Angular momentum within the nucleon

Bo-Qiang Ma (马伯强)

Peking Univ（北京大学)

International Workshop on Hadron Structure and Spectroscopy
(IWHSS 2016)

Sep.5-7, 2016，Kloster Seeon, Bavaria, Germany

Collaborators: Enzo Barone, Stan Brodsky, Jacques Soffer, Andreas Schafer, Ivan Schmidt, Jian-Jun Yang, Qi-Ren Zhang
and students: Bowen Xiao, Zhun Lu, Bing Zhang, Jun She, Jiacai Zhu, Xinyu Zhang, Tianbo Liu

The Proton "Spin Crisis"

$$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.3$$

In contradiction with the naïve quark model expectation:

Naive Quark Model:

$$\Delta u = \frac{4}{3}; \qquad \Delta d = -\frac{1}{3}; \qquad \Delta s = 0$$

$$\Sigma = \Delta u + \Delta d + \Delta s = 1$$

Why there is the proton spin puzzle/crisis?

- The quark model is very successful for the classification of baryons and mesons
- The quark model is good to explain the magnetic moments of octet baryons
- The quark model gave the birth of QCD as a theory for strong interaction

So why there is serious problem with spin of the proton in the quark model?

Many Theoretical Explanantions

- The sea quarks of the proton are largely negatively polarized
- The gluons provide a significant contribution to the proton spin

It was thought that the spin "crisis" cannot be understood within the quark model: " the lowest uud valence component of the proton is estimated to be of only a few percent." **R.L. Jaffe and Lipkin, PLB266(1991)158**

How to get a clear picture of nucleon?

- PDFs are physically defined in the IMF (infinite-momentum frame) or with space-time on the light-cone.
- Whether the physical picture of a nucleon is the same in different frames?

A physical quantity defined by matrix element is frame-independent, but its physical picture is frame-dependent.

The improvement to the parton model?

- What would be the consequence by taking into account the transversal motions of partons?
- **It might be trivial in unpolarized situation. However it brings significant influences to spin dependent quantities (helicity and transversity distributions) and transversal momentum dependent quantities (TMDs or 3dPDFs). See talk by Andrea Bressan**

The Notion of Spin

- Related to the space-time symmetry of the Poincaré group
- Generators $P^{\mu} = (H, \vec{P}),$ space-time translator

$J^{\mu\nu}$ infinitesimal Lorentz transformation

$\vec{J}$ $J^{k} = \frac{1}{2}\varepsilon_{ijk} J^{ij}$ angular momentum

$\vec{K}$ $K^{k} = J^{k0}$ boost generator

Pauli-Lubanski vertor $w_{\mu} = \frac{1}{2} J^{\rho\sigma} P^{\nu} \varepsilon_{\nu\rho\sigma\mu}$

Casimir operators: $P^{2} = P^{\mu} P_{\mu} = m^{2}$ mass

$w^{2} = w^{\mu} w_{\mu} = s^{2}$ spin

The Wigner Rotation

for a rest particle $(m,\vec{0}) = p^{\mu}$ $(0,\vec{s}) = \mathrm{w}^{\mu}$

for a moving particle $\mathrm{L}(p)p = (m,\vec{0})$ $(0,\vec{s}) = \mathrm{L}(p)w/m$

$\mathrm{L}(p) =$ ratationless Lorentz boost

Wigner Rotation

$$\vec{s}, p_{\mu} \rightarrow \vec{s'}, p'_{\mu}$$

$$\vec{s'} = R_w(\Lambda, p)\vec{s} \qquad p' = \Lambda p$$

$$R_w(\Lambda, p) = \mathrm{L}(p')\Lambda \mathrm{L}^{-1}(p) \quad \text{a pure rotation}$$

E.Wigner,
Ann.Math.40(1939)149

Melosh Rotation for Spin-1/2 Particle

The connection between spin states in the rest frame and infinite momentum frame
Or between spin states in the conventional equal time dynamics and the light-front dynamics

$$\chi^{\uparrow}(T) = w[(q^{+} + m)\chi^{\uparrow}(F) - q^{R}\chi^{\downarrow}(F)];$$

$$\chi^{\downarrow}(T) = w[(q^{+} + m)\chi^{\downarrow}(F) + q^{L}\chi^{\uparrow}(F)].$$

What is Δq measured in DIS

- Δq is defined by $\Delta q\ s_\mu = \langle p,s|\bar{q}\gamma_\mu\gamma_5 q|p,s\rangle$

$$\Delta q = \langle p,s|\bar{q}\gamma^+\gamma_5 q|p,s\rangle$$

- Using light-cone Dirac spinors

$$\Delta q = \int_0^1 \mathrm{d}x\left[q^\uparrow(x) - q^\downarrow(x)\right]$$

- Using conventional Dirac spinors

$$\Delta q = \int \mathrm{d}^3\vec{p}M_q\left[q^\uparrow(\vec{p}) - q^\downarrow(\vec{p})\right]$$

$$M_q = \frac{(p_0 + p_3 + m)^2 - \vec{p}_\perp^{\,2}}{2(p_0 + p_3)(p_0 + m)}$$

Thus Δq is the light-cone quark spin, or quark spin in IMF, not that in the rest frame of the proton

The proton spin crisis

& the Melosh-Wigner rotation

- **It is shown that the proton "spin crisis" or "spin puzzle" can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation.**
- **The quark helicity Δq measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model.**

