

# *Prospects of jet measurements for PDF determinations*

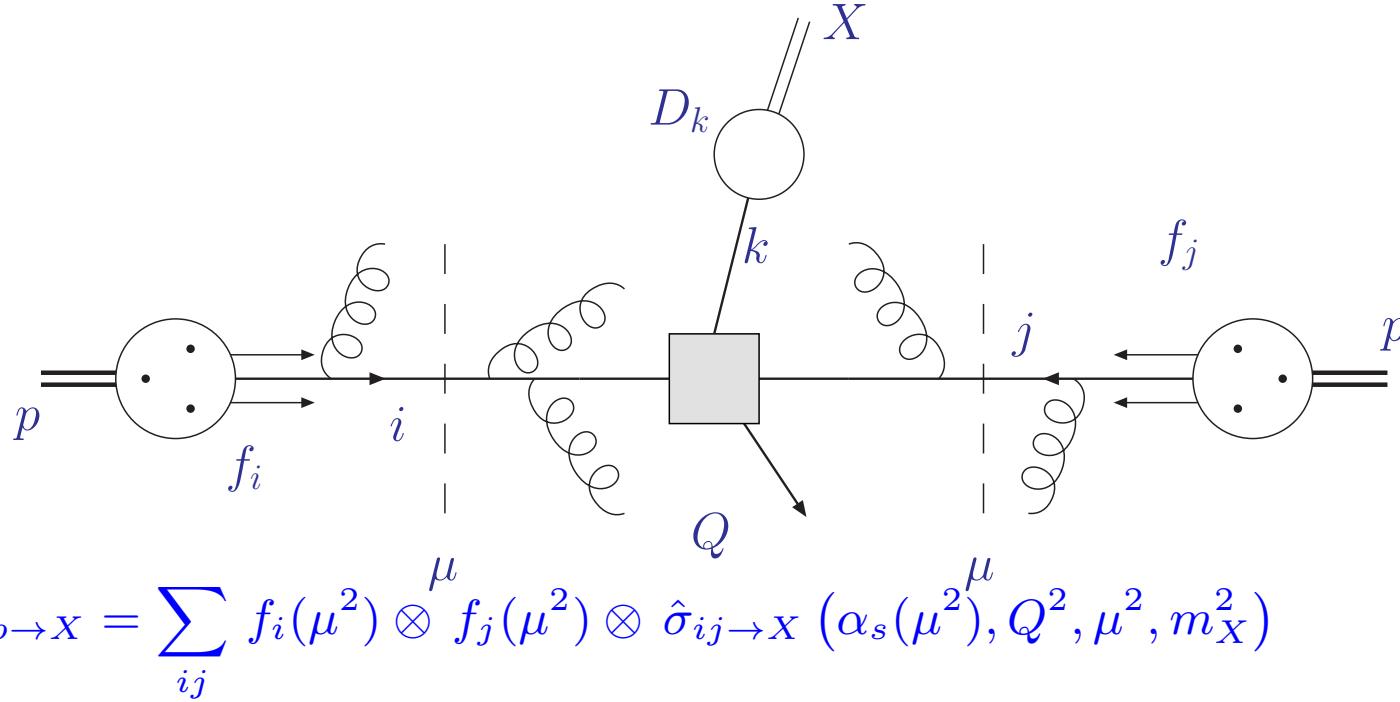
**Sven-Olaf Moch**

*Universität Hamburg*

## *Based on work done in collaboration with:*

- *Iso-spin asymmetry of quark distributions and implications for single top-quark production at the LHC*  
S. Alekhin, J. Blümlein, S. M. and R. Plačakytė [arXiv:1508.07923](#)
- *Determination of Strange Sea Quark Distributions from Fixed-target and Collider Data*  
S. Alekhin, J. Blümlein, L. Caminada, K. Lipka, K. Lohwasser, S. M.,  
R. Petti, and R. Plačakytė [arXiv:1404.6469](#)
- *The ABM parton distributions tuned to LHC data*  
S. Alekhin, J. Blümlein and S. M. [arXiv:1310.3059](#)
- Many more papers of ABM and friends ...  
2008 – ...

# QCD factorization

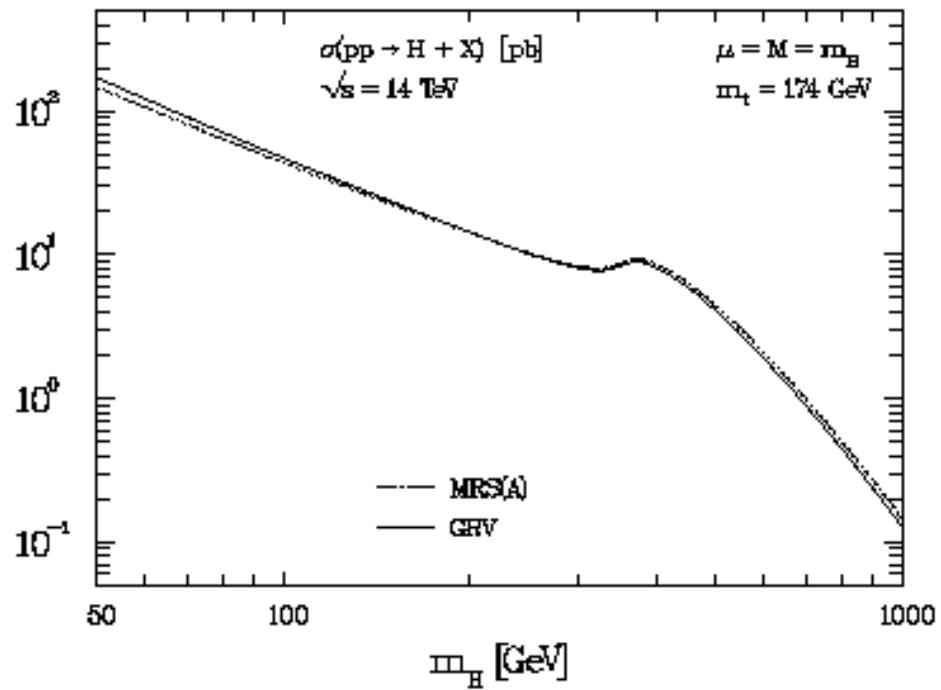


- Factorization at scale  $\mu$ 
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section  $\hat{\sigma}_{ij \rightarrow X}$  calculable in perturbation theory
  - cross section  $\hat{\sigma}_{ij \rightarrow k}$  for parton types  $i, j$  and hadronic final state  $X$
- Non-perturbative parameters: parton distribution functions  $f_i$ , strong coupling  $\alpha_s$ , particle masses  $m_X$ 
  - known from global fits to exp. data, lattice computations, ...

## *Higgs boson production*

# Higgs cross section (1995)

## NLO QCD corrections



MRS(A): Martin, Roberts and Stirling,  
Phys. Rev. D50 (1994) 6734

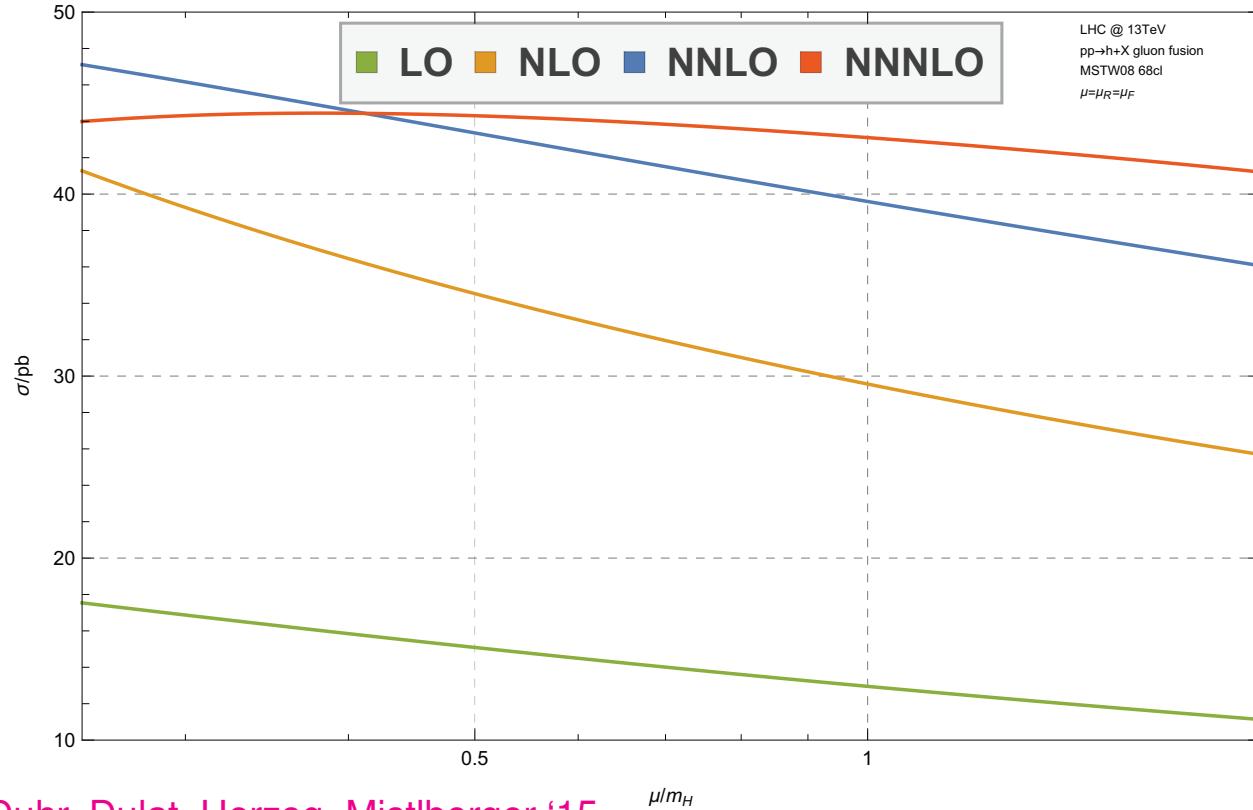
GRV: Glück, Reya and Vogt,  
Z. Phys. C53 (1992) 127

One of the main uncertainties in the prediction of the Higgs production cross section is due to the gluon density. [...] Adopting a set of representative parton distributions [...], we find a variation of about 7% between the maximum and minimum values of the cross section for Higgs masses above  $\sim 100$  GeV.

