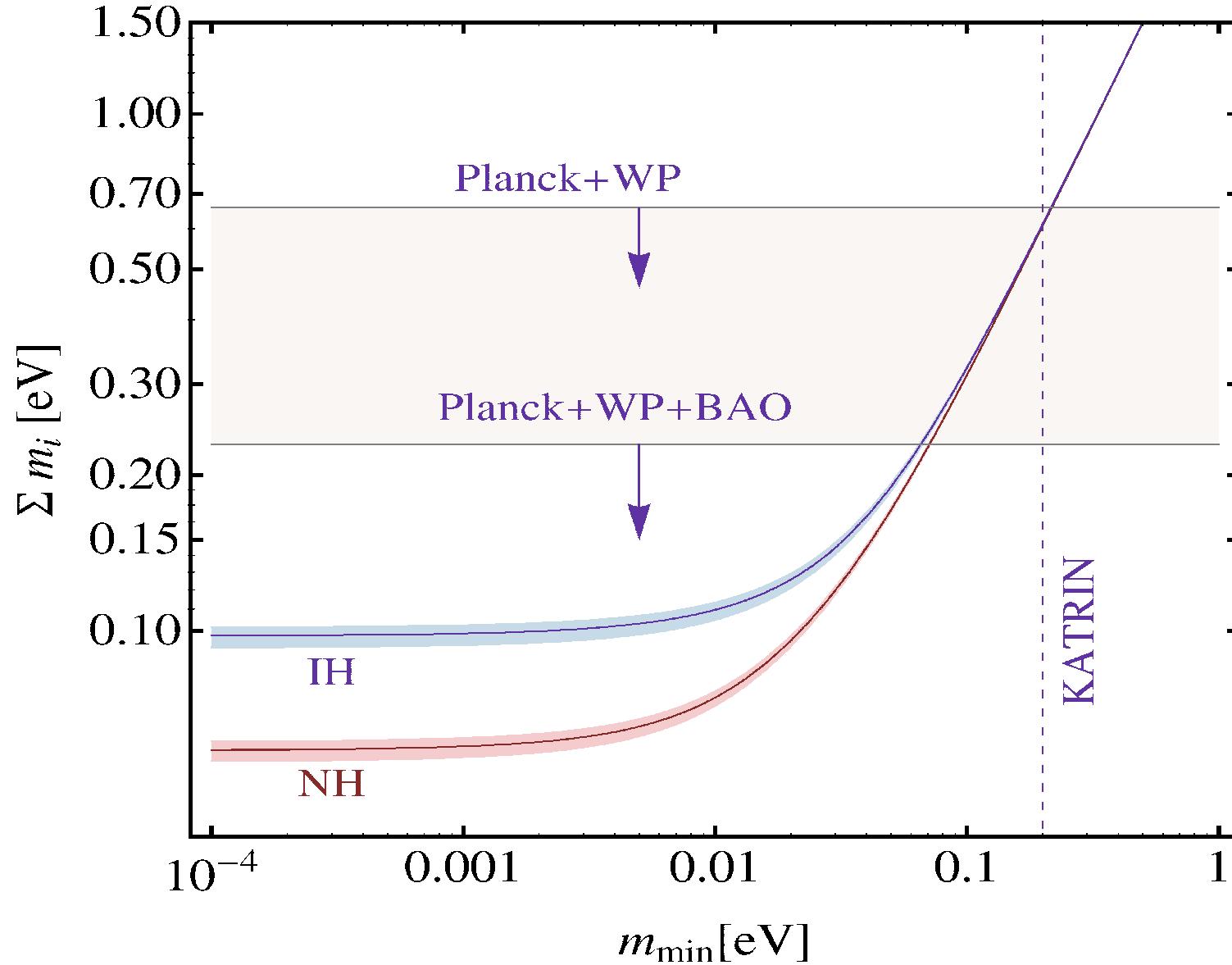
Data on weak lensing of galaxies by large scale structure, combined with data from the WMAP and Planck experiments might allow to determine

$$\sum_j m_j : \quad \delta \cong (0.01 - 0.04) \text{ eV.}$$

NH: $\sum_j m_j \leq 0.05 \text{ eV } (3\sigma);$

IH: $\sum_j m_j \geq 0.10 \text{ eV } (3\sigma).$

Mass and Hierarchy from Cosmology



Future Progress

- Determination of the nature - Dirac or Majorana, of ν_j .
- Determination of $\text{sgn}(\Delta m_{\text{atm}}^2)$, type of ν - mass spectrum

$$m_1 \ll m_2 \ll m_3, \quad \text{NH},$$

$$m_3 \ll m_1 < m_2, \quad \text{IH},$$

$$m_1 \cong m_2 \cong m_3, \quad m_{1,2,3}^2 \gg \Delta m_{\text{atm}}^2, \quad \text{QD}; \quad m_j \gtrsim 0.10 \text{ eV}.$$

- Determining, or obtaining significant constraints on, the absolute scale of ν_j -masses, or $\min(m_j)$.
- Status of the CP-symmetry in the lepton sector: violated due to δ (Dirac), and/or due to α_{21} , α_{31} (Majorana)?
- High precision determination of Δm_{\odot}^2 , θ_{12} , Δm_{atm}^2 , θ_{23} , θ_{13}
- Searching for possible manifestations, other than ν_l -oscillations, of the non-conservation of L_l , $l = e, \mu, \tau$, such as $\mu \rightarrow e + \gamma$, $\tau \rightarrow \mu + \gamma$, etc. decays.

- Understanding at fundamental level the mechanism giving rise to the ν - masses and mixing and to the L_l -non-conservation. Includes understanding
 - the origin of the observed patterns of ν -mixing and ν -masses ;
 - the physical origin of CPV phases in U_{PMNS} ;
 - Are the observed patterns of ν -mixing and of $\Delta m_{21,31}^2$ related to the existence of a new symmetry?
 - Is there any relations between q -mixing and ν - mixing? Is $\theta_{12} + \theta_c = \pi/4$?
 - Is $\theta_{23} = \pi/4$, or $\theta_{23} > \pi/4$ or else $\theta_{23} < \pi/4$?
 - Is there any correlation between the values of CPV phases and of mixing angles in U_{PMNS} ?
- Progress in the theory of ν -mixing might lead to a better understanding of the origin of the BAU.
 - Can the Majorana and/or Dirac CPVP in U_{PMNS} be the leptogenesis CPV parameters at the origin of BAU?

The next most important steps are:

- determination of the nature - Dirac or Majorana, of massive neutrinos ($(\beta\beta)_{0\nu}$ -decay exps: GERDA, CUORE, EXO, KamLAND-Zen, SNO+; SuperNEMO, MAJORANA, AMORE,...).
- determination of the neutrino mass ordering;
- determination of the status of the CP symmetry in the lepton sector (T2K, NO ν A; T2HK, DUNE)
- determination of the absolute neutrino mass scale, or $\min(m_j)$ (KATRIN, new ideas; cosmology);

The program of research extends beyond 2030 (with a total cost exceeding 2.5×10^9 Euro).

Determining the ν -Mass Ordering ($\text{sgn}(\Delta m_{\text{atm}}^2)$)

- Reactor $\bar{\nu}_e$ Oscillations in vacuum (JUNO, RENO50).

S.T.P., M. Piai, hep-ph/0112074 (PL B533 (2002) 94)

- Atmospheric ν experiments: subdominant $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ and $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ oscillations (matter effects) (ORCA, PINGU (IceCube), HK, INO).
- LBL ν -oscillation experiments T2K + NO ν A; DUNE (future); + T2HK (future), designed to search also for CP violation.
- ${}^3\text{H}$ β -decay Experiments (sensitivity to 5×10^{-2} eV) (NH vs IH).
- $(\beta\beta)_{0\nu}$ -Decay Experiments; ν_j - Majorana particles (NH vs IH).
- Cosmology: $\sum_j m_j$ (NH vs IH).
- Atomic Physics Experiments: RENP.

Solar Neutrinos ν_e , $E \sim 1$ MeV: B. Pontecorvo 1946



R. Davis et al., 1967 - 1996: 615 t C_2Cl_4 ; 0.5 Ar atoms/day, exposure 60 days.



Kamiokande (1986-1994), Super-Kamiokande (1996 -), SNO (2000 - 2006), BOREXINO (2007 -);



Super-Kamiokande: 50000t ultra-pure water;

SNO: 1000t heavy water (D_2O)



SAGE (60t), 1990-; GALLEX/GNO (30t, LNGS), 1991-2003

Atmospheric Neutrinos ν_μ , $\bar{\nu}_\mu$, ν_e , $\bar{\nu}_e$, $E \sim 1$ GeV (0.20 - 100 GeV)

$$\nu_\mu + N \rightarrow \mu^- + X, \quad \tilde{\nu}_\mu + N \rightarrow \mu^+ + X$$

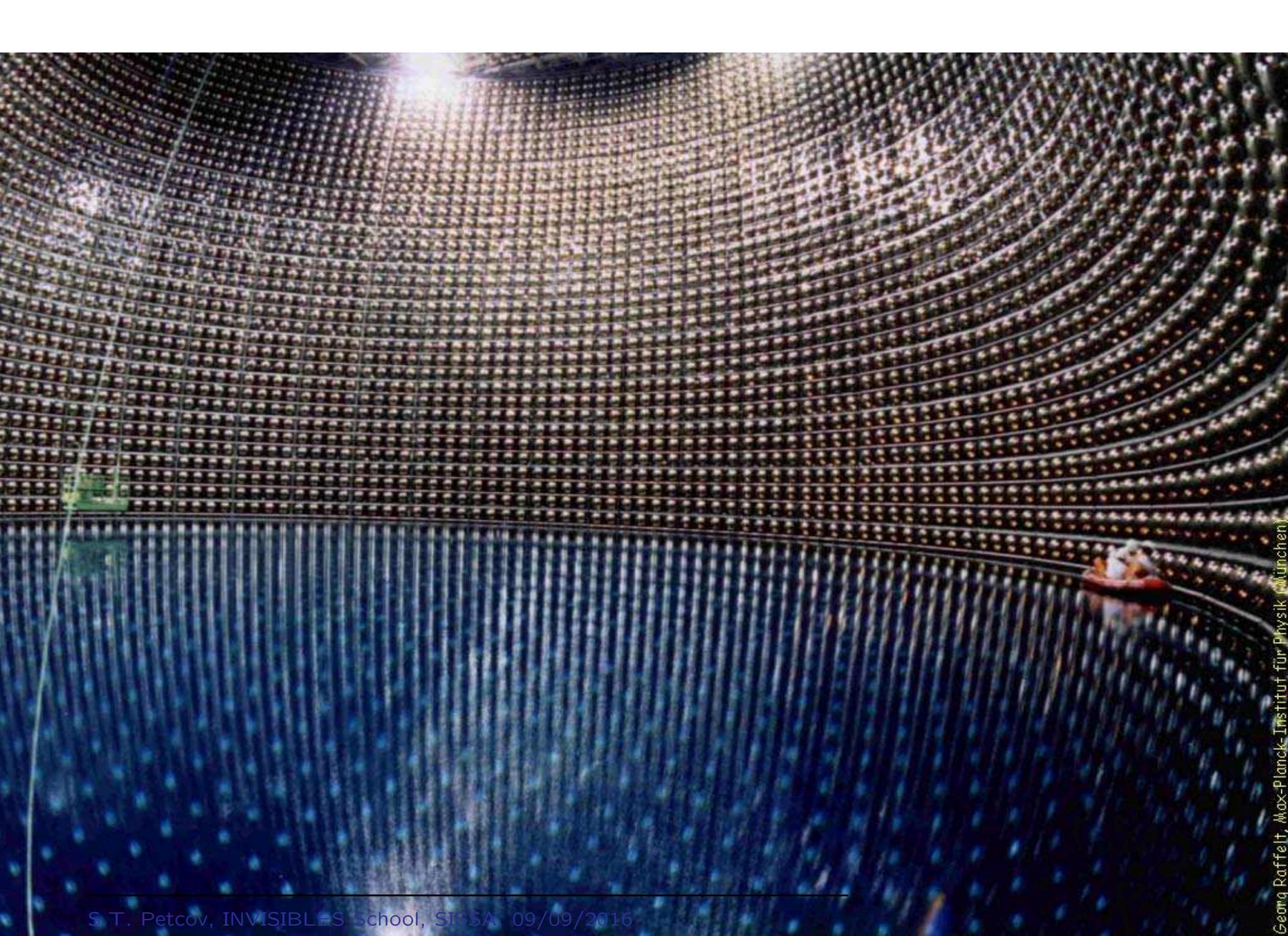
$$\nu_e + N \rightarrow e^- + X, \quad \tilde{\nu}_e + N \rightarrow e^+ + X$$

K2K, MINOS, T2K, ν_μ ($\bar{\nu}_\mu$), $E \sim 1$ GeV

$$\nu_\mu + n(N) \rightarrow \mu^- + p(X) \quad (\nu_e + n(N) \rightarrow e^- + p(X))$$

Reactor $\bar{\nu}_e$: CHOOZ, KamLAND, Double Chooz, RENO, Daya Bay ($E \cong 2 - 8$ MeV)

$$\bar{\nu}_e + p \rightarrow e^+ + n$$





Neutrino Oscillations in Vacuum

Suppose at $t = 0$ in vacuum

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta,$$

$$|\nu_{\mu(\tau)}\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta; \quad \nu_{1,2} : m_{1,2} \neq 0$$

After time t in vacuum

$$|\nu_e\rangle_t = e^{-iE_1 t} |\nu_1\rangle \cos\theta + e^{-iE_2 t} |\nu_2\rangle \sin\theta, \quad E_{1,2} = \sqrt{p^2 + m_{1,2}^2}$$

$$A(\nu_e \rightarrow \nu_\mu; t) = \langle \nu_\mu | \nu_e \rangle_t = \frac{1}{2} \sin 2\theta (e^{-iE_2 t} - e^{-iE_1 t})$$

$$P(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta (1 - \cos(E_2 - E_1)t)$$

$$P(\nu_e \rightarrow \nu_e; t) \equiv P_{ee} = 1 - P(\nu_e \rightarrow \nu_\mu; t)$$

V. Gribov, B. Pontecorvo, 1969

Neutrinos are relativistic: $t \cong L$, $E_2 - E_1 \cong (m_2^2 - m_1^2)/(2p)$

$$(E_2 - E_1)t \cong (m_2^2 - m_1^2)L/(2p) = 2\pi \frac{L}{L_{osc}^{vac}}, \quad L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$$

$$P(\nu_e \rightarrow \nu_\mu; t) = \frac{1}{2} \sin^2 2\theta \left(1 - \cos 2\pi \frac{L}{L_{osc}^{vac}}\right), \quad L_{osc}^{vac} \equiv \frac{4\pi E}{\Delta m^2}$$

$$L_{osc}^{vac} \cong 2.5 \text{ m } \frac{E[\text{MeV}]}{\Delta m^2[\text{eV}^2]}$$

$$E \cong 3 \text{ MeV}, \quad \Delta m^2[\text{eV}^2] \cong 8 \times 10^{-5} : \quad L_{osc}^{vac} \cong 100 \text{ km}$$

$$E \cong 1 \text{ GeV}, \quad \Delta m^2[\text{eV}^2] \cong 2.5 \times 10^{-3} : \quad L_{osc}^{vac} \cong 1000 \text{ km}$$

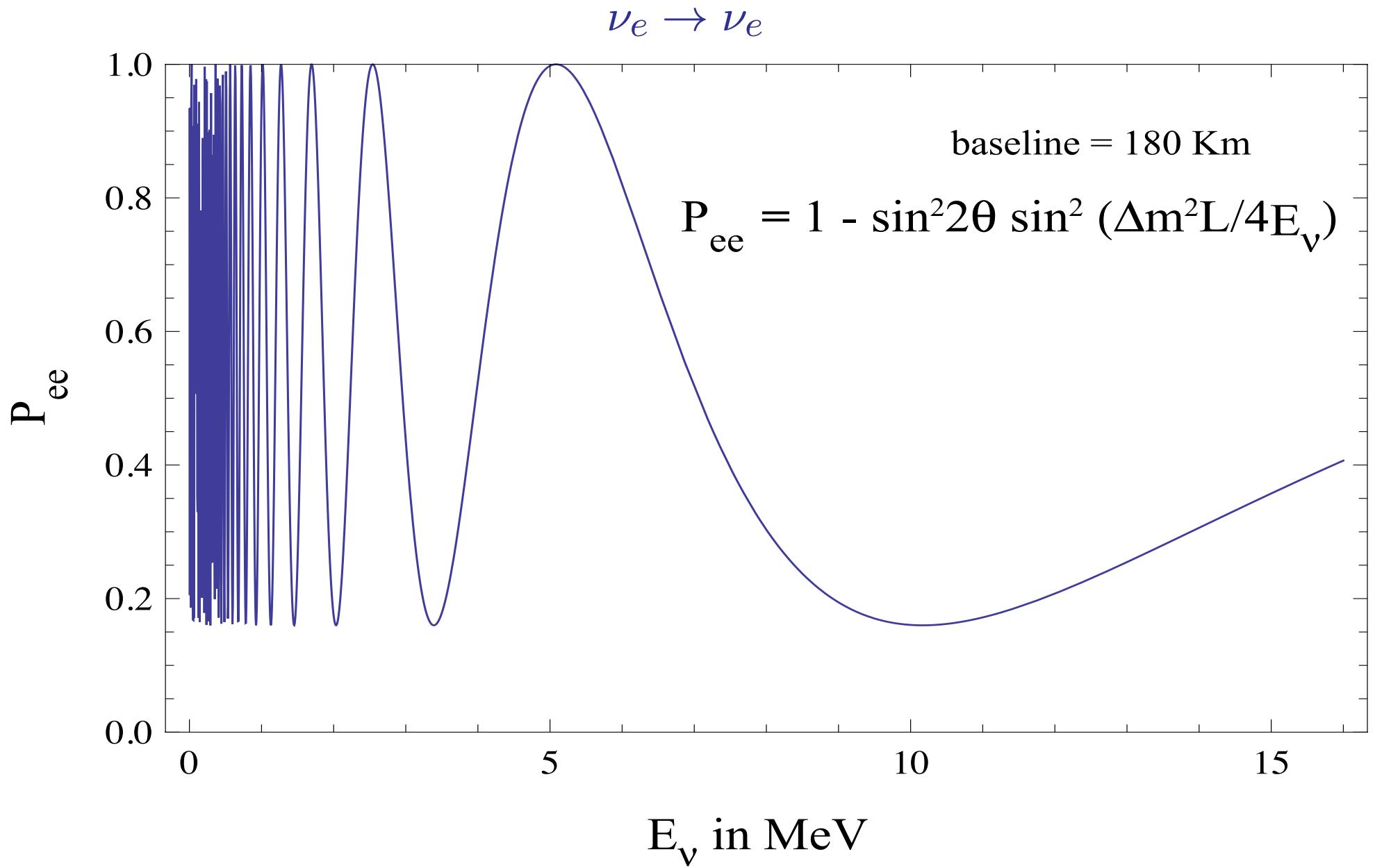
Effects of oscillations observable if

$$\sin^2 2\theta - \text{sufficiently large}, \quad L \gtrsim L_{osc}^{vac}$$

Two basic parameters: $\sin^2 2\theta$, Δm^2

SK, K2K, MINOS; CNGS (OPERA): dominant $\nu_\mu \rightarrow \nu_\tau$

KamLAND: $\bar{\nu}_e \rightarrow \bar{\nu}_e$; $\bar{\nu}_e \rightarrow (\bar{\nu}_\mu + \bar{\nu}_\tau)/\sqrt{2}$



Source	Type of ν	$\bar{E}[\text{MeV}]$	$L[\text{km}]$	$\min(\Delta m^2)[\text{eV}^2]$
Reactor	$\tilde{\nu}_e$	~ 1	1	$\sim 10^{-3}$
Reactor	$\tilde{\nu}_e$	~ 1	100	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	1	~ 1
Accelerator	$\nu_\mu, \tilde{\nu}_\mu$	$\sim 10^3$	1000	$\sim 10^{-3}$
Atmospheric ν 's	$\nu_{\mu,e}, \tilde{\nu}_{\mu,e}$	$\sim 10^3$	10^4	$\sim 10^{-4}$
Sun	ν_e	~ 1	1.5×10^8	$\sim 10^{-11}$

Correspond to: CHOOZ, Double Chooz, RENO, Daya Bay ($L \sim 1 \text{ km}$), KamLAND ($L \sim 100 \text{ km}$);
 $\tilde{\nu}_e$ disappearance; $E = (1.8 \div 8.0) \text{ MeV}$;

to accelerator experiments - past ($L \sim 1 \text{ km}$);

past, current: K2K ($L \sim 250 \text{ km}$), MINOS ($L \sim 730 \text{ km}$), ν_μ disappearance; OPERA ($L \sim 730 \text{ km}$), $\nu_\mu \rightarrow \nu_\tau$; T2K ($L \sim 295 \text{ km}$), NO ν A ($L \sim 800 \text{ km}$), ν_μ disappearance, $\nu_\mu \rightarrow \nu_e$; $E \sim 1 \text{ GeV}$;

SK experiment studying atmospheric $\nu_\mu, \tilde{\nu}_\mu, \nu_e, \tilde{\nu}_e$ ($E \cong 0.1 \div 100 \text{ GeV}$), and solar ν_e ($E \cong 5 \div 14 \text{ MeV}$) oscillations, and to the solar ν experiments ($E \cong 0.29 \div 14 \text{ MeV}$).

Two-Neutrino Oscillations in Vacuum

SK ((100-12742) km), K2K (250 km); CNGS (OPERA),
MINOS (730 km); T2K (295 km); dominant $\nu_\mu \rightarrow \nu_\tau$;

$$P(\nu_\mu \rightarrow \nu_\tau; L) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau; L) \cong \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{31}^2 L}{4E};$$
$$P(\nu_\mu \rightarrow \nu_\mu; L) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu; L) = 1 - P(\nu_\mu \rightarrow \nu_\tau; L).$$

KamLAND (~ 180 km): $\bar{\nu}_e \rightarrow \bar{\nu}_e$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L) \cong 1 - \frac{1}{2} \sin^2 2\theta_{12} (1 - \cos \frac{\Delta m_{21}^2 L}{2E}).$$

CHOOZ, Double Chooz, Daya Bay, RENO (~ 1 km):
 $\bar{\nu}_e \rightarrow \bar{\nu}_e$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e; L) \cong 1 - \frac{1}{2} \sin^2 2\theta_{13} (1 - \cos \frac{\Delta m_{31}^2 L}{2E}).$$

The equation used above corresponds to a plane wave description of the propagation of neutrinos ν_j . It accounts only for the movement of the center of the wave packet describing ν_j . In the wave packet treatment of the problem, the interference between the states of ν_j and ν_k is subject to a number of conditions, the localisation condition (in space and time) and the condition of overlapping of the wave packets of ν_j and ν_k at the detection point being the most important. For relativistic neutrinos, the localisation condition in space reads: $\sigma_{xP}, \sigma_{xD} < L_{jk}^v/(2\pi)$, $\sigma_{xP(D)}$ being the spatial width of the production (detection) wave packet. Thus, the interference will not be suppressed if the spatial width of the neutrino wave packets determined by the neutrino production and detection processes is smaller than the corresponding oscillation length in vacuum. In order for the interference to be nonzero, the wave packets describing ν_j and ν_k should also overlap in the point of neutrino detection. This requires that the spatial separation between the two wave packets at the point of neutrinos detection, caused by the two wave packets having different group velocities $v_j \neq v_k$, satisfies $|(v_j - v_k)T| \ll \max(\sigma_{xP}, \sigma_{xD})$. If the interval of time T is not measured, T in the preceding condition must be replaced by the distance L between the neutrino source and the detector.

Examples

- Spatial localisation condition

ΔL - dimensions of the ν - source (and/or detector):

$$2\pi\Delta L/L_{jk}^v \lesssim 1.$$

- Time localisation condition

ΔE - detector's energy resolution:

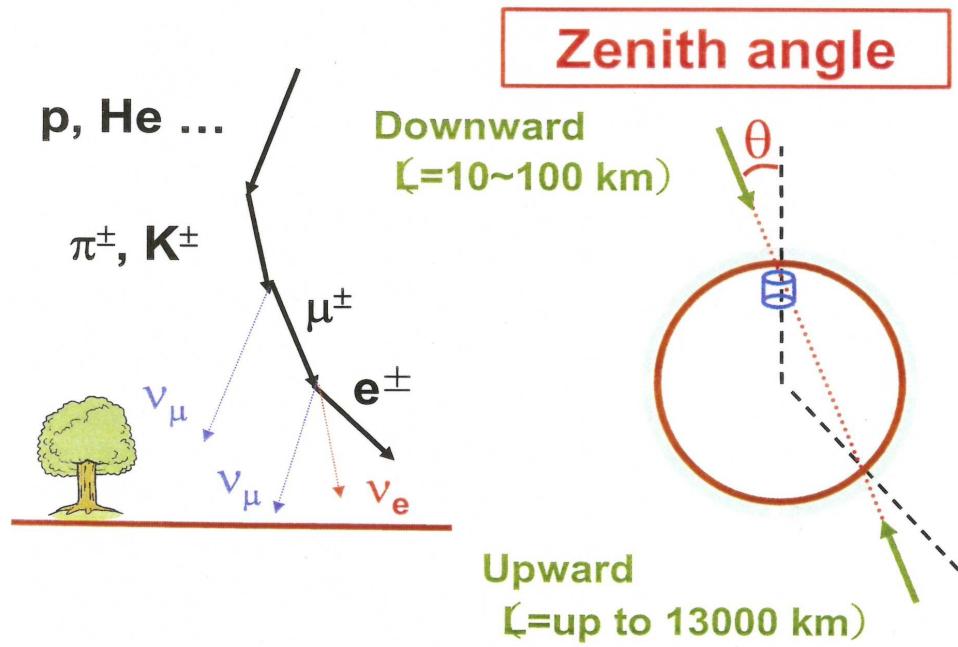
$$2\pi(L/L_{jk}^v)(\Delta E/E) \lesssim 1.$$

If $2\pi\Delta L/L_{jk}^v \gg 1$, and/or $2\pi(L/L_{jk}^v)(\Delta E/E) \gg 1$,

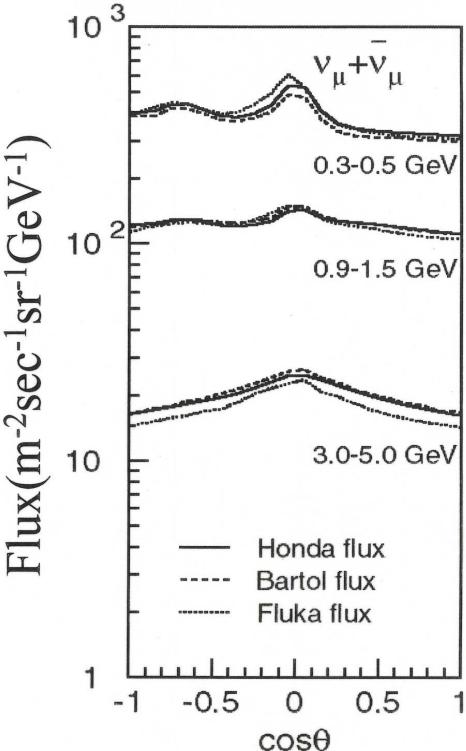
$$\bar{P}(\nu_l \rightarrow \nu_{l'}) = \bar{P}(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \cong \frac{1}{2} \sin^2 2\theta, \quad l \neq l'$$

Observing the Oscillations of Neutrinos

Atmospheric neutrinos



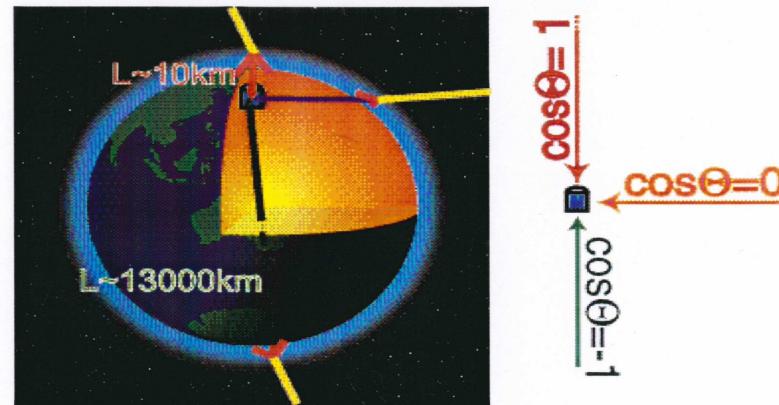
Zenith angle dist. of Atmospheric ν flux



