

Unlocking the Higgs with the matrix element method

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HiggsTools Journal Club

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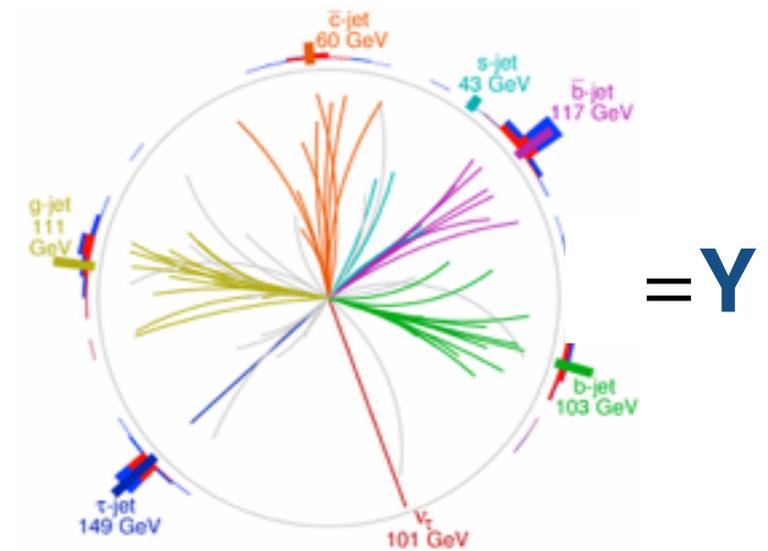
Overview

- Statistical context
- How we use the Matrix Element Method for hypothesis testing
- Why it is (specially) useful for Higgs studies
- Future developments of MEM

Not a lecture! Let's ask questions and discuss!

The statistical problem

- Given an observed event (data) \mathbf{Y} , extract interesting quantities (e.g. theory parameters Θ)

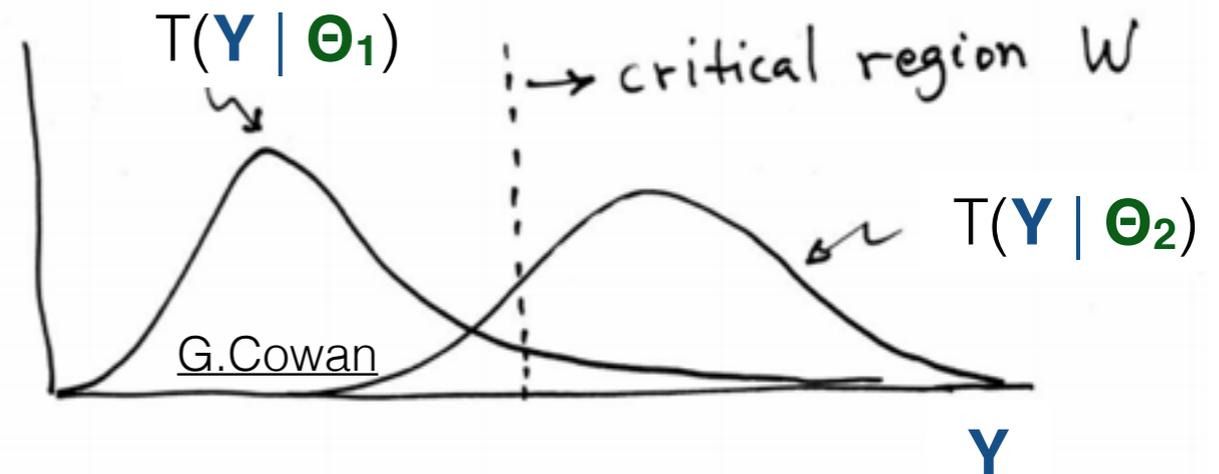


- sufficient statistic $T(\mathbf{Y})$: no loss of information about data

“bkg”

“signal”

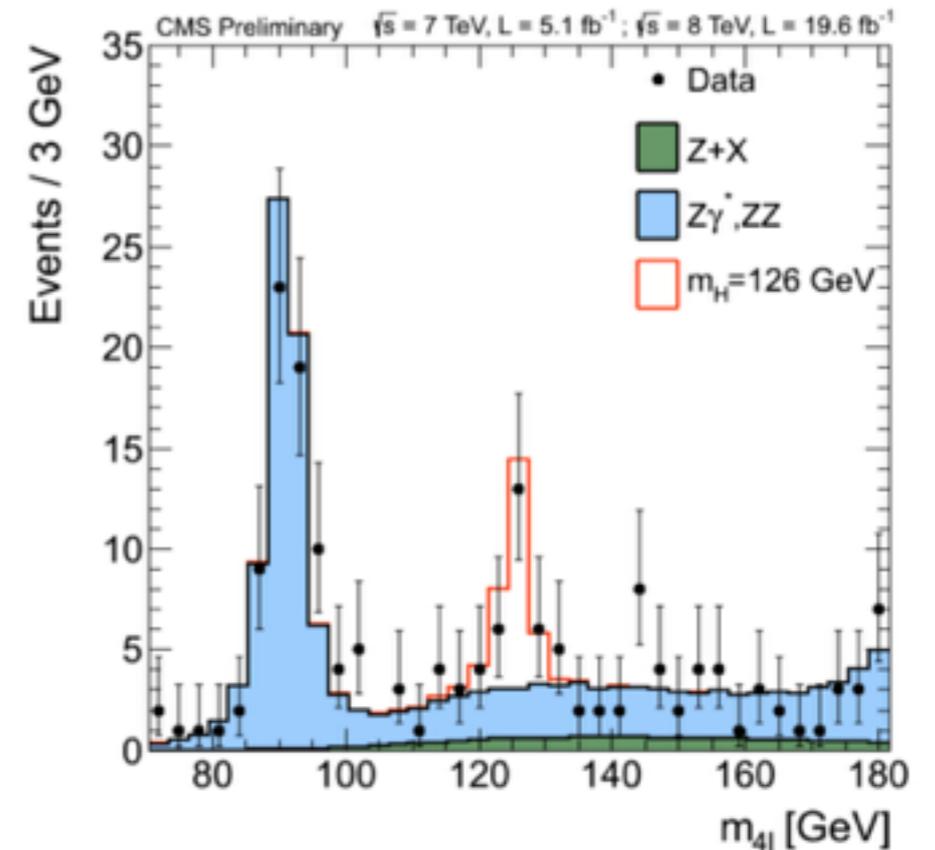
- maximize $T(\mathbf{Y} | \Theta)$ wrt. $\Theta \rightarrow$ maximum likelihood is asymptotically sufficient