Spira, Djouadi, Graudenz, Zerwas (1995)  
hep-ph/9504378

# Higgs cross section (2016)

Exact  $N^3LO$  QCD corrections



Anastasiou, Duhr, Dulat, Herzog, Mistlberger '15

- Apparent convergence of perturbative expansion
- Scale dependence of exact  $N^3LO$  prediction with residual uncertainty 3%
- Minimal sensitivity at scale  $\mu = m_H/2$

## Dependence of cross section on parton luminosity

- Cross section  $\sigma(H)$  at NNLO with uncertainties:  $\sigma(H) + \Delta\sigma(\text{PDF} + \alpha_s)$  for  $m_H = 125.0 \text{ GeV}$  at  $\sqrt{s} = 13 \text{ TeV}$  with  $\mu_R = \mu_F = m_H$  and nominal  $\alpha_s$

ABM12 Alekhin, Blümlein, S.M. '13	$39.80 \pm 0.84 \text{ pb}$
CJ12 (NLO) Owens, Accardi, Melnitchouk '12	$41.94 \pm 0.16 \text{ pb}$
CT14 Dulat et al. '15	$42.33 \pm 1.43 \text{ pb}$
HERAPDF2.0 H1+Zeus Coll.	$42.62 \pm 0.35 \text{ pb}$
JR14 (dyn) Jimenez-Delgado, Reya '14	$38.01 \pm 0.34 \text{ pb}$
MMHT14 Martin, Motylinski, Harland-Lang, Thorne '14	$42.36 \pm 0.56 \text{ pb}$
NNPDF3.0 Ball et al. '14	$42.59 \pm 0.80 \text{ pb}$
PDF4LHC15 Butterworth et al. '15	$42.42 \pm 0.78 \text{ pb}$

- Large spread for predictions from different PDFs  $\sigma(H) = 38.0 \dots 42.6 \text{ pb}$
- PDF and  $\alpha_s$  differences between sets amount to up to 11%
  - significantly larger than residual theory uncertainty due to N<sup>3</sup>LO QCD corrections

*How to explain the differences ?*

## *Parton content of the proton*

# *Data in global PDF fits*

## *Data sets considered in ABM12 analysis*

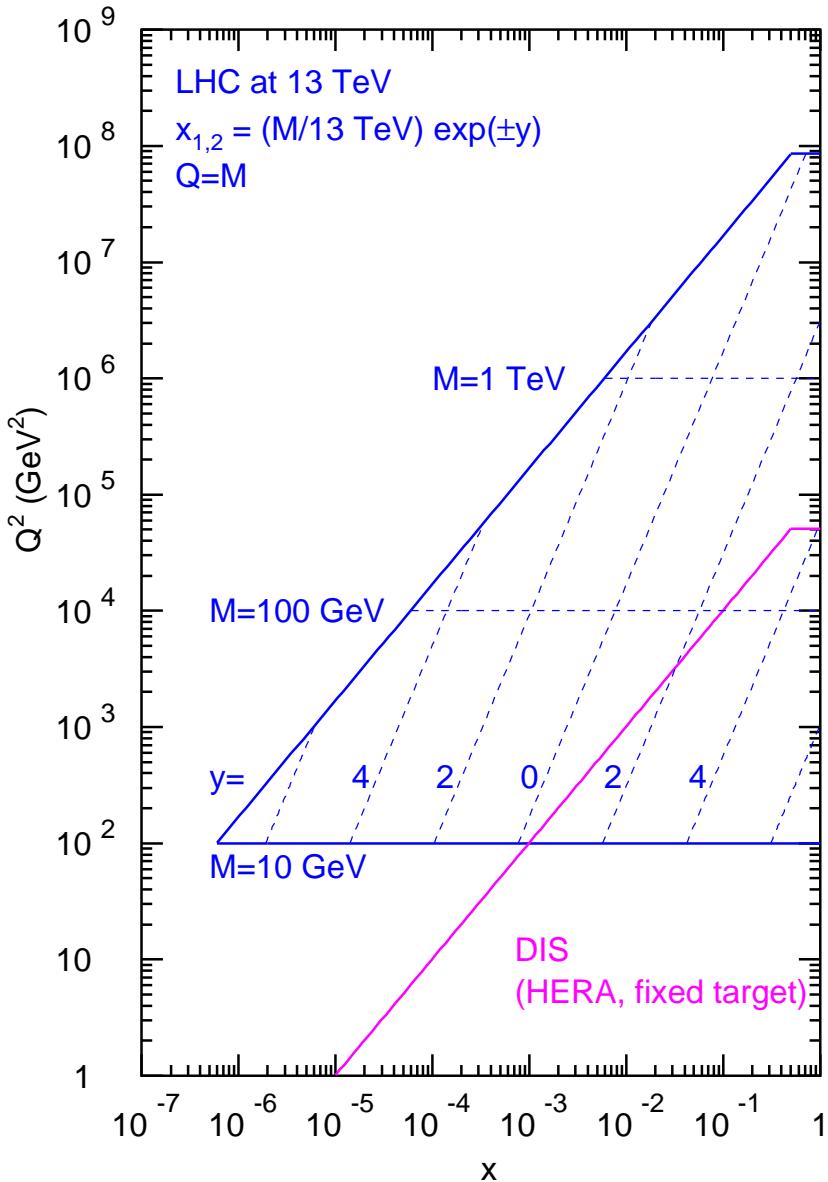
- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
  - inclusive DIS data HERA, BCDMS, NMC, SLAC  $(NDP = 2699)$
  - semi-inclusive DIS charm production data HERA  $(NDP = 52)$
  - Drell-Yan data (fixed target) E-605, E-866  $(NDP = 158)$
  - neutrino-nucleon DIS (di-muon data) CCFR/NuTeV  $(NDP = 178)$
  - LHC data for  $W^\pm$ - and  $Z$ -boson production ATLAS, CMS, LHCb  $(NDP = 60)$

## *Iterative cycle of PDF fits*

- i) check of compatibility of new data set with available world data
- ii) study of potential constraints due to addition of new data set to fit
- iii) perform high precision measurement of the non-perturbative parameters
  - parton distributions
  - strong coupling  $\alpha_s(M_Z)$
  - heavy quark masses

# Parton kinematics at LHC

- Information on proton structure depends on kinematic coverage

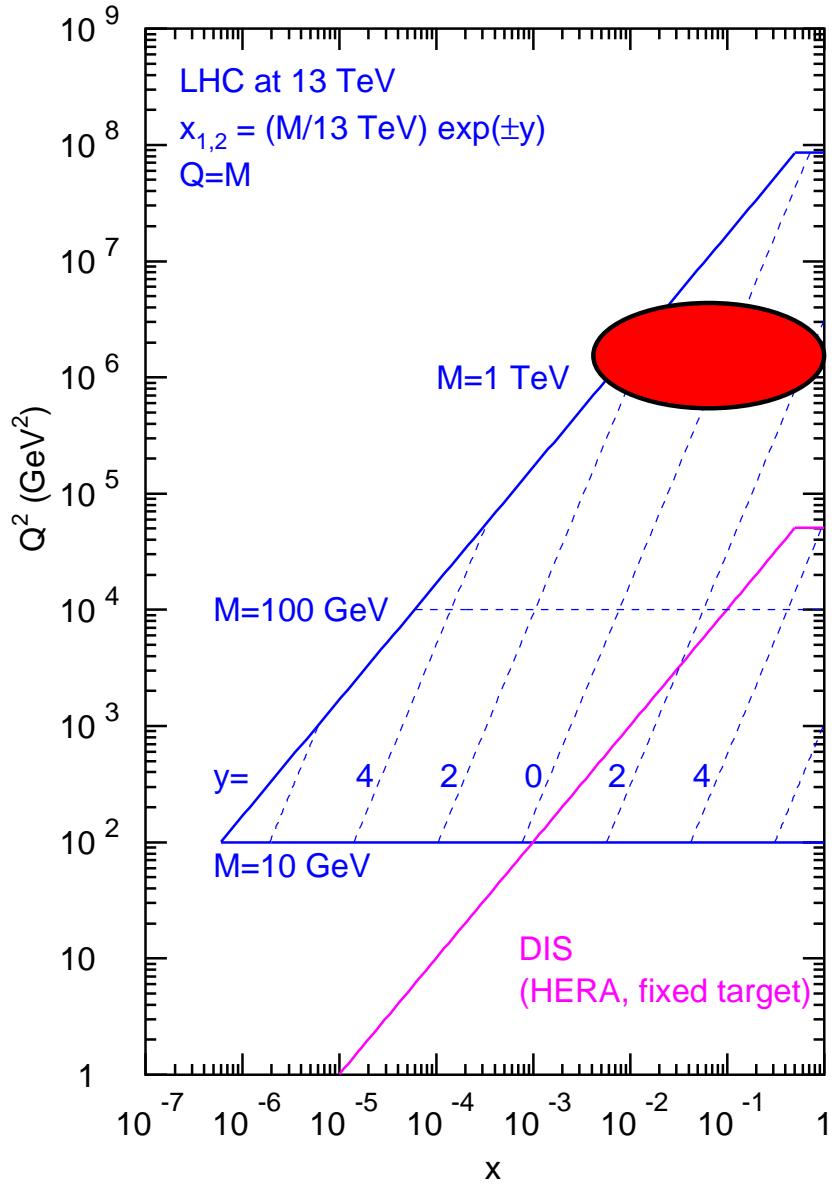


- LHC run II at  $\sqrt{s} = 13 \text{ TeV}$ 
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics with  $x_{1,2} = M/\sqrt{S} e^{\pm y}$ 
  - forward rapidities sensitive to small- $x$
- Cross section depends on convolution of parton distributions
  - small- $x$  part of  $f_i$  and large- $x$  PDFs  $f_j$

$$\sigma_{pp \rightarrow X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes [\dots]$$

# Parton kinematics at LHC

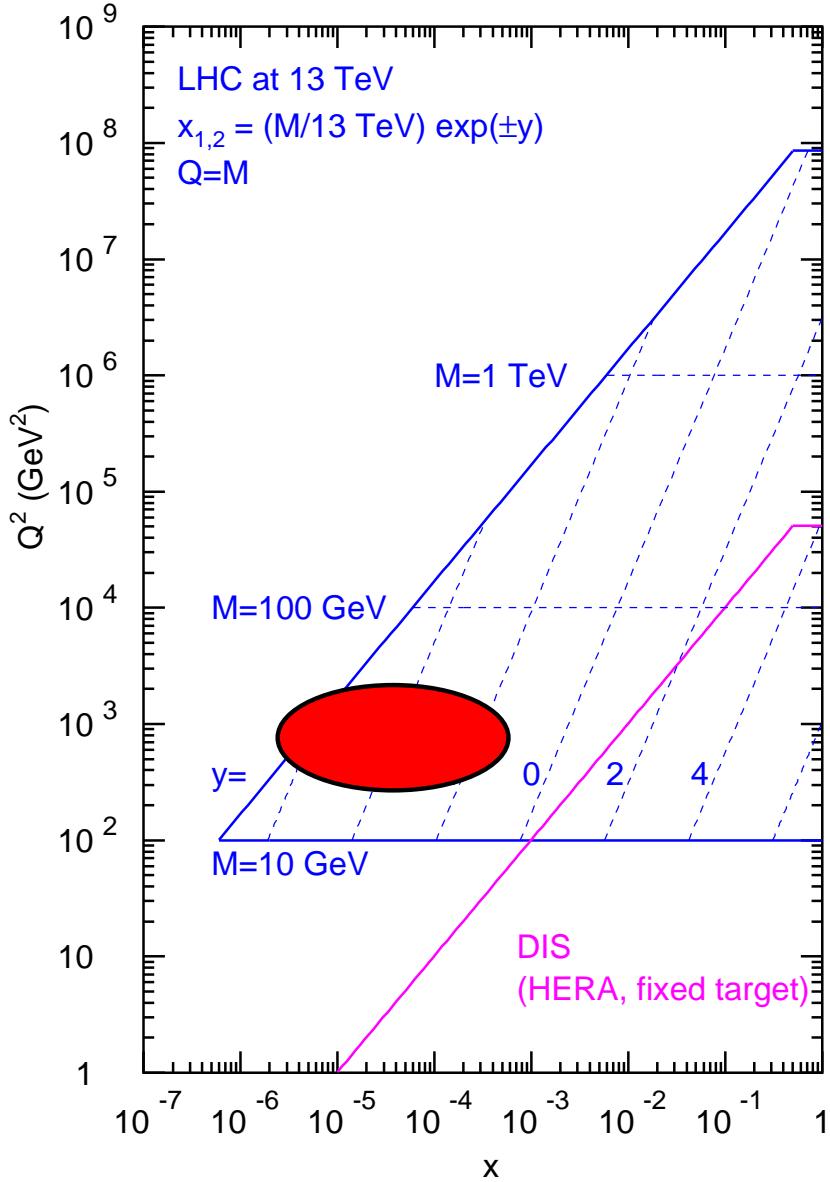
- Information on proton structure depends on kinematic coverage



- Interesting regions
  - hard high- $p_T$  jets, gluon PDF at medium to large  $x$  and  $\alpha_s$  in TeV region

# Parton kinematics at LHC

- Information on proton structure depends on kinematic coverage



- Interesting regions
  - forward jets probe small  $x$  gluon PDF at low scales

# Theory considerations in PDF fits

## Theory considerations

- Strictly NNLO QCD for determination of PDFs and  $\alpha_s$
- Consistent scheme for treatment of heavy quarks
  - $\overline{\text{MS}}$ -scheme for quark masses and  $\alpha_s$
  - fixed-flavor number scheme for  $n_f = 3, 4, 5$
- Consistent theory description for consistent data sets
  - low scale DIS data with account of higher twist
- Full account of error correlations

## Interplay with perturbation theory

- Accuracy of determination driven by precision of theory predictions
- Non-perturbative parameters sensitive to
  - radiative corrections at higher orders
  - chosen scheme (e.g. ( $\overline{\text{MS}}$  scheme))
  - renormalization and factorization scales  $\mu_R, \mu_F$
  - ...