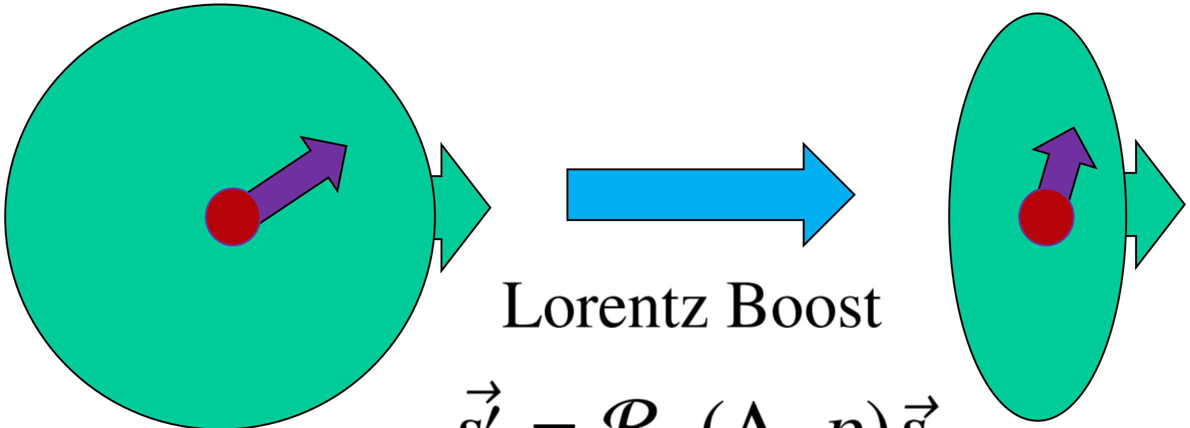
B.-Q. Ma, J.Phys. G 17 (1991) L53

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

B.-Q. Ma, J.Phys.G 17 (1991) L53-L58

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

An intuitive picture to understand the spin puzzle



$$\vec{s'} = \mathcal{R}_w(\Lambda, p)\vec{s}$$

Rest Frame

$$\Sigma \vec{s} = \vec{S}_p$$

Infinite Momentum Frame

$$\Sigma \vec{s'} \neq \vec{S}_p$$

A general consensus

The quark helicity Δq defined in the infinite momentum frame is generally not the same as the constituent quark spin component in the proton rest frame, just like that it is not sensible to compare apple with orange.

H.-Y.Cheng, hep-ph/0002157，
Chin.J.Phys.38:753,2000

Other approaches with same conclusion

Contribution from the lower component of Dirac spinors in the rest frame:

B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482

D.Qing, X.-S.Chen, F.Wang, **Phys.Rev.D58:114032,1998.**

P.Zavada, **Phys.Rev.D65:054040,2002**.

The Spin Distributions in Quark Model

The spin distribution probabilities in the quark-diquark model

$$u^{\uparrow}_{V} = \frac{1}{18};\quad u^{\downarrow}_{V} = \frac{2}{18};\quad d^{\uparrow}_{V} = \frac{2}{18};\quad d^{\downarrow}_{V} = \frac{4}{18};$$

$$u^{\uparrow}_{S} = \frac{1}{2};\quad u^{\downarrow}_{S} = 0;\quad d^{\uparrow}_{S} = 0;\quad d^{\downarrow}_{S} = 0. \tag{7}$$

Naive Quark Model:

$$\Delta u = \frac{4}{3};\qquad \Delta d = -\frac{1}{3};\qquad \Delta s = 0$$

$$\Sigma = \Delta u + \Delta d + \Delta s = 1$$

Relativistic Effect due to Melosh-Rotation

$$\Delta u_v(x) = u_v^{\uparrow}(x) - u_v^{\downarrow}(x) = -\frac{1}{18}a_V(x)W_V(x) + \frac{1}{2}a_S(x)W_S(x);$$

$$\Delta d_v(x) = d_v^{\uparrow}(x) - d_v^{\downarrow}(x) = -\frac{1}{9}a_V(x)W_V(x).$$

from

$$a_S(x) = 2u_v(x) - d_v(x);$$

$$a_V(x) = 3d_v(x).$$

We obtain

$$\Delta u_v(x) = [u_v(x) - \frac{1}{2}d_v(x)]W_S(x) - \frac{1}{6}d_v(x)W_V(x);$$

$$\Delta d_v(x) = -\frac{1}{3}d_v(x)W_V(x).$$

Relativistic SU(6) Quark Model

Flavor Symmetric Case

Relativistic Correction: $M_q = 0.75$

$\Delta u = \frac{4}{3} M_q = 1; \quad \Delta d = -\frac{1}{3} M_q = -0.25; \quad \Delta s = 0$

$\Sigma = \Delta u + \Delta d + \Delta s = 0.75$

$F_2^n(x)/F_2^p(x) \geq \frac{2}{3}$ for all x

Relativistic SU(6) Quark Model

Flavor Asymmetric Case

Relativistic Correction: $M_u \approx 0.6$; $M_d \approx 0.9$

$\Delta u = \frac{4}{3} M_u = 0.8$; $\Delta d = -\frac{1}{3} M_d = -0.3$; $\Delta s = 0$

$\Sigma = \Delta u + \Delta d + \Delta s \approx 0.5$

$F_2^n(x)/F_2^p(x) \rightarrow \frac{1}{4}$ at large x

B.-Q.Ma, Phys. Lett. **B 375** (1996) 320.

Relativistic SU(6) Quark Model

Flavor Asymmetric Case + Intrinsic Sea

For Intrinsic $d\bar{d}$ Sea ($\sim 15\%$): $\quad \Delta d_{\rm sea} \approx -0.07$

For Intrinsic $s\bar{s}$ Sea ($\sim 5\%$): $\quad \Delta s_{\rm sea} \approx -0.03$

Thus: $\quad \Sigma = \Delta u + \Delta d + \Delta s + \Delta d_{\rm sea} + \Delta s_{\rm sea} \approx 0.4$

S. J. Brodsky and B.-Q.Ma, Phys. Lett. **B 381** (1996) 317.

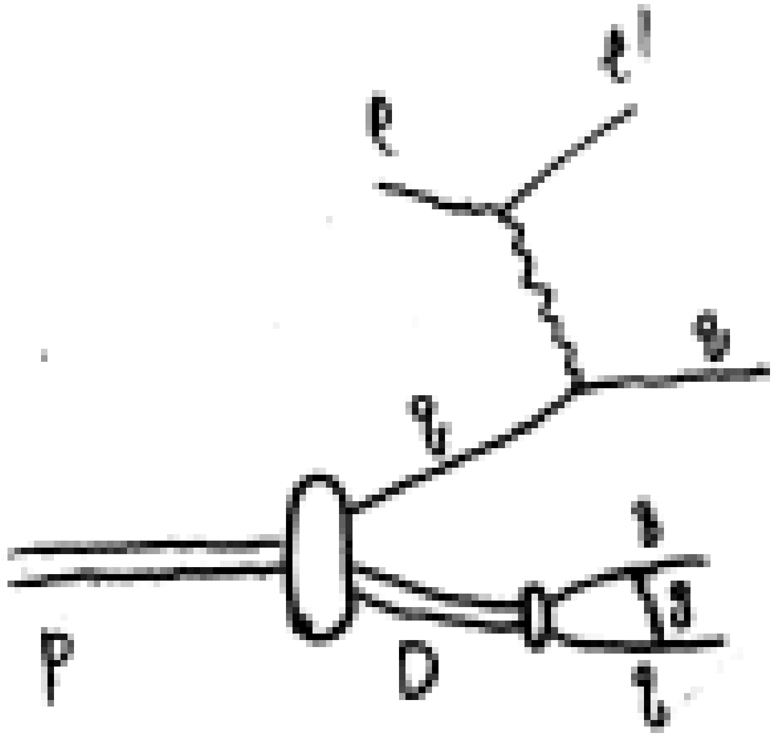
More detailed discussions, see, B.-Q.Ma, J.-J.Yang, I.Schmidt, Eur.Phys.J.A12(2001)353