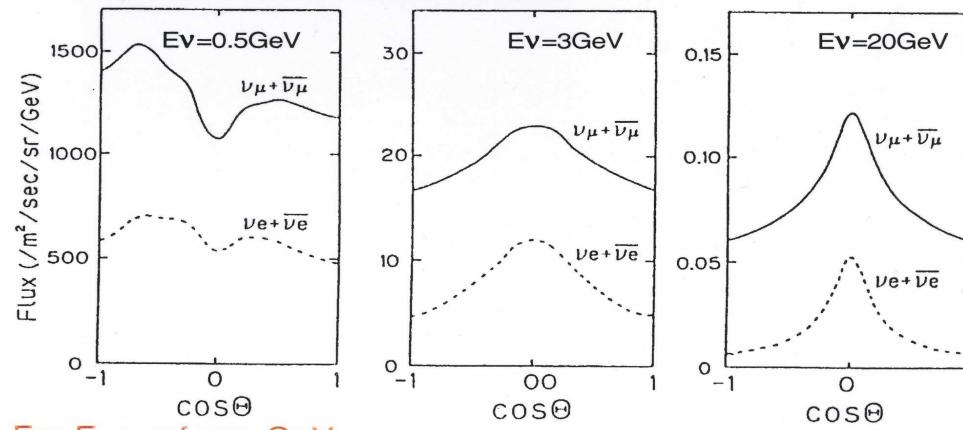
$E\nu > \text{a few GeV}$

Up/Down Symmetry

Zenith angle distribution(1D)



Calculated zenith angle distribution

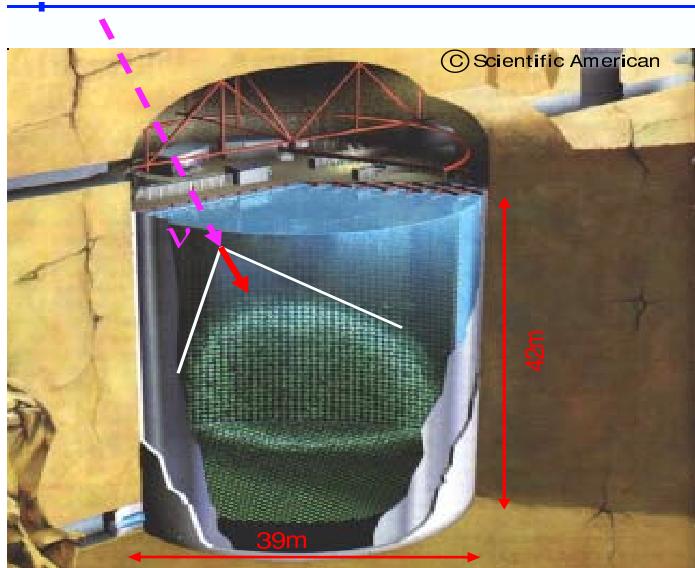


For $E_\nu >$ a few GeV,

Upward / downward = 1 (within a few %)



Up/Down asymmetry for neutrino oscillations



© Scientific American

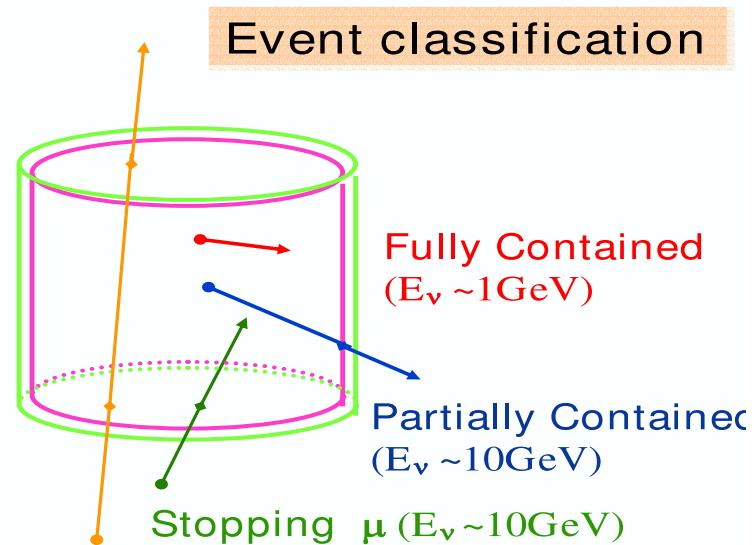
Water Cherenkov detector

1000 m underground

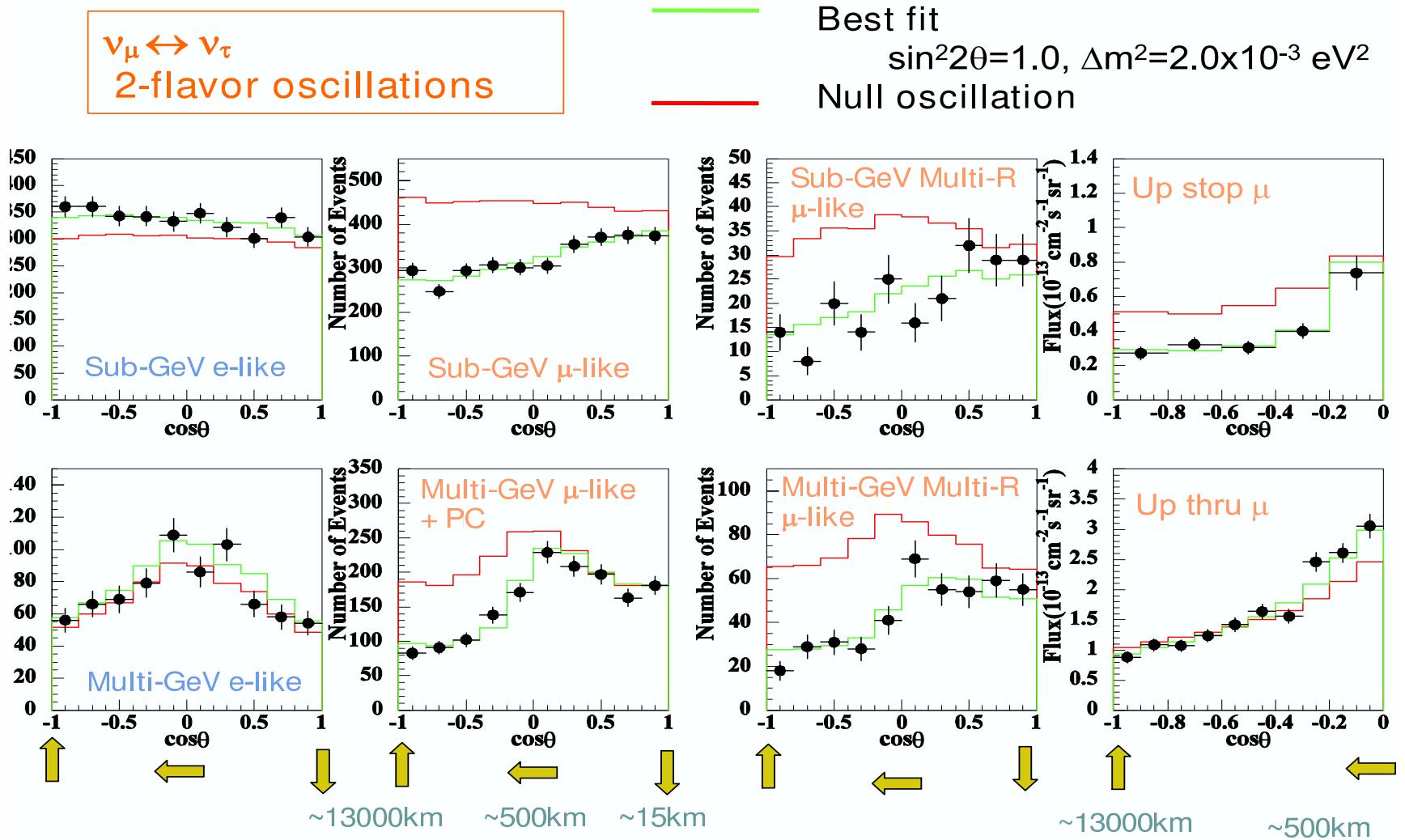
50,000 ton (22,500 ton fid.)

inner-detector(ID): 11,146 20 inch PMTs(SK-I)

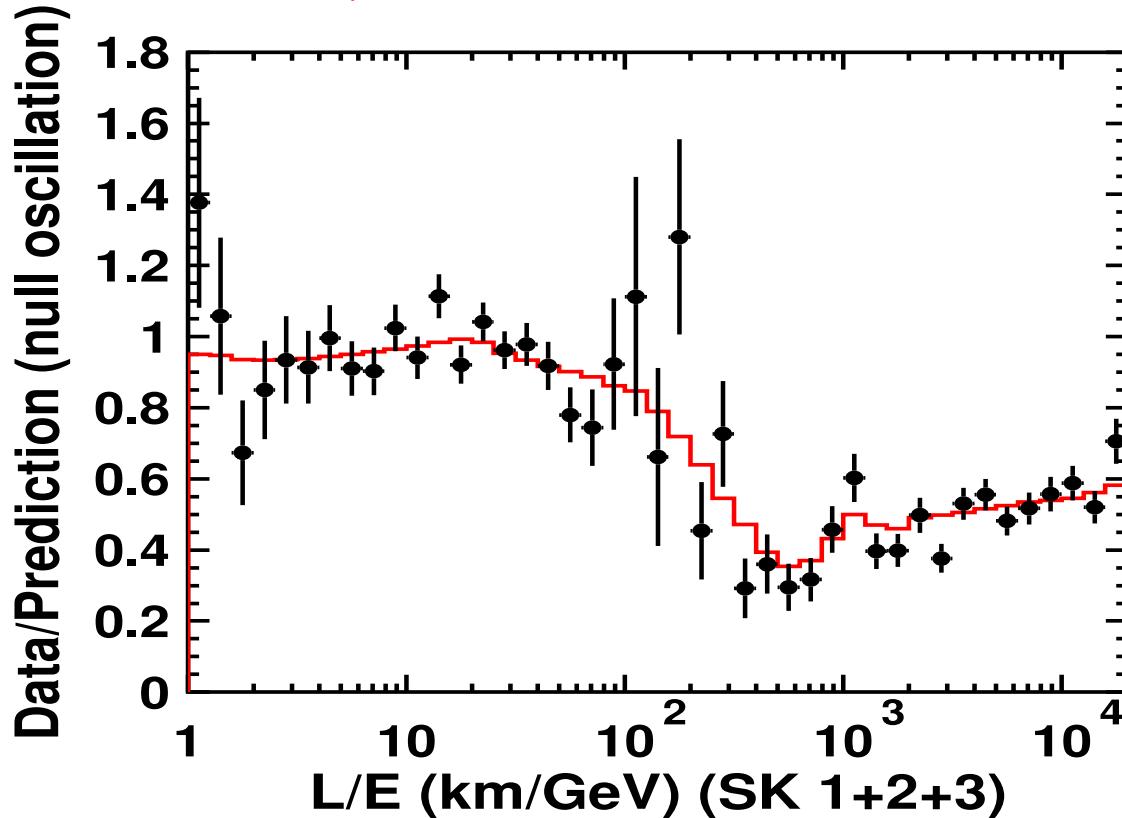
outer-detector(OD): 1,885 8 inch PMTs



Zenith angle distributions



SK: L/E Dependence, μ -Like Events



L/E analysis

Neutrino oscillation :

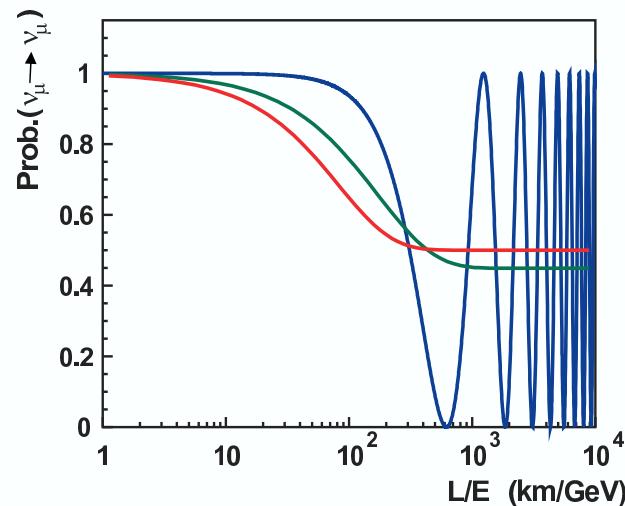
$$P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2(1.27 \frac{\Delta m^2 L}{E})$$

Neutrino decay :

$$P_{\mu\mu} = (\cos^2 \theta + \sin^2 \theta \times \exp(-\frac{m}{2\tau} \frac{L}{E}))^2$$

Neutrino decoherence :

$$P_{\mu\mu} = 1 - \frac{1}{2} \sin^2 2\theta \times (1 - \exp(-\gamma_0 \frac{L}{E}))$$



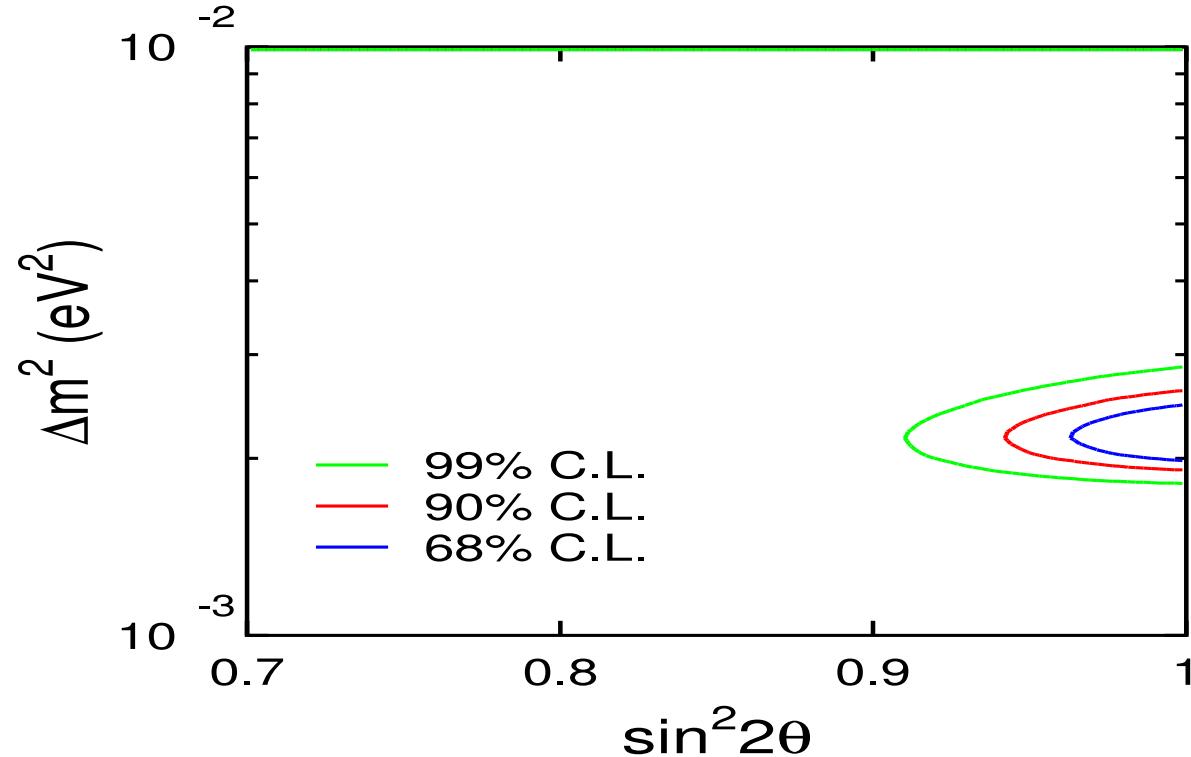
Use events with high resolution in L/E

→ The first dip can be observed

→ Direct evidence for oscillations

→ Strong constraint to oscillation parameters, especially Δm^2 value

SK: Atmospheric ν Data



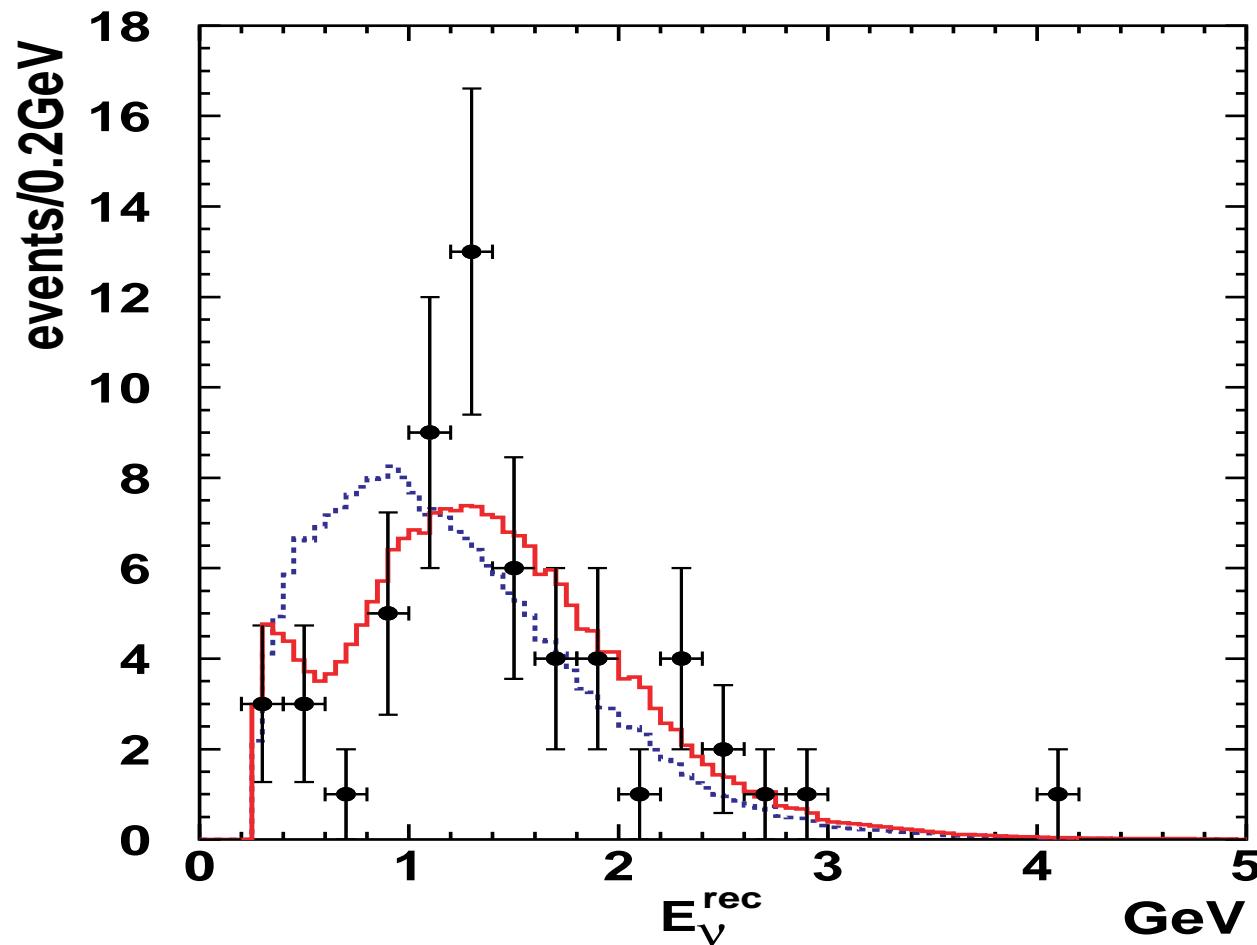
$$\Delta m_{\text{atm}}^2 \equiv \Delta m_{31}^2 = 2.4 \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{\text{atm}} \equiv \sin^2 2\theta_{23} = 1.0 ;$$

$$\Delta m_{31}^2 = (1.9 - 2.9) \times 10^{-3} \text{ eV}^2, \quad \sin^2 2\theta_{23} \geq 0.92, \quad 99\% \text{ C.L.}$$

- sign of Δm_{atm}^2 not determined. If $\theta_{23} \neq \frac{\pi}{4}$: $\theta_{23}, (\frac{\pi}{4} - \theta_{23})$ ambiguity.

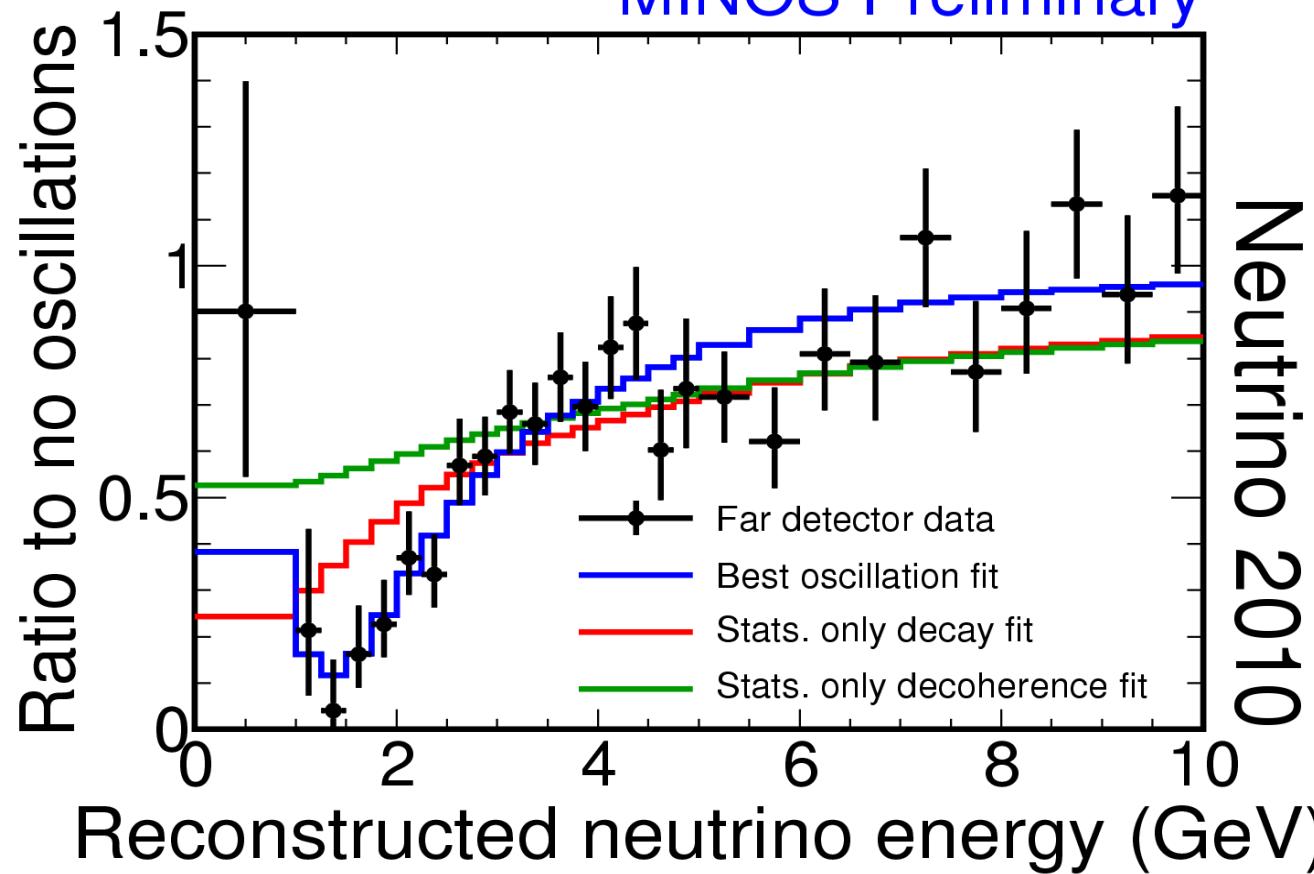
3- ν mixing: $\Delta m_{31}^2 > 0$, $m_1 < m_2 < m_3$ (NH); $\Delta m_{31}^2 < 0$, $m_3 < m_1 < m_2$ (IH).

