



MAPPING DARK MATTER IN THE MILKY WAY

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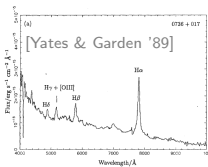
HISTORICAL PARENTHESIS: ANDROMEDA



The kinematics of an object is a prime tool to learn about its mass.

The kinematics of Andromeda has been studied since the 1930s through the Doppler shift of spectral lines in the gas.

$$\Delta\nu = -\frac{v_{\text{LOS}}}{c} \nu_0$$

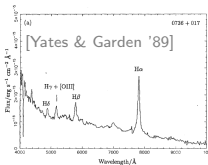


HISTORICAL PARENTHESIS: ANDROMEDA

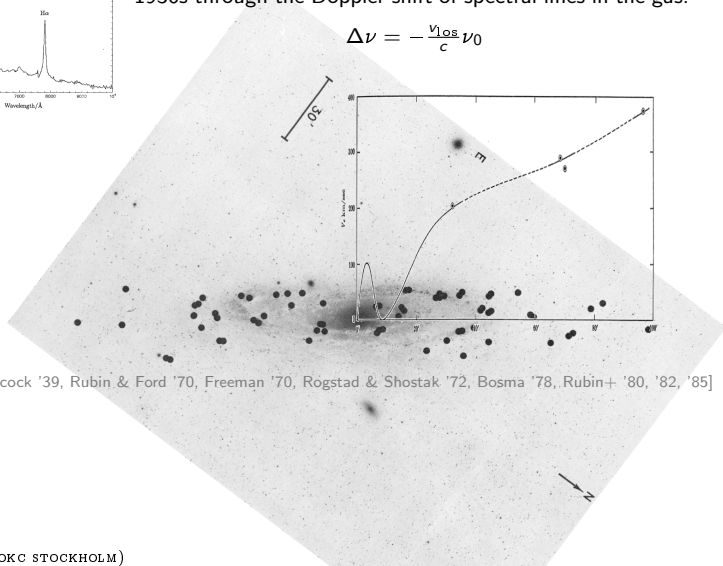


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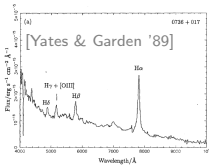


[Babcock '39, Rubin & Ford '70, Freeman '70, Røstvad & Shostak '72, Bosma '78, Rubin+ '80, '82, '85]

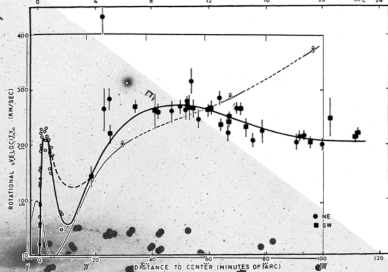
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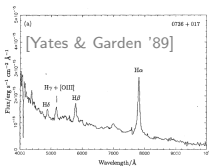
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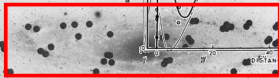
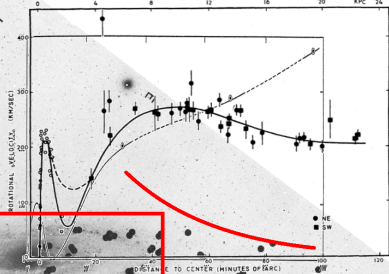
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$$\Delta\nu = -\frac{v_{\text{los}}}{c} \nu_0$$



visible matter

[Babcock '39, Rubin & Ford '70, Freeman '70, Røstgard & Shostak '72, Bosma '78, Rubin+ '80, '82, '85]

Under Newtonian gravity, a spherical mass induces

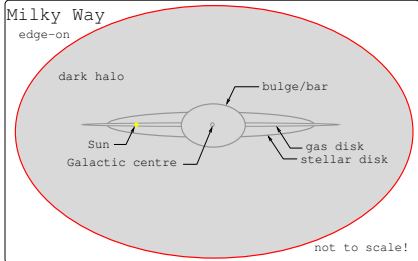
$$v_c^2 = \frac{GM(< r)}{r}$$

The rotation provided by the visible mass falls off as $v_c \propto 1/\sqrt{r}$ at large r . A flat rotation curve implies* a dark matter halo with $M(< r) \propto r$.



1. TOUR OF THE GALAXY

The Milky Way is a complex bound system of stars, gas and dark matter.



We can identify the following main components:

- supermassive black hole, with mass $4 \times 10^6 M_{\odot}$;
- stellar bulge, with barred shape of scale length $2 - 3$ kpc and mass $10^{10} M_{\odot}$;
- stellar disc, decomposed into thin and thick components of scale length 10 kpc and total mass $10^{10} M_{\odot}$ with a marked spiral structure;
- gas, in molecular, atomic and ionised phases (mainly H) with a patchy distribution towards the centre and a disc-like structure otherwise; and
- dark matter halo, extending hundreds of kpc.

$$\phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$$

how can we constrain the parameters of a galactic mass model?

The Sun is located slightly above the Galactic plane at $R_0 \simeq 8$ kpc from the Galactic centre, in between two major spiral arms, and travels together with the local standard of rest at about 220 km/s in a roughly circular orbit.

1. TOUR OF THE GALAXY

$$\phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$$

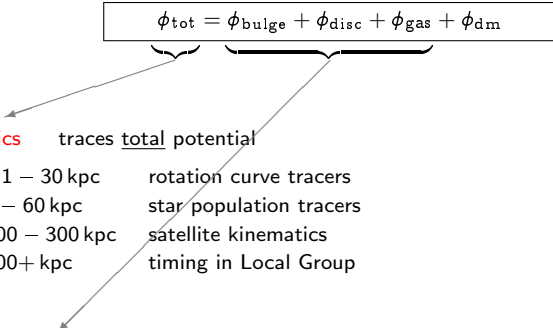
kinematics traces total potential

- $R \sim 0.1 - 30$ kpc rotation curve tracers
- $R \sim 8 - 60$ kpc star population tracers
- $R \sim 100 - 300$ kpc satellite kinematics
- $R \sim 300+$ kpc timing in Local Group

photometry traces individual baryonic components

- bulge star counts, luminosity, microlensing
- disc star counts, luminosity, stellar dynamics
- gas emission lines, dispersion measure

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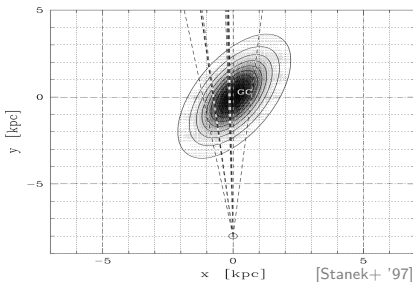
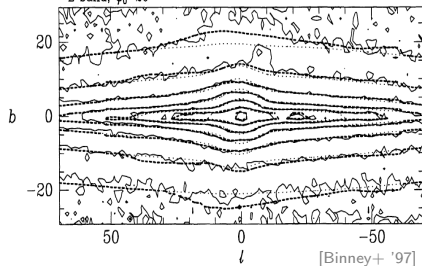
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1. TOUR OF THE GALAXY: STELLAR BULGE

$$\rho_{\text{bulge}} = \rho_0 f(x, y, z)$$

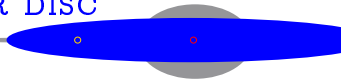
morphology $f(x, y, z)$

L band, $\phi_0=20$



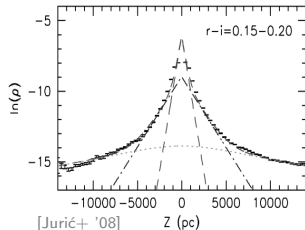
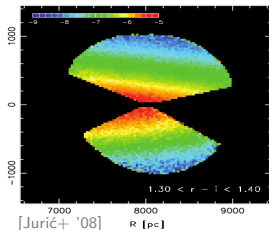
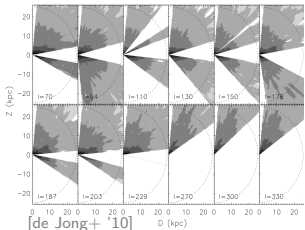
Stanek+ '97 (E2)	e^{-r}	0.9:0.4:0.3	24°	optical
Stanek+ '97 (G2)	$e^{-r_s^2/2}$	1.2:0.6:0.4	25°	optical
Zhao '96	$e^{-r_s^2/2} + r_a^{-1.85} e^{-r_a}$	1.5:0.6:0.4	20°	infrared
Bissantz & Gerhard '02	$e^{-r_s^2}/(1+r)^{1.8}$	2.8:0.9:1.1	20°	infrared
Lopez-Corredoira+ '07	Ferrer potential	7.8:1.2:0.2	43°	infrared/optical
Vanhollebecke+ '09	$e^{-r_s^2}/(1+r)^{1.8}$	2.6:1.8:0.8	15°	infrared/optical
Robin+ '12	$\text{sech}^2(-r_s) + e^{-r_s}$	1.5:0.5:0.4	13°	infrared

1. TOUR OF THE GALAXY: STELLAR DISC



$$\rho_{\text{disc}} = \rho_0 f(x, y, z)$$

morphology $f(x, y, z)$



Han & Gould '03

$$e^{-R} \text{sech}^2(z)$$

2.8:0.27

thin

optical

$$e^{-R-|z|}$$

2.8:0.44

thick

Calchi-Novati & Mancini '11

$$e^{-R-|z|}$$

2.8:0.25

thin

optical

$$e^{-R-|z|}$$

4.1:0.75

thick

de Jong+ '10

$$e^{-R-|z|}$$

2.8:0.25

thin

optical

$$e^{-R-|z|}$$

4.1:0.75

thick

$$(R^2 + z^2)^{-2.75/2}$$

1.0:0.88

halo

Jurić+ '08

$$e^{-R-|z|}$$

2.2:0.25

thin

optical

$$e^{-R-|z|}$$

3.3:0.74

thick

$$(R^2 + z^2)^{-2.77/2}$$

1.0:0.64

halo

Bovy & Rix '13

$$e^{-R-|z|}$$

2.2:0.40

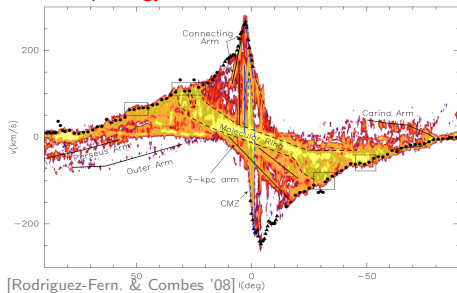
single

optical

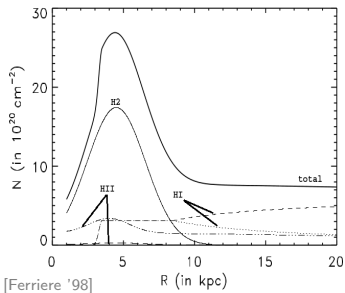
1. TOUR OF THE GALAXY: GAS

$$n_{\text{H}} = 2n_{\text{H}_2} + n_{\text{HI}} + n_{\text{HII}}$$

morphology



[Rodríguez-Fern. & Combes '08] $l(\text{deg})$

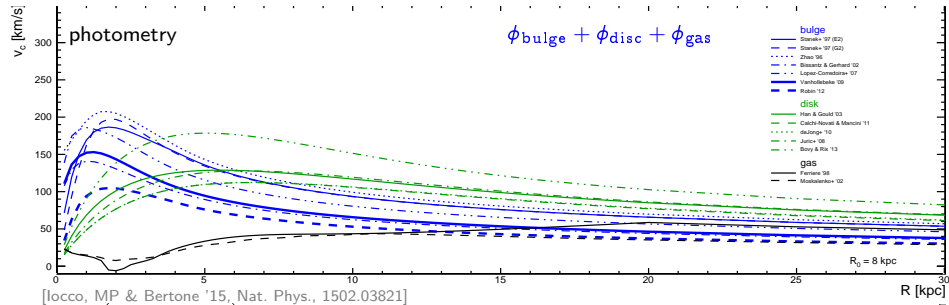


[Ferrière '98]

Ferrière '12	$r < 0.01$ kpc	$M_{\text{gas}} \sim 7 \times 10^5 M_{\odot}$	CO, 21cm, H α , ...
Ferrière+ '07	$r = 0.01 - 2$ kpc	CMZ, holed disc CMZ, holed disc warm, hot, very hot	H ₂ CO H I 21cm H II disp. meas.
Ferrière '98	$r = 3 - 20$ kpc	molecular ring cold, warm warm, hot	H ₂ CO H I 21cm H II disp. meas., H α
Moskalenko+ '02	$r = 3 - 20$ kpc	molecular ring	H ₂ CO H I 21cm H II disp. meas.