Understanding the Proton Spin "Puzzle" with a New "Minimal" Quark Model

Three quark valence component could be as large as 70% to account for the data

B.-Q. Ma, Phys.Lett. B 375 (1996) 320-326.
B.-Q. Ma, I.Schmidt, J.Soffer, Phys.Lett. B 441 (1998) 461-467.

A relativistic quark-diquark model



A relativistic quark-diquark model

- The unpolarized distribution of quark q in hadron h can be written as

$$q(x) = c_q^S a_S(x) + c_q^V a_V(x),$$

where $a_D(x)$ is

$$a_D(x) \propto \int [\mathrm{d}^2 \mathbf{k}_\perp] |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V),$$

- BHL prescription of the light-cone momentum space wave function for quark-diquark

$$\phi(x, \mathbf{k}_\perp) = A_D \exp\left\{ -\frac{1}{8\alpha_D^2} \left[\frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x} \right] \right\},$$

B.-Q. Ma, Phys.Lett. B 375 (1996) 320-326.
B.-Q. Ma, I.Schmidt, J.Soffer, Phys.Lett. B 441 (1998) 461-467.

A relativistic quark-diquark model

- longitudinally polarized quark distribution

$$\Delta q(x) = \tilde{c}_q^S \tilde{a}_S(x) + \tilde{c}_q^V \tilde{a}_V(x)$$

where

$$\tilde{a}_D(x) = \int [\mathrm{d}^2 \mathbf{k}_\perp] W_D(x, \mathbf{k}_\perp) |\phi(x, \mathbf{k}_\perp)|^2 \quad (D = S \text{ or } V)$$

- Melosh-Winger rotation factor

Longitudinally polarized

$$W_D(x, \mathbf{k}_\perp) = \frac{(k^+ + m_q)^2 - \mathbf{k}_\perp^2}{(k^+ + m_q)^2 + \mathbf{k}_\perp^2}$$

where $k^+ = x\mathcal{M}$, $\mathcal{M}^2 = \frac{m_q^2 + \mathbf{k}_\perp^2}{x} + \frac{m_D^2 + \mathbf{k}_\perp^2}{1-x}$.

The Melosh-Wigner rotation

in pQCD based parametrization of quark helicity distributions

“The helicity distributions measured on the light-cone are related by a Wigner rotation (Melosh transformation) to the ordinary spin S_i^z of the quarks in an equal-time rest-frame wavefunction description. Thus, due to the non-collinearity of the quarks, one cannot expect that the quark helicities will sum simply to the proton spin.”

S.J.Brodsky, M.Burkardt, and I.Schmidt,

Nucl.Phys.B441 (1995) 197-214, p.202

pQCD counting rule

$$q_{\mathrm{h}}^{\pm} \propto (1-x)^{p}$$

$$p = 2n - 1 + 2\,|\,\Delta s_z\,| \qquad\qquad \Delta s_z = s_q - s_N$$

- **Based on the minimum connected tree graph of hard gluon exchanges.**
- **"Helicity retention" is predicted -- The helicity of a valence quark will match that of the parent nucleon.**

Parameters in pQCD counting rule analysis

In leading term

$$q_i^{+} = \frac{\tilde{A}_{q_i}}{B_3} x^{-\frac{1}{2}} (1-x)^3$$

$$q_i^{-} = \frac{\tilde{C}_{q_i}}{B_5} x^{-\frac{1}{2}} (1-x)^5$$

Baryon	q_1	q_2	$\tilde{A}_{q_1}$	$\tilde{C}_{q_1}$	$\tilde{A}_{q_2}$	$\tilde{C}_{q_2}$
p	u	d	1.375	0.625	0.275	0.725

B.-Q. Ma, I. Schmidt, J.-J. Yang, Phys.Rev.D63(2001) 037501.

New Development: H. Avakian, S.J.Brodsky, D.Boer, F.Yuan, Phys.Rev.Lett.99:082001,2007.

Two different sets of parton distributions

- SU(6) quark-diquark model

$$\Delta u_v(x) = [u_v(x) - \frac{1}{2} d_v(x)] W_S(x) - \frac{1}{6} d_v(x) W_V(x),$$
$$\Delta d_v(x) = -\frac{1}{3} d_v(x) W_V(x).$$