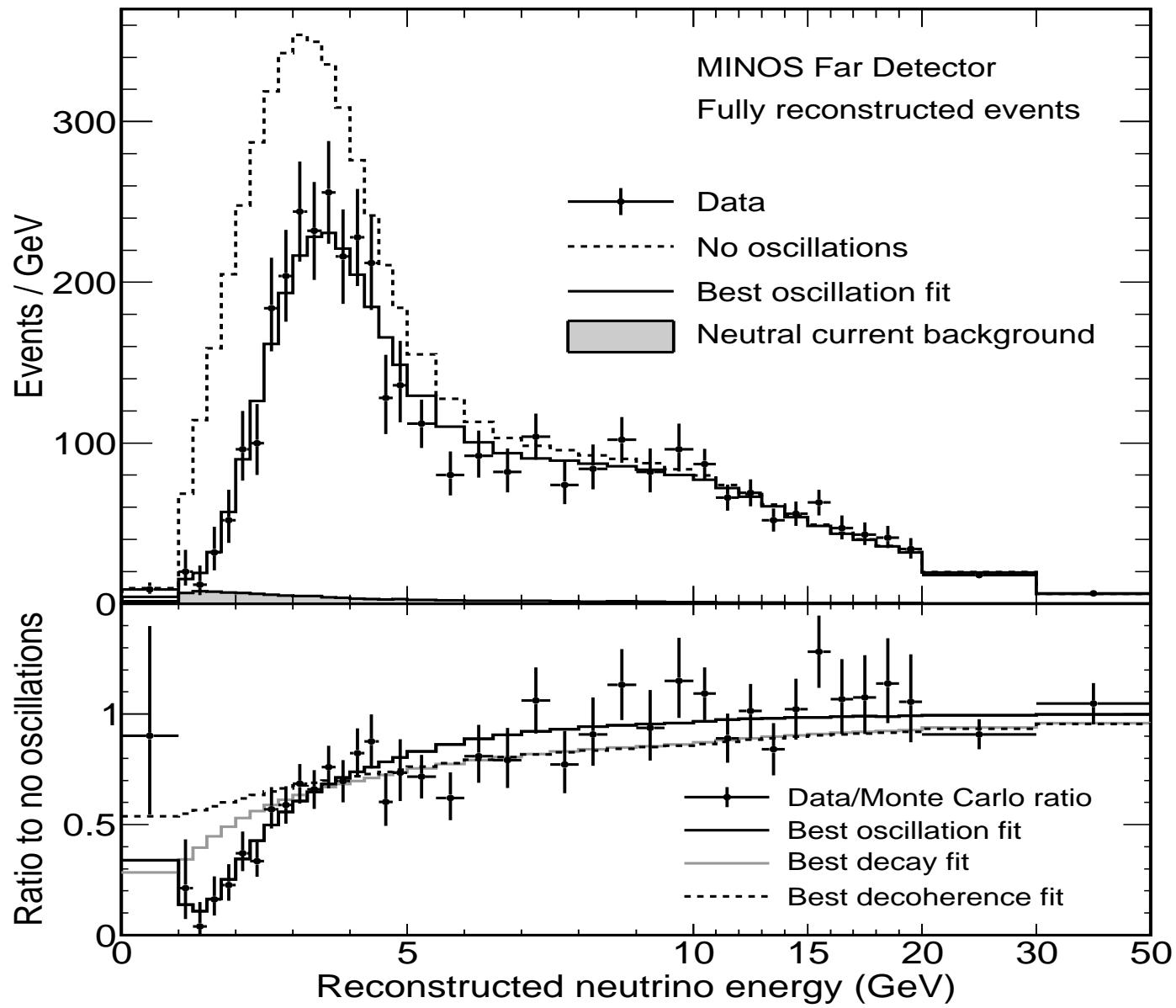
K2K: ν_μ Spectrum (ν_μ “disappearance”)

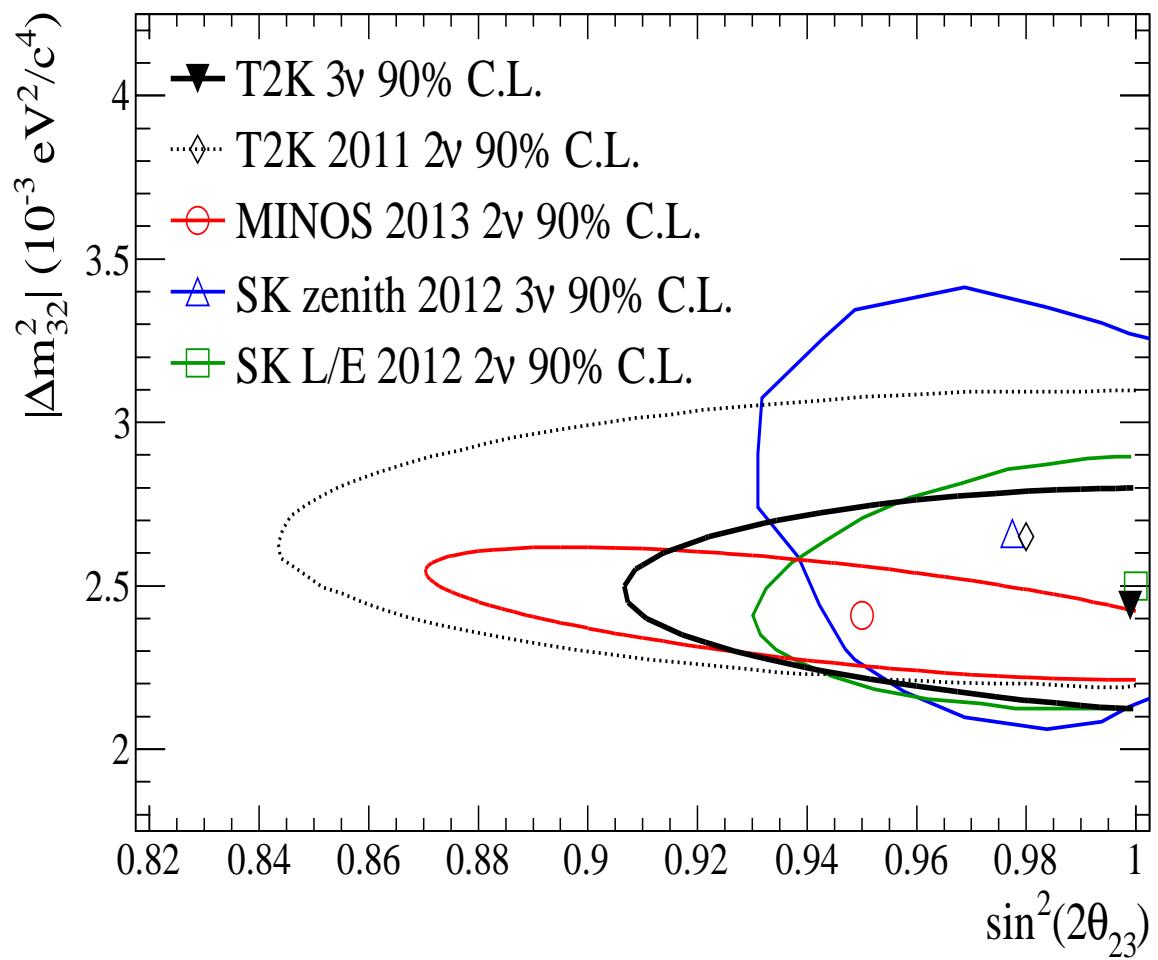


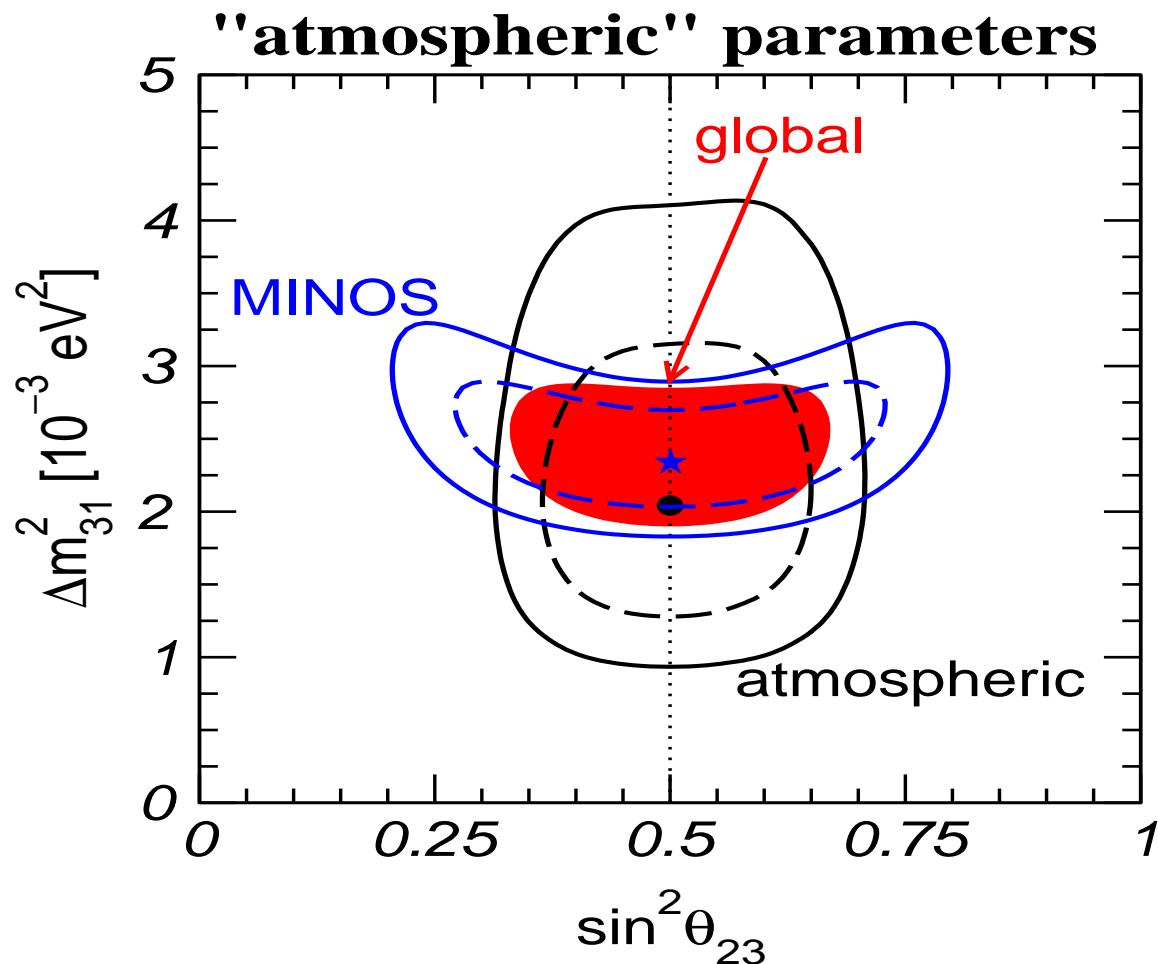
MINOS: ν_μ Spectrum (ν_μ “disappearance”)

MINOS Preliminary









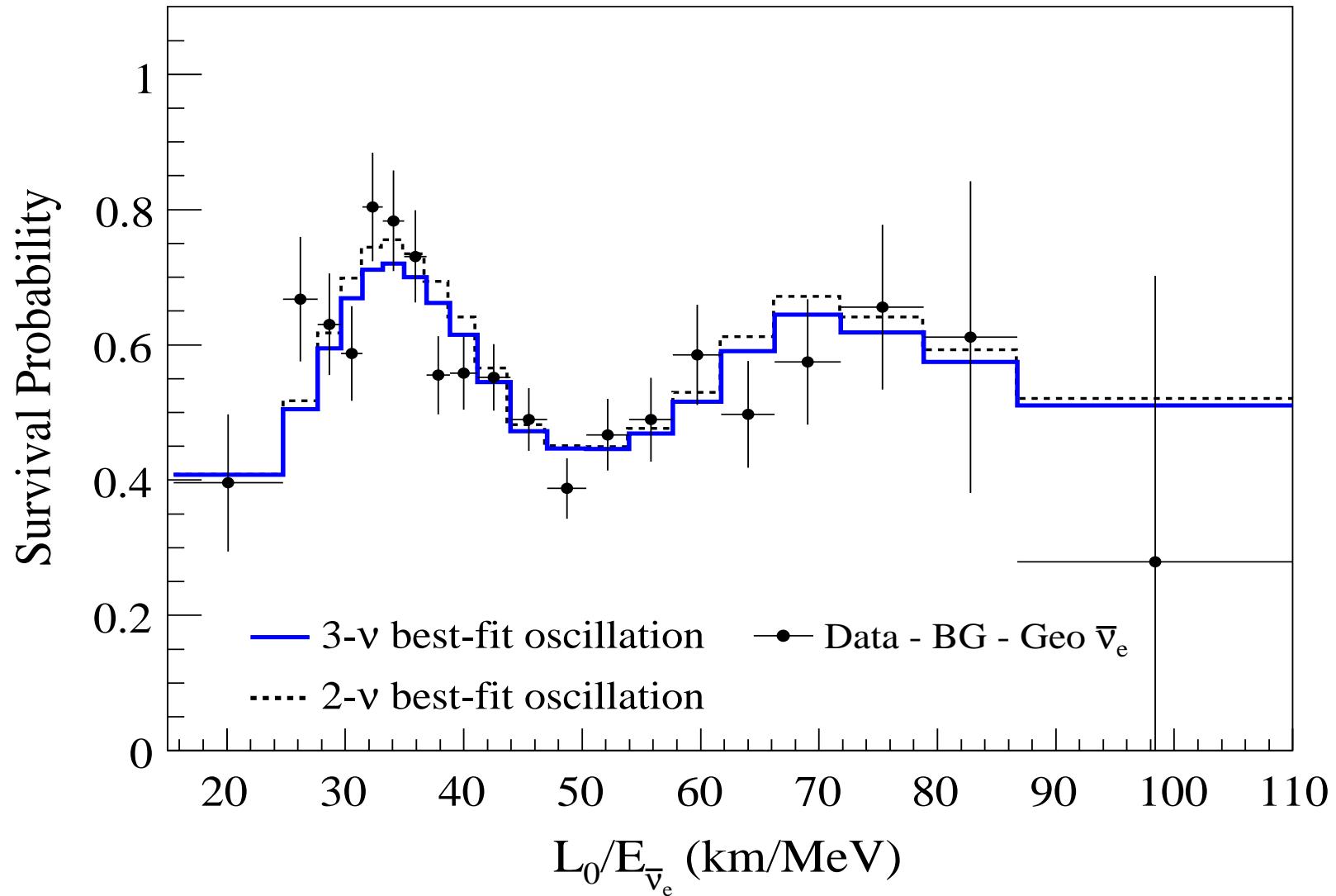
- sign of Δm_{atm}^2 not determined;

T. Schwetz, arXiv:0710.5027[hep-ph]

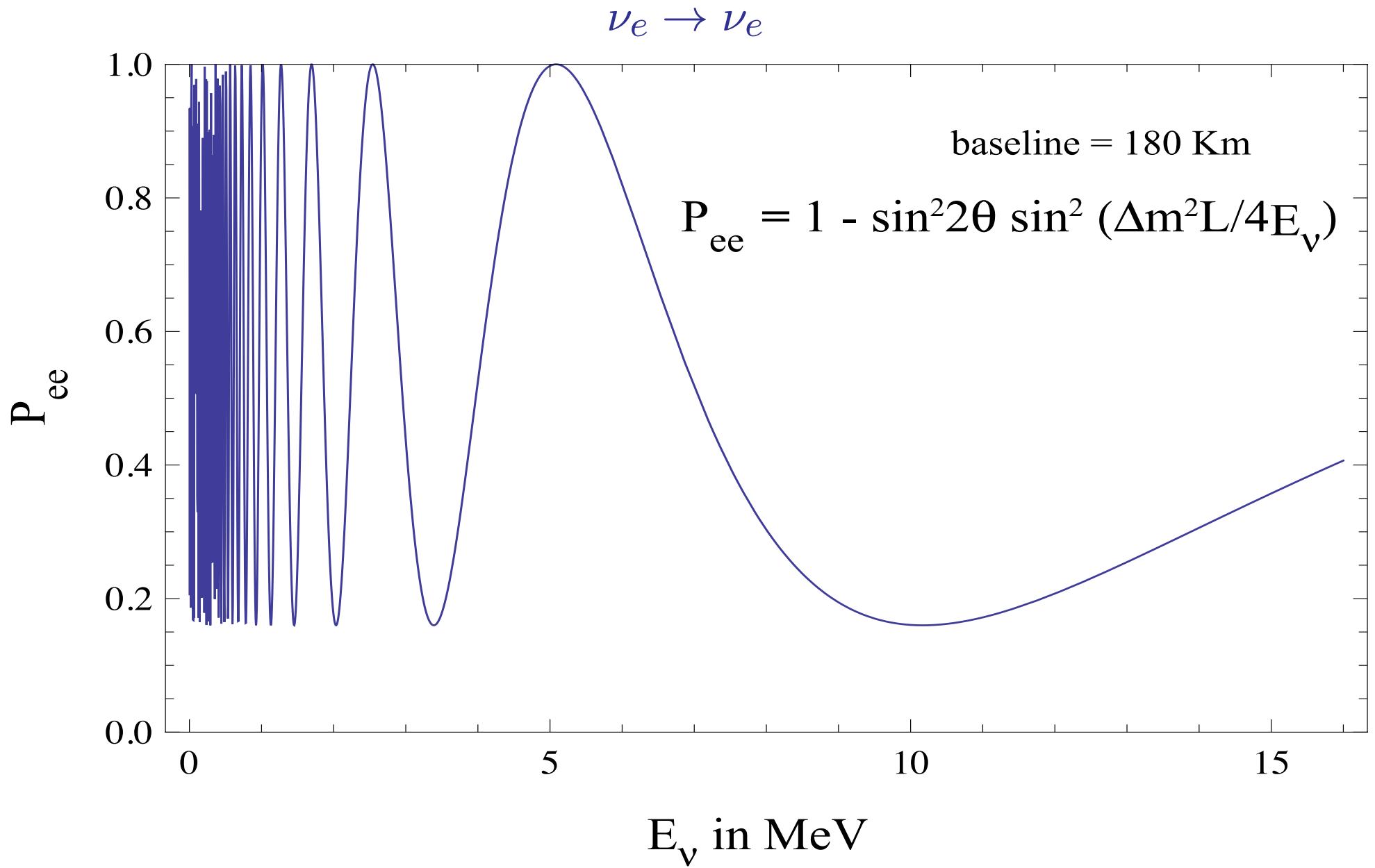
3- ν mixing: $\Delta m_{31}^2 > 0$, $m_1 < m_2 < m_3$ (normal ordering (NO));

$\Delta m_{31}^2 < 0$, $m_3 < m_1 < m_2$ (inverted ordering (IO)).

- If $\theta_{23} \neq \frac{\pi}{4}$: θ_{23} , $(\frac{\pi}{4} - \theta_{23})$ ambiguity.



KamLAND: L/E -Dependence (reactor $\bar{\nu}_e$, $\bar{L} = 180$ km, $E = (1.8 - 10)$ MeV)



Solar Neutrinos: ν_e , $E \sim (0.26 - 14.4)$ MeV

Super-Kamiokande, $E \cong (5.0 - 14.4)$ MeV

$$\begin{aligned} R(SK) &\propto \Phi_E^0(\nu_e) \sum_{l=e,\mu,\tau} P(\nu_e \rightarrow \nu_l) \sigma(\nu_l e^- \rightarrow \nu_l e^-) \\ &= \sigma(\nu_e e^- \rightarrow \nu_e e^-) [\Phi_E^0(\nu_e) P(\nu_e \rightarrow \nu_e) \\ &\quad + \Phi_E^0(\nu_e) (1 - P(\nu_e \rightarrow \nu_e)) \frac{\sigma(\nu_{\mu(\tau)} e^- \rightarrow \nu_{\mu(\tau)} e^-)}{\sigma(\nu_e e^- \rightarrow \nu_e e^-)}] \\ &= \sigma(\nu_e e^- \rightarrow \nu_e e^-) [\Phi_E(\nu_e) + 0.16(\Phi_E(\nu_\mu) + \Phi_E(\nu_\tau))] \end{aligned}$$

$$\begin{aligned} P(\nu_e \rightarrow \nu_e) + P(\nu_e \rightarrow \nu_\mu) + P(\nu_e \rightarrow \nu_\tau) &= 1, \\ \sigma(\nu_\mu e^- \rightarrow \nu_\mu e^-) &= \sigma(\nu_\tau e^- \rightarrow \nu_\tau e^-). \end{aligned}$$

SNO, CC: $E \cong (5.0 - 14.4) \text{ MeV}$



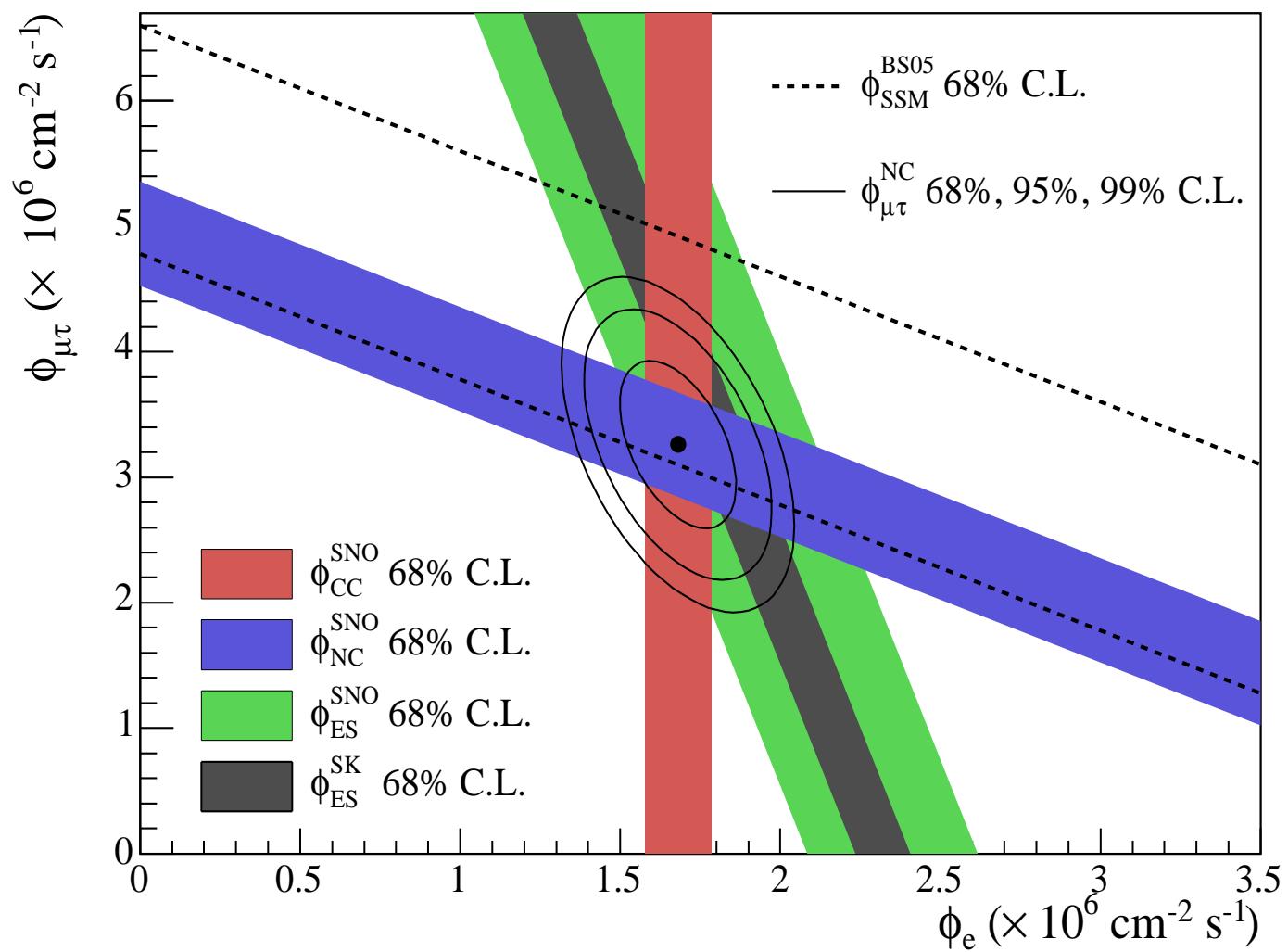
$$R(SNO) \propto \sigma(\nu_e + D \rightarrow e^- + p + p) \Phi_E^0(\nu_e) P(\nu_e \rightarrow \nu_e)$$

$$= \sigma(\nu_e + D \rightarrow e^- + p + p) \Phi_E(\nu_e)$$

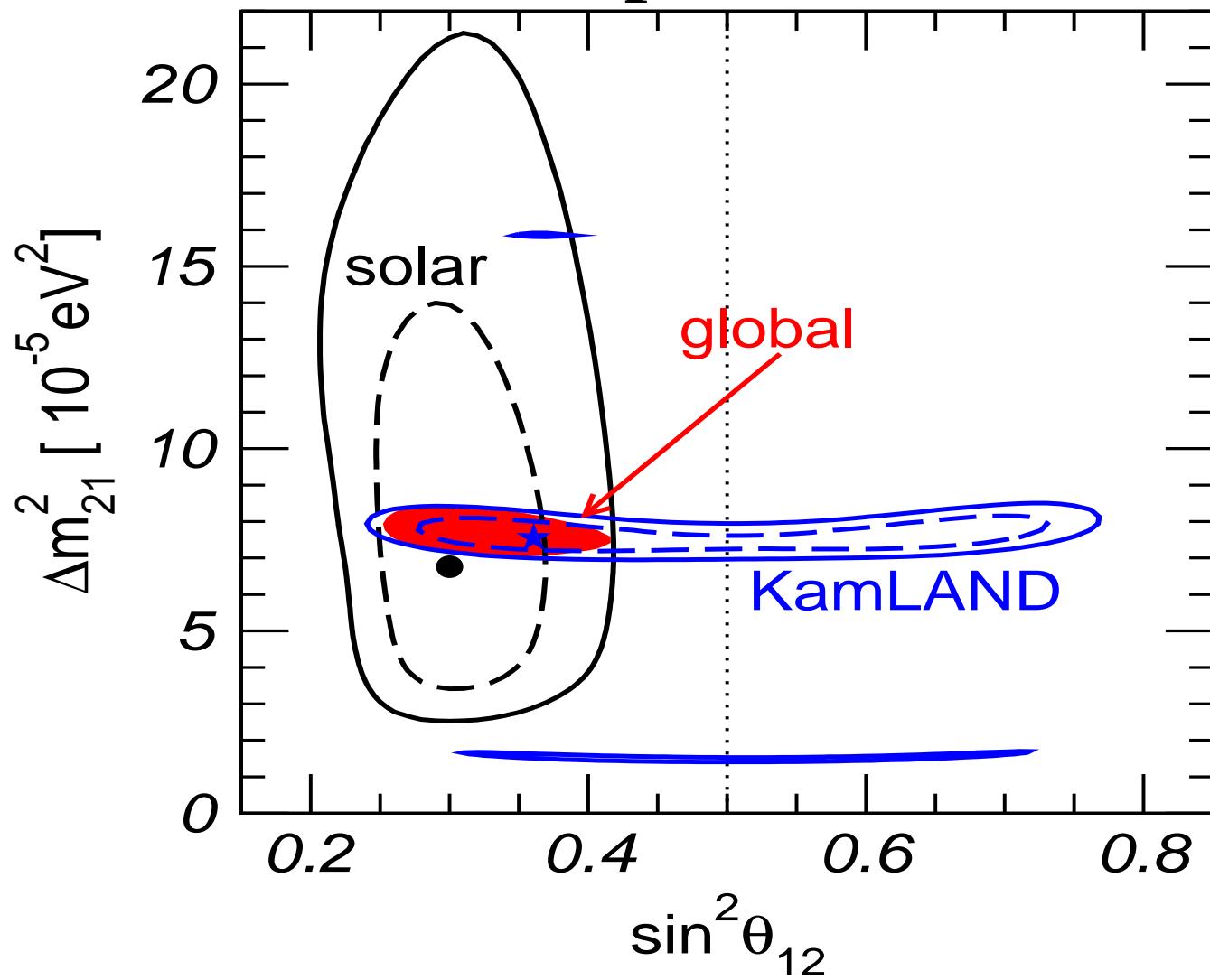
$$SK: \Phi^{SK}(\nu_\odot) = \Phi_E(\nu_e) + 0.16(\Phi_E(\nu_\mu) + \Phi_E(\nu_\tau))$$

$$SNO \text{ CC: } \Phi^{SNO}(\nu_\odot) = \Phi_E(\nu_e)$$

No oscillations: $\Phi_E(\nu_\mu) + \Phi_E(\nu_\tau) = 0$, $\Phi^{SK}(\nu_\odot) = \Phi^{SNO}(\nu_\odot)$



"solar" parameters



T. Schwetz, arXiv:0710.5027[hep-ph]

Leptonic CP Violation

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'}, \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

CP-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' = e, \mu, \tau$$

N. Cabibbo, 1978
S.M. Bilenky, J. Hosek, S.T.P., 1980;
V. Barger, S. Pakvasa et al., 1980.

CPT-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$$

$$l = l': \quad P(\nu_l \rightarrow \nu_l) = P(\bar{\nu}_l \rightarrow \bar{\nu}_l)$$

T-invariance:

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

3 ν -mixing:

$$A_{\text{CP}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}), \quad l \neq l' = e, \mu, \tau$$

$$A_{\text{T}}^{(l,l')} \equiv P(\nu_l \rightarrow \nu_{l'}) - P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

$$A_{\text{T(CP)}}^{(e,\mu)} = A_{\text{T(CP)}}^{(\mu,\tau)} = -A_{\text{T(CP)}}^{(e,\tau)}$$

P.I. Krastev, S.T.P., 1988; V. Barger, S. Pakvasa et al., 1980

3-Neutrino Oscillations in Vacuum

$$|\nu_l\rangle = \sum_{j=1}^n U_{lj}^* |\nu_j\rangle, \quad |\bar{\nu}_l\rangle = \sum_{j=1}^n U_{lj} |\bar{\nu}_j\rangle;$$

$$\nu_j : m_j \neq 0; \quad l = e, \mu, \tau; \quad n = 3.$$

$$A(\nu_l \rightarrow \nu_{l'}) = \sum_j U_{l'j} D_j U_{jl}^\dagger, \quad A(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) = \sum_j U_{l'j}^* D_j U_{jl}^T,$$

$$l, l' = e, \mu, \tau,$$

$D_j = D_j(p_j; L, T)$ describes the propagation of ν_j between the source and the detector, U_{jl}^\dagger (U_{jl}^T) and $U_{l'j}$ ($U_{l'j}^*$) are the amplitudes to find ν_j ($\bar{\nu}_j$) in the initial and in the final flavour neutrino state, respectively. In the plane wave (relativistics) formalism,

$$D_j \equiv D_j(\tilde{p}_j; L, T) = e^{-i\tilde{p}_j(x_f - x_0)} = e^{-i(E_j T - p_j L)}, \quad p_j \equiv |\mathbf{p}_j|,$$

x_0 and x_f - the space-time coordinates of the points of neutrino production and detection, $T = (t_f - t_0)$ and $L = \mathbf{k}(\mathbf{x}_f - \mathbf{x}_0)$, \mathbf{k} being the unit vector in the direction of neutrino momentum, $\mathbf{p}_j = \mathbf{k}p_j$.

$$P(\nu_l \rightarrow \nu_{l'}) = \sum_j |U_{l'j}|^2 |U_{lj}|^2$$

$$+ 2 \sum_{j>k} |U_{l'j} U_{lj}^* U_{lk} U_{l'k}^*| \cos(\frac{\Delta m_{jk}^2}{2p} L - \phi_{l'l;jk}) , \quad l, l' = e, \mu, \tau ,$$

$$P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) = \sum_j |U_{l'j}|^2 |U_{lj}|^2$$

$$+ 2 \sum_{j>k} |U_{l'j} U_{lj}^* U_{lk} U_{l'k}^*| \cos(\frac{\Delta m_{jk}^2}{2p} L + \phi_{l'l;jk}) , \quad l, l' = e, \mu, \tau ,$$

$$\phi_{l'l;jk} = \arg(U_{l'j} U_{lj}^* U_{lk} U_{l'k}^*) .$$

- Spatial localisation condition

ΔL - dimensions of the ν - source (and/or detector):

$$2\pi \Delta L / L_{jk}^v \lesssim 1.$$

- Time localisation condition

ΔE - detector's energy resolution:

$$2\pi (L/L_{jk}^v) (\Delta E/E) \lesssim 1.$$

If $2\pi \Delta L / L_{jk}^v \gg 1$, and/or $2\pi (L/L_{jk}^v) (\Delta E/E) \gg 1$,

$$\bar{P}(\nu_l \rightarrow \nu_{l'}) = \bar{P}(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) \cong \sum_j |U_{l'j}|^2 |U_{lj}|^2$$

In vacuum: $A_{CP(T)}^{(e,\mu)} = J_{CP} F_{osc}^{vac}$

$$J_{CP} = \text{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta$$

$$F_{osc}^{vac} = \sin\left(\frac{\Delta m_{21}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{32}^2}{2E}L\right) + \sin\left(\frac{\Delta m_{13}^2}{2E}L\right)$$

P.I. Krastev, S.T.P., 1988

In matter: Matter effects violate

CP : $P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_l \rightarrow \bar{\nu}_{l'})$

CPT : $P(\nu_l \rightarrow \nu_{l'}) \neq P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l)$

P. Langacker et al., 1987

Can conserve the T-invariance (**Earth**)

$$P(\nu_l \rightarrow \nu_{l'}) = P(\nu_{l'} \rightarrow \nu_l), \quad l \neq l'$$

In matter with constant density: $A_T^{(e,\mu)} = J_{CP}^{\text{mat}} F_{osc}^{\text{mat}}$

$$J_{CP}^{\text{mat}} = J_{CP}^{\text{vac}} R_{CP}$$

R_{CP} does not depend on θ_{23} and δ ; $|R_{CP}| \lesssim 2.5$

P.I. Krastev, S.T.P., 1988

Neutrino Oscillations in Matter

When neutrinos propagate in matter, they interact with the background of electrons, protons and neutrinos, which generates an effective potential in the neutrino Hamiltonian: $H = H_{vac} + V_{eff}$.

