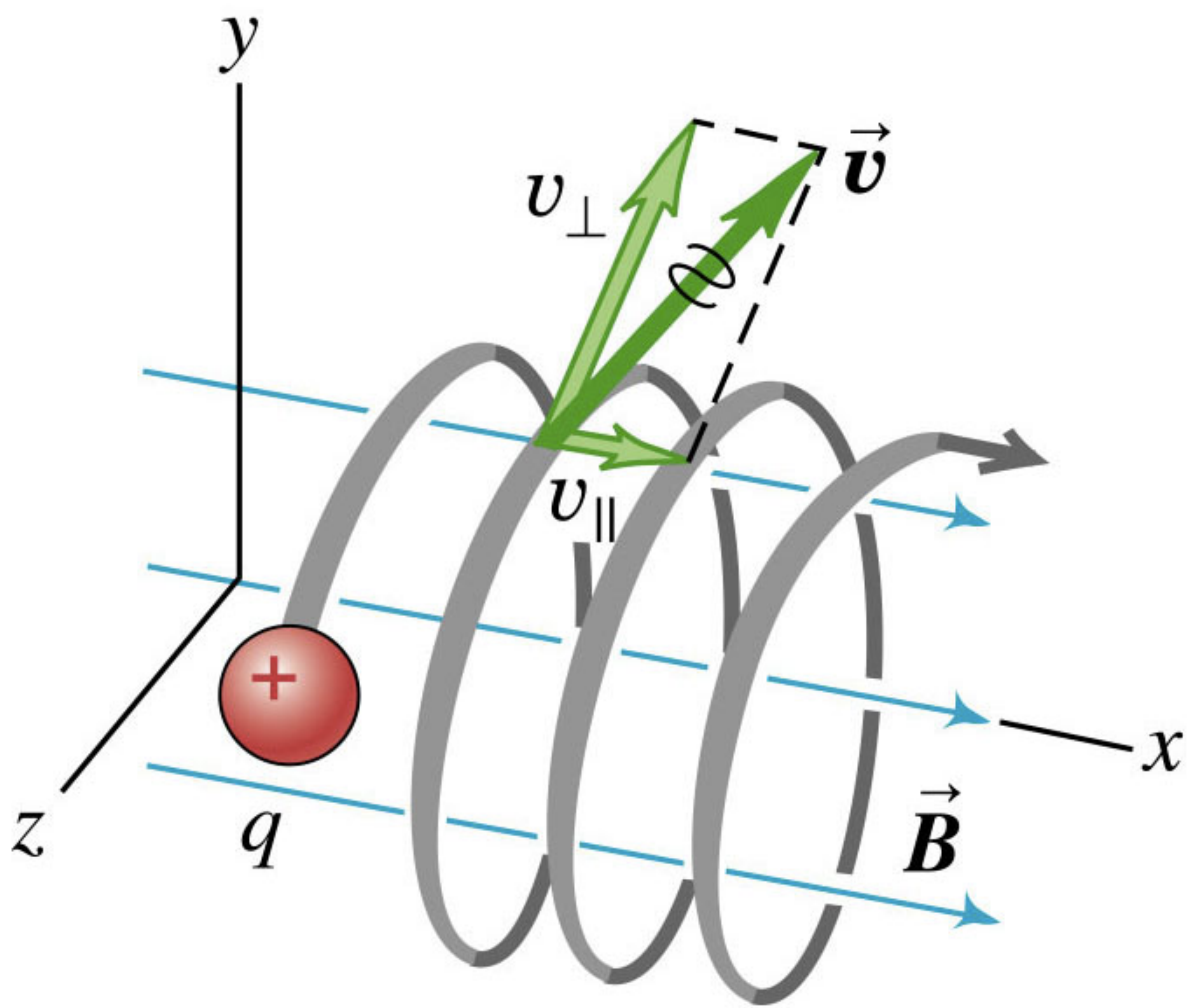
Propagation of cosmic rays in the turbulent interstellar magnetic field

Stefano Gabici
APC, Paris

Charged particle in uniform magnetic field

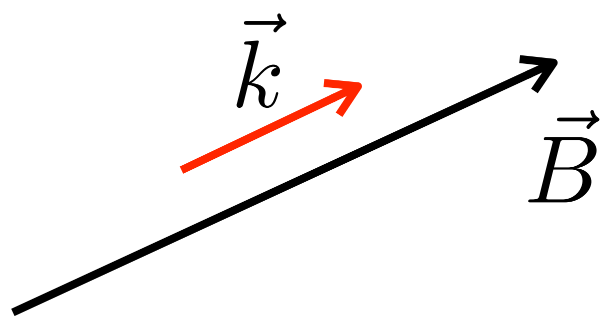


MHD waves in one slide

momentum eqn. $\varrho \left(\frac{\partial}{\partial t} + \mathbf{v} \, \nabla \right) \mathbf{v} = -\nabla P$

sound waves

particle motion unaffected along B



sound speed -> $$c_s^2 = \gamma \frac{P}{\varrho}$$

MHD waves in one slide

momentum eqn. $$\varrho\left(\frac{\partial}{\partial t}+\mathbf{v}\,\nabla\right)\mathbf{v}=-\cancel{\nabla P}+\frac{1}{c}\,\mathbf{j}\times\mathbf{B}=-\nabla\underbrace{\left(\frac{\mathbf{B}}{8\pi}\right)}_{\text{pressure}}+\underbrace{\frac{(\mathbf{B}\nabla)\mathbf{B}}{4\pi}}_{\text{tension}}$$

sound waves

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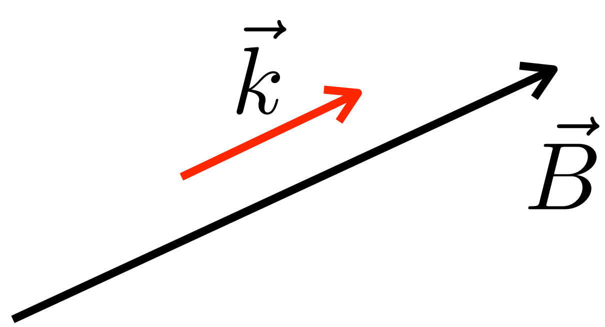
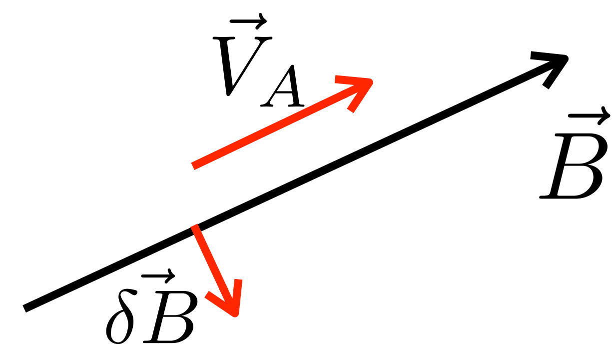


sound speed -> $$\boxed{c_s^2=\gamma\frac{P}{\varrho}}$$

Alfven waves

incompressible mode $\delta\varrho=0$



Alfven speed -> $$\boxed{v_A^2=\frac{B^2}{4\pi\varrho}}$$

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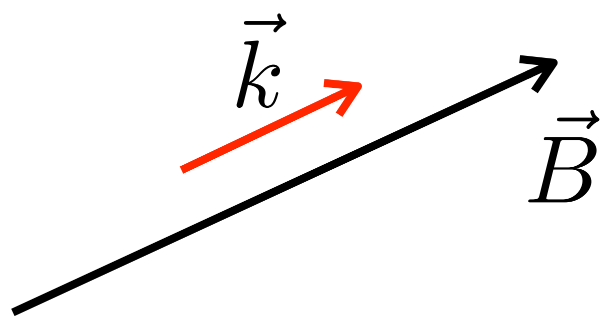

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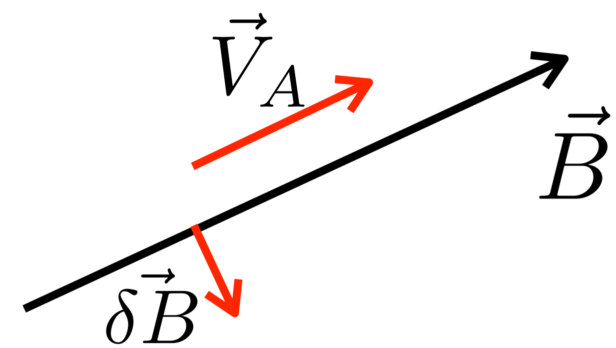

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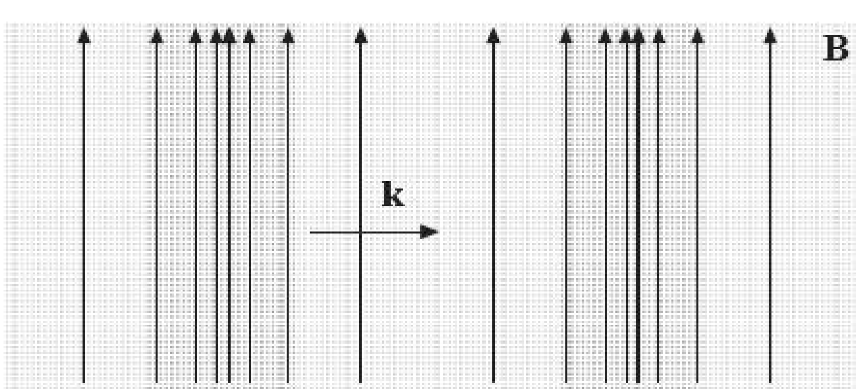

compressible mode

magnetosonic waves

magnetosonic speed -> $$v_{ms}^2=c_s^2+v_A^2$$

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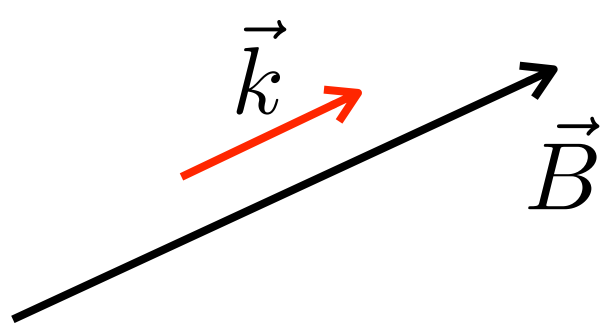

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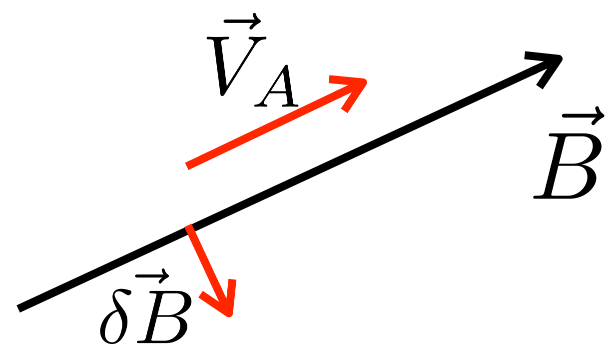
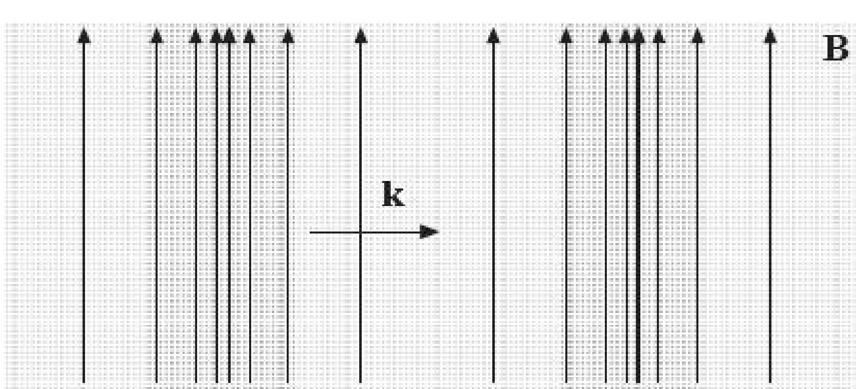


$\boxed{\omega=k_z V_A}$

Alfven speed -> $v_A^2=\frac{B^2}{4\pi\varrho}$



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Resonant scattering with Alfven waves

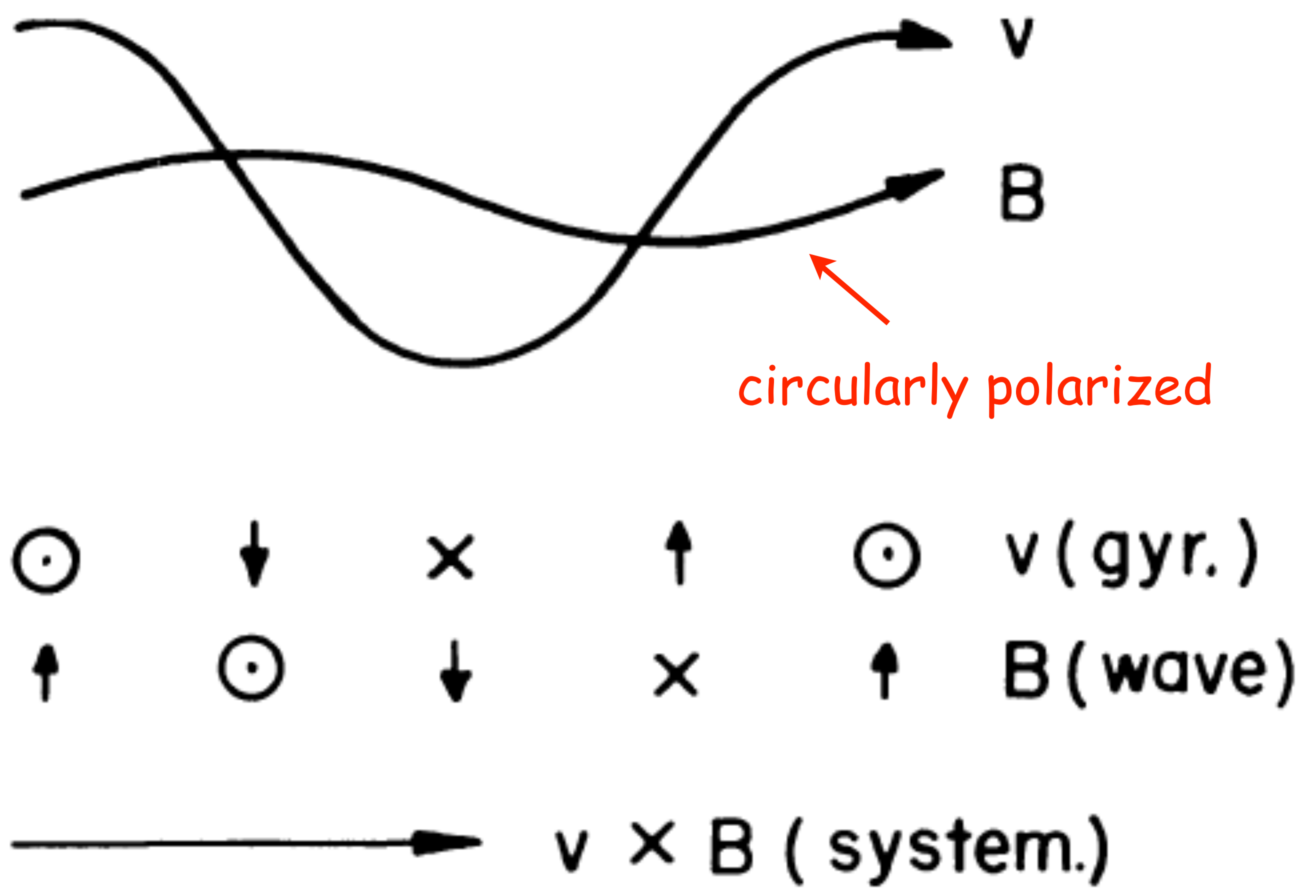
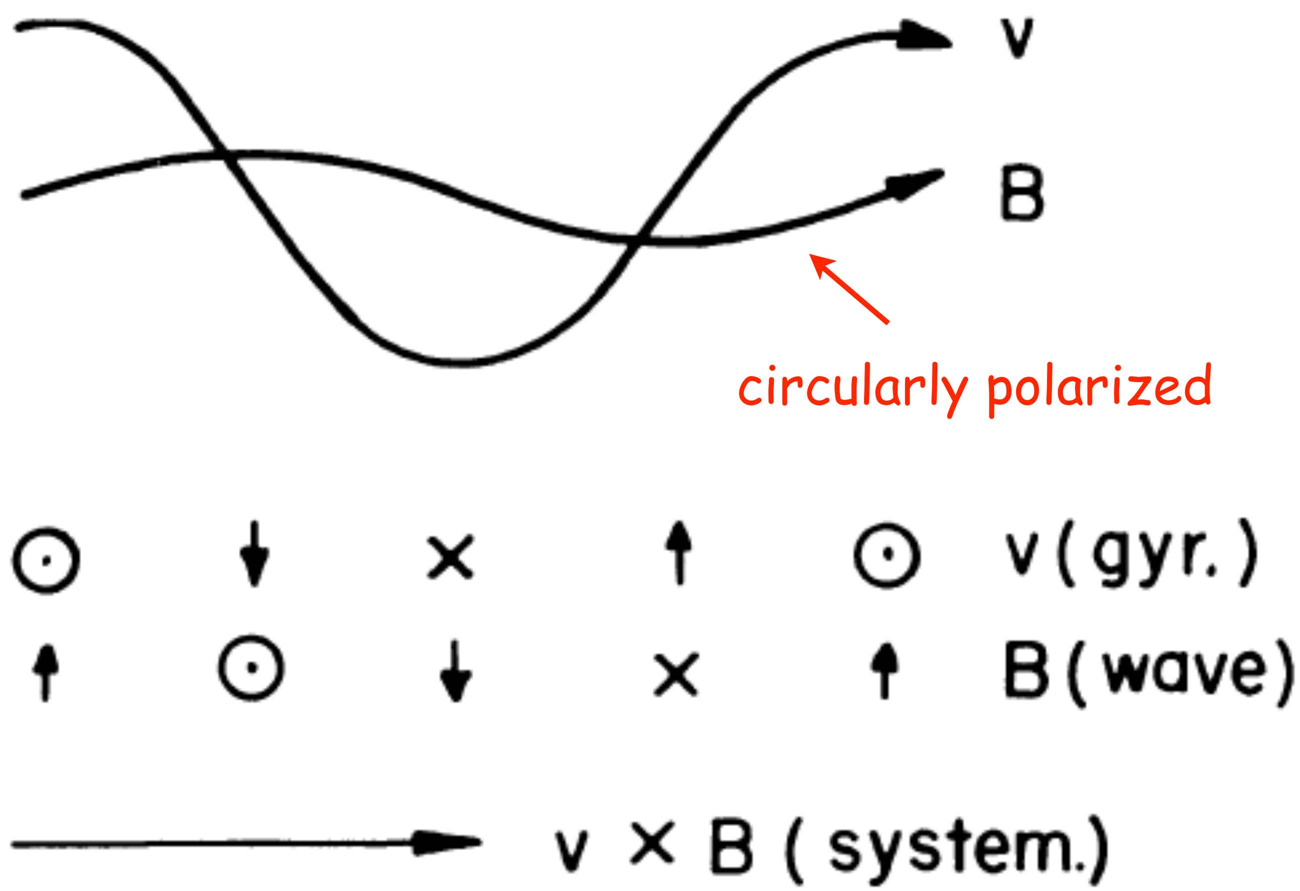