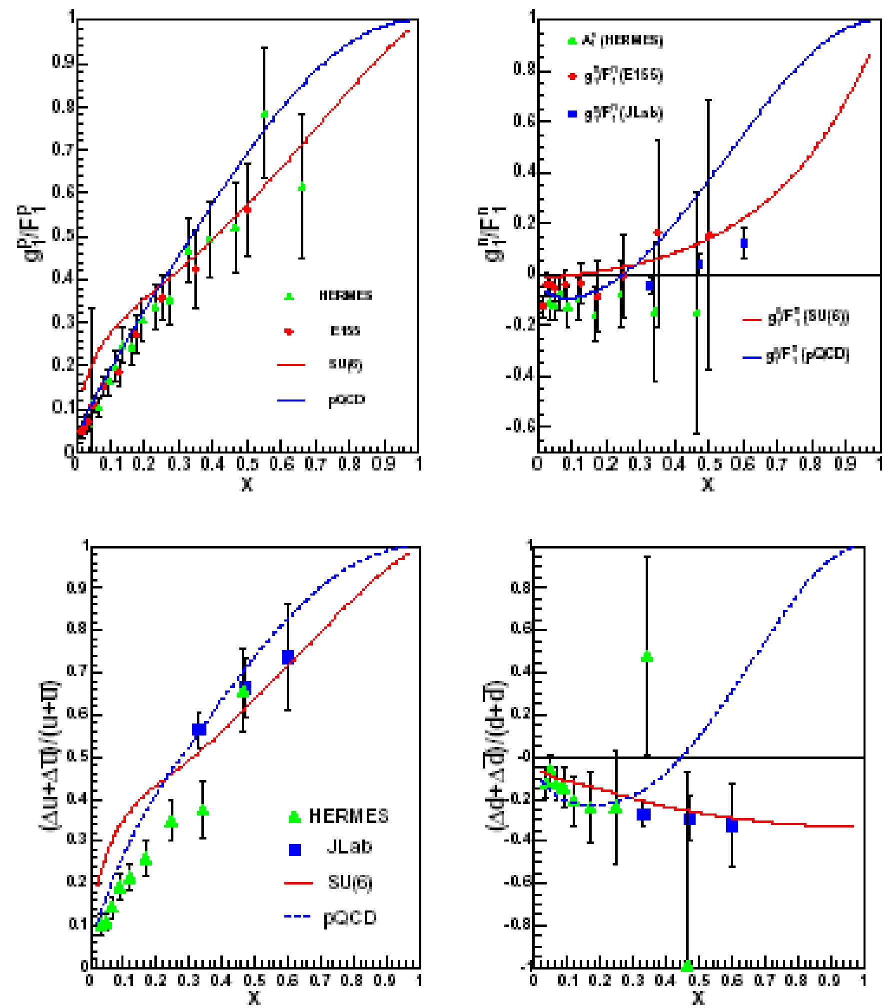
- pQCD based counting rule analysis

$$\begin{aligned}
u_v^{\text{pQCD}}(x) &= u_v^{\text{para}}(x), \\
d_v^{\text{pQCD}}(x) &= \frac{d_v^{\text{th}}(x)}{u_v^{\text{th}}(x)} u_v^{\text{para}}(x), \\
\Delta u_v^{\text{pQCD}}(x) &= \frac{\Delta u_v^{\text{th}}(x)}{u_v^{\text{th}}(x)} u_v^{\text{para}}(x), \\
\Delta d_v^{\text{pQCD}}(x) &= \frac{\Delta d_v^{\text{th}}(x)}{u_v^{\text{th}}(x)} u_v^{\text{para}}(x),
\end{aligned}$$

- CTEQ5 set 3 as input.

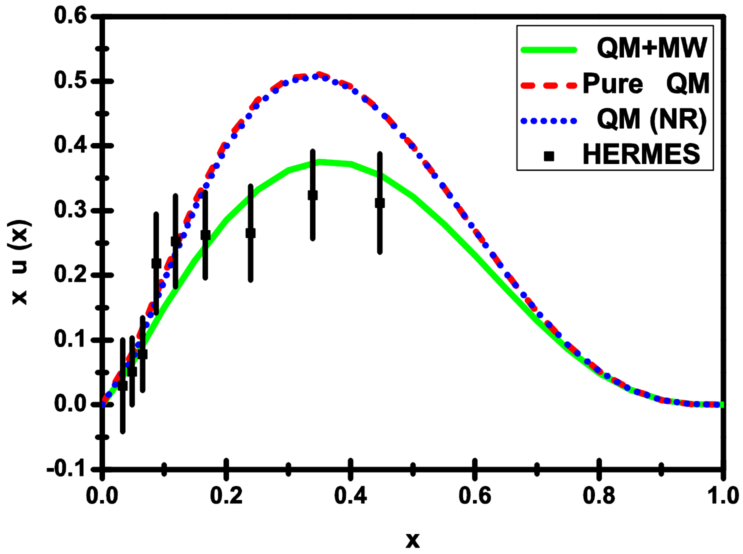
Different predictions in two models

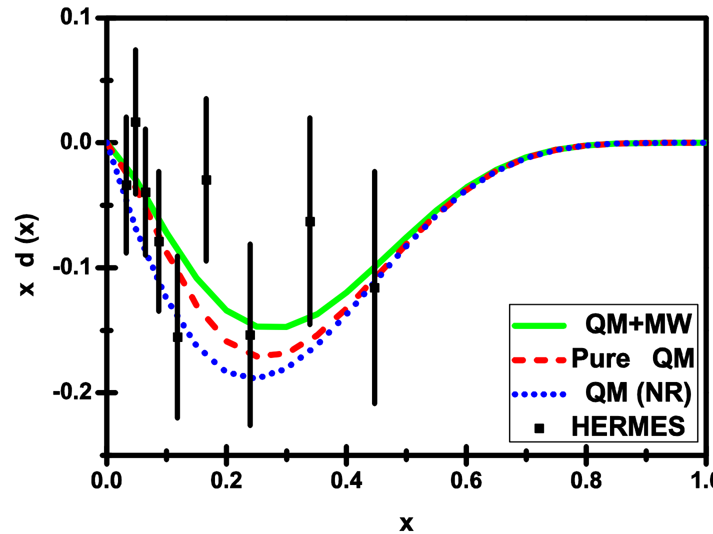
- Helicity distribution
  - SU(6) quark-diquark model:
    $\Delta u(x)/u(x) \rightarrow 1$ as $x \rightarrow 1.$
    $\Delta d(x)/d(x) \rightarrow -\frac{1}{3}$ as $x \rightarrow 1.$
  - pQCD based counting rule analysis:
    $\Delta u(x)/u(x) \rightarrow 1$ as $x \rightarrow 1.$
    $\Delta d(x)/d(x) \rightarrow 1$ as $x \rightarrow 1.$



X.Zhang, B.-Q. Ma, PRD 85 (2012) 114048.