This modifies the neutrino mixing since the eigenstates and the eigenvalues of H_{vac} and of $H = H_{vac} + V_{eff}$ are different, leading to a different oscillation probability w.r.t to that in vacuum.

Typically the matter background is not CP and CPT symmetric, e.g., the Earth and the Sun contain only electrons, protons and neutrons, and the resulting oscillations violate CP and CPT symmetries.

$$P_{3\nu}(\nu_\mu \rightarrow \nu_e) \cong \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta M_{31}^2 L}{4E}$$

$\sin^2 2\theta_{13}^m$, ΔM_{31}^2 depend on the matter potential
 $V_{eff} = \sqrt{2} G_F N_e$,

For antineutrinos V_{eff} has the opposite sign:

$$V_{eff} = -\sqrt{2} G_F N_e.$$

$\Delta m_{31}^2 > 0$ (NO): $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ matter enhanced,
 $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ - suppressed

$\Delta m_{31}^2 < 0$ (IO): $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ matter enhanced,
 $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ - suppressed

$$\sin^2 2\theta_{13}^m = \frac{\tan^2 2\theta_{13}}{(1 - \frac{N_e}{N_e^{res}})^2 + \tan^2 2\theta_{13}} (= 1, \text{ if } N_e = N_e^{res}),$$

$$\cos 2\theta_{13}^m = \frac{1 - N_e/N_e^{res}}{\sqrt{(1 - \frac{N_e}{N_e^{res}})^2 + \tan^2 2\theta_{13}}},$$

$$N_e^{res} = \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2E\sqrt{2}G_F},$$

$$N_e^{res} \cong 6.56 \times 10^6 \frac{\Delta m^2 [\text{eV}^2]}{E [\text{MeV}]} \cos 2\theta \text{ cm}^{-3} \text{ N}_A,$$

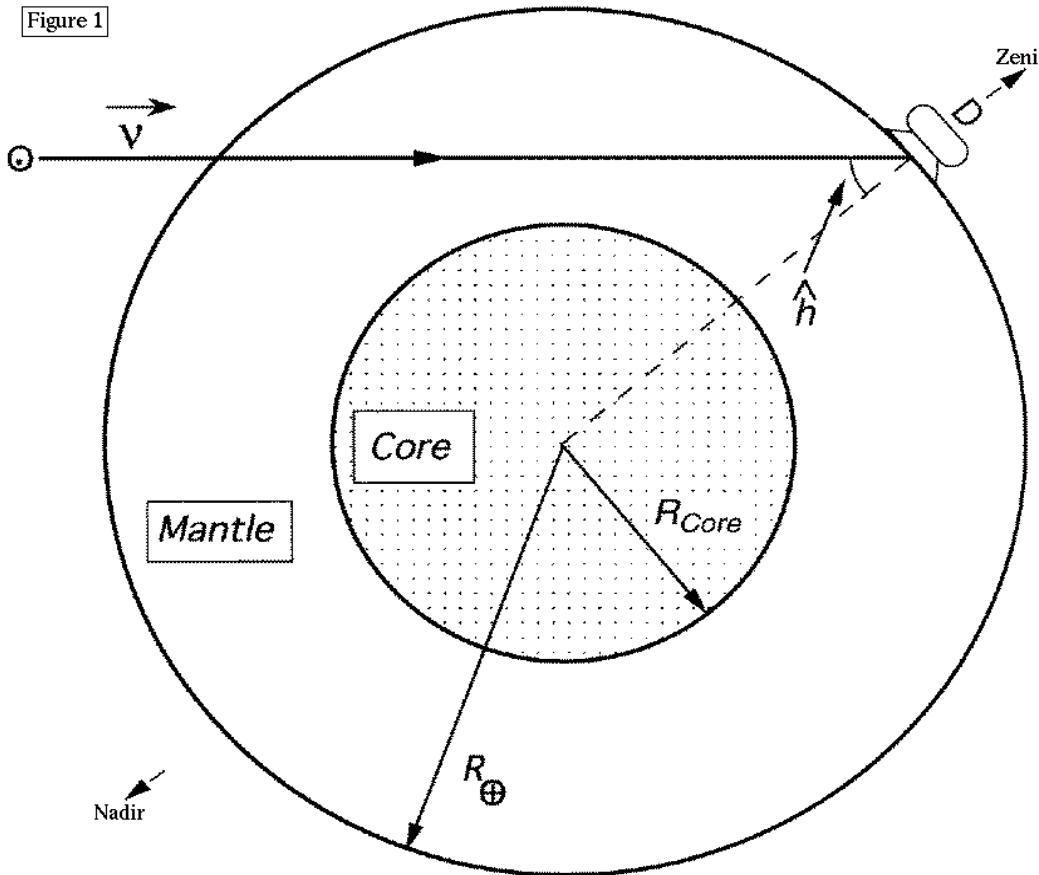
$$\frac{\Delta M_{31}^2}{2E} \equiv \frac{\Delta m_{31}^2}{2E} \left((1 - \frac{N_e}{N_e^{res}})^2 \cos^2 2\theta_{13} + \sin^2 2\theta_{13} \right)^{\frac{1}{2}}$$

For $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$: $N_e \rightarrow (-N_e)$.

$\Delta m_{31}^2 > 0$ (NO): $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ matter enhanced,
 $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ - suppressed

$\Delta m_{31}^2 < 0$ (IO): $\bar{\nu}_{\mu(e)} \rightarrow \bar{\nu}_{e(\mu)}$ matter enhanced,
 $\nu_{\mu(e)} \rightarrow \nu_{e(\mu)}$ - suppressed

The Earth



Earth: $R_{core} = 3446 \text{ km}$, $R_{mant} = 2885 \text{ km}$

Earth: $\bar{N}_e^{mant} \sim 2.3 \text{ } N_A \text{ cm}^{-3}$, $\bar{N}_e^{core} \sim 5.7 \text{ } N_A \text{ cm}^{-3}$

The Earth

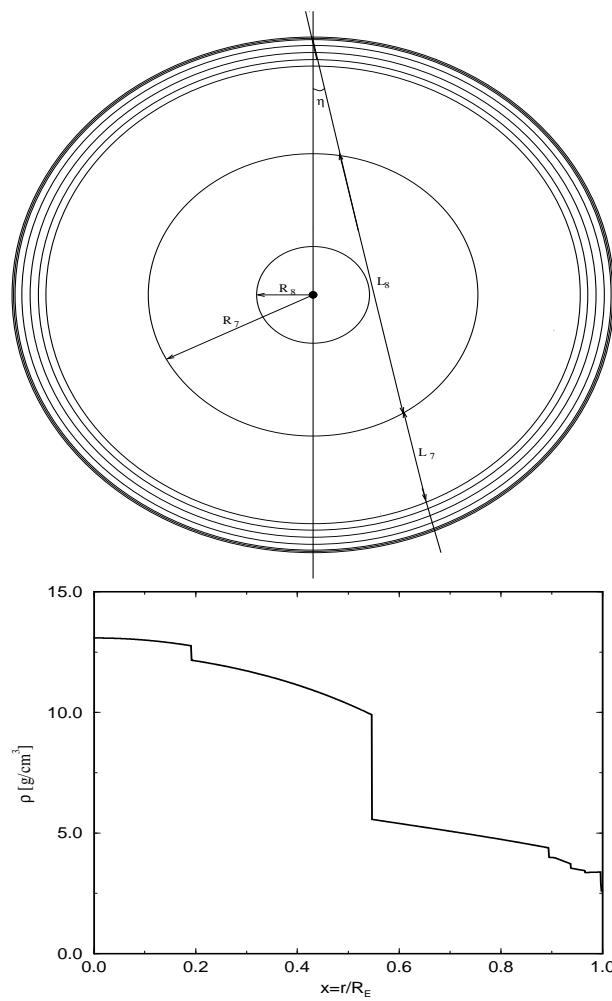
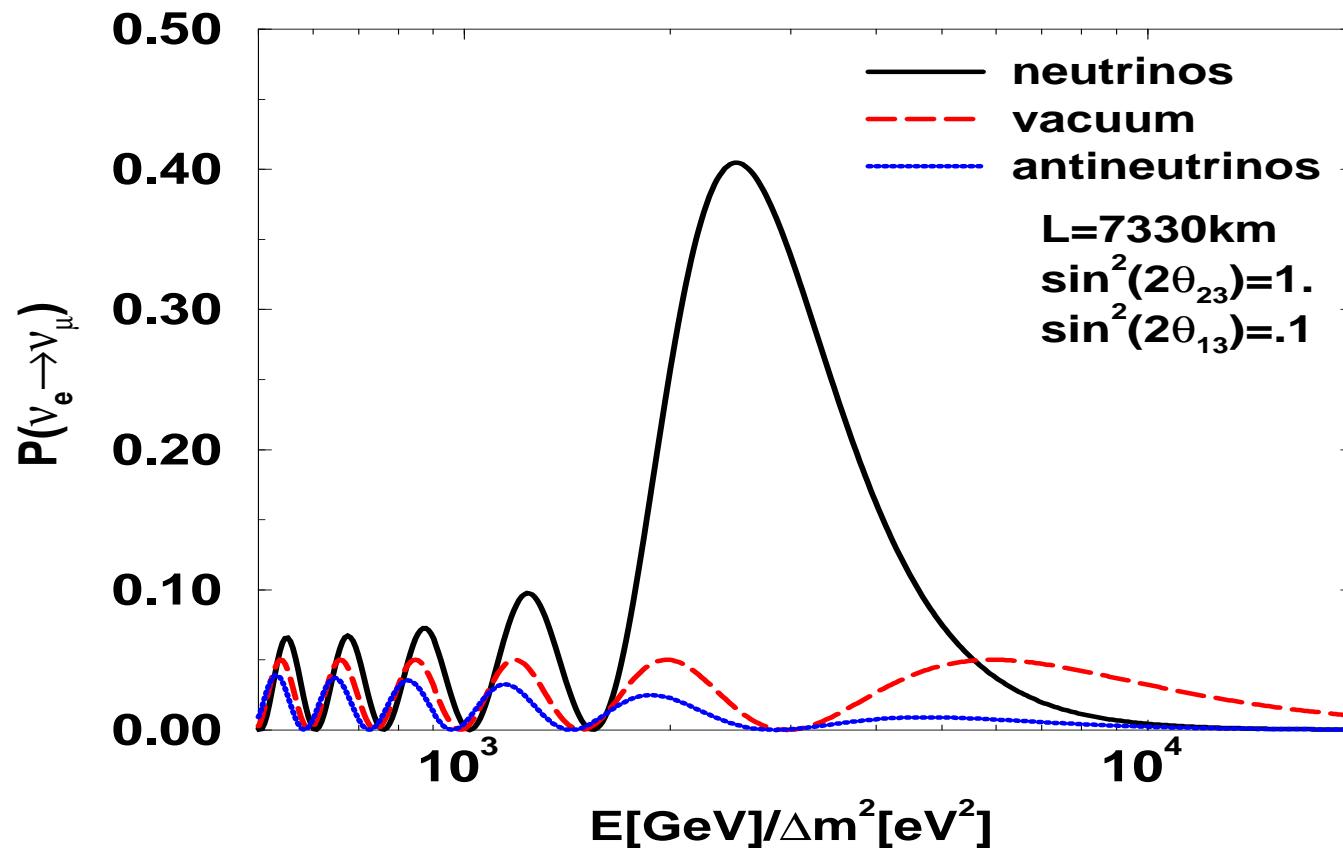


FIG. 1. Density profile of the Earth.

$$R_c = 3446 \text{ km}, R_m = 2885 \text{ km}; \bar{N}_e^{mant} \sim {}^{16}2.3 N_A \text{ cm}^{-3}, \bar{N}_e^{core} \sim 5.7 N_A \text{ cm}^{-3}$$

Earth matter effect in $\nu_\mu \rightarrow \nu_e$, $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (MSW)



$$\Delta m^2 = 2.5 \times 10^{-3} \text{ eV}^2, E^{res} = 6.25 \text{ GeV}; P^{3\nu} = \sin^2 \theta_{23} P_m^{2\nu} = 0.5 P_m^{2\nu}; N_e^{res} \cong 2.3 \text{ cm}^{-3} \text{ N_A}; L_m^{res} = L^v / \sin 2\theta_{13} \cong 6250 / 0.32 \text{ km}; 2\pi L / L_m \cong 0.75\pi (\neq \pi).$$

Earth mantle: up to 2nd order in $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \cong \frac{1}{30}$ and $\sin^2 \theta_{13} \cong 0.0214$:

$$P_m^{3\nu \text{ man}}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3 ,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{\text{man}} \frac{2E}{\Delta m_{31}^2} \cong \frac{N_e^{\text{man}}}{N_e^{\text{res}}}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

Rephasing Invariants Associated with CPVP

Dirac phase δ :

$$J_{CP} = \text{Im} \left\{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \right\} .$$

C. Jarlskog, 1985 (for quarks)

CP-, T- violation effects in neutrino oscillations

P. Krastev, S.T.P., 1988

Majorana phases α_{21} , α_{31} :

$$S_1 = \text{Im} \left\{ U_{e1} U_{e3}^* \right\}, \quad S_2 = \text{Im} \left\{ U_{e2} U_{e3}^* \right\} \quad (\text{not unique}); \quad \text{or}$$

$$S'_1 = \text{Im} \left\{ U_{\tau 1} U_{\tau 2}^* \right\}, \quad S'_2 = \text{Im} \left\{ U_{\tau 2} U_{\tau 3}^* \right\}$$

J.F. Nieves and P. Pal, 1987, 2001

G.C. Branco et al., 1986

J.A. Aguilar-Saavedra and G.C. Branco, 2000

CP-violation: both $\text{Im} \left\{ U_{e1} U_{e3}^* \right\} \neq 0$ and $\text{Re} \left\{ U_{e1} U_{e3}^* \right\} \neq 0$.

S_1 , S_2 appear in $| \langle m \rangle |$ in $(\beta\beta)_{0\nu}$ -decay.

In general, J_{CP} , S_1 and S_2 are independent.

$$\delta \cong 3\pi/2?$$

$$\begin{aligned}
J_{CP} &= \text{Im} \{ U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^* \} \\
&= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta
\end{aligned}$$

Status and prospects of global analyses of neutrino mass-mixing parameters

A. Marrone
Univ. of Bari & INFN



4-9 July 2016 — London — United Kingdom

Our "pre-London" reference analysis in the standard 3ν mixing scenario:

Bari group, arXiv:1601.07777 (NPB Special Issue on ν Oscillations)

Updated in a preliminary way with some data presented here at Neutrino 2016 (thanks in particular to F. Capozzi):

- New NOVA neutrino data in appearance and disappearance channels
- New T2K anti-neutrino data in appearance channel

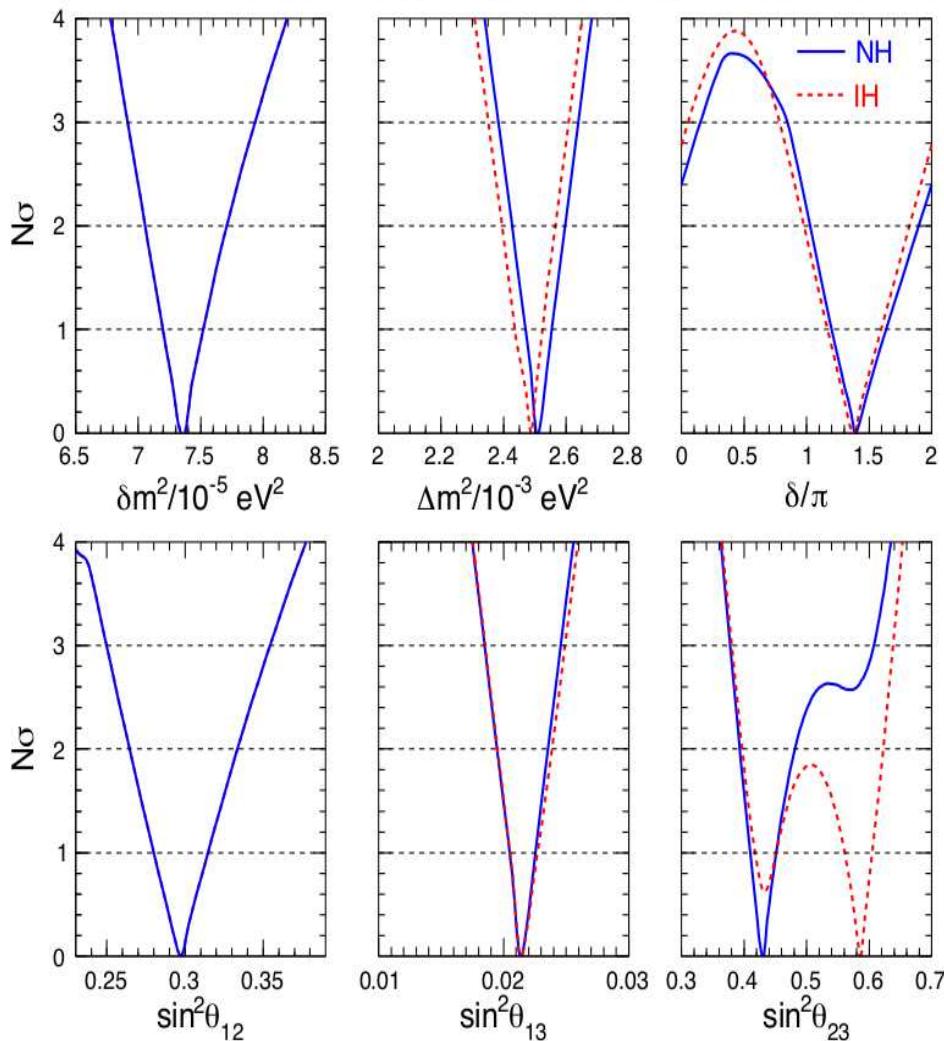
Other updates not (yet) included

Please focus only on "trends" of the global analysis: numbers may change when a more refined and proper analysis of the new data will be performed in due time

Bounds on single oscillation parameters

(preliminary update)

LBL Acc + Solar + KamLAND + SBL Reactors + Atmos



CP phase trend:

- $\delta \sim 1.4\pi$ at best fit
- CP-conserving cases ($\delta = 0, \pi$) disfavored at $\sim 2\sigma$ level or more
- Significant fraction of the $[0, \pi]$ range disfavored at $> 3\sigma$

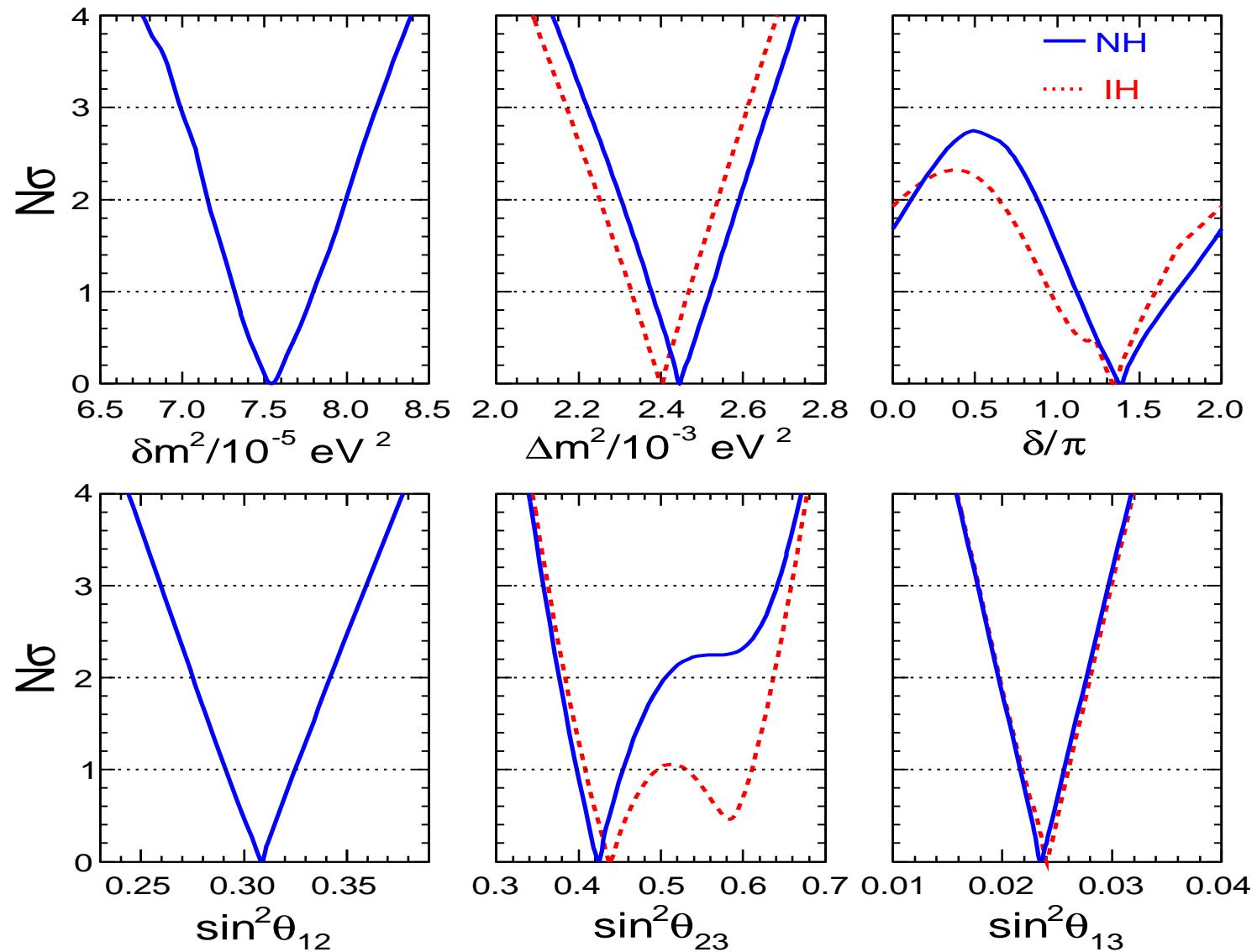
θ_{23} trend:

- maximal mixing disfavored at about $\sim 2\sigma$ level
- best-fit octant flips with mass ordering

$$\Delta\chi^2_{\text{IO-NO}} = 3.1$$

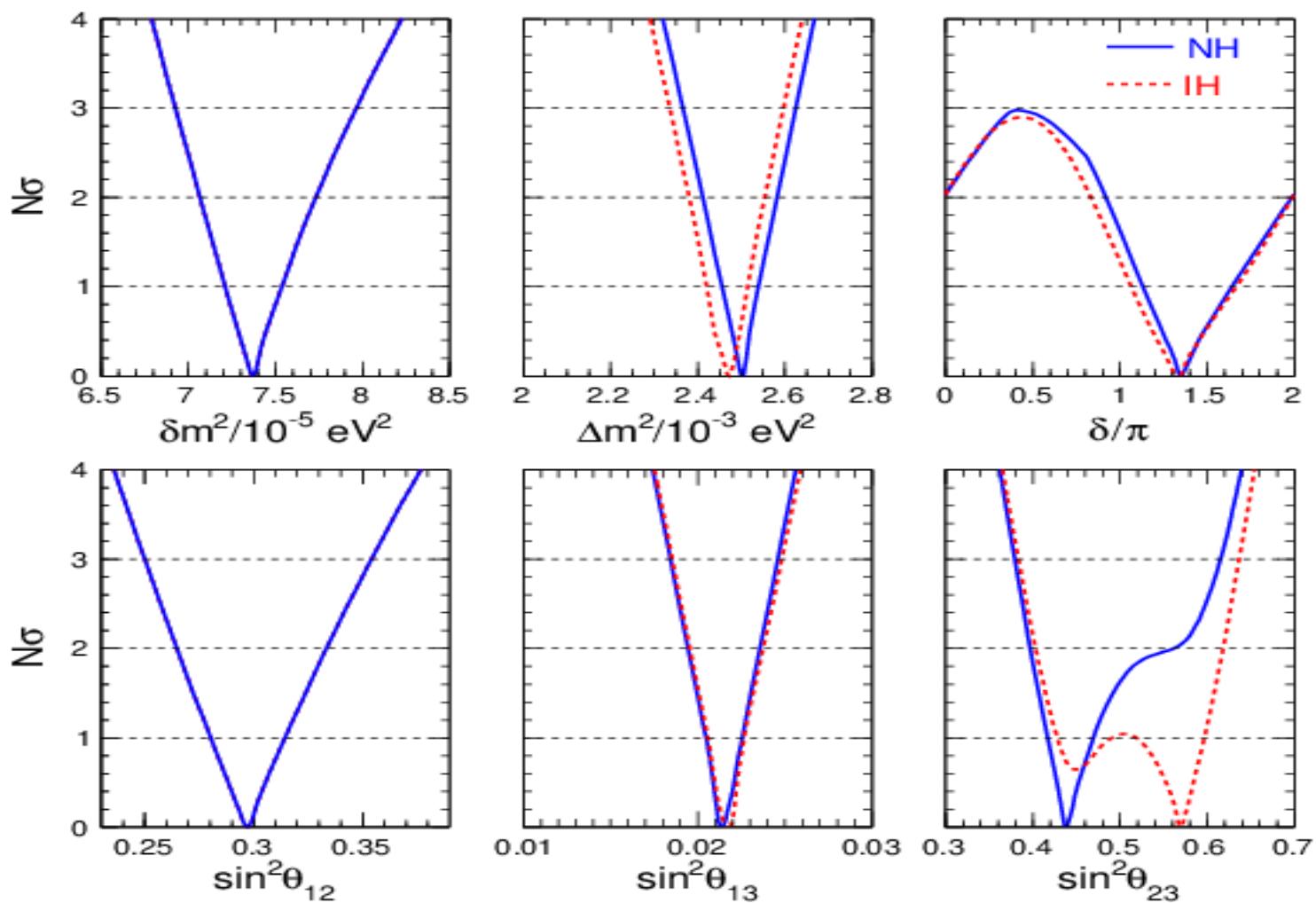
inverted ordering slightly disfavored

LBL Acc + Solar + KL + SBL Reactors + SK Atm

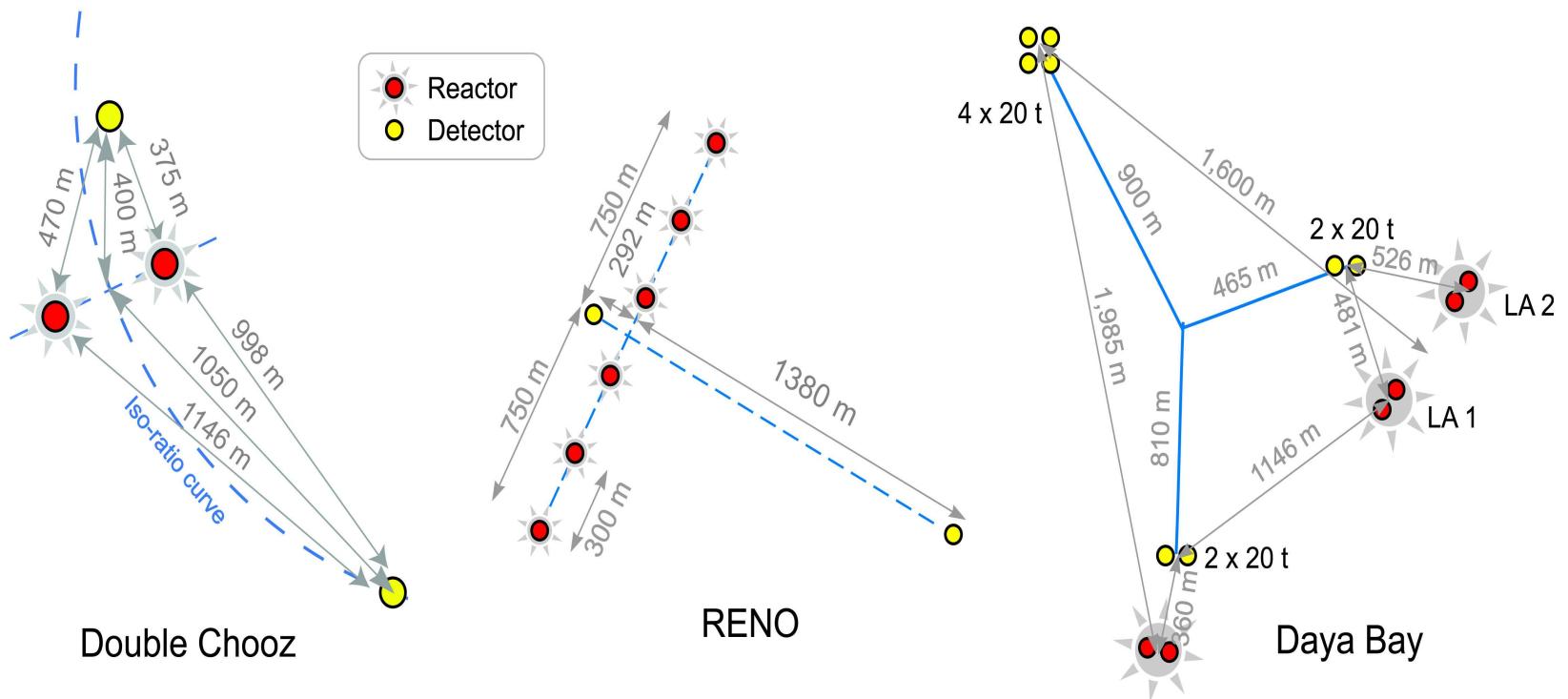


F. Capozzi, E. Lisi *et al.*, arXiv:1312.2878

LBL Acc + Solar + KamLAND + SBL Reactors + Atmos



F. Capozzi, E. Lisi *et al.*, arXiv:1601.01777v1



M. Mezzetto, T. Schwetz, arXiv:1003.5800[hep-ph]





LBL Oscillation Experiments NO ν A, T2K (Detector=SuperKamiokande (SK))

T2K: Tokai - Kamioka; off-axis ν , $\bar{\nu}_\mu$ beams, $E \cong 0.6$ GeV, $L \cong 295$ km, SK (50 kt water Cherenkov).

