Lattice QCD:
light-meson spectroscopy/dynamics

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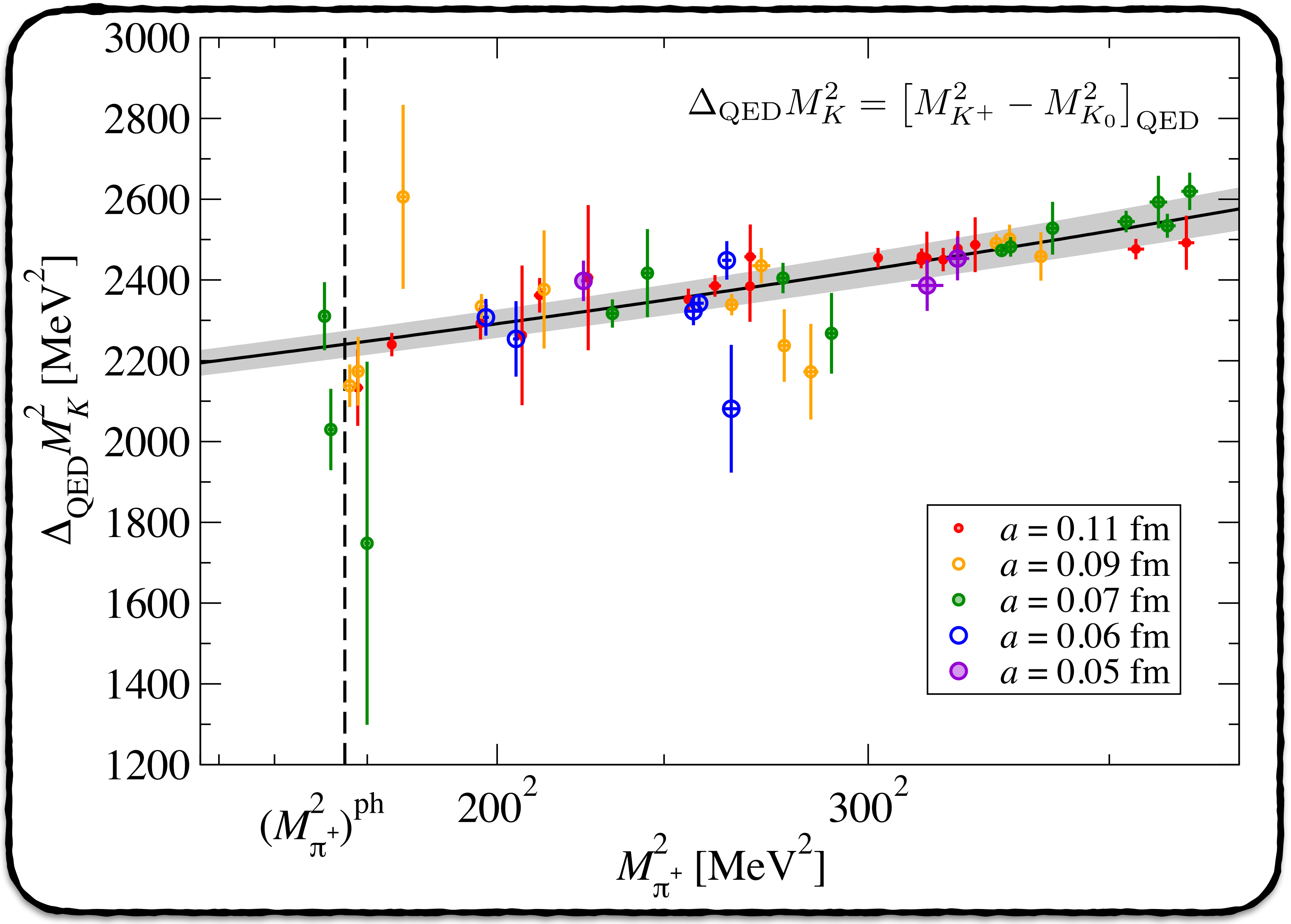
Spectroscopy motivation

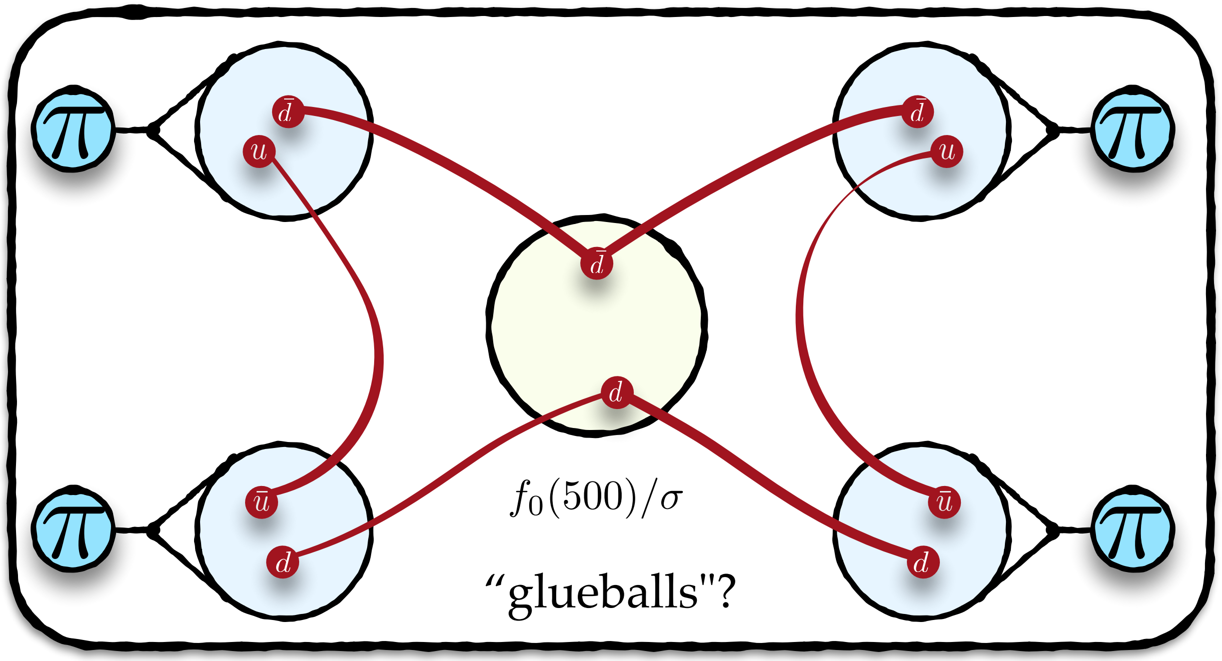
- *Vanilla* spectroscopy - QCD stable states [non-composite states]
  - Physical or lighter quark masses [down to m_π~120 MeV] ✓
  - Non-degenerate light-quark masses: N_f=1+1+1+1 ✓
  - Dynamical QED ✓

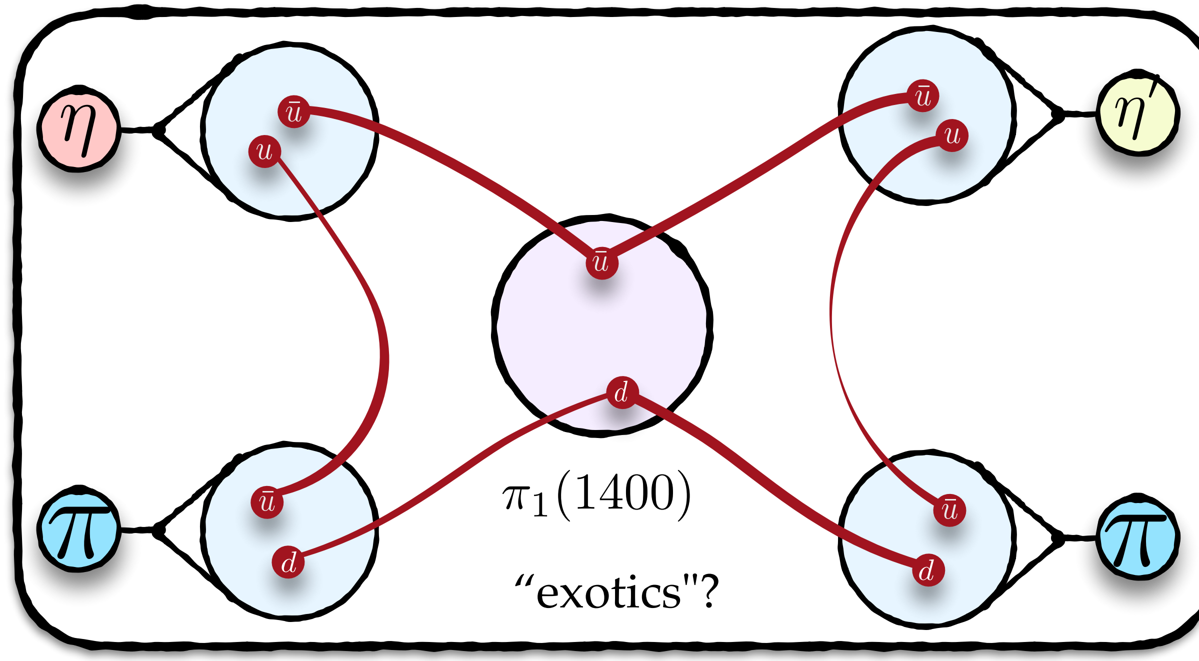


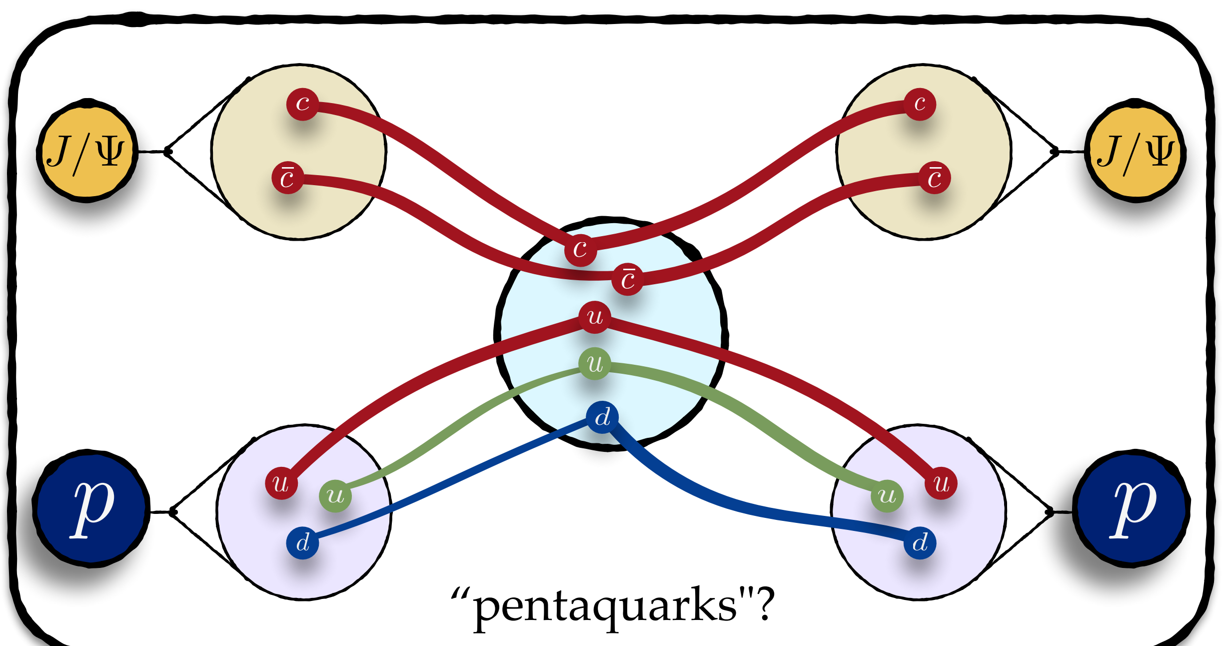
Fodor et al. [BMWc] (2016)

Spectroscopy motivation

- *Vanilla* spectroscopy - QCD stable states [non-composite states]
- the frontier of spectroscopy - hadronic resonances [composite states]







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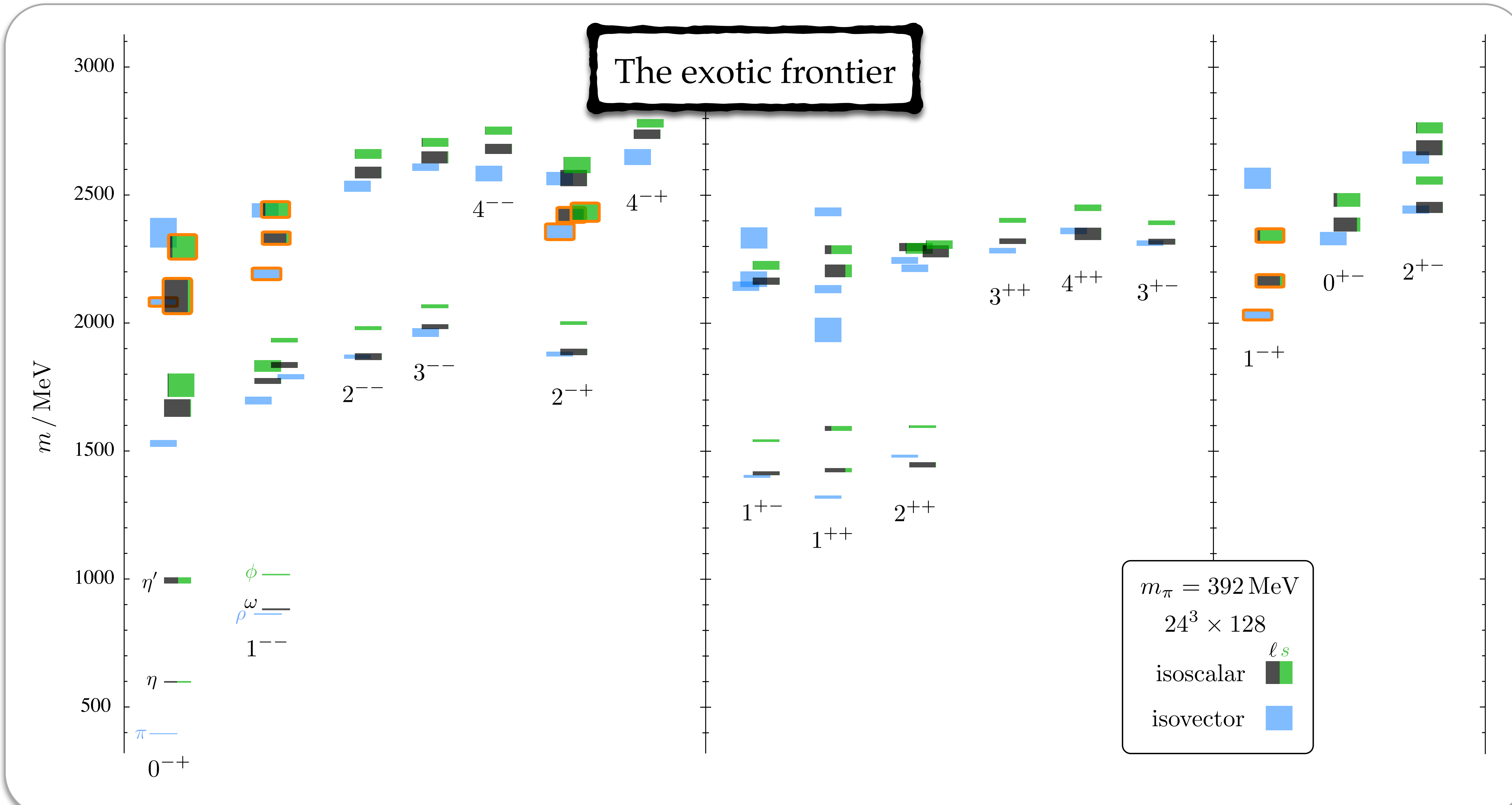

Dudek, Edwards, Guo, Thomas [Hadspec Collab.] (2013)

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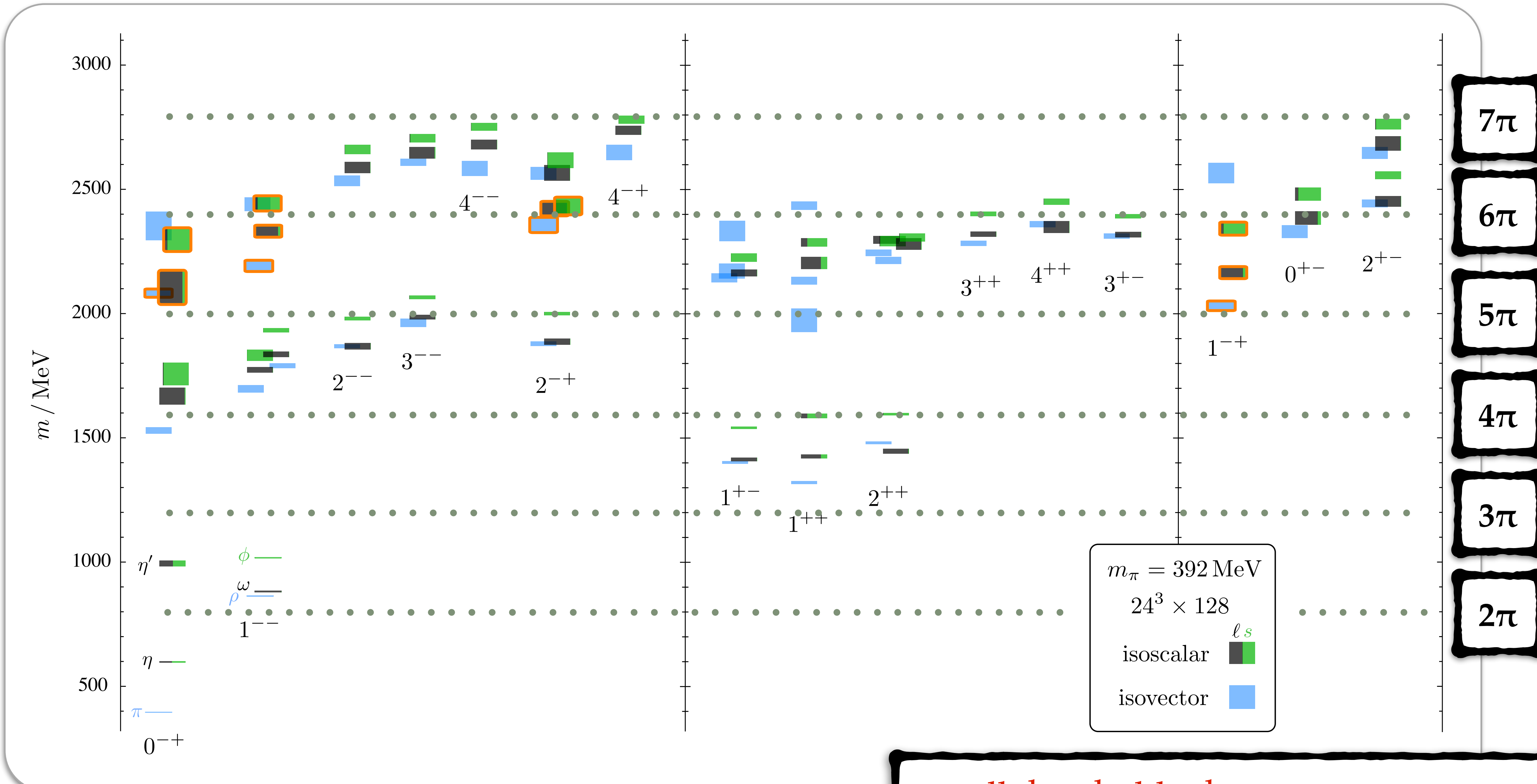
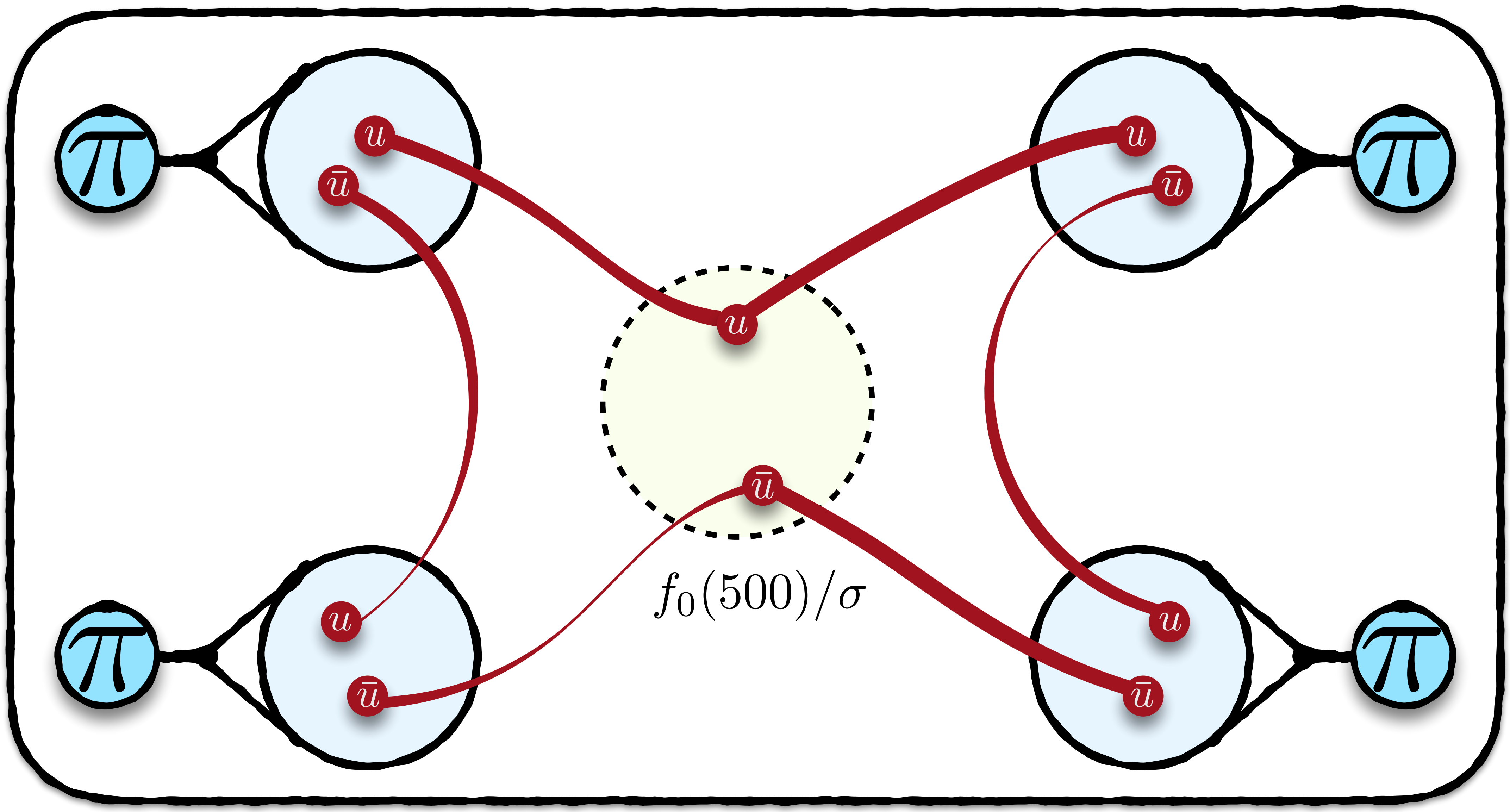