The proton spin in a light-cone chiral quark model





An upgrade of previous work by including Melosh-Wigner rotation: T. P. Cheng and L. F. Li, PRL 74 (1995) 2872

Chances：New Research Directions

- New quantities：Transversity, Generalized Parton Distributions, Collins Functions, Sivers Functions, Boer-Mulders Functions, Pretzelosity, Wigner Distributions

- Hyperon Physics：The spin structure of Lambda and Sigma hyperons

B.-Q. Ma, I. Schmidt, J.-J. Yang, PLB 477 (2000) 107, PRD 61 (2000) 034017
B.-Q. Ma, J. Soffer, PRL 82 (1999) 2250

The Melosh-Wigner Rotation in Transversity

$$2\,\delta q = \langle p, \uparrow | \bar{q}_{\lambda} \gamma^{\perp} \gamma^{+} q_{-\lambda} | p, \downarrow \rangle$$

$$\delta q(x) = \int [\mathrm{d}^2 \boldsymbol{k}_{\perp}] \tilde{M}_q(x, \boldsymbol{k}_{\perp}) \Delta q_{\mathrm{RF}}(x, \boldsymbol{k}_{\perp})$$

$$\tilde{M}_q(x, \boldsymbol{k}_{\perp}) = \frac{(k^{+} + m)^2}{(k^{+} + m)^2 + \boldsymbol{k}_{\perp}^2}$$

I.Schmidt&J.Soffer, Phys.Lett.B 407 (1997) 331

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

The Melosh-Wigner Rotation in Quark Orbital Angular Moment

$$\hat{L}_q = -i\left(k_1 \frac{\partial}{\partial k_2} - k_2 \frac{\partial}{\partial k_1}\right).$$

$$L_q(x) = \int [d^2 k_\perp] M_L(x, k_\perp) \Delta q_{QM}(x, k_\perp)$$

$$M_L(x, k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

Spin and orbital sum in light-cone formalism

$$\frac{1}{2}M_q + M_L = \frac{1}{2}$$

$$M_q(x,k_\perp) = \frac{(k^+ + m)^2 - k_\perp^2}{(k^+ + m)^2 + k_\perp^2} \qquad M_L(x,k_\perp) = \frac{k_\perp^2}{(k^+ + m)^2 + k_\perp^2}$$

$$\frac{1}{2}\Delta q(x) + L_q(x) = \frac{1}{2}\Delta q_{QM}(x)$$

Ma&Schmidt, Phys.Rev.D 58 (1998) 096008

Leading-Twist TMD PDFs

Nucleon Spin
Quark Spin

		Quark polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1		$h_1^{\perp}$ Boer-Mulders
	L		g_1 Helicity	$h_{1L}^{\perp}$ Long-Transversity
	T	$f_{1T}^{\perp}$ Sivers	g_{1T} Trans-Helicity	h_1 Transversity $h_{1T}^{\perp}$ Pretzelosity

The Melosh-Wigner Rotation in "Pretzelosity"

$$g_1^q(x, k_\perp) - h_1^q(x, k_\perp) = h_{1T}^{\perp(1)q}(x, k_\perp) \ .$$

$$\frac{(k^+ + m)^2 - \mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} - \frac{(k^+ + m)^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} = -\frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$$

$$\mathrm{Pretzelosity} = \Delta \mathrm{q} - \delta \mathrm{q} = -\mathrm{L_q}$$

$$\mathrm{Pretzelosity} = -\int [\mathrm{d}^2\mathbf{k}_\perp] \frac{\mathbf{k}_\perp^2}{(\mathrm{k}^+ + \mathrm{m})^2 + \mathbf{k}_\perp^2} \Delta \mathrm{q_{QM}}(\mathrm{x}, \mathbf{k}_\perp)$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

New Sum Rule of Physical Observables

$$g_1^q(x, k_\perp) - h_1^q(x, k_\perp) = h_{1T}^{\perp(1)q}(x, k_\perp) \; .$$

$$\frac{(k^+ + m)^2 - \mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} - \frac{(k^+ + m)^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2} = -\frac{\mathbf{k}_\perp^2}{(k^+ + m)^2 + \mathbf{k}_\perp^2}$$

$$\mathrm{Pretzelosity} = \Delta \mathrm{q} - \delta \mathrm{q} = -\mathrm{L_q}$$

$$\mathrm{Pretzelosity} = -\int [\mathrm{d}^2\mathbf{k}_\perp] \frac{\mathbf{k}_\perp^2}{(\mathrm{k}^+ + \mathrm{m})^2 + \mathbf{k}_\perp^2} \Delta \mathrm{q}_{\mathrm{QM}}(\mathrm{x}, \mathbf{k}_\perp)$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

The Melosh-Wigner Rotation in five 3dPDFs

分布函数	Melosh转动因子（$W_D(D=V,\ S)$）
g_{1L}	$[(x\mathscr{M}_D+m_q)^2-p_\perp^2]/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
g_{1T}	$2M_N(x\mathscr{M}_D+m_q)/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
h_1	$(x\mathscr{M}_D+m_q)^2/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
$h_{1L}^\perp$	$-2M_N(x\mathscr{M}_D+m_q)/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$
$h_{1T}^\perp$	$-2M_N^2/[(x\mathscr{M}_D+m_q)^2+p_\perp^2]$

$\mathscr{M}_D^2=\frac{m_q^2+p_\perp^2}{x}+\frac{m_D^2+p_\perp^2}{1-x}$是旁观双夸克的不变质量。

Names for New (tmd) PDF: g_{1T} and $h_{1L}^{\perp}$

g_{1T} trans-helicity 横纵度

$h_{1L}^{\perp}$ longi-transversity / heli-transversity 纵横度

Physics Letters B 696 (2011) 246–251

ELSEVIER

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

PHYSICS LETTERS B

Proposal for measuring new transverse momentum dependent parton distributions g_{1T} and $h_{1L}^{\perp}$ through semi-inclusive deep inelastic scattering

Jiacai Zhu [a], Bo-Qiang Ma [a,b,*]

[a] School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
[b] Center for High Energy Physics, Peking University, Beijing 100871, China

The Necessity of Polarized p pbar Collider

The polarized proton antiproton Drell-Yan process

is ideal to measure

the pretzelosity distributions of the nucleon.

PHYSICAL REVIEW D **82,** 114022 (2010)

Probing the leading-twist transverse-momentum-dependent parton distribution function $h_{1T}^{\perp}$ via the polarized proton-antiproton Drell-Yan process

Jiacai Zhu

School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

Bo-Qiang Ma*

School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
Center for High Energy Physics, Peking University, Beijing 100871, China

(Received 10 October 2010; published 22 December 2010)

Probing Pretzelosity in pion p Drell-Yan Process

COMPASS pion p Drell-Yan process

can also measure

the pretzelosity distributions of the nucleon.