Fig. from Wentzel 1972

Resonant scattering with Alfven waves



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Resonance condition:

to stay in phase with the wave

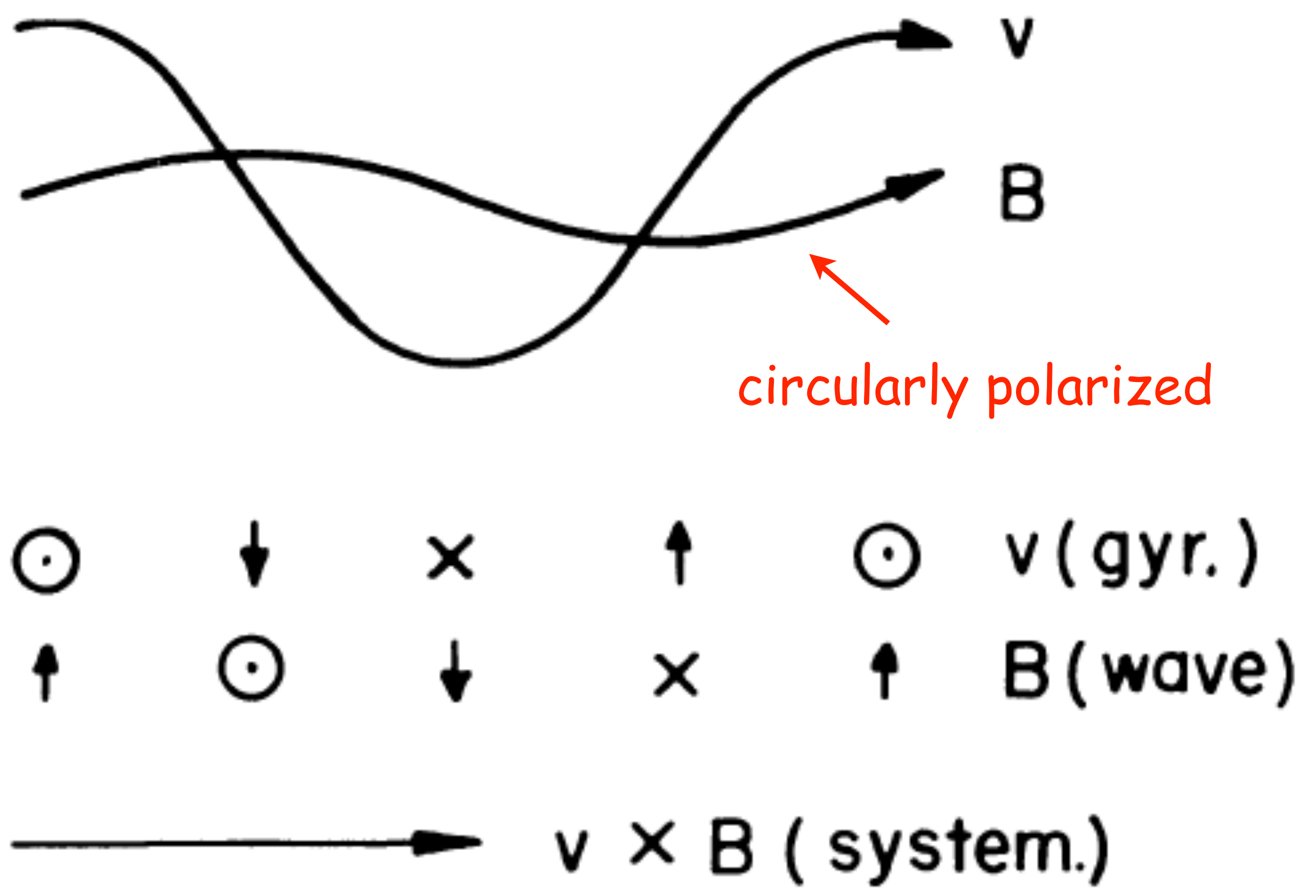
left/right moving CRs

$$\lambda = \frac{2\pi}{k_z} = \left(\frac{2\pi}{\Omega_g}\right)(v_z - V_A) \longrightarrow \boxed{k_z v_z - \omega \pm \Omega_g = 0}$$

particle gyrofrequency

Resonant scattering with Alfven waves

$$v_z >> V_A$$



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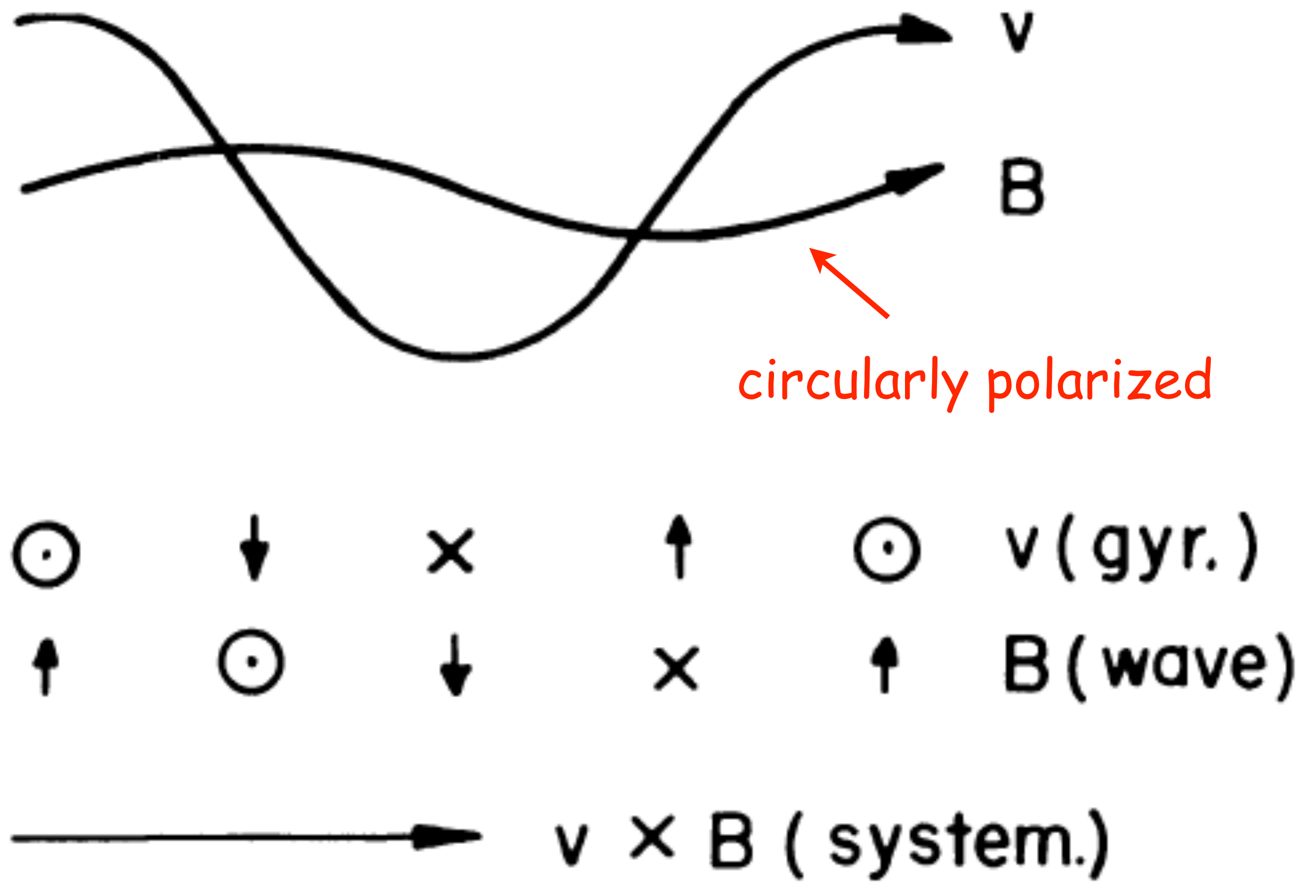

circularly polarized

$$v_z >> V_A$$

- -> "static" B-field
- -> no E field
- -> no work done
- -> $E_{before} = E_{after}$

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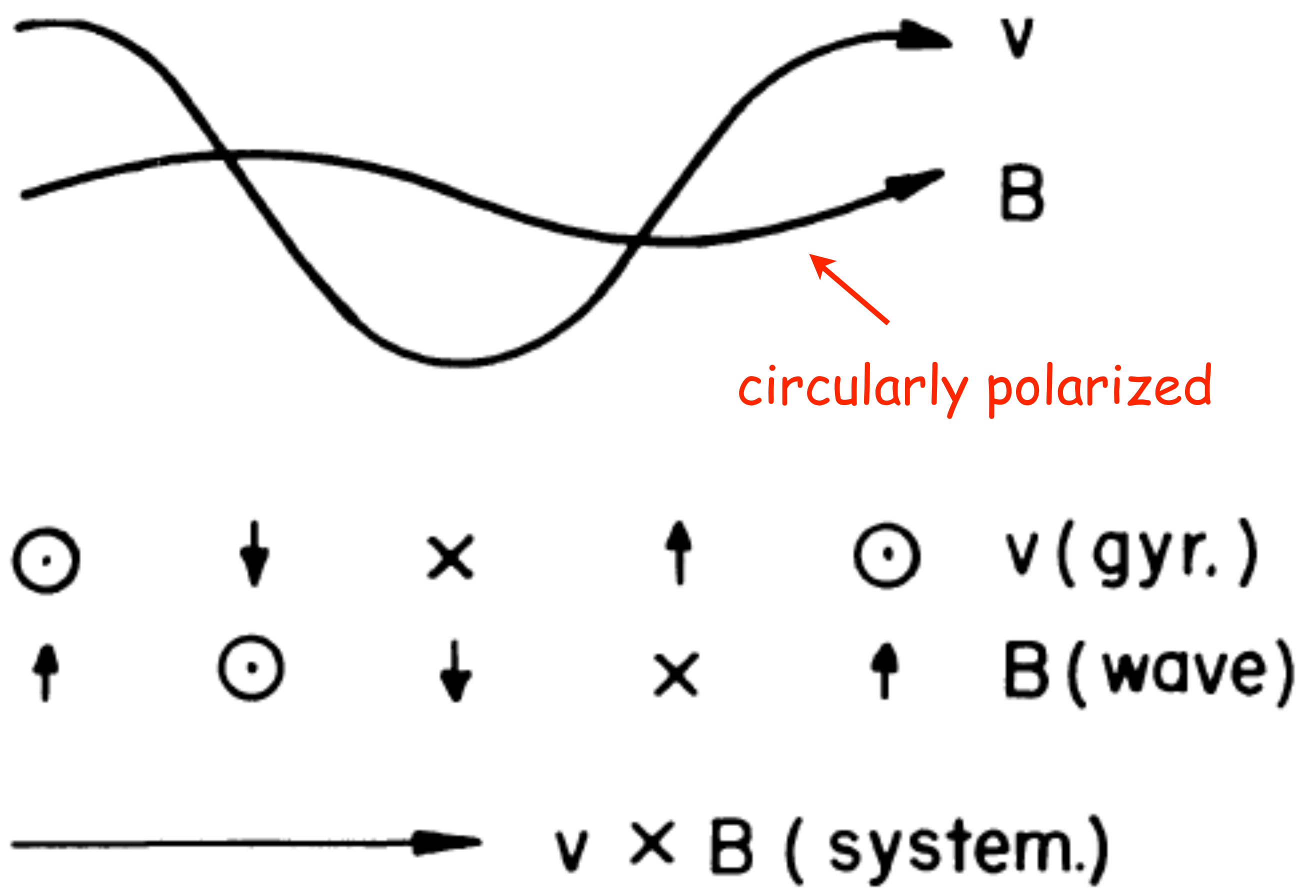

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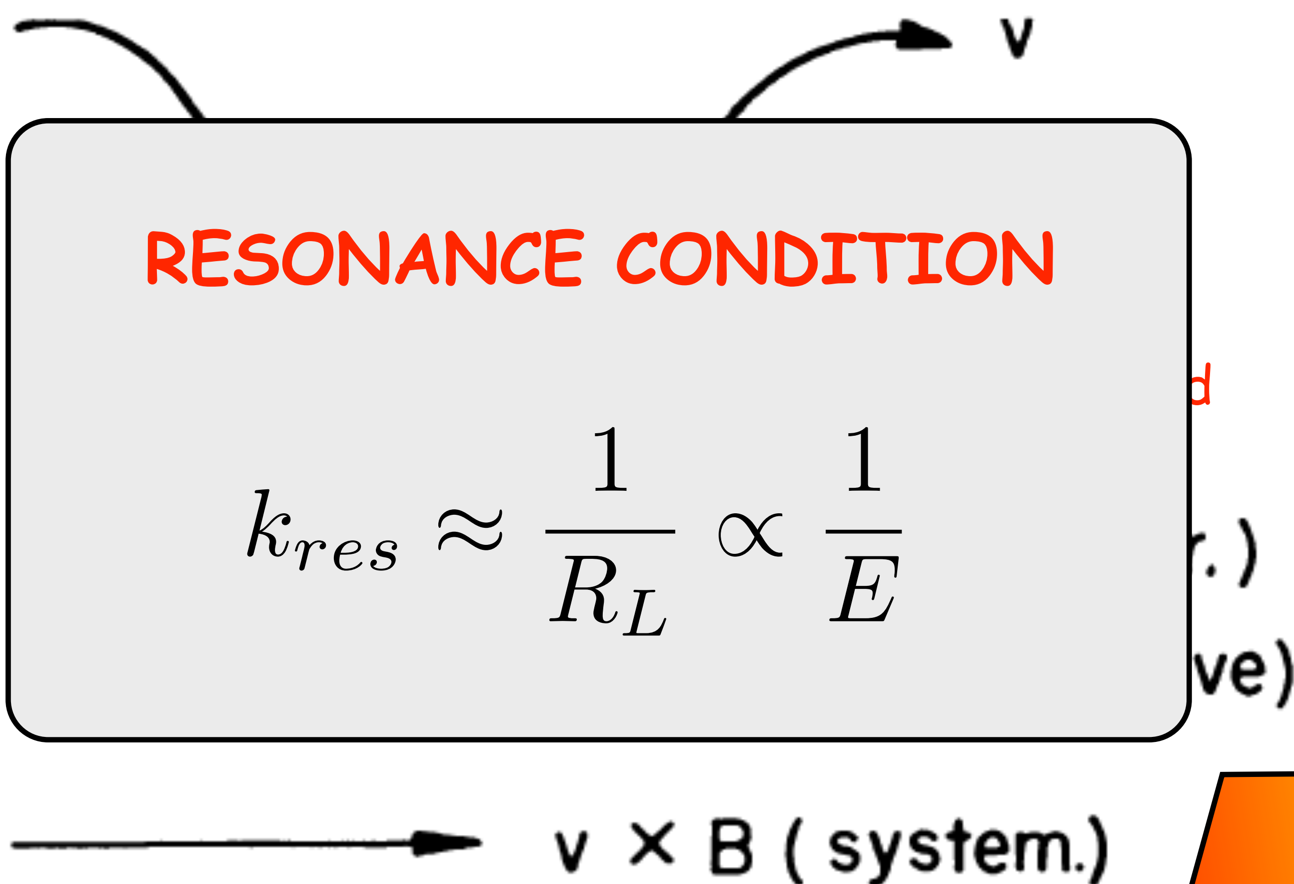