1. TOUR OF THE GALAXY: PHOTOMETRY

$$\phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$$



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$$\phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$$

The equation $\phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$ is shown in a box. A bracket under ϕ_{bulge} has an arrow pointing to the kinematics box. A bracket under $\phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$ has an arrow pointing to the photometry box.

kinematics traces total potential

$R \sim 0.1 - 30$ kpc	rotation curve tracers
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$R \sim 100 - 300$ kpc	satellite kinematics
$R \sim 300+$ kpc	timing in Local Group

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bulge	star counts, luminosity, microlensing
disc	star counts, luminosity, stellar dynamics
gas	emission lines, dispersion measure

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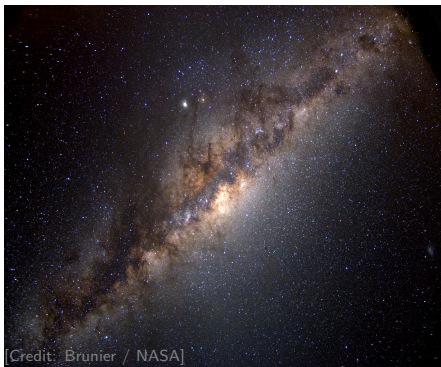
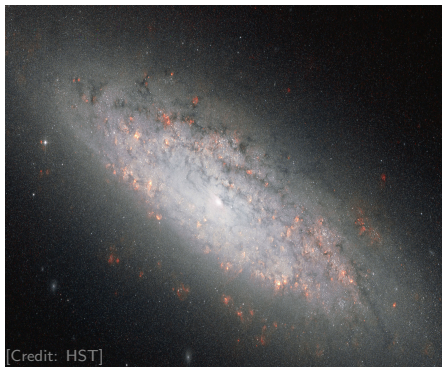
disc star counts, luminosity, stellar dynamics

gas emission lines, dispersion measure

1. TOUR OF THE GALAXY: ROTATION CURVE

$$v_c^2 = r \frac{d\phi_{\text{tot}}}{dr} \stackrel{\text{sph.}}{=} \frac{G M_{\text{tot}}(< r)}{r}$$

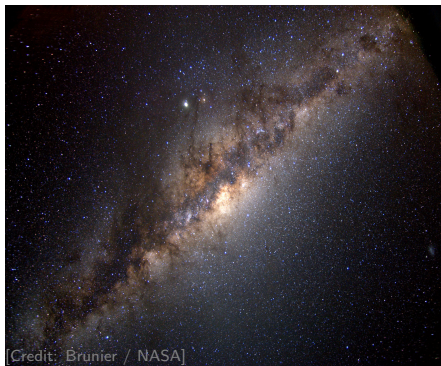
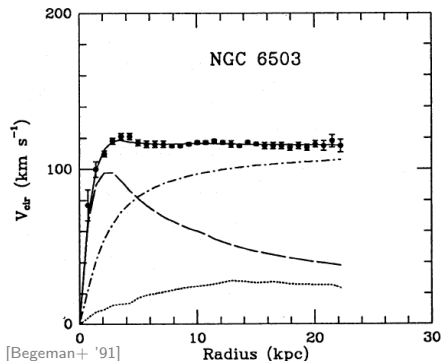
Rotation curve tracers are young objects or regions that track galactic rotation. In external galaxies the only available tracer is the gas, while in our Galaxy we can use also some stars and star-forming regions. However, the case of our Galaxy is much more challenging due to our position.



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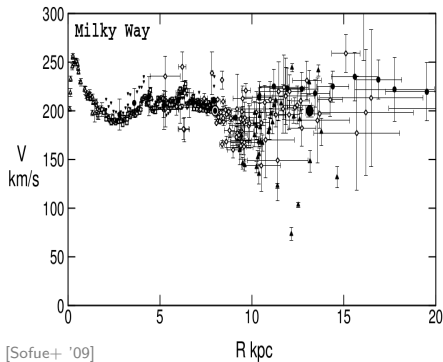
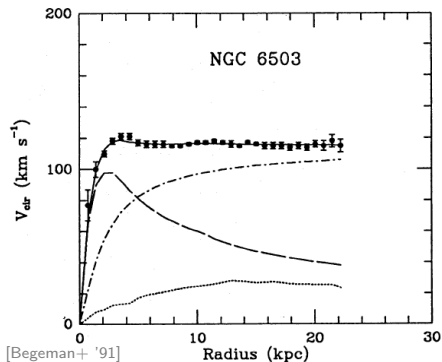
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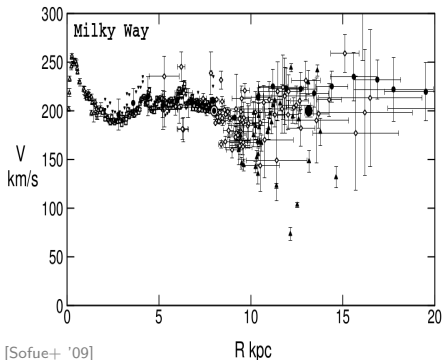
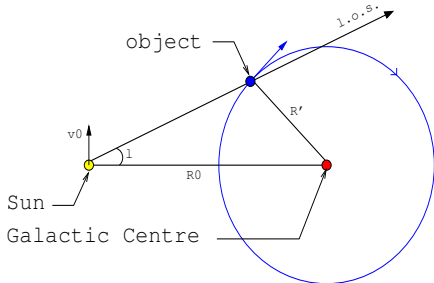
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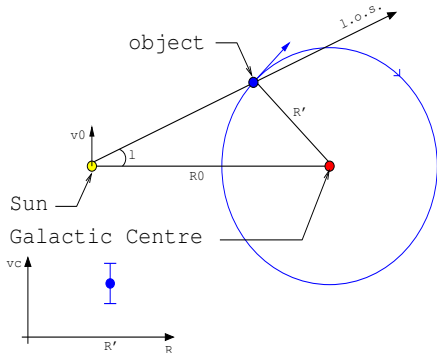


$$v_{\text{lsr}}^{\text{los}} = \left(\frac{v_c(R')}{R'/R_0} - v_0 \right) \cos b \sin l$$

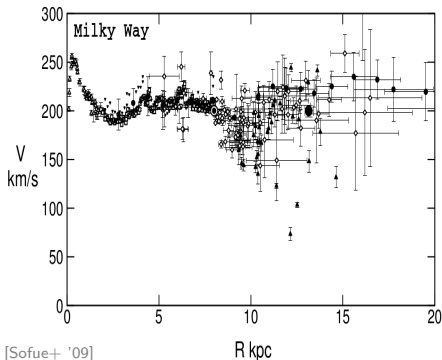
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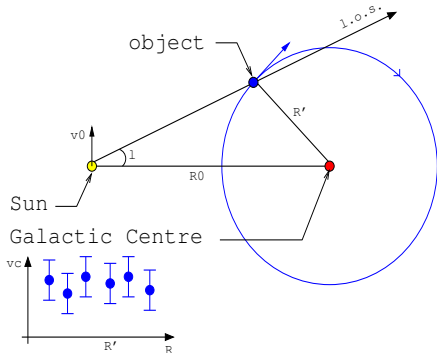
$$v_{\text{l sr}}^{\text{los}} = \left(\frac{v_c(R')}{R'/R_0} - v_0 \right) \cos b \sin \ell$$



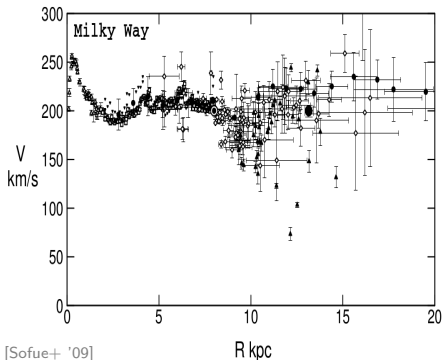
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1. TOUR OF THE GALAXY: ROTATION CURVE

gas

Object type	R [kpc]	quadrants	# objects
HI terminal velocities			
Fich+ '89	2.1 – 8.0	1,4	149
Malhotra '95	2.1 – 7.5	1,4	110
McClure-Griffiths & Dickey '07	2.8 – 7.6	4	701
HI thickness method			
Honma & Sofue '97	6.8 – 20.2	–	13
CO terminal velocities			
Burton & Gordon '78	1.4 – 7.9	1	284
Clemens '85	1.9 – 8.0	1	143
Knapp+ '85	0.6 – 7.8	1	37
Luna+ '06	2.0 – 8.0	4	272
HII regions			
Blitz '79	8.7 – 11.0	2,3	3
Fich+ '89	9.4 – 12.5	3	5
Turbide & Moffat '93	11.8 – 14.7	3	5
Brand & Blitz '93	5.2 – 16.5	1,2,3,4	148
Hou+ '09	3.5 – 15.5	1,2,3,4	274
giant molecular clouds			
Hou+ '09	6.0 – 13.7	1,2,3,4	30
open clusters			
Frinchaboy & Majewski '08	4.6 – 10.7	1,2,3,4	60
planetary nebulae			
Durand+ '98	3.6 – 12.6	1,2,3,4	79
classical cepheids			
Pont+ '94	5.1 – 14.4	1,2,3,4	245
Pont+ '97	10.2 – 18.5	2,3,4	32
carbon stars			
Demers & Battinelli '07	9.3 – 22.2	1,2,3	55
Battinelli+ '13	12.1 – 24.8	1,2	35
masers			
Reid+ '14	4.0 – 15.6	1,2,3,4	80
Honma+ '12	7.7 – 9.9	1,2,3,4	11
Stepanishchev & Bobylev '11	8.3	3	1
Xu+ '13	7.9	4	1
Bobylev & Bajkova '13	4.7 – 9.4	1,2,4	7

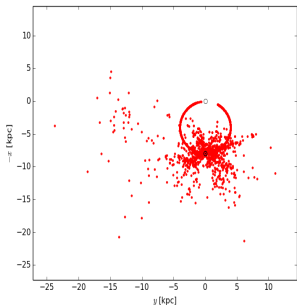
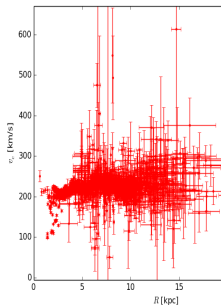
stars

masers

(GALKIN)

coming soon: galkin, public code in python

```
#####  
# galkin, version 1.2, by Miguel Pato and Fabio Iocco  
# Last update: MP 30 Jun 2015.  
#####  
# A tool to handle the available data on the rotation  
#####  
  
### read input ###  
launching window...
```



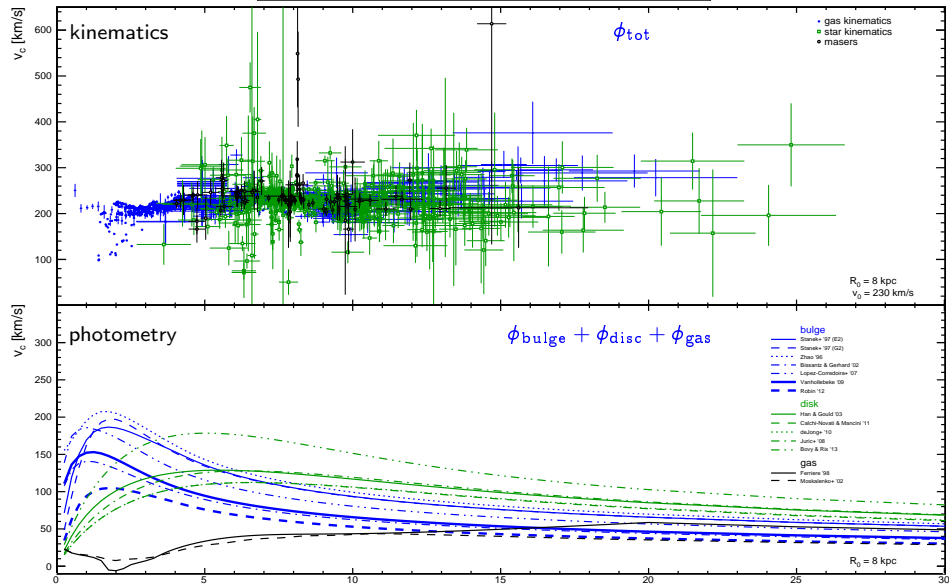
The screenshot shows a dialog box titled "enter input parameters" with the following sections:

- galactic parameters**
 - RO [kpc]= 8.0
 - V0 [km/s]= 230.0
 - syst [km/s]= 0.0
 - Usun [km/s]= 11.10
 - Vsun [km/s]= 12.24
 - Wsun [km/s]= 07.25
- data to use**
 - HI terminal velocities
 - Fich+ 89 (Table 2)
 - Malhotra 95
 - McClure-Griffiths & Dickey 07
 - HI thickness
 - Honma & Sofue 97
 - CO terminal velocities
 - Burton & Gordon 78
 - Clemens 85
 - Knapp+ 85
 - Luna+ 06
 - HII regions
 - Blitz 79
 - Fich+ 89 (Table 1)
 - Turbide & Moffat 93
 - Brand & Blitz 93
 - Hou+ 09 (Table A1)
 - giant molecular clouds
 - Hou+ 09 (Table A2)
 - open clusters
 - Frinchaboy & Majewski 08
 - planetary nebulae
 - Durand+ 98
 - classical cepheids
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 - Pont+ 97
 - carbon stars
 - Demers & Battinelli 07
 - Battinelli+ 12
 - masers
 - Reid+ 14
 - Honma+ 12
 - Stepanishchev & Bobylev 11
 - Xu+ 13
 - Bobylev & Bajkova 13

user-friendly interface
data & parameter selection
output rotation curve
output positional data

1. TOUR OF THE GALAXY: SUMMARY

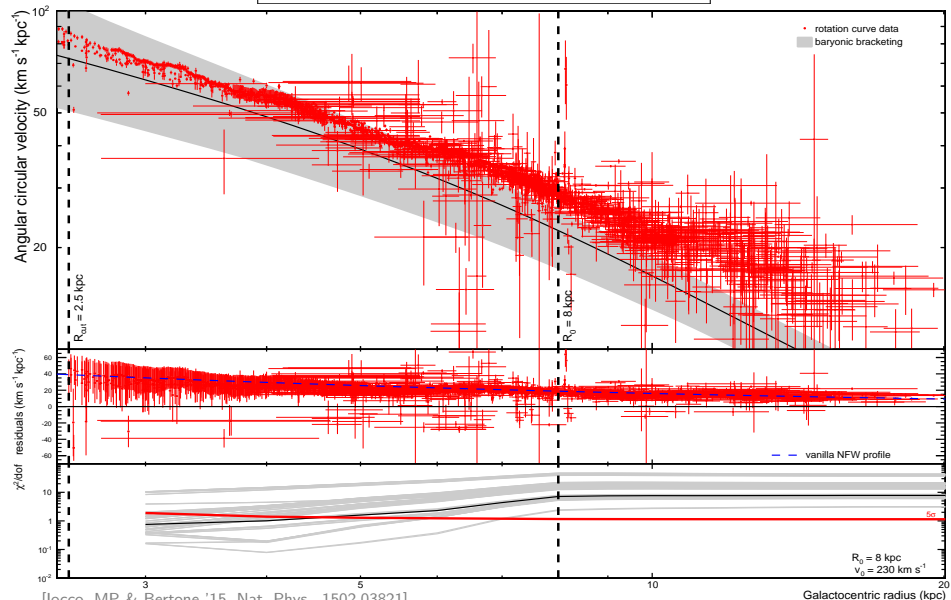
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[Iocco, MP & Bertone '15, Nat. Phys., 1502.03821]

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$$\phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$$



2. DARK MATTER: LOCALISE OR GLOBALISE?

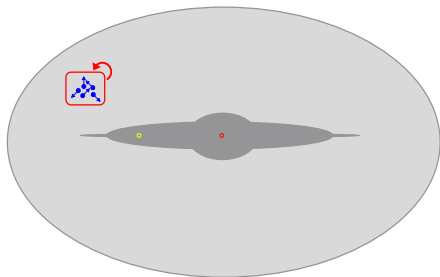
local methods

vs

global methods

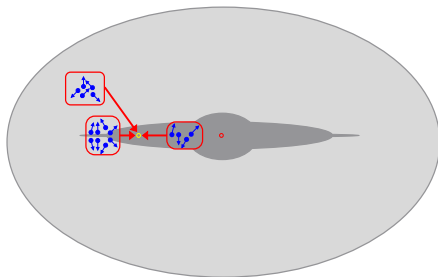
aim: use data from a patch of the sky to derive dynamics there.