# Benchmark measurements

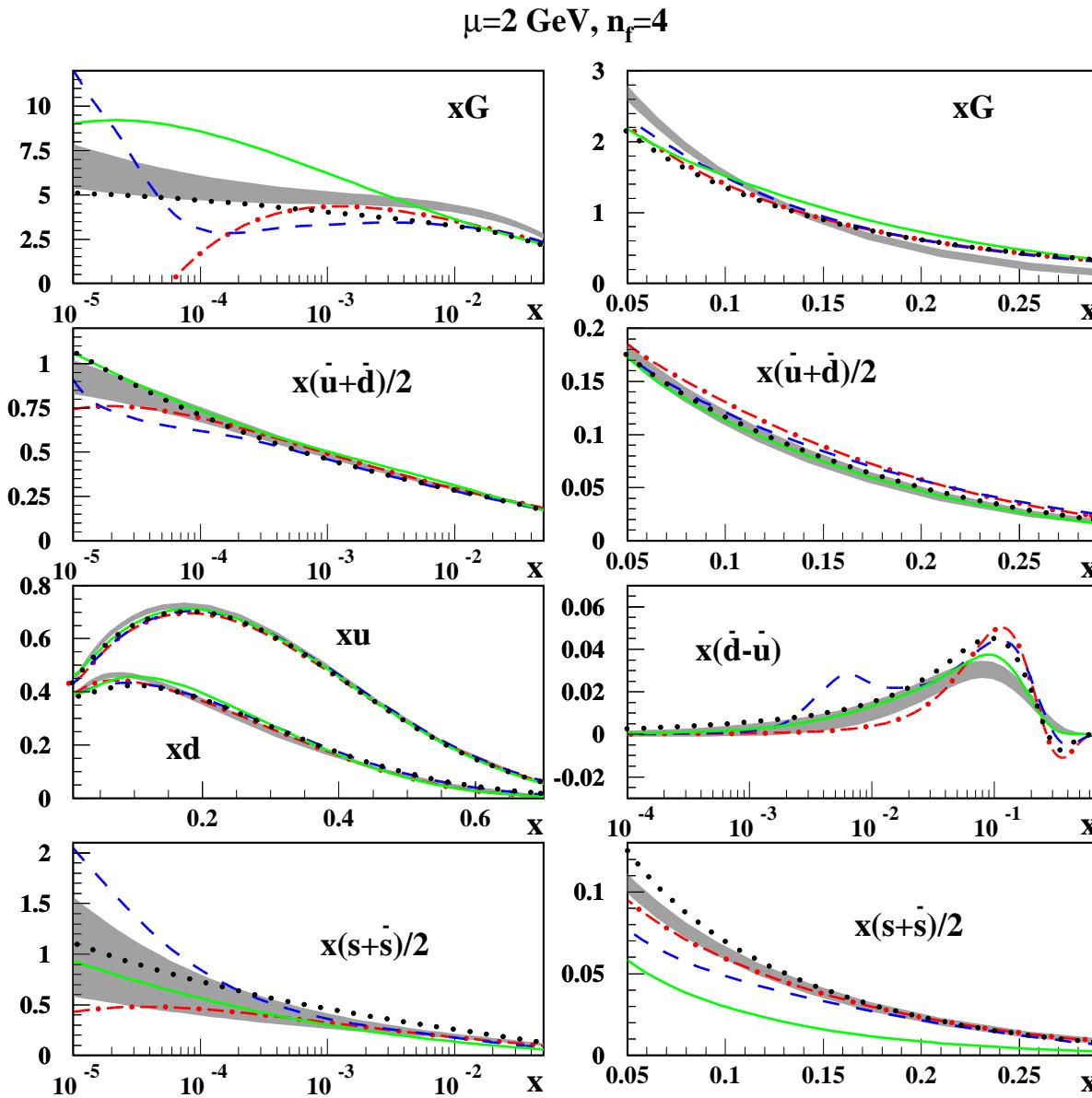
## DIS

- Structure functions for neutral and charged current known to  $\mathcal{O}(\alpha_s^3)$   
S.M. Vermaseren, Vogt '04–'08
  - $F_2, F_3$ , known N<sup>3</sup>LO,  $F_L$  known NNLO
- Heavy-quark structure functions
  - asymptotic NNLO terms at large  $Q^2 \gg m^2$  Bierenbaum, Blümlein, Klein '09;  
Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock '14
  - approximate NNLO expressions for neutral and charged current  
Lo Presti, Kawamura, S.M., Vogt '12, Blümlein, A. Hasselhuhn, and T. Pfoh '14

## LHC

- Complete NNLO QCD corrections available for
  - $W^\pm$ - and  $Z$ -boson production  
Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02
  - hadro-production of top-quark pairs Czakon, Fiedler, Mitov '13
  - single top-quark production ( $t$ -channel) Brucherseifer, Caola, Melnikov '14

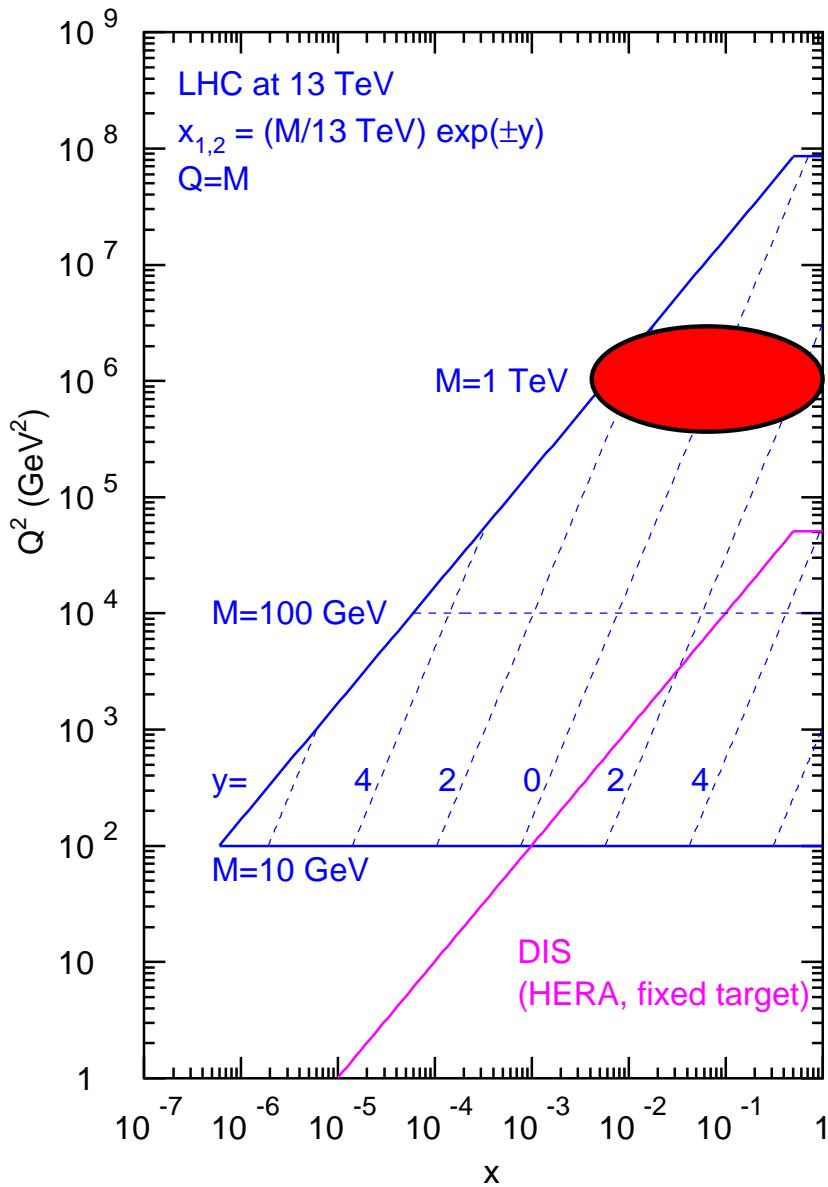
# Parton distributions tuned to LHC data



- $1\sigma$  band for ABM12 PDFs (NNLO, 4-flavors) at  $\mu = 2 \text{ GeV}$   
Alekhin, Blümlein, S.M.'13
- comparison with:  
JR09 (solid lines),  
MSTW (dashed dots),  
NN23 (dashes) and  
CT10 (dots)
- Some interesting observations to be made ...

# *Heavy quarks in deep-inelastic scattering*

# Parton kinematics at LHC



# Treatment of heavy-quarks

## Light quarks

- Neglect “light quark” masses  $m_u, m_d \ll \Lambda_{QCD}$  and  $m_s < \Lambda_{QCD}$  in hard scattering process
  - scale-dependent  $u, d, s, g$  PDFs from mass singularities

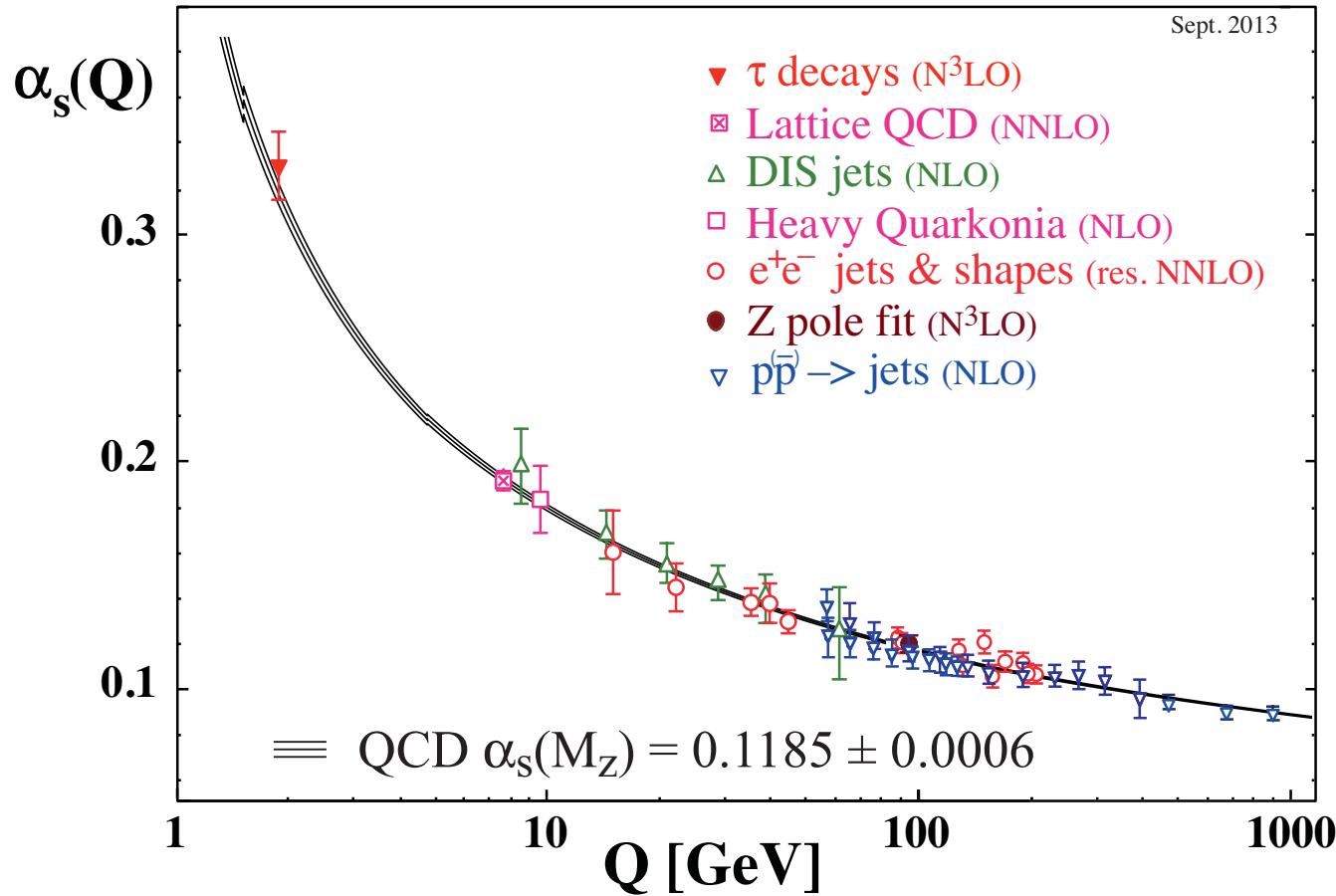
## Heavy quarks

- No mass singularities for  $m_c, m_b, m_t \gg \Lambda_{QCD}$ , no (evolving) PDFs
  - $c$  and  $b$  PDFs for  $Q \ggg m_c, m_b$  generated perturbatively
  - matching of two distinct theories
    - $n_f$  light flavors + heavy quark of mass  $m$  at low scales
    - $n_f + 1$  light flavors at high scales