Fig. from Wentzel 1972

RESONANCE CONDITION

$$k_{res} \approx \frac{1}{R_L} \propto \frac{1}{E}$$

$$\boxed{v_z >> V_A}$$

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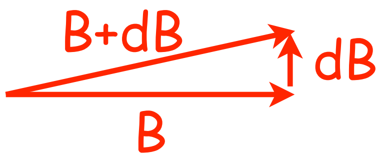
particle gyrofrequency

Diffusion in pitch angle

$$\delta\vartheta \approx \pm \frac{\delta B}{B}$$

angle between B+dB and B



see e.g. Kulsrud's book

Diffusion in pitch angle

change in pitch angle after one scattering

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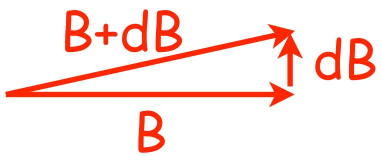

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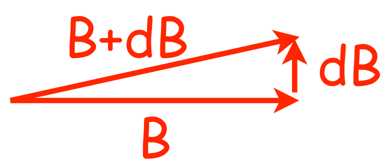

$$\langle(\Delta\vartheta)^2\rangle$$

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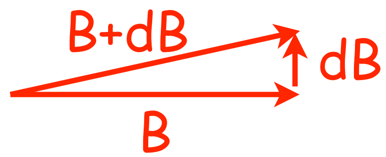

$$\langle(\Delta\vartheta)^2\rangle = \Sigma\langle(\delta\vartheta)^2\rangle$$

train of waves

see e.g. Kulsrud's book

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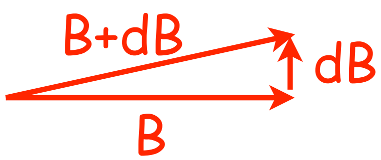

N identical scatterings

$$\langle(\Delta\vartheta)^2\rangle = \Sigma\langle(\delta\vartheta)^2\rangle = N\,\langle(\delta\vartheta)^2\rangle$$

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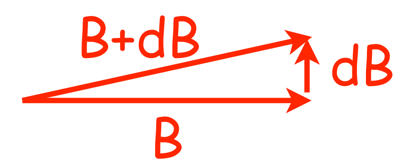

this number comes from the exact calculation

N identical scatterings

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train of waves

duration of one interaction $\tau = \dfrac{2\pi}{\Omega_g}$

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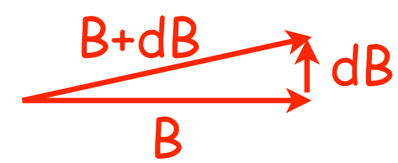

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train of waves

duration of one interaction $\tau = \dfrac{2\pi}{\Omega_g}$

$$\boxed{D_\vartheta = \frac{\langle(\Delta\vartheta)^2\rangle}{2t} = \frac{\pi}{8}\,\Omega_g\left\langle\left(\frac{\delta B}{B}\right)^2\right\rangle}$$

see e.g. Kulsrud's book

From pitch angle to space diffusion

Particle loses memory of initial pitch angle after a time:

$$\tau_s \approx \frac{1}{D_\vartheta} \approx \left(\Omega_g \left\langle \left(\frac{\delta B}{B} \right)^2 \right\rangle \right)^{-1}$$

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scattering time for space diffusion (along B-field)

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e.g. Kolmogorov -> s = 5/3 -> $D \propto E^{1/3}$

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$$\boxed{D_{\|} \approx v^2 \tau_s \propto E^{2-s}}$$

increases with energy

e.g. Kolmogorov -> s = 5/3 -> $D \propto E^{1/3}$

Perpendicular diffusion

e.g. Drury 1983

$$D_{\parallel} = \frac{1}{3}c^2\tau_s \approx \frac{1}{3}\frac{c^2}{\Omega_g}\mathcal{F}(k)^{-1} = \frac{1}{3}R_L c\ \mathcal{F}(k)^{-1}$$

$[I(k_{res})\ k_{res}]/B^2$

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perpendicular displacement after time τ_s

$$\lambda_{\perp} \approx \sqrt{N} \; (\delta\vartheta \; R_L)$$

random walk

displacement after one scattering

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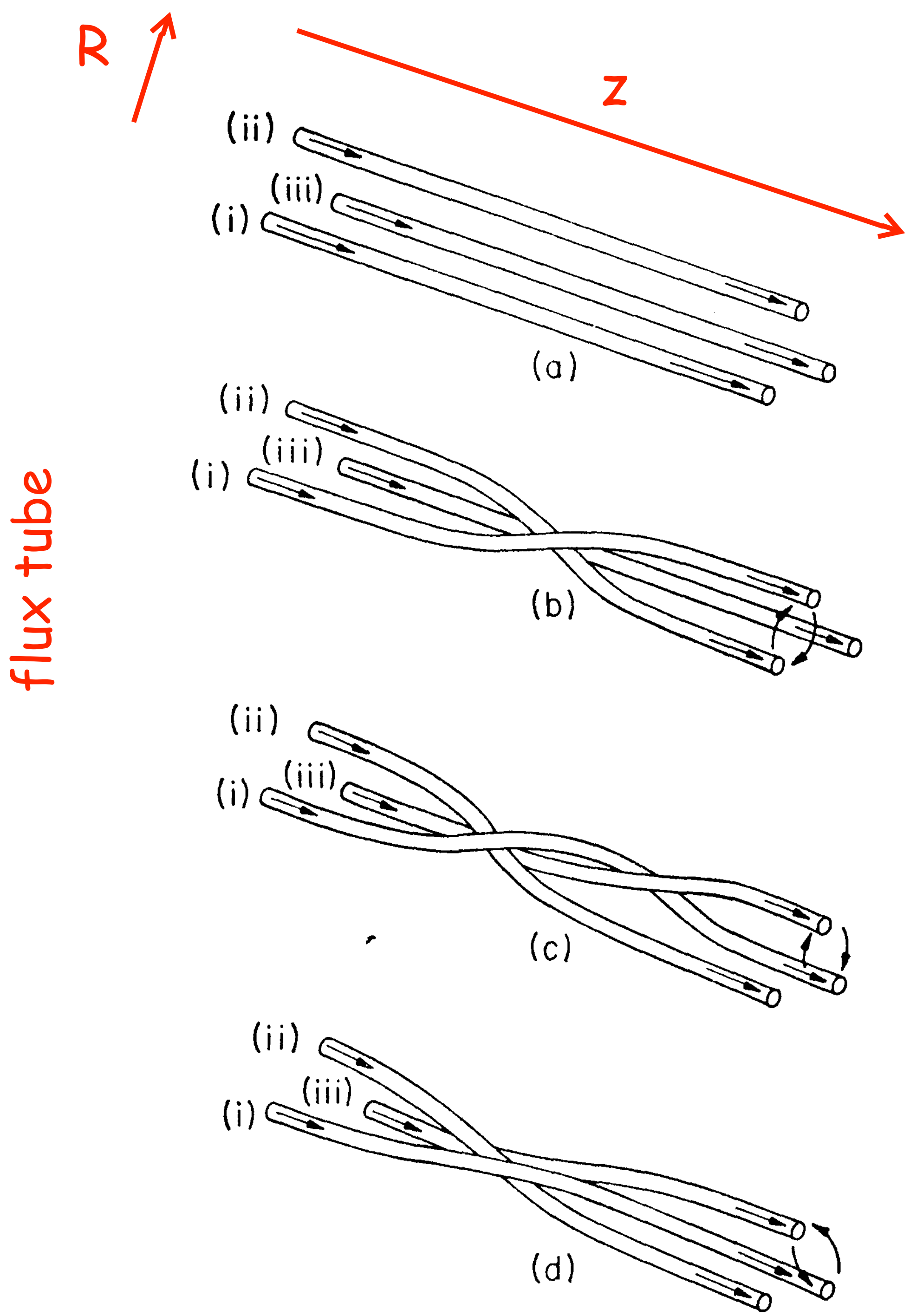
$$D_{\perp} \approx \frac{1}{3}\frac{\lambda_{\perp}^2}{\tau_s} \approx \frac{1}{3}R_L c\mathcal{F}(k) \propto E^s \quad \Rightarrow \quad D_{\parallel}D_{\perp} = \left(\frac{1}{3}R_L c\right)^2 = D_B^2$$

Bohm -> minimum possible diffusion coefficient, totally random field on scale R_L

Diffusion of magnetic field lines

turbulent velocity field -> turbulent B field -> random walk of lines of force



Jokipii & Parker 1969

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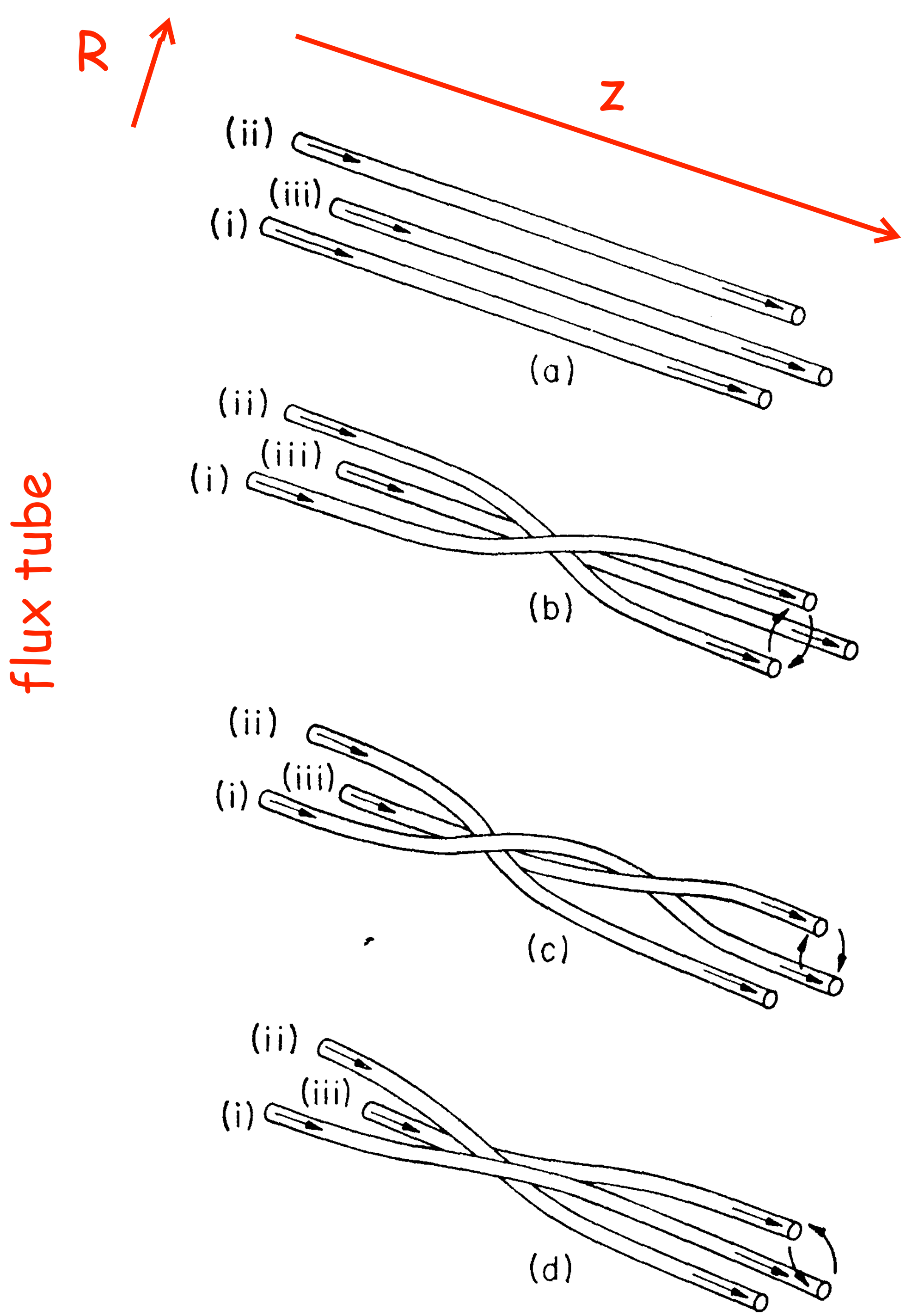

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diffusion coefficient of field lines

$$(\Delta R)^2 \approx 2 D_m \Delta z$$

$\delta B/B < 1$

turbulent field coherence length

$$D_m = \frac{b^2 \lambda_B}{4}$$

Isichenko 1991

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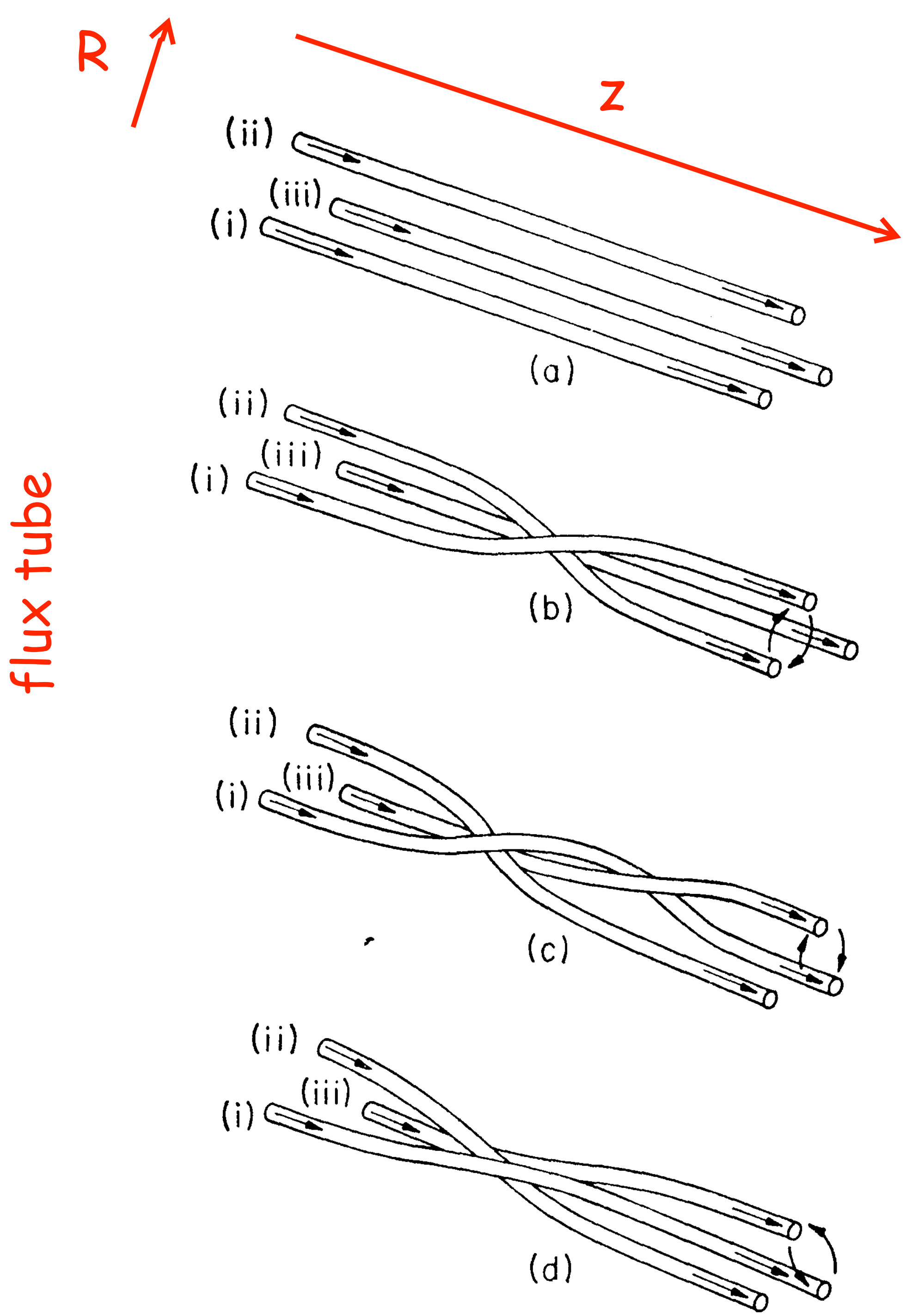