How to find a good test statistic $T(\mathbf{Y} | \Theta)$?

The usual approach

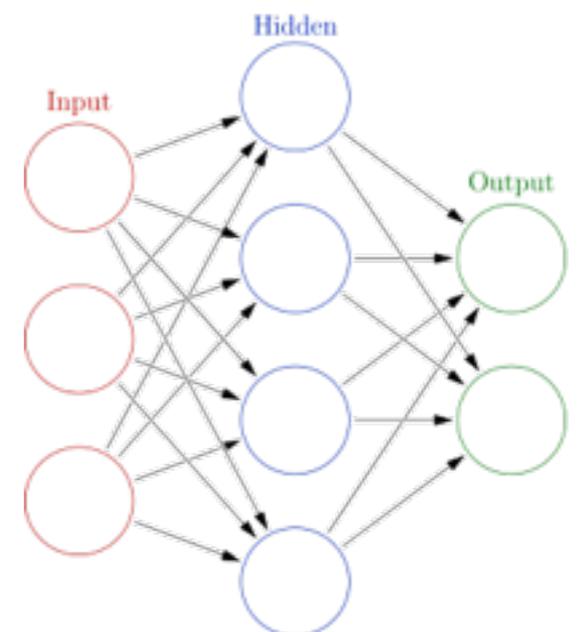
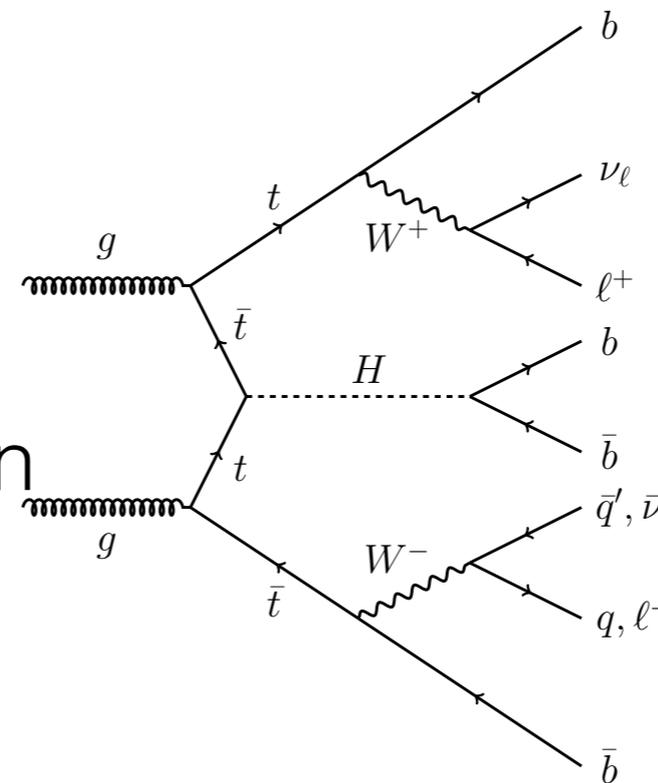
- Construct a statistic T that is sensitive to Θ , e.g Higgs candidate mass M if Θ is the theory mass
- Estimate the distribution by generating MC events
 - detector simulation \rightarrow cuts \rightarrow differential distribution $T(M | \Theta)$
 - maximize wrt. Θ (template method), but need new templates (simulation) for each Θ and nuisance