NO ν A: Fermilab - site in Minnesota; off-axis ν beam, $E = 2$ GeV, $L \cong 810$ km, 14 kt liquid scintillator; 2014.

- November 11, 2011, Double Chooz: 2σ evidence for $\theta_{13} \neq 0$.
- March 8, 2012, Daya Bay: 5.2σ evidence for $\theta_{13} \neq 0$,
 $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$.
- April 4, 2012, RENO: 4.9σ evidence for $\theta_{13} \neq 0$,
 $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$.
- Daya Bay, May 2015:
 $\sin^2 2\theta_{13} = 0.084 \pm 0.005$.
- RENO, April 2015 (WIN 2015):
 $\sin^2 2\theta_{13} = 0.087 \pm 0.008$ (*stat.*) ± 0.008 .
- Double Chooz, 2014:
 $\sin^2 2\theta_{13} = 0.090^{+0.032}_{-0.029}$ ($0.092^{+0.033}_{-0.029}$).

$$P^{3\nu}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P^{3\nu}(\theta_{13}, \Delta m^2_{31(32)}; \theta_{12}, \Delta m^2_{21}) \cong$$
$$1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31(32)}}{4E} L\right), \text{ no dependence on } \theta_{23}, \delta.$$

T2K: Search for $\nu_\mu \rightarrow \nu_e$ oscillations

T2K: first results March 2011 (2 events);
June 14, 2011 (6 events): evidence for $\theta_{13} \neq 0$ at 2.5σ ;
July, 2013 (28 events).

For $|\Delta m_{23}^2| = 2.4 \times 10^{-3}$ eV 2 , $\sin^2 2\theta_{23} = 1$, $\delta = 0$, NO
(IO) spectrum:

$\sin^2 2\theta_{13} = 0.14$ (1.7), best fit.

This value is by a factor of ~ 1.6 (1.9) bigger than the value obtained in the Daya Bay and RENO experiments.

$$P_m^{3\nu}(\nu_\mu \rightarrow \nu_e) = P_m^{3\nu}(\theta_{13}, \Delta m_{31(32)}^2, \theta_{12}, \Delta m_{21}^2, \theta_{23}, \delta).$$

T2K, NO ν A: up to 2nd order in $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \cong \frac{1}{30}$ and $\sin^2 \theta_{13} \cong 0.0214$:

$$P_m^{3\nu \text{ man}}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3 ,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{\text{man}} \frac{2E}{\Delta m_{31}^2} \cong \frac{N_e^{\text{man}}}{N_e^{\text{res}}}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

The results on $\nu_\mu \rightarrow \nu_e$ oscillations from NO ν A are compatible with, and strengthened, the hint that $\delta \cong 3\pi/2$ (A. Marone, talk at Nu2016 (July 8); F. Capozzi *et al.*, arXiv:1601.01777v1).

Large $\sin \theta_{13} \cong 0.15 + \delta = 3\pi/2$ - far-reaching implications:

- For the searches for CP violation in ν -oscillations; for the b.f.v. one has $J_{CP} \cong -0.035$;
- Important implications also for the “flavoured” leptogenesis scenario of generation of the baryon asymmetry of the Universe (BAU).

If all CPV, necessary for the generation of BAU is due to δ , a necessary condition for reproducing the observed BAU is

$$|\sin \theta_{13} \sin \delta| \gtrsim 0.09$$

S. Pascoli, S.T.P., A. Riotto, 2006.

Future LBL Neutrino Oscillation Experiments on $\text{sgn}(\Delta m_{31}^2)$ and CP Violation

LBL Oscillation Experiments T2K, NO ν A, DUNE, T2HK (HK=Hyper-Kamiokande: water-Cherenkov, ~ 1 Mton, fiducial ~ 0.5 Mton).

NO ν A: Fermilab - site in Minnesota; off-axis ν beam, $E = 2$ GeV, $L \cong 810$ km, 14 kt liquid scintillator; 2014.

T2HK: $L = 295$ km, 2.5° off-axis (narrow band) ν_μ beam (from 750 kW proton) beam, maximum at $E \cong 0.6$ GeV (the first osc. maximum).

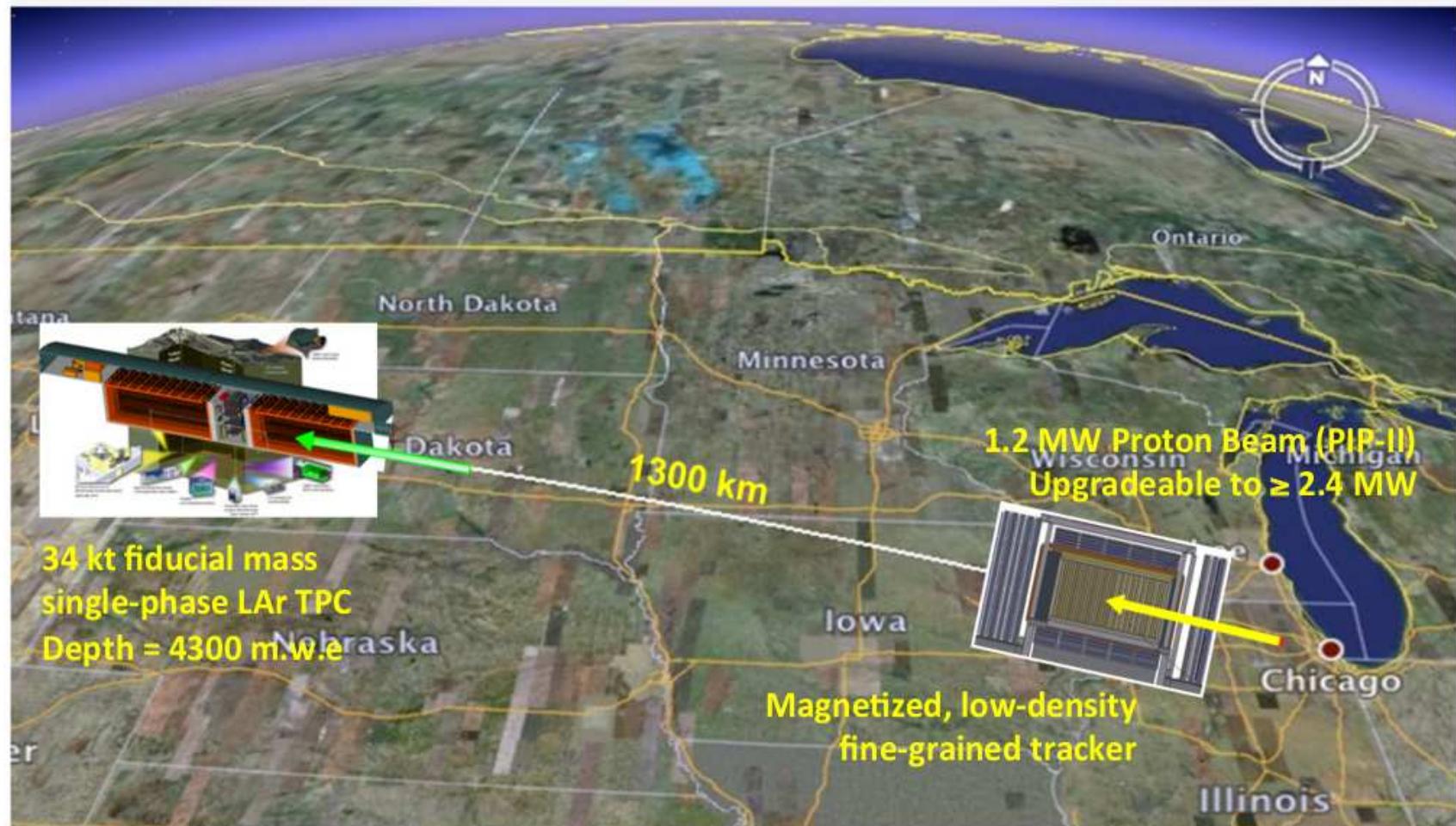
DUNE: Fermilab-DUSEL, $L = 1290$ km, 1.2 MW (2.3 MW) proton beam, wide band ν beam (first and second osc. maxima at $E = 2.4$ GeV and 0.8 GeV); 34 kt fiducial volume LAr detectors; plans to run 5 years with ν_μ and 5 years with $\bar{\nu}_\mu$; 2025 (?)



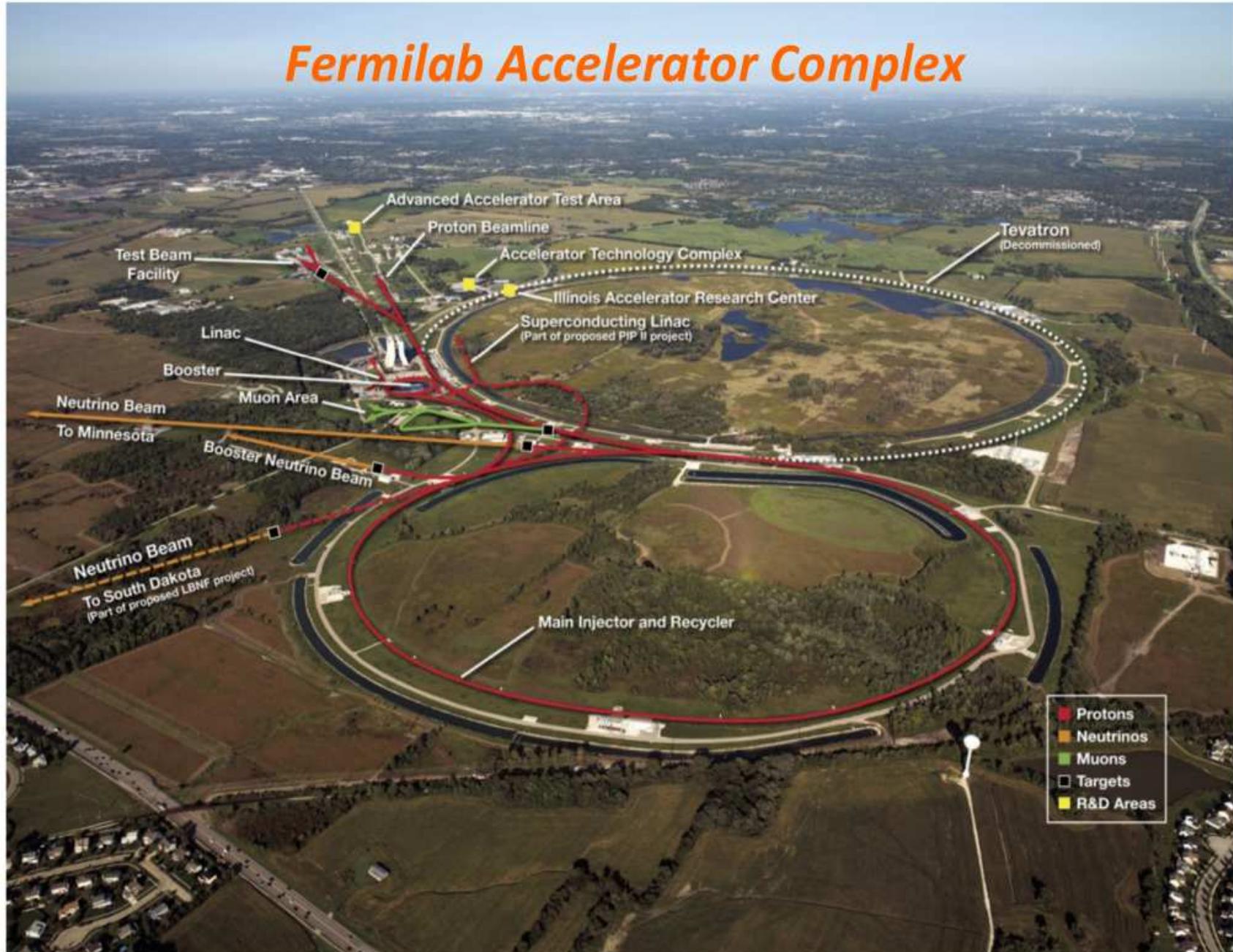




LBNE Design



Fermilab Accelerator Complex





LBNE Parameters



- Wide band neutrino beam from FNAL
 - protons: 60-120 GeV, 1.2 MW; upgradable to 2.3 MW
 - $10 \mu\text{s}$ pulses every 1.0 to 1.33 sec depending on P energy&power.
 - Neutrinos: sign selected, horn focused, 0.5 - 5 GeV
 - 1300 km thru the Earth to Sanford Underground Research Facility.
- Liquid argon TPC parameters
 - 34 kt fiducial (50kt tot) at 4850 ft level. cosmics $\sim 0.1\text{Hz}$, beam $\sim 9\text{k CC/yr}$
 - drift ~ 3.5 m, field: 500 V/cm, 2 mods = (14m(H)X 22m(W)X45m(L))
 - readout: x,u,v, pitch: 5 mm, wrapped wires, 2X108 APAs, 2X(275k ch)
 - Max Yield: $\sim 9000 \text{ e/mm/MIP}$, 10000 ph/mm/MIP
- near detector parameters
 - distance ~ 450 m, $\sim 3\text{M events/ton/MW/yr}$
 - Magnetized Fine Grained Tracker (8 ton) with ECAL, and muon id.
 - Supplemented by a small LARTPC (few tons) or gas TPC.

Scale of project is dictated by physics. Beam and ND and FD detectors require high technology. Project can be done in phases with international partners.

Up to 2nd order in the two small parameters $|\alpha| \equiv |\Delta m_{21}^2|/|\Delta m_{31}^2| \ll 1$ and $\sin^2 \theta_{13} \ll 1$:

$$P_m^{3\nu \text{ man}}(\nu_\mu \rightarrow \nu_e) \cong P_0 + P_{\sin \delta} + P_{\cos \delta} + P_3,$$

$$P_0 = \sin^2 \theta_{23} \frac{\sin^2 2\theta_{13}}{(A-1)^2} \sin^2[(A-1)\Delta],$$

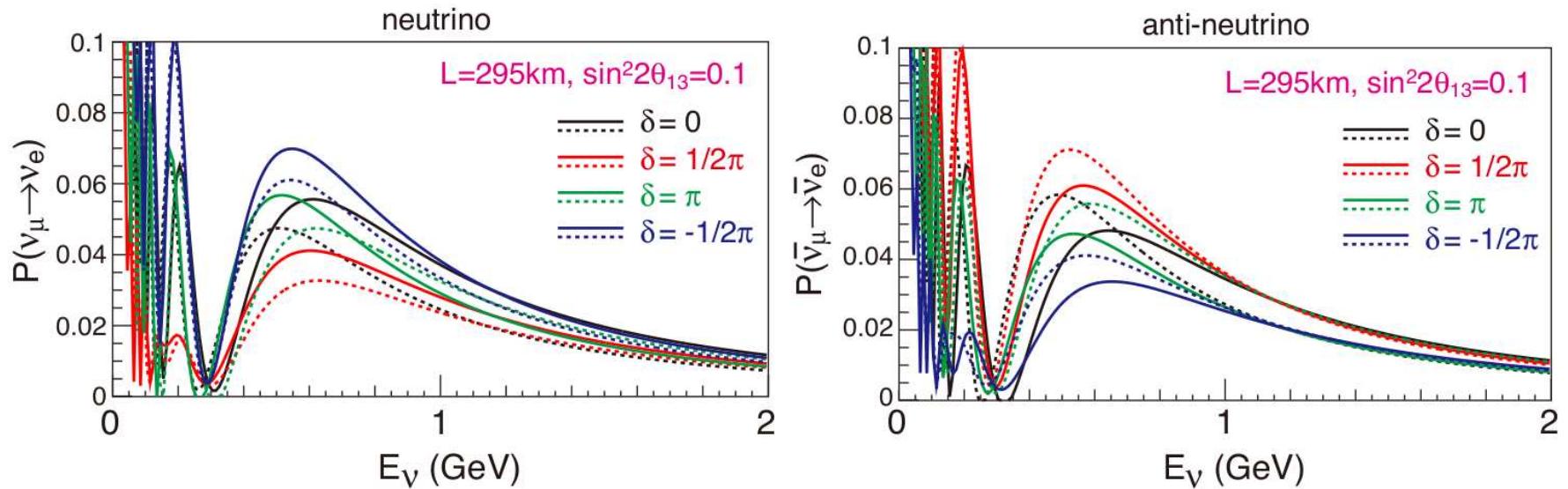
$$P_3 = \alpha^2 \cos^2 \theta_{23} \frac{\sin^2 2\theta_{12}}{A^2} \sin^2(A\Delta),$$

$$P_{\sin \delta} = -\alpha \frac{8 J_{CP}}{A(1-A)} (\sin \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

$$P_{\cos \delta} = \alpha \frac{8 J_{CP} \cot \delta}{A(1-A)} (\cos \Delta) (\sin A\Delta) (\sin[(1-A)\Delta]),$$

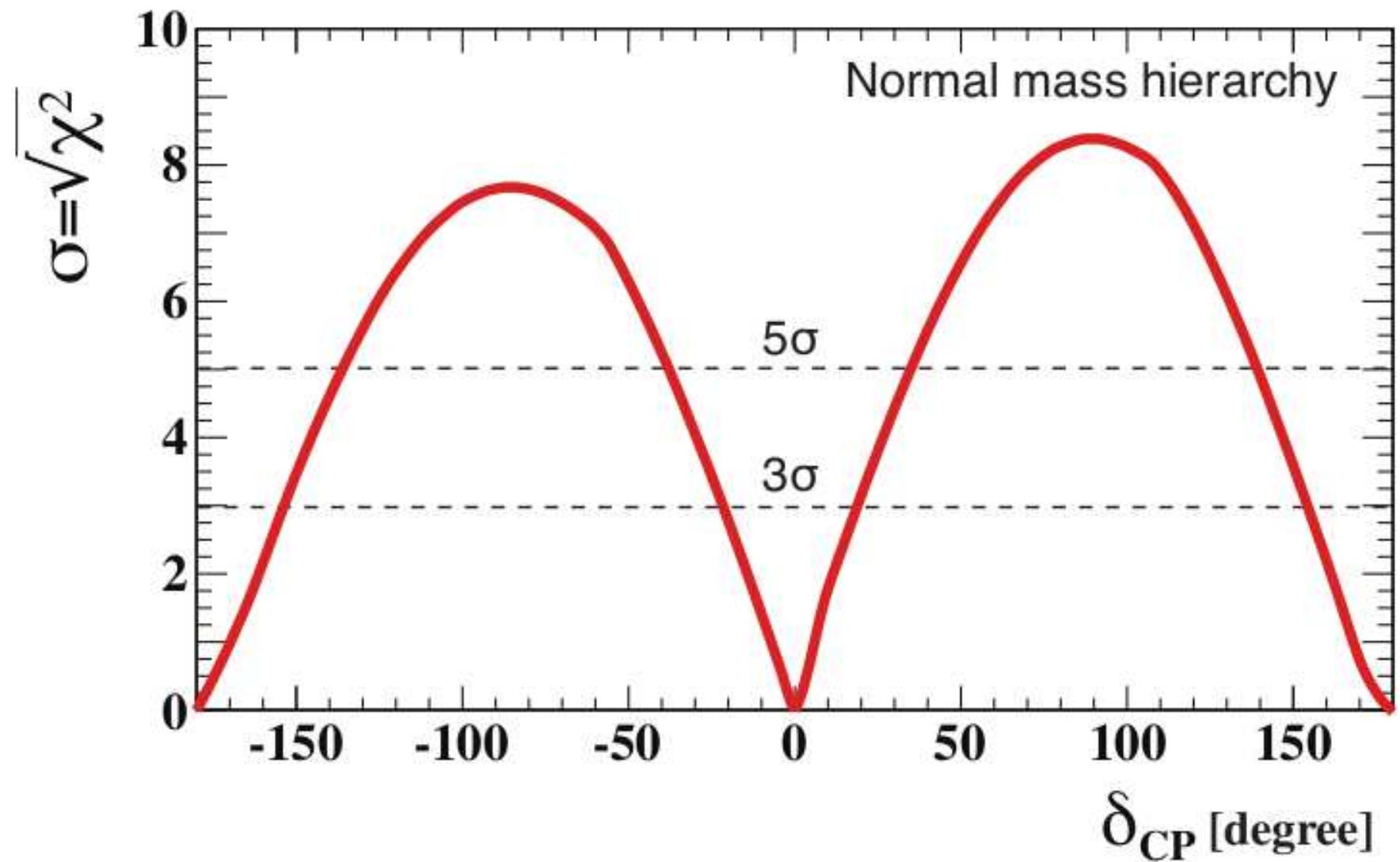
$$\Delta = \frac{\Delta m_{31}^2 L}{4E}, \quad A = \sqrt{2} G_F N_e^{\text{man}} \frac{2E}{\Delta m_{31}^2}.$$

$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e: \delta, \quad A \rightarrow (-\delta), \quad (-A)$$

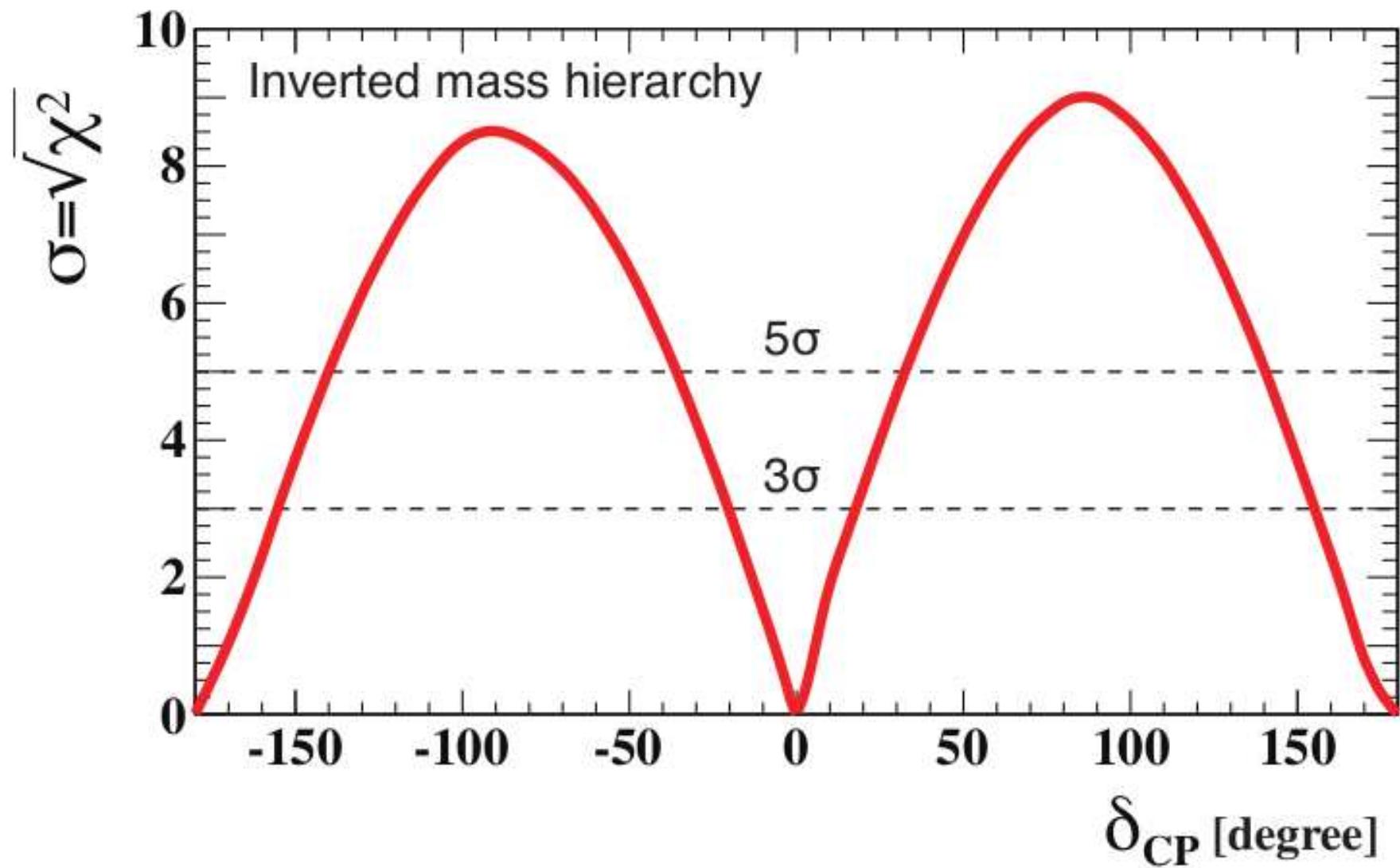


The $\nu_\mu \rightarrow \nu_e$ (left) and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ (right) oscillation probabilities as a function of the neutrino energy for $L=295$ km (T2K, T2HK) and $\sin^2 2\theta_{13} = 0.1$. Black, red, green, and blue lines correspond to $\delta = 0, \frac{1}{2}\pi, \pi$, and $-\frac{1}{2}\pi$, respectively. Solid (dashed) line represents the case for a NO (IO) mass spectrum.