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Isichenko 1991

assumption: no cross field diffusion

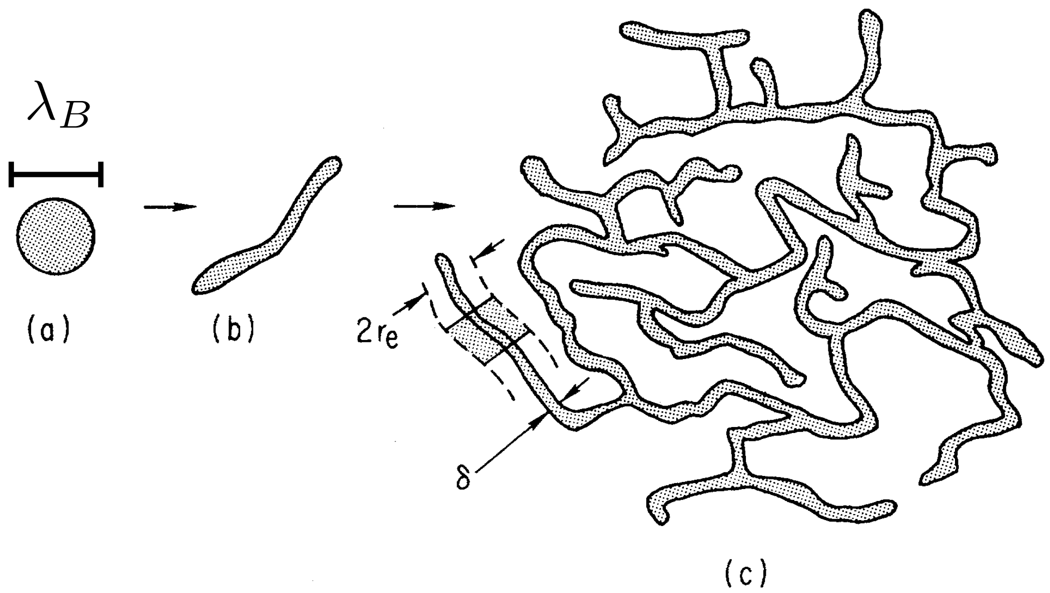
diffusion along B -> $(\Delta z)^2 = 2 \; D_{\parallel} \; t$

$$(\Delta R)^2 \propto t^{1/2}$$

sub-diffusion

Compound diffusion

evolution in z of a coherent patch of B-field lines



Rechester & Rosenbluth 1978

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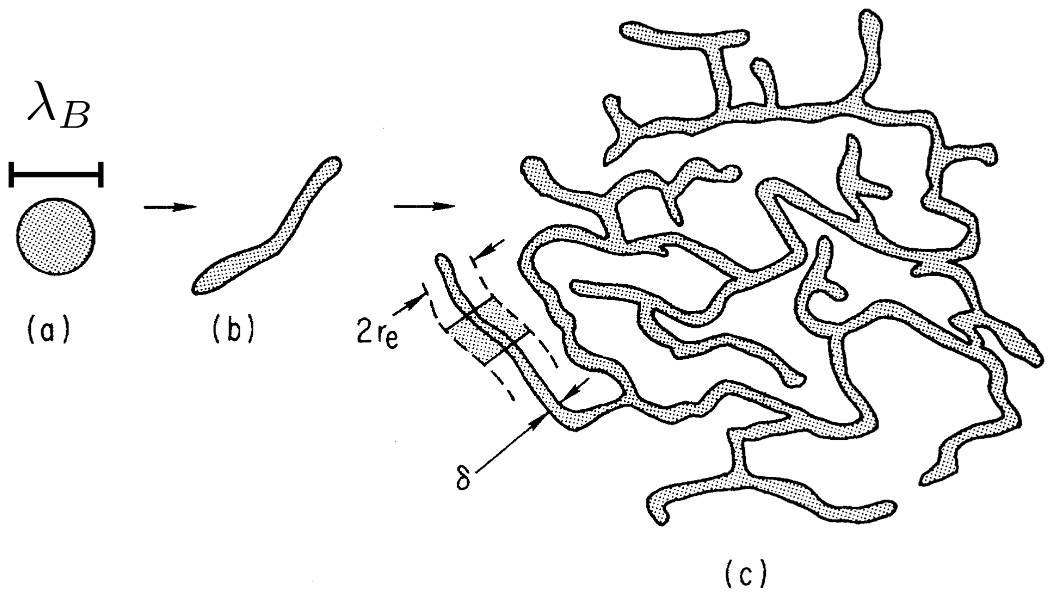

chaotic behavior of B: exponential separation of adiacent field lines

$$l(z) \sim \lambda_B e^{\left(\frac{z}{\lambda_L}\right)}$$

Lyapunov index -> $\lambda_L = \lambda_B / b^2$

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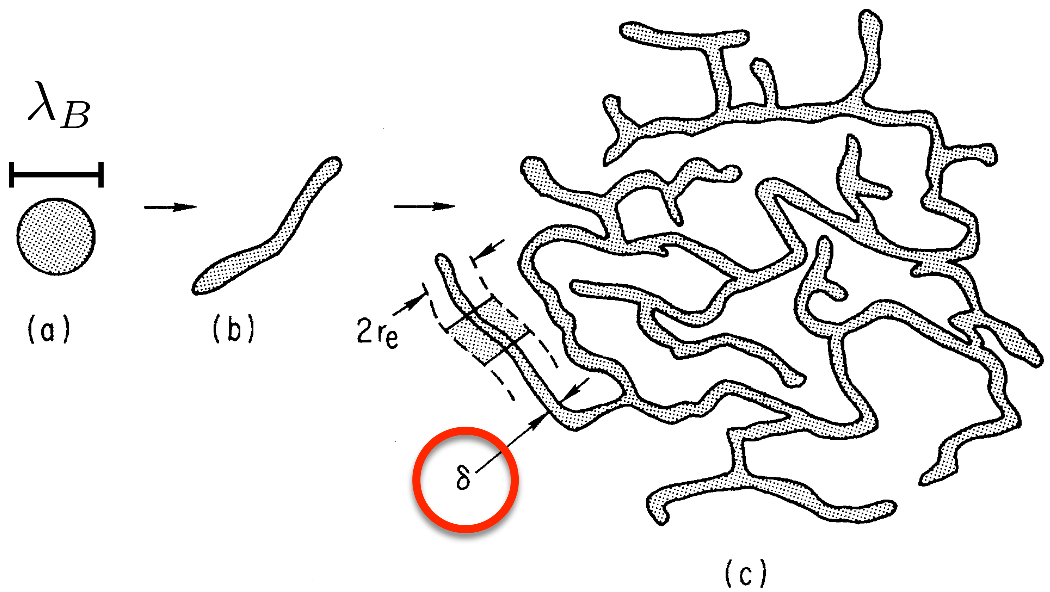

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constant B -> constant area $\longrightarrow$ $\delta(z) \sim \lambda_B e^{-\left(\frac{z}{\lambda_L}\right)}$

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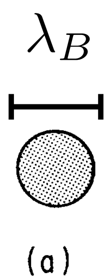
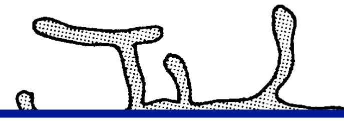
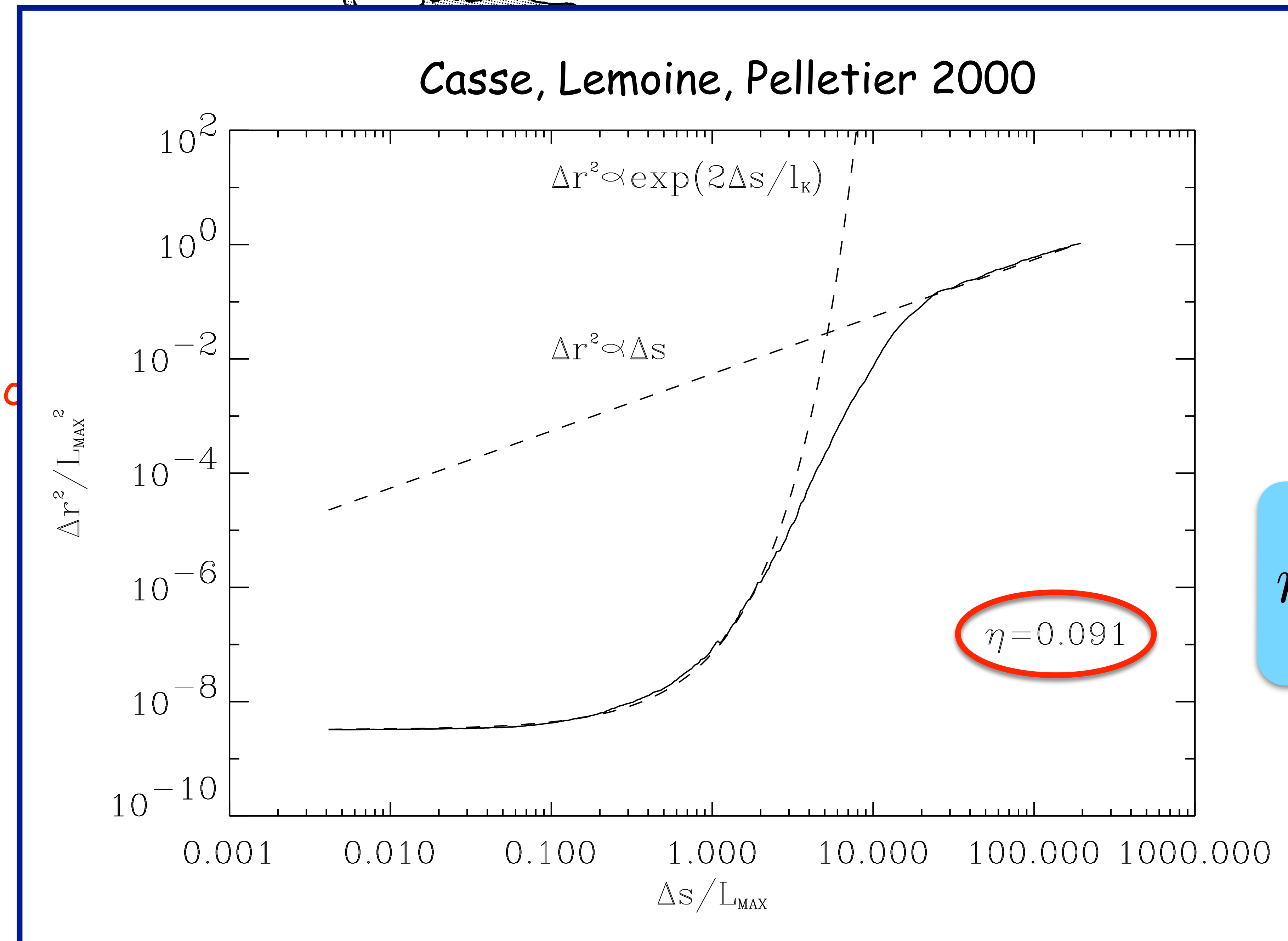
chaotic behavior of B: exponential separation of adiacent field lines







$$_B e^{\left(\frac{z}{\lambda_L}\right)}$$

$$\lambda_L = \lambda_B/b^2$$

$$_B e^{-\left(\frac{z}{\lambda_L}\right)}$$

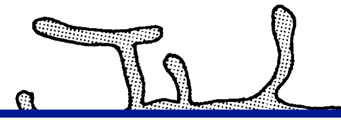
$$\eta = \frac{(\delta B)^2}{(B+\delta B)^2}$$