- + “assumption-free”
- low precision



aim: use data across the Galaxy to derive dynamics somewhere.

- global assumptions
- + high precision



[Kapteyn '22, Jeans '22, Oort '32, Hill '60, Oort '60, Bahcall '84, Bienaymé+ '87, Kuijken & Gilmore '91, Bahcall+ '92, Creze+ '98, Holmberg & Flynn '00, Holmberg & Flynn '04, Bienaymé+ '06, Garbari+ '11 '12, Moni Bidin+ '12, Bovy & Tremaine '12, Smith+ '12, Zhang+ '13, Bovy & Rix '13, Loebman+ '14, Moni Bidin+ '14]

[Caldwell & Ostriker '81, Gates+ '95, Dehnen & Binney '98, Sakamoto+ '03, Dehnen+ '06, Xue+ '08, Sofue+ '09, Strigari & Trotta '09, Catena & Ullio '10, Weber & de Boer '10, Salucci+ '10, Iocco+ '11, McMillan '11, Nesti & Salucci '13, Bhattacharjee+ '14, Kafle+ '14, MP & Iocco '15, MP, Iocco & Bertone '15, Sofue '15]

2. LOCAL METHODS

In a galaxy star encounters are rare and stars feel on average the smooth gravitational potential. We can therefore treat a set of stars as a collisionless gas and apply the collisionless Boltzmann equation, whose first momentum gives the **Jeans equations**:

$$-\rho_s \frac{\partial \phi_{\text{tot}}}{\partial x_j} = \frac{\partial(\rho_s \bar{v}_j)}{\partial t} + \sum_i \frac{\partial(\rho_s \bar{v}_i \bar{v}_{ij})}{\partial x_i} \quad , \quad j = 1, 2, 3 \text{ (cartesian)} .$$

We can couple this to the **Poisson equation**: $4\pi G \rho_{\text{tot}} = \nabla^2 \phi_{\text{tot}}$.

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$$\phi_{\text{tot}}(R, z) \quad \partial/\partial t \rightarrow 0 \quad -F_R = \partial\phi_{\text{tot}}/\partial R \quad -F_z = \partial\phi_{\text{tot}}/\partial z$$

$$\begin{aligned} -4\pi G \rho_{\text{tot}} &= \frac{1}{R} \frac{\partial}{\partial R} (R F_R) + \frac{\partial F_z}{\partial z} & F_R &= \frac{1}{\rho_s} \left(\frac{\partial(\rho_s \bar{v}_R^2)}{\partial R} + \frac{\partial(\rho_s \bar{v}_R \bar{v}_z)}{\partial z} \right) + \frac{\bar{v}_R^2 - \bar{v}_\phi^2}{R} \\ & & F_z &= \frac{1}{\rho_s} \left(\frac{\partial(\rho_s \bar{v}_R \bar{v}_z)}{\partial R} + \frac{\partial(\rho_s \bar{v}_z^2)}{\partial z} \right) + \frac{\bar{v}_R \bar{v}_z}{R} \end{aligned}$$

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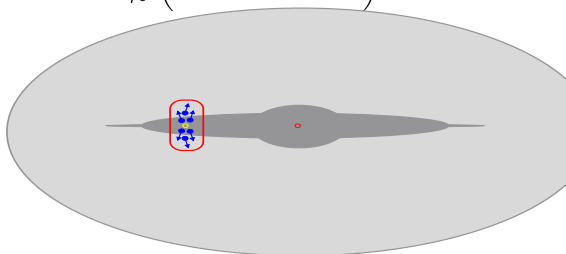
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$$F_z = \frac{1}{\rho_s} \left(\frac{\partial(\rho_s \cancel{v_R v_z})}{\partial R} + \frac{\partial(\rho_s \bar{v}_z^2)}{\partial z} \right) + \frac{\cancel{v_R v_z}}{R}$$

$$-4\pi G \rho_{\text{tot}} = \frac{\partial}{\partial z} \left(\frac{1}{\rho_s} \frac{\partial(\rho_s \bar{v}_z^2)}{\partial z} \right)$$



This is the so-called Oort limit.

2. LOCAL METHODS

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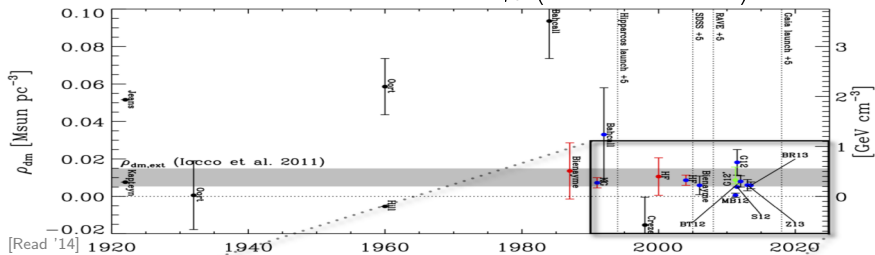
$$-\rho_s \frac{\partial \phi_{\text{tot}}}{\partial x_j} = \frac{\partial(\rho_s \bar{v}_j)}{\partial t} + \sum_i \frac{\partial(\rho_s \bar{v}_i \bar{v}_j)}{\partial x_i} \quad , \quad j = 1, 2, 3 \text{ (cartesian).}$$

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2. DARK MATTER: LOCALISE OR GLOBALISE?

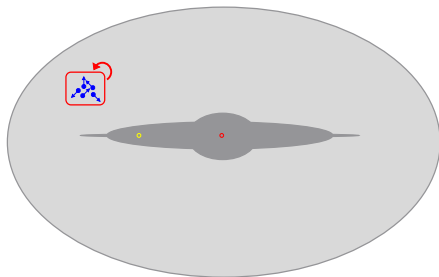
local methods

vs

global methods

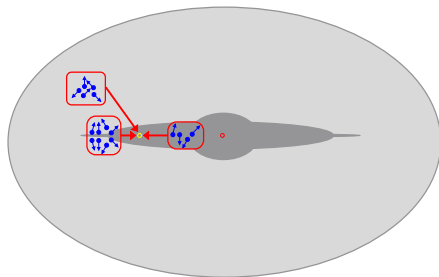
aim: use data from a patch of the sky to derive dynamics there.

- + “assumption-free”
- low precision



aim: use data across the Galaxy to derive dynamics somewhere.

- global assumptions
- + high precision

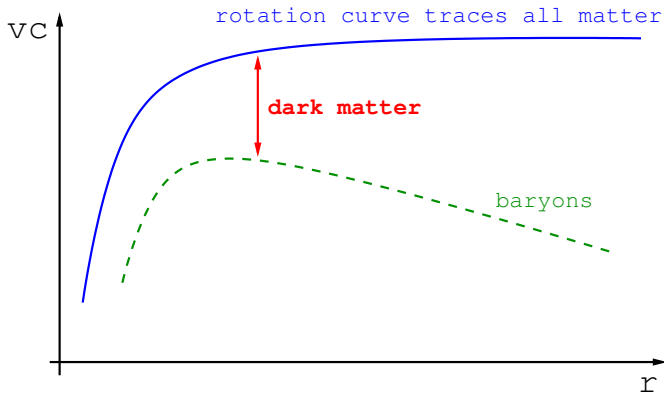


[Kapteyn '22, Jeans '22, Oort '32, Hill '60, Oort '60, Bahcall '84, Bienaymé+ '87, Kuijken & Gilmore '91, Bahcall+ '92, Creze+ '98, Holmberg & Flynn '00, Holmberg & Flynn '04, Bienaymé+ '06, Garbari+ '11 '12, Moni Bidin+ '12, Bovy & Tremaine '12, Smith+ '12, Zhang+ '13, Bovy & Rix '13, Loebman+ '14, Moni Bidin+ '14]

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2. GLOBAL METHODS

$$\phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$$

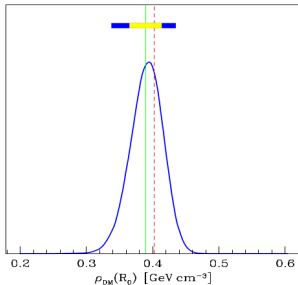


$$v_c^2 = v_b^2 + v_{\text{dm}}^2$$

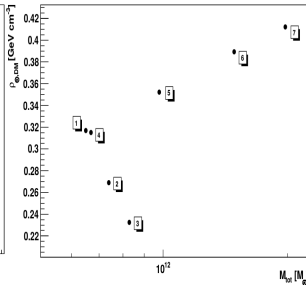
$$v_{\text{dm}}^2 \stackrel{\text{sph.}}{=} G M_{\text{dm}}(< r)/r \rightarrow \rho_{\text{dm}}$$

[Dehnen & Binney '98, Sofue+ '09, Catena & Ullio '10, Weber & de Boer '10, Salucci+ '10, McMillan '11, Iocco+ '11, Nesti & Salucci '13, Sofue '15]

2. PROFILE FITTING



[Catena & Ullio '10]

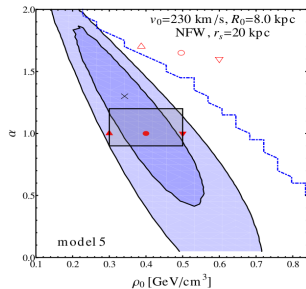


[Weber & de Boer '10]

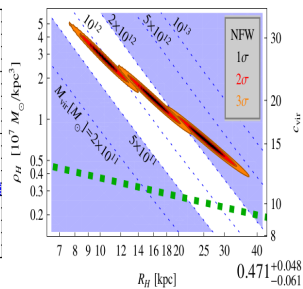
$$\rho_{\odot} = \left(0.430 \pm 0.113_{(a_{\odot})} \pm 0.096_{(r_{\odot D})} \right) \frac{\text{GeV}}{\text{cm}^3}$$

[Salucci '10]

$$0.40 \pm 0.04 \text{ GeV cm}^{-3}$$



[Iocco+ '11]

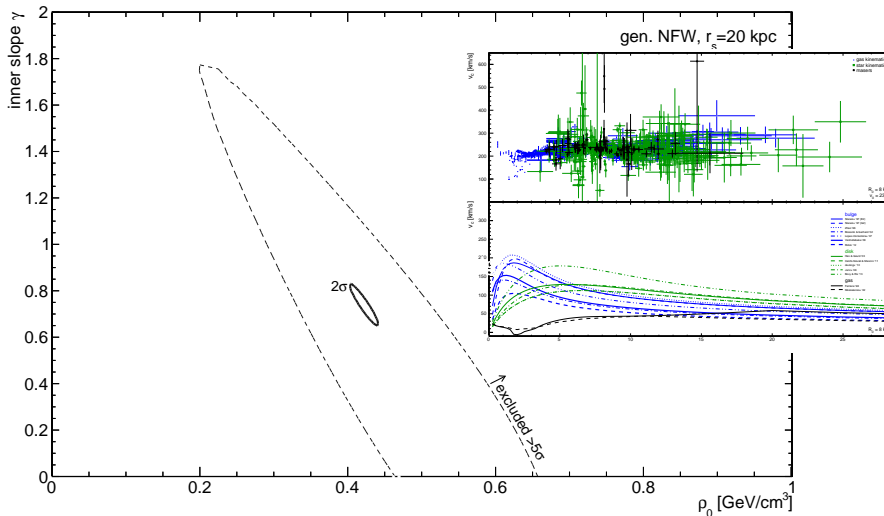


[Nesti & Salucci '13]

[McMillan '11]

2. PROFILE FITTING

$$\rho_{\text{dm}} \propto (r/r_s)^{-\gamma} (1 + r/r_s)^{-3+\gamma} \quad [\text{MP, Iocco \& Bertone '15, 1504.06324}]$$