Physics Letters B 696 (2011) 513–517

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Single spin asymmetry in πp Drell–Yan process

Zhun Lu [a,b], Bo-Qiang Ma [c,*], Jun She [c]

[a] Department of Physics, Southeast University, Nanjing 211189, China
[b] Departamento de Física, Universidad Técnica Federico Santa María, and Centro Científico-Tecnológico de Valparaíso Casilla 110-V, Valparaíso, Chile
[c] School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

Eur. Phys. J. C (2012) 72:2037
DOI 10.1140/epjc/s10052-012-2037-7

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Azimuthal asymmetries in lepton-pair production at a fixed-target experiment using the LHC beams (AFTER)

Tianbo Liu[1], **Bo-Qiang Ma**[1,2,a]

[1]School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
[2]Center for High Energy Physics, Peking University, Beijing 100871, China

unpolarized and single polarized *pp* and *pd* processes. We conclude that it is feasible to measure these azimuthal asymmetries, consequently the three-dimensional or transverse momentum dependent parton distribution functions (3dPDFs or TMDs), at this new AFTER facility.

PHYSICAL REVIEW D **91,** 034019 (2015)

Quark Wigner distributions in a light-cone spectator model

Tianbo Liu[1] and Bo-Qiang Ma[1,2,3,*]

[1]*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

[2]*Collaborative Innovation Center of Quantum Matter, Beijing, China*

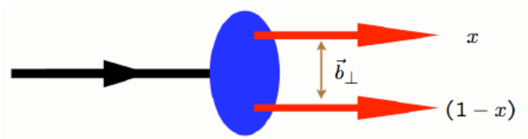
[3]*Center for High Energy Physics, Peking University, Beijing 100871, China*

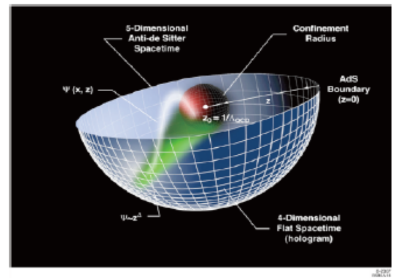
(Received 31 March 2014; published 19 February 2015)

We investigate the quark Wigner distributions in a light-cone spectator model. The Wigner distribution, as a quasidistribution function, provides the most general one-parton information in a hadron. Combining the polarization configurations, unpolarized, longitudinal polarized, or transversal polarized, of the quark and the proton, we can define 16 independent Wigner distributions at leading twist. We calculate all these Wigner distributions for the u quark and the d quark, respectively. In our calculation, both the scalar and the axial-vector spectators are included, and the Melosh–Wigner rotation effects for both the quark and the axial-vector spectator are taken into account. The results provide us a very rich picture of the quark structure in the proton.

Light Front Holography of Nucleon

S.J. Brodsky, G.F. de Téramond, PRD 77, 056007 (2008).





5-D coordinate

Light-front variable

$$\zeta = \sqrt{x(1-x)}|b_\perp| \qquad\qquad z$$

Eigenfunction: Massless Quarks

$$\left(-\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta} - V(\zeta)\right)\phi_- = M\phi_+$$

$$\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta} - V(\zeta)\right)\phi_+ = M\phi_-$$

G.F. de Téramond, S.J. Brodsky, Subnucl.Ser. 45 (2009) 139-183.

Light Front Holography of Nucleon

- An ultra-relativistic description with massless quarks.
- The sum of quark spins is zero.
- The angular momentum of the nucleon is totally from the orbital angular momentum of the quark and the spectator.

It is far away from a realistic picture of the nucleon spin.

PHYSICAL REVIEW D **92,** 096003 (2015)

Baryon properties from light-front holographic QCD

Tianbo Liu[1,*] and Bo-Qiang Ma[1,2,3,†]

[1]*School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*

[2]*Collaborative Innovation Center of Quantum Matter, Beijing, China*

[3]*Center for High Energy Physics, Peking University, Beijing 100871, China*

(Received 4 June 2015; published 3 November 2015)

$$\psi_+(x,\boldsymbol{k}_\perp) \sim \frac{4\pi}{\sqrt{\lambda x(1-x)}} e^{-\frac{1}{2\lambda}\left(\frac{\boldsymbol{k}_\perp^2+m_1^2}{x}+\frac{\boldsymbol{k}_\perp^2+m_2^2}{1-x}\right)},$$

$$\psi_-(x,\boldsymbol{k}_\perp) \sim \frac{4\pi|\boldsymbol{k}_\perp|}{\lambda x(1-x)} e^{-\frac{1}{2\lambda}\left(\frac{\boldsymbol{k}_\perp^2+m_1^2}{x}+\frac{\boldsymbol{k}_\perp^2+m_2^2}{1-x}\right)}.$$

$$\psi_{L=0}(x,\boldsymbol{k}_\perp) = N\frac{4\pi[m_1+\sqrt{\lambda x(1-x)}]}{\lambda x(1-x)} \times e^{-\frac{1}{2\lambda}\left(\frac{\boldsymbol{k}_\perp^2+m_1^2}{x}+\frac{\boldsymbol{k}_\perp^2+m_2^2}{1-x}\right)},$$

$$\psi_{L=1}(x,\boldsymbol{k}_\perp) = N\frac{-4\pi(k^1+ik^2)}{\lambda x(1-x)} \times e^{-\frac{1}{2\lambda}\left(\frac{\boldsymbol{k}_\perp^2+m_1^2}{x}+\frac{\boldsymbol{k}_\perp^2+m_2^2}{1-x}\right)},$$