Expected T2HK sensitivity to CP violation



K. Abe et al., arXiv:1502.05199.

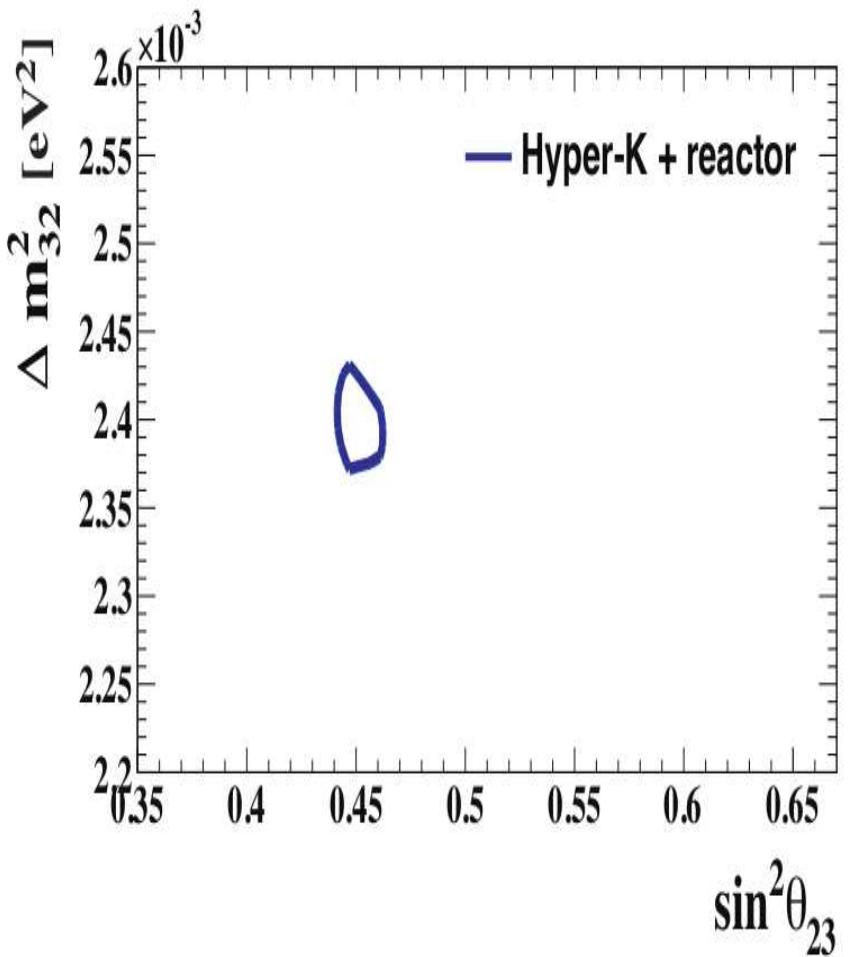
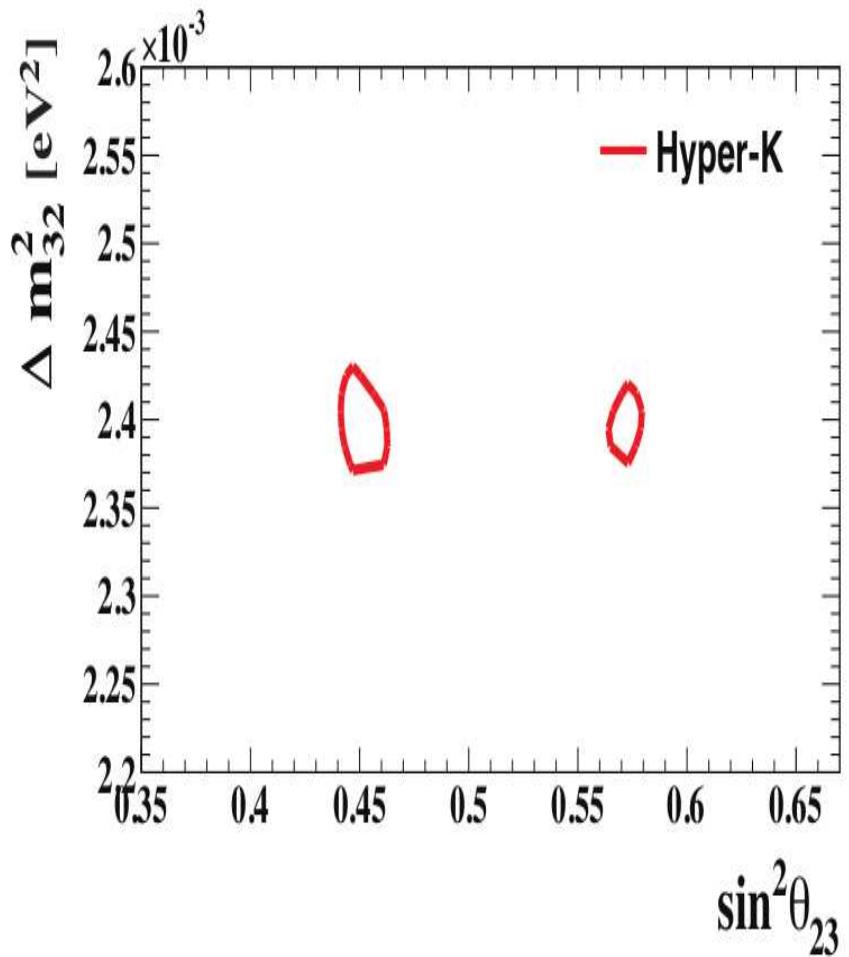


K. Abe et al., arXiv:1502.05199.

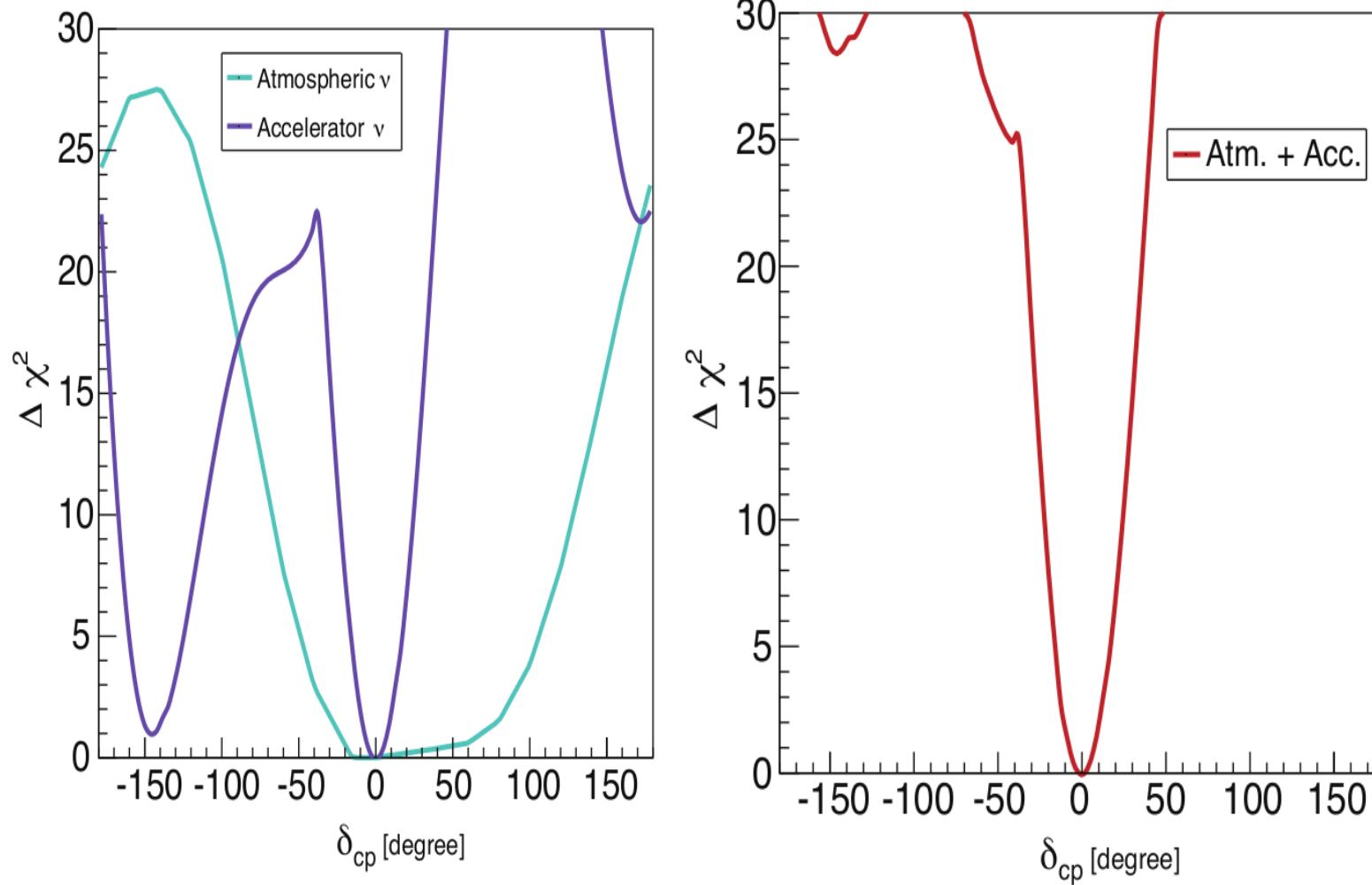
Expected 1σ uncertainty of Δm_{32}^2 and $\sin^2 \theta_{23}$ for true $\sin^2 \theta_{23} = 0.45, 0.50, 0.55$.
 Reactor constraint on $\sin^2 2\theta_{13} = 0.1 \pm 0.005$ is imposed.

True $\sin^2 \theta_{23}$	0.45	0.50	0.55			
Parameter	Δm_{32}^2 (eV 2)	$\sin^2 \theta_{23}$	Δm_{32}^2 (eV 2)	$\sin^2 \theta_{23}$	Δm_{32}^2 (eV 2)	$\sin^2 \theta_{23}$
NO	1.4×10^{-5}	0.006	1.4×10^{-5}	0.015	1.5×10^{-5}	0.009
IO	1.5×10^{-5}	0.006	1.4×10^{-5}	0.015	1.5×10^{-5}	0.009

K. Abe et al., arXiv:1502.05199.



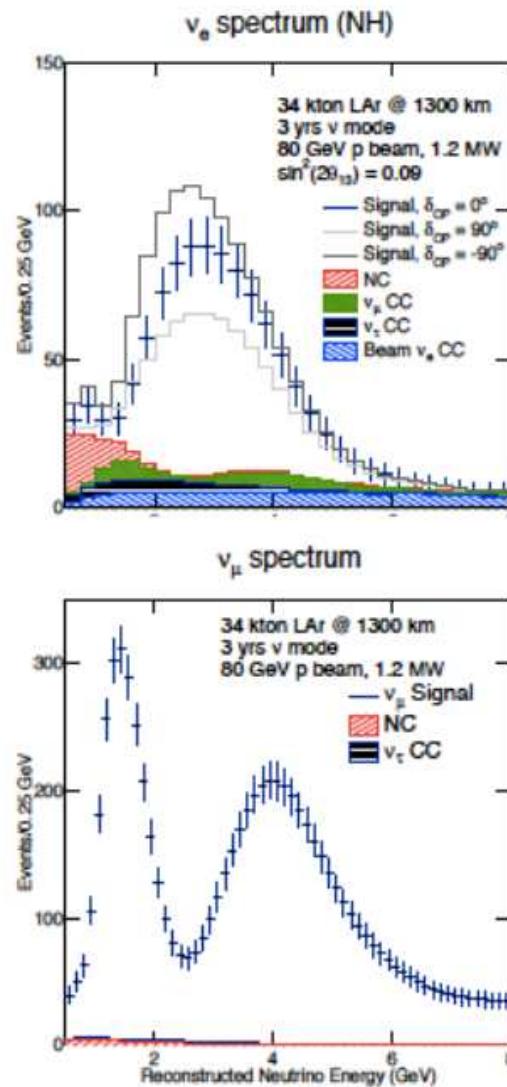
90% CL allowed regions in the $\sin^2\theta_{23}$ – Δm_{32}^2 plane. The true values are $\sin^2\theta_{23} = 0.45$ and $\Delta m_{32}^2 = 2.4 \times 10^{-3}$ eV 2 . Effect of systematic uncertainties is included. Left: T2HK only; right: with a reactor constraint.



Combination of the T2HK and HK ν_{atm} data. Expected $\Delta\chi^2$ values for a) accelerator and atmospheric neutrino measurements (left), and b) for combining the two measurements (right), assuming that the mass ordering is unknown. The true mass ordering and value of δ are assumed to be NO and $\delta = 0$. In this example study, the $\Delta\chi^2$ is simply added.



LBNE Event Rate



34 kton Lar
1.2 MW beam
NH, $\delta_{CP} = 0$
Fit parameters
from 2012

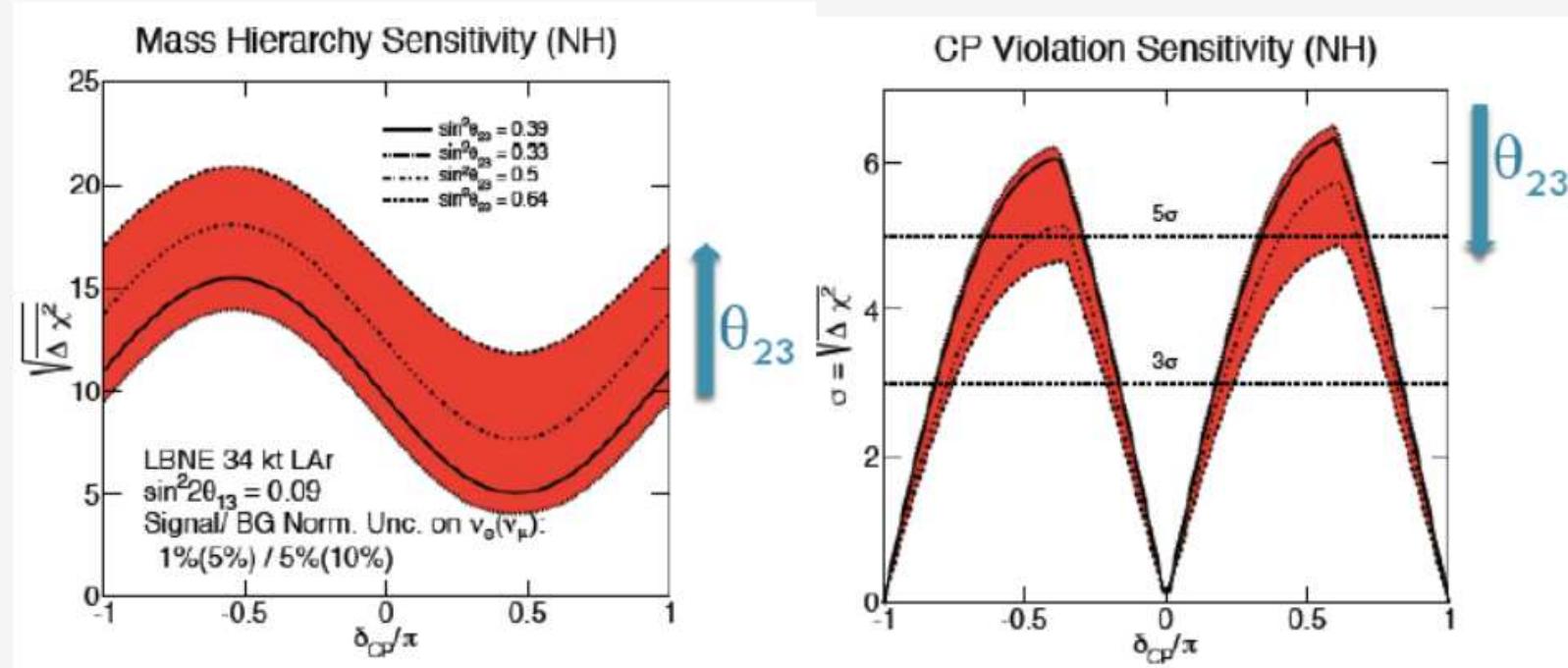
80 GeV Beam	ν mode	$\bar{\nu}$ mode
Signal: $\nu_e + \bar{\nu}_e$	777	189
BG: NC	67	39
BG: $\nu_\mu + \bar{\nu}_\mu$ CC	84	39
BG: Beam $\nu_e + \bar{\nu}_e$	147	81
BG: $\nu_\tau + \bar{\nu}_\tau$ CC	49	32

Full oscillation structure visible in energy spectrum. A combined fit to the spectrum to provide unambiguous parameter sensitivity in one experiment

[arXiv:1307.7335 – LBNE Document](https://arxiv.org/abs/1307.7335)



LBNE Sensitivity to MH & CPV



Width of the band indicates variation within the 2013 allowed rage for ϑ_{23} .

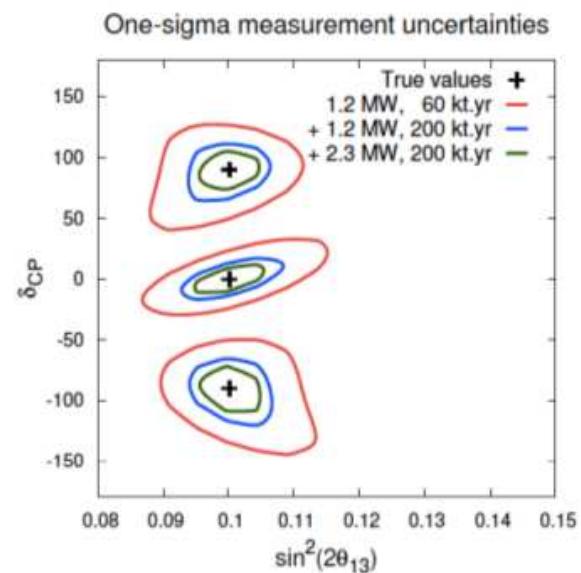
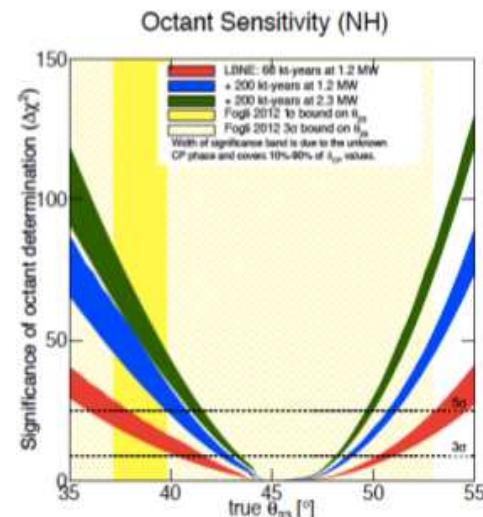
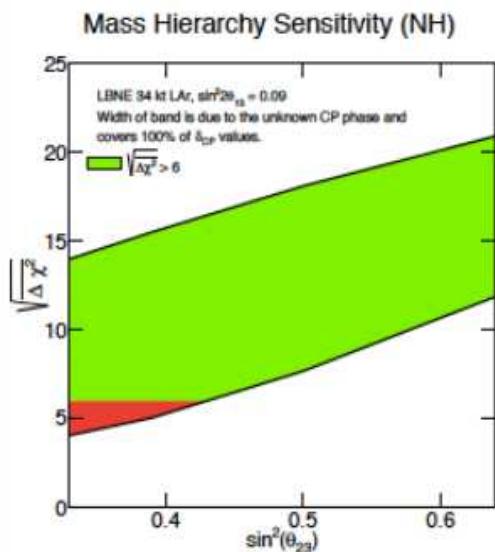
Exposure $\sim 245\text{kTon} \sim 34\text{ kT} \times 1.2\text{ MW} \times (3v + 3vbar)$ years

Elizabeth Worcester – NOW 2014

arXiv:1307.7335 – LBNE Document



Other Measurements for a Comprehensive Program



34kton, 6 yrs, 1.2MW

DUNE could also have very good sensitivity to CP-violation with a 60% coverage at 3σ in the allowed range of values of $\sin^2 2\theta_{13}$, for a 200 kton Water Cherenkov or 34 kton LAr detectors (assuming it will run for 5 years in neutrinos and 5 years in antineutrinos).

DUNE, for example, could achieve the determination of the mass ordering at 3σ in less than a year.

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12} , θ_{13} and θ_{23} , can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

The Quest for Nature's Message

With the observed pattern of neutrino mixing Nature is sending us a message. The message is encoded in the values of the neutrino mixing angles, leptonic CP violation phases and neutrino masses. The message can have two completely different contents: it can read

ANARCHY or SYMMETRY.

ANARCHY:

A. De Gouvea, H. Murayama, hep-ph/0301050; PLB, 2015.

L. Hall, H. Murayama, N. Weiner, hep-ph/9911341.

Understanding the Pattern of Neutrino Mixing. Predictions for the CPV Phase δ .

Neutrino Mixing: New Symmetry?

- $\theta_{12} = \theta_\odot \cong \frac{\pi}{5.4}$, $\theta_{23} = \theta_{\text{atm}} \cong \frac{\pi}{4} (?)$, $\theta_{13} \cong \frac{\pi}{20}$

$$U_{\text{PMNS}} \cong \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \epsilon \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}}(?) \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}}(?) \end{pmatrix};$$

Very different from the CKM-matrix!

- $\theta_{12} \cong \pi/4 - 0.20$, $\theta_{13} \cong 0 + \pi/20$, $\theta_{23} \cong \pi/4 \mp 0.10$.
- U_{PMNS} due to new approximate symmetry?

A Natural Possibility (vast literature):

$$U = U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \delta^\ell) \ Q(\psi, \omega) U_{\text{TBM}, \text{BM}, \text{LC}, \dots} \ \bar{P}(\xi_1, \xi_2),$$

with

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \pm\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \pm\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \mp\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $U_{\text{lep}}^\dagger(\theta_{ij}^\ell, \delta^\ell)$ - from diagonalization of the l^- mass matrix;
- $U_{\text{TBM}, \text{BM}, \text{LC}, \dots} \bar{P}(\xi_1, \xi_2)$ - from diagonalization of the ν mass matrix;
- $Q(\psi, \omega)$, - from diagonalization of the l^- and/or ν mass matrices.

P. Frampton, STP, W. Rodejohann, 2003

U_{LC} , U_{GRAM} , U_{GRBM} , U_{HGM} :

$$U_{LC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{c_{23}^\nu}{\sqrt{2}} & \frac{c_{23}^\nu}{\sqrt{2}} & s_{23}^\nu \\ \frac{s_{23}^\nu}{\sqrt{2}} & -\frac{s_{23}^\nu}{\sqrt{2}} & c_{23}^\nu \end{pmatrix}; \quad \mu - \tau \text{ symmetry : } \theta_{23}^\nu = \mp\pi/4;$$

$$U_{GR} = \begin{pmatrix} c_{12}^\nu & s_{12}^\nu & 0 \\ -\frac{s_{12}^\nu}{\sqrt{2}} & \frac{c_{12}^\nu}{\sqrt{2}} & -\sqrt{\frac{1}{2}} \\ -\frac{s_{12}^\nu}{\sqrt{2}} & \frac{c_{12}^\nu}{\sqrt{2}} & \sqrt{\frac{1}{2}} \end{pmatrix}; \quad U_{HGM} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \theta_{12}^\nu = \pi/6.$$

U_{GRAM} : $\sin^2 \theta_{12}^\nu = (2+r)^{-1} \cong 0.276$, $r = (1+\sqrt{5})/2$
(GR: $r/1$; $a/b = a + b/a$, $a > b$)

U_{GRBM} : $\sin^2 \theta_{12}^\nu = (3-r)/4 \cong 0.345$.

- U_{TBM} : $s_{12}^2 = 1/3$, $s_{23}^2 = 1/2$, $s_{13}^2 = 0$; $s_{13}^2 = 0$ must be corrected; if $\theta_{23} \neq \pi/4$, $s_{23}^2 = 0.5$ must be corrected.
- U_{BM} : $s_{12}^2 = 1/2$, $s_{23}^2 = 1/2$, $s_{13}^2 = 0$; $s_{13}^2 = 0$, $s_{12}^2 = 1/2$ and possibly $s_{23}^2 = 1/2$ must be corrected.

$U_{\text{TBM(BM)}}$: Groups A_4 , T' (S_4), ... (vast literature)

(Reviews: G. Altarelli, F. Feruglio, arXiv:1002.0211; M. Tanimoto et al., arXiv:1003.3552; S. King and Ch. Luhn, arXiv:1301.1340)

- U_{GRA} : Group A_5, \dots ; $s_{13}^2 = 0$ and possibly $s_{12}^2 = 0.276$ and $s_{23}^2 = 1/2$ must be corrected.

L. Everett, A. Stuart, arXiv:0812.1057; ...

- U_{LC} : alternatively $U(1)$, $L' = L_e - L_\mu - L_\tau$

S.T.P., 1982

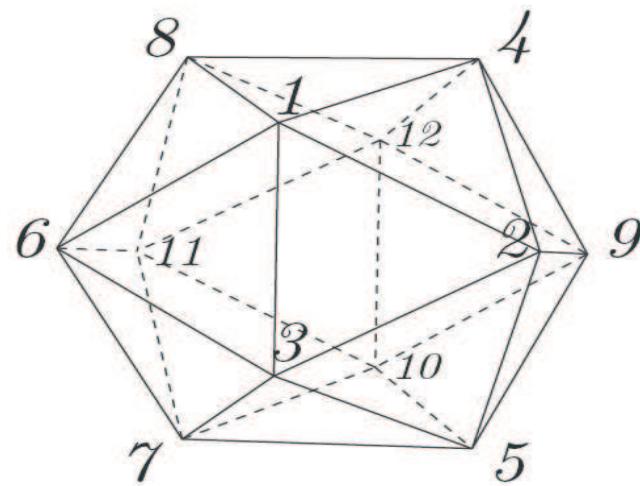
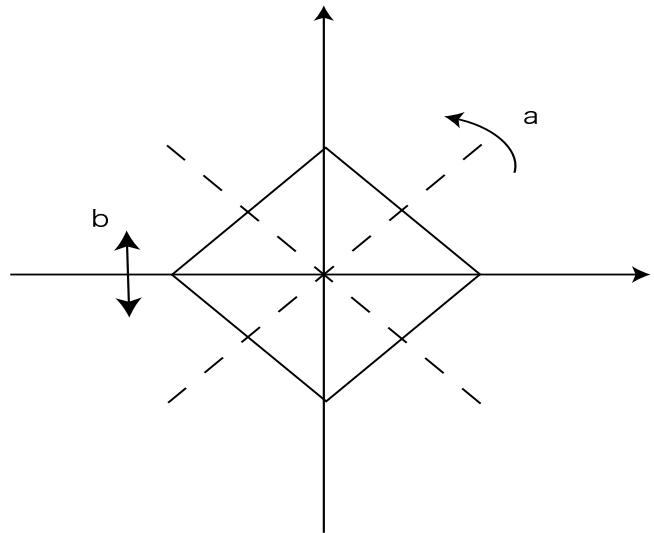
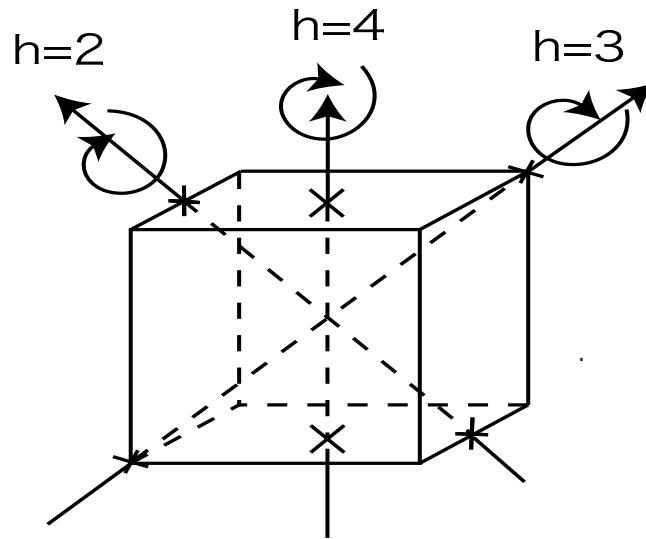
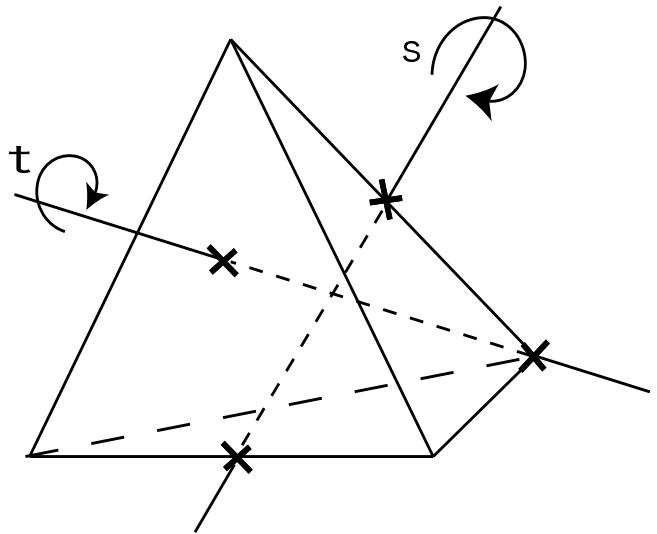
- U_{LC} : $s_{12}^2 = 1/2$, $s_{13}^2 = 0$, s_{23}^ν - free parameter; $s_{13}^2 = 0$ and $s_{12}^2 = 1/2$ must be corrected.