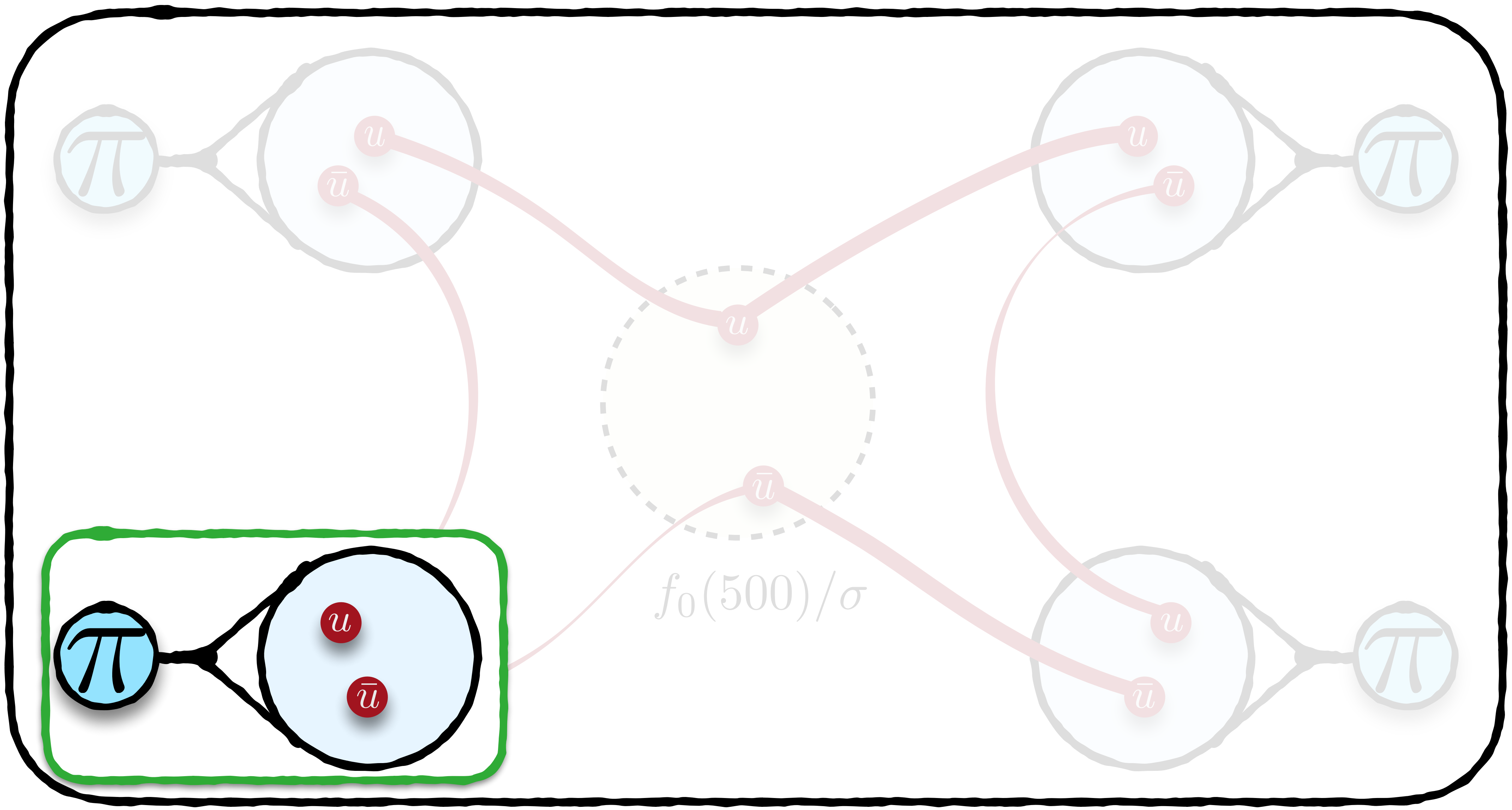
Dudek, Edwards, Guo, Thomas [Hadspec Collab.] (2013)

not all thresholds shown
not all threshold are expected to matter

Need for lattice QCD

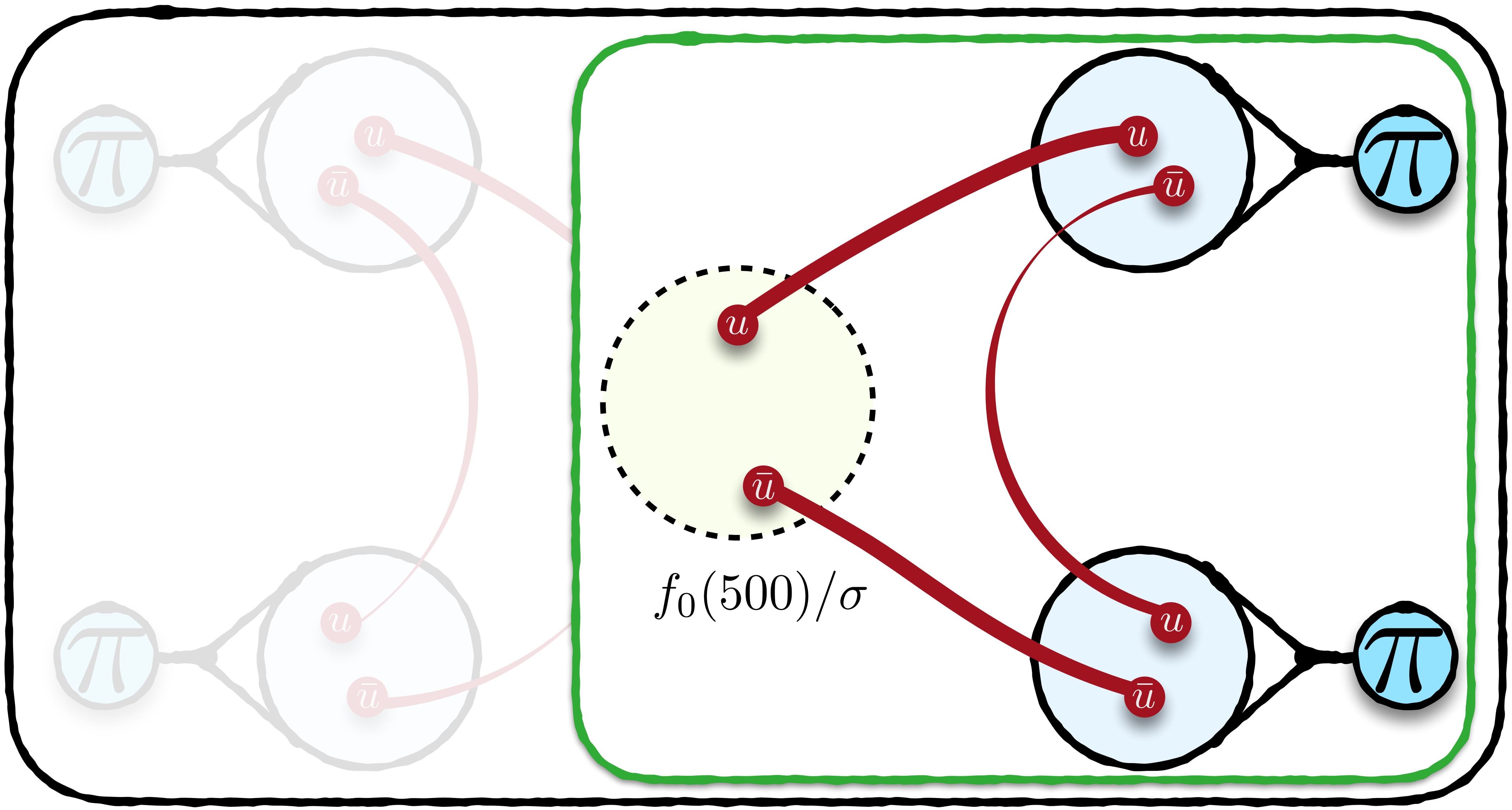


Need for lattice QCD



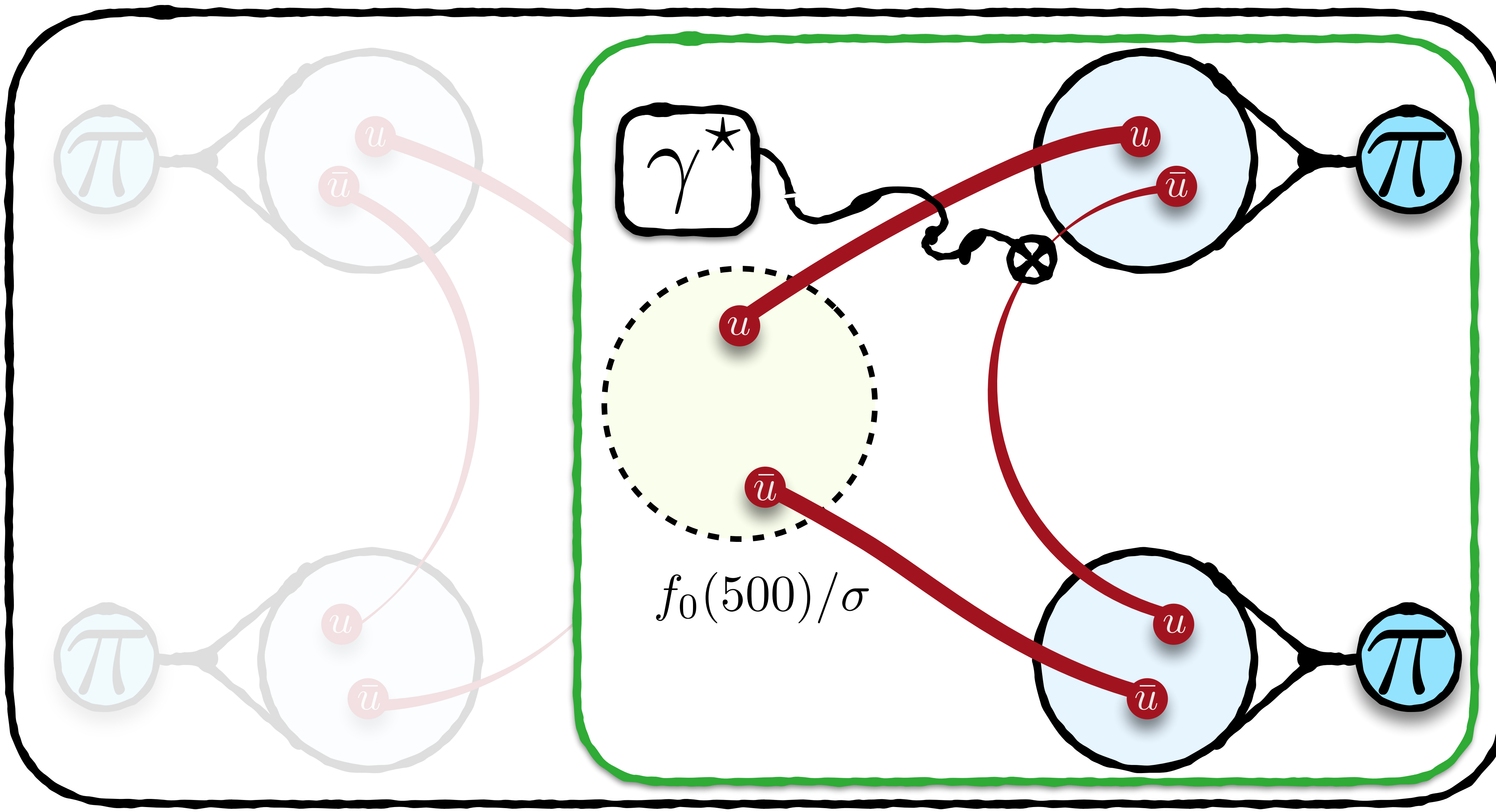
- QCD-stable states are generated exactly

Need for lattice QCD



- QCD-stable states are generated exactly
- Resonances are generated and decay in accordance to QCD

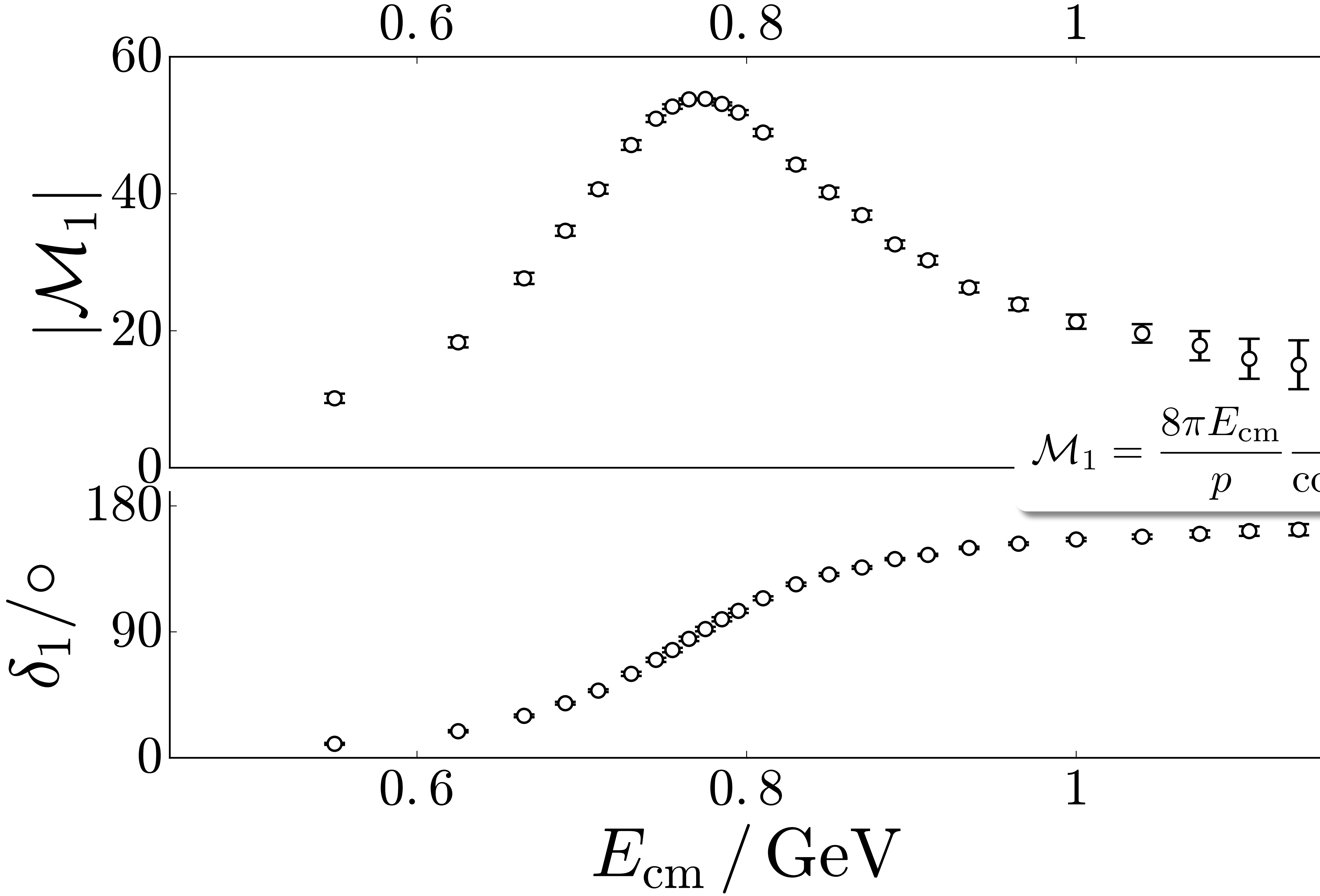
Need for lattice QCD



- QCD-stable states are generated exactly
- Resonances are generated and decay in accordance to QCD
- QED / weak sector can be treated perturbatively or non-perturbatively

A pseudo-quantitative definition

(bump in cross sections/amplitude - e.g., $\pi\pi$ scattering in ϱ-channel)

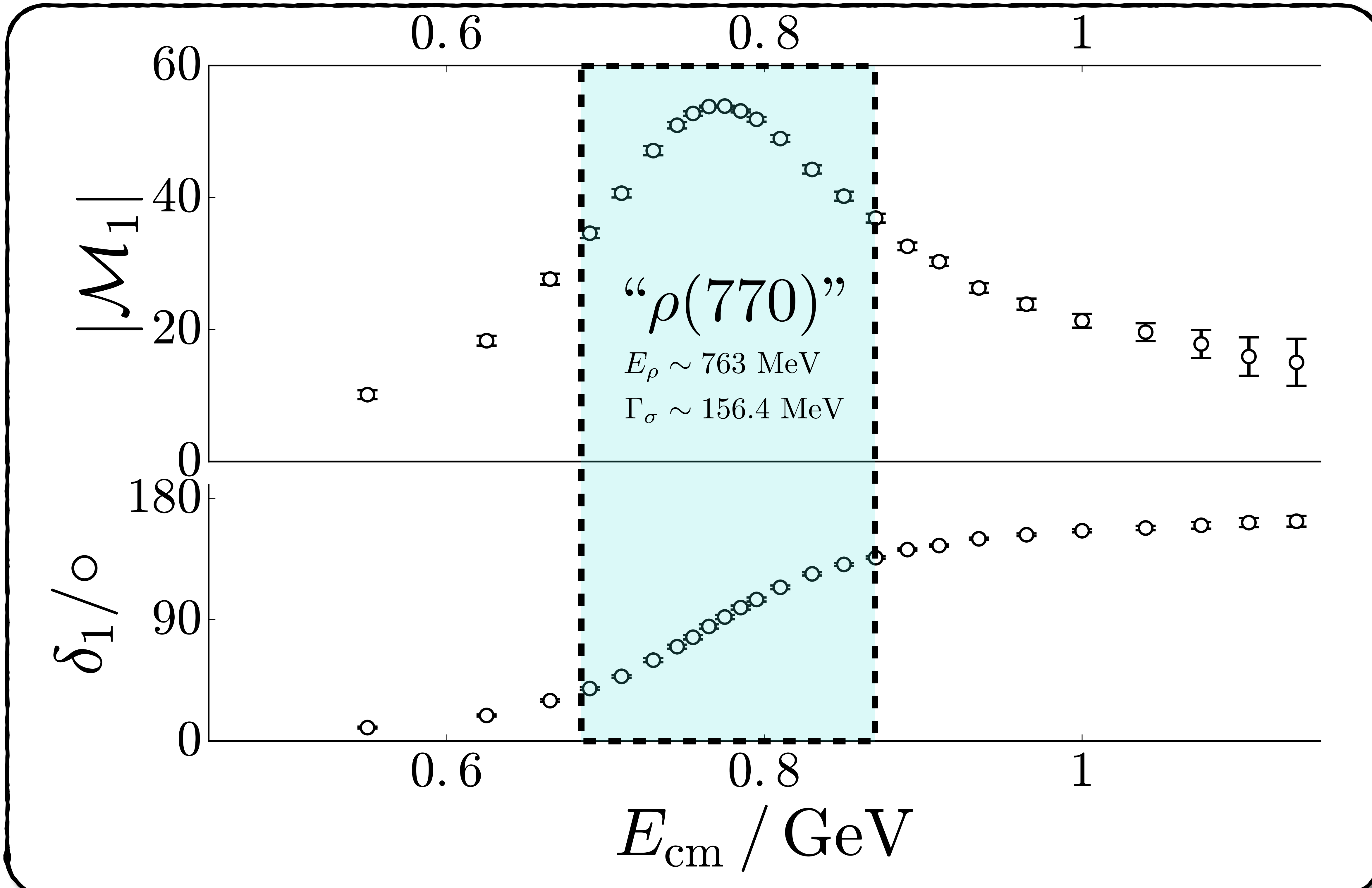


$$\mathcal{M}_1 = \frac{8\pi E_{\rm cm}}{p} \frac{1}{\cot \delta_1 - i}$$

Protopopescu et al. (1972)

A pseudo-quantitative definition

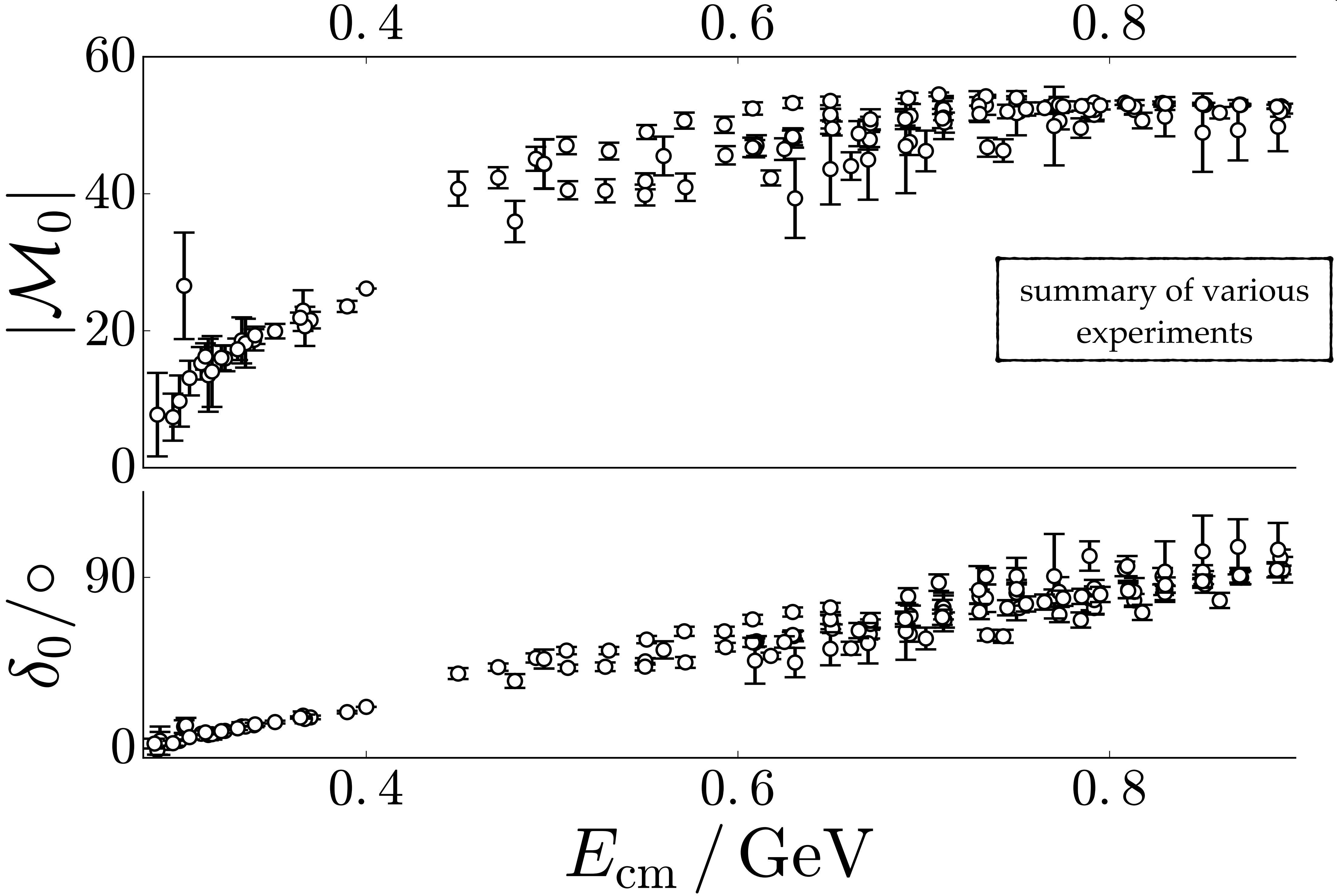
(bump in cross sections/amplitude - e.g., ππ scattering in ϱ-channel)



Protopopescu et al. (1972)

A counter example

(Isoscalar, scalar $\pi\pi$ scattering)



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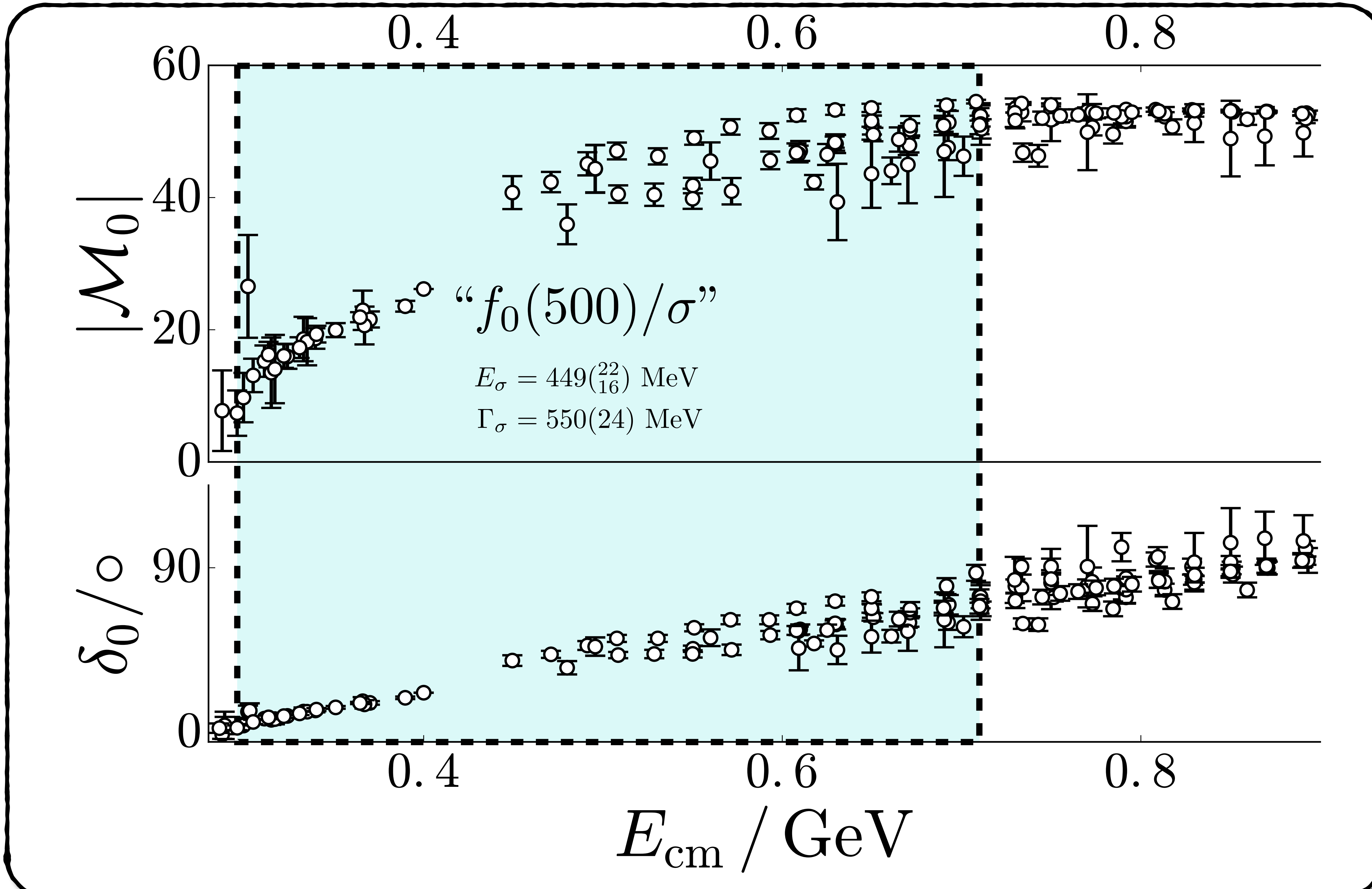
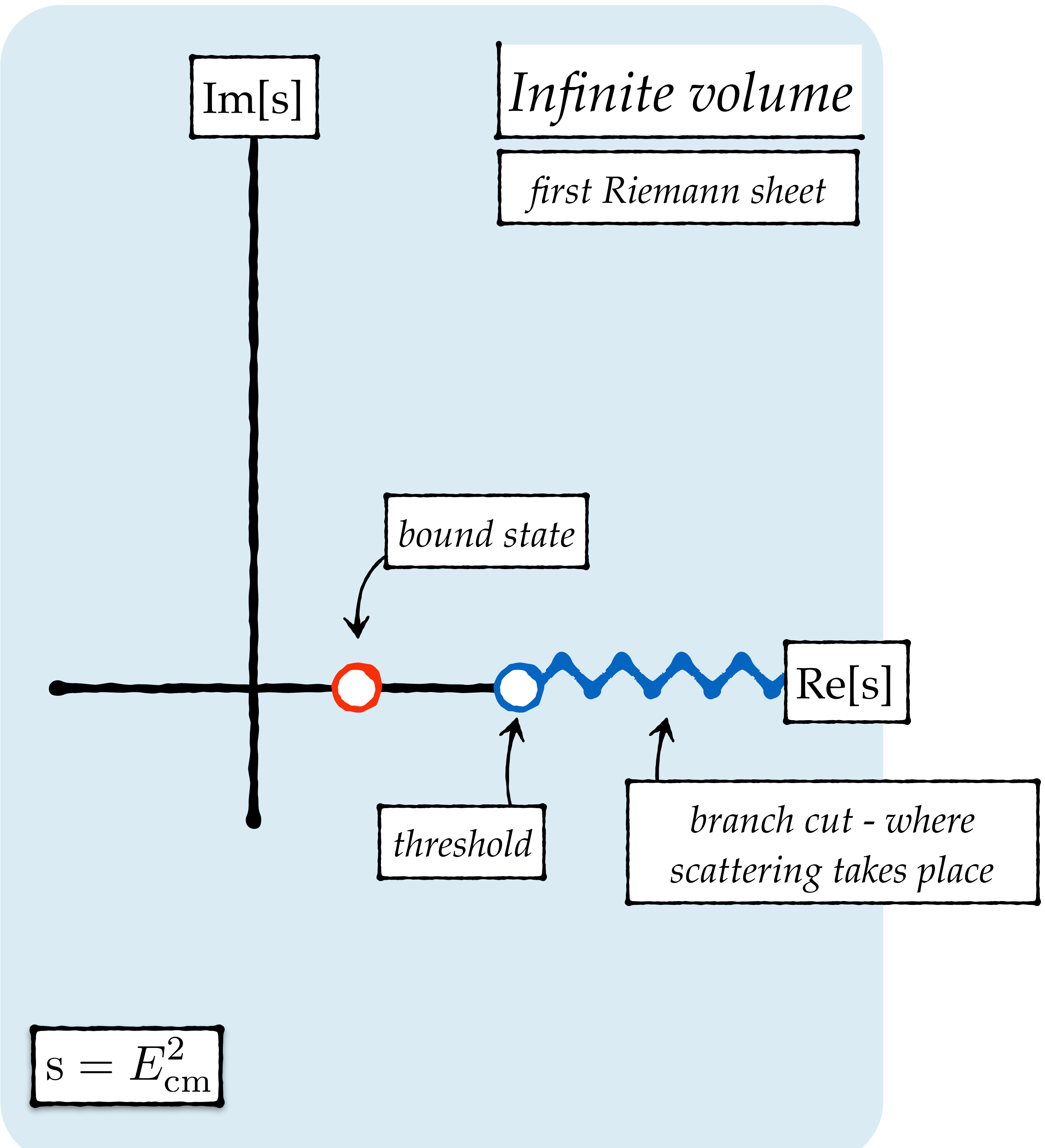
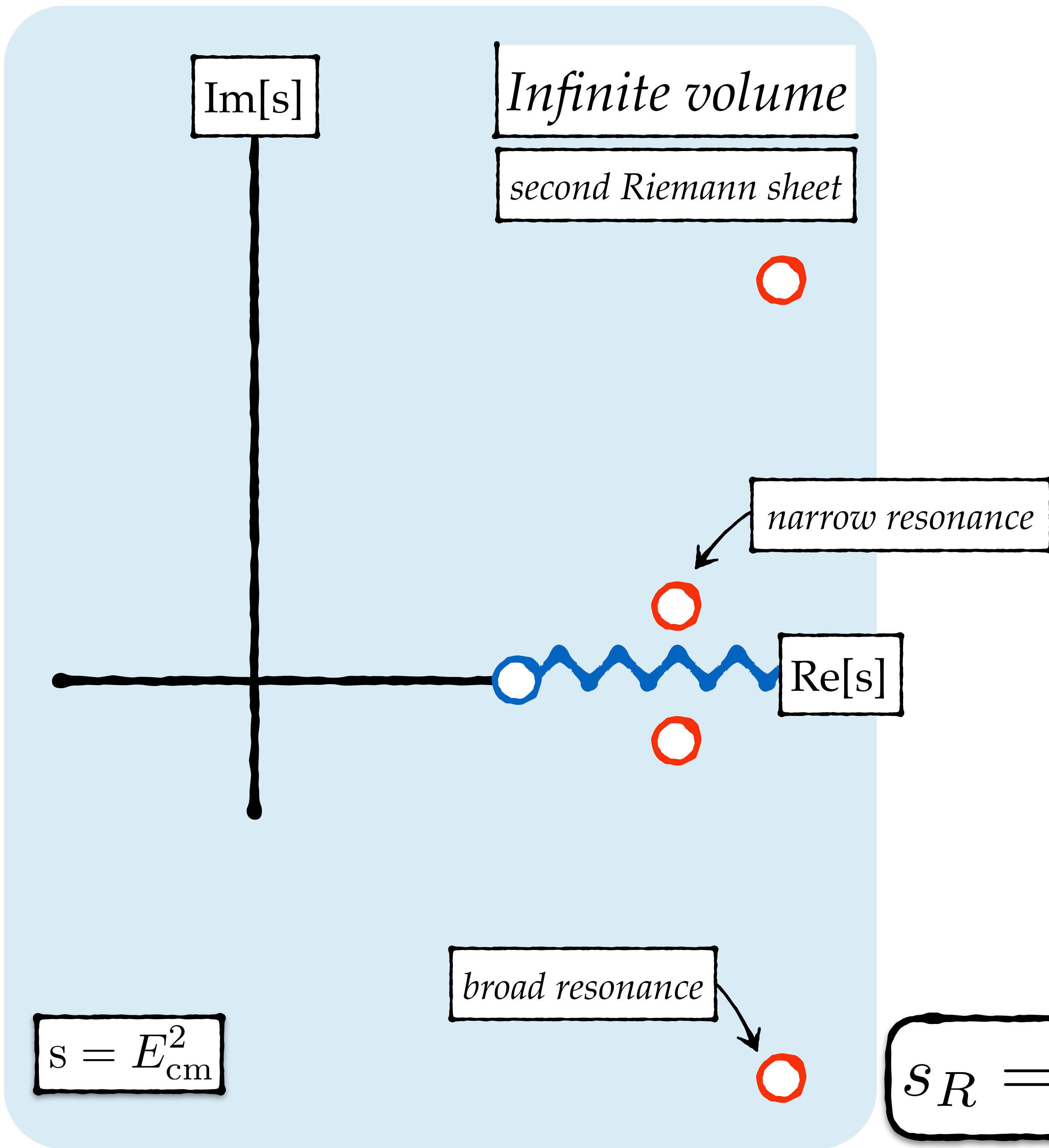