# Strong coupling with flavor thresholds

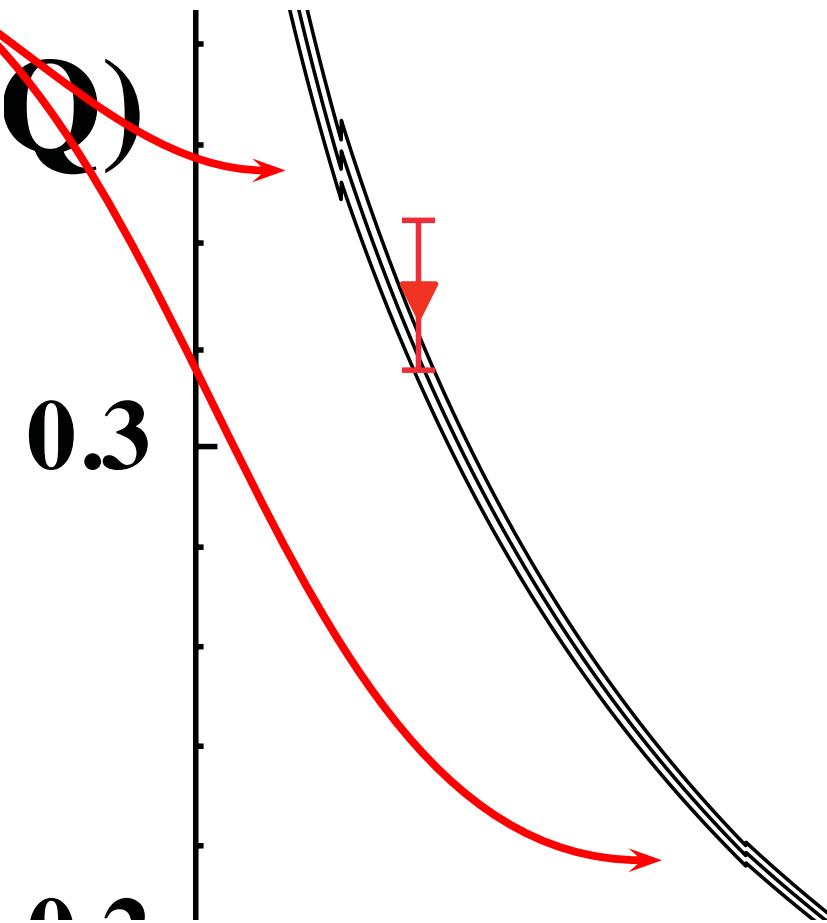
- Solution of QCD  $\beta$ -function for  $\alpha_s^{n_l} \longrightarrow \alpha_s^{(n_l+n_h)}$ 
  - discontinuities for  $n_f = 3 \rightarrow n_f = 4 \rightarrow n_f = 5$
- Big picture

Bethke for PDG 2014



# *Strong coupling with flavor thresholds*

- Solution of QCD  $\beta$ -function for  $\alpha_s^{n_l} \rightarrow \alpha_s^{(n_l+n_h)}$ 
  - discontinuities for  $n_f = 3 \rightarrow n_f = 4 \rightarrow n_f = 5$
- Zoom

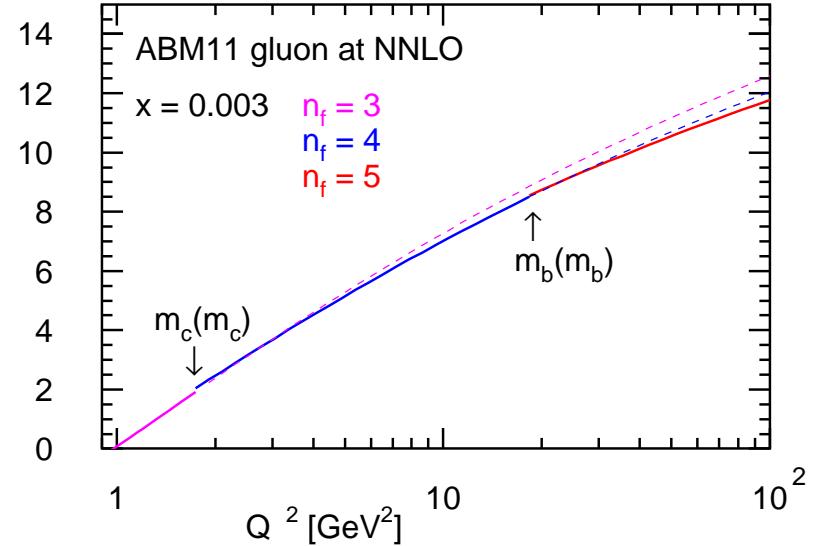
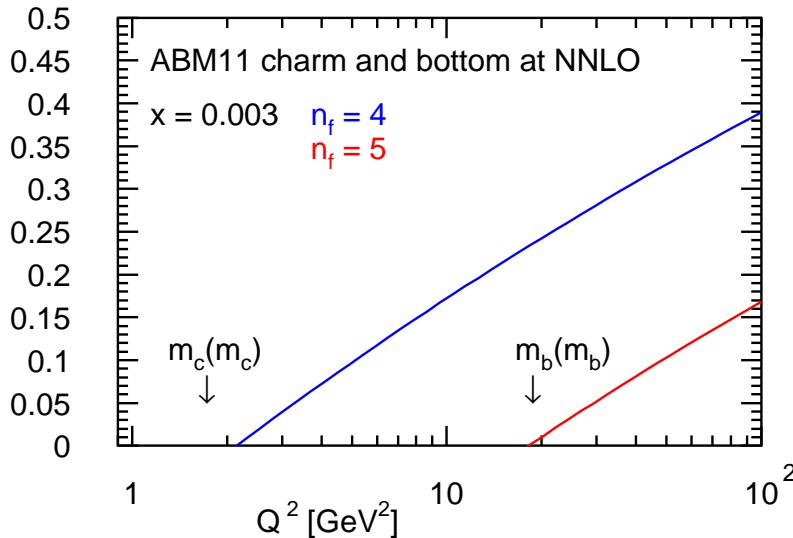


# PDFs with flavor thresholds

- Generate heavy-quark PDFs  $h^{(n_f+1)}$  from light-flavor PDFs
  - heavy-quark operator matrix elements (OMEs)  $A_{ji}$  at three loops  
Bierenbaum, Blümlein, Klein '09; Ablinger, Behring, Blümlein, De Freitas, von Manteuffel, Schneider '14

$$h^{(n_f+1)}(x, \mu) + \bar{h}^{(n_f+1)}(x, \mu) = A_{hq}(x) \otimes \Sigma^{(n_f)}(x, \mu) + A_{hq}(x) \otimes g^{(n_f)}(x, \mu)$$

- likewise light-quark PDFs  $l_i^{(n_f)} \rightarrow l_i^{(n_f+1)}$  and gluon and the quark singlet PDFs  $(\Sigma^{(n_f)}, g^{(n_f)}) \rightarrow (\Sigma^{(n_f+1)}, g^{(n_f+1)})$
- Solution of evolution equations between thresholds for  $n_f \rightarrow (n_f + 1)$  with fixed  $n_f = 3 \rightarrow n_f = 4 \rightarrow n_f = 5$



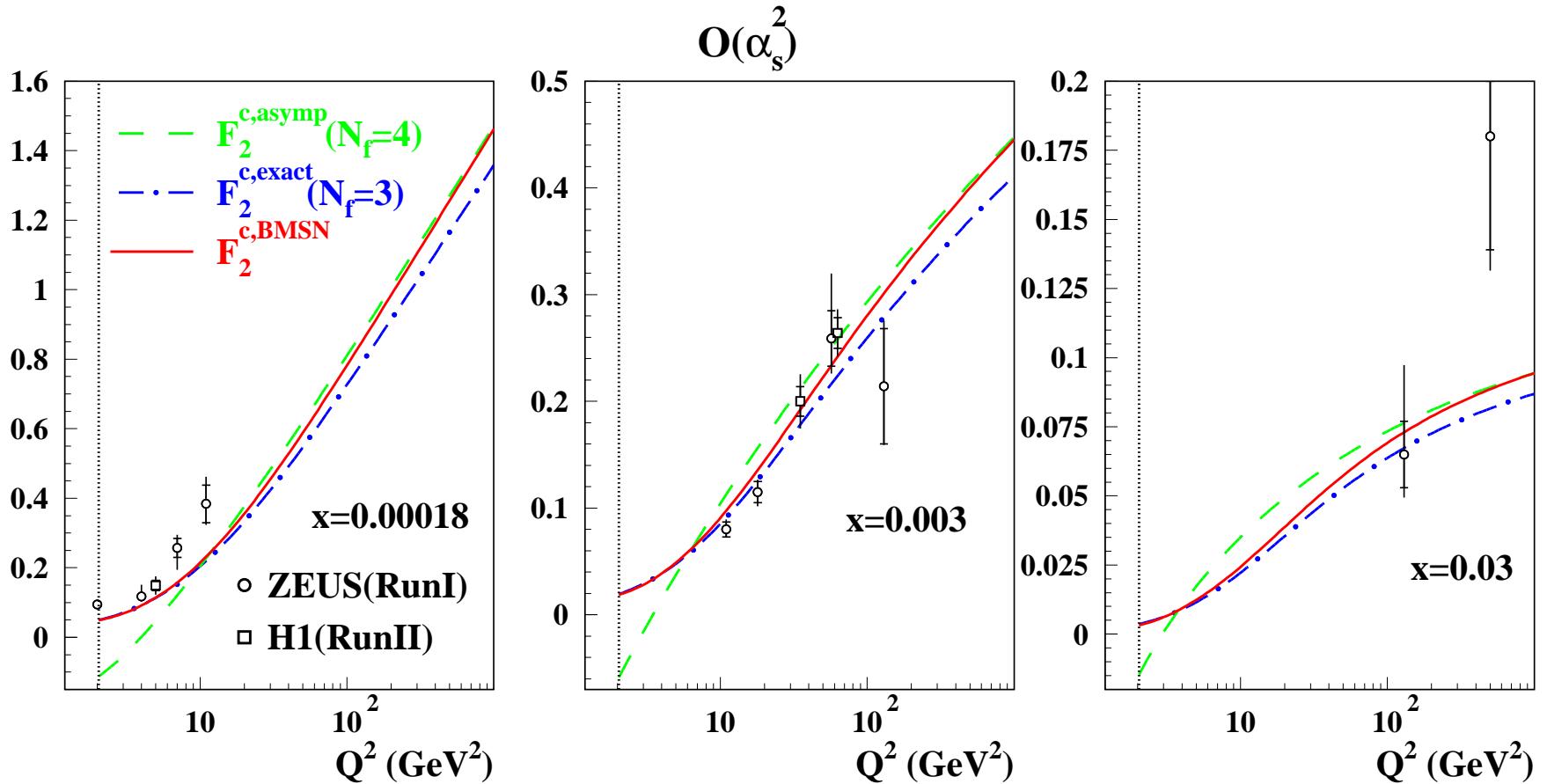
# Cross sections with flavor thresholds

## Fixed flavor number scheme (FFNS) (“do nothing”)

- Cross section with massive quarks at scales  $Q \gg m_c$ 
  - top-quark hadro-production ( $t\bar{t}$  pairs, single top in 4FS or 5FS, ... ]
- $F_2^c$  at HERA with  $u, d, s, g$  partons and massive charm coeff. fcts.
  - complete NLO predictions Laenen, Riemersma, Smith, van Neerven ‘92
  - approximations at NNLO Bierenbaum, Blümlein, Klein ‘09; Lo Presti, Kawamura, S.M., Vogt ‘12; Behring, Bierenbaum, Blümlein, De Freitas, Klein, Wissbrock ‘14