Light-Front Wanfunctions

generic ansatz,

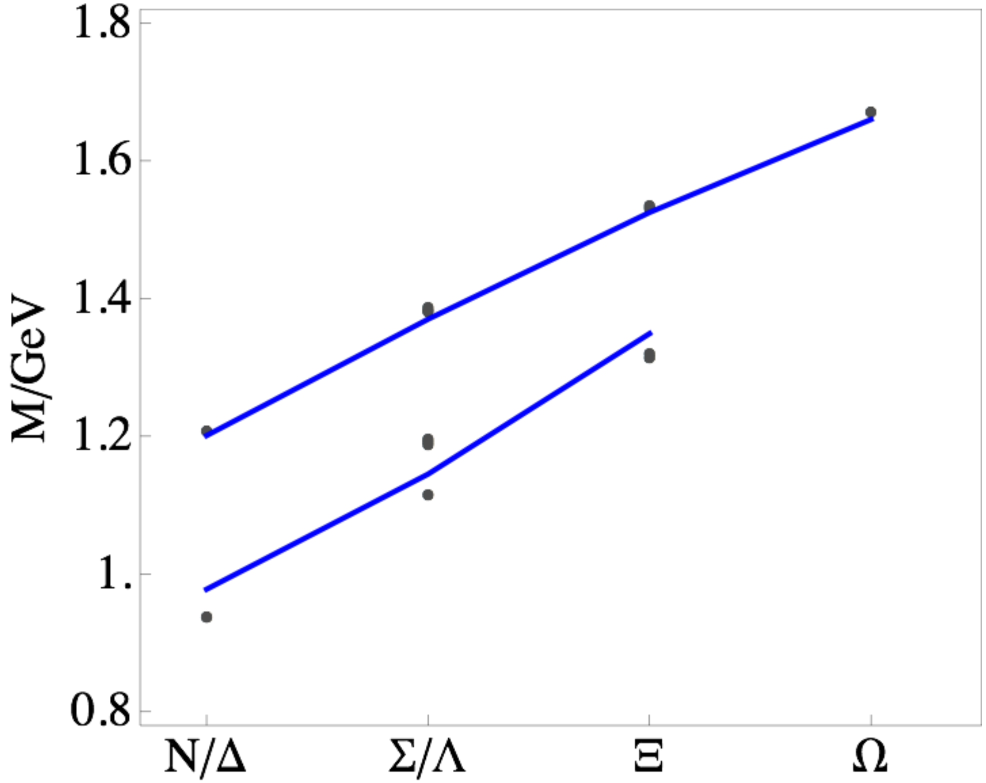
$$\frac{|\psi_{L=0}|^2}{|\psi_{L=1}|^2} = \frac{(m_1 + x\mathcal{M})^2}{\boldsymbol{k}_\perp^2}$$

$$\psi_{L=0}(x, \boldsymbol{k}_\perp) \sim \frac{4\pi[m_1 + \sqrt{\lambda x(1-x)}]}{\lambda x(1-x)} \exp\left[-\frac{1}{2\lambda}\left(\frac{\boldsymbol{k}_\perp^2 + m_1^2}{x} + \frac{\boldsymbol{k}_\perp^2 + m_2^2}{1-x}\right)\right]$$

$$\psi_{L=1}(x, \boldsymbol{k}_\perp) \sim \frac{-4\pi(k^1 + ik^2)}{\lambda x(1-x)} \exp\left[-\frac{1}{2\lambda}\left(\frac{\boldsymbol{k}_\perp^2 + m_1^2}{x} + \frac{\boldsymbol{k}_\perp^2 + m_2^2}{1-x}\right)\right]$$