NFW: $\rho_0 = 0.420^{+0.021}_{-0.018} (2\sigma)$

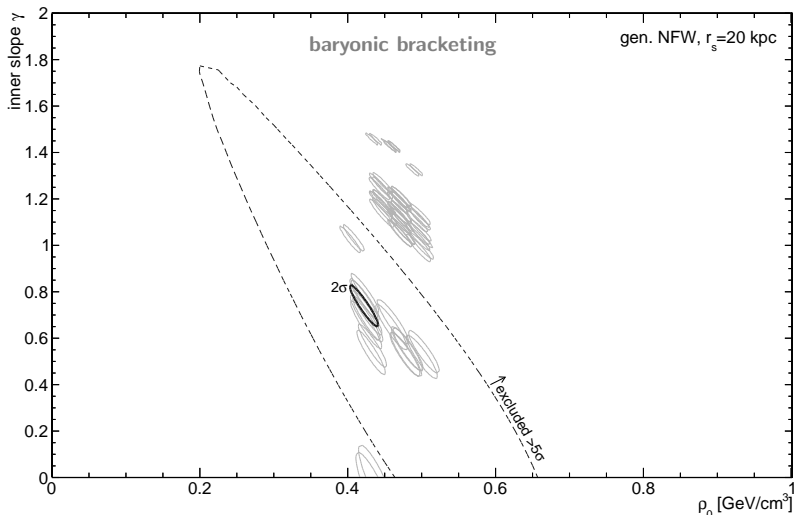
GeV/cm³

Einstein: $\rho_0 = 0.420^{+0.019}_{-0.021} (2\sigma)$

GeV/cm³

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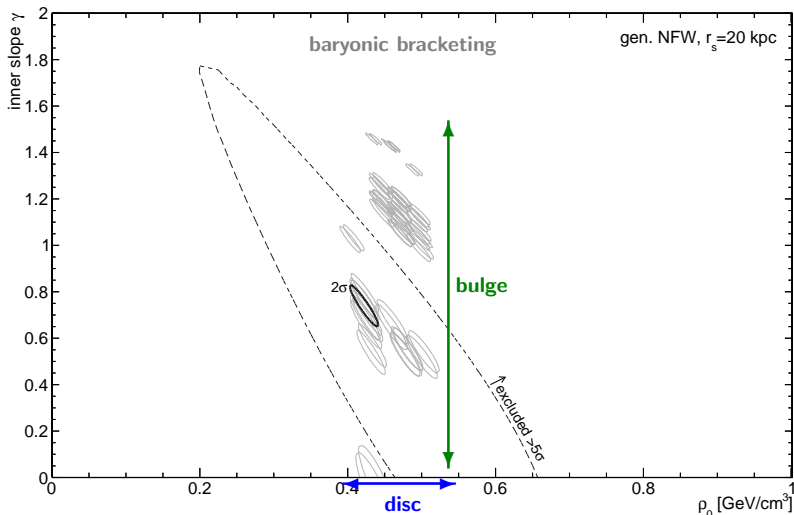


$$\text{NFW: } \rho_0 = 0.420^{+0.021}_{-0.018} (2\sigma) \pm 0.025 \text{ GeV}/\text{cm}^3$$

$$\text{Einasto: } \rho_0 = 0.420^{+0.019}_{-0.021} (2\sigma) \pm 0.026 \text{ GeV}/\text{cm}^3$$

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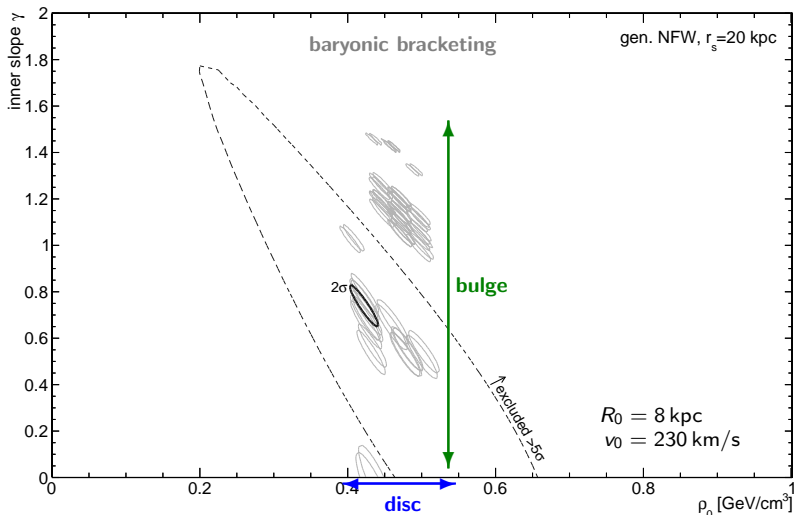


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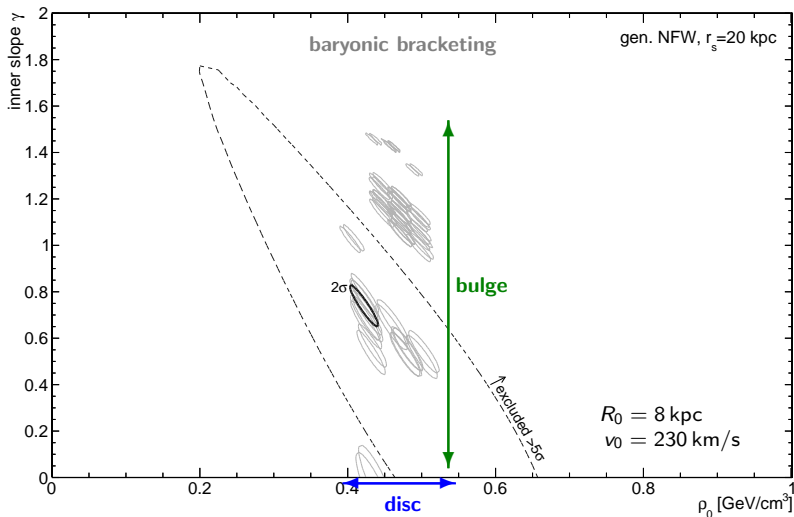


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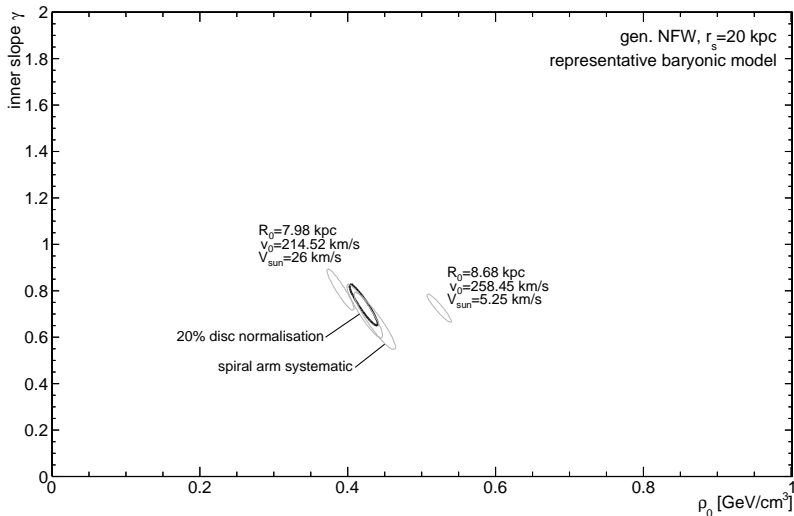


S-stars orbits: $R_0 = 8.33 \pm 0.35$ kpc

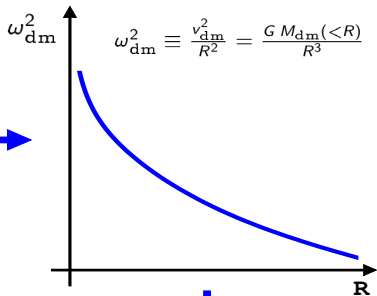
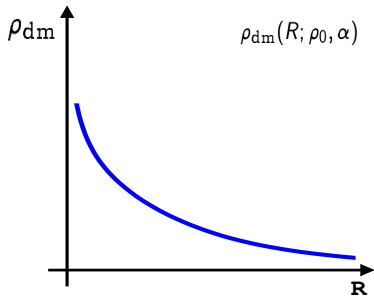
Sgr A* proper motion: $\Omega_{\odot} \equiv (v_0 + V_{\odot})/R_0 = 30.26 \pm 0.12$ km/s/kpc

2. PROFILE FITTING

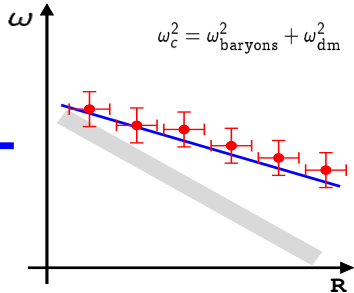
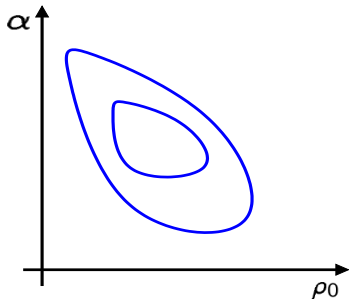
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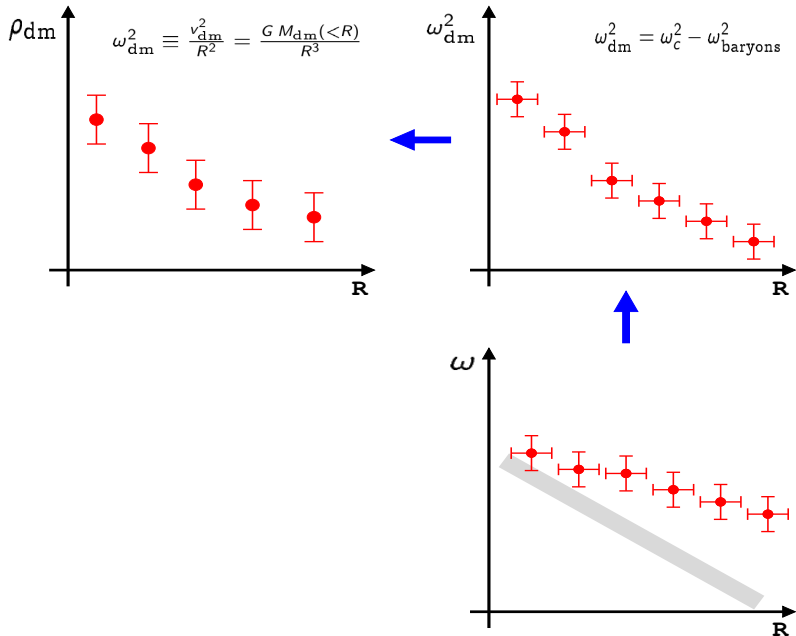
2. PROFILE FITTING



$$\omega_c^2 = \omega_{\text{baryons}}^2 + \omega_{\text{dm}}^2$$



2. PROFILE RECONSTRUCTION



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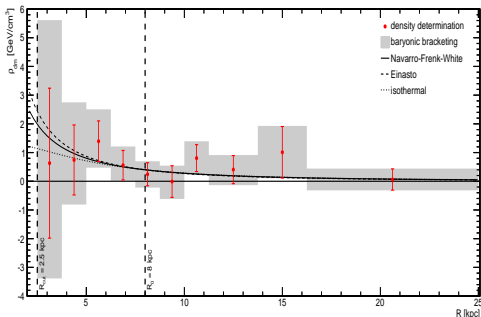
Let us take a spherical dark matter distribution. Then,

$$\omega_{\text{dm}}^2 = \omega_c^2 - \omega_{\text{baryons}}^2 \quad , \quad \omega_{\text{dm}}^2 = \frac{GM_{\text{dm}}(< R)}{R^3} = \frac{4\pi G}{R^3} \int_0^R dr r^2 \rho_{\text{dm}} .$$

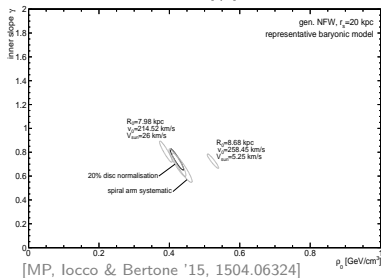
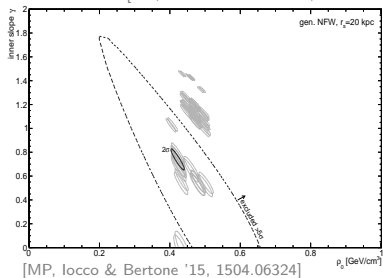
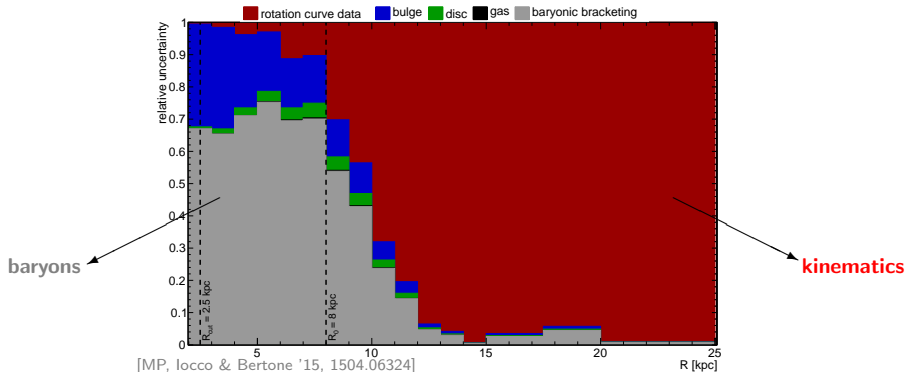
Solving for ρ_{dm} ,

$$\rho_{\text{dm}}(R) = \frac{1}{4\pi G} \left(3\omega_{\text{dm}}^2 + R \frac{d\omega_{\text{dm}}^2}{dR} \right) = \frac{\omega_{\text{dm}}^2}{4\pi G} \left(3 + \frac{d \ln \omega_{\text{dm}}^2}{d \ln R} \right) .$$

That is, the deviation from $\omega_{\text{dm}}^2 \propto R^{-3}$ (or $v_{\text{dm}} \propto R^{-1/2}$) measures the dark matter density at each R . No assumption has been made on the functional form of $\rho_{\text{dm}}(R)$.