## Variable flavor number scheme (VFNS) (“match something”)

- (Smooth) matching of two distinct theories:  
 $n_f$  light + heavy quark at low scales  $\longrightarrow n_f + 1$  light flavors at high scales
  - Higgs boson production in  $b\bar{b}$ -annihilation (“Santander matching” Harlander, Krämer, Schumacher ‘11)
- $F_2^c$  at HERA with ACOT Aivazis, Collins, Olness, Tung ‘94, BMSN Buza, Matiounine, Smith, van Neerven ‘98, RT Thorne, Roberts ‘98, FONNL Forte, Laenen, Nason, Rojo ‘10
  - model assumptions in matching conditions
  - details of implementation matter in global fits

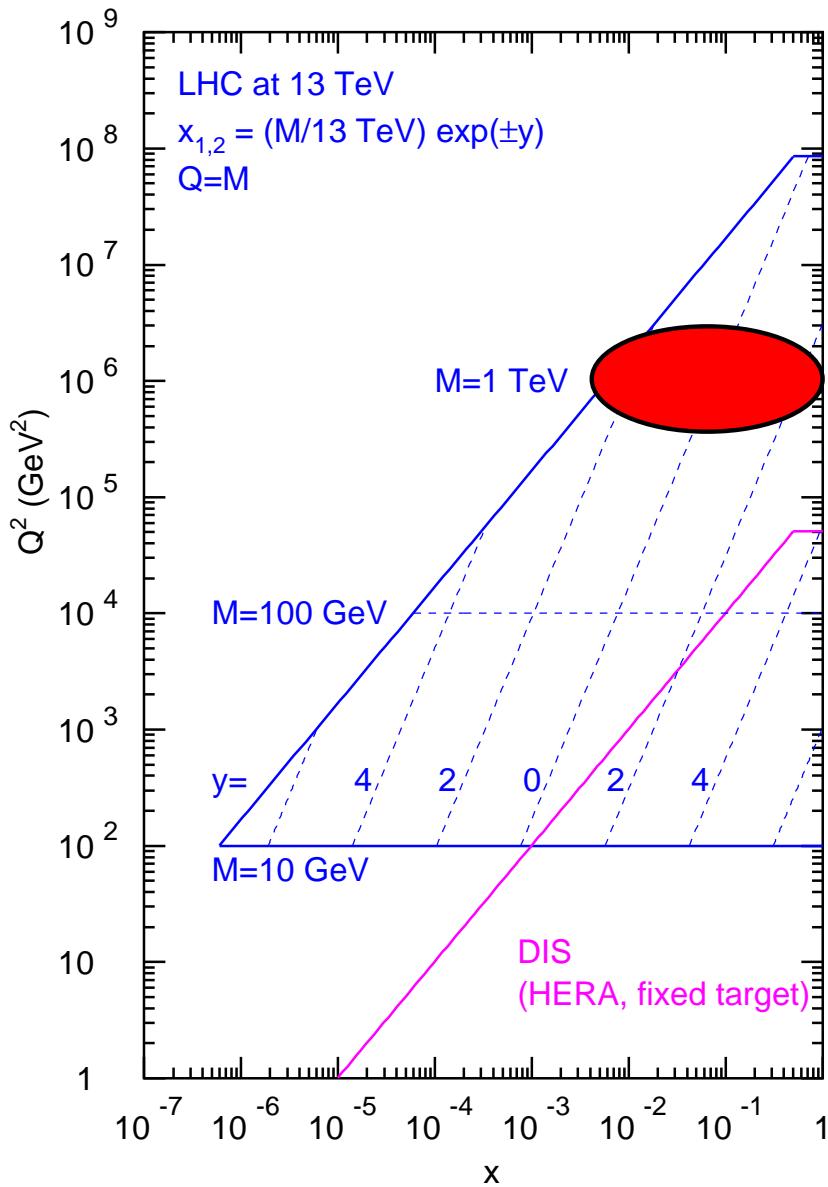


Alekhin, Blümlein, Klein, S.M. '09

- $F_2^c$  in different schemes compared to H1- and ZEUS-data
  - GMVFN scheme in BMSN prescription (solid lines)
  - 3-flavor scheme (dash-dotted lines)
  - 4-flavor scheme (dashed lines)
  - charm-quark mass  $m_c = 1.43$  GeV (vertical dotted line)

## *Threshold logarithms at large $p_T$*

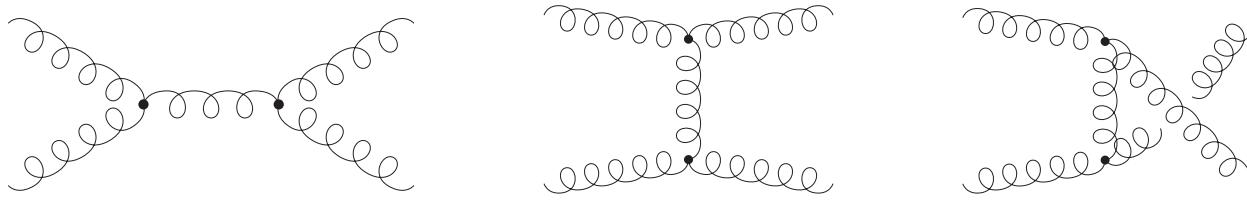
# Parton kinematics at LHC



# Jet hadro-production at NNLO

## Status of QCD theory for jet cross sections

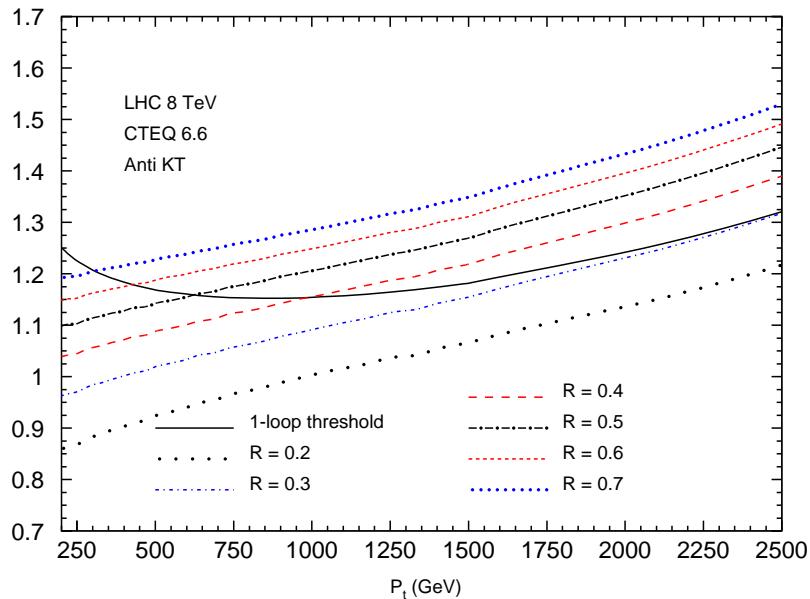
- Hadro-production of jets (single jet inclusive, di-jets)
  - NLO known since long
  - partial results at NNLO (purely gluonic contributions)  
Currie, Gehrmann-De Ridder, Glover, Pires, '13



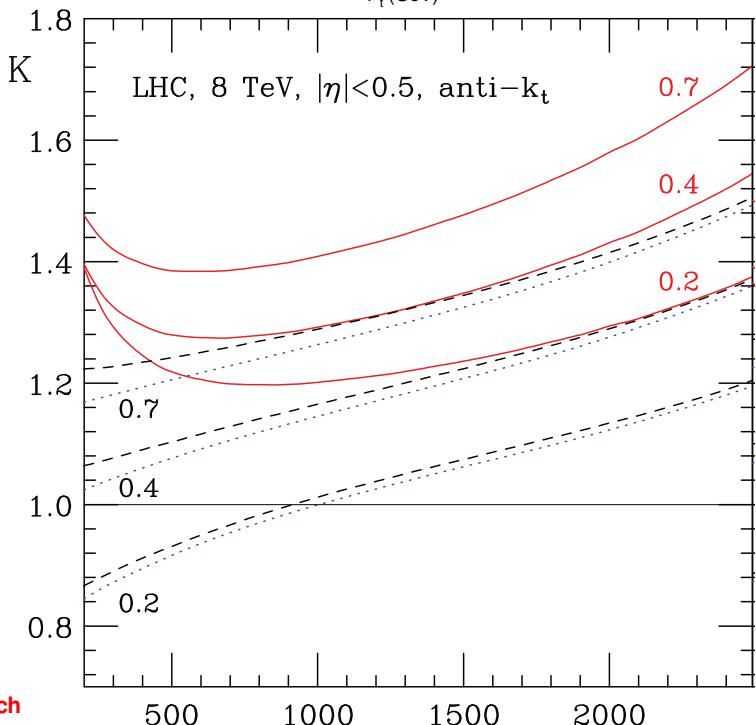
## Threshold logarithms

- One-jet inclusive jets hadro-production  $P + P(\bar{P}) \rightarrow J(R) + X(s_4)$ 
  - large threshold corrections of type  $\alpha_s^l [\ln^{2l-1}(s_4/p_T^2)/s_4]_+$  from soft/collinear gluon radiation Kidonakis, Owens '00
  - $\ln R$  dependence on jet's cone size  $R$  in small cone approximation de Florian, Vogelsang '07
- Threshold approximation needs to account for Sudakov logarithms  $\ln(p_T^2/s)$  and  $\ln R$  terms  $\rightarrow$  numerically of similar magnitude

# Threshold logarithms at NNLO



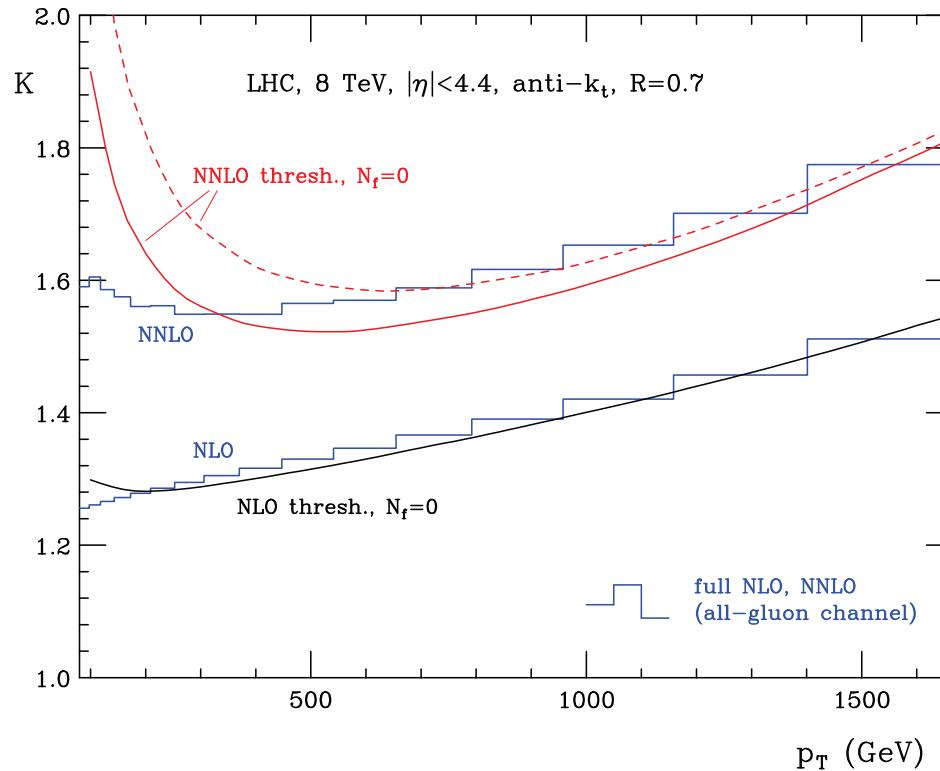
- threshold logarithms alone (w/o  $\ln R$ ) at NLO fail to describe exact results  
Kumar, Moch '13



