

CP violation in the Quark and Lepton sectors.

Gustavo C. Branco

CFTP / IIST , Lisboa

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Open Questions in CP Violation

- What is the Origin of CP Violation?
 - Explicitly broken as in the Standard Model through the Kobayashi-Maskawa mechanism (1973) or
 - Spontaneously broken as suggested by T. D. Lee (1973)
- Pure gauge interactions conserve CP
W. Grimus, M.N. Rebelo

- How to conceive an experiment that could distinguish between spontaneous and explicit CP violation?

An important but very difficult question!

Some interesting work was done using CP-odd invariants relevant for the scalar sector

G.C.B, M. N. Rebelo, J. Silva-Marcos

F. Gunion and H. Haber

B. Grzakoski, O.M. Ogreid, P. Osland

M. krawczyk, D. Sokolowska

- What is the connection between CP Violation and Family Symmetries?
W. Grimus, G. Ecker

- Can one have Geometrical CP Violation?
GCB, J.M. Gérard, W. Grimus
(1984)

Recent developments :

I. de Medeiros Varzielas

S. King

M. Lindner, M. Holthausen, M. Lindner, M. Schmidt

I. P. Ivanov, L. Lavoura

F. Feruglio

M.C. Chen, M. Fallbacher, T. Mahaithappa, M. Ratz
etc

- Are there New Sources of CP Violation beyond those present in the K. M. mechanism?

The answer is Yes!, since the SM cannot generate sufficient BAU.

But then the next question is :

- How to generate adequate BAU? Leptogenesis?
- Extended Higgs sector? Many scenarios have been proposed.
- How to test, experimentally, Leptogenesis?
- How relate CP violation relevant for Leptogenesis from leptonic CP violation, detectable at low energies through neutrino oscillations?

- 6) Answer : In general, it is not possible to establish this connection. It is conceivable to establish the connection, if leptonic flavour symmetries are introduced.
- How to solve the Strong CP problem?
Peccei - Quinn provided an elegant solution ... but Axions have not been found. Are there other plausible solutions?

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There is clear experimental evidence that V_{CKM} is complex.

Does this mean that CP is explicitly violated by complex Yukawa couplings?

Answer: NO! One may have a realistic models where CP is a good symmetry of the Lagrangian, but a vacuum phase breaks CP spontaneously and is capable of a complex CKM matrix.

Realistic models involve the introduction of vector like ... and a complex $SU(2)$ singlet.

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The Yukawa interactions are the most general terms of the Lagrangian allowed by the SM gauge group and renormalizability, involving fermions and the Higgs doublet.

$$-\mathcal{L}_Y = (Y_u)_{ij} \bar{q}_{iL} \tilde{\phi} u_{iR} + (Y_d)_{ij} \bar{q}_{iL} \phi d_{iR} \\ + (Y_e)_{ij} \bar{e}_{iL} \phi e_{iR} + h.c.$$

with $\tilde{\phi} \equiv c \tau_2 \phi^*$. The Yukawa matrices Y_u, Y_d, Y_e are arbitrary complex matrices.

After spontaneous gauge symmetry breaking, one obtains :

$$-\mathcal{L}_Y = \left\{ (Y_u)_{ij} \frac{1}{\sqrt{2}} \bar{u}_{iL}^{\circ} u_{jR}^{\circ} H + (Y_d)_{ij} \frac{1}{\sqrt{2}} \bar{d}_{iL}^{\circ} d_{jR}^{\circ} H + (Y_e)_{ij} \frac{1}{\sqrt{2}} \bar{e}_{iL}^{\circ} e_{jR}^{\circ} H + h.c. \right\}$$

$$\mathcal{L}_{\text{mass}} = (m_u)_{ij} \bar{u}_{iL}^{\circ} u_{jR}^{\circ} + (m_d)_{ij} \bar{d}_{iL}^{\circ} d_{jR}^{\circ} + (m_e)_{ij} \bar{e}_{iL}^{\circ} e_{jR}^{\circ}$$

$$m_u = \frac{v}{\sqrt{2}} Y_u ; \quad m_d = \frac{v}{\sqrt{2}} Y_d ; \quad m_e = \frac{v}{\sqrt{2}} Y_e$$

$u^{\circ}, d^{\circ}, e^{\circ} \rightarrow$ fermions in the Weak-basis

• Important feature of the SM:

fermion mass matrices are proportional to the corresponding Yukawa matrices.