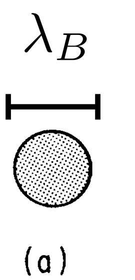
Rechester & Rosenbluth 1978

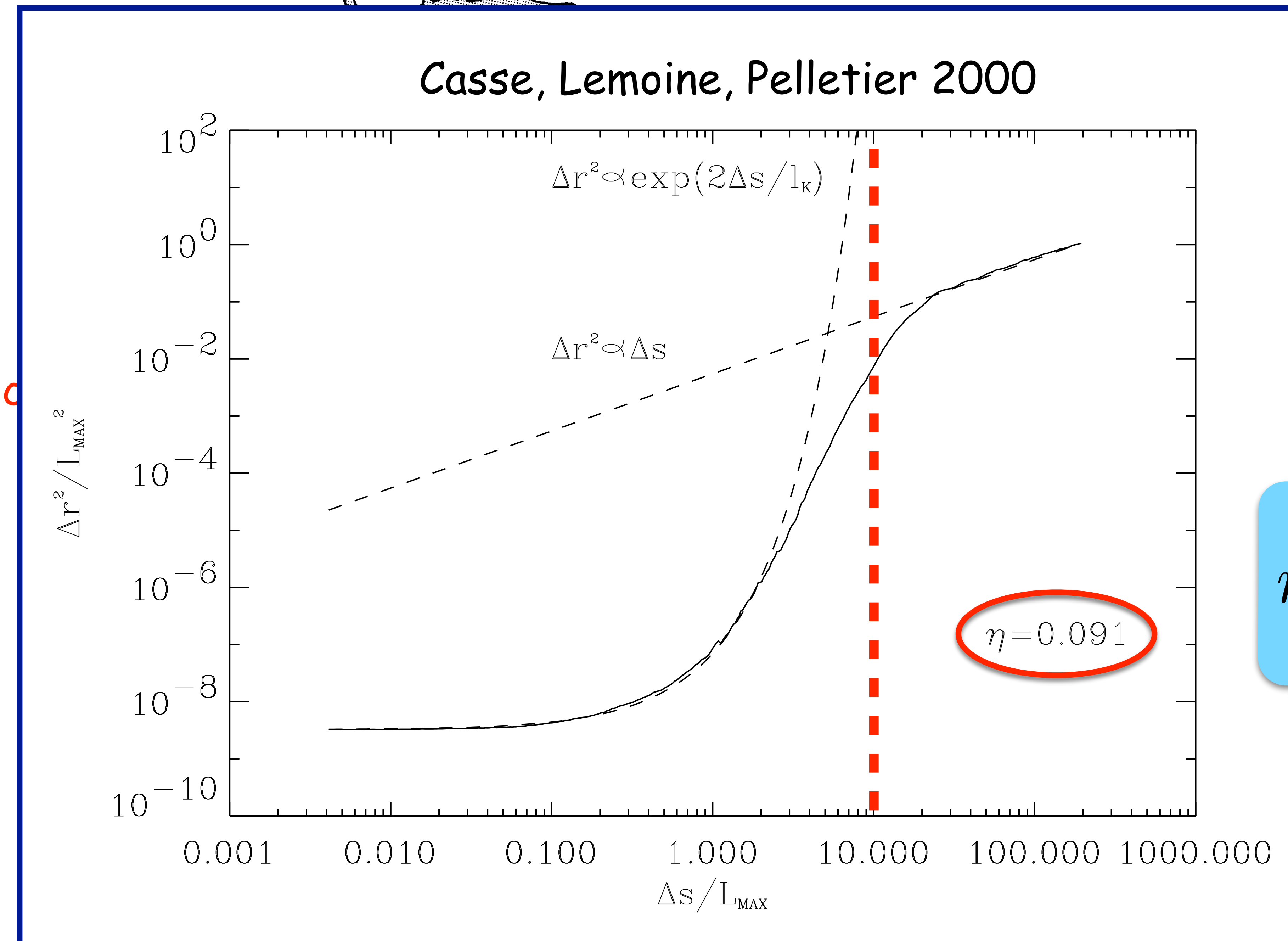
Compound diffusion

evolution in z of a coherent patch of B-field lines

chaotic behavior of B: exponential separation of adiacent field lines







$$_B e^{\left(\frac{z}{\lambda_L}\right)}$$

$$\lambda_L = \lambda_B/b^2$$

$$_B e^{-\left(\frac{z}{\lambda_L}\right)}$$

$$\eta = \frac{(\delta B)^2}{(B+\delta B)^2}$$

Rechester & Rosenbluth 1978

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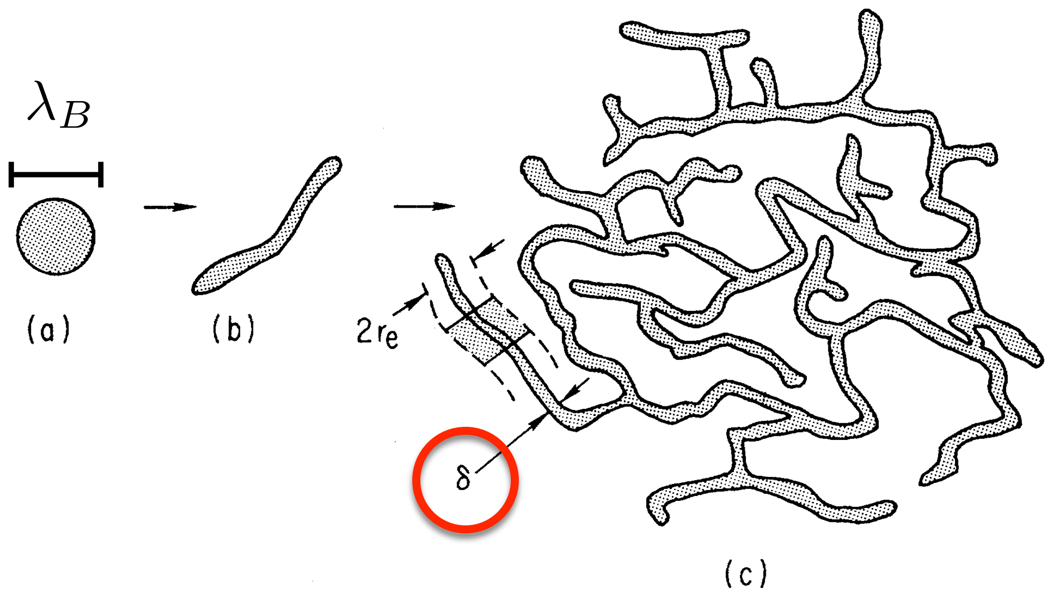

constant B -> constant area

chaotic behavior of B: exponential separation of adiacent field lines

$$l(z) \sim \lambda_B e^{\left(\frac{z}{\lambda_L}\right)}$$

Lyapunov index -> $\lambda_L = \lambda_B / b^2$

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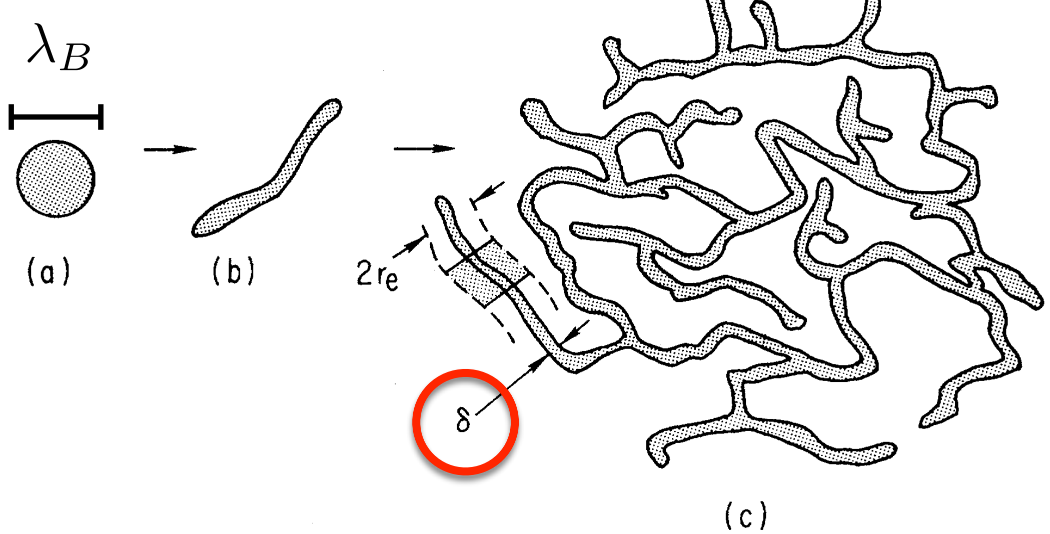

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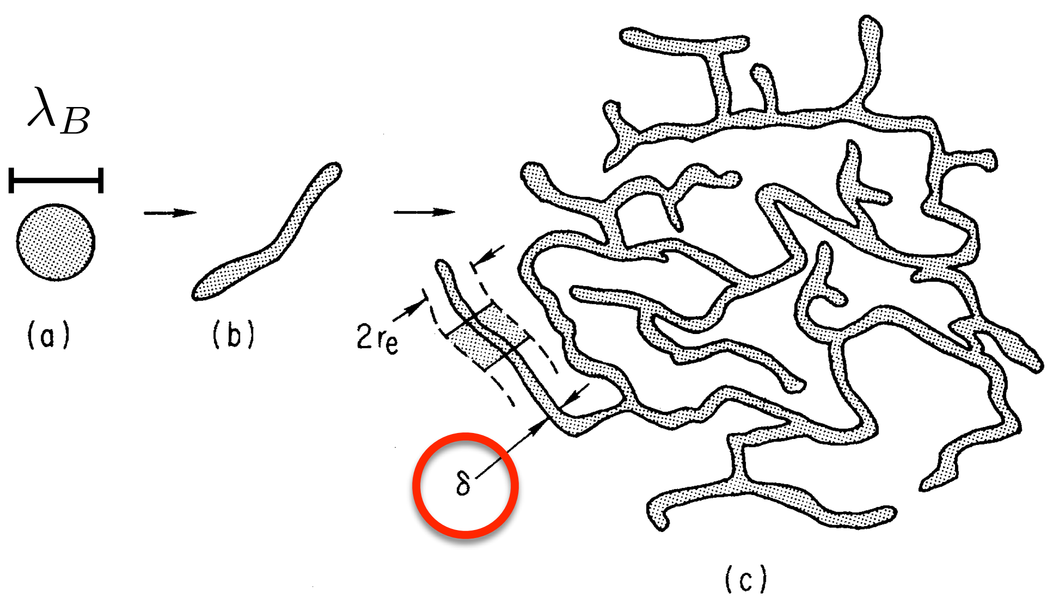

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random walk across B with mean free path -> $L_\perp$

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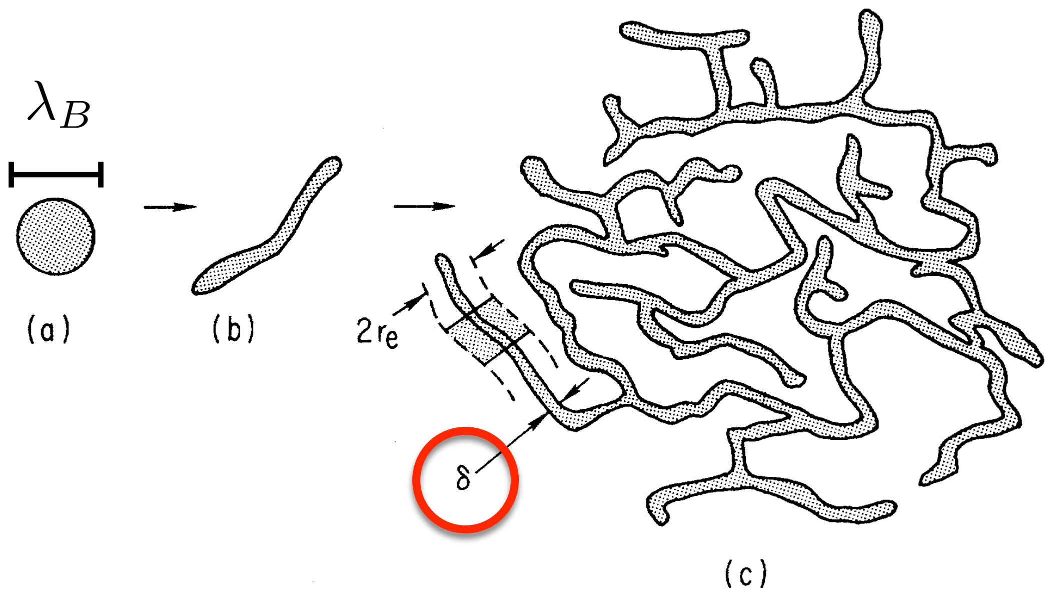

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random walk across B with mean free path -> $L_\perp$

perpendicular diffusion -> $\kappa_\perp \approx \dfrac{L_\perp^2}{t_d}$

Rechester & Rosenbluth 1978

Numerical simulations

implicit assumption done so far: $$\frac{\delta B}{B} \ll 1$$

Casse, Lemoine, Pelletier 2000

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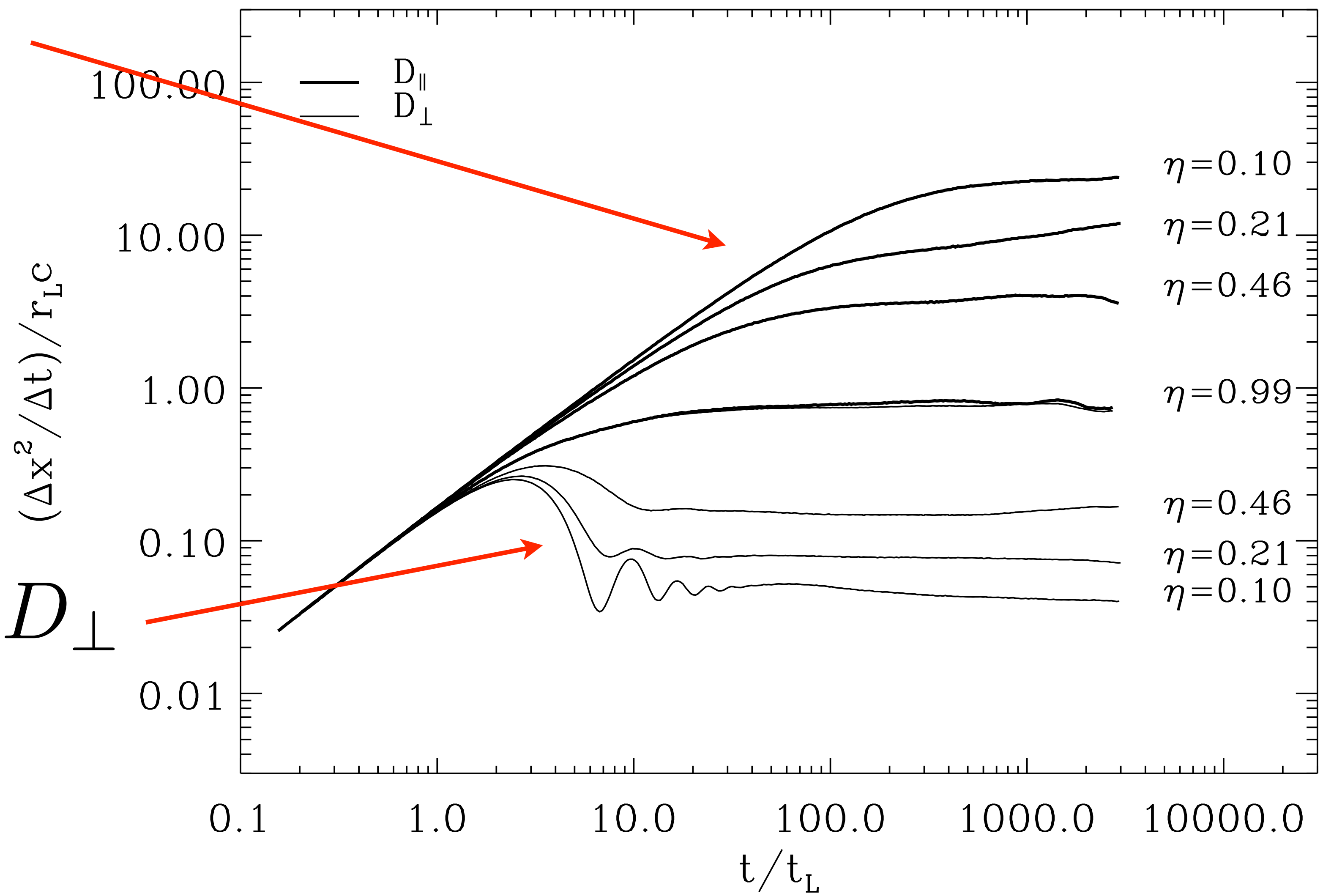
$D_{\parallel}$



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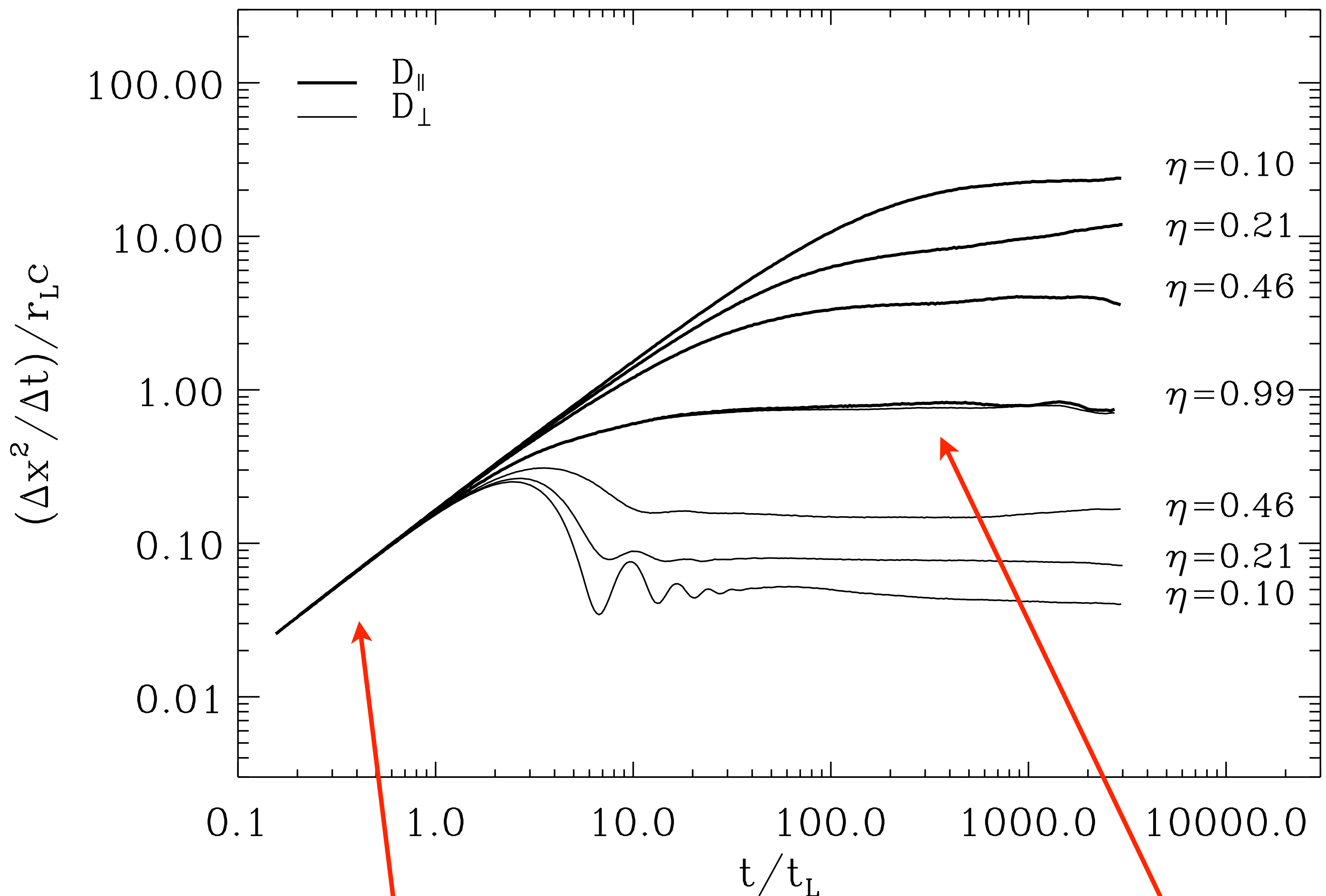