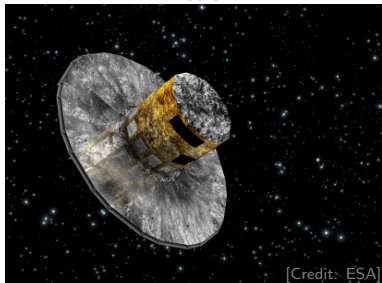


3. FUTURE DIRECTIONS?



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Gaia



[Credit: ESA]

fact sheet

2013-2018

space-based

$\lambda = 320 - 1000 \text{ nm}$

10^9 stars $G < 20 \text{ mag}$

parallax $\pm 10 \mu\text{as}$

proper motion $\pm 10 \mu\text{as/yr}$

radial velocity $\pm 1 \text{ km/s}$

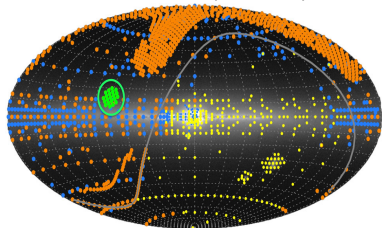
wish list

disc modelling

Oort's constants

local density

APOGEE-2 (SDSS-IV)



[Credit: SDSS]

fact sheet

2014-2020

south+north hemispheres

$\lambda = 1510 - 1700 \text{ nm}$

300,000 stars

radial velocity $\pm 0.1 \text{ km/s}$

wish list

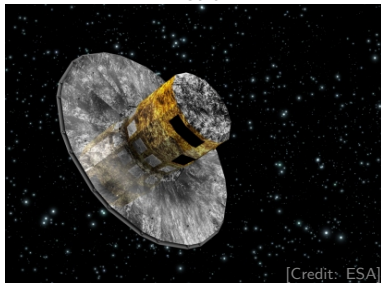
bulge modelling

disc modelling

kinematics all across

3. FUTURE DIRECTIONS?

Gaia



fact sheet

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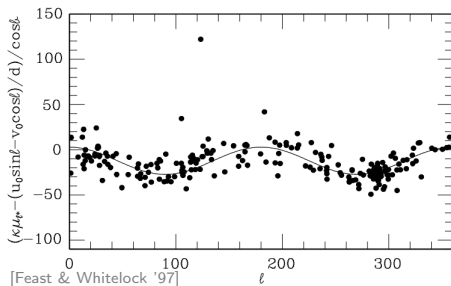
radial velocity $\pm 1 \text{ km/s}$

wish list

disc modelling

Oort's constants

local density



Use local kinematics to constrain Oort constants:

$$A = \frac{1}{2} \left(\frac{v_0}{R_0} - v'_0 \right) \quad B = -\frac{1}{2} \left(\frac{v_0}{R_0} + v'_0 \right)$$

$$v_{\text{los}} \simeq A d \sin 2\ell$$

$$\mu_\ell = \frac{v_t}{d} \simeq B + A \cos 2\ell$$

Hipparcos:

$A = +14.82 \pm 0.84 \text{ km/s/kpc}$

$B = -12.37 \pm 0.64 \text{ km/s/kpc}$

Gaia will largely outperform Hipparcos.

[work in progress with Mattia Fornasa]

4. SUMMARY & CONCLUSION

photometry vs kinematics

photometry: tracks baryonic matter

kinematics: tracks total matter

kinematics – photometry: tracks dark matter

local vs global methods

local methods: robust but low precision

global methods: model-dependent but high precision

both are complementary

current uncertainties

inner Galaxy: baryons

outer Galaxy: kinematics

bottomline

The distribution of dark matter in the Milky Way remains largely unconstrained, but Gaia and other surveys will shrink current uncertainties, leading to a new precision era in mapping dark matter in the Galaxy.

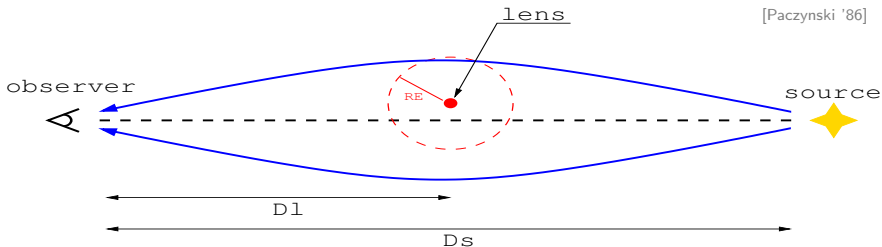
BACKUP SLIDES

1. TOUR OF THE GALAXY: STELLAR BULGE

$$\rho_{\text{bulge}} = \rho_0 f(x, y, z)$$

normalisation ρ_0

One possibility to normalise bulge models is to use microlensing.

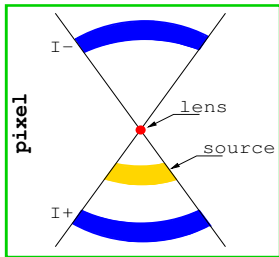


Microlensing is simply a regime of gravitational lensing where the multiple images are not resolved.

$$\text{Einstein radius } R_E^2 = \frac{4GM_l}{c^2} D_l \left(1 - \frac{D_l}{D_s}\right)$$

$$\text{unresolved images } A(t) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$

$$M_l \sim [10^{-6}, 10^2] M_\odot; t_E \sim \text{hr} - \text{days}$$

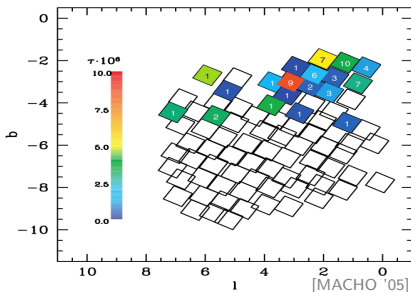
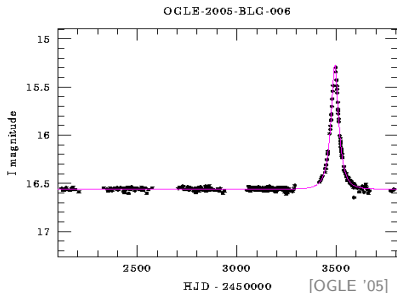


1. TOUR OF THE GALAXY: STELLAR BULGE

$$\rho_{\text{bulge}} = \rho_0 f(x, y, z)$$

normalisation ρ_0

Microensing in our Galaxy was predicted in 1986 and observed for the first time in 1993 by MACHO and EROS.



The microlensing optical depth, i.e. the probability for observing a microlensing event,

$$\tau = \int_0^{D_s} dD_l \int dM_l (\pi R_E^2) \times \left(\frac{d^2 N_l}{dV dM_l} \right) = \frac{4\pi G}{c^2} \int_0^{D_s} dD_l \rho_l D_l \left(1 - \frac{D_l}{D_s} \right)$$

is particularly convenient since it depends on ρ_l only, not on M_l .

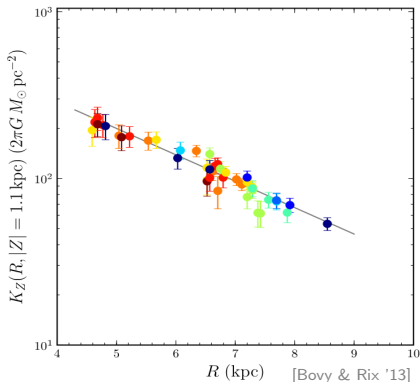
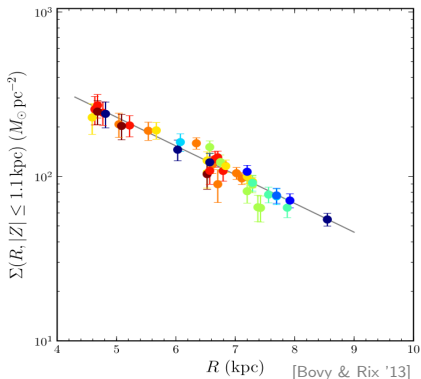
$$\langle \tau \rangle = 2.17_{-0.38}^{+0.47} \times 10^{-6}, (l, b) = (1.50^\circ, -2.68^\circ) \quad [\text{MACHO '05}]$$

1. TOUR OF THE GALAXY: STELLAR DISC

$$\rho_{\text{disc}} = \rho_0 f(x, y, z)$$

normalisation ρ_0

The normalisation of the stellar disc can be pinned down with the kinematics of specific stars.



The latest dynamical measurement uses G dwarfs from SEGUE and fixes the stellar local surface density to

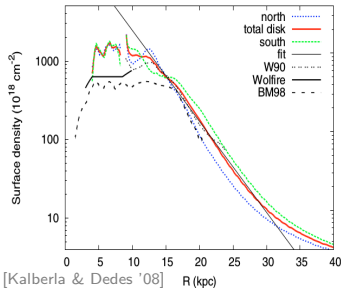
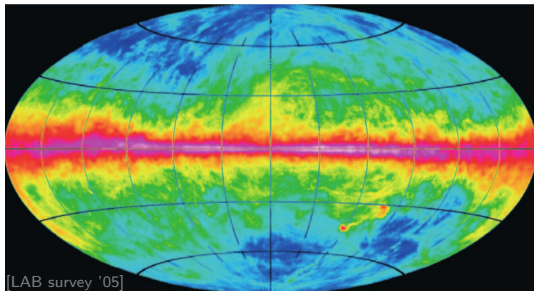
$$\Sigma_{\star} = 38 \pm 4 M_{\odot} / \text{pc}^2. \quad [\text{Bovy \& Rix '13}]$$

1. TOUR OF THE GALAXY: GAS

$$n_{\text{H}} = 2n_{\text{H}_2} + n_{\text{HI}} + n_{\text{HII}}$$

normalisation

The gas content is dominated by H_2 in the inner Galaxy and H I in the outer Galaxy.



For H_2 , the main normalisation uncertainty arises from the CO -to- H_2 factor,

$$X_{\text{CO}} = 0.25 - 1.0 \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1} \text{ km}^{-1} \text{ s} \quad (r < 2 \text{ kpc})$$

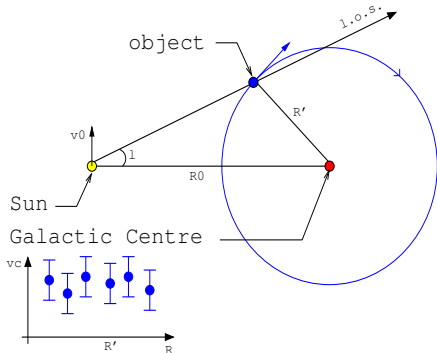
$$X_{\text{CO}} = 0.50 - 3.0 \times 10^{20} \text{ cm}^{-2} \text{ K}^{-1} \text{ km}^{-1} \text{ s} \quad (r > 2 \text{ kpc}) . \quad [\text{Ferrière+ '07, Ackermann '12}]$$

For H I , different surveys disagree by up to a factor ~ 2 in the inner 15 kpc.

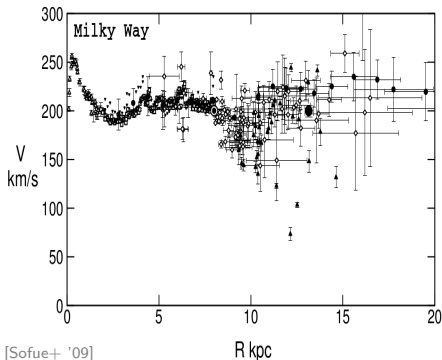
1. TOUR OF THE GALAXY: ROTATION CURVE

$$v_c^2 = r \frac{d\phi_{\text{tot}}}{dr} \stackrel{\text{sph.}}{=} \frac{G M_{\text{tot}}(< r)}{r}$$

Rotation curve tracers are young objects or regions that track galactic rotation. In external galaxies the only available tracer is the gas, while in our Galaxy we can use also some stars and star-forming regions. However, the case of our Galaxy is much more challenging due to our position.



$$v_{\text{lsr}}^{\text{los}} = \left(\frac{v_c(R')}{R'/R_0} - v_0 \right) \cos b \sin \ell$$



[Sofue+ '09]

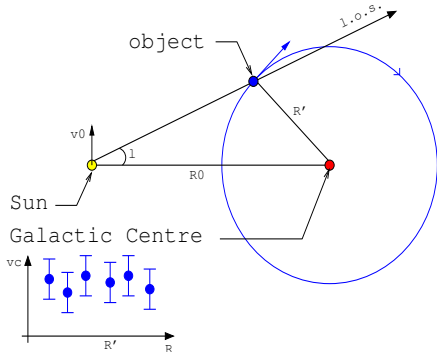
Doppler shift

1. gas (21cm, H α , CO)
2. stars (H, He, O, ...)
3. masers (H₂O, CH₃OH, ...)