Fermion mass matrices are diagonalized by bi-unitary transformations:

$$u_L^o = U_L^u u_L; d_L^o = U_L^d d_L; e_L^o = U_L^e e_L$$

$$u_R^o = U_R^u u_R; d_R^o = U_R^d d_R; e_R^o = U_R^e e_R$$

$U_{L,R}^{u,d,e}$ are unitary matrices such that:

$$U_L^u + m_u U_R^u = \text{diag.}(m_u, m_e, m_t)$$

With analogous relations for diagonalization of m_d, m_e

These bi-unitary transformations affect the charged current interactions. In the Weak-basis, one had :

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left[\bar{u}_{iL}^0 \gamma^\mu d_{iL}^0 + \bar{\nu}_{iL}^0 \gamma^\mu e_{iL}^0 \right] W_\mu^{+ + h.c.}$$

In the mass-eigenstate basis:

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \left[\bar{u}_L \gamma^\mu \underbrace{(u_L^u u_L^d)}_{d_L} + \bar{\nu}_L^0 \gamma^\mu u_L^e e_L \right]$$

$V_{CKM} = (U_L^u U_L^d)$; Since in the SM neutrinos are massless, one can redefine $\nu_L^0 \rightarrow \nu_L^0 = U_L^e \nu_L^0$

\Rightarrow no leptonic mixing !!

The electromagnetic and neutral current interactions remain invariant when one changes from the weak-basis to the mass eigenstate basis. In the weak-basis one has:

$$J_{em}^\mu = \frac{2}{3} \left[\bar{u}_L^0 \gamma^\mu u_L^0 + \bar{u}_R^0 \gamma^\mu u_R^0 \right] - \frac{1}{3} \left[\bar{d}_L^0 \gamma^\mu d_L^0 + \bar{d}_R^0 \gamma^\mu d_R^0 \right] - [\bar{e}_L \gamma^\mu e_L^0 + \bar{e}_R \gamma^\mu e_R^0]$$

In the mass eigenstate basis:

$$J_{em}^\mu = \frac{2}{3} \left[\bar{u}_L \gamma^\mu \underbrace{U_L^{u+}}_1 U_L^u u_L + \bar{u}_R U_R^{u+} U_R^u u_R \right] - \frac{1}{3} \left[\bar{d}_L U_L^{d+} U_L^d \gamma^\mu d_L + \bar{d}_R U_R^{d+} U_R^d d_R \right] - \dots$$

For the neutral current interactions:

$$\mathcal{L}_{NC} = \frac{g}{\cos \theta_W} \cdot \left[\bar{u}_L^0 \gamma^\mu u_L^0 - \bar{d}_L^0 \gamma^\mu d_L^0 + \bar{\nu}_L^0 \gamma^\mu \nu_L^0 - \bar{e}_L^0 \gamma^\mu e_L^0 - \sin^2 \theta_W J_{em}^\mu \right] \epsilon_\mu$$

In the mass eigenstate basis:

$$\mathcal{L}_{NC} = \frac{g}{\cos \theta_W} \left[\bar{u}_L \underbrace{\tilde{u}_L^+}_{1} u_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L + \bar{\nu}_L \gamma^\mu \nu_L - \bar{e} \gamma^\mu e_L - \sin^2 \theta_W J_{em}^\mu \right] \epsilon_\mu$$

Fundamental Properties of the V_{CKM} matrix

$$\mathcal{L}_{CC} = (\bar{u} \bar{c} \bar{s})_L \gamma^\mu \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix} W_\mu^+ + h.c.$$

V_{CKM} is complex, but the individual phases of its elements have no meaning, due to the freedom to rephase the mass eigenstate quark fields.

$$u_\alpha = e^{i\gamma_\alpha} u'_\alpha \quad d_K = e^{i\gamma_K} d'_K$$

Under rephasing :

$$V'_{\alpha K} = e^{i(\gamma_K - \gamma_\alpha)} V_{\alpha K}$$

It is useful to consider rephasing invariant quantities, which do not change under rephasing

Simpler examples: moduli and quartets:

$$Q_{\alpha i \beta j} = V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^*$$

with $\alpha \neq \beta, i \neq j$

Invariants of higher order can be written as functions of the quartets and moduli.