Infinite-volume spectroscopy 101

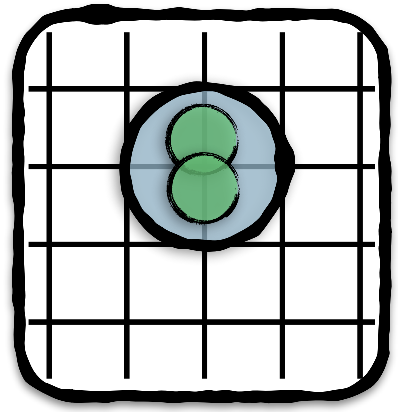


Infinite-volume spectroscopy 101

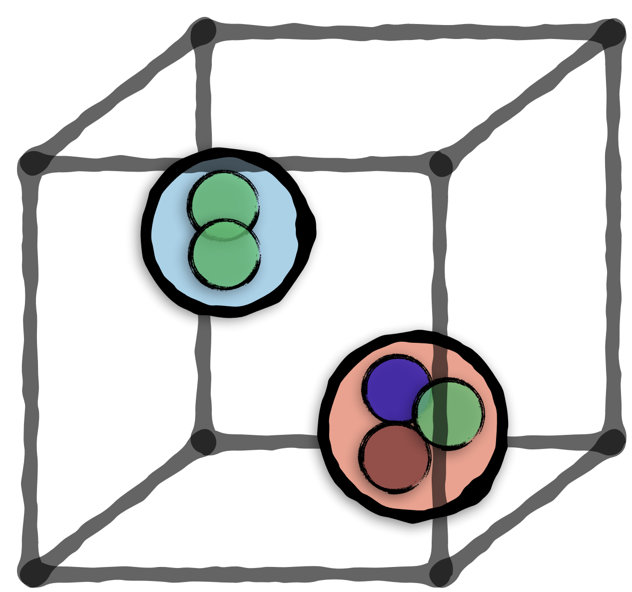


$$s_R = \left(E_R - \frac{i}{2}\Gamma_R\right)^2$$

Lattice QCD

- Lattice spacing:  $\} \, a \sim 0.03 - 0.1 \text{ fm}$

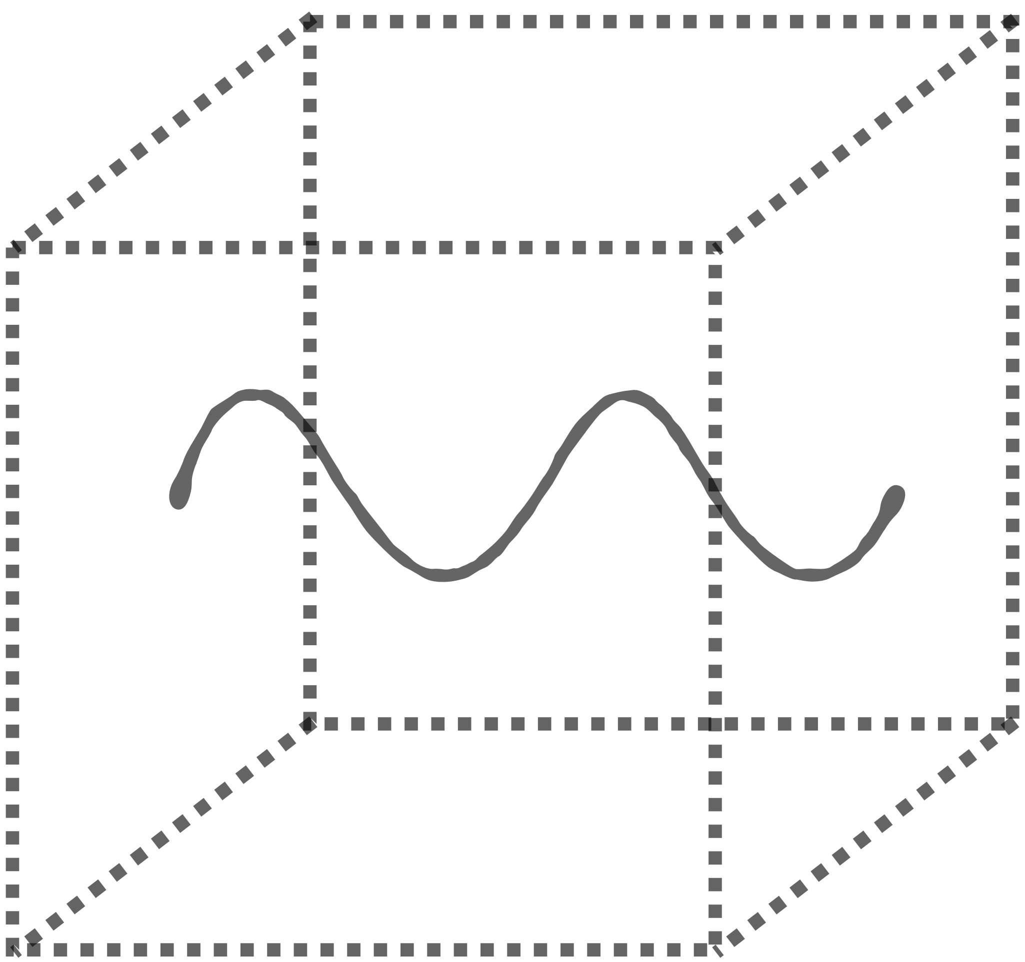
- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$

- Finite volume: 

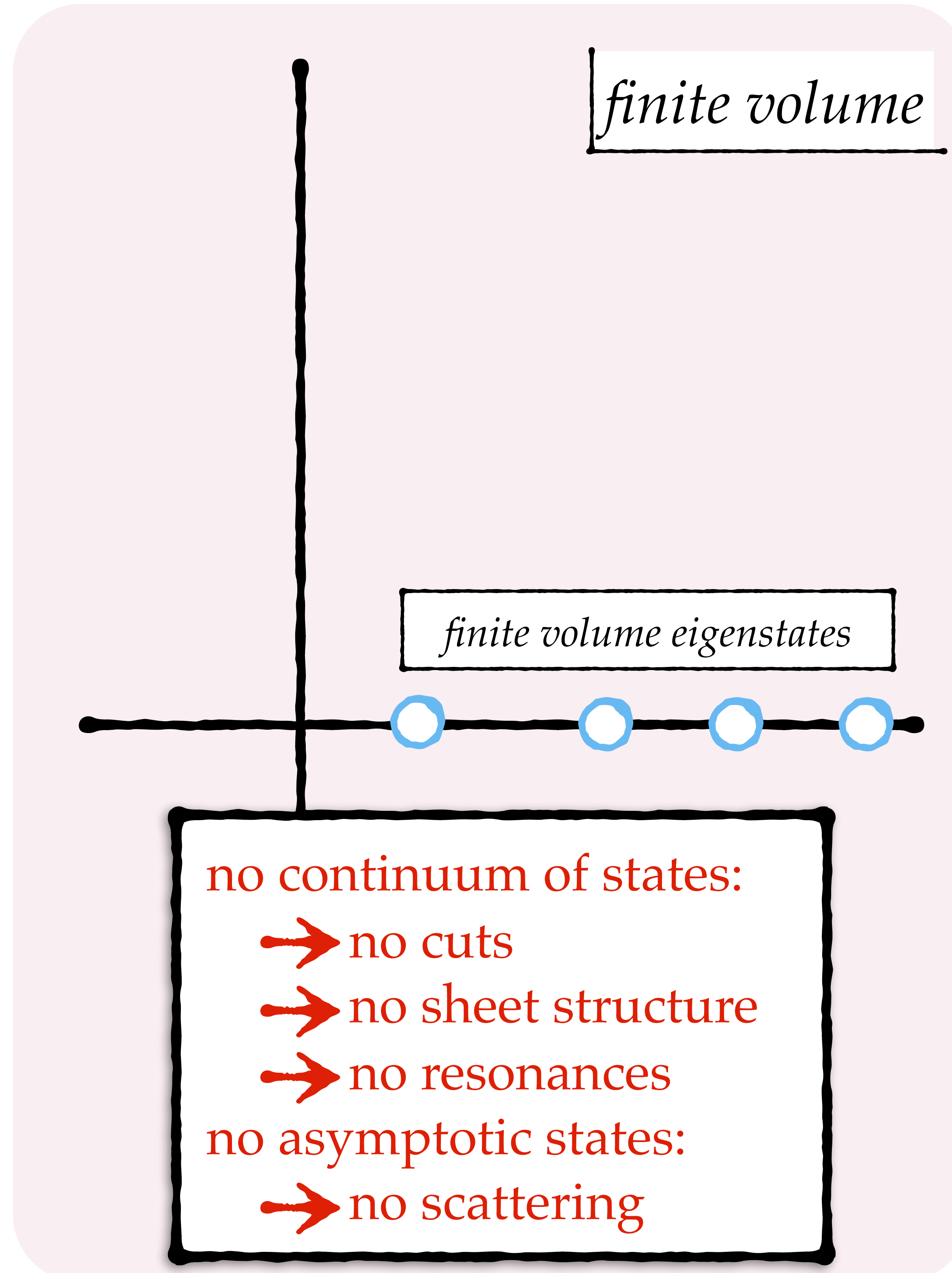
- Quark masses: $m_q \to m_q^{\text{phys.}}$

Have we '*mangled*' QCD too much?

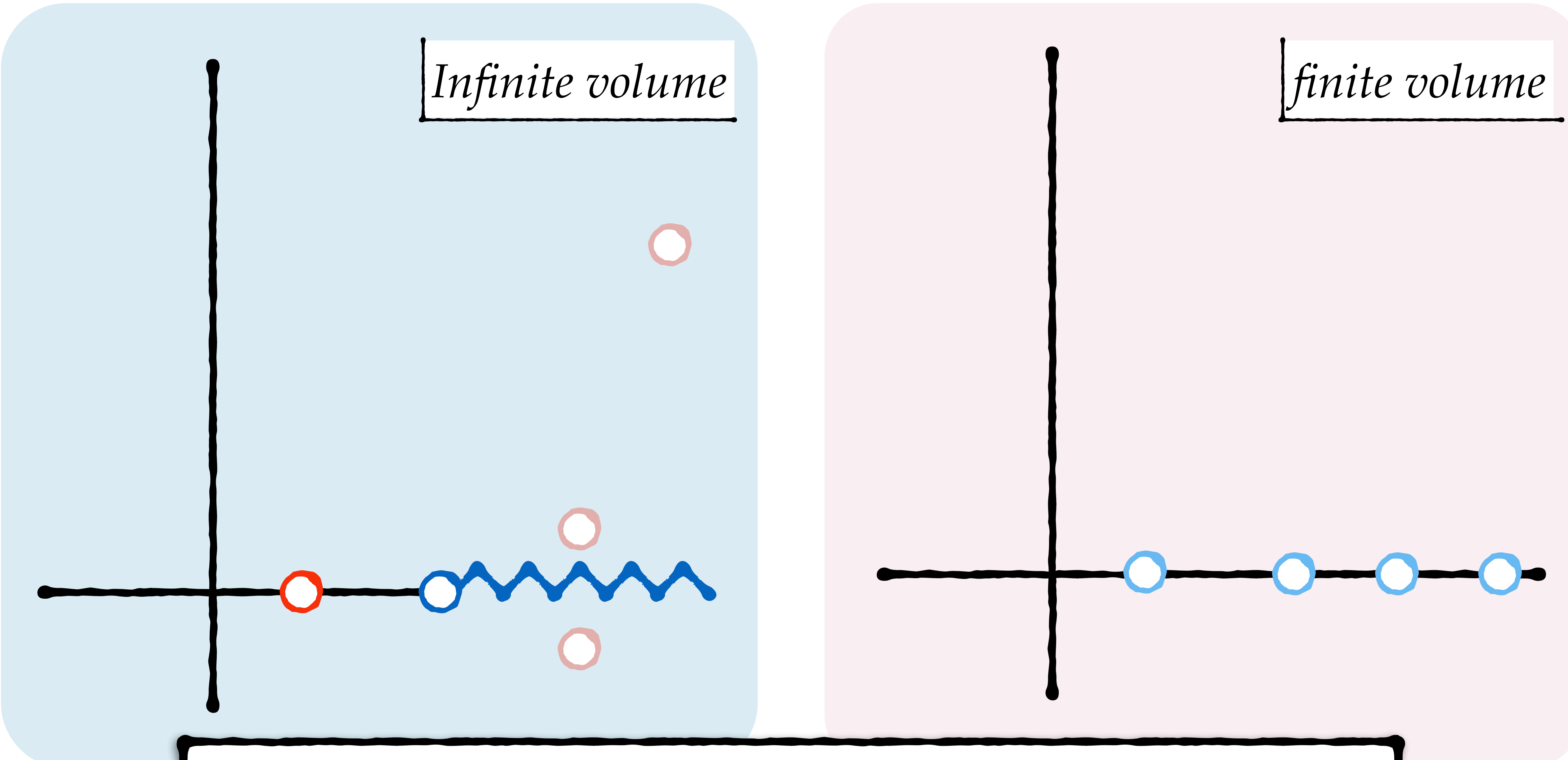
Finite vs. infinite volume spectrum



"only a discrete number of modes can exist in a finite volume"



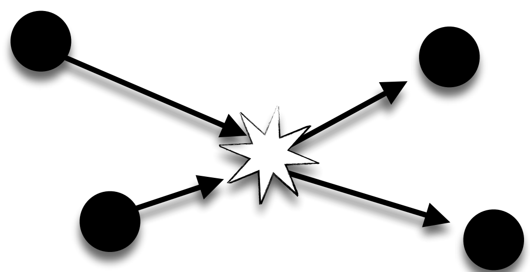
Finite vs. infinite volume spectrum



both pictures *are* QCD

the connection is perhaps not obvious since we have historically been "confined" to thinking about infinite volume physics

Experiment



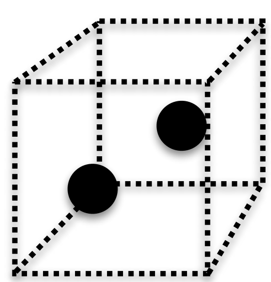
scattering data

amplitude analysis →

partial wave amplitudes

Lattice QCD

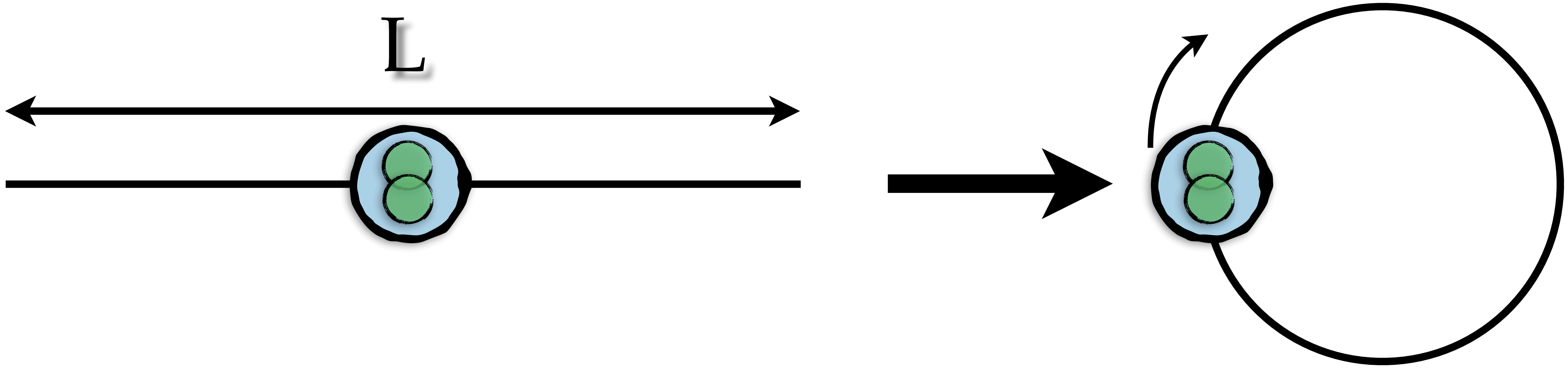
FV spectrum



Lüscher formalism →

partial wave amplitudes

Physics in a 1D-box



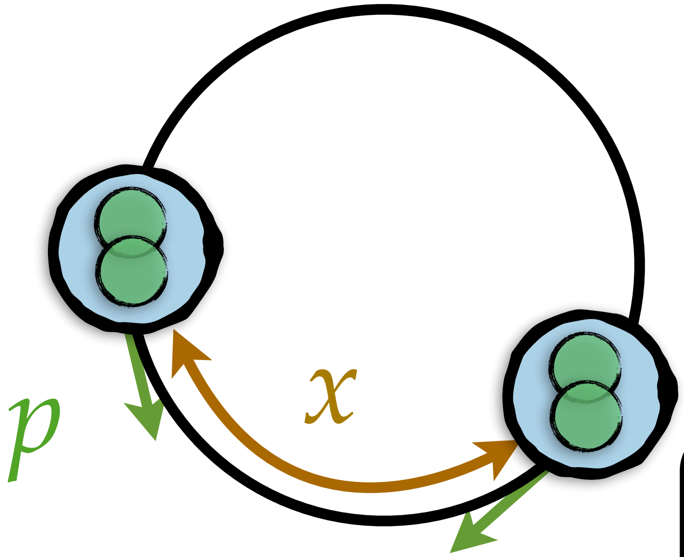
$$\phi(x) \sim e^{ipx}$$

Periodicity: $$L\, p_n = 2\pi n$$

Physics in a 1D-box

Two identical particles:

infinite volume scattering phase shift



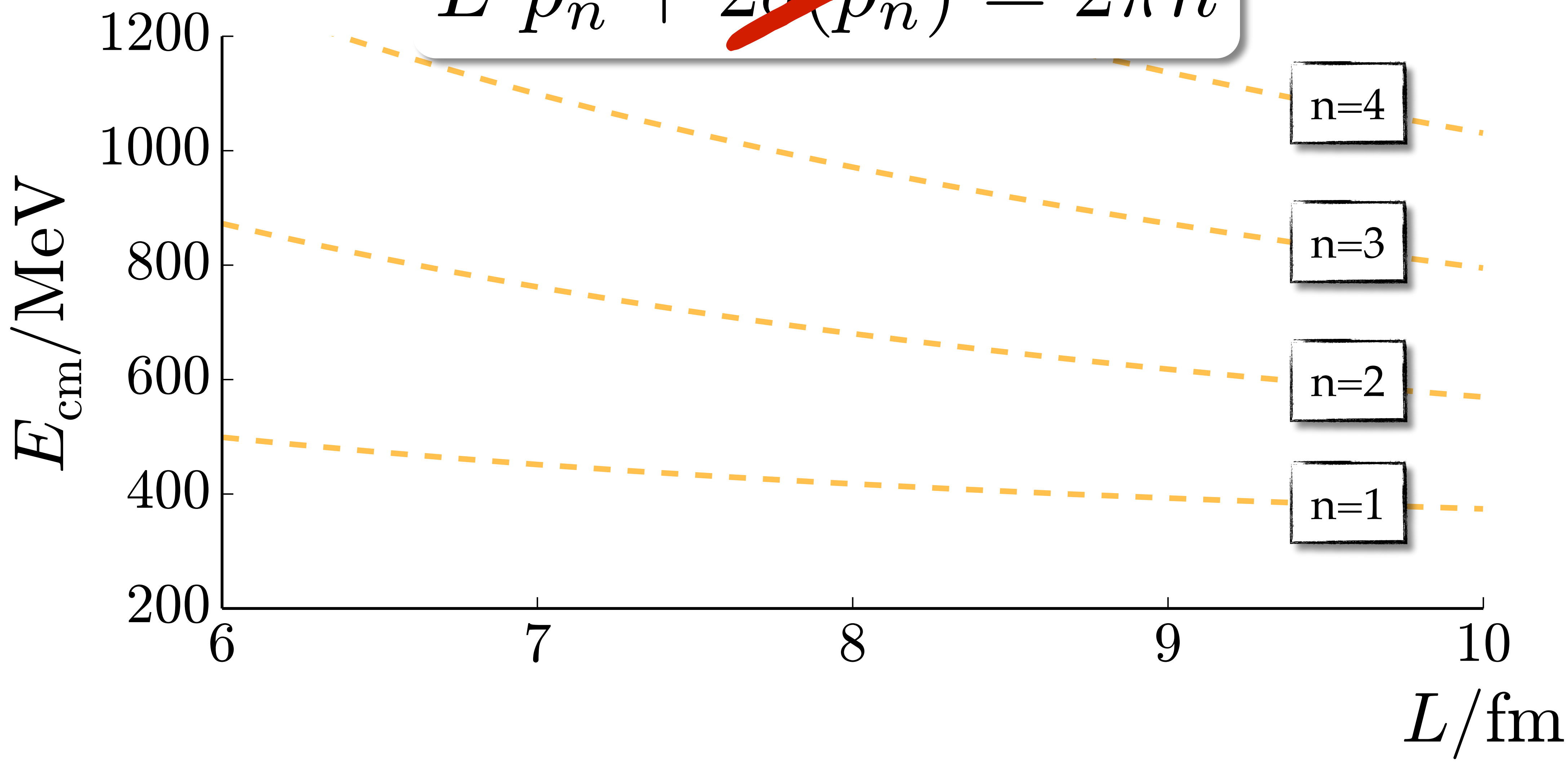
$$\psi(x) \sim \cos(p|x| + \delta(p))$$

Asymptotic wavefunction

Periodicity: $$L\ p_n + 2\delta(p_n) = 2\pi n$$

Physics in a 1D-box

$$L\, p_n + 2\delta(p_n) = 2\pi n$$



Physics in a 1D-box

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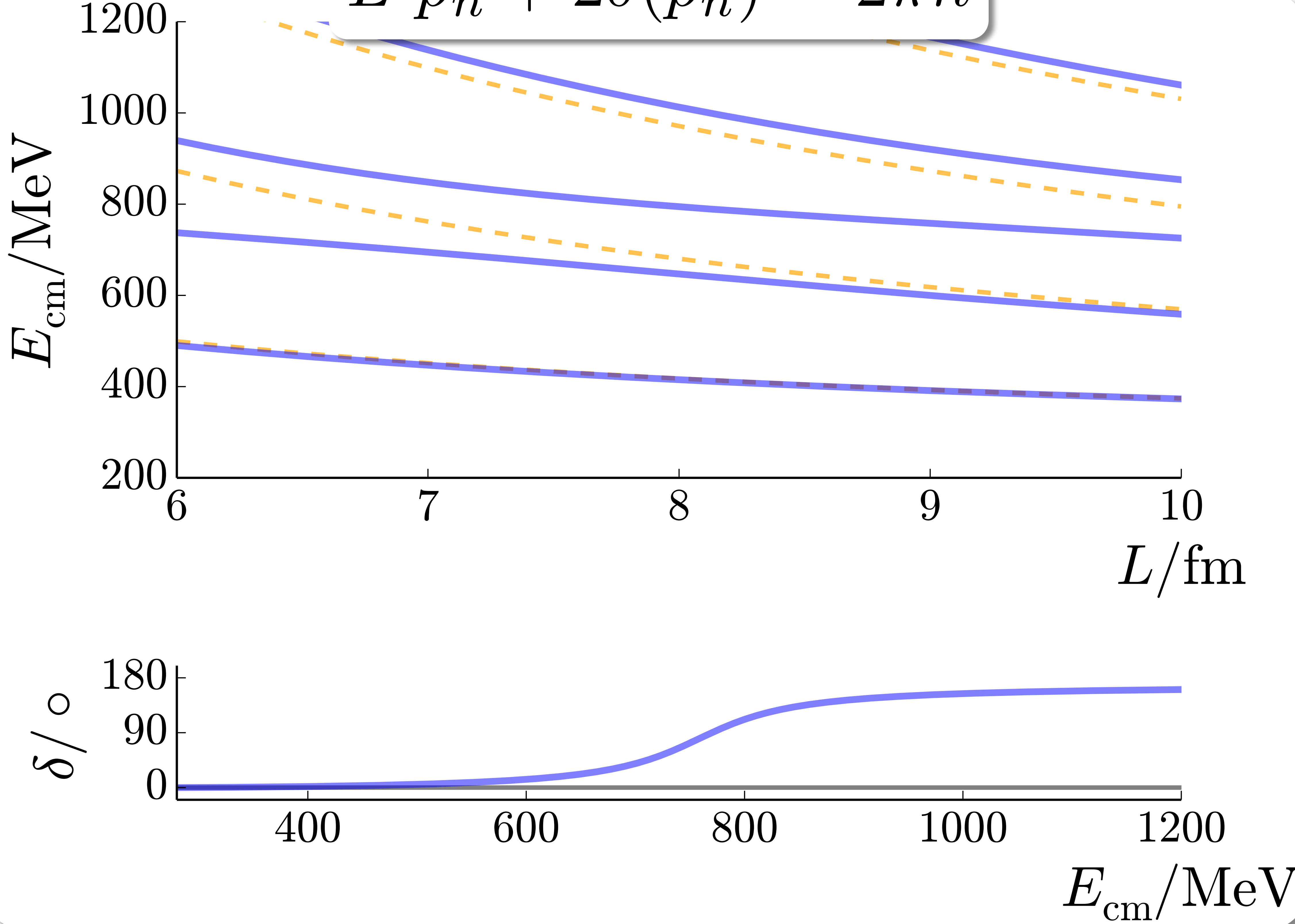