- U_{GRB} : Group D_{10}, \dots ; $s_{13}^2 = 0$ and possibly $s_{12}^2 = 0.345$ and $s_{23}^2 = 1/2$ must be corrected.
- U_{HG} : Group D_{12}, \dots ; $s_{13}^2 = 0$, $s_{12}^2 = 0.25$ and possibly $s_{23}^2 = 1/2$ must be corrected.

For all symmetry forms considered we have: $\theta_{13}^\nu = 0$, $\theta_{23}^\nu = \mp\pi/4$.

They differ by the value of θ_{12}^ν :

TBM, BM, GRA, GRB and HG forms correspond to $\sin^2 \theta_{12}^\nu = 1/3; 0.5; 0.276; 0.345; 0.25$.



Examples of symmetries: A_4 , S_4 , D_4 , A_5

From M. Tanimoto et al., arXiv:1003.3552

For arbitrary fixed θ_{12}^ν and any θ_{23}
("minimal" and "next-to-minimal" cases):

$$\cos \delta = \frac{\tan \theta_{23}}{\sin 2\theta_{12} \sin \theta_{13}} [\cos 2\theta_{12}^\nu + (\sin^2 \theta_{12} - \cos^2 \theta_{12}^\nu) (1 - \cot^2 \theta_{23} \sin^2 \theta_{13})].$$

S.T.P., arXiv:1405.6006

This results is exact.

"Minimal" case: $\sin^2 \theta_{23} = \frac{1}{2} \frac{1 - 2 \sin^2 \theta_{13}}{1 - \sin^2 \theta_{13}}$.

In all cases TBM, BM (LC), GRA, GRB, HG:

- New sum rules relating $\theta_{12}, \theta_{13}, \theta_{23}$ and δ ;
- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta_{12}^\nu)$.

S.T.P., arXiv:1405.6006

- $J_{CP} = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = J_{CP}(\theta_{12}, \theta_{23}, \theta_{13}; \theta_{12}^\nu)$.
- TBM case: $\delta \cong 3\pi/2$ or $\pi/2$; b.f.v. of θ_{ij} :
 $\delta \cong 263.5^\circ$ or 96.5° , $\cos \delta = -0.114$, $J_{CP} \cong \mp 0.034$.
- GRAM case, b.f.v. of θ_{ij} : $\delta \cong 286.8^\circ$ or 73.2° ;
 $\cos \delta = 0.289$, $J_{CP} \cong \mp 0.0327$.
- GRBM case, b.f.v. of θ_{ij} : $\delta \cong 258.5^\circ$ or 101.5° ;
 $\cos \delta = -0.200$, $J_{CP} \mp 0.0333$.
- HGM case, b.f.v. of θ_{ij} : $\delta \cong 298.4^\circ$ or 61.6° ;
 $\cos \delta = 0.476$, $J_{CP} \cong \mp 0.0299$.
- BM, LC cases: $\delta \cong \pi$, $\cos \delta \cong -0.978$, $J_{CP} \cong \mp 0.008$

The results shown - for NO neutrino mass spectrum; the results are practically the same for IO spectrum. (Best fit values of θ_{ij} : F. Capozzi et al., arXiv:1312.2878v1.)

S.T.P., arXiv:1405.6006

By measuring $\cos \delta$ or δ one can distinguish between different symmetry forms of \tilde{U}_ν !

Relatively high precision measurement of δ will be performed at the future planned neutrino oscillation experiments, (DUNE, T2HK) see, e.g., A. de Gouvea *et al.*, arXiv:1310.4340; P. Coloma *et al.*, arXiv:1203.5651; R. Acciarri *et al.* [DUNE Collab.], arXiv:1512.06148, 1601.05471 and 1601.02984.

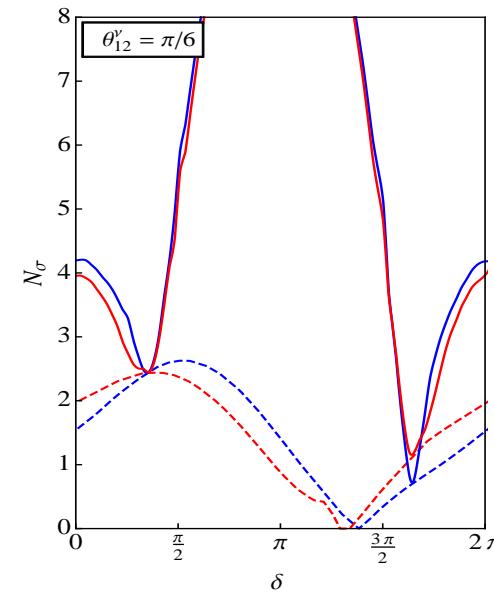
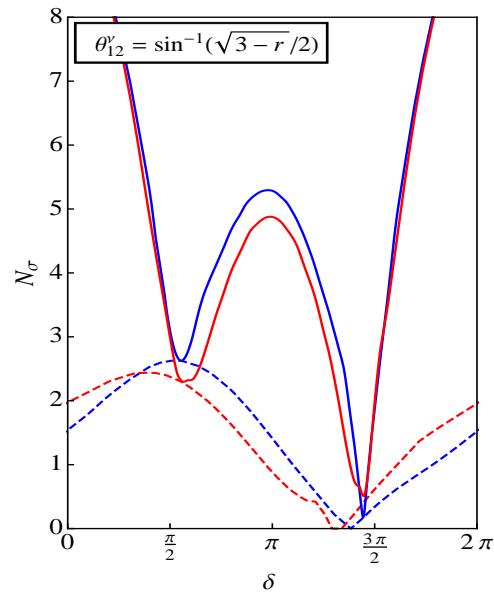
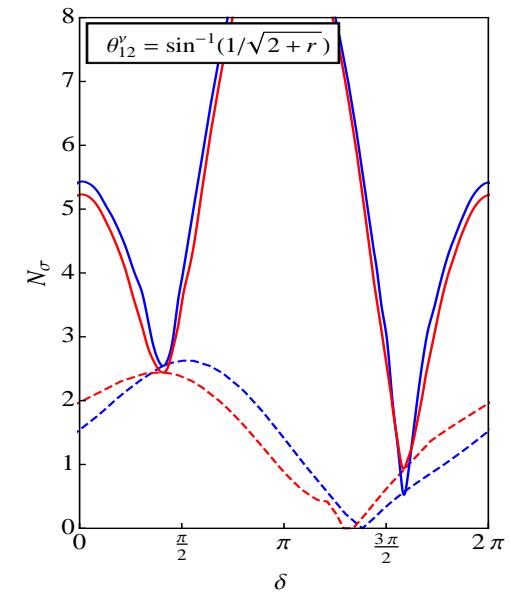
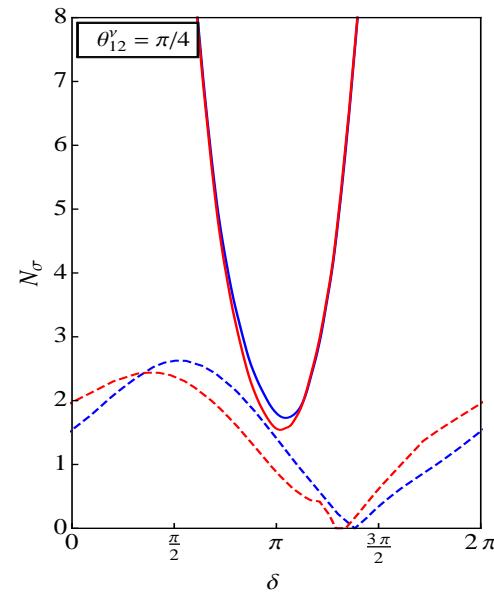
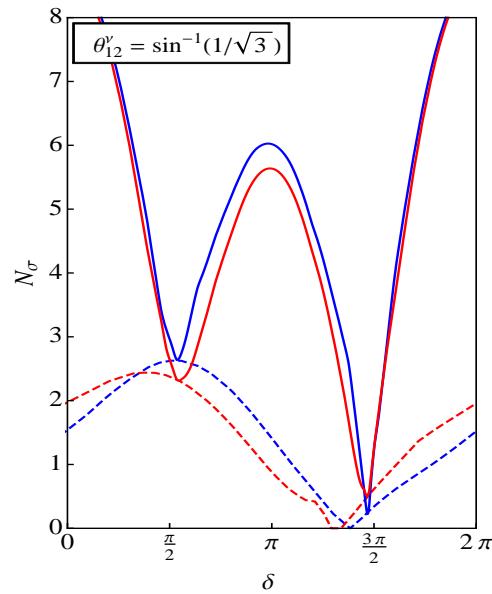
**Statistical analysis, likelihood method;
input “data”: $\sin^2 \theta_{13}$, $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$, δ
from F. Capozzi et al., arXiv:1312.2878v2 (May 5,
2014).**

$$L(\cos \delta) \propto \exp \left(-\frac{\chi^2(\cos \delta)}{2} \right)$$

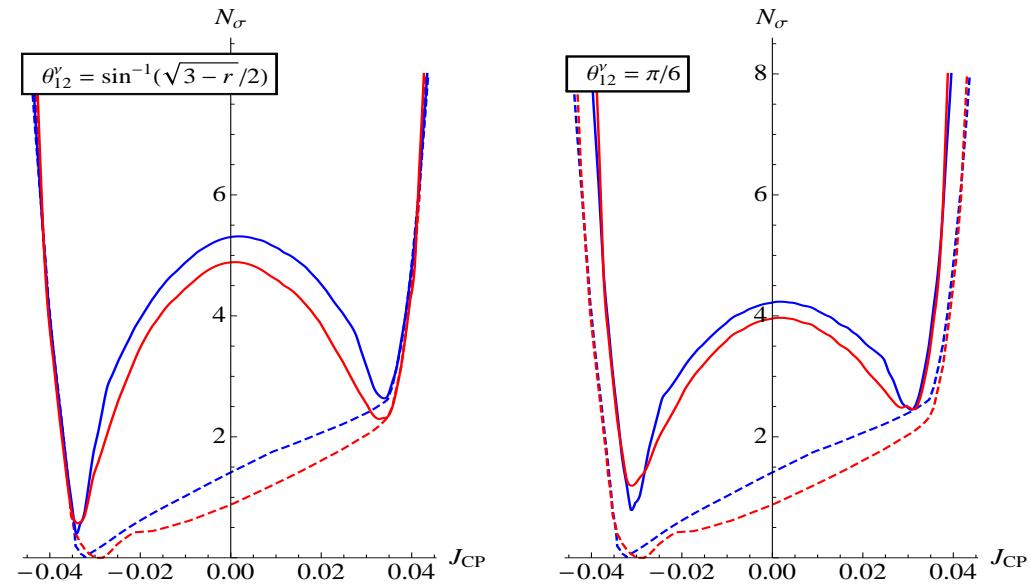
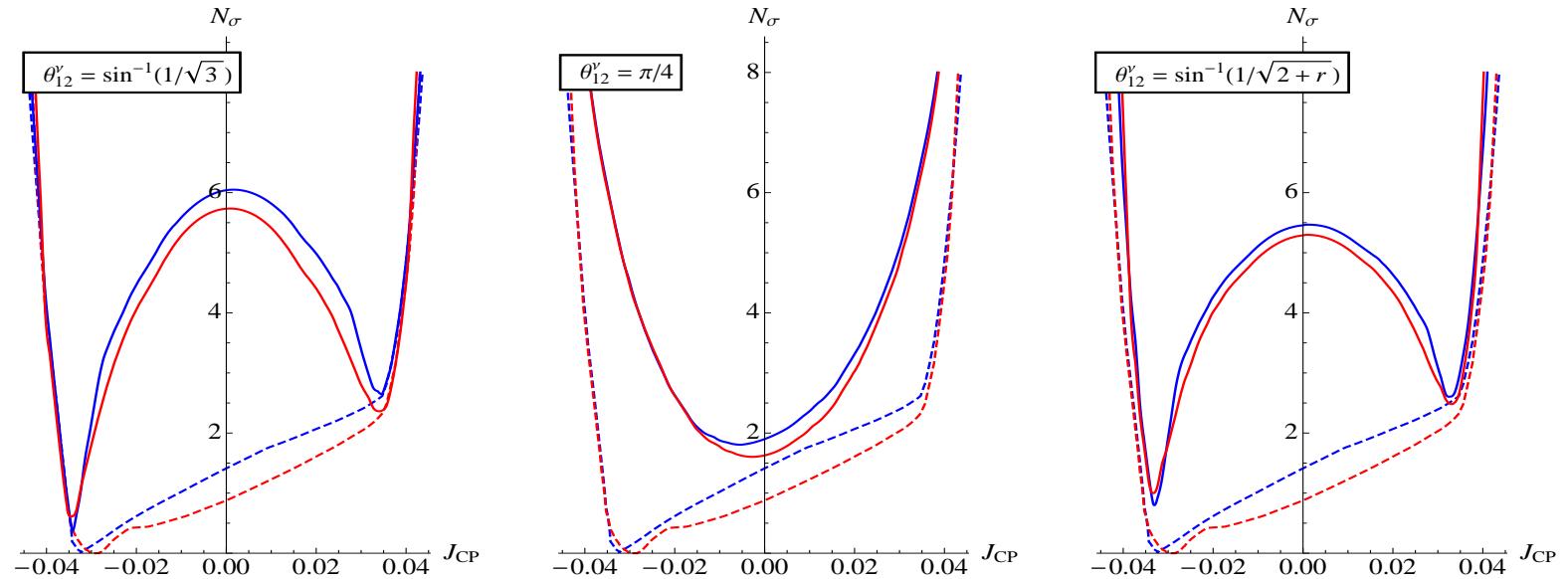
$n\sigma$ confidence level interval of values of $\cos \delta$:

$$L(\cos \delta) \geq L(\chi^2_{\min}) \cdot L(\chi^2 = n^2)$$

I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

TBM, GRA, GRB, HG: $J = 0$ excluded at 5σ , 4σ , 4σ , 3σ confidence level.

At 3σ : $0.020 \leq |J_{CP}| \leq 0.039$.

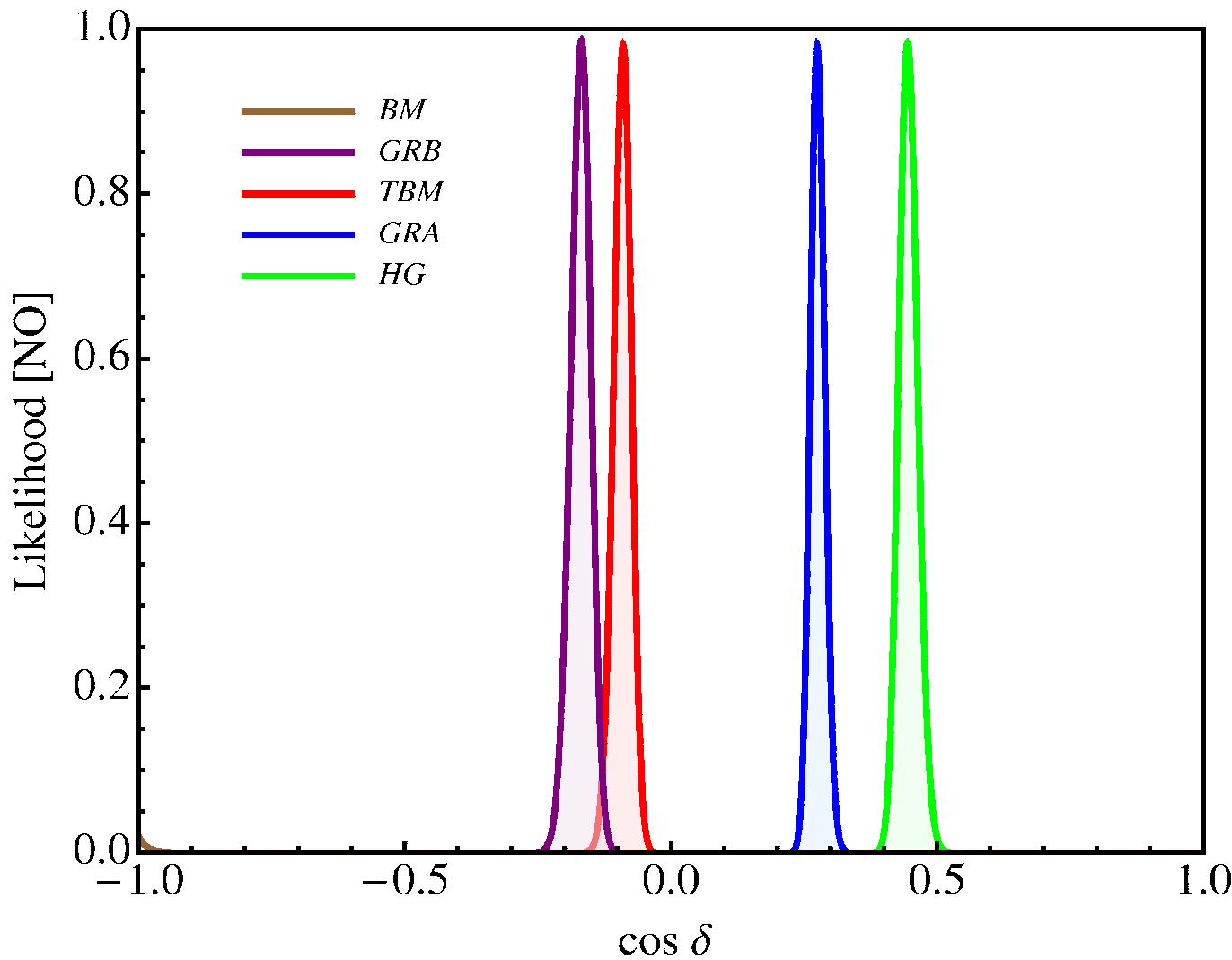
BM (LC), b.f.v.: $J_{CP} = 0$;
at 3σ : -0.026 (-0.025) $\leq J_{CP} \leq 0.021$ (0.023) **for NO (IO) neutrino mass spectrum.**

Prospective precision:

$\delta(\sin^2 \theta_{12}) = 0.7\% \text{ (JUNO),}$

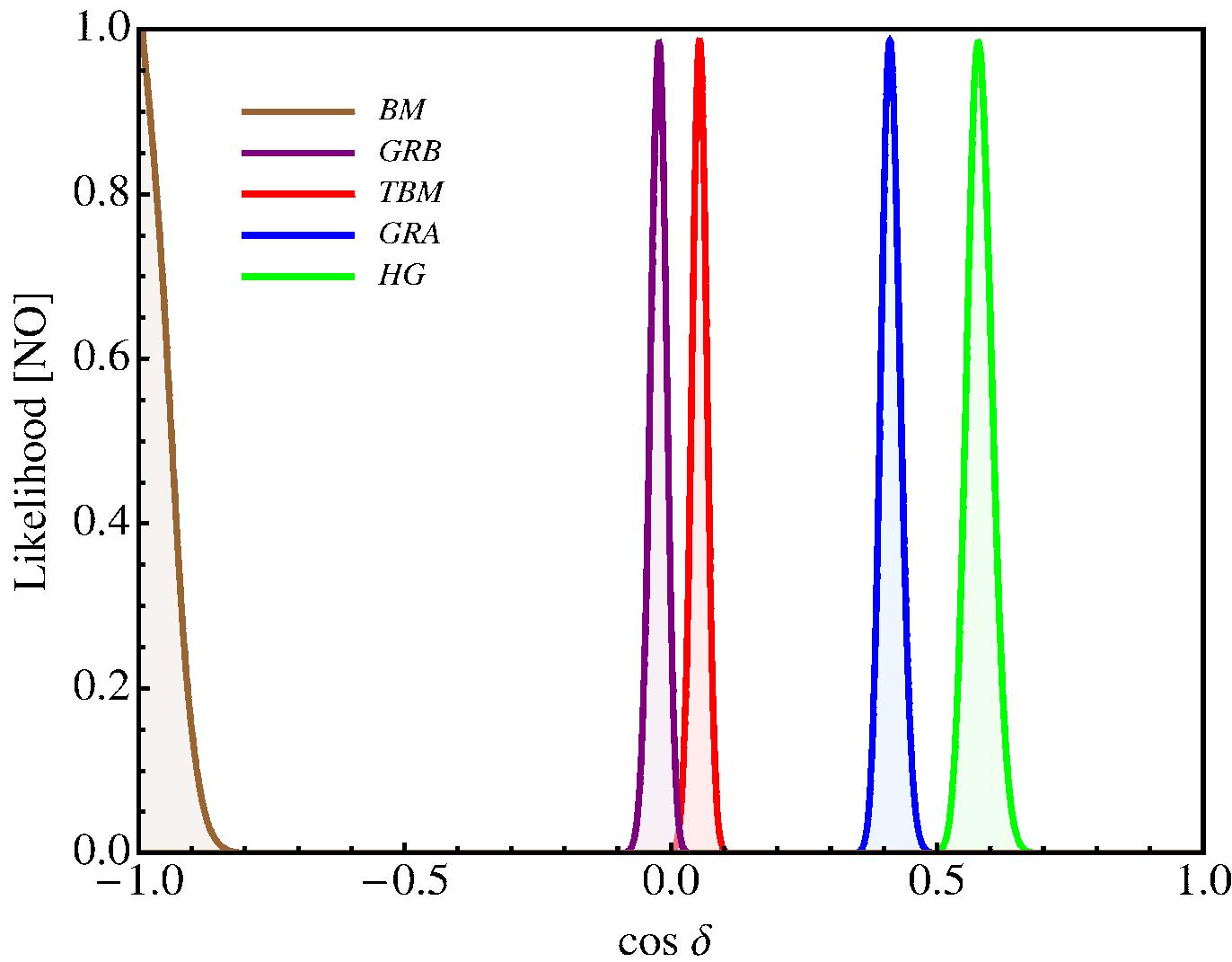
$\delta(\sin^2 \theta_{13}) = 3\% \text{ (Daya Bay),}$

$\delta(\sin^2 \theta_{23}) = 5\% \text{ (T2K, NO}\nu\text{A combined).}$



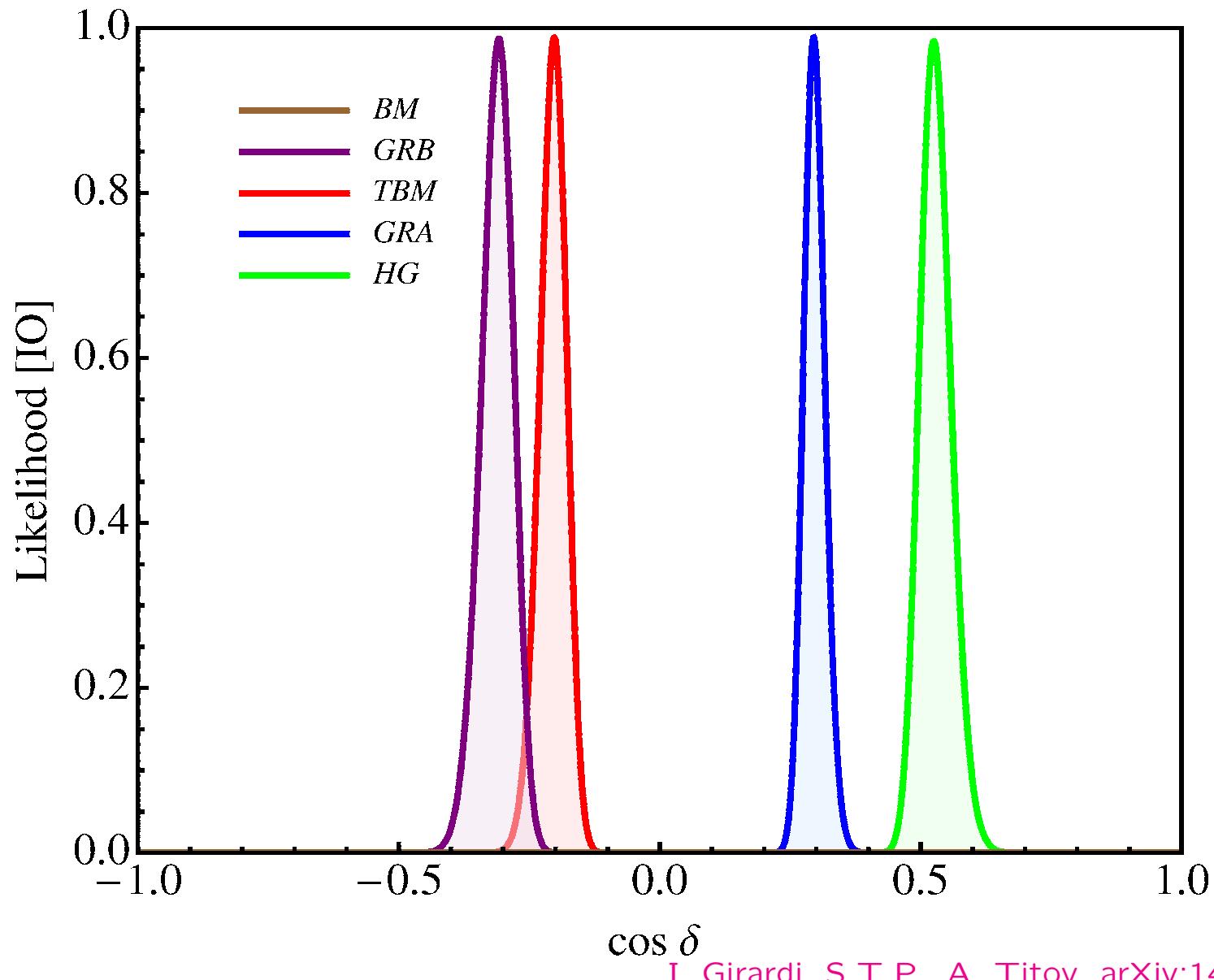
I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

b.f.v. of $\sin^2 \theta_{ij}$ (Capozzi et al., 2014) + the prospective precision used.



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

The same, but for $\sin^2 \theta_{12} = 0.33$ (the BM prediction dependence on $\sin^2 \theta_{12}$).



I. Girardi, S.T.P., A. Titov, arXiv:1410.8056

$$\sin^2 \theta_{23} = 0.557 \text{ (b.f.v.: C. Gonzales-Garcia et al., 2014, IO case).}$$

For, e.g., $|\cos \delta| < 0.93$ (76% of values of δ), and
 $\Delta(\cos \delta) = 0.10(0.08)$:

$$\Delta\delta \Delta(\cos \delta) / \sqrt{1 - 0.93^2} = 16^\circ (12^\circ).$$

Planned to be reached, e.g., in T2HK.

Thus, a measurement of $\cos \delta$ in the quoted range will allow to distinguish between the TBM/GRB, BM (LC) and GRA/HG forms at $\sim 3\sigma$ C.L., if $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ are measured with the prospective precisions.

Distinguishing between **GRA** and **HG** forms at 3σ C.L. requires $1\sigma(\cos \delta) \cong 0.03$ (if b.f.v. of $\cos \delta$ at one of the two maxima).

TBM and GRB cannot be distinguished at 3σ C.L. with the prospective uncertainties on \sin_{ij}^θ ; for zero uncertainties on \sin_{ij}^θ (infinite precision), can be distinguished if $1\sigma(\cos \delta) \cong 0.03$.

I. Girardi, S.T.P., A. Titov, arXiv:1504

The predictions obtained for $\cos \delta$ are valid in a large class of theoretical models of (lepton) flavour based on discrete symmetries.

J. Gehrlein *et al.*, “An $SU(5) \times A_5$ Golden Ratio Flavour Model”, arXiv:1410.2095;

I. Girardi *et al.*, “Generalised Geometrical CP Violation in a T' Lepton Flavour Model”, arXiv:1312.1966, JHEP 1402 (2014) 050.