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ballistic $\Delta x \propto t$

$\Delta x \propto t^{1/2}$ diffusive

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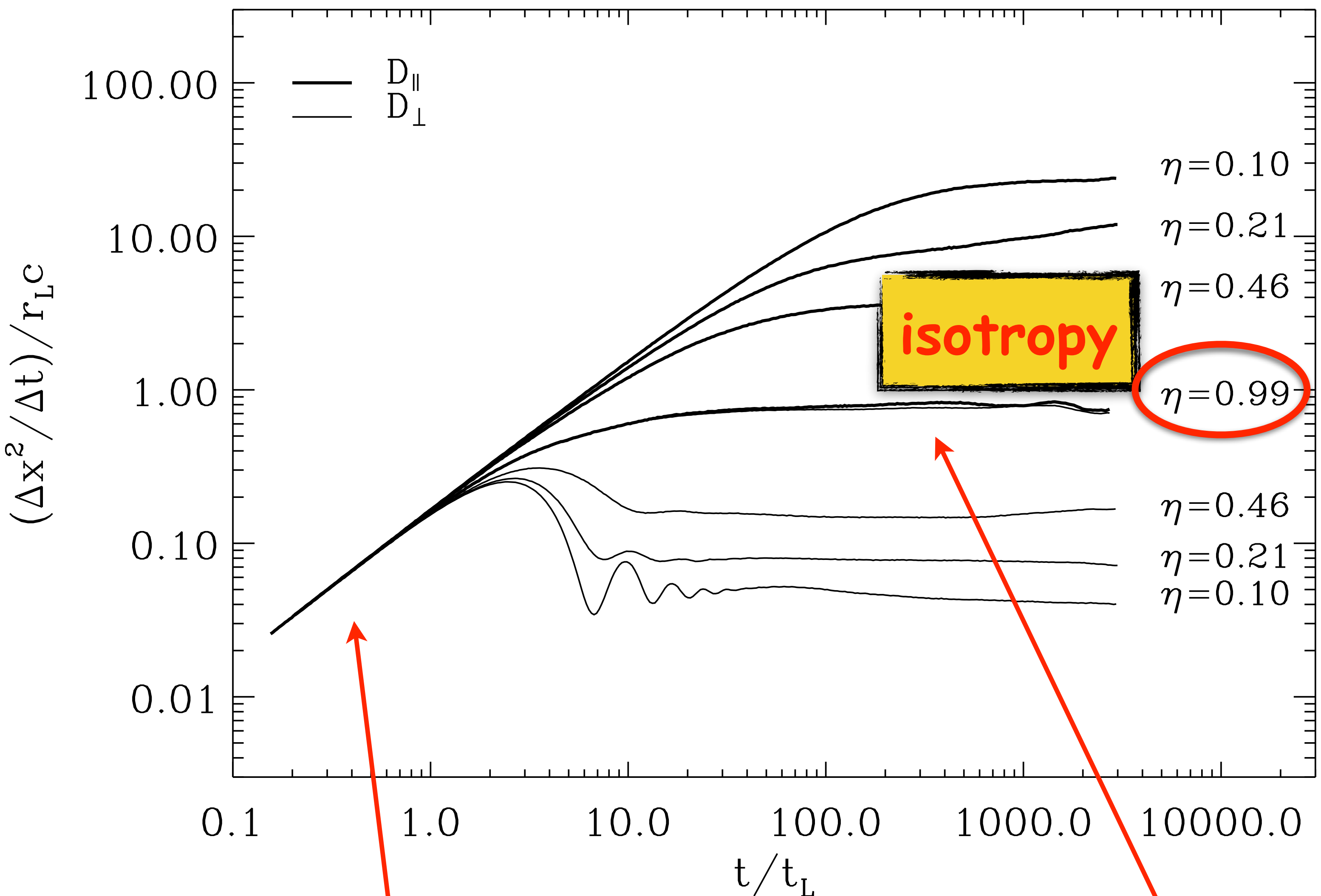

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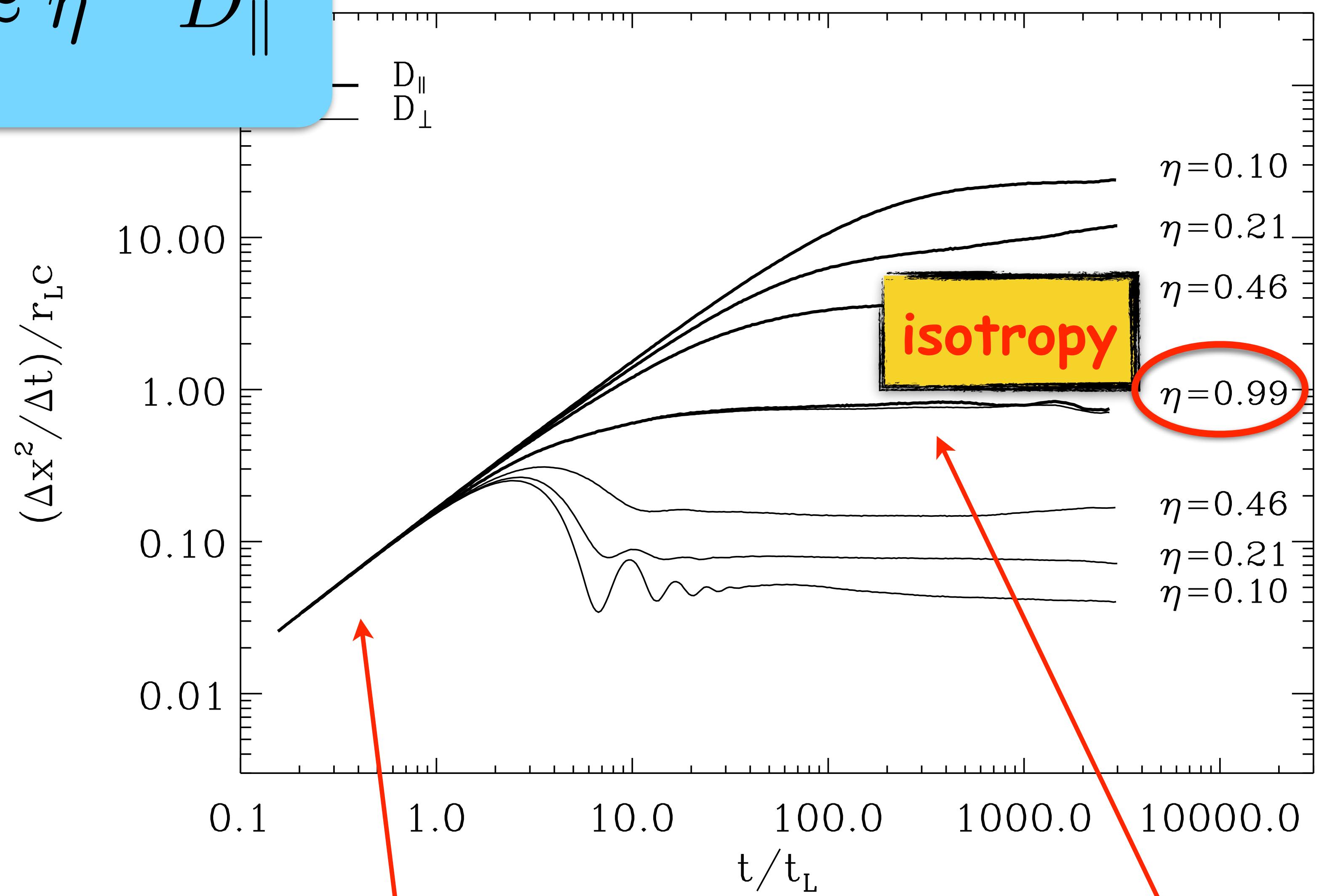
$$D_\perp \approx \eta^{2.3} D_\parallel$$



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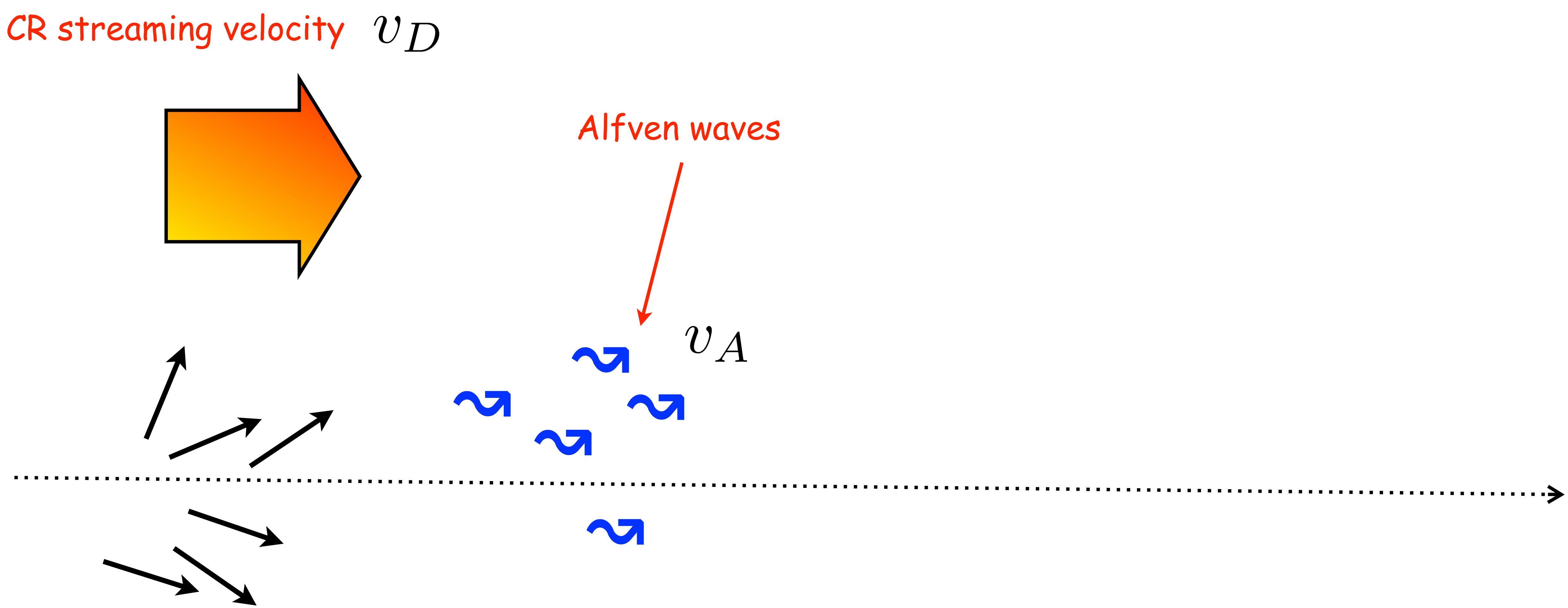
Alfven waves from streaming instability

Naive treatment

Kulsrud's book



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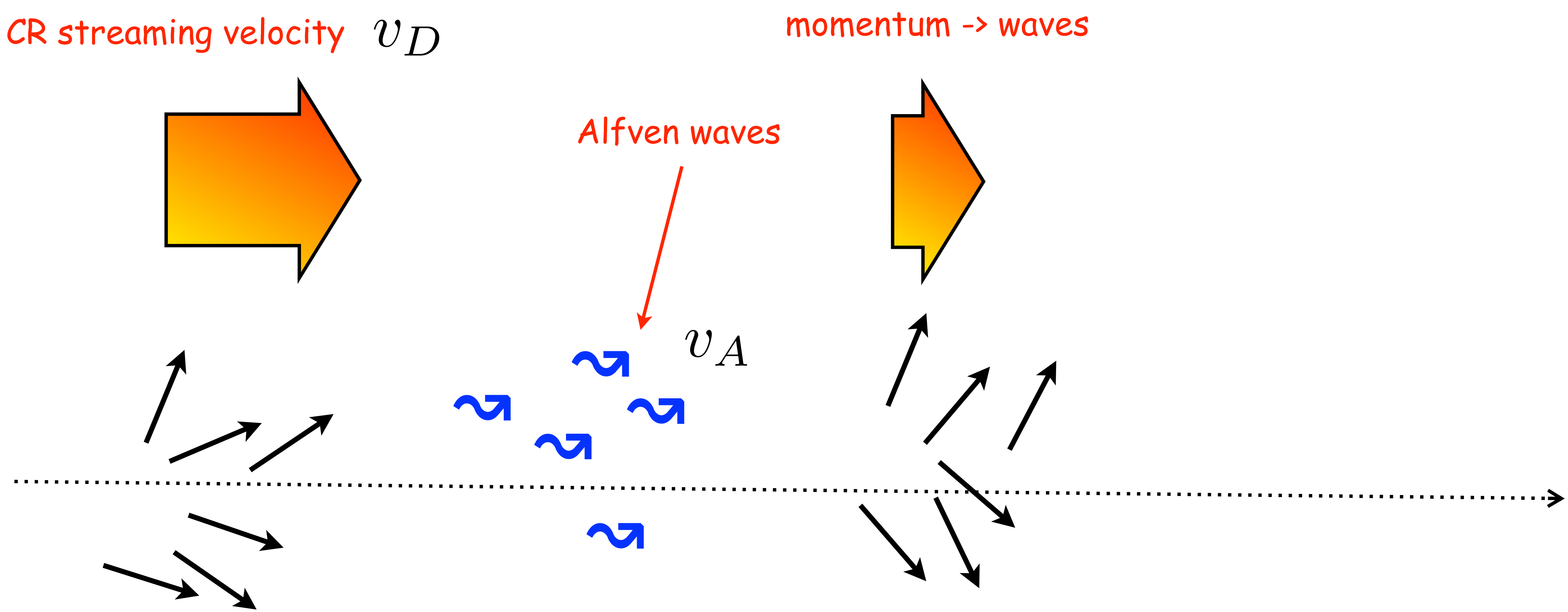

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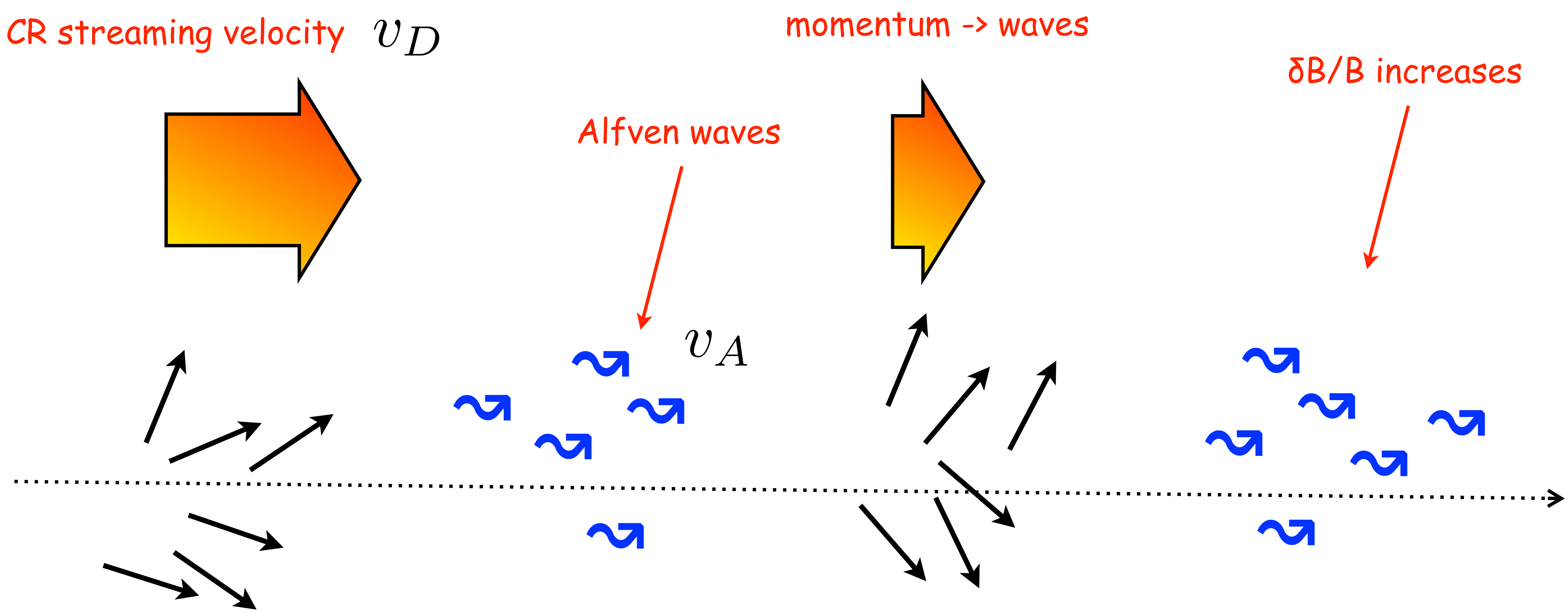