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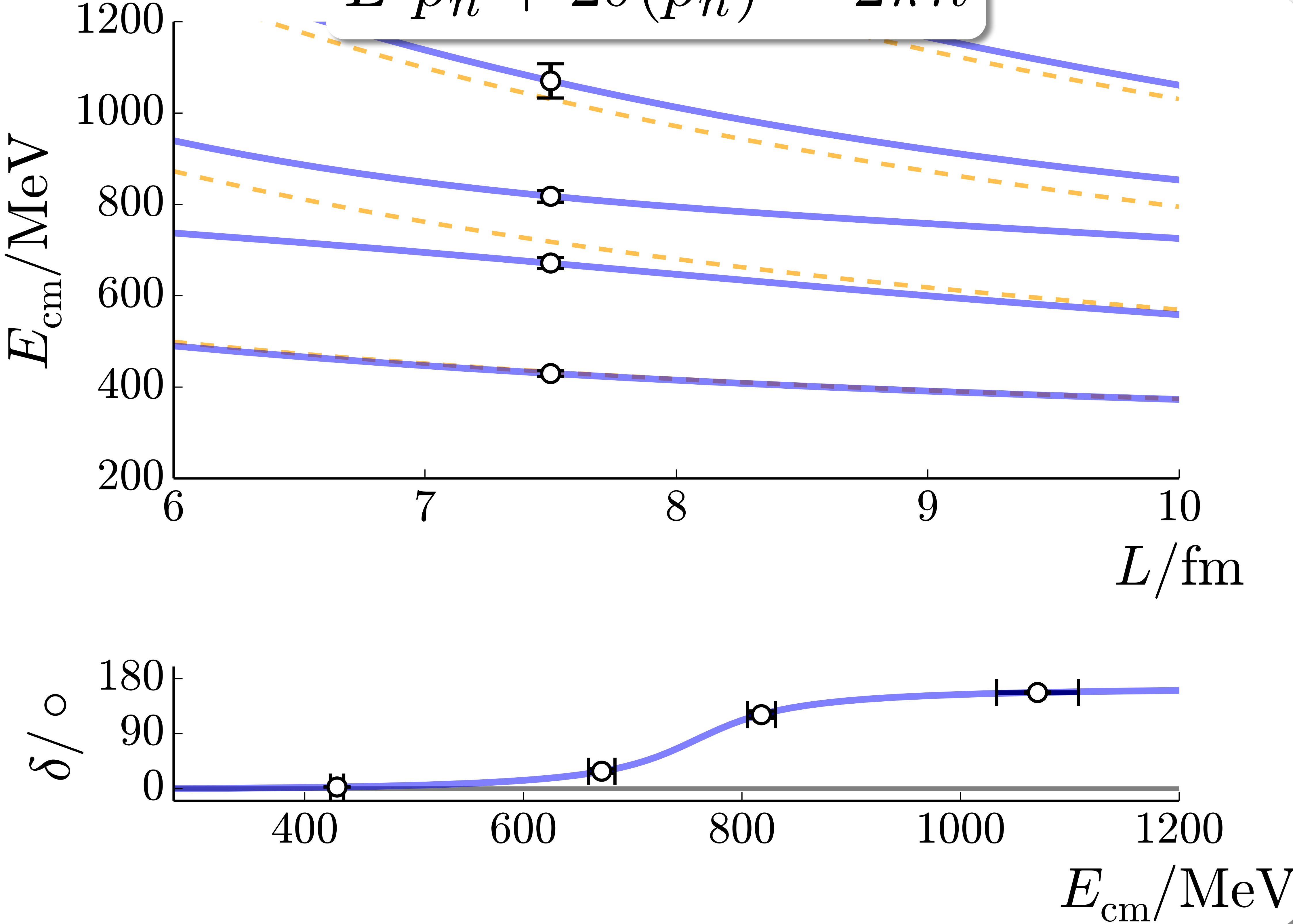

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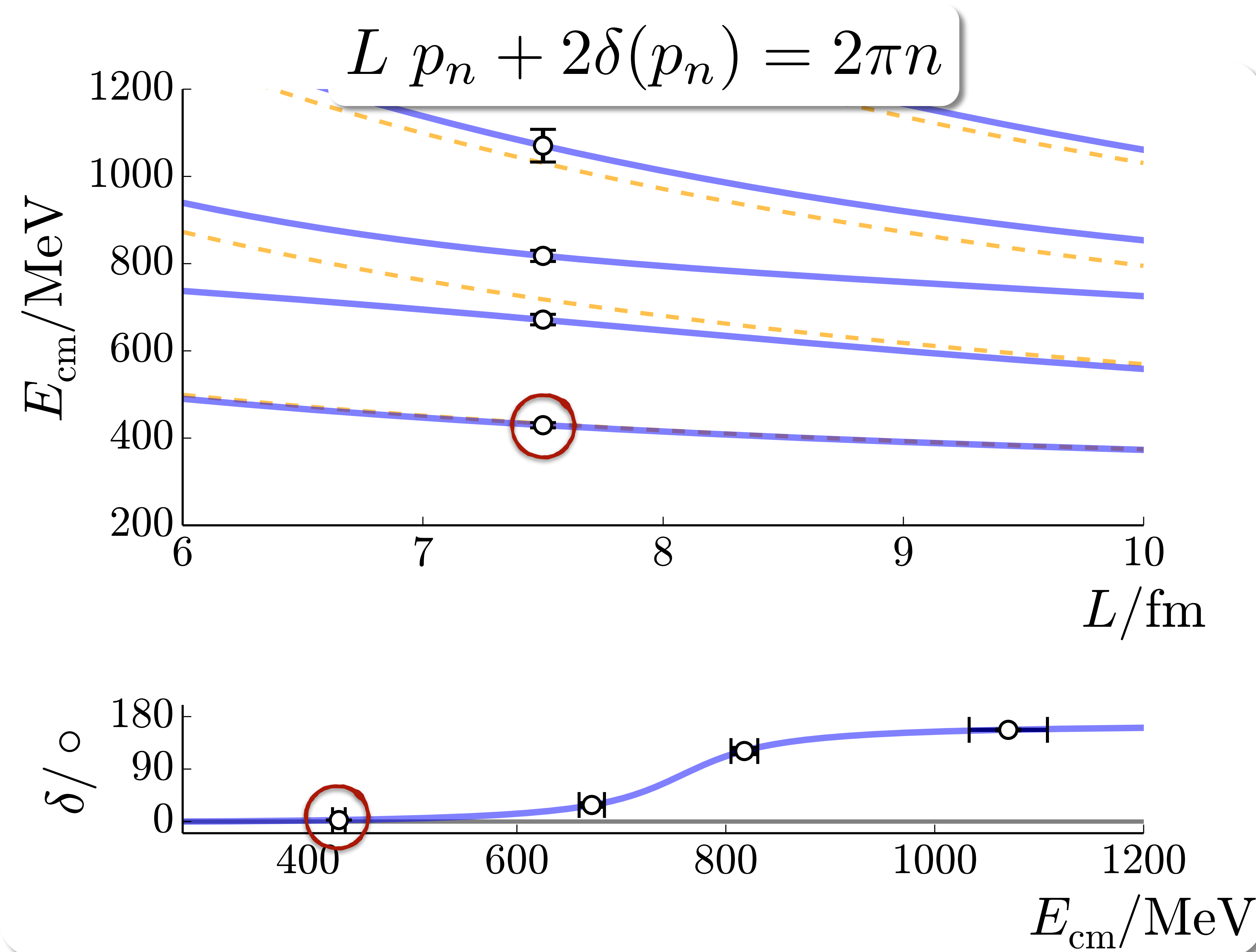

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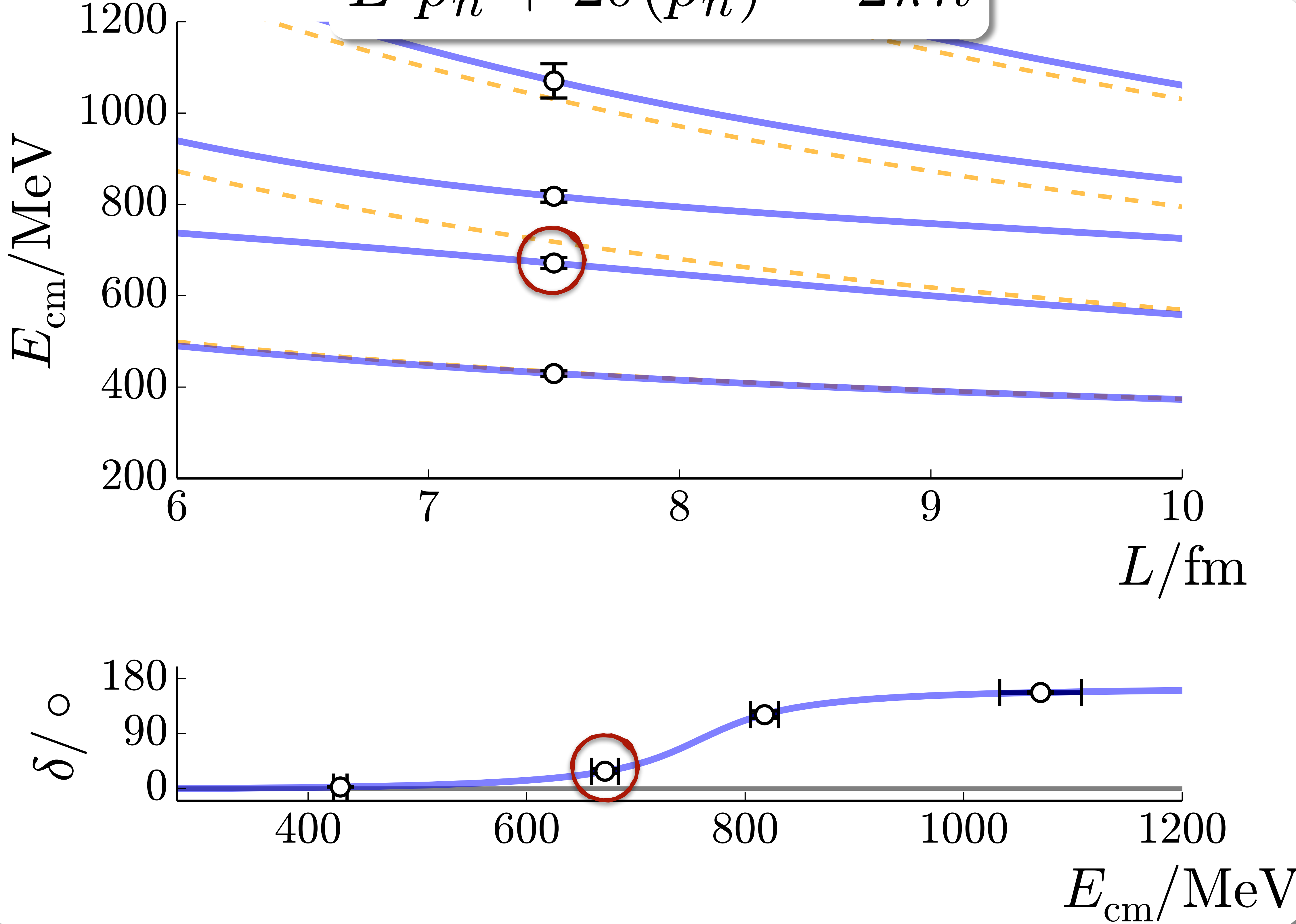

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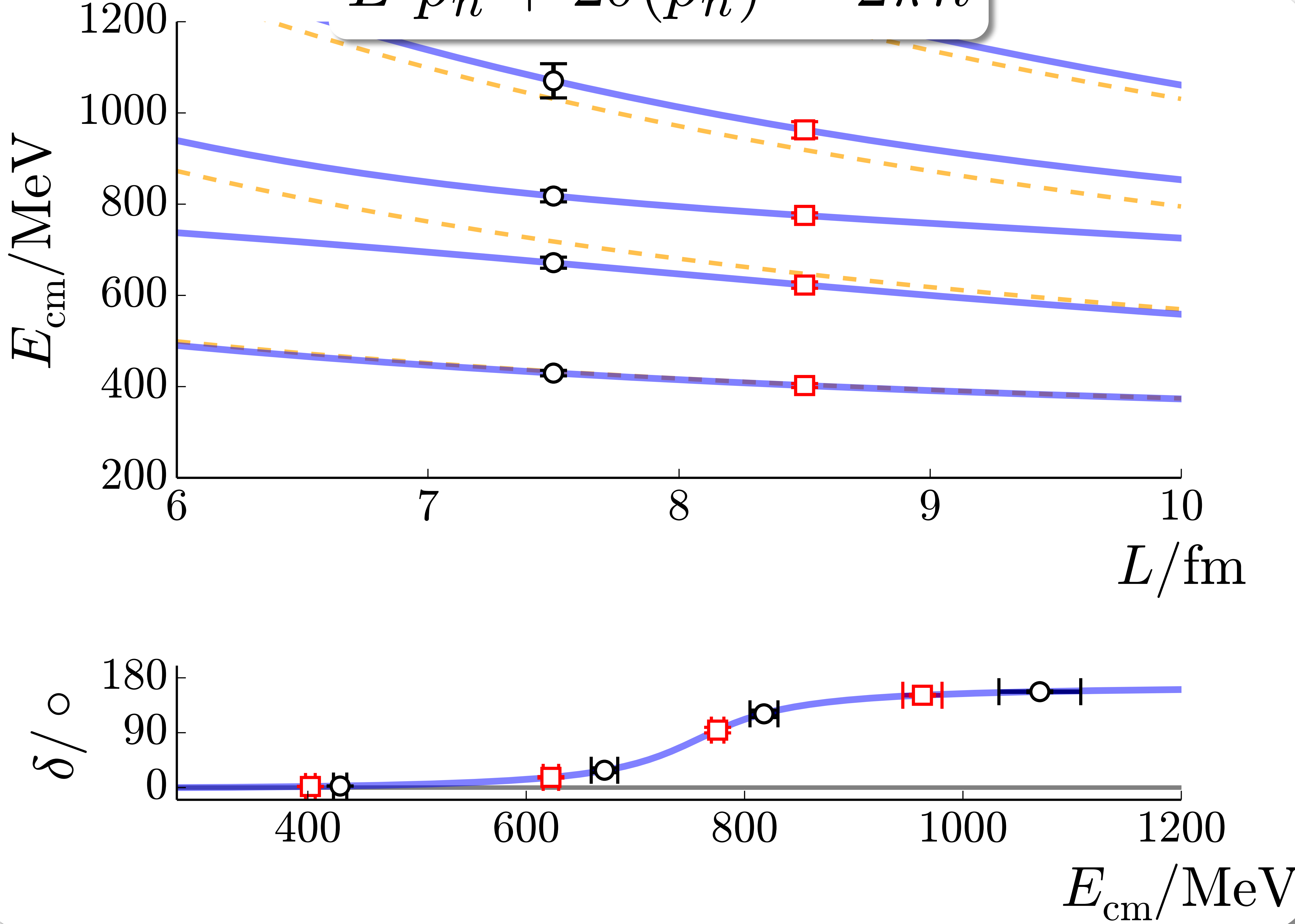
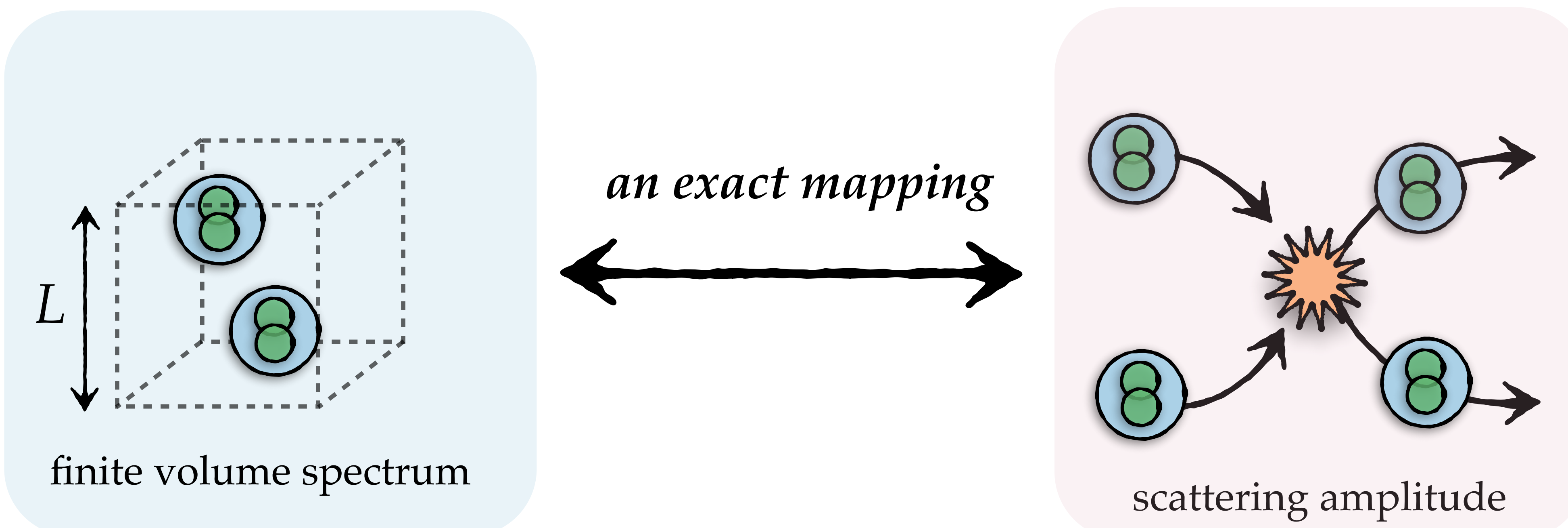


Lüscher formalism

spectrum satisfy: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$



E_L = finite volume spectrum
L = finite volume
F = known function

$\mathcal{M}$ = scattering amplitude

$$\mathcal{M} = \frac{8\pi E_{\text{cm}}}{p} \frac{1}{\cot\delta - i}$$

Lüscher formalism

spectrum satisfy: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Bernard, Lage, Meißner & Rusetsky (2008) [Nπ systems]
- Gockeler, Horsley, Lage, Meißner, Rakow, Rusetsky, Schierholz, & Zanotti (2012) [Nπ systems]
- RB, Davoudi, Luu & Savage (2013) [generic spinning systems]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- RB (2014) / RB & Hansen (2015) [moving inelastic spinning particles]

Extracting the spectrum

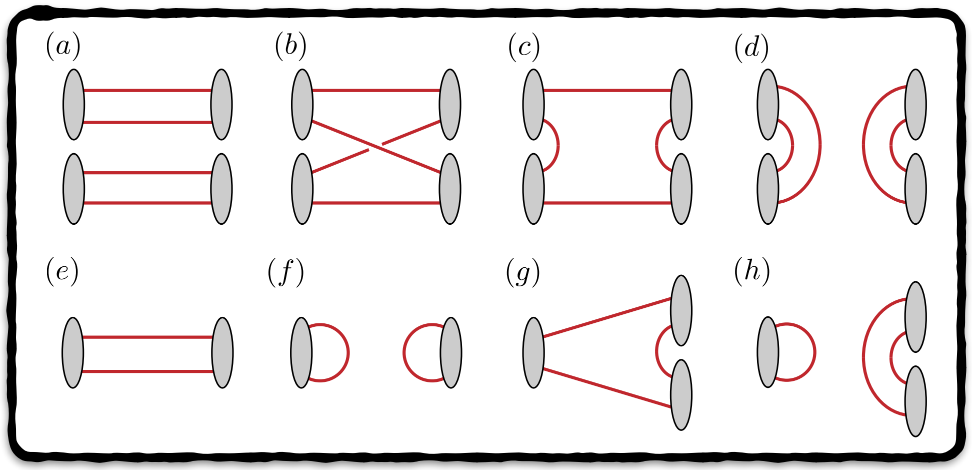
Two-point correlation functions:

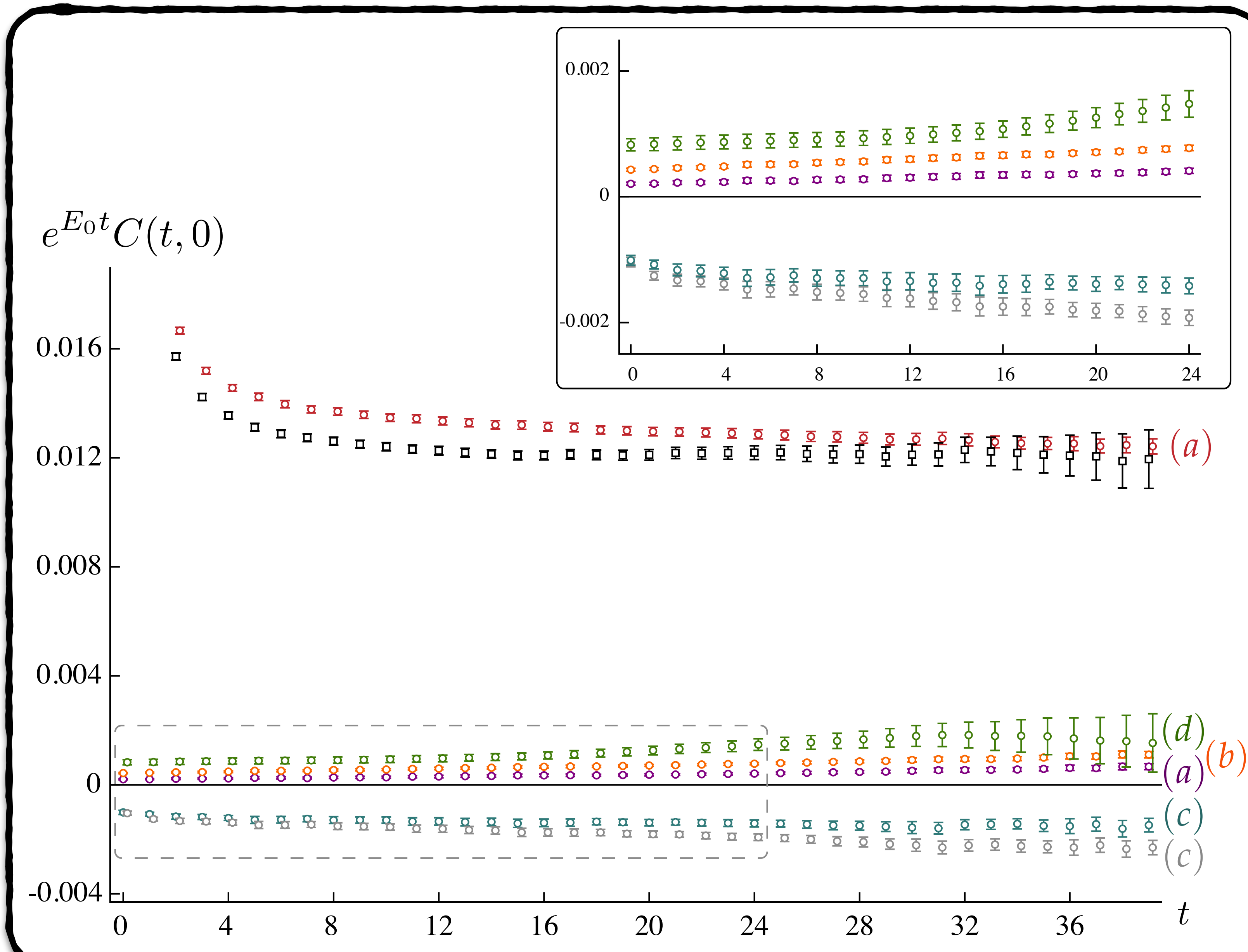
$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