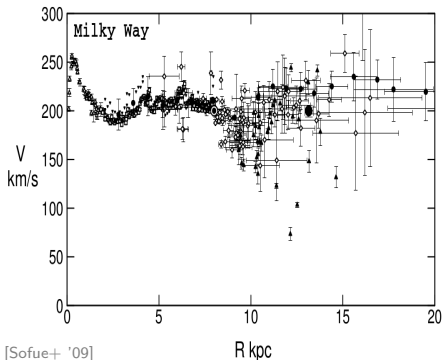
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Rotation curve tracers are young objects or regions that track galactic rotation. In external galaxies the only available tracer is the gas, while in our Galaxy we can use also some stars and star-forming regions. However, the case of our Galaxy is much more challenging due to our position.



$$v_{\text{l sr}}^{\text{los}} = \left(\frac{v_c(R')}{R'/R_0} - v_0 \right) \cos b \sin l$$



[Sofue+ '09]

distance

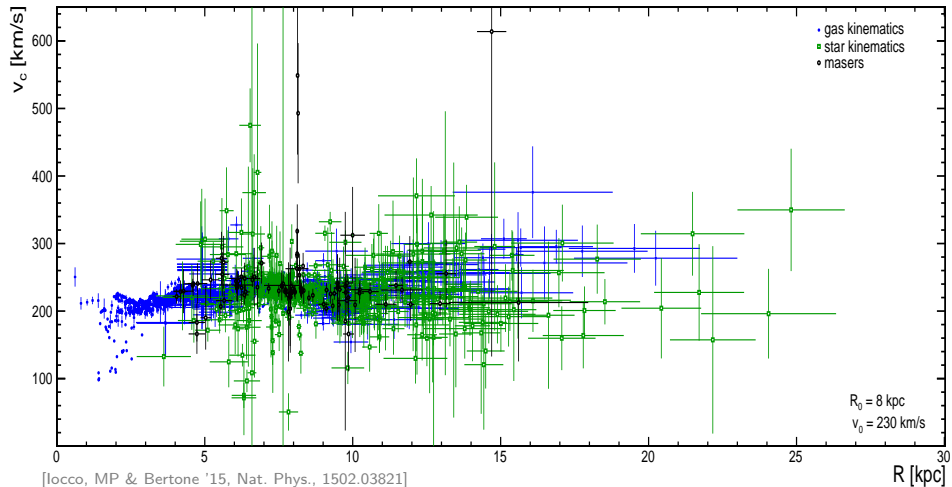
1. terminal velocities (gas)
2. photo-spectroscopy (stars)
3. parallax (masers)

1. TOUR OF THE GALAXY: ROTATION CURVE

optimised to $R = 3 - 20$ kpc

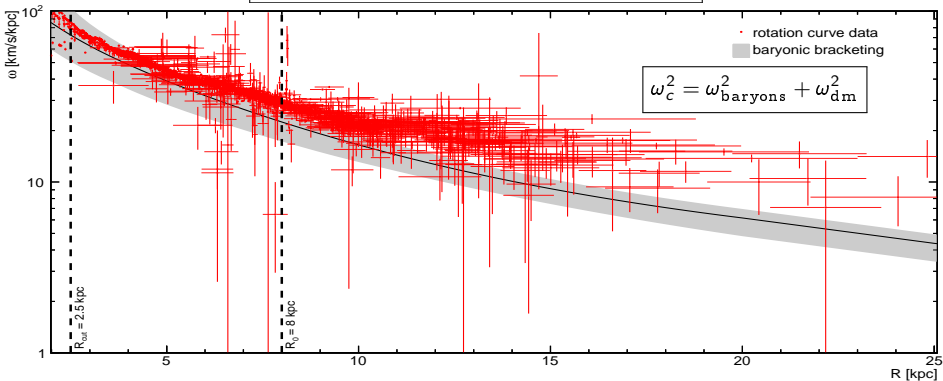
2780 individual measurements

2174/506/100 from gas/stars/masers



1. EVIDENCE FOR DARK MATTER

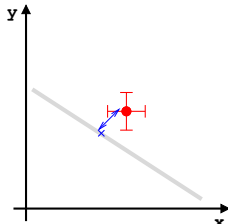
$$\phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$$



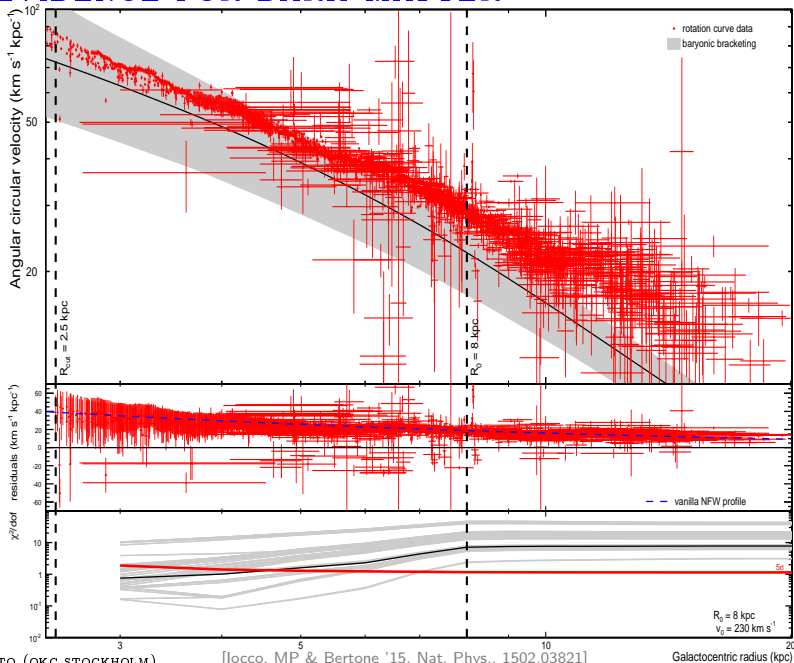
How bad is a baryon-only fit?

$$x = R/R_0 \quad y = \omega/\omega_0 - 1$$

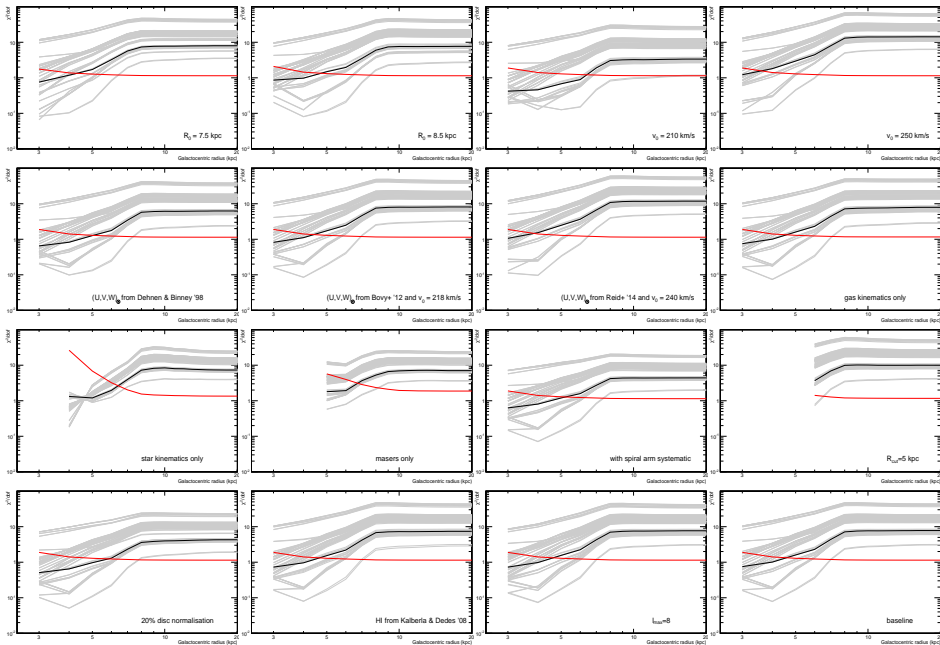
$$\chi^2 = \sum_{i=1}^N d_i^2 \equiv \sum_{i=1}^N \left[\frac{(y_i - y_{b,i})^2}{\sigma_{y,i}^2} + \frac{(x_i - x_{b,i})^2}{\sigma_{x,i}^2} \right]$$



1. EVIDENCE FOR DARK MATTER



1. EVIDENCE FOR DARK MATTER



1. TOUR OF THE GALAXY

$$\phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$$

dynamics

traces total potential

$R \sim 0.1 - 30$ kpc rotation curve tracers

$R \sim 8 - 60$ kpc star population tracers

$R \sim 100 - 300$ kpc satellite kinematics

$R \sim 300+$ kpc timing in Local Group

"photometry"

traces individual baryonic components

bulge star counts, luminosity, microlensing

disc star counts, luminosity, stellar dynamics

gas emission lines, dispersion measure

1. TOUR OF THE GALAXY: STAR POPULATION

In a galaxy star encounters are rare and stars feel on average the smooth gravitational potential. We can therefore treat a set of stars as a collisionless gas and apply the collisionless Boltzmann equation, whose first momentum gives the **Jeans equations**:

$$-\rho_s \frac{\partial \phi_{\text{tot}}}{\partial x_j} = \frac{\partial(\rho_s \bar{v}_j)}{\partial t} + \sum_i \frac{\partial(\rho_s \bar{v}_i \bar{v}_{ij})}{\partial x_i} \quad , \quad j = 1, 2, 3 \text{ (cartesian)} .$$

ρ_s : star density v_j : velocity of stars

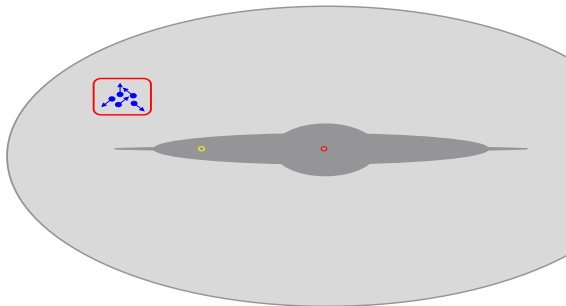
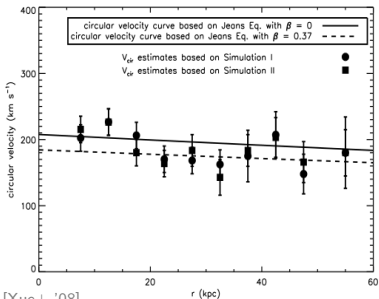
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[Xue+ '08]

[Sakamoto+ '03, Dehnen+ '06, Xue+ '08, Bhattacharjee+ '14, Kafle+ '14]

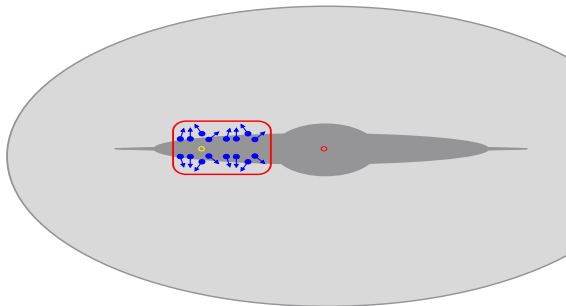
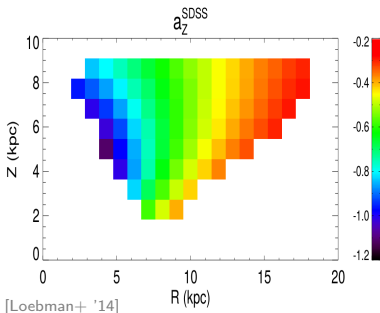
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[Bovy & Rix '13, Loebman+ '14]

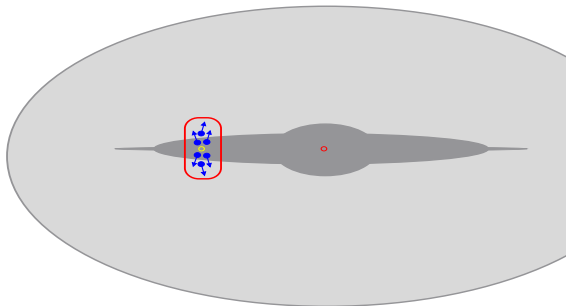
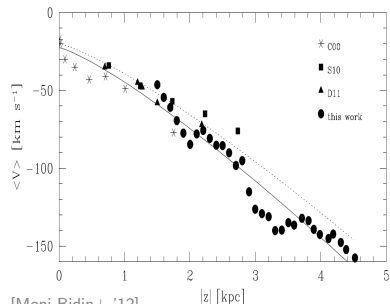
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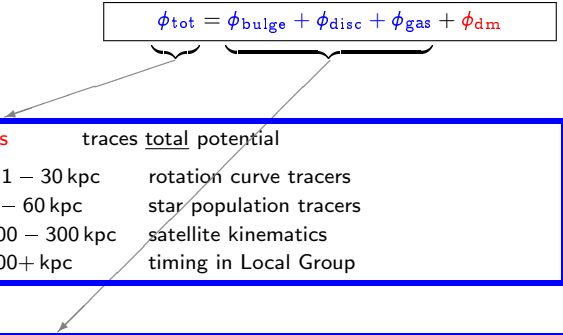
v_j : velocity of stars



[Moni Bidin+ '12]

[Kuijken & Gilmore '91, Holmberg & Flynn '04, Moni Bidin+ '12, Bovy & Tremaine '12, Moni Bidin+ '14]

2. DARK MATTER CONTENT

$$\phi_{\text{tot}} = \phi_{\text{bulge}} + \phi_{\text{disc}} + \phi_{\text{gas}} + \phi_{\text{dm}}$$


dynamics

traces total potential

$R \sim 0.1 - 30$ kpc	rotation curve tracers
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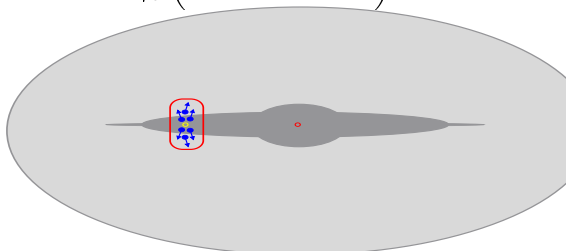
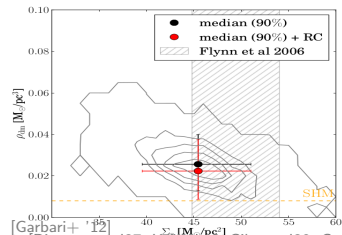
We can couple this to the **Poisson equation**: $4\pi G \rho_{\text{tot}} = \nabla^2 \phi_{\text{tot}}$.

$$\phi_{\text{tot}}(R, z) \quad \partial/\partial t \rightarrow 0 \quad -F_R = \partial\phi_{\text{tot}}/\partial R \quad -F_z = \partial\phi_{\text{tot}}/\partial z$$

$$-4\pi G \rho_{\text{tot}} = \frac{1}{R} \frac{\partial}{\partial R} (R F_R) + \frac{\partial F_z}{\partial z}$$

$$F_R = \frac{1}{\rho_s} \left(\frac{\partial(\rho_s \bar{v}_R^2)}{\partial R} + \frac{\partial(\rho_s \bar{v}_R \bar{v}_z)}{\partial z} \right) + \frac{\bar{v}_R^2 - \bar{v}_\phi^2}{R}$$

$$F_z = \frac{1}{\rho_s} \left(\frac{\partial(\rho_s \bar{v}_R \bar{v}_z)}{\partial R} + \frac{\partial(\rho_s \bar{v}_z^2)}{\partial z} \right) + \frac{\bar{v}_R \bar{v}_z}{R}$$