The measurement of the Dirac phase in the PMNS mixing matrix, together with an improvement of the precision on the mixing angles θ_{12} , θ_{13} and θ_{23} , can provide unique information about the possible existence of new fundamental symmetry in the lepton sector.

Determining the Nature of Massive Neutrinos

This is one of the most challenging and pressing problems in neutrino physics.

- Massive Dirac Neutrinos: $U(1)$, Conserved (Additive) Charge, e.g., L .
- Massive Majorana Neutrinos: No Conserved (Additive) Charge(s).

The type of massive neutrinos in a given theory is determined by the type of (effective) mass term $\mathcal{L}_m^\nu(x)$ neutrinos have, more precisely, by the symmetries $\mathcal{L}_m^\nu(x)$ and the total Lagrangian $\mathcal{L}(x)$ of the theory have.

Mass Term: any by-linear in fermion (neutrino) fields invariant under the proper Lorentz transformations.

Seesaw Mechanisms of Neutrino Mass Generation

- Predict light massive Majorana neutrinos
- Explain the smallness of ν -masses.
- Through leptogenesis theory link the ν -mass generation to the generation of baryon asymmetry of the Universe.

S. Fukugita, T. Yanagida, 1986.

Dirac CP-Nonconservation: δ in U_{PMNS}

Observable manifestations in

$$\nu_l \leftrightarrow \nu_{l'} , \quad \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'} , \quad l, l' = e, \mu, \tau$$

- not sensitive to Majorana CPVP α_{21}, α_{31}

S.M. Bilenky, J. Hosek, S.T.P., 1980;
P. Langacker et al., 1987

$$A(\nu_l \leftrightarrow \nu_{l'}) = \sum_j U_{l'j} e^{-i(E_j t - p_j x)} U_{jl}^\dagger$$

$$U = VP : P_j e^{-i(E_j t - p_j x)} P_j^* = e^{-i(E_j t - p_j x)}$$

P - diagonal matrix of Majorana phases.

The result is valid also in the case of oscillations in matter: ν_l oscillations are not sensitive to the nature of ν_j .

ν_j — Dirac or Majorana particles, fundamental problem

ν_j —Dirac: **conserved lepton charge exists**, $L = L_e + L_\mu + L_\tau$, $\nu_j \neq \bar{\nu}_j$

ν_j —Majorana: **no lepton charge is exactly conserved**, $\nu_j \equiv \bar{\nu}_j$

The observed patterns of ν —mixing and of Δm_{atm}^2 and Δm_{\odot}^2 can be related to Majorana ν_j and an **approximate symmetry**:

$$L' = L_e - L_\mu - L_\tau$$

S.T.P., 1982

See-saw mechanism: ν_j — Majorana

Establishing that ν_j are Majorana particles would be as important as the discovery of ν — oscillations.

If ν_j – Majorana particles, U_{PMNS} contains (3- ν mixing)

δ -Dirac, α_{21} , α_{31} - Majorana physical CPV phases

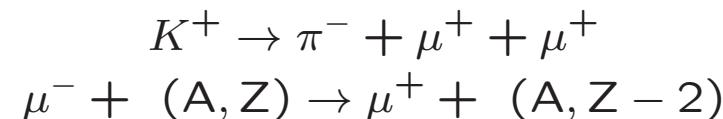
ν -oscillations $\nu_l \leftrightarrow \nu_{l'}, \bar{\nu}_l \leftrightarrow \bar{\nu}_{l'}, l, l' = e, \mu, \tau,$

- are not sensitive to the nature of ν_j ,

S.M. Bilenky et al., 1980;
P. Langacker et al., 1987

- provide information on $\Delta m_{jk}^2 = m_j^2 - m_k^2$, but not on the absolute values of ν_j masses.

The Majorana nature of ν_j can manifest itself in the existence of $\Delta L = \pm 2$ processes:



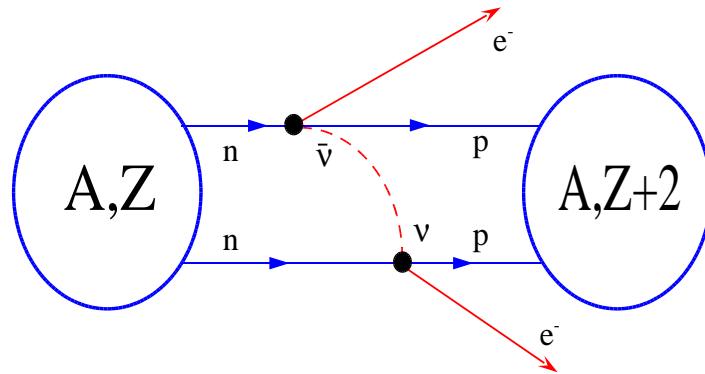
The process most sensitive to the possible Majorana nature of ν_j – $(\beta\beta)_{0\nu}$ -decay



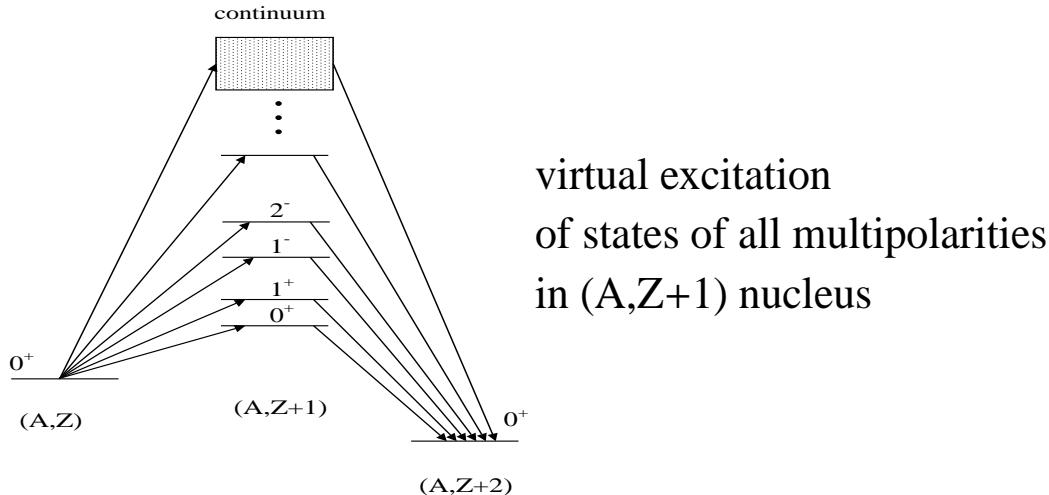
of even-even nuclei, ^{48}Ca , ^{76}Ge , ^{82}Se , ^{100}Mo , ^{116}Cd , ^{130}Te , ^{136}Xe , ^{150}Nd .

$2n$ from (A, Z) exchange a virtual Majorana ν_j (via the CC weak interaction) and transform into $2p$ of $(A, Z + 2)$ and two free e^- .

Nuclear $0\nu\beta\beta$ -decay



strong in-medium modification of the basic process
 $dd \rightarrow uue^-e^-(\bar{\nu}_e\bar{\nu}_e)$



V. Rodin, talk at Gran Sasso, 2006

$(\beta\beta)_{0\nu}$ -Decay Experiments:

- Majorana nature of ν_j
- Type of ν -mass spectrum (NH, IH, QD)
- Absolute neutrino mass scale

${}^3\text{H}$ β -decay, cosmology: m_ν (QD, IH)

- CPV due to Majorana CPV phases

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle \text{ M(A,Z)}, \quad \text{M(A,Z) - NME},$$

$$\begin{aligned} |\langle m \rangle| &= |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}} + m_3|U_{e3}|^2 e^{i\alpha_{31}}| \\ &= |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i\alpha_{31}}|, \quad \theta_{12} \equiv \theta_\odot, \theta_{13} \text{- CHOOZ} \end{aligned}$$

α_{21}, α_{31} - the two Majorana CPVP of the PMNS matrix.

CP-invariance: $\alpha_{21} = 0, \pm\pi, \alpha_{31} = 0, \pm\pi$;

$$\eta_{21} \equiv e^{i\alpha_{21}} = \pm 1, \quad \eta_{31} \equiv e^{i\alpha_{31}} = \pm 1$$

relative CP-parities of ν_1 and ν_2 , and of ν_1 and ν_3 .

L. Wolfenstein, 1981;

S.M. Bilenky, N. Nedelcheva, S.T.P., 1984;

B. Kayser, 1984.

$$A(\beta\beta)_{0\nu} \sim \langle m \rangle \text{ M(A,Z)}, \quad \text{M(A,Z) - NME},$$

$$|\langle m \rangle| \cong \sqrt{m_3^2 + \Delta m_{13}^2} |\cos^2 \theta_{12} + e^{i\alpha_{21}} \sin^2 \theta_{12}|, \quad m_3 < (\ll) m_1 < m_2 \text{ (IH)},$$

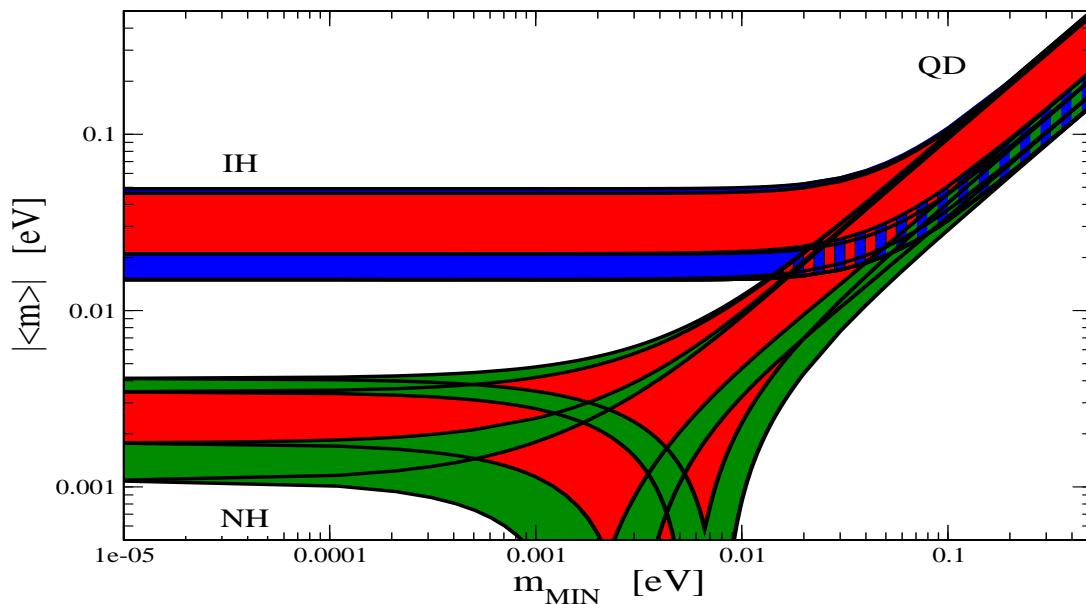
$$|\langle m \rangle| \cong m |\cos^2 \theta_{12} + e^{i\alpha} \sin^2 \theta_{12}|, \quad m_{1,2,3} \cong m \gtrsim 0.10 \text{ eV (QD)},$$

CP-invariance: $\alpha_{21} = 0, \pm\pi, \alpha_{31} = 0, \pm\pi;$

$$|\langle m \rangle| \lesssim 5 \times 10^{-3} \text{ eV, NH};$$

$$\sqrt{\Delta m_{13}^2} \cos 2\theta_{12} \cong 0.013 \text{ eV} \lesssim |\langle m \rangle| \lesssim \sqrt{\Delta m_{13}^2} \cong 0.055 \text{ eV, IH};$$

$$m \cos 2\theta_{12} \lesssim |\langle m \rangle| \lesssim m, \quad m \gtrsim 0.10 \text{ eV, QD}.$$



S. Pascoli, PDG, 2014

$$\sin^2 \theta_{13} = 0.0234 \text{ (0.0240)} \pm 0.0021; \delta = 0.$$

$$1\sigma(\Delta m_{21}^2) = 2.6\%, \quad 1\sigma(\sin^2 \theta_{12}) = 5.4\%, \quad 1\sigma(|\Delta m_{31(23)}^2|) = 3\%.$$

From F. Capozzi *et al.*, arXiv:1312.5254v2 (May 5, 2014)

$2\sigma(|\langle m \rangle|)$ used.

Best sensitivity: GERDA (^{76}Ge), EXO (^{136}Xe), KamLAND-ZEN (^{136}Xe).

Claim for a positive signal at $> 3\sigma$:

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$|\langle m \rangle| = (0.1 - 0.9) \text{ eV} \text{ (99.73% C.L.)}$; b.f.v.: $|\langle m \rangle| = 0.33 \text{ eV}$.

IGEX ^{76}Ge : $|\langle m \rangle| < (0.33 - 1.35) \text{ eV} \text{ (90% C.L.)}$.

Recent data - NEMO3 (^{100}Mo), CUORICINO (^{130}Te):

$|\langle m \rangle| < (0.45 - 0.96) \text{ eV}$, $|\langle m \rangle| < (0.18 - 0.64) \text{ eV} \text{ (90% C.L.)}$.

H. Klapdor-Kleingrothaus et al., PL B586 (2004),

$$\tau(^{76}\text{Ge}) = 2.23_{-0.31}^{+0.44} \times 10^{25} \text{ yr at 90% C.L.}$$

Results from 2012-2016:

$$\tau(^{136}\text{Xe}) > 1.6 \times 10^{25} \text{ yr at 90% C.L., EXO}$$

$$\tau(^{136}\text{Xe}) > 1.07 \times 10^{26} \text{ yr at 90% C.L., KamLAND – Zen}$$

$$|\langle m \rangle| < (0.016 - 0.165) \text{ eV}.$$

$$\tau(^{76}\text{Ge}) > 2.1 \times 10^{25} \text{ yr at 90% C.L., GERDA.}$$

$$\tau(^{76}\text{Ge}) > 3.0 \times 10^{25} \text{ yr at 90% C.L., GERDA + IGEX + HdM.}$$

Large number of experiments: $|\langle m \rangle| \sim (0.01\text{-}0.05) \text{ eV}$

CUORE - ^{130}Te ,

GERDA - ^{76}Ge ,

KamLAND-ZEN - ^{136}Xe ;

EXO - ^{136}Xe ;

SNO+ - ^{130}Te ;

AMoRE - ^{100}Mo (S. Korea);

CANDLES - ^{48}Ca ;

SuperNEMO - ^{82}Se , ...;

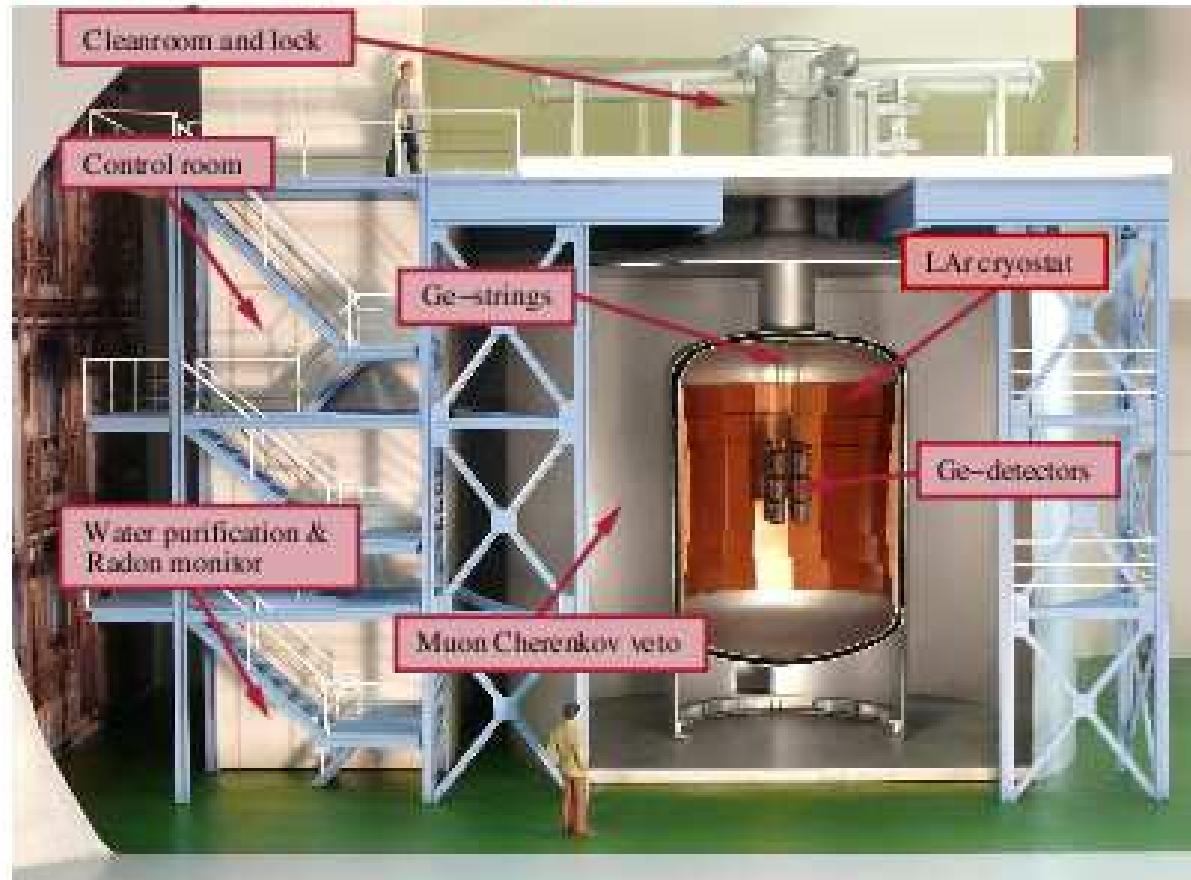
MAJORANA - ^{76}Ge ;

COBRA - ^{116}Cd ;

MOON - ^{100}Mo .



GERDA: Experimental Setup



GRADUAT
SITAT
AACHEN



S.T. Petcov, INVISIBLES School, SISSA, 09/09/2016

Majorana CPV Phases and $|<m>|$

CPV can be established provided

- $|<m>|$ measured with $\Delta \lesssim 15\%$;
- Δm_{atm}^2 (IH) or m_0 (QD) measured with $\delta \lesssim 10\%$;
- $\xi \lesssim 1.5$;
- α_{21} (QD): in the interval $\sim [\frac{\pi}{4} - \frac{3\pi}{4}]$, or $\sim [\frac{5\pi}{4} - \frac{3\pi}{2}]$;
- $\tan^2 \theta_\odot \gtrsim 0.40$.

S. Pascoli, S.T.P., W. Rodejohann, 2002

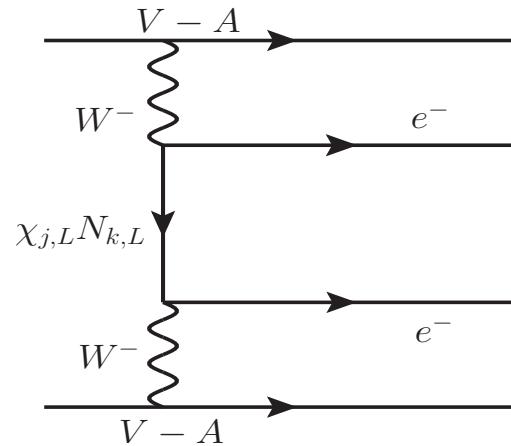
S. Pascoli, S.T.P., L. Wolfenstein, 2002

S. Pascoli, S.T.P., T. Schwetz, hep-ph/0505226

No “No-go for detecting CP-Violation via $(\beta\beta)_{0\nu}$ -decay”

V. Barger *et al.*, 2002

Different Mechanisms of $(\beta\beta)_{0\nu}$ -Decay



Light Majorana Neutrino Exchange

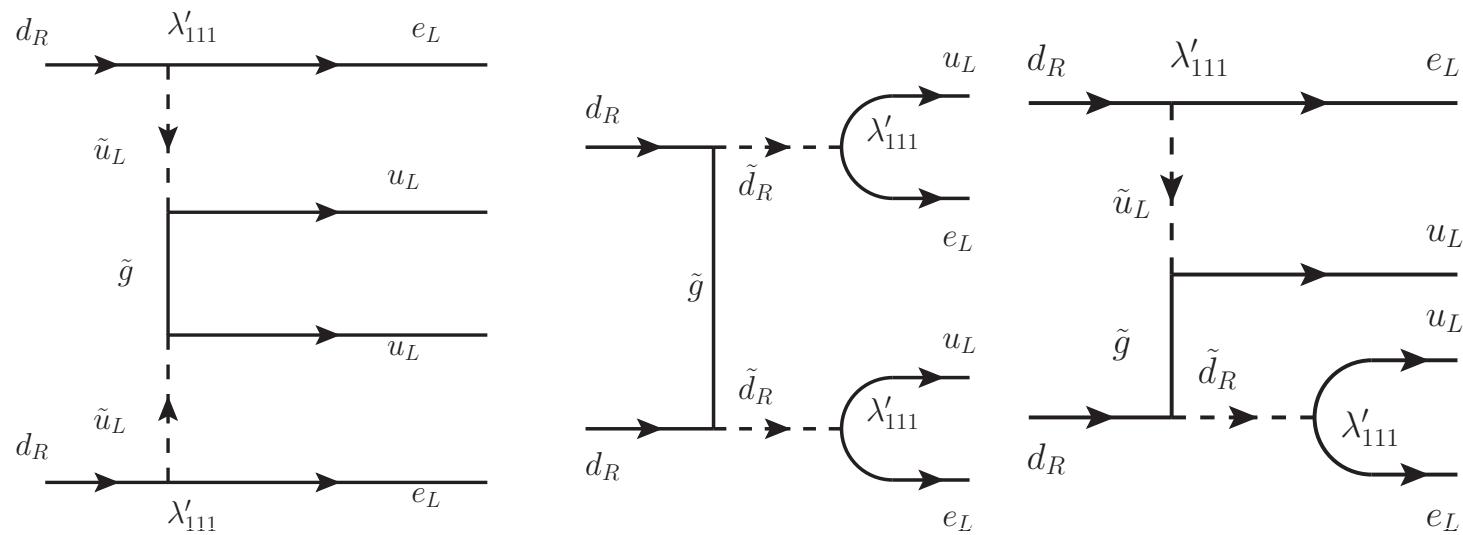
$$\eta_\nu = \frac{\langle m \rangle}{m_e} .$$

Heavy Majorana Neutrino Exchange Mechanisms

(V-A) Weak Interaction, LH $N_k, M_k \gtrsim 10$ GeV:

$$\eta_N^L = \sum_k^{heavy} U_{ek}^2 \frac{m_p}{M_k}, \text{ } m_p \text{ - proton mass, } U_{ek} \text{ - CPV} .$$

SUSY Models with R-Parity Non-conservation



$$\begin{aligned} \mathcal{L}_{R_p} = & \lambda'_{111} \left[(\bar{u}_L \bar{d}_L) \begin{pmatrix} e_R^c \\ -\nu_{eR}^c \end{pmatrix} \tilde{d}_R + (\bar{e}_L \bar{\nu}_{eL}) d_R \begin{pmatrix} \tilde{u}_L^* \\ -\tilde{d}_L^* \end{pmatrix} \right. \\ & \left. + (\bar{u}_L \bar{d}_L) d_R \begin{pmatrix} \tilde{e}_L^* \\ -\tilde{\nu}_{eL}^* \end{pmatrix} \right] + h.c. \end{aligned}$$

The problem of distinguishing between different sets of multiple (e.g., two) mechanisms being operative in $(\beta\beta)_{0\nu}$ -decay was studied in

1. A. Faessler, A. Meroni, S.T.P., F. Simkovic and J. Vergados, “Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$ -Decay”, arXiv:1103.2434, Phys. Rev. D83 (2011) 113003.
2. A. Meroni, S.T.P. and F. Simkovic, “Multiple CP Non-conserving Mechanisms of bb0nu-Decay and Nuclei with Largely Different Nuclear Matrix Elements”, (arXiv:1212.1331, JHEP **1302** (2013) 025.

Earlier studies include:

A. Halprin, S.T.P., S.P. Rosen, “Effects of Mixing of Light and Heavy Majorana Neutrinos in Neutrinoless Double Beta Decay”, Phys. Lett. 125B (1983) 335).

Instead of Conclusions

We are at the beginning of the Road...

**The next 5, 10, 15,... years will be very exciting in
Neutrino (Lepton) Physics!**

Supporting Slides

Three Types of Seesaw Mechanisms

Require the existence of new degrees of freedom (particles) beyond those present in the ST

Type I seesaw mechanism: ν_{lR} - RH $\nu s'$ (heavy).

Type II seesaw mechanism: $H(x)$ - a triplet of H^0, H^-, H^{--} Higgs fields (HTM).

Type III seesaw mechanism: $T(x)$ - a triplet of fermion fields.

The scale of New Physics determined by the masses of the New Particles.

Massive neutrinos ν_j - Majorana particles.

All three types of seesaw mechanisms have TeV scale versions, predicting rich low-energy phenomenology ($(\beta\beta)_{0\nu}$ -decay, LFV processes, etc.) and New Physics at LHC.

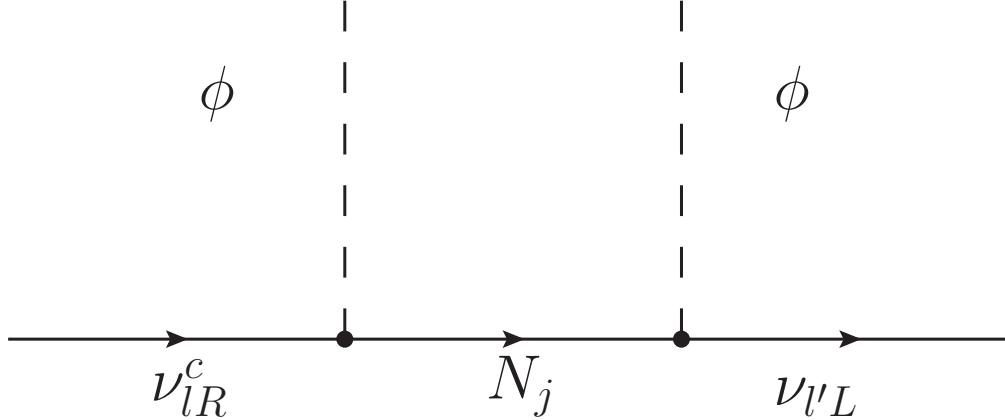
Type I Seesaw Mechanism

- Requires both $\nu_{lL}(x)$ and $\nu_{l'R}(x)$.
- Dirac+Majorana Mass Term: $M^{LL} = 0$, $|M_D| = v Y^\nu / \sqrt{2} | << |M^{RR}|$.
- Diagonalising M^{RR} : N_j - heavy Majorana neutrinos, $M_j \sim \text{TeV}$; or $(10^9 - 10^{13}) \text{ GeV}$ in GUTs.

For sufficiently large M_j , Majorana mass term for $\nu_{lL}(x)$:

$$M_\nu \cong v_u^2 (Y^\nu)^T M_j^{-1} Y^\nu = U_{\text{PMNS}}^* m_\nu^{\text{diag}} U_{\text{PMNS}}^\dagger.$$

$$v_u Y^\nu = M_D, \quad M_D \sim 1 \text{ GeV}, \quad M_j = 10^{10} \text{ GeV}; \quad M_\nu \sim 0.1 \text{ eV}.$$



- $\nu_{l'R}(x)$: Majorana mass term at “high scale” (\sim TeV; or $(10^9 - 10^{13})$ GeV in $SO(10)$ GUT)

$$\mathcal{L}_M^\nu(x) = + \frac{1}{2} \nu_{l'R}^\top(x) C^{-1} (M^{RR})_{ll}^\dagger \nu_{lR}(x) + h.c. = - \frac{1}{2} \sum_j \bar{N}_j M_j N_j ,$$

- Yukawa type coupling of $\nu_{lL}(x)$ and $\nu_{l'R}(x)$ involving $\Phi(x)$:

$$\begin{aligned} \mathcal{L}_Y(x) &= \bar{Y}_{ll}^\nu \overline{\nu_{l'R}}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c. , \\ &= Y_{jl}^\nu \overline{N_{jR}}(x) \Phi^T(x) (i\tau_2) \psi_{lL}(x) + h.c. , \\ M_D &= \frac{v}{\sqrt{2}} Y^\nu , \quad v = 246 \text{ GeV} . \end{aligned}$$