- Evaluate **all** Wick contraction - [distillation - Peardon, *et al.* (Hadron Spectrum, 2009)]

e.g. $\pi_{[000]}\pi_{[110]}$

$m_\pi = 236$ MeV





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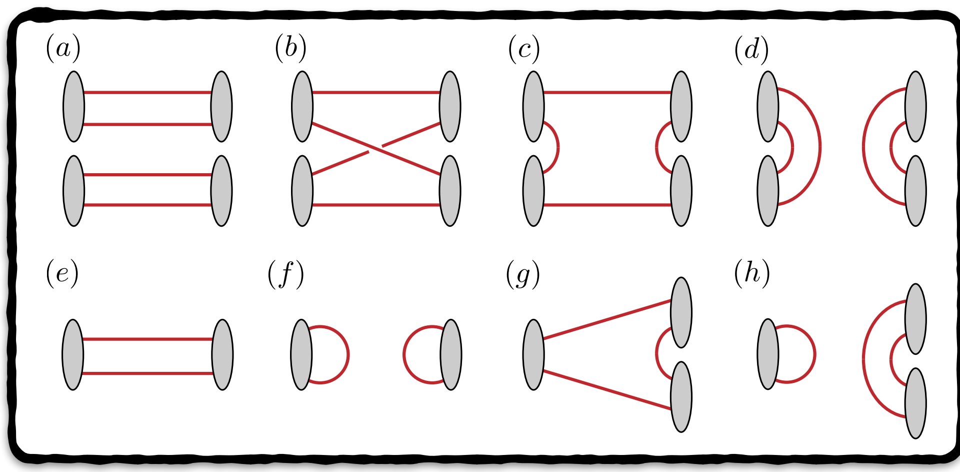

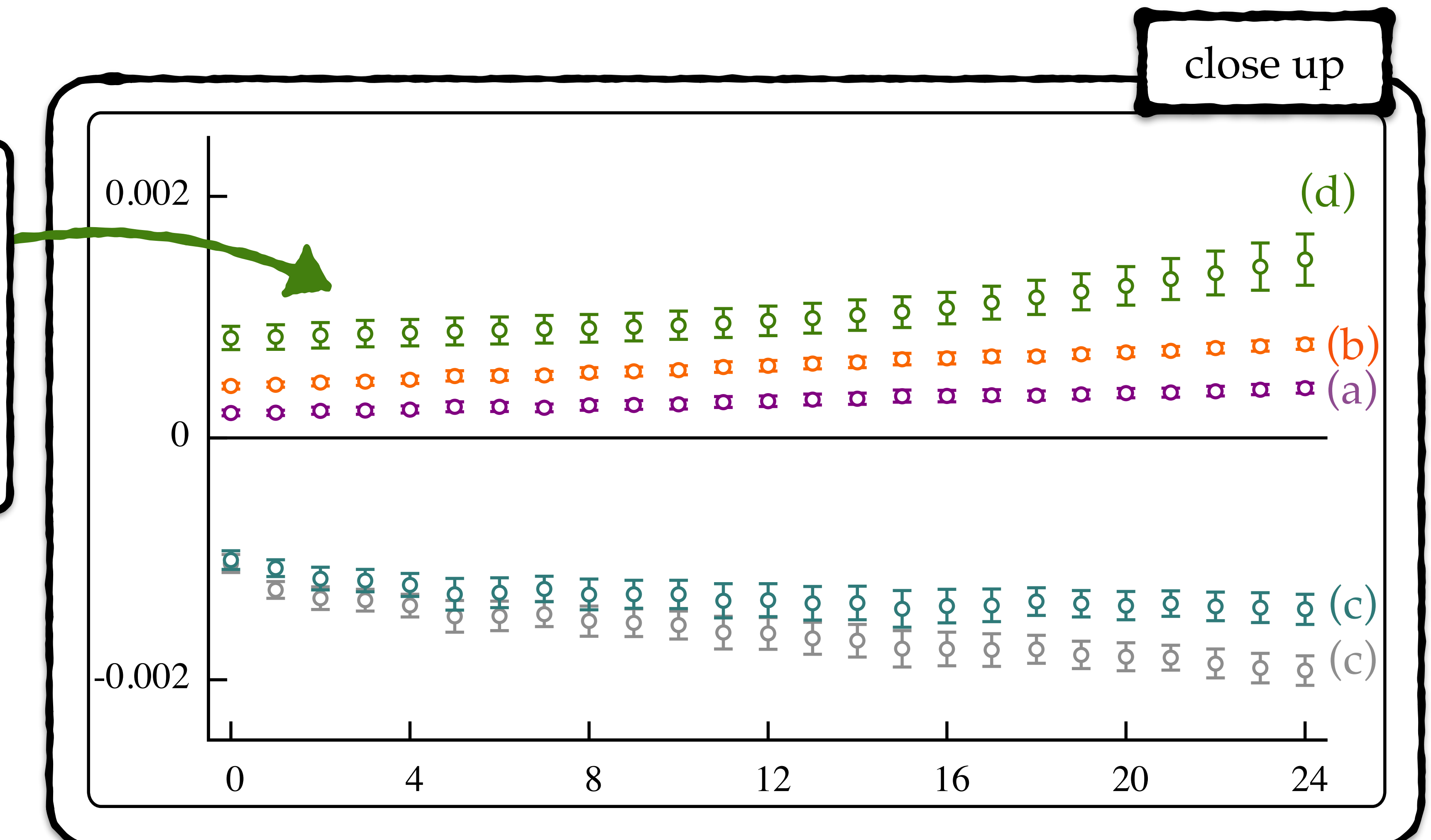

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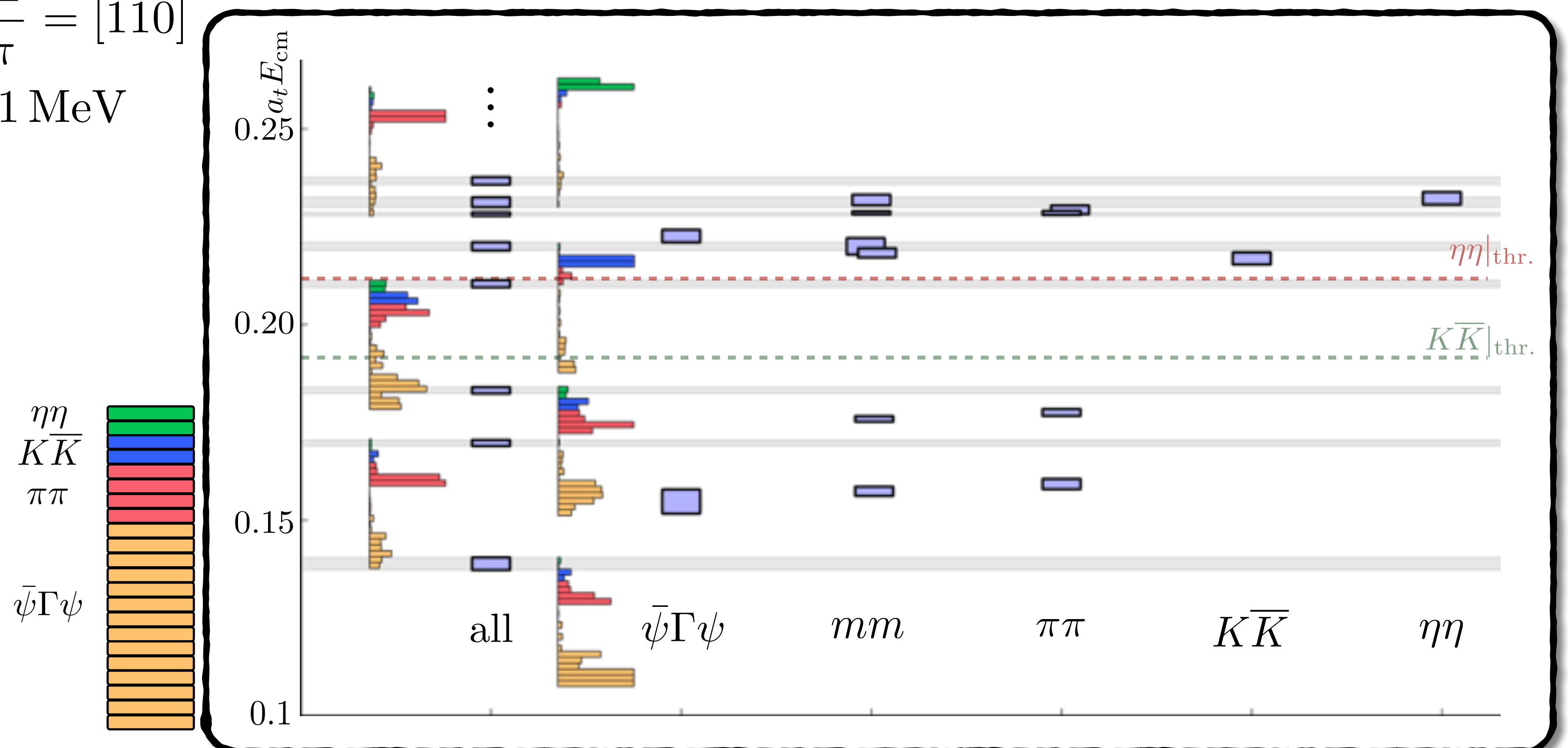
- Evaluate **all** Wick contraction - [distillation - Peardon, *et al.* (Hadron Spectrum, 2009)]
- Use a large basis of operators with the same quantum numbers
- 'Diagonalize' correlation function *variationally*

e.g. $\vec{d} = \frac{\vec{P} L}{2\pi} = [110]$

$m_\pi = 391\,\text{MeV}$

$L/a_s = 24$



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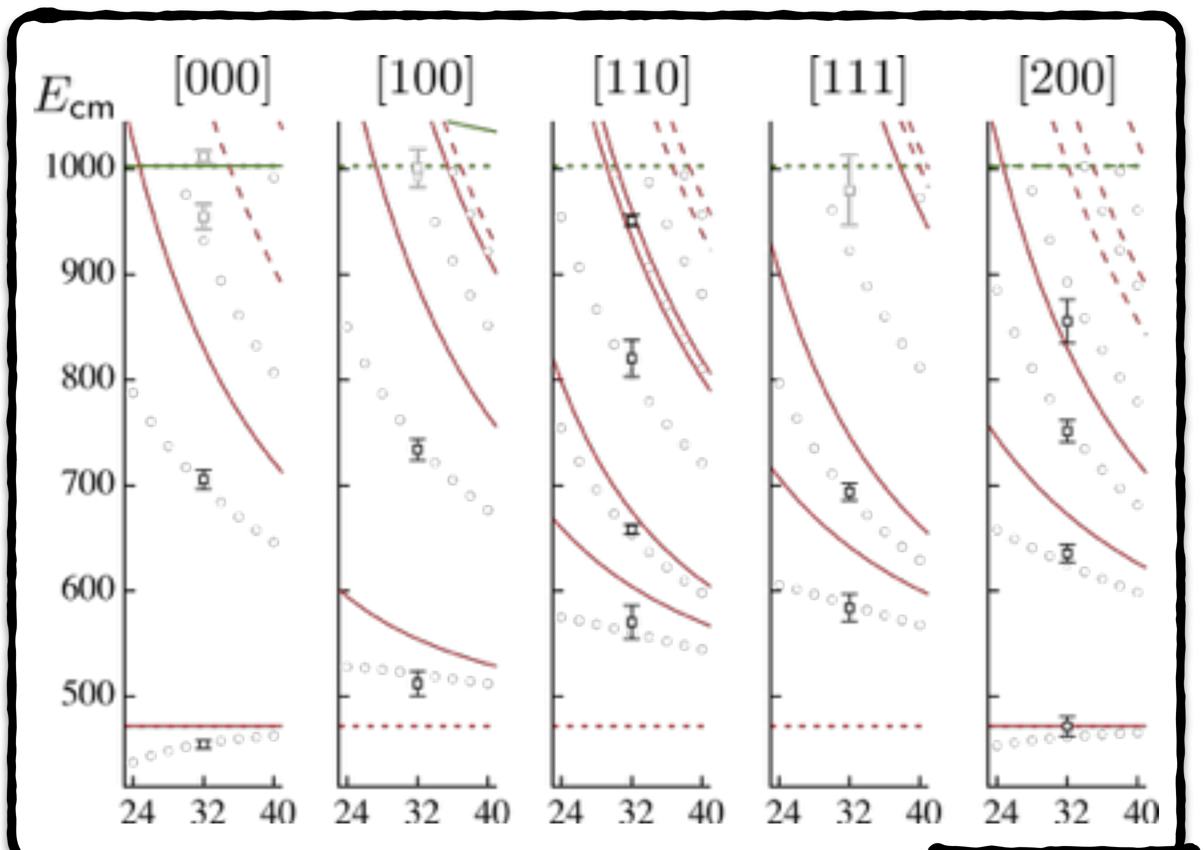
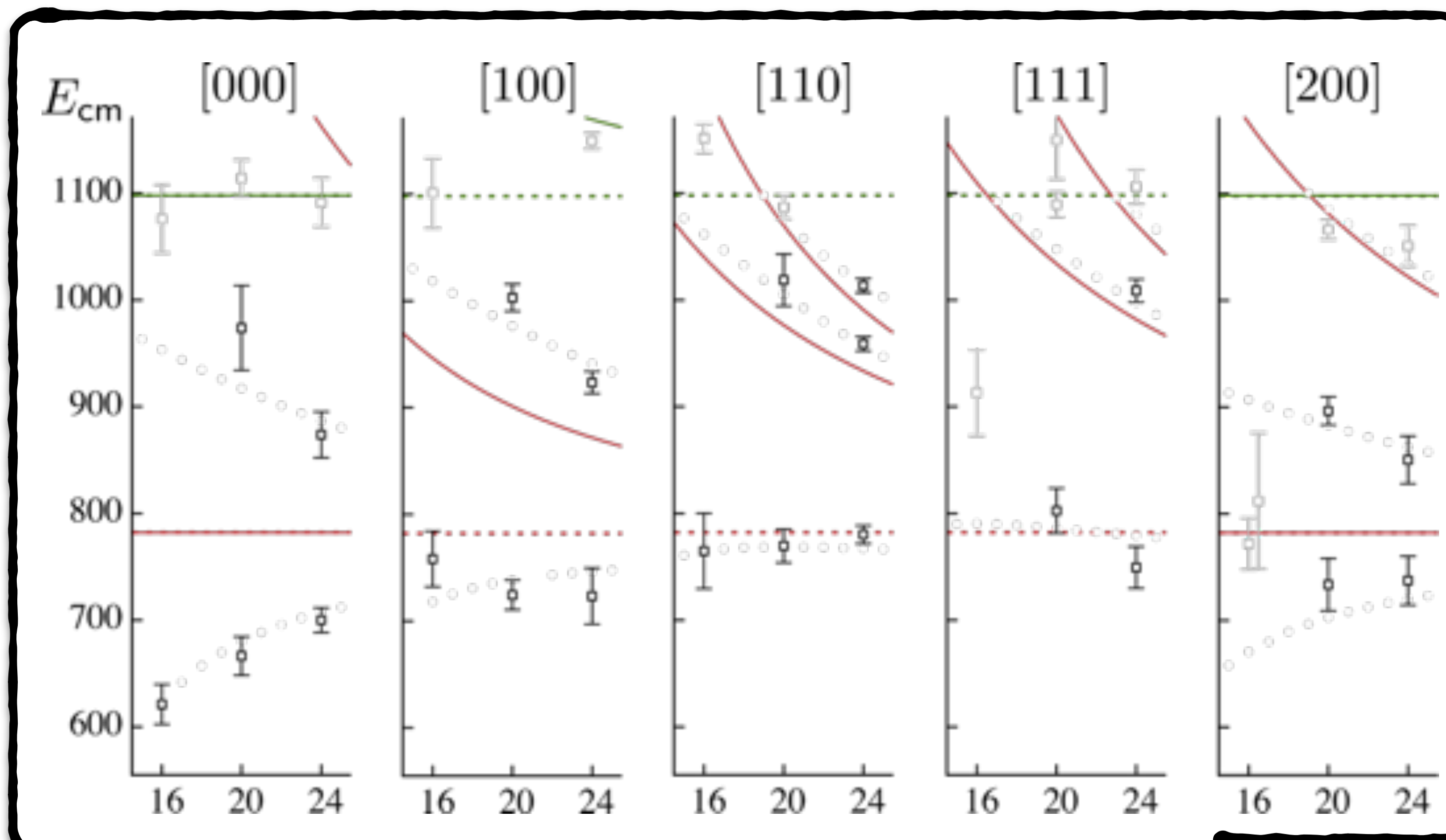
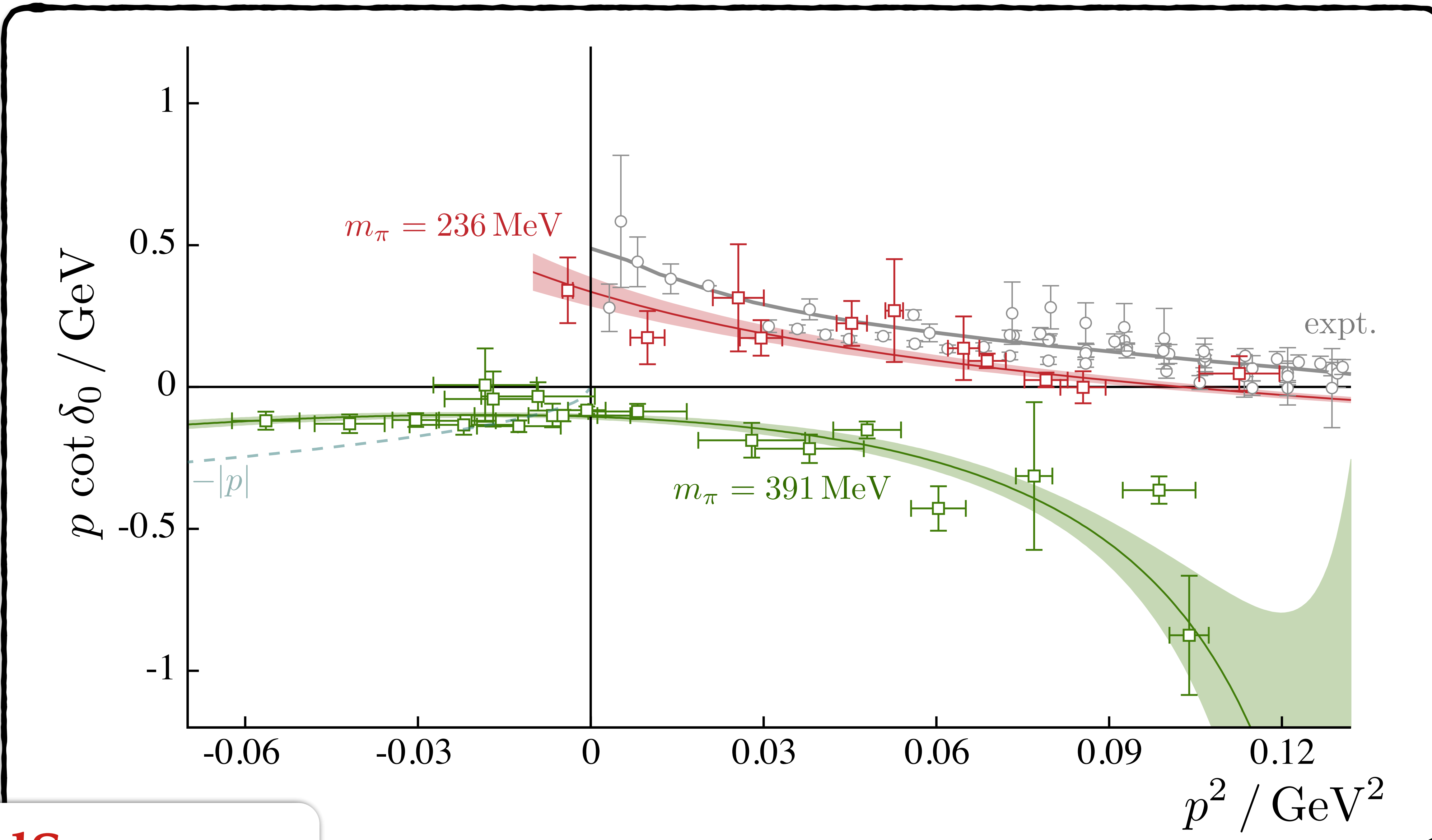