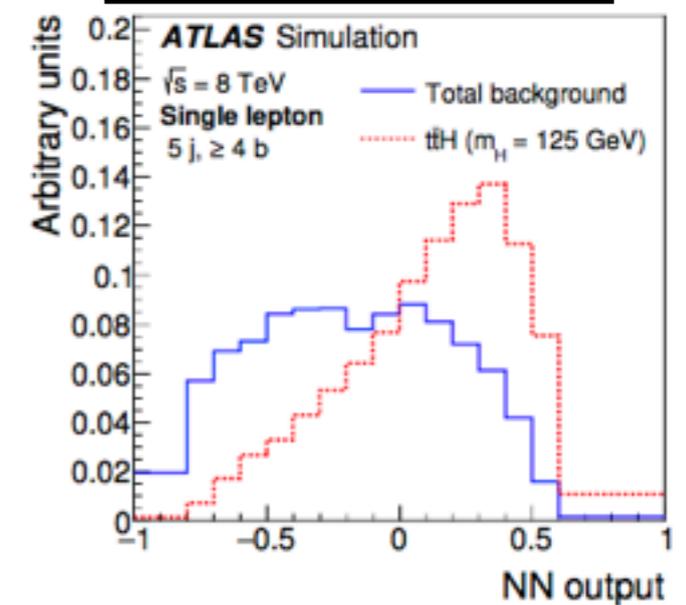
The difficulty

- complicated final states, high dimension, simple N-dimensional likelihood fails
- Combine measured quantities $\mathbf{Y} \rightarrow T(\mathbf{Y})$ to minimize information loss: MVA
- High MC simulation statistics required to estimate $T(\mathbf{Y})!$
- **Can we calculate an event weight directly?**

ttH(bb): 2 \rightarrow 8 scattering



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building the MEM

- Parton-level \mathbf{X} : 4-momenta of the scattering

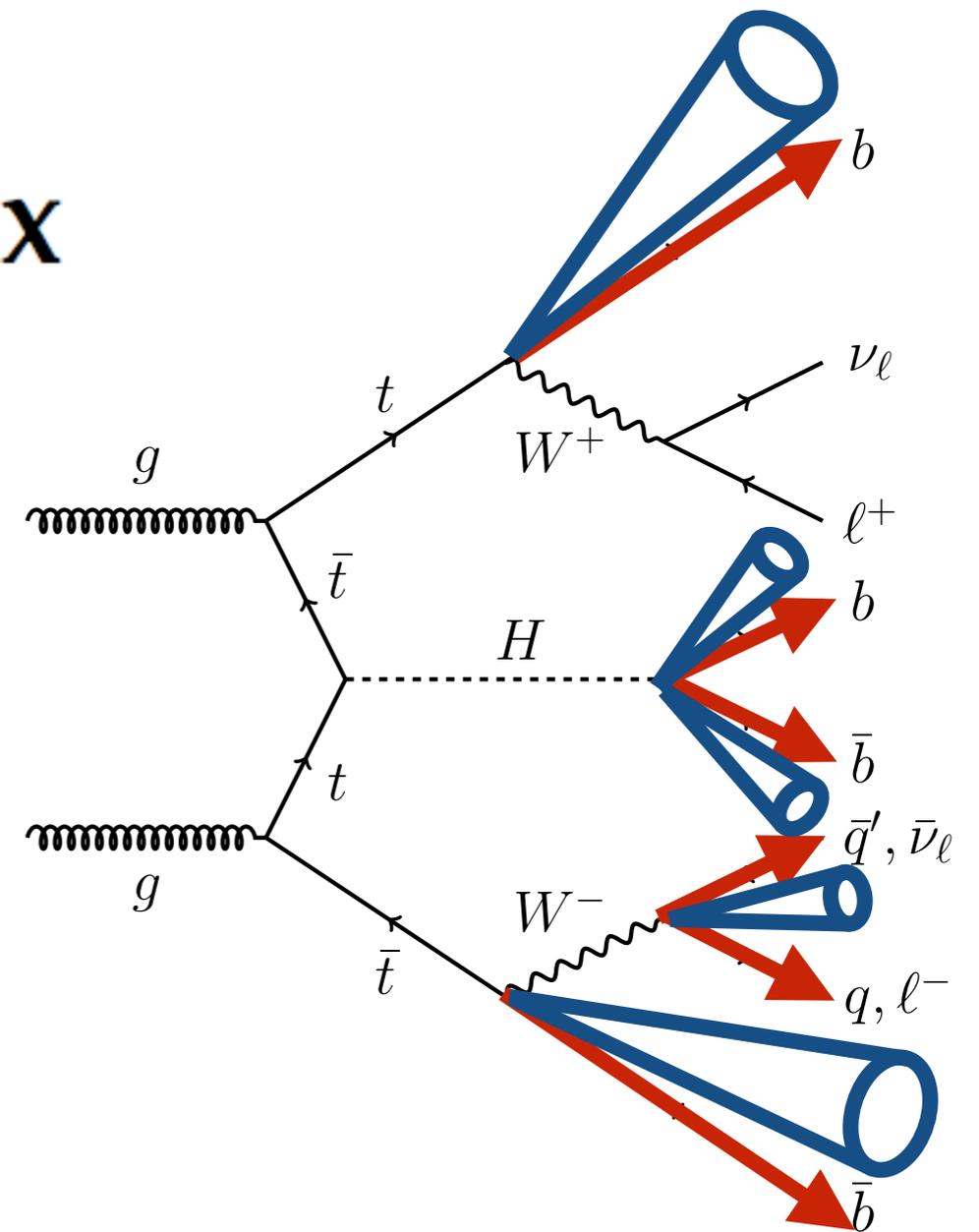
$$dP_{\mathbf{X}}(H_{\theta}) \propto f_a(x_1) f_b(x_2) |\mathcal{M}(\mathbf{X}|H_{\theta})|^2 d\mathbf{X}$$