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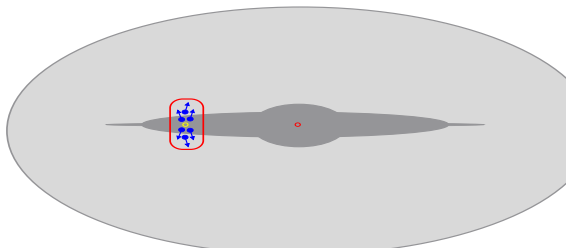
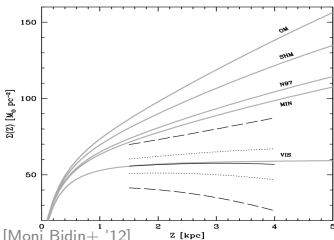
$$-\rho_s \frac{\partial \phi_{\text{tot}}}{\partial x_j} = \frac{\partial(\rho_s \bar{v}_j)}{\partial t} + \sum_i \frac{\partial(\rho_s \bar{v}_i \bar{v}_j)}{\partial x_i} \quad , \quad j = 1, 2, 3 \text{ (cartesian)} .$$

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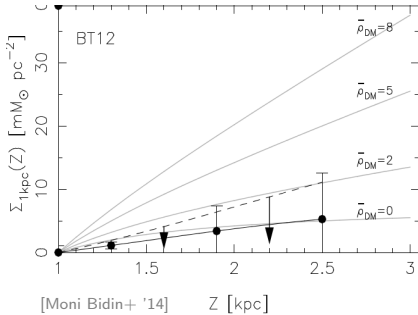
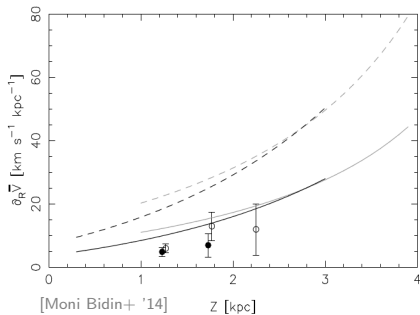
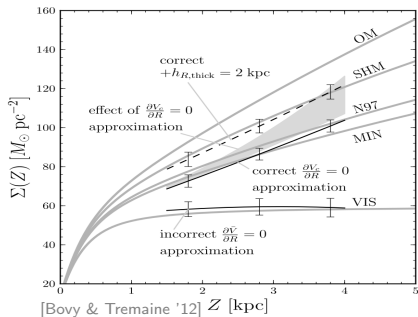
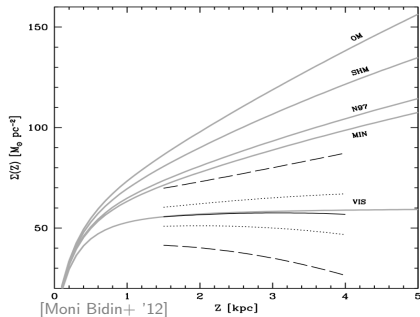
$$-4\pi G \rho_{\text{tot}} = \frac{1}{R} \frac{\partial}{\partial R} (R F_R) + \frac{\partial F_z}{\partial z}$$

$$-4\pi G \Sigma_{\text{tot}}(z) = \int_{-z}^z dz \frac{1}{R} \frac{\partial}{\partial R} (R F_R) + F_z(z) - F_z(-z)$$



[Moni Bidin+ '12, Bovy & Tremaine '12, Moni Bidin+ '14]

2. LOCAL METHODS



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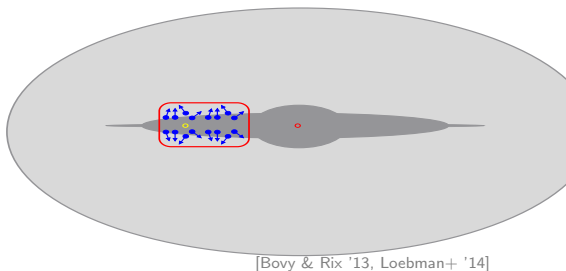
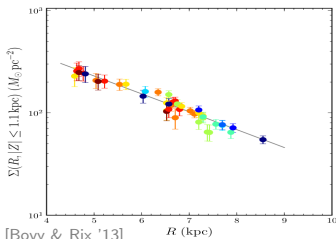
$$-\rho_s \frac{\partial \phi_{\text{tot}}}{\partial x_j} = \frac{\partial(\rho_s \bar{v}_j)}{\partial t} + \sum_i \frac{\partial(\rho_s \bar{v}_i \bar{v}_j)}{\partial x_i} \quad , \quad j = 1, 2, 3 \text{ (cartesian)} .$$

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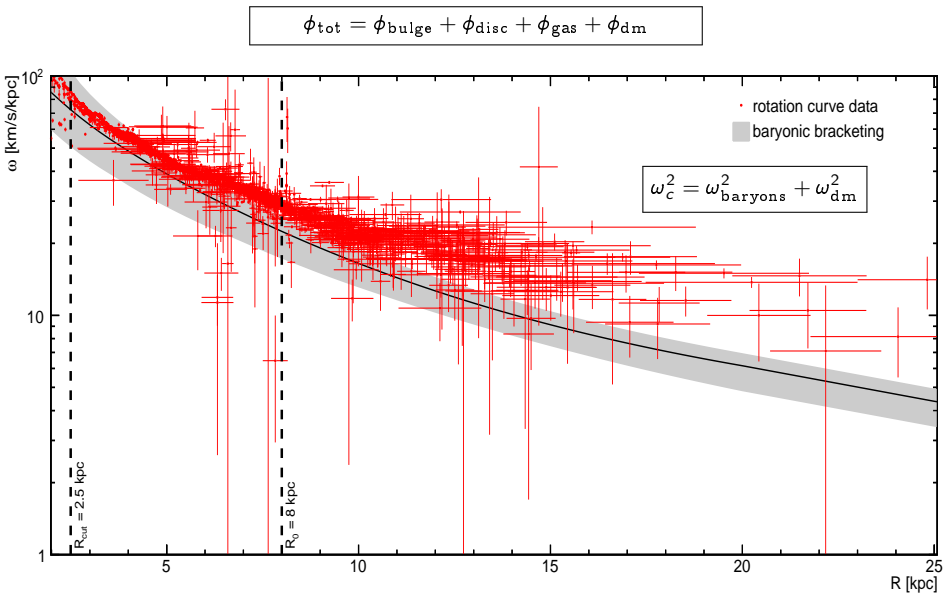
$$\phi_{\text{tot}}(R, z) \quad \partial/\partial t \rightarrow 0 \quad -F_R = \partial\phi_{\text{tot}}/\partial R \quad -F_z = \partial\phi_{\text{tot}}/\partial z$$

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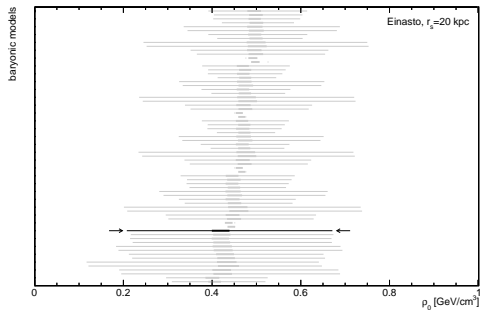
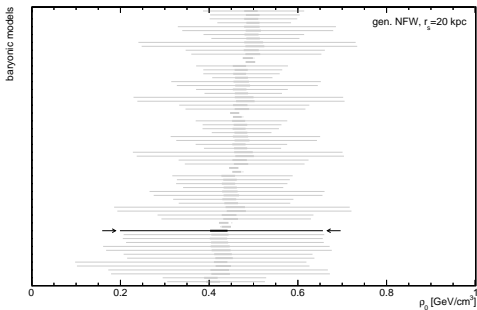
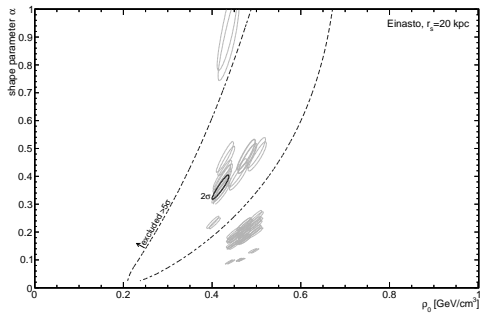
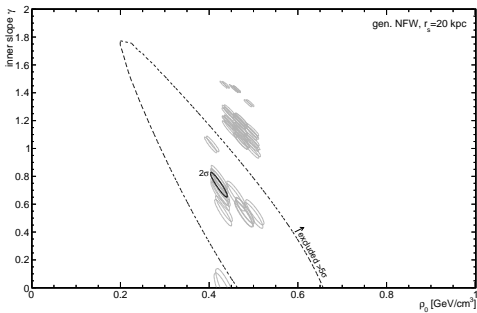
$$-4\pi G \Sigma_{\text{tot}}(z) = \int_{-z}^z dz \frac{1}{R} \frac{\partial}{\partial R} (R F_R) + F_z(z) - F_z(-z)$$



2. GLOBAL METHODS



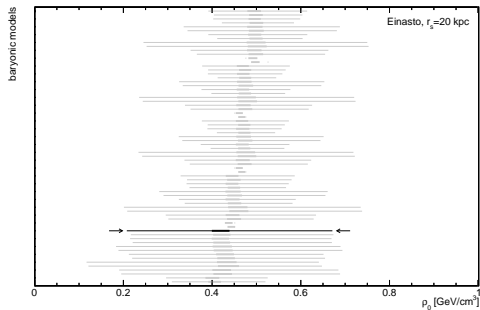
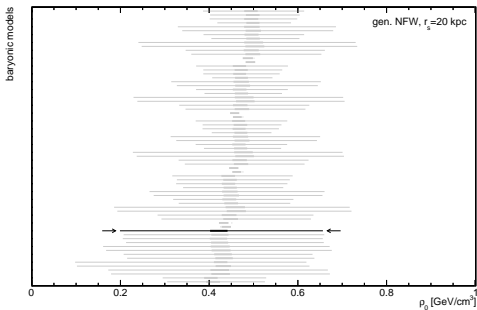
2. GLOBAL METHODS $\rho_{\text{dm}} \propto (r/r_s)^{-\gamma} (1 + r/r_s)^{-3+\gamma}, \exp(-2((r/r_s)^\alpha - 1)/\alpha)$



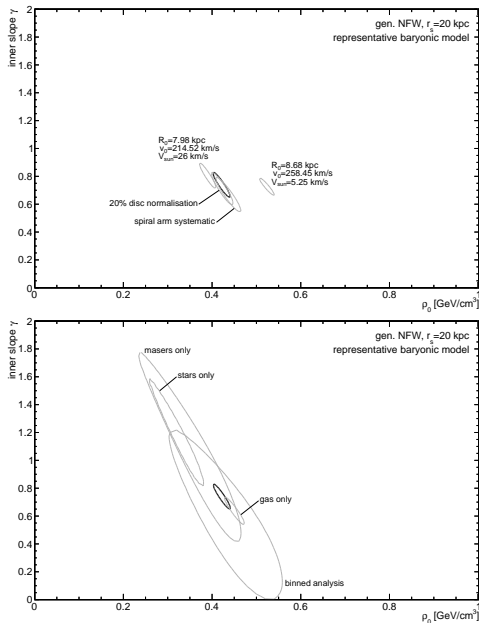
2. GLOBAL METHODS $\rho_{\text{dm}} \propto (r/r_s)^{-\gamma} (1 + r/r_s)^{-3+\gamma}, \exp(-2((r/r_s)^\alpha - 1)/\alpha)$

$$\text{NFW: } \rho_0 = 0.420_{-0.018}^{+0.021} (2\sigma) \pm 0.025 \text{ GeV/cm}^3$$

$$\text{Einasto: } \rho_0 = 0.420_{-0.021}^{+0.019} (2\sigma) \pm 0.026 \text{ GeV/cm}^3$$



2. GLOBAL METHODS $\rho_{\text{dm}} \propto (r/r_s)^{-\gamma} (1 + r/r_s)^{-3+\gamma}$



2. MODIFIED NEWTONIAN DYNAMICS

wait, what about MoND?

$$\mu \left(\frac{a}{a_0} \right) a = a_N$$

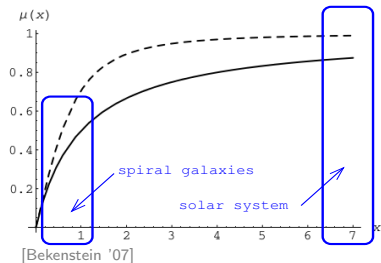
$$a_0 \simeq 10^{-10} \text{ m/s}^2$$

[Milgrom x3 '83]

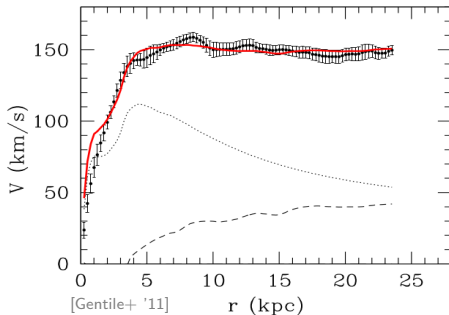
$$\lim_{x \ll 1} \mu(x) = x$$

$$\lim_{x \gg 1} \mu(x) = 1$$

$$\mu_{\text{std}}(x) = \frac{x}{\sqrt{1+x^2}}, \quad \mu_{\text{sim}}(x) = \frac{x}{1+x}$$