Example:

$$V_{\alpha i} V_{\beta j} V_{\gamma k} V_{\delta l}^* V_{\beta k}^* V_{\delta i}^* = \frac{Q_{\alpha i \beta j} Q_{\beta k \delta l}}{|V_{\beta i}|^2}$$

Quartets can be easily constructed using the following scheme:

$$V = \begin{bmatrix} V_{ud} & & V_{us} & V_{ub} \\ V_{cd} & & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & & V_{tb} \end{bmatrix}$$

The two quartets :

$$Q_{uscb} \equiv V_{us} V_{cb} V_{ub}^* V_{cs}^* ; Q_{cdts} \equiv V_{cd} V_{ts} V_{td}^* V_{cs}^*$$

Counting of parameters :

In the SM with n_g generations, the CKM matrix is unitary. Therefore it has in general n_g^2 parameters. But some of them can be eliminated by rephasing:

eliminated by rephasing
↑

$$N_{\text{parameters}} = n_g^2 - (2n_g - 1) = (n_g - 1)^2$$

A $n_g \times n_g$ orthogonal matrix is parametrized by

$$N_{\text{angle}} = \frac{1}{2} n_g (n_g - 1)$$

$$N_{\text{phases}} = N_{\text{par.}} - N_{\text{angle}} = \frac{1}{2} (n_g - 1)(n_g - 2)$$

Therefore, one has in the SM:

$$n_g = 1, 2 \rightarrow N_{\text{phase}} = 0$$

$$n_g = 3 \rightarrow N_{\text{phase}} = 1$$

One can show that $N_{\text{phase}} \neq 0$ is associated to CP violation in the SM. Kobayashi - Maskawa (1973)

Let us consider rephasing invariant quantity (RIQ)
For 2 generations there is only one RIQ:

$$Q_{udcs} \equiv V_{ud} V_{cd}^* V_{us}^* V_{cs}$$

But from orthogonality of V_{CKM} , one has:

$$V_{ud} V_{cd}^* + V_{us} V_{cs}^* = 0 ; \text{ Multiplying by } V_{us}^* V_{cs}$$

one obtains

$$Q_{udcs} = - |V_{us}|^2 |V_{cs}|^2 \rightarrow \text{real!}$$

Unitarity is crucial !!

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For 3 generations, orthogonality of the first two rows lead to :

$$V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0$$

Multiplying by $V_{us}^* V_{cs}$ and taking imaginary parts, one has :

$$\text{Im } Q_{udcs} = -\text{Im } Q_{ubcs}$$

For $n_g = 3$, one can show that

$$I \equiv |\text{Im } Q|$$

has the same value for all invariant quartets.
It gives the strength of CP violation in the SM

- A very special feature of the SM:
 From 4 independent moduli (for example
 $|V_{ub}|, |V_{cb}|, |V_{us}|, |V_{cd}|$) one can determine $|\text{Im } Q|$
 and therefore the
 strength of CP viol.
 in the SM.
- New Physics may
contribute !!

- This special feature of the SM is not valid in many other models. For example, does not hold true in $SU(2)_L \times SU(2)_R \times U(1)$

CP Violation

In order to study the CP properties of a Lagrangian, it is convenient to separate the \mathcal{L} in two parts :

$$\mathcal{L} = \mathcal{L}_{(CP)} + \mathcal{L}'$$

where $\mathcal{L}_{(CP)}$ is the part of the Lagrangian which one knows that conserves CP. One should allow for the most general CP transformation allowed by $\mathcal{L}_{(CP)}$.

CP can be investigated in the fermion mass eigenstate basis and in a weak basis. We shall do the analysis in both cases.

Mass-eigenstate basis - Let us consider the SM, after gauge symmetry breaking and after diagonalization of the quark mass matrices:

$$m_u = \text{diag}(m_u, m_c, m_t); \quad m_d = \text{diag}(m_d, m_s, m_b)$$

Taking into account that the quark masses are non-degenerate, the most general CP transformation is:

$$(CP) W^{+\mu}(t, \vec{r})(CP)^{-1} = -e^{i\beta_W} W^{-\mu}(t, -\vec{r})$$

$$(CP) W^{-\mu}(t, \vec{r})(CP)^{-1} = -e^{i\beta_W} W^{+\mu}(t, -\vec{r})$$

$$(CP) u_\alpha(t, \vec{r})(CP)^{-1} = e^{i\beta_\alpha} \gamma^0 C \bar{u}_\alpha^\top(t, -\vec{r})$$

$$(CP) d_K(t, \vec{r})(CP)^{-1} = e^{i\beta_K} \gamma^0 C \bar{u}^\top(t, -\vec{r})$$