- detector transfer function $\mathcal{W}(\mathbf{Y}, \mathbf{X})$ from simulation

$$dP_{\mathbf{Y}|\mathbf{X}} = \mathcal{W}(\mathbf{Y}, \mathbf{X}) d\mathbf{Y}$$

- Measured quantities \mathbf{Y} : jet, lepton momenta, MET

$$dP_{\mathbf{Y}}(H_{\theta}) = \int \left[dP_{\mathbf{X}}(H_{\theta}) \times \mathcal{W}(\mathbf{Y}, \mathbf{X}) \right] d\mathbf{Y} = p_{H_{\theta}}(\mathbf{Y}) d\mathbf{Y}$$



Interpreting as weight

- Given differential probability

$$dP_{\mathbf{Y}}(H_{\theta}) = \int \left[dP_{\mathbf{X}}(H_{\theta}) \times \mathcal{W}(\mathbf{Y}, \mathbf{X}) \right] d\mathbf{Y} = p_{H_{\theta}}(\mathbf{Y}) d\mathbf{Y}$$

- Interpret as differential cross-section

$$p_{H_{\theta}}(\mathbf{Y}) = \frac{1}{\sigma_{H_{\theta}}} \frac{d\sigma_{H_{\theta}}}{d\mathbf{Y}}$$

- Which is just an event weight, given hypothesis H_{θ}

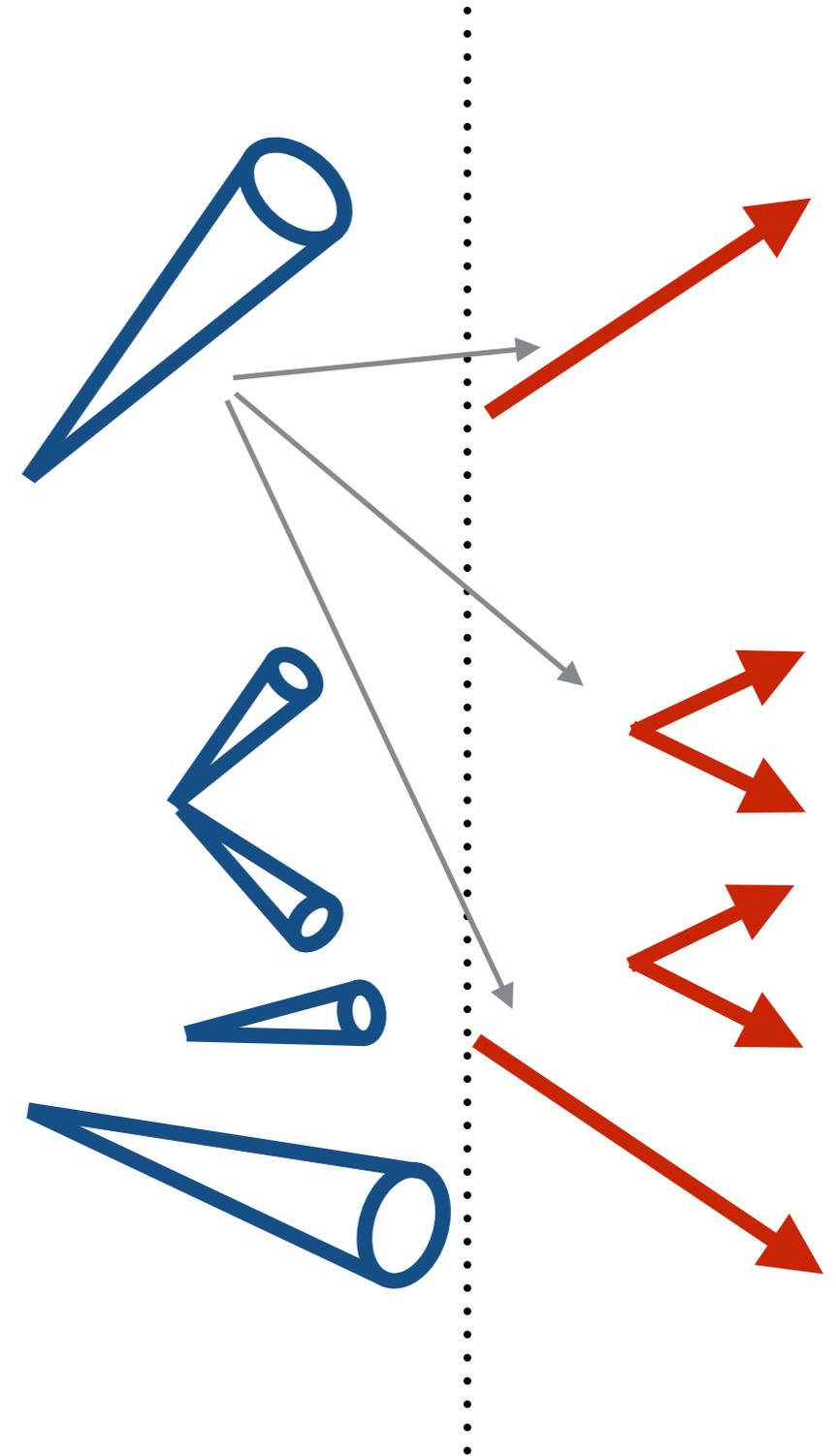
$$\mathcal{P}(\mathbf{Y}|H_{\theta}) = \frac{1}{\sigma_{H_{\theta}}} \int d\mathbf{X} \int dx_a dx_b f(x_a, x_b) |\mathcal{M}(\mathbf{X}|H_{\theta})|^2 \mathcal{W}(\mathbf{Y}, \mathbf{X}).$$