NGC 3198



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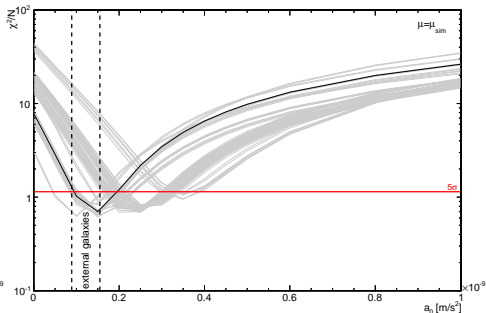
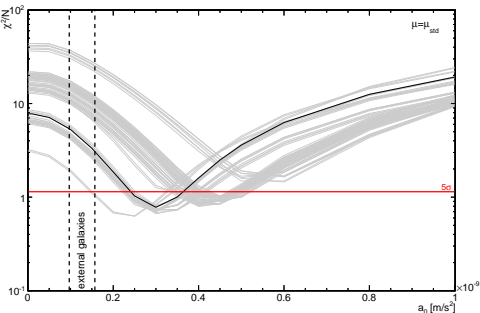
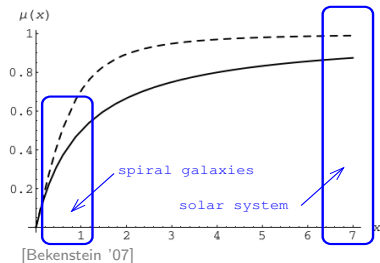
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[Milgrom x3 '83]

$$a \rightarrow R\omega_c^2$$

$$a_N = R\omega_b^2$$

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[locco, MP & Bertone '15, 1505.05181]

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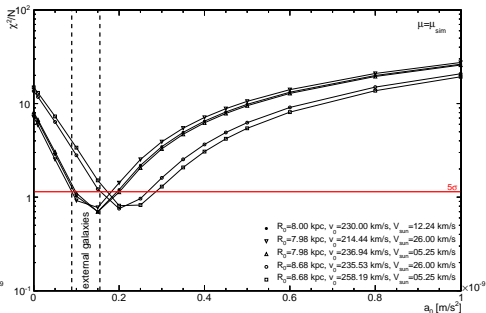
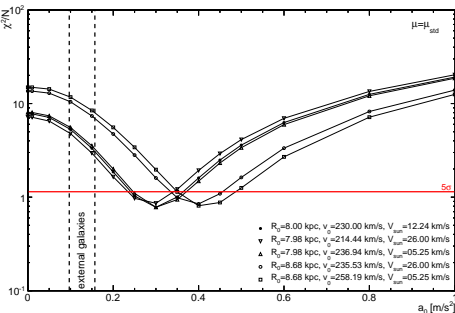
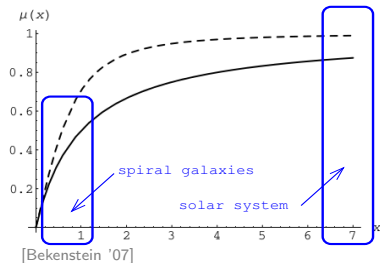
$$\mu \left(\frac{a}{a_0} \right) a = a_N$$

$$a_0 \simeq 10^{-10} \text{ m/s}^2$$

[Milgrom x3 '83]

$$a \rightarrow R\omega_c^2 \quad a_N = R\omega_b^2$$

$$\mu_{\text{std}}(x) = \frac{x}{\sqrt{1+x^2}} \quad , \quad \mu_{\text{sim}}(x) = \frac{x}{1+x}$$



[locco, MP & Bertone '15, 1505.05181]

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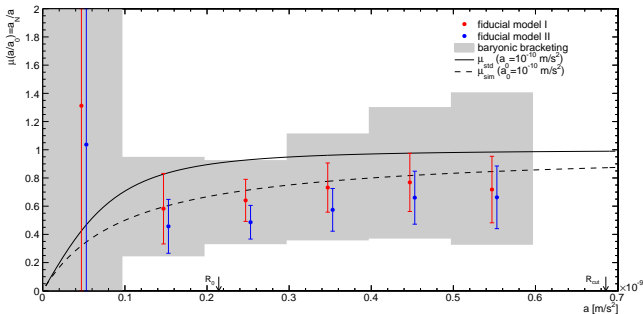
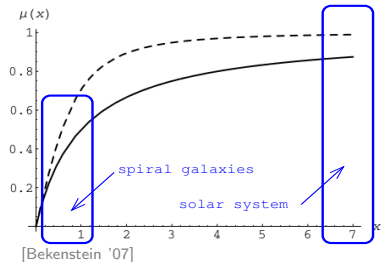
$$\mu\left(\frac{a}{a_0}\right) a = a_N$$

$$a_0 \simeq 10^{-10} \text{ m/s}^2$$

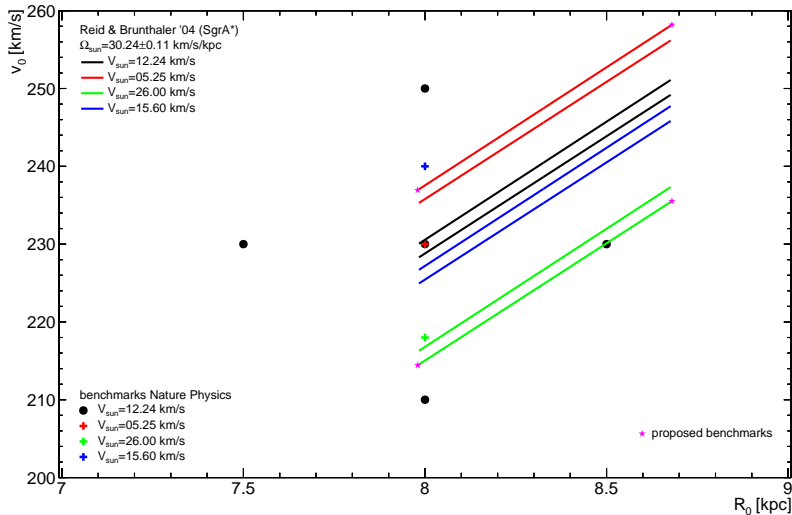
[Milgrom x3 '83]

$$a \rightarrow R\omega_c^2$$

$$a_N = R\omega_b^2$$



2. GALACTIC PARAMETERS



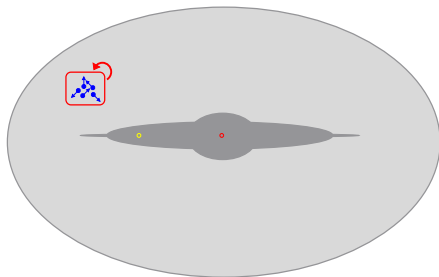
[locco, MP & Bertone '15, 1505.05181]

3. FUTURE DIRECTIONS?

local methods

aim: use data from a patch of the sky to derive dynamics there.

- + “assumption-free”
- low precision



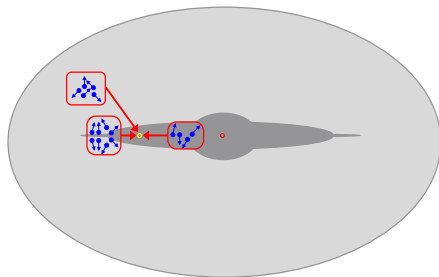
[Kapteyn '22, Jeans '22, Oort '32, Hill '60, Oort '60, Bahcall '84, Bienaymé+ '87, Kuijken & Gilmore '91, Bahcall+ '92, Creze+ '98, Holmberg & Flynn '00, Holmberg & Flynn '04, Bienaymé+ '06, Garbari+ '11 '12, Moni Bidin+ '12, Bovy & Tremaine '12, Smith+ '12, Zhang+ '13, Bovy & Rix '13, Loebman+ '14, Moni Bidin+ '14]

vs

global methods

aim: use data across the Galaxy to derive dynamics somewhere.

- global assumptions
- + high precision



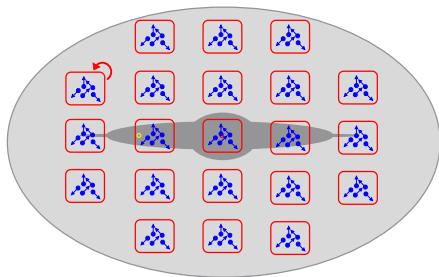
[Caldwell & Ostriker '81, Gates+ '95, Dehnen & Binney '98, Sakamoto+ '03, Dehnen+ '06, Xue+ '08, Sofue+ '09, Strigari & Trotta '09, Catena & Ullio '10, Weber & de Boer '10, Salucci+ '10, Iocco+ '11, McMillan '11, Nesti & Salucci '13, Bhattacharjee+ '14, Kafle+ '14, MP & Iocco '15, MP, Iocco & Bertone '15, Sofue '15]

3. FUTURE DIRECTIONS?

local methods

aim: use data from a patch of the sky to derive dynamics there.

- + “assumption-free”
- low precision



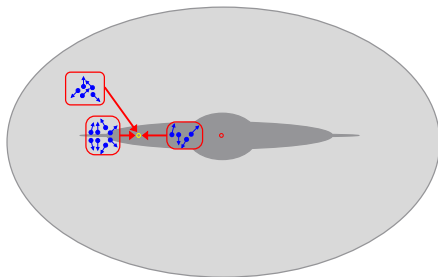
[Kapteyn '22, Jeans '22, Oort '32, Hill '60, Oort '60, Bahcall '84, Bienaymé+ '87, Kuijken & Gilmore '91, Bahcall+ '92, Creze+ '98, Holmberg & Flynn '00, Holmberg & Flynn '04, Bienaymé+ '06, Garbari+ '11 '12, Moni Bidin+ '12, Bovy & Tremaine '12, Smith+ '12, Zhang+ '13, Bovy & Rix '13, Loebman+ '14, Moni Bidin+ '14]

vs

global methods

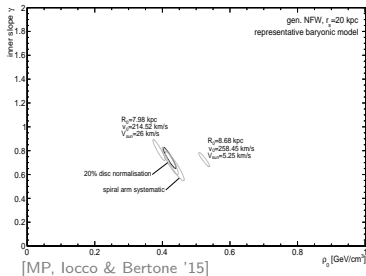
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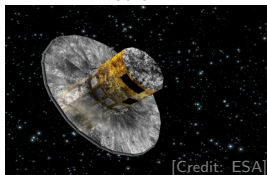


[Caldwell & Ostriker '81, Gates+ '95, Dehnen & Binney '98, Sakamoto+ '03, Dehnen+ '06, Xue+ '08, Sofue+ '09, Strigari & Trotta '09, Catena & Ullio '10, Weber & de Boer '10, Salucci+ '10, Iocco+ '11, McMillan '11, Nesti & Salucci '13, Bhattacharjee+ '14, Kafle+ '14, MP & Iocco '15, MP, Iocco & Bertone '15, Sofue '15]

3. FUTURE DIRECTIONS?



Gaia



fact sheet

2013-2018

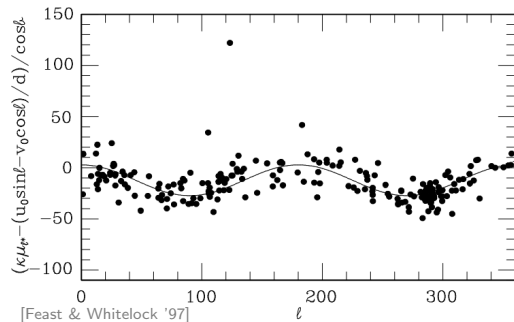
$\lambda = 320 - 1000$ nm

10^9 stars $G < 20$ mag

parallax $\pm 10 \mu\text{as}$

proper motion $\pm 10 \mu\text{as/yr}$

radial velocity ± 1 km/s



Oort constants:

$$A = \frac{1}{2} \left(\frac{v_0}{R_0} - v'_0 \right) \quad B = -\frac{1}{2} \left(\frac{v_0}{R_0} + v'_0 \right)$$

Hipparcos:

$$A = +14.82 \pm 0.84 \text{ km/s/kpc}$$

$$B = -12.37 \pm 0.64 \text{ km/s/kpc}$$

Gaia will improve proper motions, radial velocities and parallaxes by a factor 10-200 wrt Hipparcos.

3. FUTURE DIRECTIONS?

The terminal velocity method is very powerful, but it is only applicable in the inner Galaxy at $R < R_0$. In the solar neighbourhood alternative tracers are needed.

We could for instance measure the line-of-sight velocities of local stars. We have then the same expression as before,

$$v_{\text{los}} = \left(v(R) \frac{R_0}{R} - v_0 \right) \sin \ell,$$

but now we are interested in the regime $R \sim R_0$.

Problem #9: Taylor-expand the expression above in powers of the distance to the star.

First, let us expand $\Omega \equiv v/R$: $\Omega(R) \simeq \Omega_0 + (R - R_0)\Omega'_0$, and therefore

$$v_{\text{los}} = \left(\Omega_0 R_0 + (R - R_0)\Omega'_0 R_0 - v_0 \right) \sin \ell = (R - R_0)R_0\Omega'_0 \sin \ell \equiv -2A(R - R_0) \sin \ell,$$

having defined the Oort's constants $A = -\frac{1}{2}R_0\Omega'_0 = \frac{1}{2}\left(\frac{v_0}{R_0} - v'_0\right)$, $B = -\frac{1}{2}\left(\frac{v_0}{R_0} + v'_0\right)$.

From simple geometry, $\vec{R} = \vec{R}_0 + \vec{d} \Rightarrow R^2 = R_0^2 + d^2 - 2dR_0 \cos \ell \simeq R_0^2 - 2dR_0 \cos \ell$, so

$$R - R_0 = \frac{R^2 - R_0^2}{R + R_0} \simeq \frac{R^2 - R_0^2}{2R_0} \simeq \frac{-2dR_0 \cos \ell}{2R_0} = -d \cos \ell.$$

Replacing in the expression of v_{los} , we obtain finally $v_{\text{los}} = Ad \sin 2\ell$.

Problem #10*: Do the same as in #9 but for the transverse velocity.

Follow Binney & Merrifield, Sec. 10.3.3 to obtain $\mu_\ell = B + A \cos 2\ell$.

The bottomline is that by measuring the motion of local stars we can constrain A, B and thus the local circular velocity: $v_0 = R_0(A - B) \simeq 220 \text{ km/s}$.