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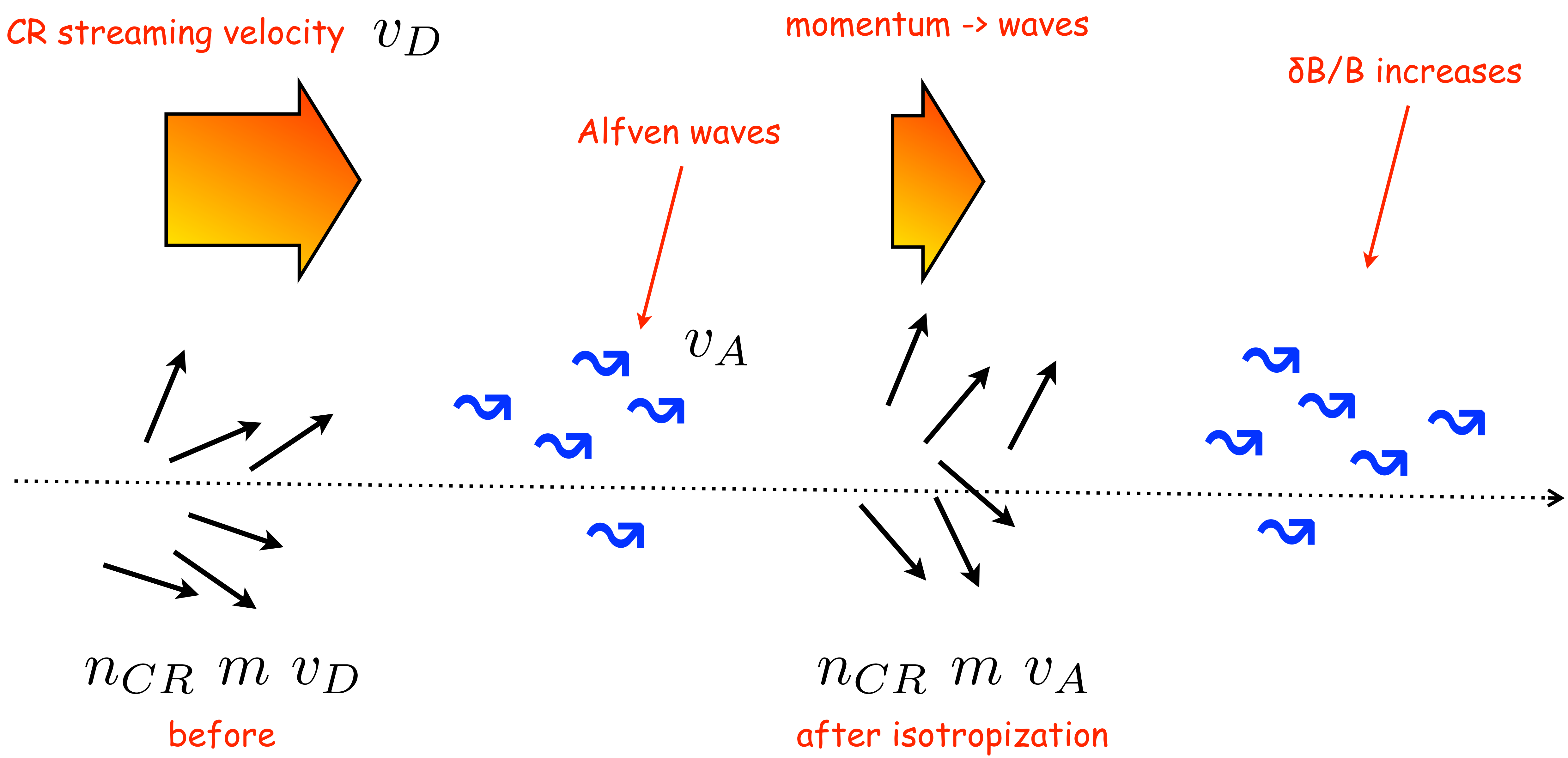

CRs are isotropized in the frame of waves

$$\frac{dP_{CR}}{dt} = \frac{n_{CR}\ m\ (v_D - v_A)}{\tau} \qquad \tau \approx \frac{1}{D_\vartheta}$$

Alfven waves from streaming instability

$$\frac{dP_{CR}}{dt} = \frac{n_{CR}\, m\, (v_D - v_A)}{\tau} = 2\,\Gamma_{CR}\,\frac{(\delta B)^2}{8\pi\, v_A}$$

$$m = \gamma M$$

growth rate of waves

$$\boxed{\Gamma_{CR} \sim \frac{n_{CR}}{n_{gas}} \left(\frac{v_D - v_A}{v_A} \right) \Omega_0}$$

does NOT depend on dB/B !!!

10^{-10} cm^{-3}

$$\Gamma_{CR} \approx \frac{n_{CR}}{n_{gas}}\, \Omega_0 \approx 10^{-4}\ \mathrm{yr}^{-1}$$

1 cm^{-3}

much shorter than the CR lifetime !!!

Kulsrud's book

Problem #1: neutrals

Ion-neutral damping

$$\Gamma_{i-n} \approx 8.4 \times 10^{-9} \left(\frac{n_n}{\mathrm{cm}^{-3}}\right) \left(\frac{T}{10^4 \mathrm{K}}\right)^{0.4} \mathrm{yr}^{-1} \approx (40...250)^{-1} \left(\frac{n_n}{0.1\ \mathrm{cm}^{-3}}\right) \mathrm{yr}^{-1}$$

Kulsrud & Cesarsky 1971

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@ ~1 GeV -> very effective confinement

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2 way-outs:

1) go to a fully ionized region (Galactic halo?)

2) external source of turbulence (other than CRs)

Kulsrud & Cesarsky 1971

Problem #2: anisotropy

Way-out -> “external” Alfvenic turbulence

Goldreich & Sridhar 1995, 1997

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Goldreich&Sridhar (1995,1997) -> strong Alfvenic turbulence is anisotropic!

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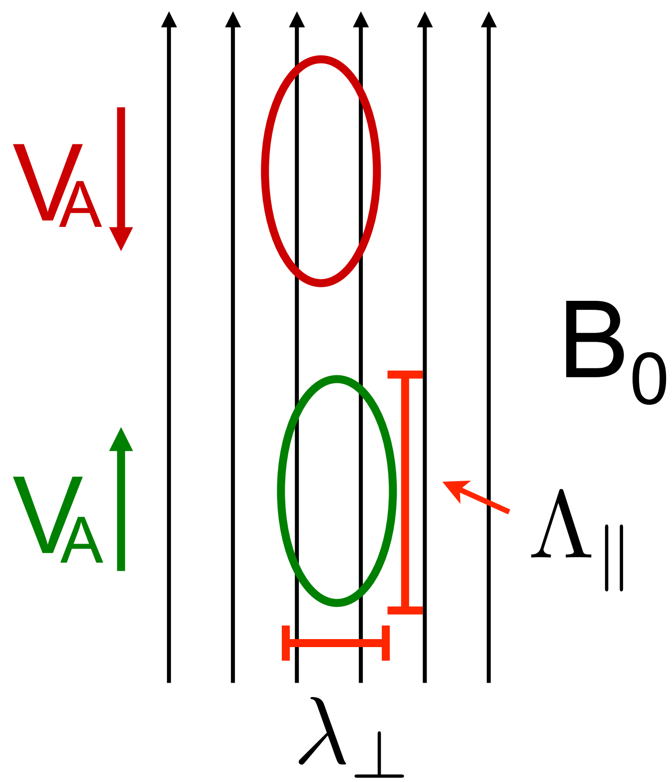

interaction of wave packets
moving in opposite directions

Goldreich & Sridhar 1995, 1997

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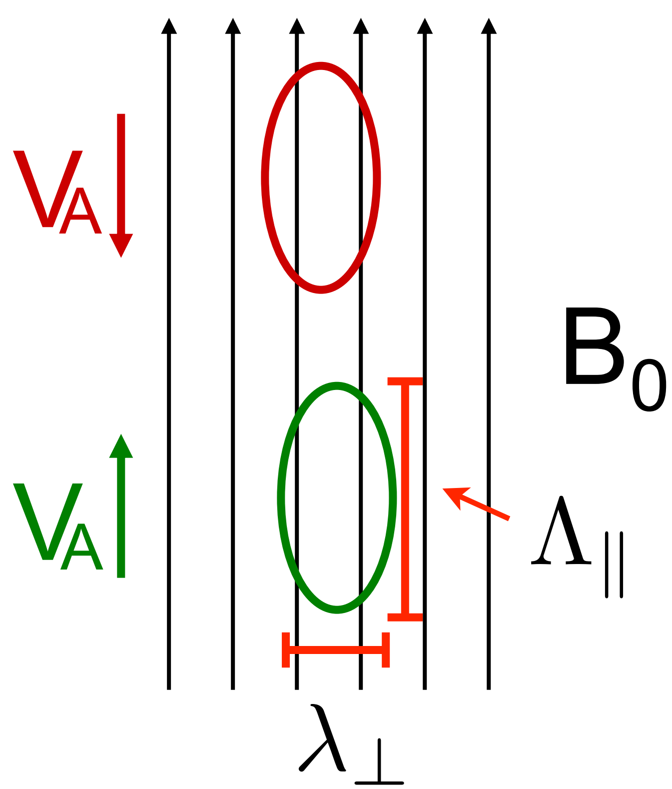

interaction of wave packets
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critical balance

$$\omega = v_A k_{||} \sim \frac{v_A}{\Lambda_{||}}$$

(wave period)$^{-1}$

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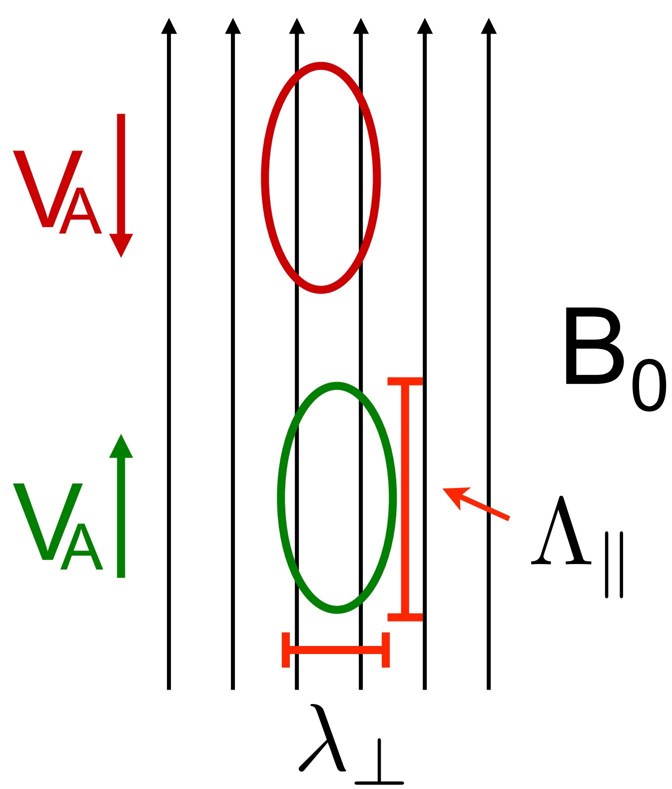

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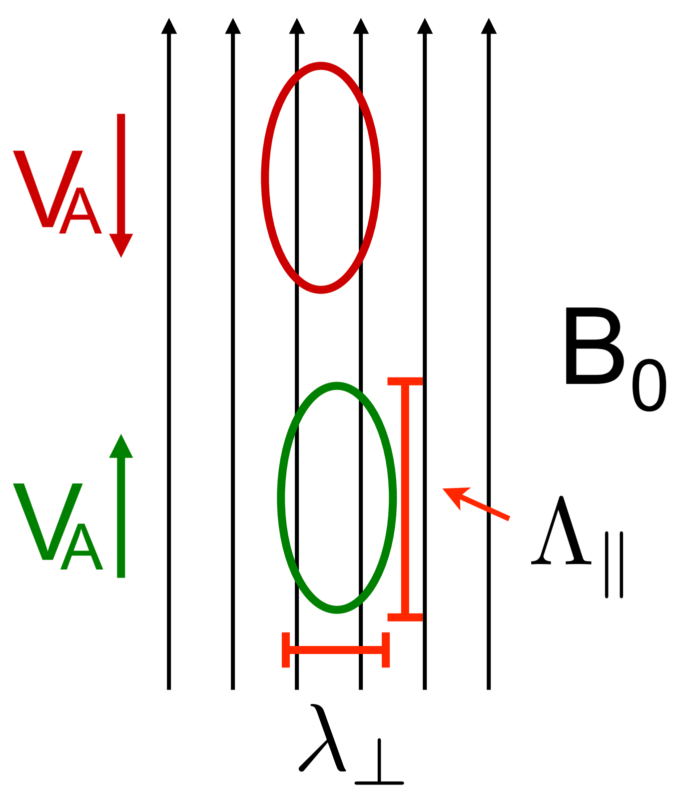

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(wave period)$^{-1}$ (cascade time)$^{-1}$

constant energy cascade rate (Kolmogorov)

$$\epsilon \sim \frac{(\delta v)^2}{t_c} \sim \frac{(\delta v_{\lambda_\perp})^3}{\lambda_\perp} \equiv \frac{v_A^3}{L_{inj}}$$

Goldreich & Sridhar 1995, 1997

Problem #2: anisotropy

putting things together:

$$\delta v_{\lambda_\parallel} = \left(\frac{\lambda_\perp}{L_{inj}}\right)^{1/3}$$

$$\Lambda_\parallel = \lambda_\perp^{2/3} L_{inj}^{1/3}$$

same relations hold for δB

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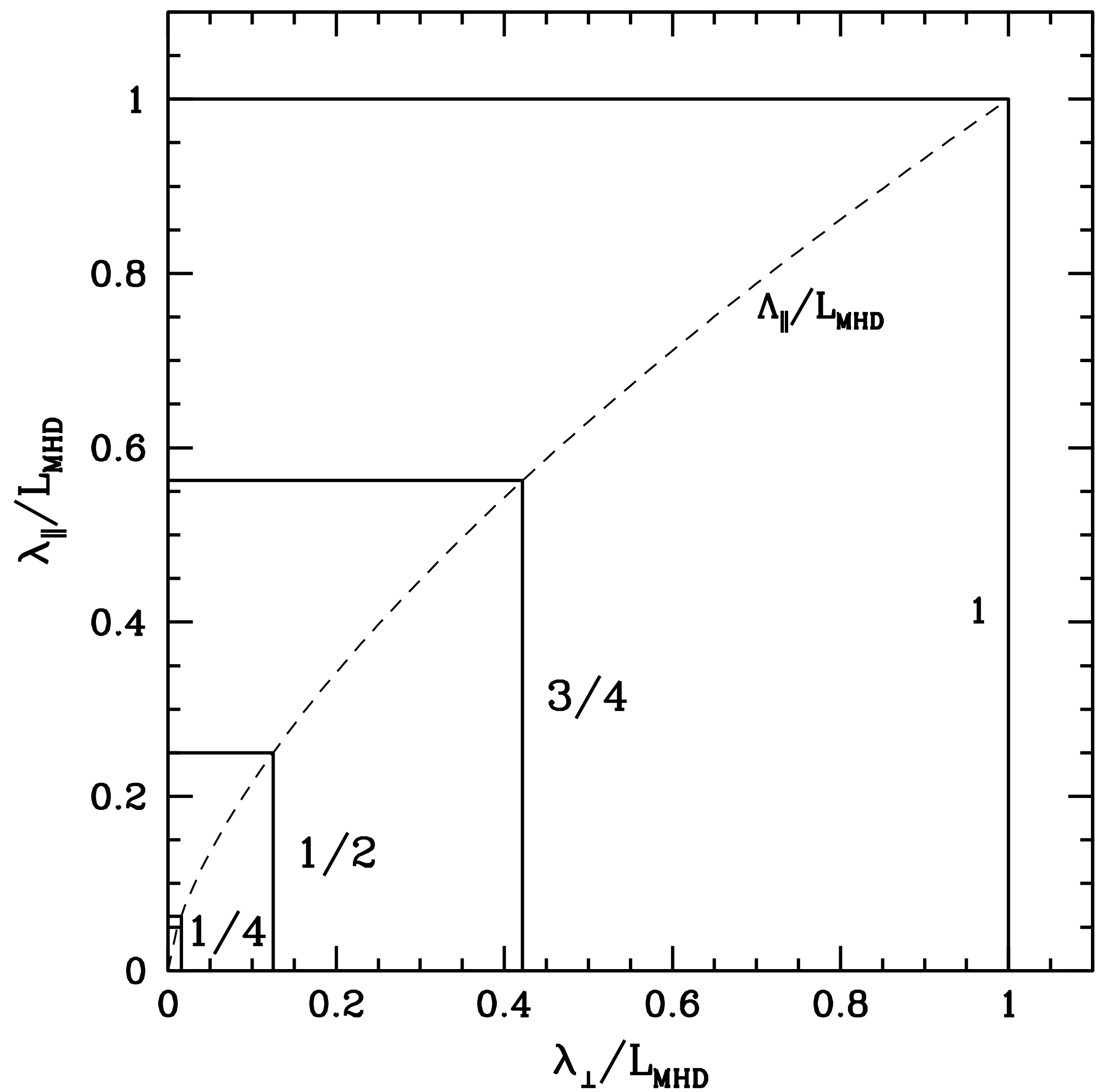