Detector information

- Measure detector response to tree-level process
- For jets: parton shower and hadronization: $E_q, \Omega_q \rightarrow E_j, \Omega_j$, energy-dependent double-gaussian
- Factorize objects, energy/direction, assume continuity
- Capture simplified detector & simulation properties in a closed (albeit parametric) form
- **Question: didn't we just move the simulation problem? Which hadronization stage to correct to?**

Association

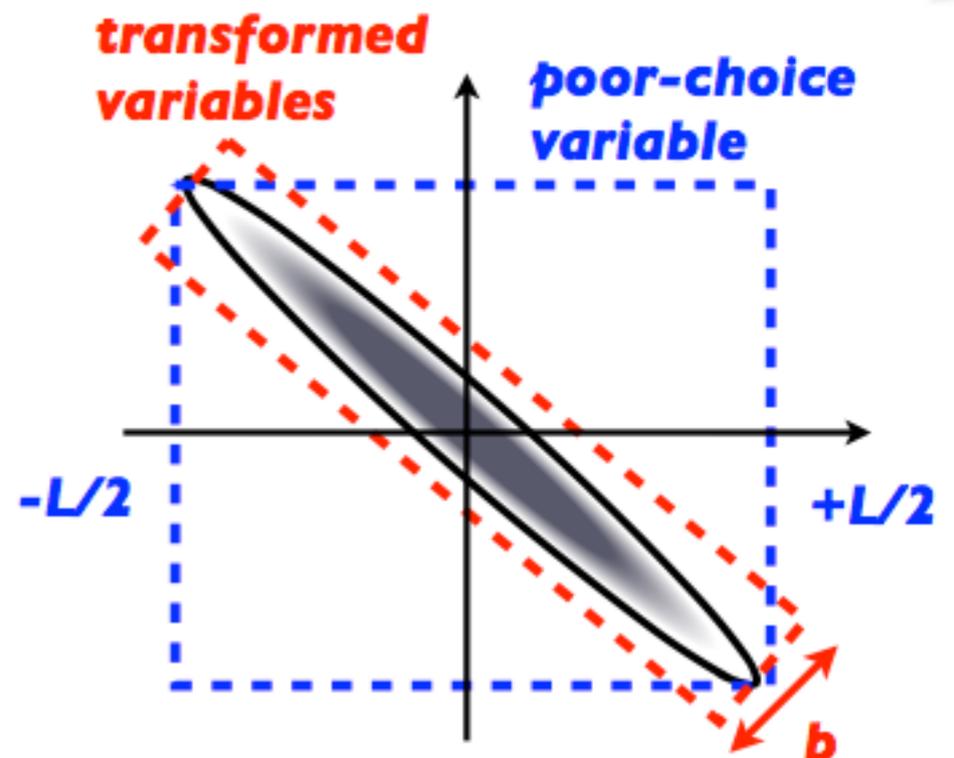
- Associating parton-level (**N_q quarks**) to detector-level (**N_j jets**): permutations (up to hundreds)
- Can (and should!) use additional information to restrict
 - e.g. is jet from b-quark or light quark?
- **Question: what is the optimal way of associating?**



Integration

- Integrand an oscillating function: transfer functions, resonant propagators, 4-momentum conservation
- Derive Jacobian factor for each topology: currently artigianal
- **Question: how to benefit from existing technology?**

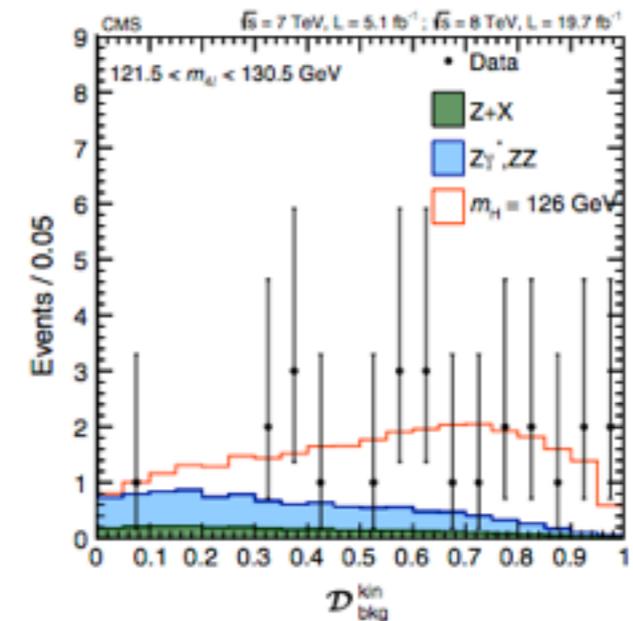
$$\frac{1}{(q_k^2 - m_k^2)^2 + m_k^2 \Gamma_k^2},$$