Invariance of the charged current weak interactions under CP, constrains $V_{\alpha K}$ to satisfy the constraint:

$$V_{\alpha K}^* = e^{i(\beta_W + \beta_K - \beta_\alpha)} V_{\alpha K}$$

It can be easily shown that this constrains all rephasing invariant functions of V_{CKM} to be real

$$Im Q \neq 0 \Rightarrow CP \text{ violation}$$

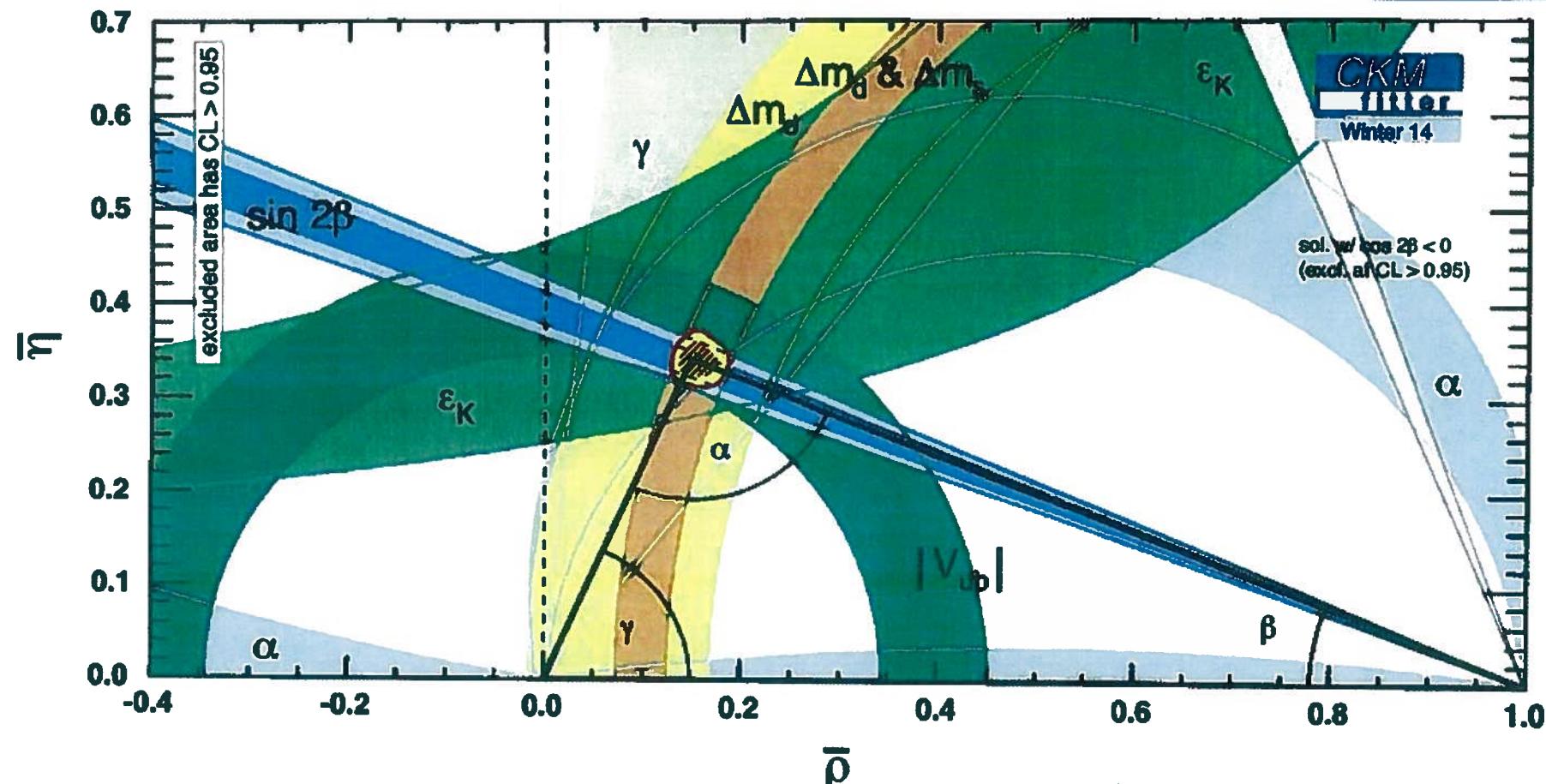
Unitarity triangles

Consider orthogonality of the first and third columns of V_{CKM} :