m$_\pi$=236 MeV



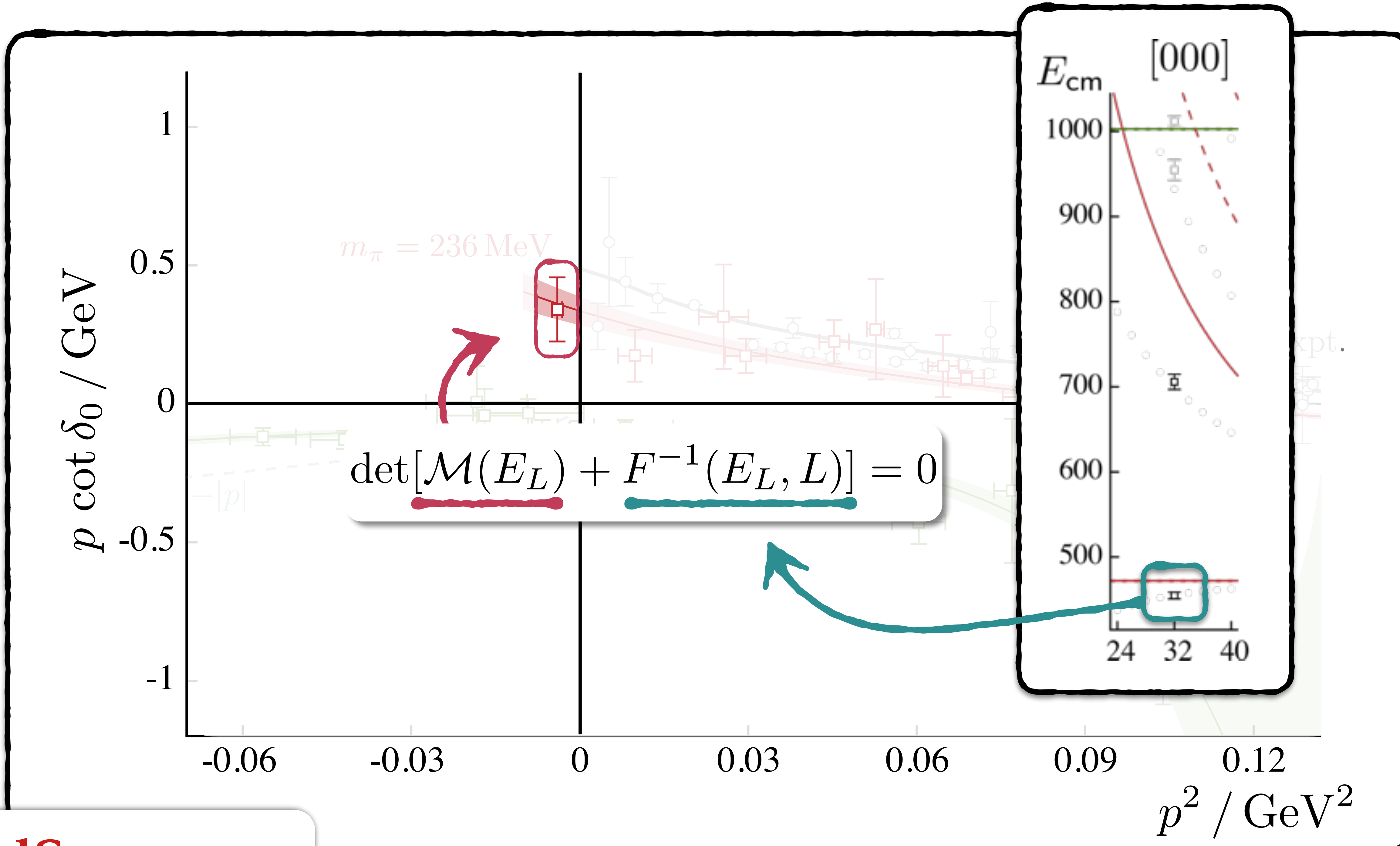
m$_\pi$=391 MeV

Isoscalar $\pi\pi$ scattering



HadSpec Collaboration

Isoscalar $\pi\pi$ scattering

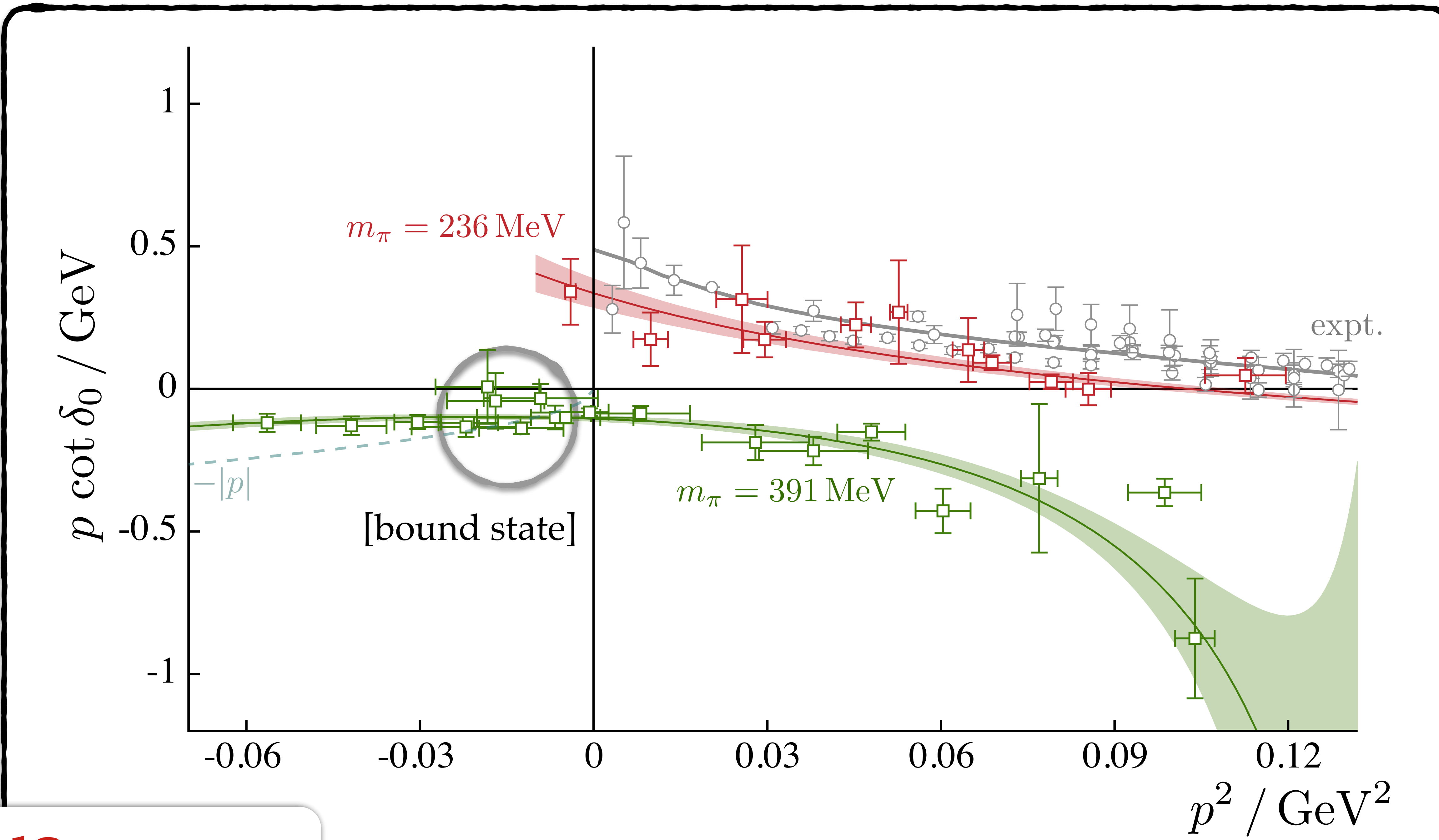


$$\det[\mathcal{M}(E_L) + F^{-1}(E_L, L)] = 0$$

HadSpec Collaboration

$$\mathcal{M}_0 = \frac{16\pi E_{\mathrm{cm}}}{p\cot\delta_0 - ip}$$

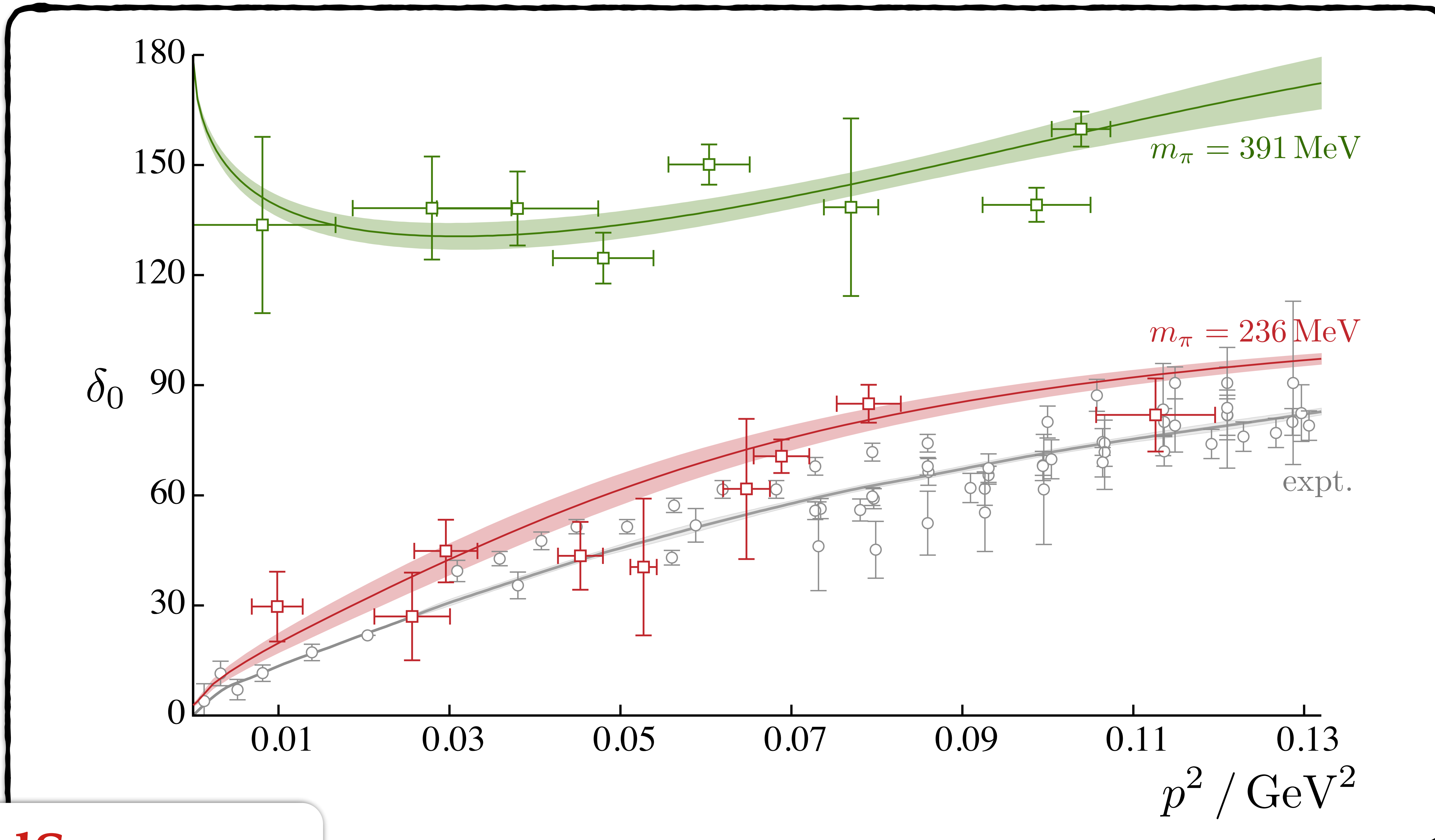
Isoscalar $\pi\pi$ scattering



HadSpec Collaboration

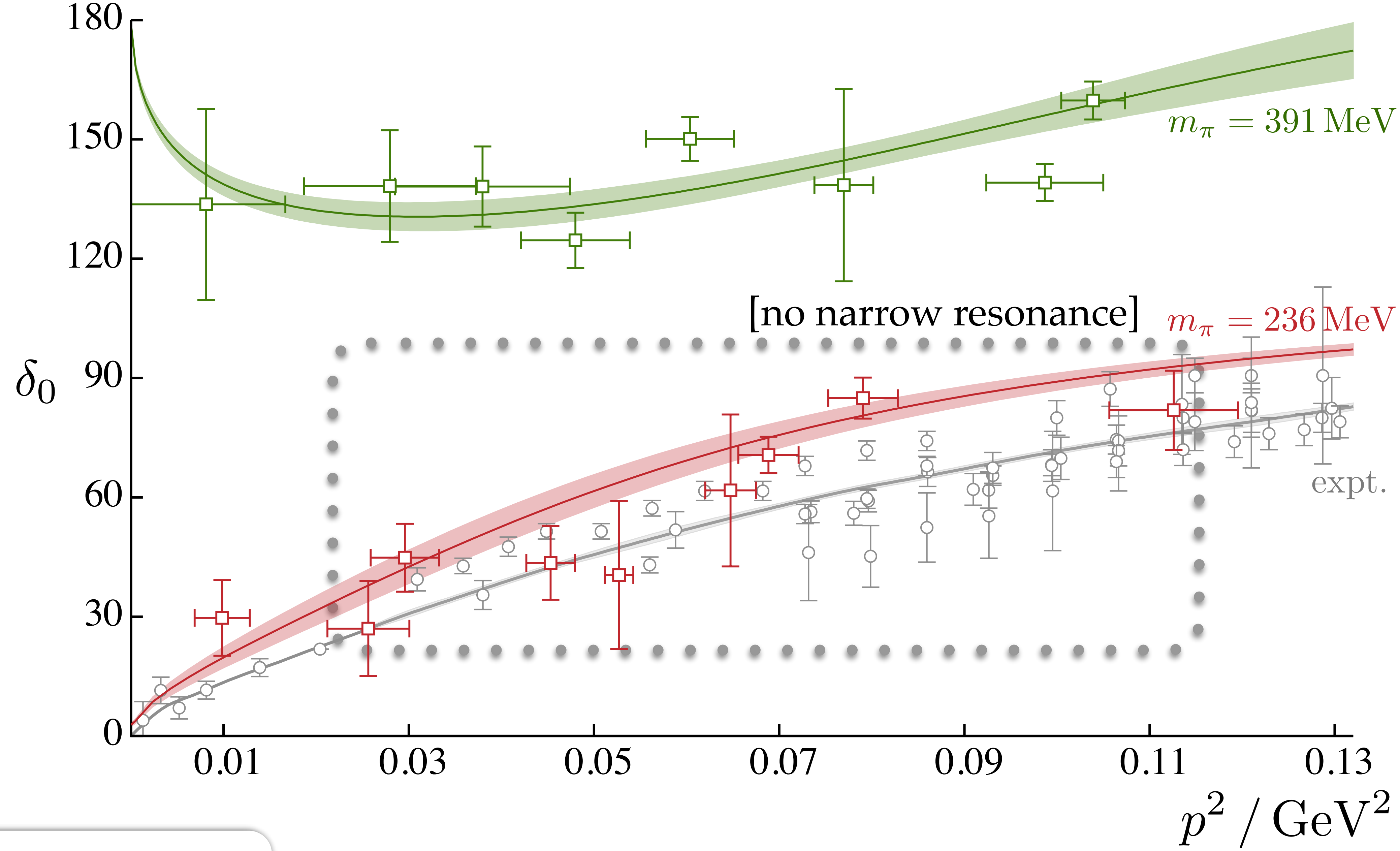
$$\mathcal{M} \sim \frac{1}{p \cot \delta_0 - ip} \to \frac{1}{p \cot \delta_0 + |p|}$$

Isoscalar $\pi\pi$ scattering



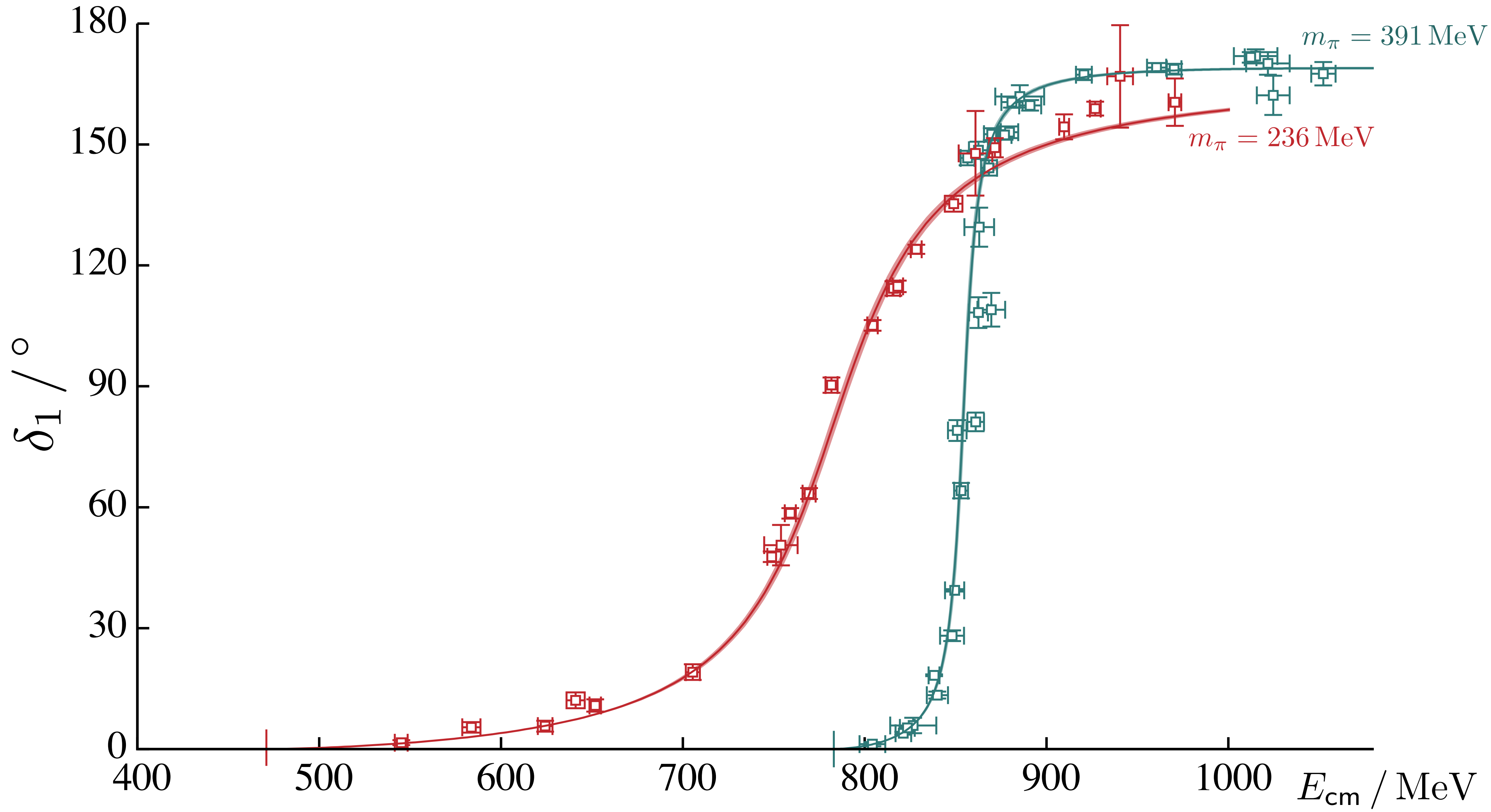
HadSpec Collaboration

Isoscalar $\pi\pi$ scattering



HadSpec Collaboration

Isovector $\pi\pi$ scattering



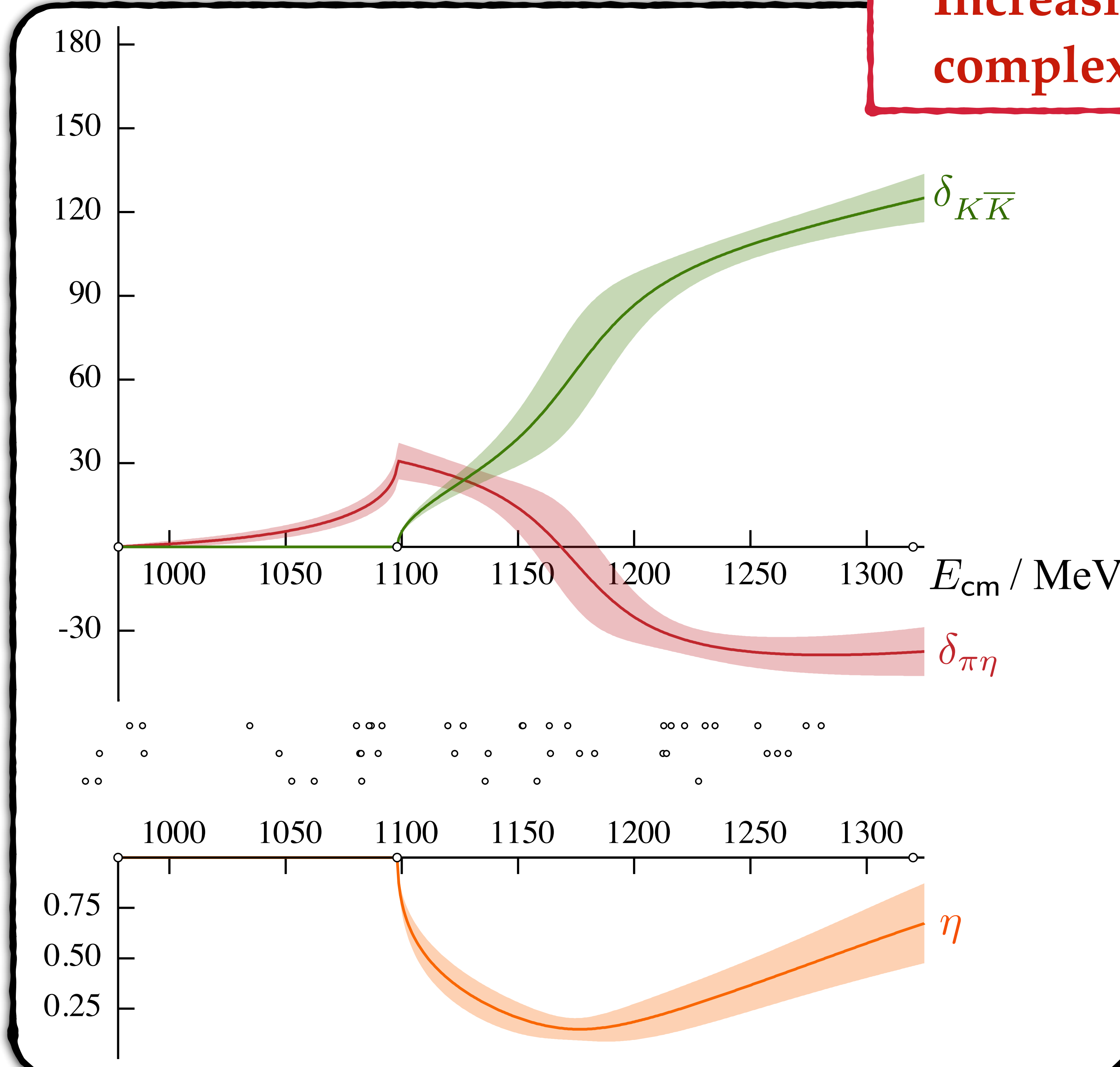
HadSpec Collaboration

Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

$\pi\eta$–$K\overline{K}$ scattering

(S-wave, I=1 channel)

Increasingly complex systems



RB

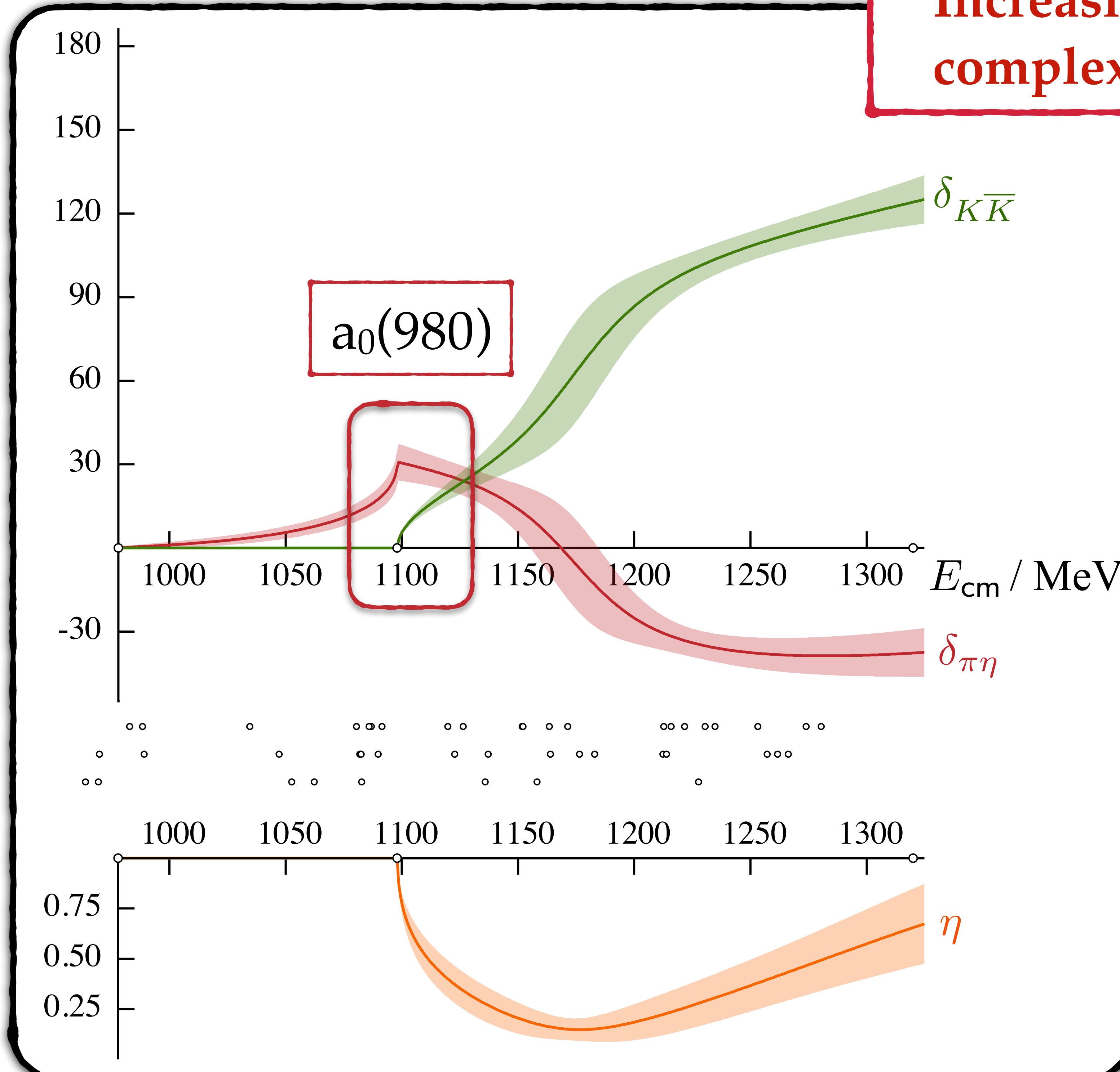
formalism: Hansen & Sharpe / RB & Davoudi (2012)

Dudek, Edwards & Wilson (2016)

$\pi\eta$–$K\overline{K}$ scattering

(S-wave, I=1 channel)

Increasingly complex systems



RB

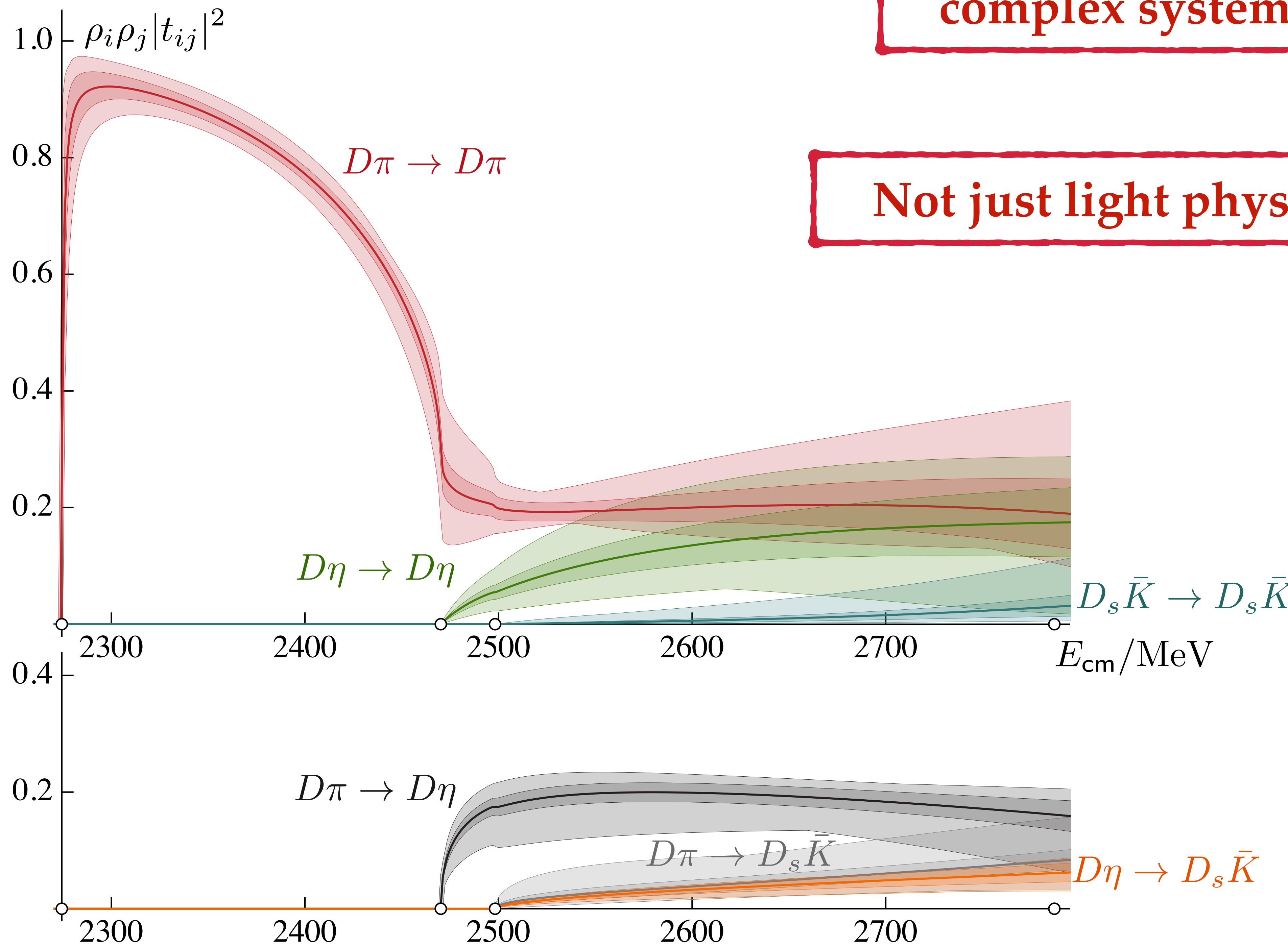
Dudek, Edwards & Wilson (2016)

$D\pi$–$D\eta$–$D_s\overline{K}$ scattering

(S-wave, I=1/2 channel)

Increasingly complex systems

Not just light physics!