- cone size dependence  $\ln R$  numerically important
- $\ln R$  terms improve agreement between exact result and threshold approximation at NLO de Florian, Hinderer, Mukherjee, Ringer, Vogelsang '13

# Jet hadro-production at NNLO

- Nice match at NNLO
  - threshold approximation de Florian, Hinderer, Mukherjee, Ringer, Werner Vogelsang '13
  - exact NNLO (purely gluonic) computation Currie, Gehrmann-De Ridder, Glover, Pires '13



- in principle ready for applications in PDF analyses or experimental analyses of jet data Carrazza, Pires '14

## *Strong coupling constant*

# Strong coupling constant (1992)

	$\alpha_s(M_Z^2)$
$R_\tau$	$0.117^{+0.010}_{-0.016}$
DIS	$0.112 \pm 0.007$
$\Upsilon$ Decays	$0.110 \pm 0.010$
$R_{e^+e^-}(s < 62\text{GeV})$	$0.140 \pm 0.020$
$p\bar{p} \rightarrow W + jets$	$0.121 \pm 0.024$
$\Gamma(Z \rightarrow \text{hadrons})/\Gamma(Z \rightarrow l\bar{l})$	$0.132 \pm 0.012$
Jets at LEP	$0.122 \pm 0.009$
Average	$0.118 \pm 0.007$

G. Altarelli (1992)  
in QCD - 20 Years Later,  
CERN-TH-6623-92

## Essential facts

- World average 1992  $\alpha_s(M_Z) = 0.118 \pm 0.007$
- Central value at NLO QCD
  - still right, but for very different reasons
- Error at NLO QCD
  - now down to  $\sim 0.0050 - 0.0040$  (theory scale uncertainty)

# Strong coupling constant (2016)

## Measurements at NNLO

- Values of  $\alpha_s(M_Z)$  at NNLO from PDF fits

SY	$0.1166 \pm 0.013$	$F_2^{ep}$	Santiago, Yndurain '01
	$0.1153 \pm 0.063$	$xF_3^{\nu N}$ (heavy nucl.)	
A02	$0.1143 \pm 0.013$	DIS	Alekhin '01
MRST03	$0.1153 \pm 0.0020$		Martin, Roberts, Stirling, Thorne '03
BBG	$0.1134^{+0.0019}_{-0.0021}$	valence analysis, NNLO	Blümlein, Böttcher, Guffanti '06
GRS	0.112	valence analysis, NNLO	Glück, Reya, Schuck '06
A06	$0.1128 \pm 0.015$		Alekhin '06
JR08	$0.1128 \pm 0.0010$	dynamical approach	Jimenez-Delgado, Reya '08
	$0.1162 \pm 0.0006$	including NLO jets	
ABKM09	$0.1135 \pm 0.0014$	HQ: FFNS $n_f = 3$	Alekhin, Blümlein, Klein, S.M. '09
	$0.1129 \pm 0.0014$	HQ: BSMN	
MSTW	$0.1171 \pm 0.0014$		Martin, Stirling, Thorne, Watt '09
Thorne	0.1136	[DIS+DY, HT*] (2013)	Thorne '13
ABM11 <sub>J</sub>	$0.1134 \dots 0.1149 \pm 0.0012$	Tevatron jets (NLO) incl.	Alekhin, Blümlein, S.M. '11
NN21	$0.1173 \pm 0.0007$	(+ heavy nucl.)	NNPDF '11
ABM12	$0.1133 \pm 0.0011$		Alekhin, Blümlein, S.M. '13
	$0.1132 \pm 0.0011$	(without jets)	
CT10	0.1140	(without jets)	Gao et al. '13
CT14	$0.1150^{+0.0060}_{-0.0040}$	$\Delta\chi^2 > 1$ (+ heavy nucl.)	Dulat et al. '15
MMHT	$0.1172 \pm 0.0013$	(+ heavy nucl.)	Martin, Motylinski, Harland-Lang, Thorne '15

# *Strong coupling constant (2016)*

## *Other measurements of $\alpha_s$ at NNLO*

- Values of  $\alpha_s(M_Z)$  at NNLO from measurements at colliders

3-jet rate	$0.1175 \pm 0.0025$	Dissertori et al. 2009	<a href="#">arXiv:0910.4283</a>
$e^+e^-$ thrust	$0.1131^{+0.0028}_{-0.0022}$	Gehrman et al.	<a href="#">arXiv:1210.6945</a>
$e^+e^-$ thrust	$0.1140 \pm 0.0015$	Abbate et al.	<a href="#">arXiv:1204.5746</a>
$C$ -parameter	$0.1123 \pm 0.0013$	Hoang et al.	<a href="#">arXiv:1501.04111</a>
CMS	$0.1151 \pm 0.0033$	$t\bar{t}$	<a href="#">arXiv:1307.1907</a>
NLO Jets ATLAS	$0.111^{+0.0017}_{-0.0007}$		<a href="#">arXiv:1312.5694</a>
NLO Jets CMS	$0.1148 \pm 0.0055$		<a href="#">arXiv:1312.5694</a>

# Differences in $\alpha_s$ determinations

## Why $\alpha_s$ values from MSTW and NNPDF are large

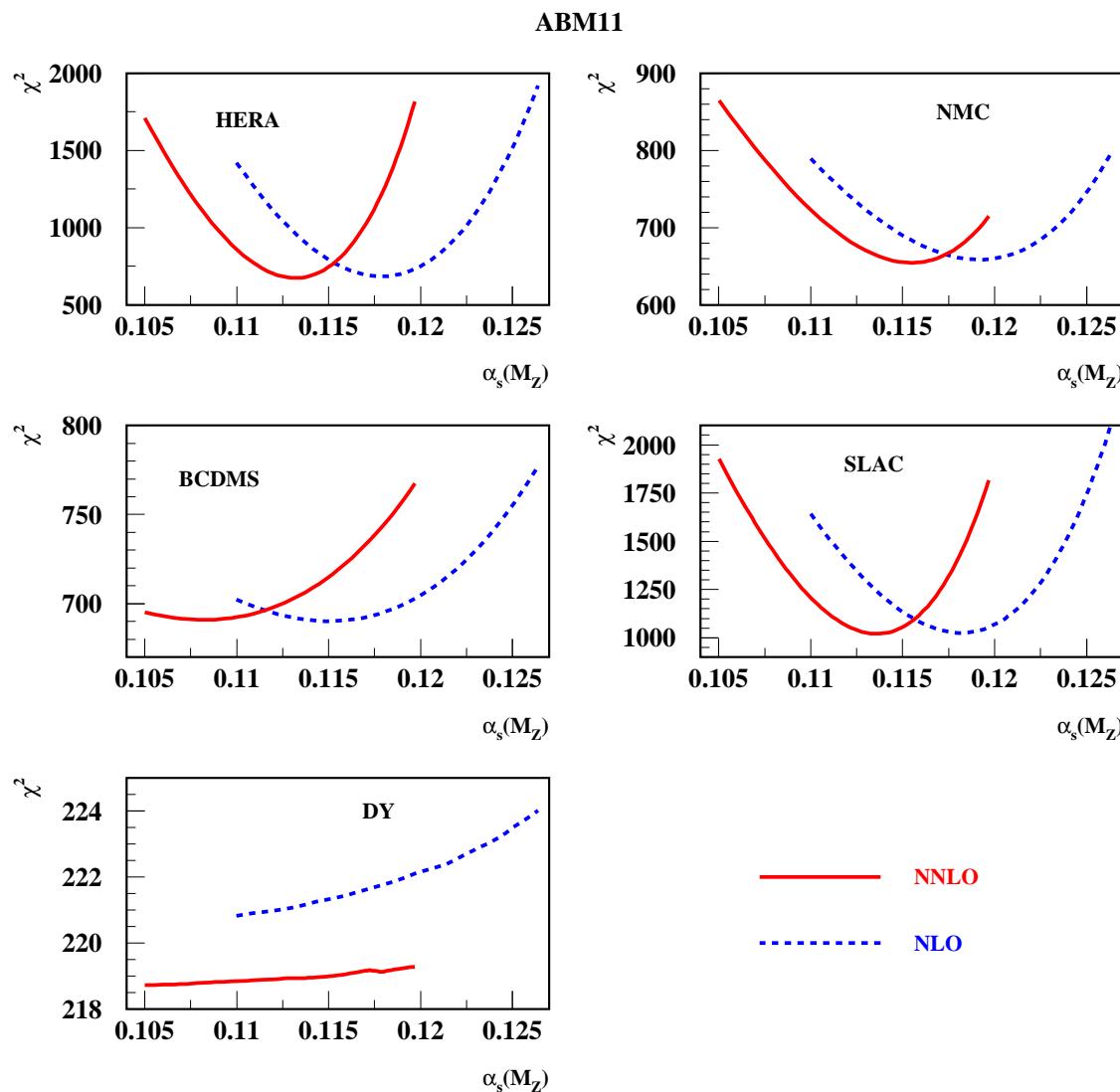
- Differences result from different physics models and analysis procedures
  - target mass corrections (powers of nucleon mass  $M_N^2/Q^2$ )
  - higher twist  $F_2^{\text{ht}} = F_2 + \text{ht}^{(4)}(x)/Q^2 + \text{ht}^{(6)}(x)/Q^4 + \dots$
  - error correlations
- Hadroproduction of jets known NLO only
  - strictly speaking  $\alpha_s(M_Z)$  value only NLO (systematically larger)

	$\alpha_s$	NNLO	target mass corr.	higher twist	error correl.
ABM12	$0.1132 \pm 0.0011$	yes	yes	yes	yes
NNPDF21	$0.1173 \pm 0.0007$	(yes)	yes	no	yes
MSTW	$0.1171 \pm 0.0014$	(yes)	no	no	no
MMHT	$0.1172 \pm 0.0013$	(yes)	no	no	—

- Effects for differences are understood
  - variants of ABM with no higher twist etc. reproduce larger  $\alpha_s$  values  
Alekhin, Blümlein, S.M. '11