$$\frac{V_{ud} V_{ub}^*}{\lambda^3} + \frac{V_{cd} V_{cb}^*}{\lambda^3} + \frac{V_{td} V_{tb}^*}{\lambda^3} = 0$$

$$|V_{CKM}| \approx \begin{bmatrix} 1 & 1 & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix}$$

CP violation in the SM quark sector



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

- ▶ Current world average: $\sin 2\beta = 0.679 \pm 0.020$

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The internal angles of the triangle are rephasing invariant:

$$\alpha \equiv \arg [-V_{td} V_{ub} V_{ud}^* V_{tb}^*]$$

$$\beta \equiv \arg [-V_{cd} V_{tb} V_{cb}^* V_{td}^*]$$

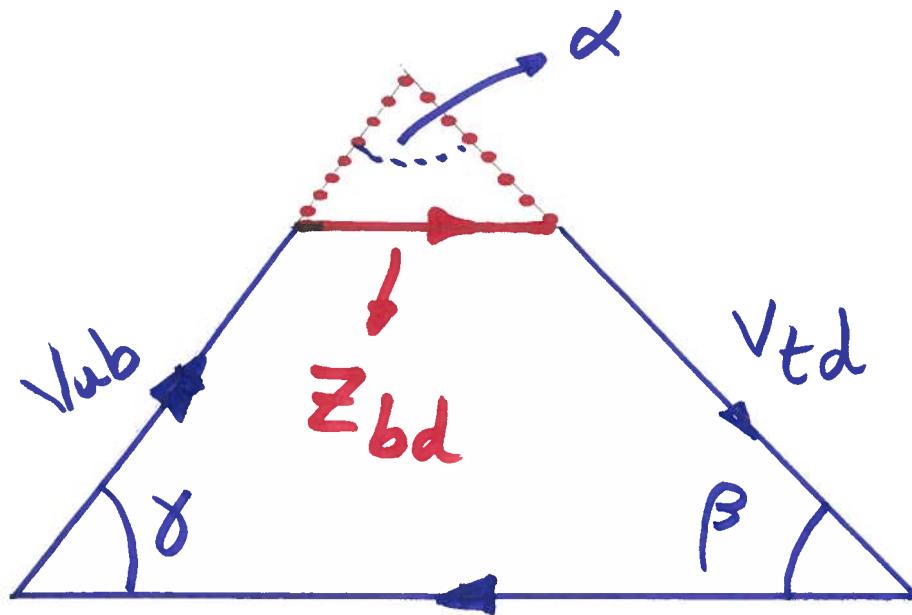
$$\gamma \equiv \arg [-V_{ud} V_{cb} V_{ub}^* V_{cd}^*]$$

one gets then:

$$\alpha + \beta + \gamma = \pi$$

This is true by definition and no test of unitarity !!

Even if 3×3 unitarity of V_{CKM} is violated one still has (of course !!) $\alpha + \beta + \gamma = \pi$



Z_{bd} reflects deviations of unitarity and the existence of **FCNC** (naturally suppressed in the case of vector-like quark models)

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The strength of CP violation in the SM is small due to the smallness of some $|V_{ij}|$ like V_{ub}, V_{cb} .

$$\lambda^3 \quad \lambda \quad \lambda^2$$

$$|Im Q| = |V_{ud} V_{ub} V_{cd} V_{cb}| \sin \delta$$

Therefore $|Im Q| \approx \lambda^6 \sin \delta$

What would be the maximal possible value for $Im Q$?