Lithwick & Goldreich 2001

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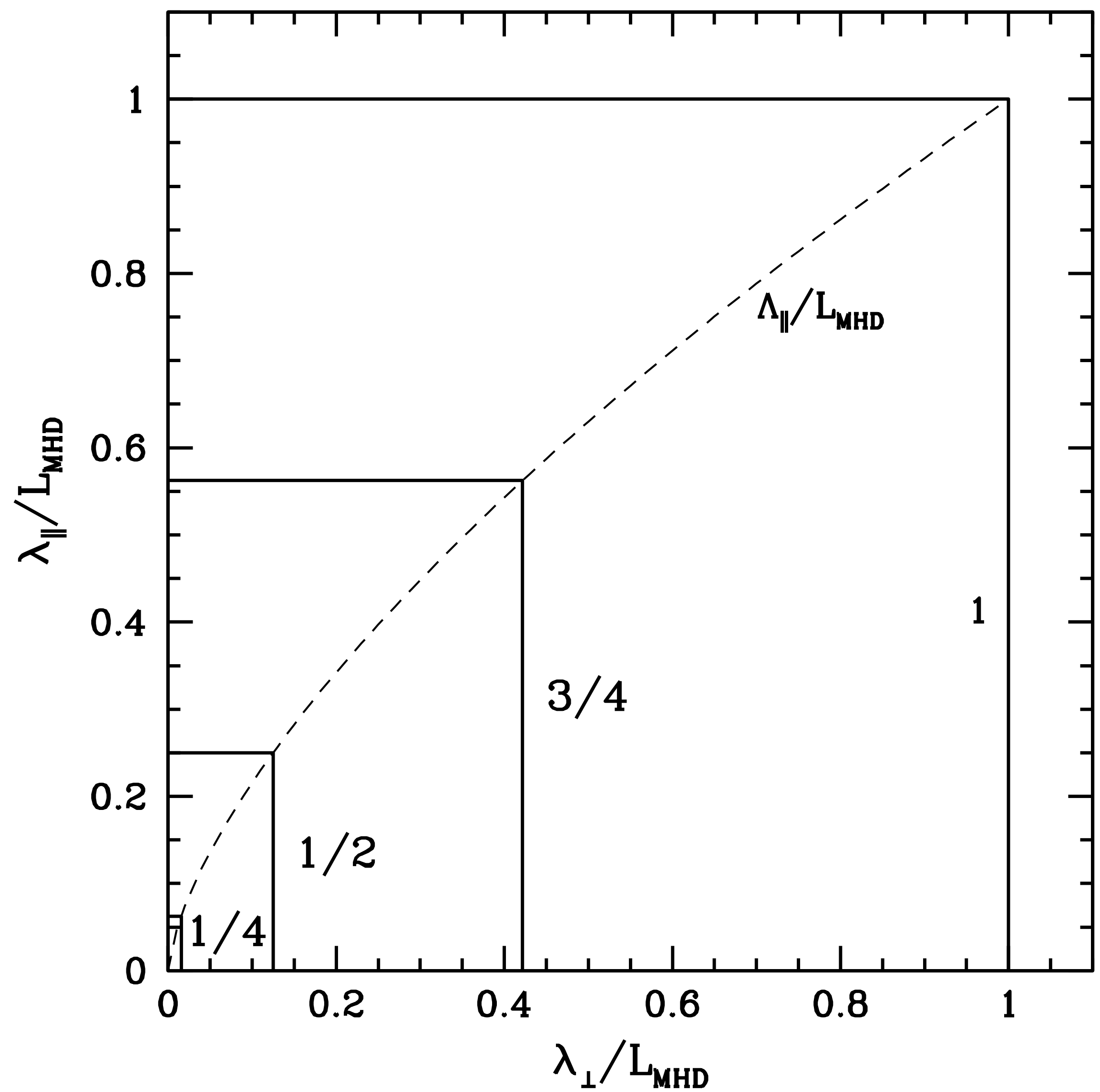

Lithwick & Goldreich 2001

CR-wave resonance condition

no power here

$$k_z v_z \approx \Omega_g$$

scattering is very ineffective!

Chandran 2000

Problem #3: turbulent damping

Way-out -> fully ionized medium

Farmer & Goldreich 2004

Problem #3: turbulent damping

Way-out -> fully ionized medium

Parallel Alfven waves generated by CR streaming instability interact with (mostly) perpendicular background Alfven waves -> introduction of perpendicular component to the CR-generated waves -> waves are integrated within the cascade -> damping mechanism

Farmer & Goldreich 2004

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It can be shown that:

$$\Gamma_d \sim \left(\frac{\epsilon}{R_L\ v_A}\right)^{\frac{1}{2}} \sim (600)^{-1}\left(\frac{\epsilon}{0.06\mathrm{erg/s/g}}\right)^{\frac{1}{2}}\left(\frac{v_A}{200\mathrm{km/s}}\right)^{-\frac{1}{2}}\left(\frac{E}{\mathrm{GeV}}\right)^{-\frac{1}{2}}\ \mathrm{s}^{-1}$$

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streaming instability does not work for $E \gtrsim 100 \ \mathrm{GeV}$

Farmer & Goldreich 2004

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streaming inst[illegible] work for $E \gtrsim 100 \ \mathrm{GeV}$

might work in the vicinity of CR sources (Ptuskin+, Malkov+, Nava+)

Farmer & Goldreich 2004

Magnetosonic waves

Alfven waves $\omega = k_z V_A$ incompressible

magnetosonic w. $\omega = k v_{\pm}$ compressible

fast mode

$$v_{\pm} = \left\{ \frac{1}{2} \left[v_A^2 + c_s^2 \pm \sqrt{(v_A^2 + c_s^2)^2 - 4 v_a^2 c_s^2 \cos^2 \vartheta} \right] \right\}^{1/2}$$

slow mode

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slow mode

	high beta-plasma $c_s \gg v_a$	low beta-plasma $c_s \ll v_a$
fast wave	✗	$\omega = k v_a$
slow wave	$\omega = k_z v_a$	$\omega = k_z c_s$

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slow mode

	high beta-plasma $c_s \gg v_a$	low beta-plasma $c_s \ll v_a$	cascade
fast wave	✗	$\omega = k v_a$	isotropic
slow wave	$\omega = k_z v_a$	$\omega = k_z c_s$	anisotropic

e.g. Cho & Lazarian 2003

Magnetosonic waves: fast modes

Yan & Lazarian 2002, 2004, 2008

resonance condition $\omega - k_z v_z - n\Omega_g = 0$

harmonics

$n = \pm 1, \pm 2, \pm 3...$ gyroresonance

$n = 0 \ \rightarrow \dfrac{\omega}{k_z} = v_z$ transit time damping (or resonant mirror interaction)

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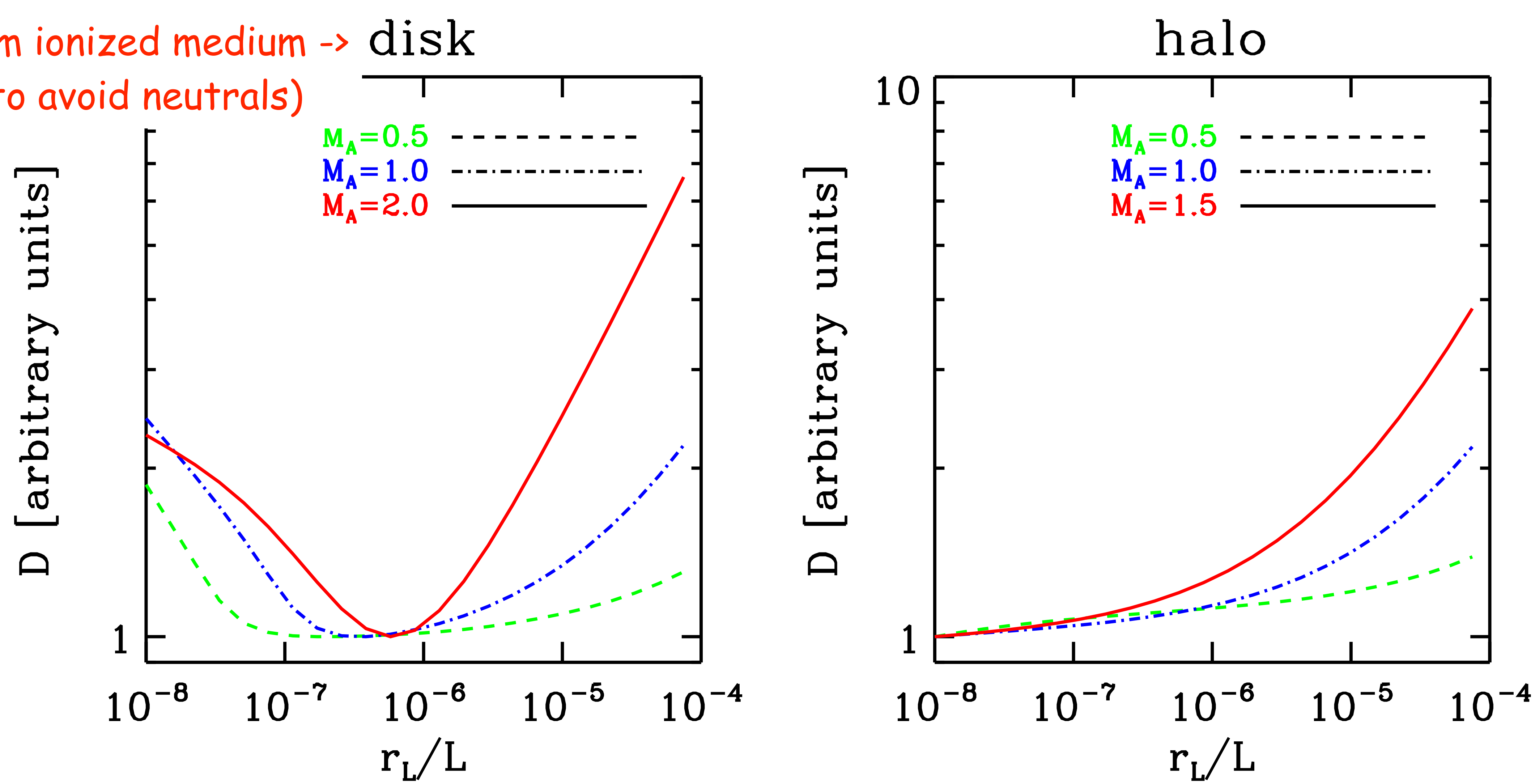
warm ionized medium ->
(to avoid neutrals)



Evoli & Yan 2013

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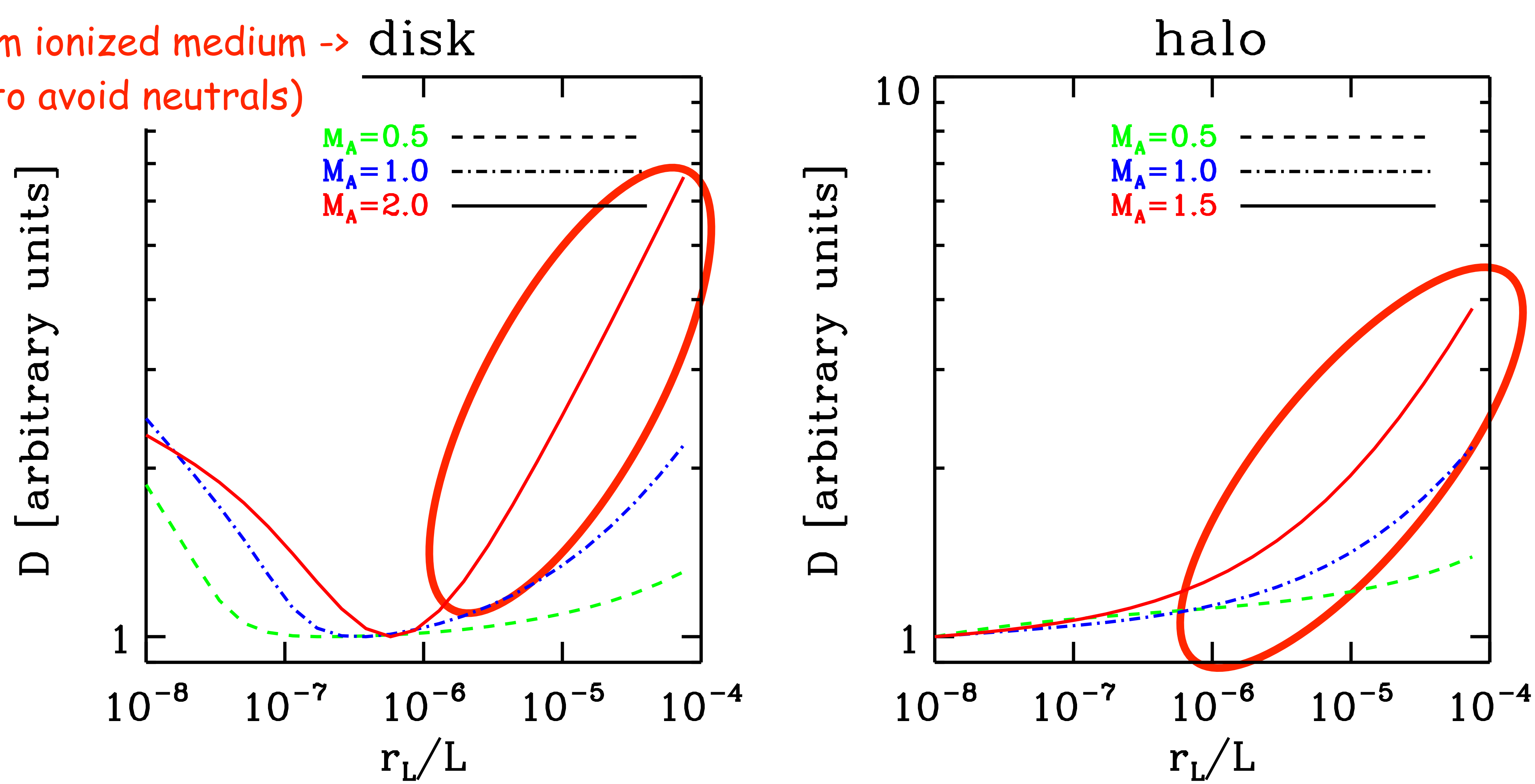
warm ionized medium -> disk
(to avoid neutrals)



Evoli & Yan 2013

Propagation codes

- first attempts: D is a (broken) power law in rigidity, uniform throughout the Galaxy (disk+halo), isotropic…
- other ingredients: distribution of sources, wind, CR reacceleration, gas and photon fields spatial distribution, cross sections, etc…
- recent additions: D depends on position, accounts for possible anisotropic diffusion, magnetic field structure…

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even the most sophisticated approaches on the market have to rely onto quite drastic simplifying assumptions and significant astrophysical uncertainties

-> first attempts to introduce into propagation codes some results from realistic (with many caveats) theoretical studies (Evoli & Yan 2013)

-> should we put more physics into propagation codes or keep them simple and use them to guide theoretical calculations?

-> any claim based on the use of diffuse maps obtained from propagation codes should be very cautious

Conclusions

- a lot of progresses in our understanding of the propagation of CRs in the turbulent ISM magnetic field
- still quite far from a reliable and realistic picture
- most likely, CRs do not trap themselves in the galaxy via streaming instability (maybe at ~GeV energies, but definitely not at energies >> GeV, but there is no evidence for a distinction between high and low energy CRS…)
- streaming instability might work in localized regions surrounding CR sources
- Alfvenic cascade is anisotropic -> very inefficient scattering
- magnetosonic waves?
- problems with neutrals! density of neutral H must be quite small in order to avoid strong damping… -> diffusion in the halo?
- …