# Zooming in on ABM

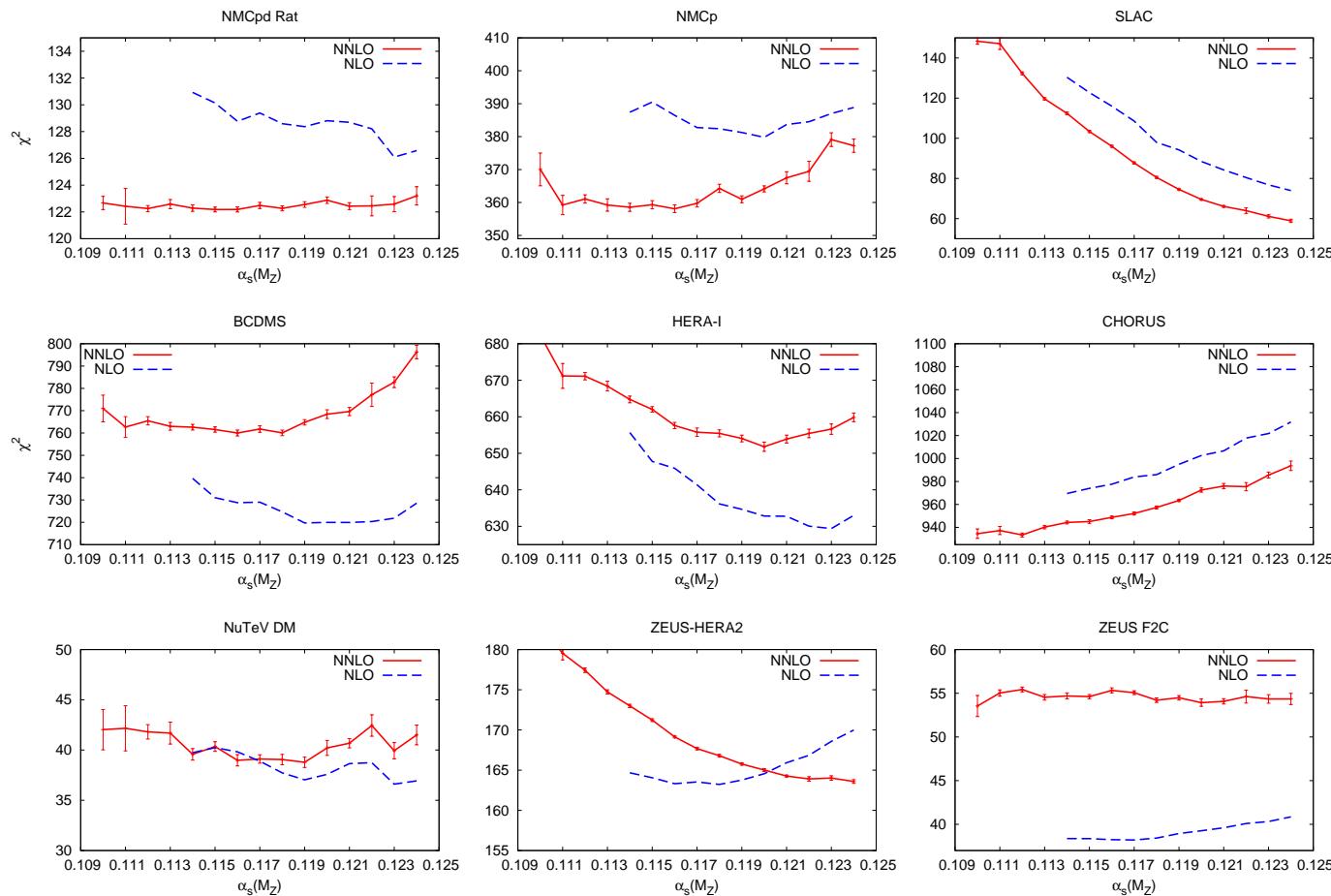
$\alpha_s$  from DIS and PDFs



- Profile of  $\chi^2$  for different data sets in ABM11 PDF fit Alekhin, Blümlein, S.M. '12

# Zooming in on NNPDF

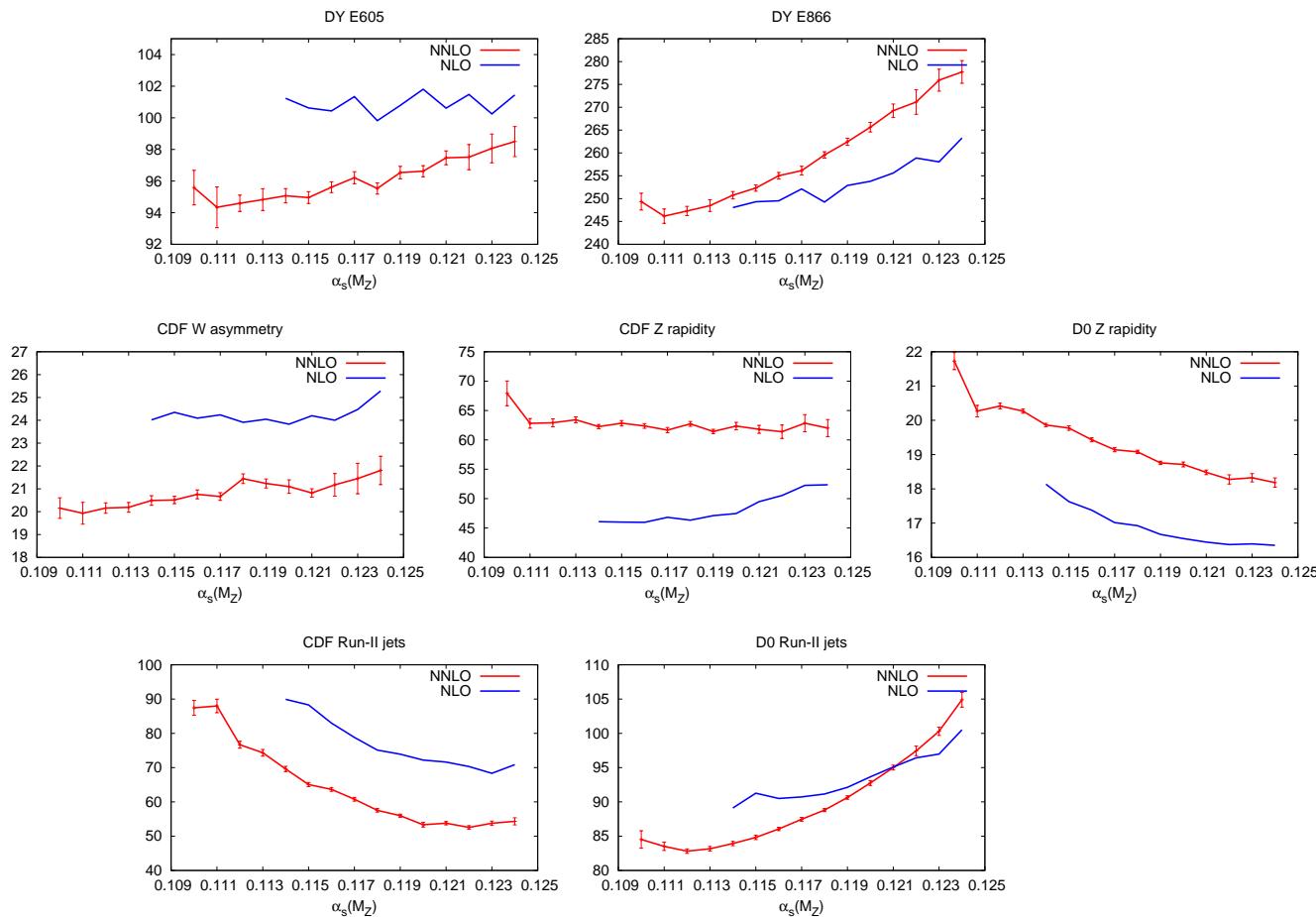
## $\alpha_s$ from DIS and PDFs



- Profile of  $\chi^2$  for different data sets in NNPDF21 fit Ball et al. '11

# Zooming in on NNPDF

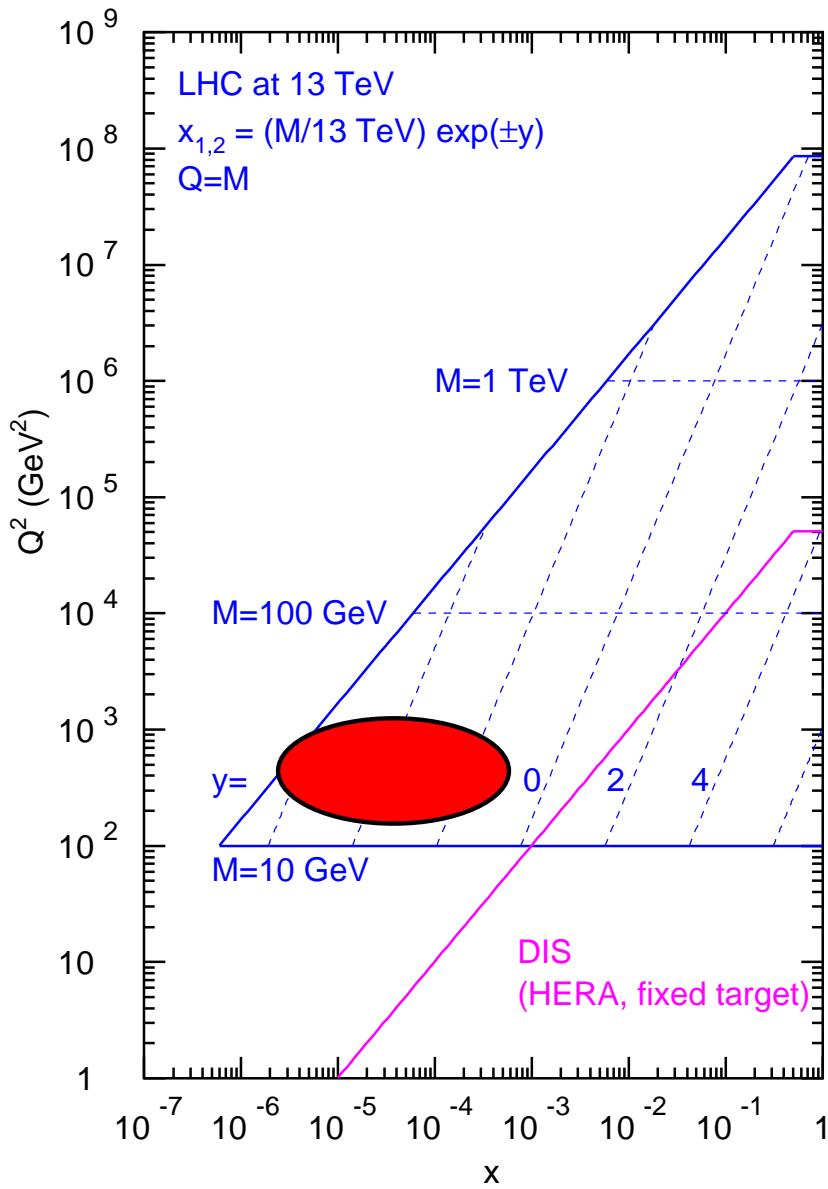
## $\alpha_s$ from DIS and PDFs



- Profile of  $\chi^2$  for different data sets in NNPDF21 fit Ball et al. '11

## *Forward region*

# Parton kinematics at LHC



# Gluon PDF at small $x$

## Facts

- Gluon PDF at small- $x$ 
  - fits yield  $xg(x) \simeq x^a$  with  $a \simeq -0.2$
  - kinematic coverage of data down to  $x \simeq 10^{-5}$

## Issues

- Extrapolation of gluon PDF towards smaller  $x$ 
  - some PDFs feature large uncertainties for extrapolation to unmeasured regions —> this invalidates predictive potential
- Some PDFs predict negative gluon PDF at small- $x$  and low scales
  - unphysical feature as consequence of modelling in variable flavor number schemes applied
  - large differences between gluon PDFs fitted at NLO and NNLO