T.Liu, B.-Q.Ma, Phys.Rev.D92 (2015) 096003

Baryon spectra: octet and decuplet

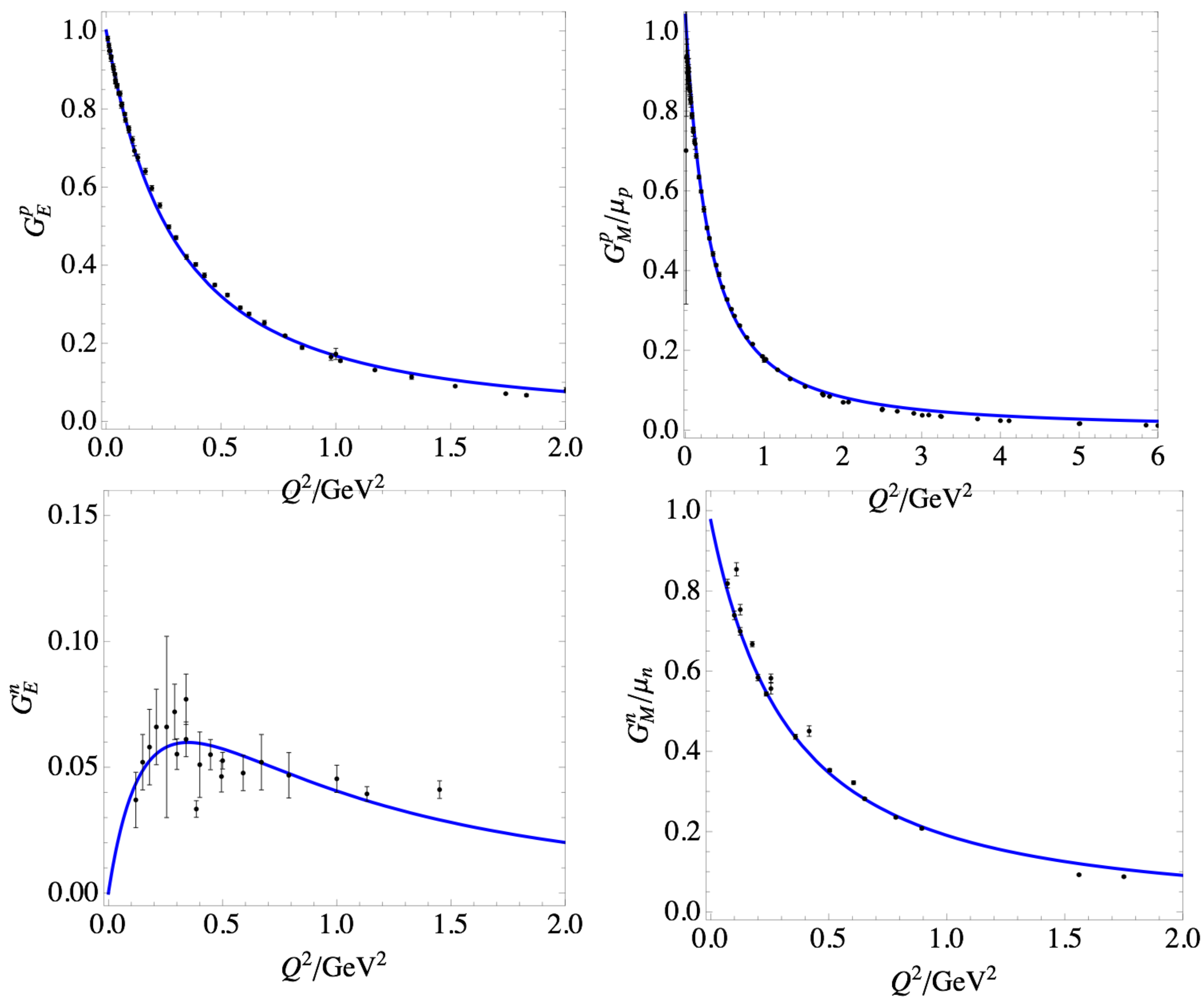


T.Liu, B.-Q.Ma, Phys.Rev.D92 (2015) 096003

Form factors\

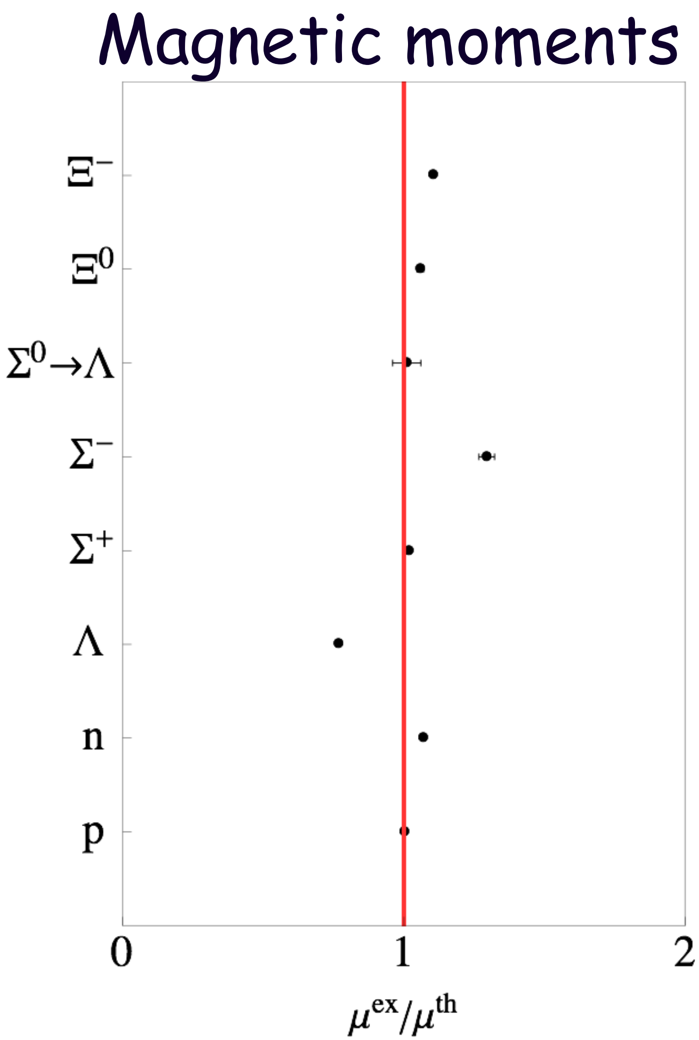
Proton:

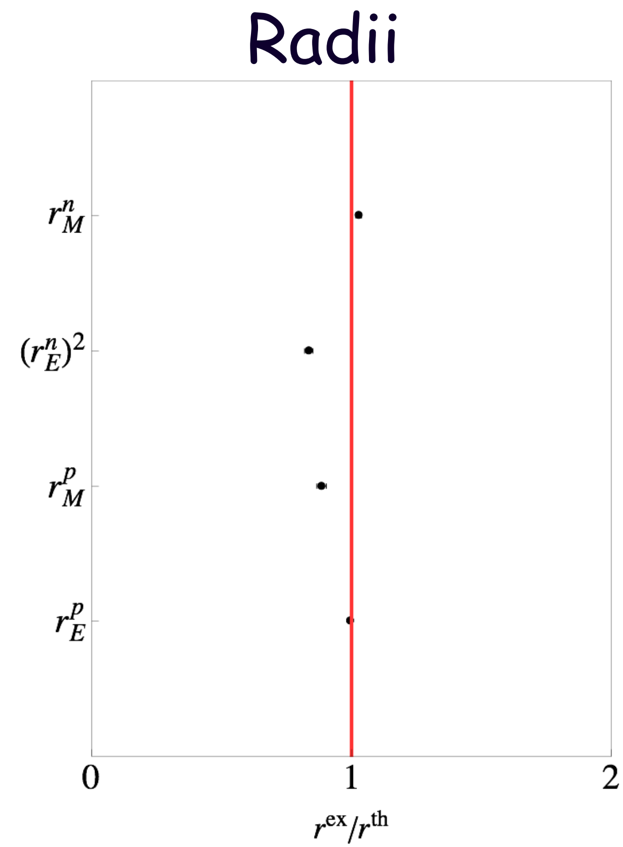
Neutron:



T.Liu, B.-Q.Ma, Phys.Rev.D92 (2015) 096003

Magnetic moments, Radii





Axial charge: 0.308

$0.330 \pm 0.011(\text{theo.}) \pm 0.025(\text{exp.}) \pm 0.028(\text{evol.})$

T.Liu, B.-Q.Ma, Phys.Rev.D92 (2015) 096003

The Spin of Nucleon from Holographic QCD

Axial charge: 0.308

$$0.330 \pm 0.011(\text{theo.}) \pm 0.025(\text{exp.}) \pm 0.028(\text{evol.})$$

The light-front holographic model with nonzero quark mass is essential to understand the spin structure with other low energy properties reproduced.

T.Liu, B.-Q.Ma, Phys.Rev.D92 (2015) 096003

Conclusions (I)

- **The Melosh-Wigner rotation plays a key role to understand the proton spin puzzle from the light-front quark model.**

non-relativistic ➡ **relativistic**

- **The light-front holographic model with nonzero quark mass is essential to understand the spin structure with other low energy properties reproduced.**

ultra-relativistic (m=0) ➡ **relativistic (with m)**

Conclusions (II)

- The relativistic motion of quarks is important for a realistic picture of the nucleon.
- Measurements of new TMDs or 3dPDFs can provide information concerning quark relativistic motions

→ to enrich our knowledge concerning the quark orbital angular momentum