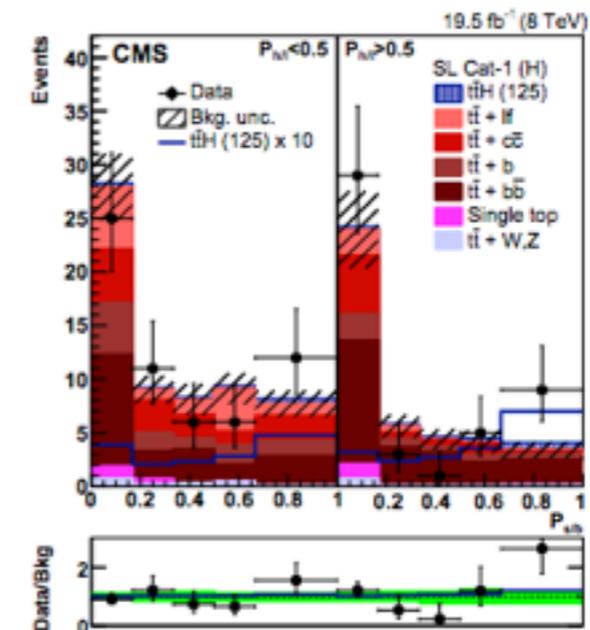
Applications

- **Point estimation:** mass, spin, parity
- **Searches:** $tt+H$, single-top, WH
- perfect resolution \rightarrow simplify transfer functions, PDFs: kinematic discriminant (MELA, MEKD)
- Framework for extending theory concepts to experiment

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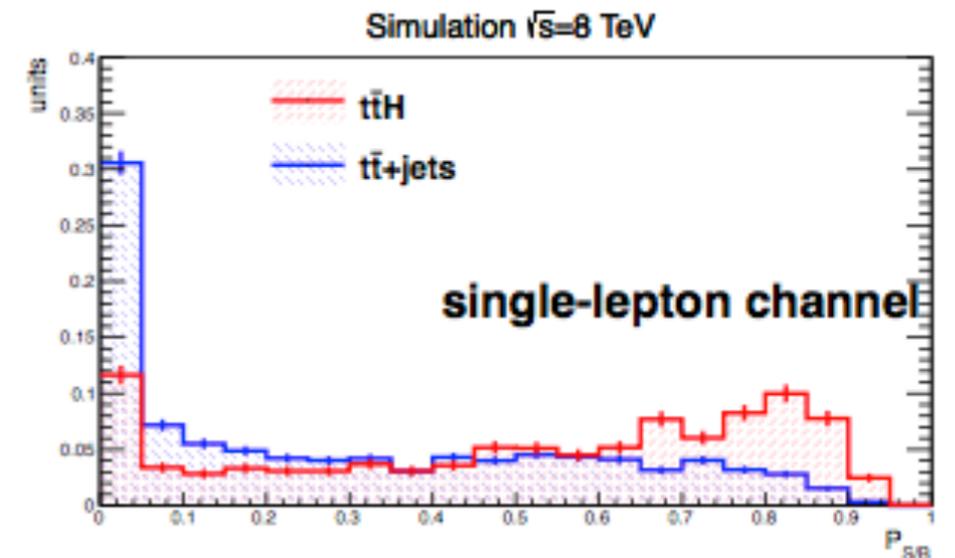
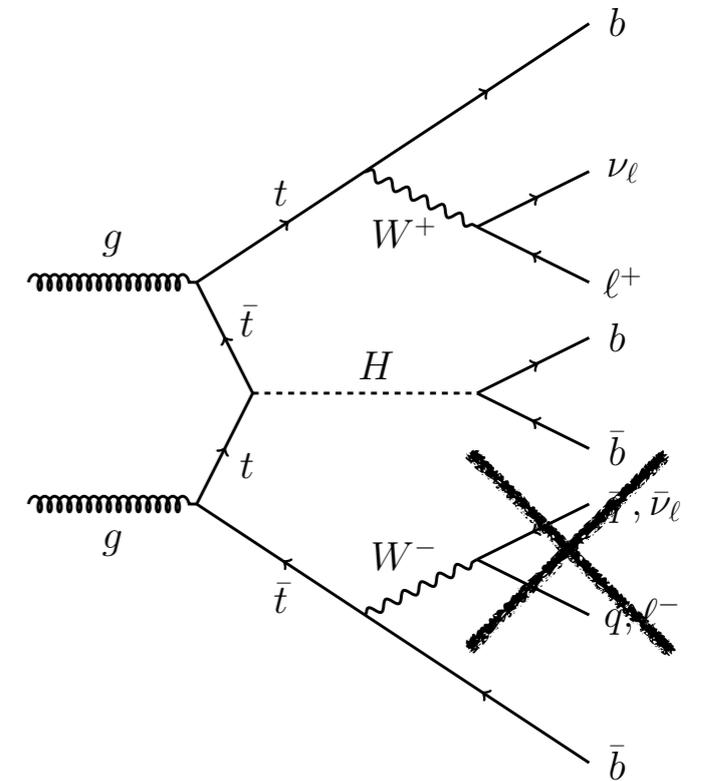
H \rightarrow ZZ \rightarrow 4l