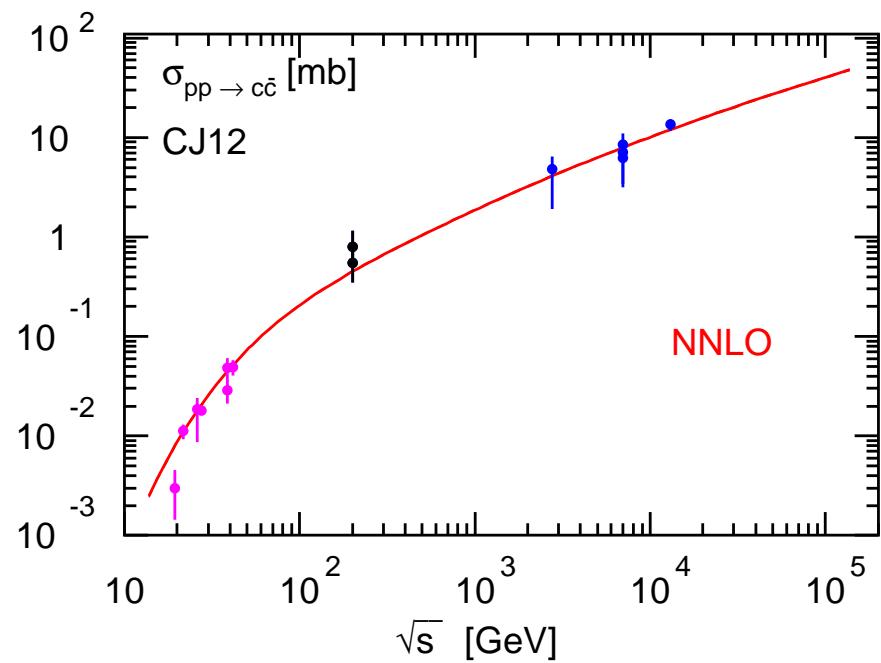
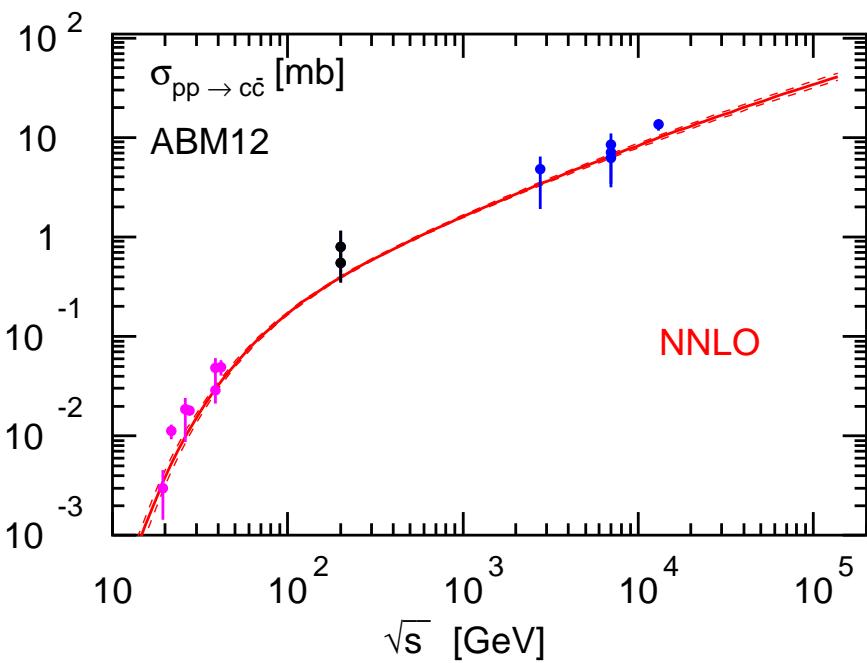
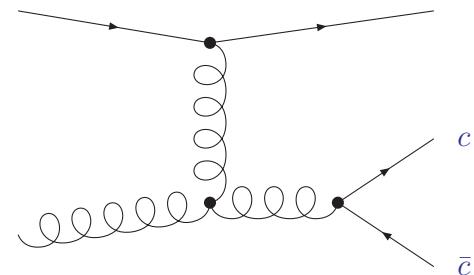
## Challenge

- Test of perturbative QCD in forward region (large rapidities)

# *Charm-quark hadro-production at NNLO*

## *Diagnostic tool*

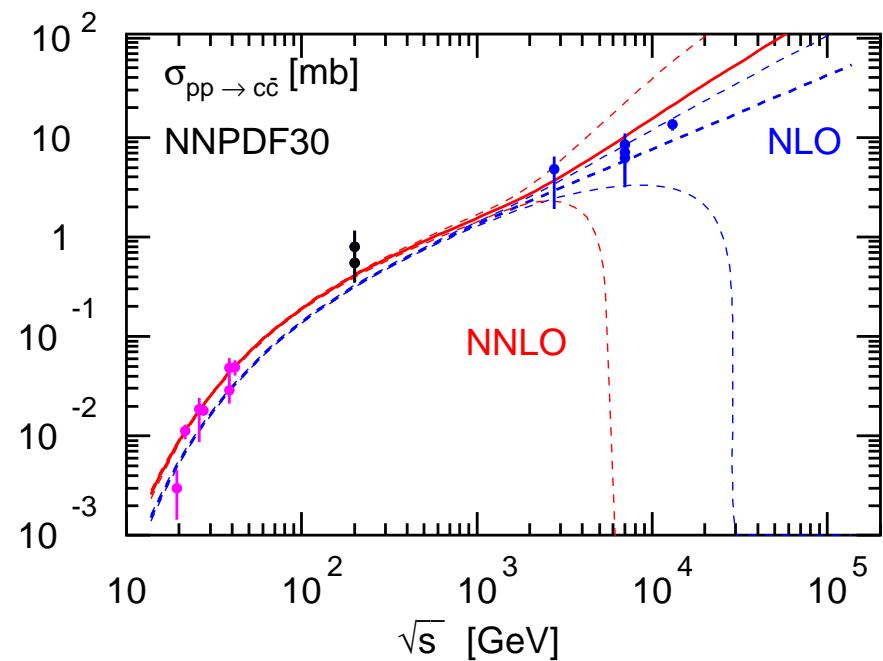
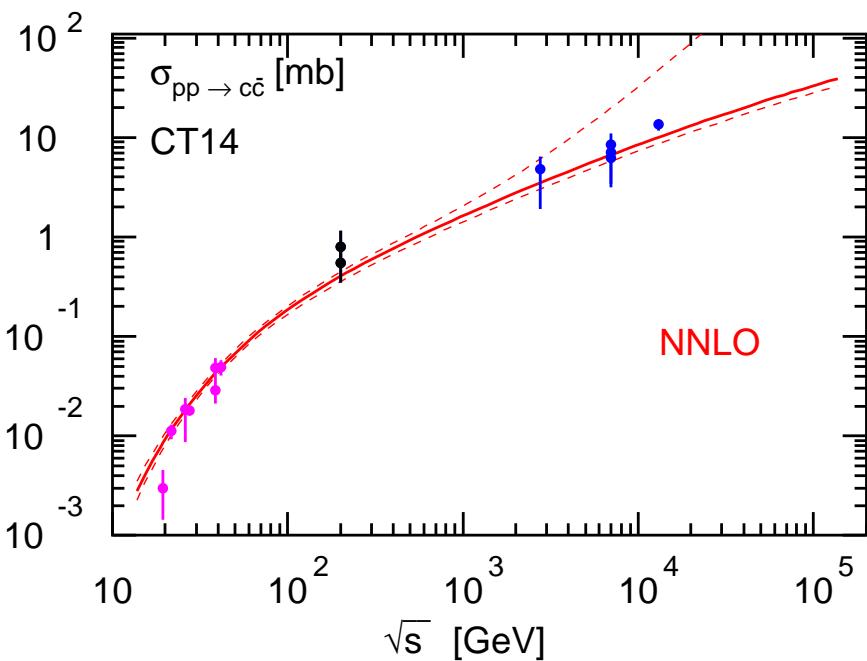
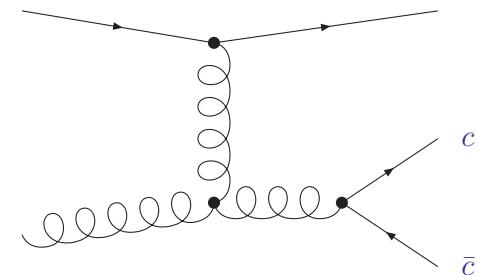
- Charm-quark hadro-production at high energies
  - quark-gluon parton luminosity dominates



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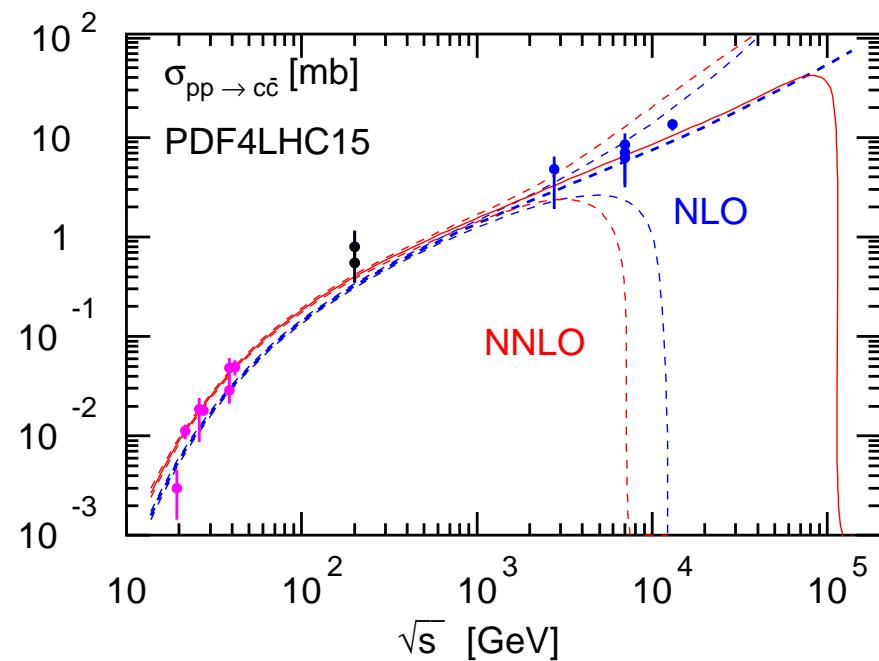
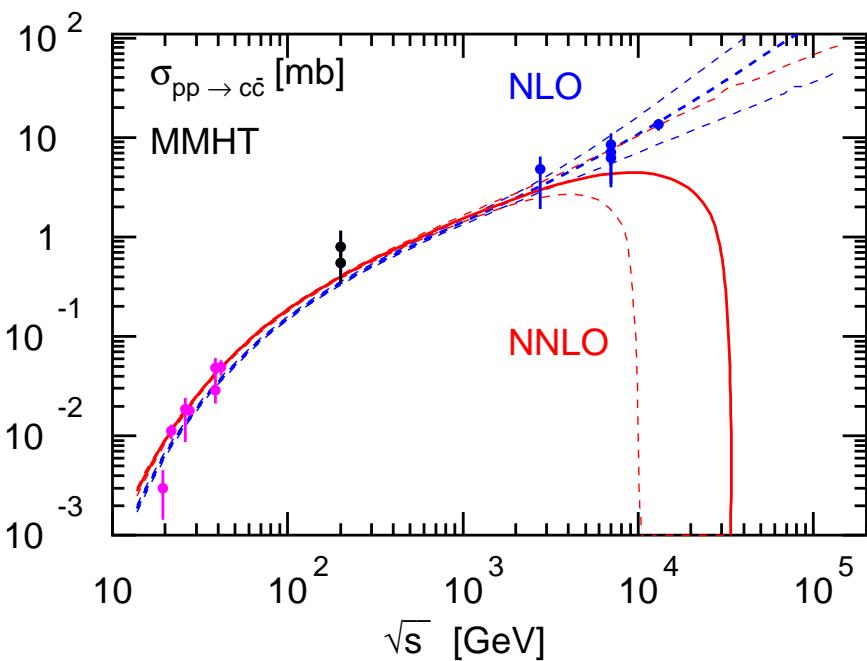
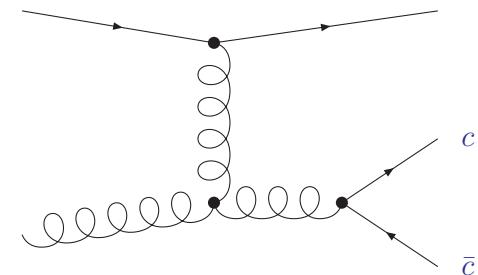
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# Summary

## *Prospects of jet measurements for PDF determinations*

### *High $p_T$*

- High  $p_T$  jet data allow for precision determinations of ...
  - ... strong coupling constant  $\alpha_s(M_Z)$
  - ... gluon PDF at medium to large- $x$
- Precision of  $\lesssim 1\%$  makes theoretical predictions at NNLO in QCD mandatory

### *Forward region*

- Jet data from forward region ...
  - ... test perturbative QCD at small- $x$
  - ... rule out negative gluon PDF at small- $x$
  - ... test perturbative QCD at small- $x$
- Because of large spread in PDFs theoretical predictions at NLO in QCD suffice