It corresponds to :

$$V = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^* \\ 1 & \omega^* & \omega \end{bmatrix}$$

$$Im Q = \frac{1}{6\sqrt{3}} \approx 0.096.$$

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A convenient parametrization of V_{CKM} is:

$$V = \begin{bmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{bmatrix}$$

$$s_{13} = |V_{ub}| ; \quad s_{12} = \frac{|V_{us}|}{\sqrt{1 - |V_{ub}|^2}} ; \quad s_{23} = \frac{|V_{cb}|}{\sqrt{1 - |V_{ub}|^2}}$$

Once s_{ij} are fixed, all other experimental data has to be fit with a single parameter δ .
The SM is very predictive !!

Invariant Approach to CP Violation

Let us consider the \mathcal{L}^{SM} , written in a weak-basis where all gauge currents are flavour diagonal. The most general CP transformation which leaves \mathcal{L}_{CP} invariant is :

$$(CP) \quad u_L^o (CP)^{-1} = e^{i\beta_W} K_L \gamma^o C \bar{u}_L^o {}^T$$

$$(CP) \quad d_L^o (CP)^{-1} = K_L \gamma^o C \bar{d}_L^o {}^T$$

$$(CP) \quad u_R^o (CP)^{-1} = K_R^u \gamma^o C \bar{u}_R^o {}^T$$

$$(CP) \quad d_R^o (CP)^{-1} = K_R^d \gamma^o C \bar{d}_R^o {}^T$$

K_L, K_R^u, K_R^d are unitary matrices acting in flavour space

It can be readily shown that in order for the Yukawa interactions (or equivalently the mass terms in m_u, m_d) to be invariant the following relations have to be satisfied : J. Bernabéu, GCB and M. Gronau

$$K_L^+ m_u K_R^u = m_u^* \quad ; \quad K_L^+ m_d K_R^d = m_d^*$$

The existence of the matrices K_L, K_R^u, K_R^d is a necessary and sufficient condition for CP invariance in the SM, for any number of generations! From the above Eqs one derives :

$$K_L^+ H_u K_L = H_u^* = H_u^T \quad ; \quad K_L^+ H_d K_L = H_d^* = H_d^T ; \quad H_{d,u}^* = M_d M_u^T$$

$$\text{So: } K_L^+ [H_u, H_d] K_L = [H_u^T, H_d^T] = - [H_u, H_d]^T$$

and one obtains

$$\boxed{\text{Tr} [H_u, H_d]^r = 0 \quad (\text{r odd})}$$

The minimal non-trivial case corresponds to $r=3$.

For two generations the invariant automatically vanishes

For 3 generations one obtains :

$$\text{tr} [H_u, H_d]^3 = 6 i (m_t^2 - m_c^2) (m_t^2 - m_u^2) \times (m_c^2 - m_u^2) \times \\ (m_b^2 - m_s^2) (m_b^2 - m_d^2) (m_s^2 - m_d^2) \text{Im } Q$$

$\text{tr} [H_u, H_d]^3$ is a necessary condition for CP invariance for any number of generations. For $n_g = 3$, it is a necessary and sufficient condition for CP invariance. For $n_g = 3$ one has also :

$$\text{tr} [H_u, H_d]^3 = 3 \det [H_u, H_d] \quad \text{Jarlsson}$$

The question of Neutrino mass

In the SM neutrinos are strictly massless

- Neutrinos cannot have a Dirac mass because ν_R is not introduced in the SM
- Neutrinos in the SM cannot have a Direct mass because it is not gauge invariant:

$$\nu_L^T C M_L \nu_L$$

Also at tree-level no mass, since ^{no} Higgs triplet is introduced

- A non-vanishing neutrino mass cannot be generated in the SM in higher orders of perturbation or through non-perturbative effects due to exact B-L conservation

Therefore, the discovery of neutrino masses and oscillations, rules out the SM!!

(Um) Fortunately a minimal extension of the SM solves the problem

Why Glashow, Weinberg and Salam
did not introduce ν_R ?

Probable reason : They wanted to avoid
non-vanishing neutrino masses ! Seesaw mechanism
 was not known. The mechanism was intro-
 duced in more ambitious extensions of the
 SM, like

$SU(5)$; $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$
 etc., where ν_R could not be avoided.

Minimal extension of the SM :

$$SM \rightarrow \check{V}SM$$

Just add ν_R to the spectrum of the SM
and write the most general Lagrangian
consistent with gauge invariance. This
includes the term :

$$\nu_R^T C M_R \nu_R$$

Since ν_R is a singlet of $SU(2) \times U(1)$,
this term is gauge invariant.