MEKD: 1210.0896

MELA: 1208.4018

Ongoing in run II

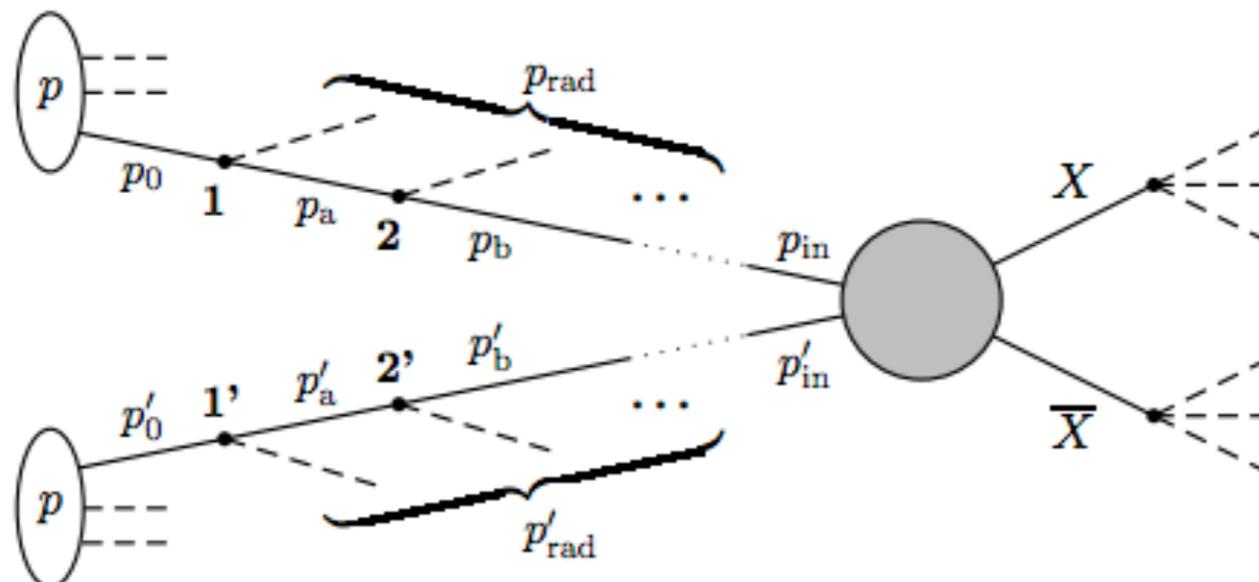
- Extending to unreconstructed final states (resolve $P \sim 0$)
- Multiple event hypotheses: integrate over unknowns
- Leveraging machine-learning: MEM/ML based on final state topology
- $t\bar{t}H(bb)$ fully-hadronic, substructure
- NLO extensions
- distributed computing



Moving to NLO

- **ISR**: kinematics changed, possible bias in e.g. measured mass
 - boost with ISR momentum to “MEM frame”
 - Sudakov factors @ parton level: $\mathcal{P}_j(p_{T,E}^2, z) \quad z \approx \frac{p_{in,z}}{p_{in,z} + p_{rad,z}}$,
 - transfer function for ISR at hadron level $W_{ISR}(p_T, p_T^{vis})$

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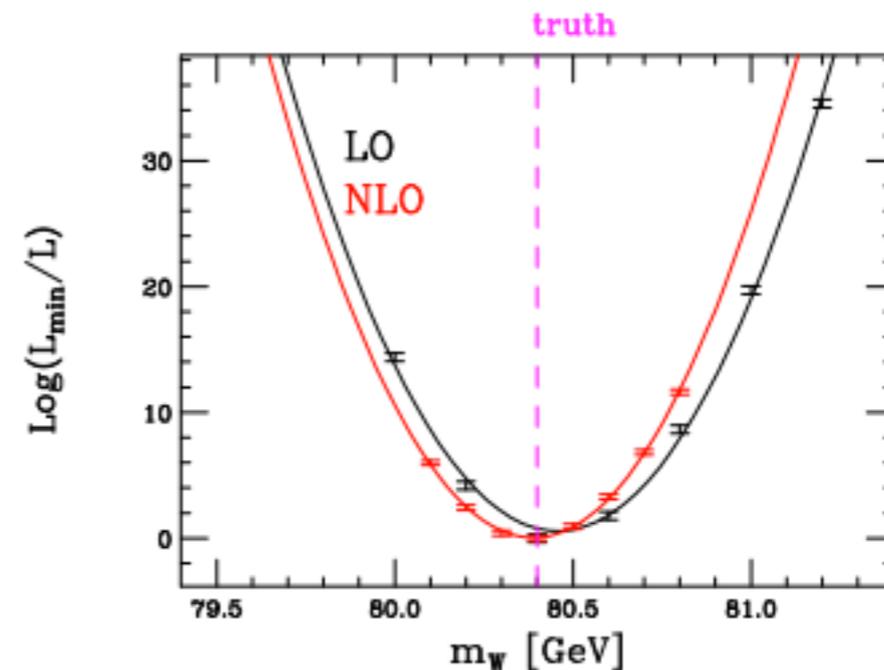


Question: how to benefit from automated NLO?

Real & virtual

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1301.7086

- If NLO known, can “just” integrate real & virtual
- Real emissions boost events back to acceptance region
- NLO covers larger phase space, sensitivity improved by $\sim O(10\%)$
- But phase space is non-trivial



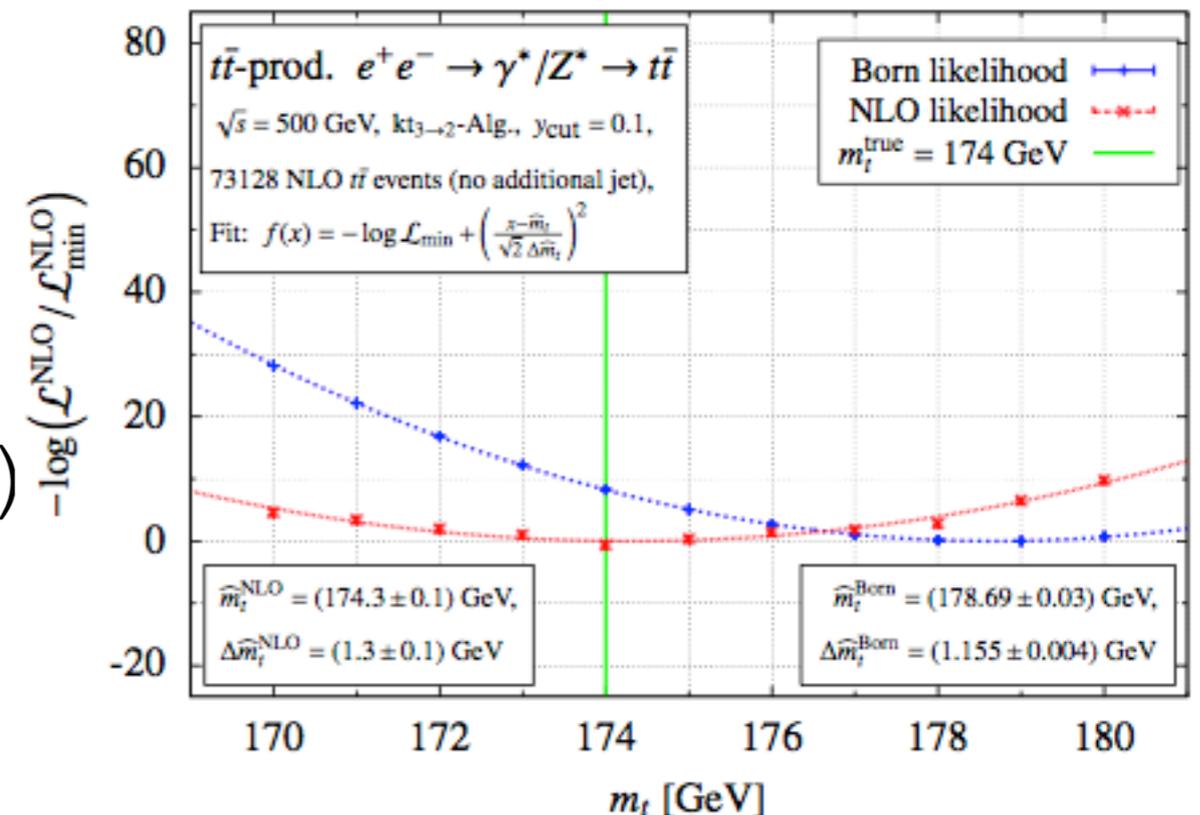
$$\tilde{P}_{LO}(\tilde{\phi}) = \frac{1}{\sigma_{LO}} \int dx_1 dx_2 d\phi \delta(x_1 x_2 s - Q^2) \times f^j(x_1) f^i(x_2) \mathcal{B}_{ij}(x_1, x_2, \phi) W(\phi, \tilde{\phi}).$$

$$\tilde{P}_{NLO}(\tilde{\phi}) = \frac{1}{\sigma_{NLO}} \int d\phi (V(\phi) + R(\phi)) W(\phi, \tilde{\phi}).$$

Jet algorithms

- Real: n jets in final state, $n+1$ parton phase space
- recombine jets: massive jets from massless partons!
- need a modified clustering algorithm (3- \rightarrow 2 instead of 2- \rightarrow 1)
- factorize PS as n -jet + unresolved

1506.08798



$$\frac{d\sigma_{n\text{-jet}}^{\text{NLO}}(\Omega)}{dJ_1 \dots dJ_n} = \frac{d\sigma^{\text{B+V}}(\Omega)}{dJ_1 \dots dJ_n} + \int dR_{\text{unres}}(\Phi) \frac{d\sigma^{\text{R}}(\Omega)}{dp_1 \dots dp_{n+1}}.$$

Machine learning

- **Standard ML**: throw simulation to a BDT/ANN
- **Standard MEM**: build best possible (N)LO discriminant, simulation in TF-s
- Combined approach: ML to choose MEM hypothesis based on reconstructed properties, combine with best possible simulation of processes

or

- Use ML to identify and calibrate objects, feed to best analytical discriminant

Variable	Definition	NN rank			
		$\geq 6j, \geq 4b$	$\geq 6j, 3b$	$5j, \geq 4b$	$5j, 3b$
$D1$	Neyman–Pearson MEM discriminant (Eq. (4))	1	10	-	-
Centrality	Scalar sum of the p_T divided by sum of the E for all jets and the lepton	2	2	1	-
p_T^{jet5}	p_T of the fifth leading jet	3	7	-	-
$H1$	Second Fox–Wolfram moment computed using all jets and the lepton	4	3	2	-

Summary

- MEM can replace “black-box” multivariate thinking
- Can be more sensitive, less simulation needed to test different hypotheses
- But there are subtleties to be discussed and understood with theorists
- Finding an optimal discriminant is hard
- No single automatic state-of-the-art package, many home-made implementations