Furthermore, one expects M_R to be large, since it is not "protected" by gauge invariance. This automatically leads to the seesaw mechanism:

3 light neutrinos which enter in neutrino oscillations with a mass

$$m_\nu \approx \frac{M_D^2}{M_R}$$

3 have neutrinos with mass of order M_R

CP Violation in the leptonic Sector

If neutrinos are Dirac particles, there is essential no modification with respect to the quark sector. If neutrinos are Majorana particles, some subtleties arise.

Let us assume that nature chooses Majorana Neutrinos. Without loss of generality, one can choose to work in a weak-basis where the charged-lepton mass matrix is diagonal:

$$M_L = d_L = \text{diag.}(m_e, m_\mu, m_\tau)$$

In this basis, the Majorana neutrino mass matrix is a 3×3 complex symmetric mass matrix. One can still make the rephasing:

$$\ell'_{L,R} = K_L \ell_{L,R} \quad \nu'_L = K_L \nu_L$$

with $K_L = \text{diag.}(e^{i\varphi_1} e^{i\varphi_2} e^{i\varphi_3})$. Under

this rephasing :

$$(m'_\nu)_{ij} = e^{i(\varphi_i + \varphi_j)} (m_\nu)_{ij}$$

Through this rephasing one can eliminate n phases from m_ν .

The total number of phases in m_V is

$$N_{\text{phases}} = \frac{1}{2} n(n+1) - n = \frac{1}{2} n(n-1)$$


 number of phases
 in a complex symmetric matrix

So for $n=3$, one has 3 CP violating phases in m_V . The individual phases of m_V have no physical meaning. But one can construct rephasing invariant polynomials:

$$P_1 = (m_V^*)_{11} (m_V^*)_{22} (m_{12})^2$$

$$P_2 = (m_V^*)_{11} (m_V^*)_{33} (m_{13})^2$$

$$P_3 = (m_V^*)_{33} (m_V^*)_{12} (m_{13}) (m_{12})_{23}$$

In the mass eigenstate basis the charged currents interactions can be written:

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} (\bar{e} \bar{\mu} \bar{\tau}) \overrightarrow{\delta^m} \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

How many CP violating phases. Recall that in V_{CKM} one could eliminate 5 phases through rephasing. In this case one can only rephase the 3 charged lepton fields, so one can only eliminate 3 phases. So altogether one has 3 CP violating phases.

So one can write

$$U = \underbrace{V}_{\text{one phase}} K \xrightarrow{\text{PMNS}} \text{Majorana type phases}$$

$$K = \text{diag}(1, e^{i\alpha_1}, e^{i\alpha_2})$$

Within See-saw models

$\sqrt{V_{\text{PMNS}}}$ is non-unitary

Gavela, B et al

Recall that in V^{CKM} the only physically meaningful full phases were the arguments of quark fields.

A novel feature in the leptonic sector with Majorana neutrinos: one has rephasing invariant bi-linears:

$$\arg(U_{\ell\alpha} U_{\ell\beta}^*)$$

Majorana-type
phases

GCB, M.N. Rebelo

Invariant approach to Majorana type CP violation.

One can derive CP-odd WB invariants sensitive to Majorana type CP violation :

$$I_{\text{Majorana}}^{\text{CP}} = \text{Im} \text{Tr}(m_1 m_1^\dagger m_2 m_2^\dagger m_1^\ast m_2^\dagger m_1^\ast m_2)$$

In the case of $n_g = 2$

$$I_{\text{Majorana}}^{\text{CP}} = \frac{1}{4} m_1 m_2 \Delta m_{21}^2 \left(\frac{m_\mu^2 - m_e^2}{m_\mu^2 + m_e^2} \right) \times \sin^2 \Theta \sin 2\delta$$

where PMNS matrix is :

$$U = \begin{bmatrix} \cos \theta & -\sin \theta e^{i\delta} \\ \sin \theta e^{-i\delta} & \cos \theta \end{bmatrix}$$

$\delta = \pi/2$
CP invariant

Conclusions

- It is clear that there are new sources of CP violation. They are needed (e.g.) to generate sufficient BAU. The Question is: where will they appear **experimentally**?
- Flavours and CP violation are intimately connected. We need one good idea from theory and to continue the fantastic work from experiment
- Message for young people: remember Bell inequalities !!