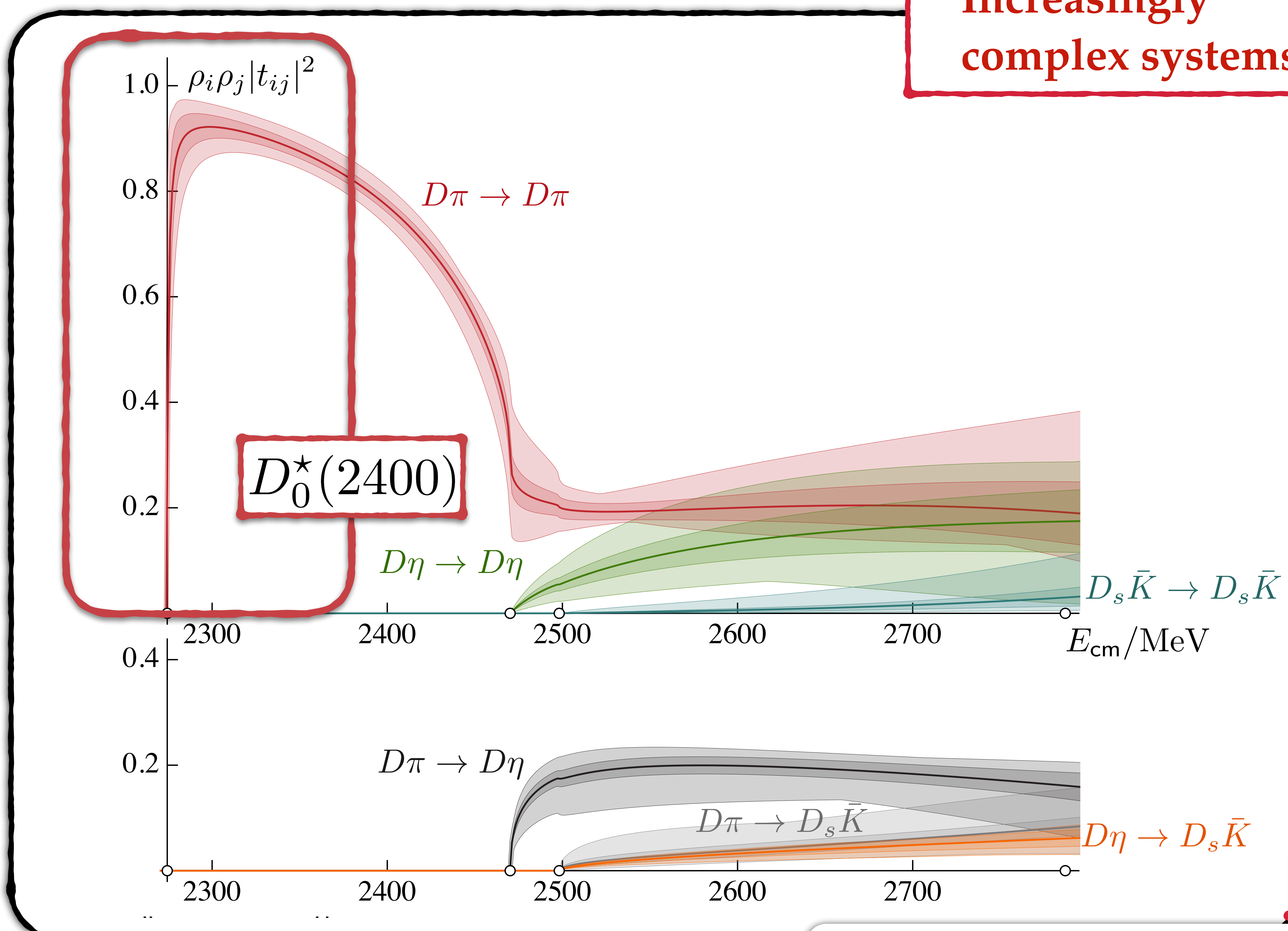
RB

Moir, Peardon, Ryan, Thomas, Wilson

$D\pi$–$D\eta$–$D_s\overline{K}$ scattering

(S-wave, I=1/2 channel)

Increasingly complex systems



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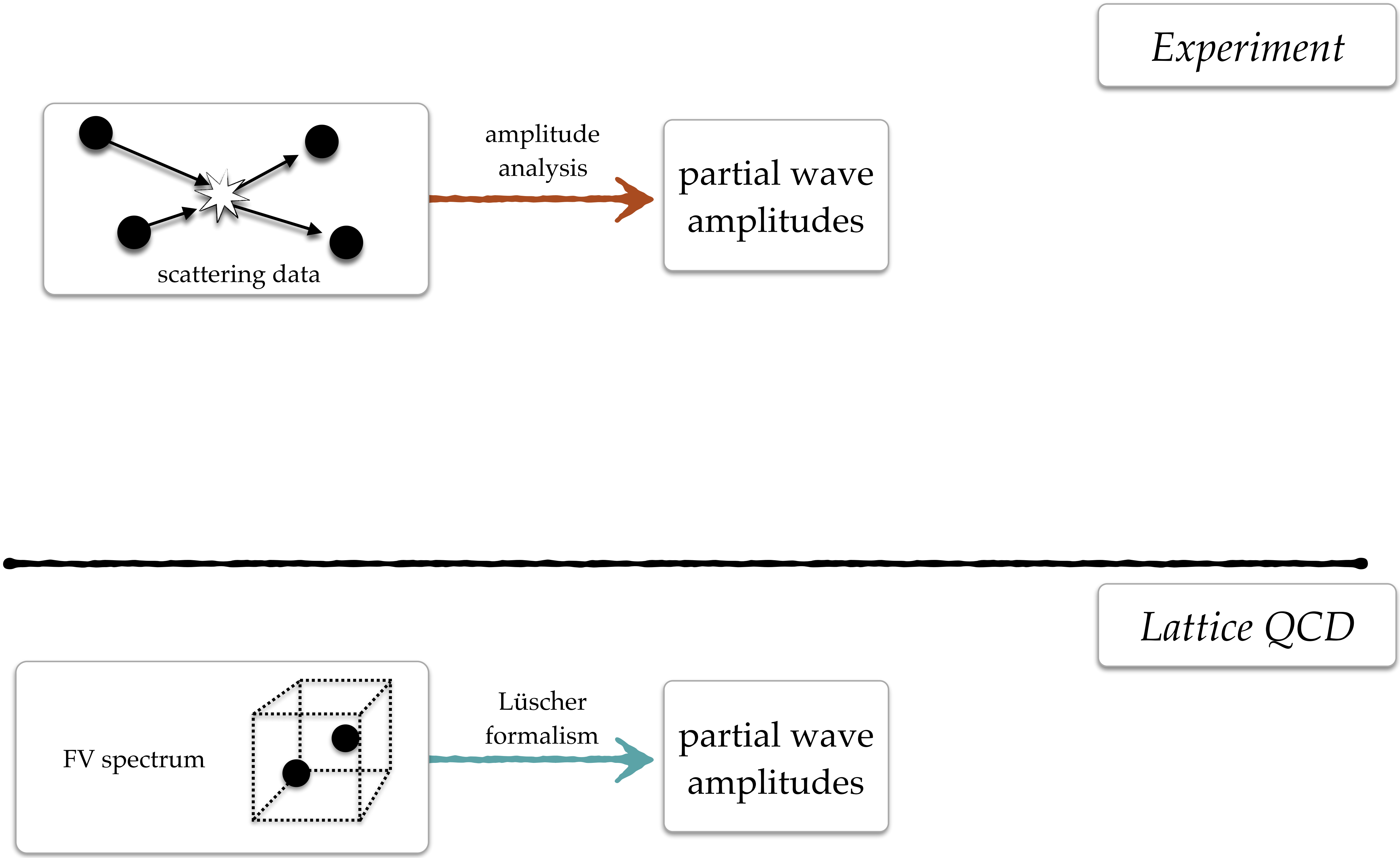
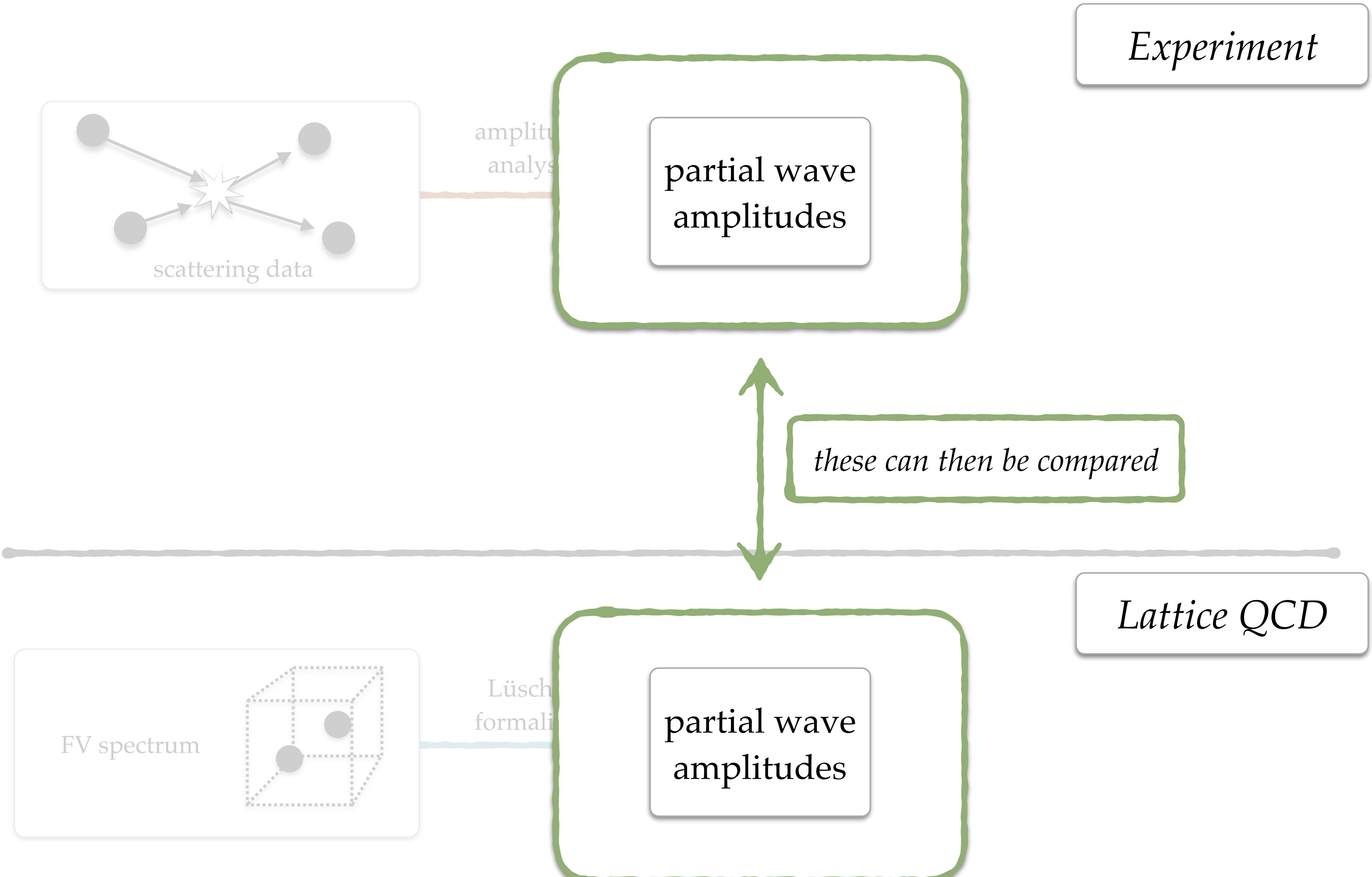
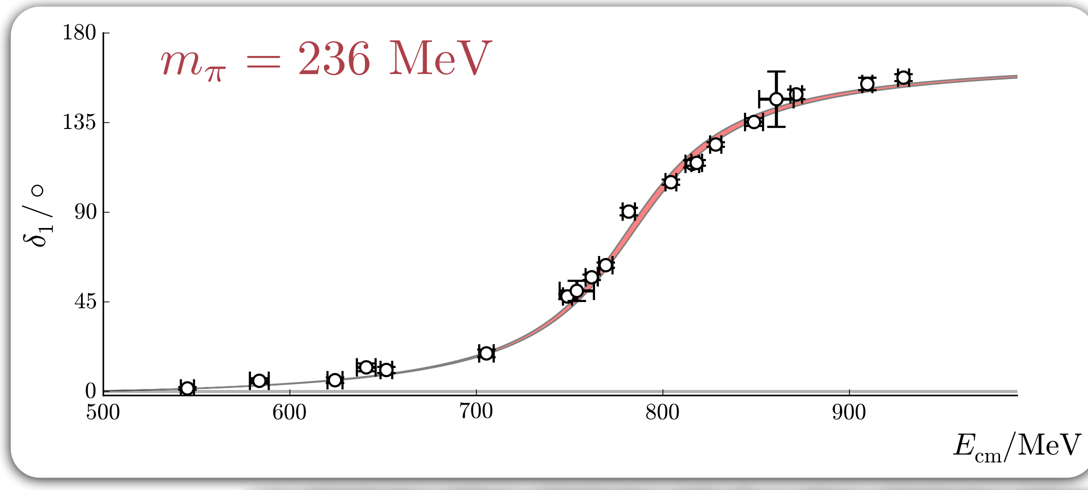
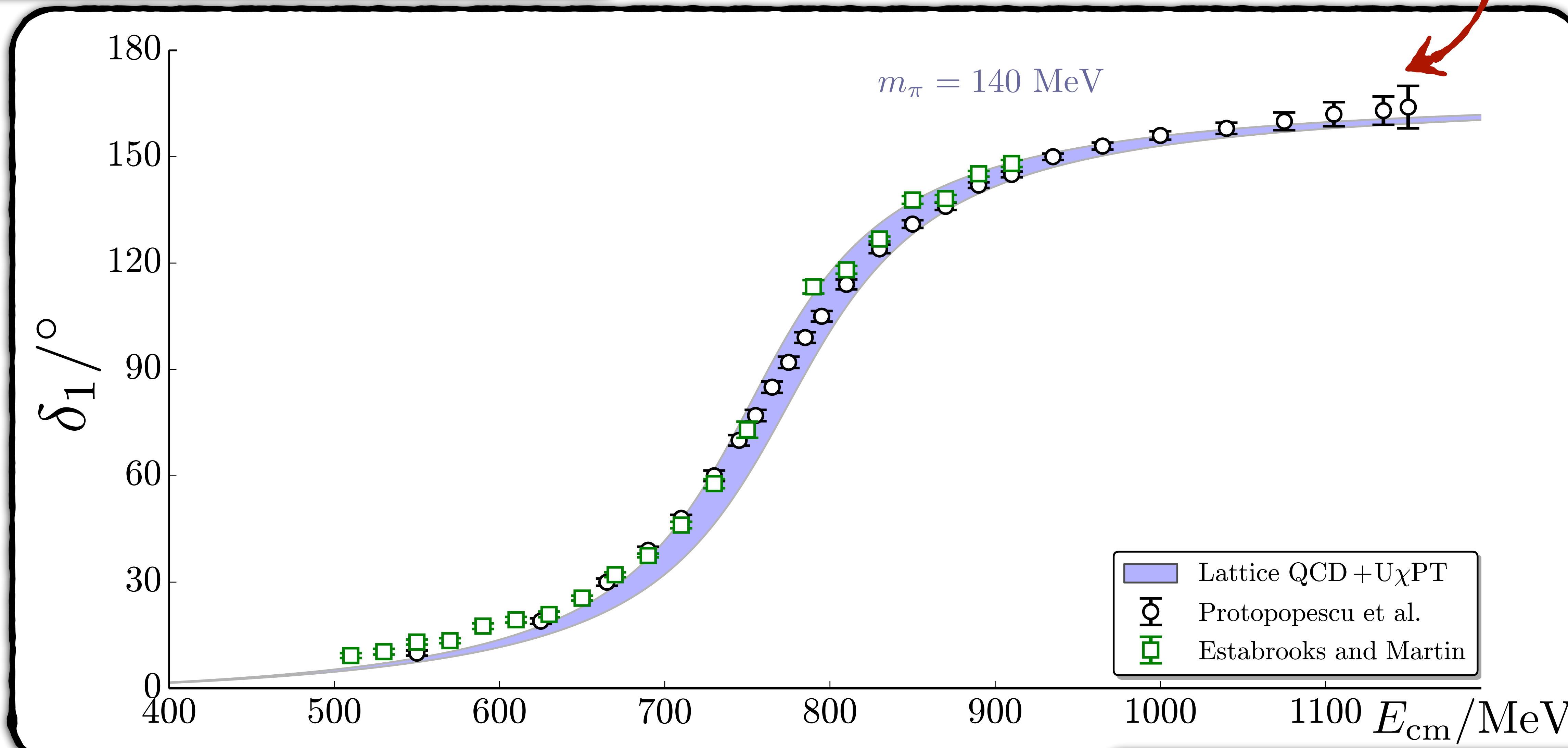

Experiment
scattering data
amplitude analysis
partial wave amplitudes
Lattice QCD
FV spectrum
Lüscher formalism
partial wave amplitudes


Experiment
scattering data
partial wave amplitudes
these can then be compared
Lattice QCD
FV spectrum
partial wave amplitudes

Comparing with experiment

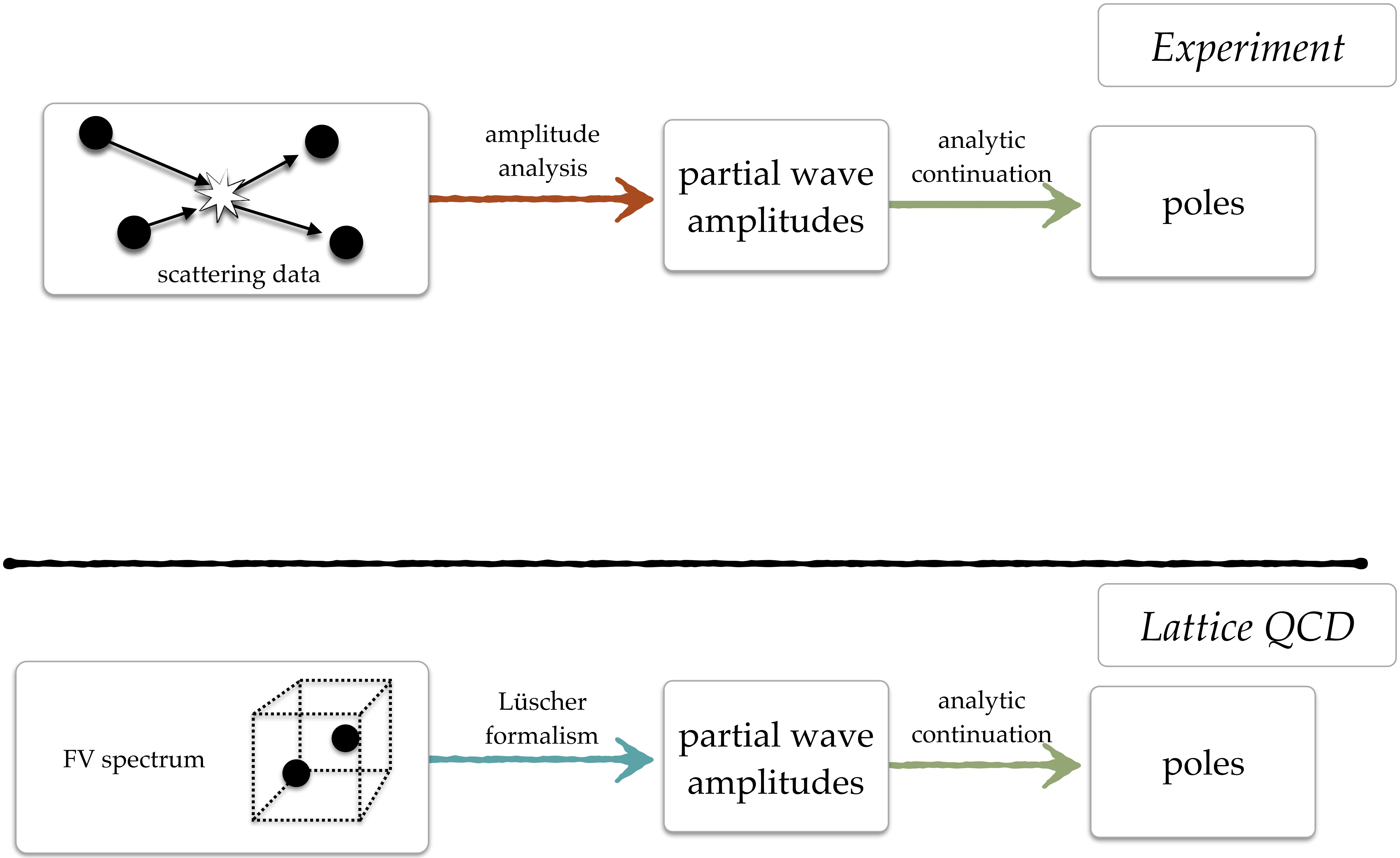


$$\det\left[F^{-1} + \mathcal{M}_{U\chi PT}\left(m_\pi, \{\alpha\}\right)\right] = 0$$



Bolton, RB & Wilson (2015)
UχPT - Dobado and Pelaez (1997)

First chiral extrapolation of a resonant amplitude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amplitude analysis
partial wave amplitudes
analytic continuation
poles
Lattice QCD
FV spectrum
Lüscher formalism
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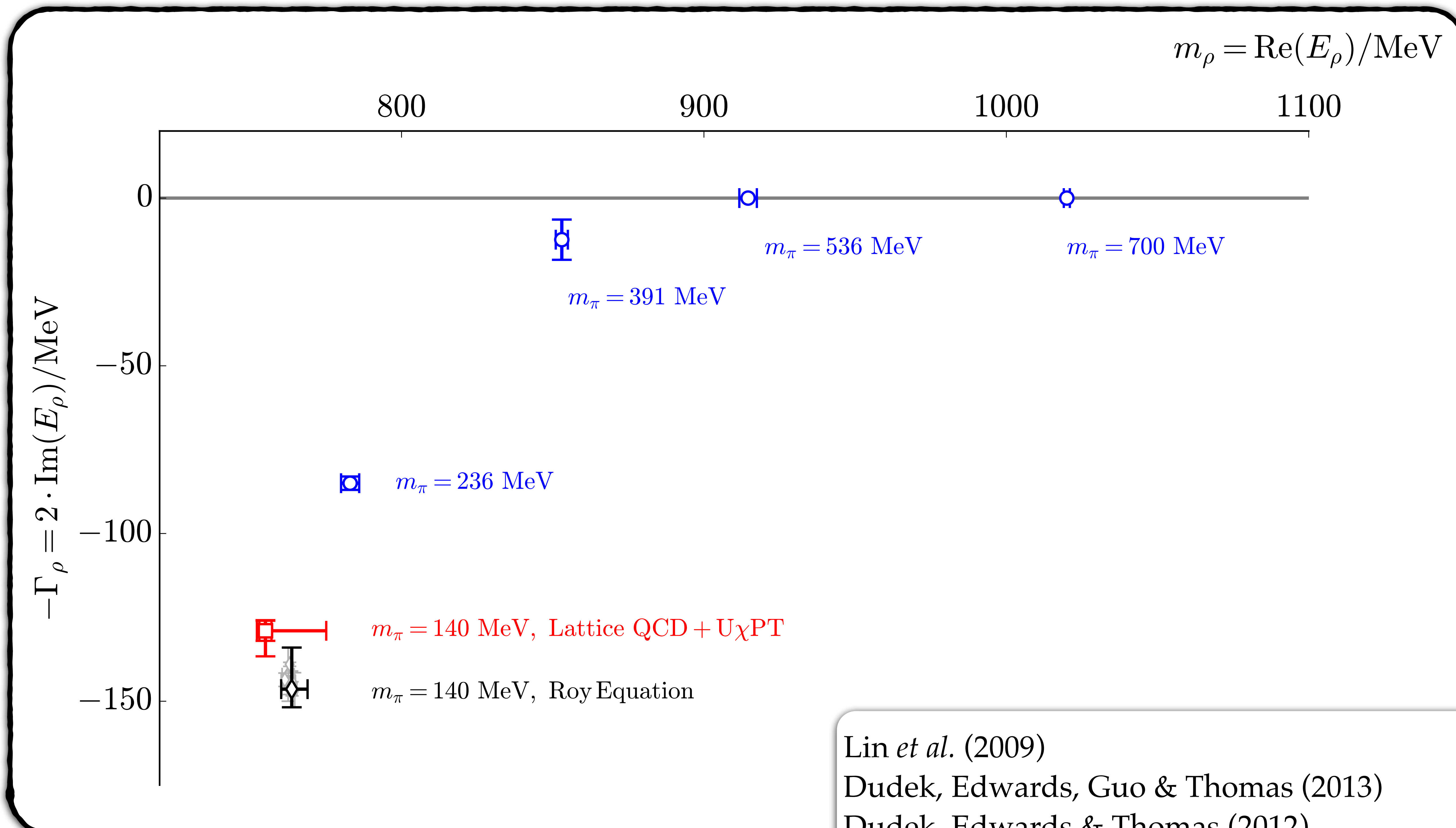
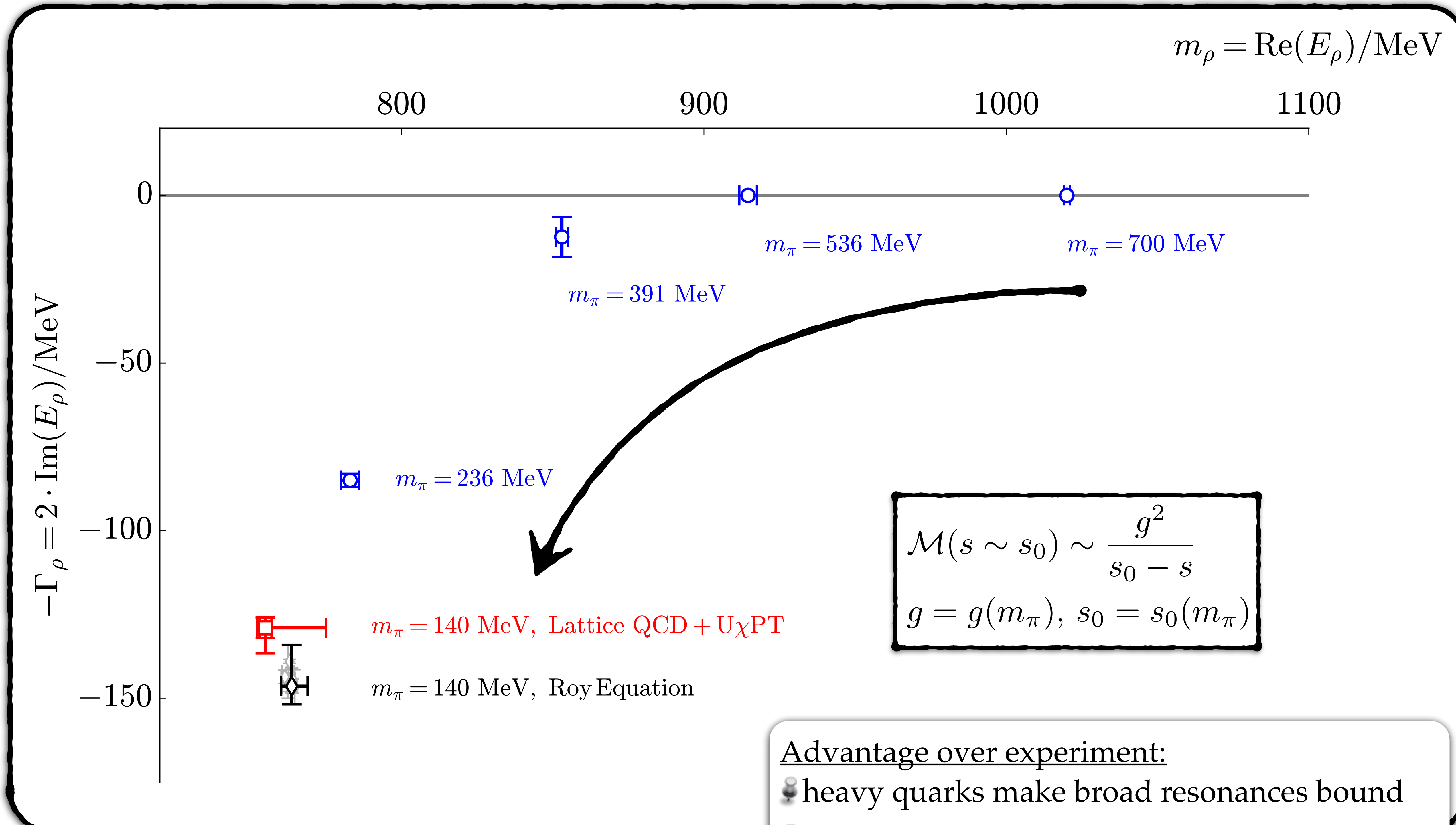
The ϱ vs m_π



Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

The ϱ vs m_π

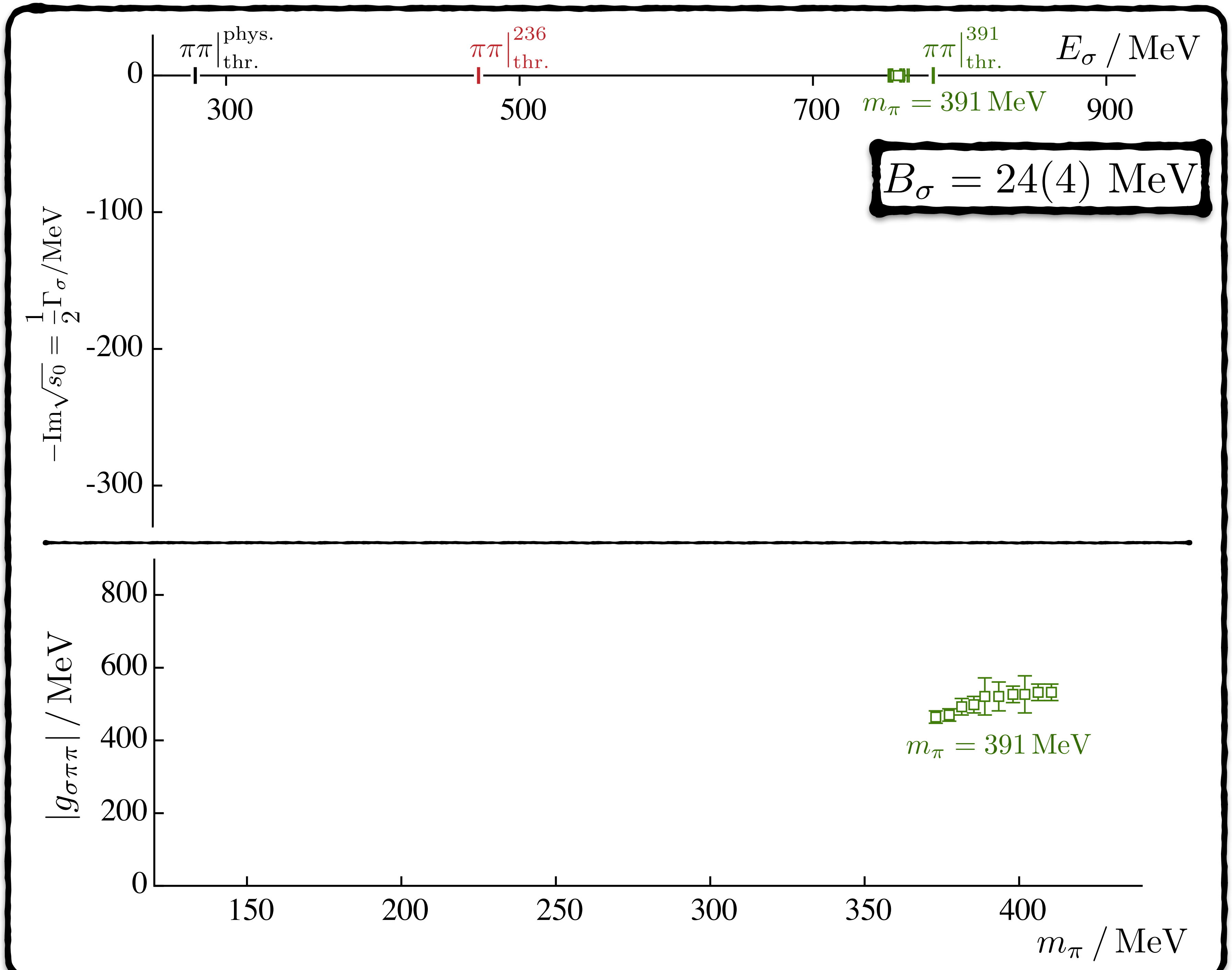


<u>Advantage over experiment:</u>

- heavy quarks make broad resonances bound
- **unambiguously track poles in complex plane**

The σ / $f_0(500)$ vs m_π

$$s_0 = (E_\sigma - \tfrac{i}{2}\Gamma_\sigma)^2, \qquad g^2_{\sigma\pi\pi} = \lim_{s\to s_0} (s_0 - s)\, t(s)$$



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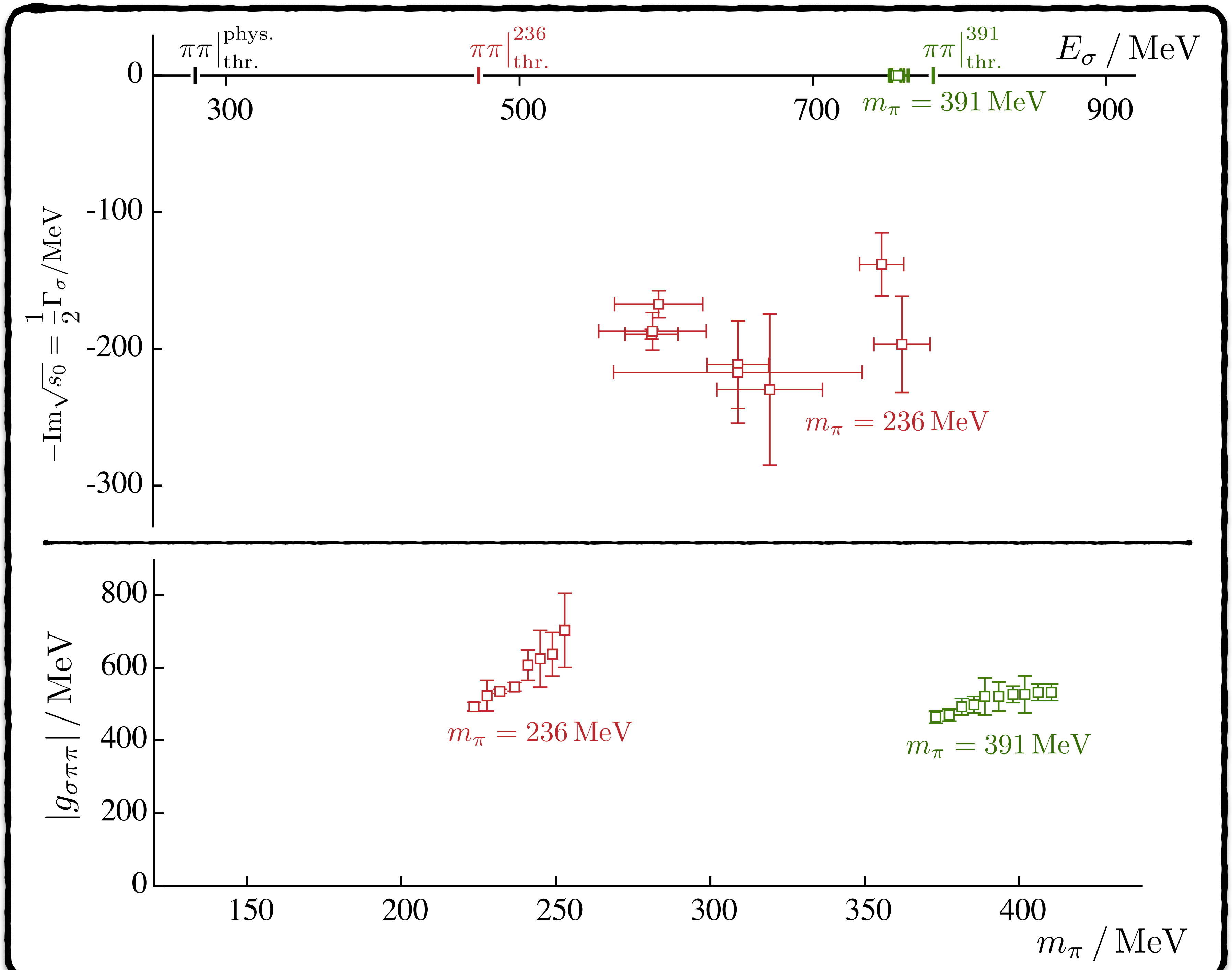

The σ / f$_0$(500) vs m_π

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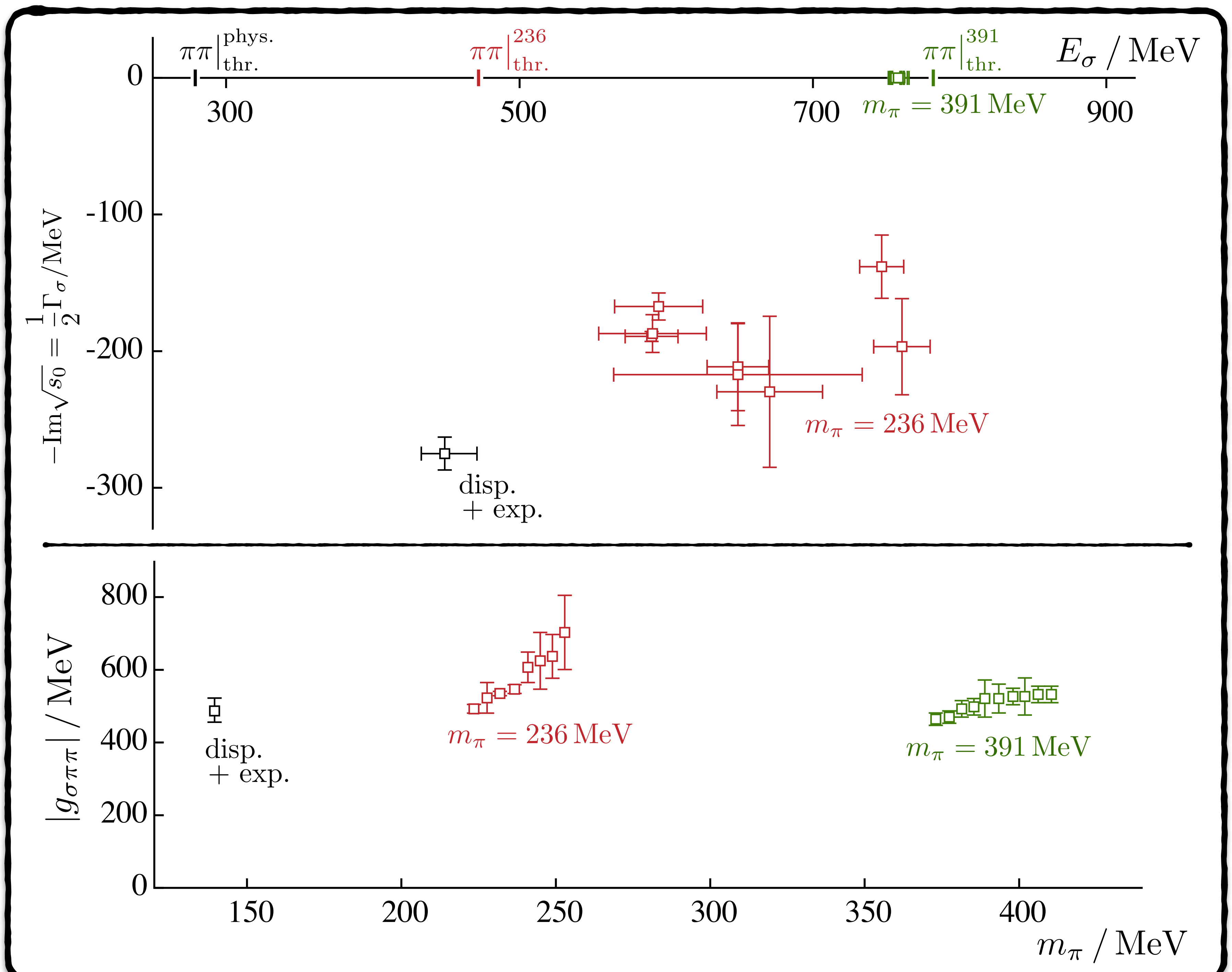
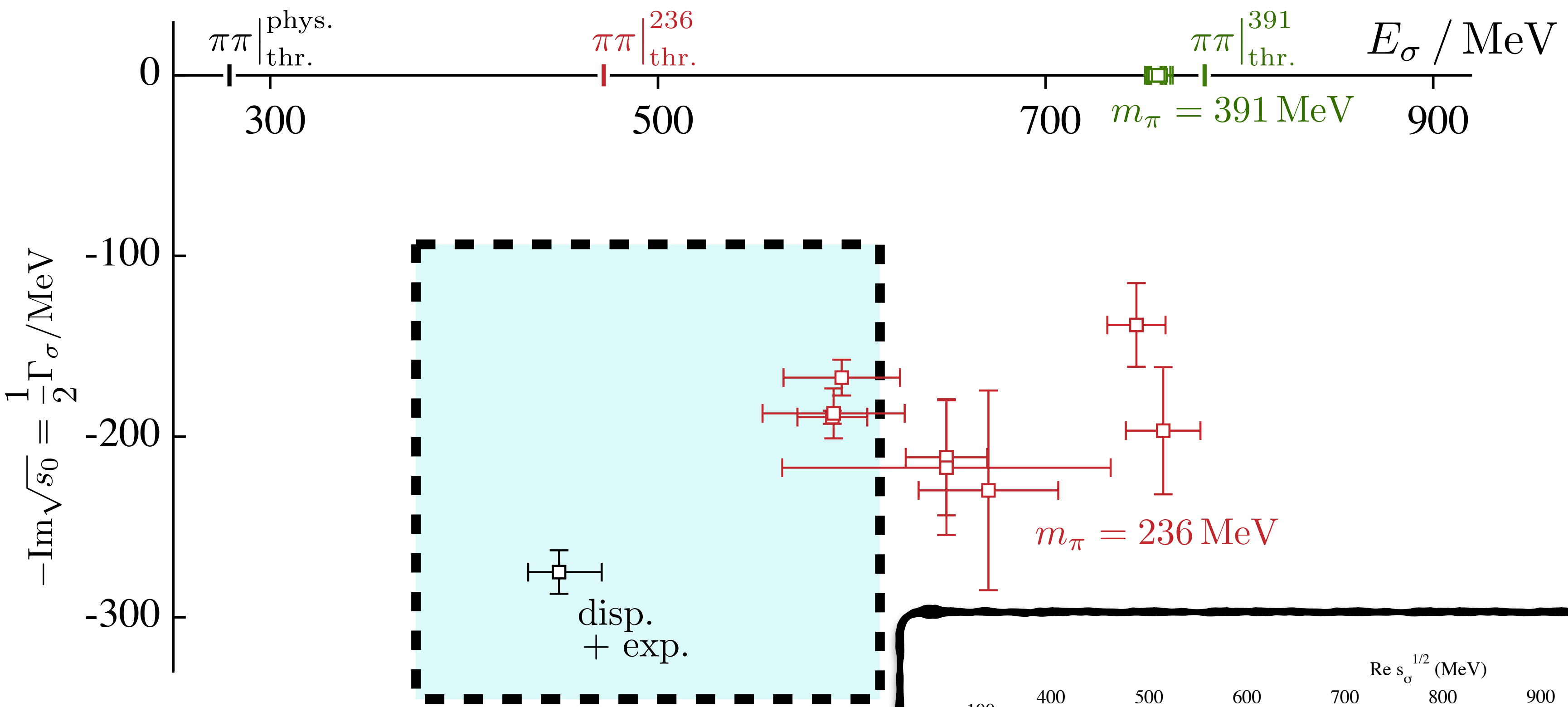


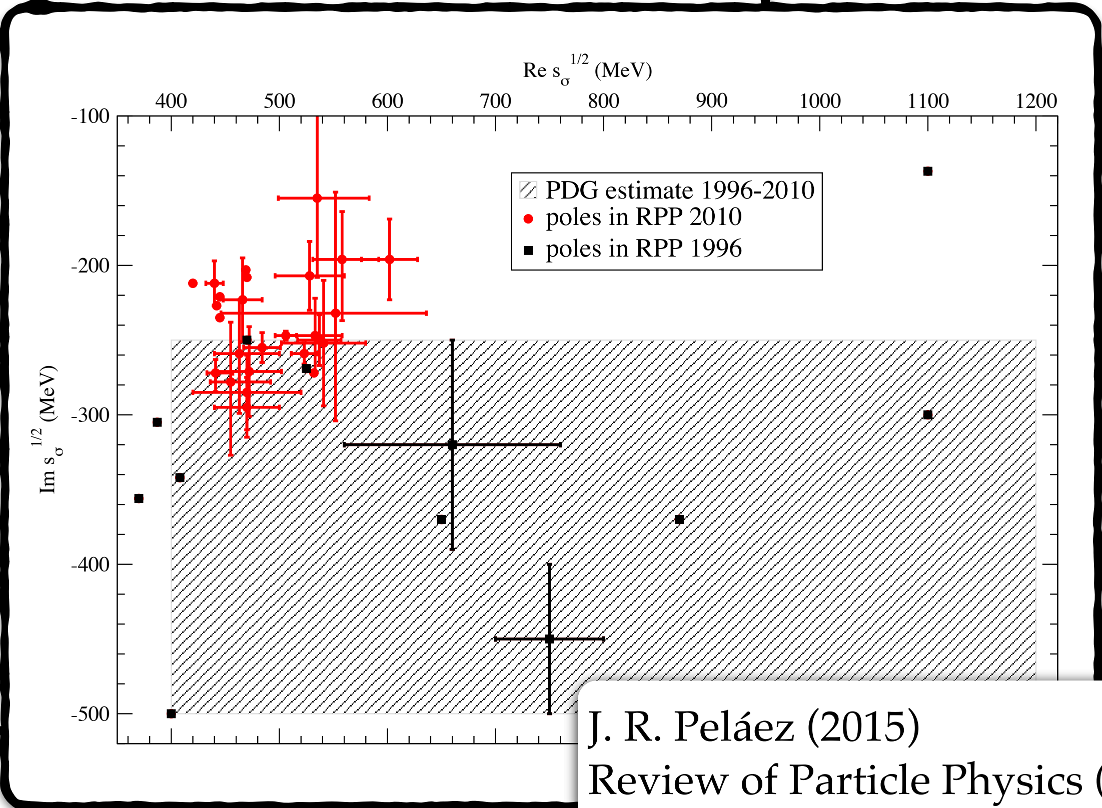
disp. +exp. = Peláez (2015), Caprini, et al. (2006), & Garcia-Martin et al. (2011)

The σ / f₀(500) vs m_π

$$s_0 = (E_\sigma - \tfrac{i}{2}\Gamma_\sigma)^2, \qquad g^2_{\sigma\pi\pi} = \lim_{s\to s_0}(s_0 - s)\, t(s)$$



Historical perspective



J. R. Peláez (2015)
Review of Particle Physics (RPP)

The σ / f_0(500) vs m_π

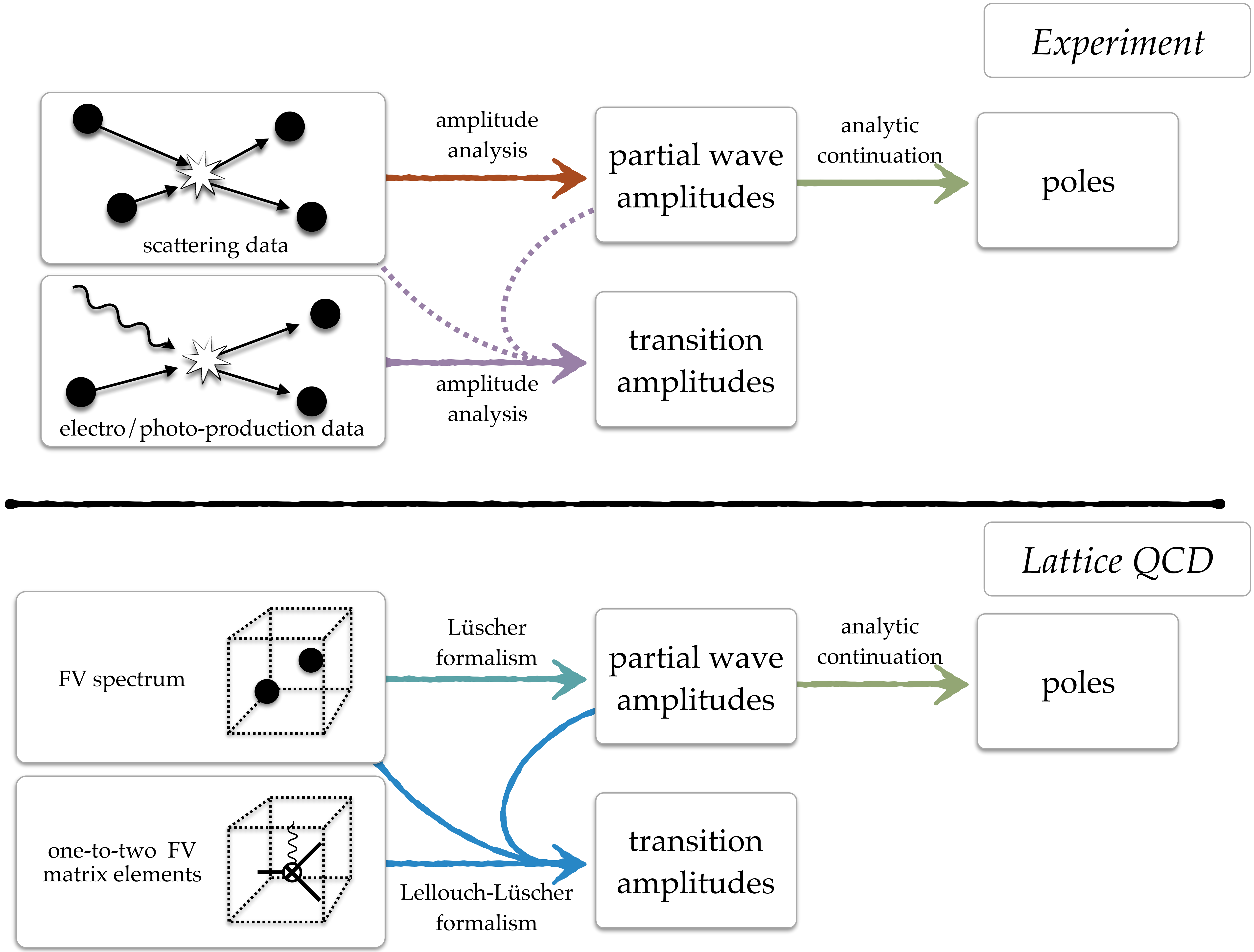
$$s_0 = (E_\sigma - \tfrac{i}{2}\Gamma_\sigma)^2, \qquad g^2_{\sigma\pi\pi} = \lim_{s\to s_0} (s_0 - s)\, t(s)$$

Redefinition

Composite states: those whose nature strongly depends on the values of the quark mass.

e.g., the σ can be a real bound state, virtual bound state or a resonance if you dial the coupling of the Higgs to light quarks

a challenge for experimentalist!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Lellouch-Lüscher formalism
transition amplitudes

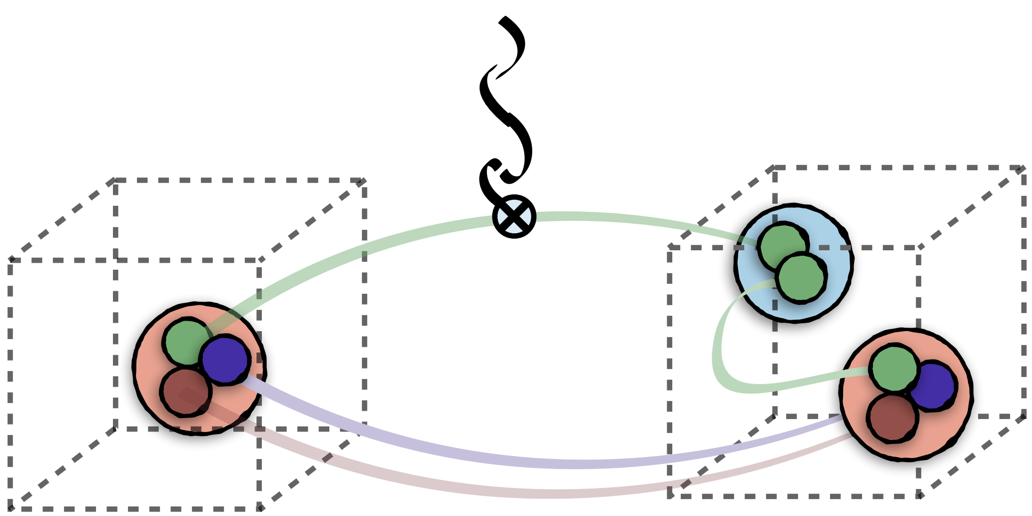
Matrix elements

1) Access matrix elements:

$$C^{3pt.}_{\mathbf{2}\to\mathbf{1}\mathcal{J}} = \langle \mathcal{O}_1(\delta t)\mathcal{J}(t)\mathcal{O}_2^\dagger(0)\rangle \longrightarrow \langle \mathbf{1}|\mathcal{J}|\mathbf{2}\rangle_L Z_1 Z_2^* e^{-(\delta t - t)E_1} e^{-tE_2} + \cdots$$

2) Interpret matrix elements:

$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\,\mathcal{R}\,\mathcal{H}$$



RB, Hansen & Walker-Loud (2014)
RB & Hansen (2015)
RB & Hansen (2015)

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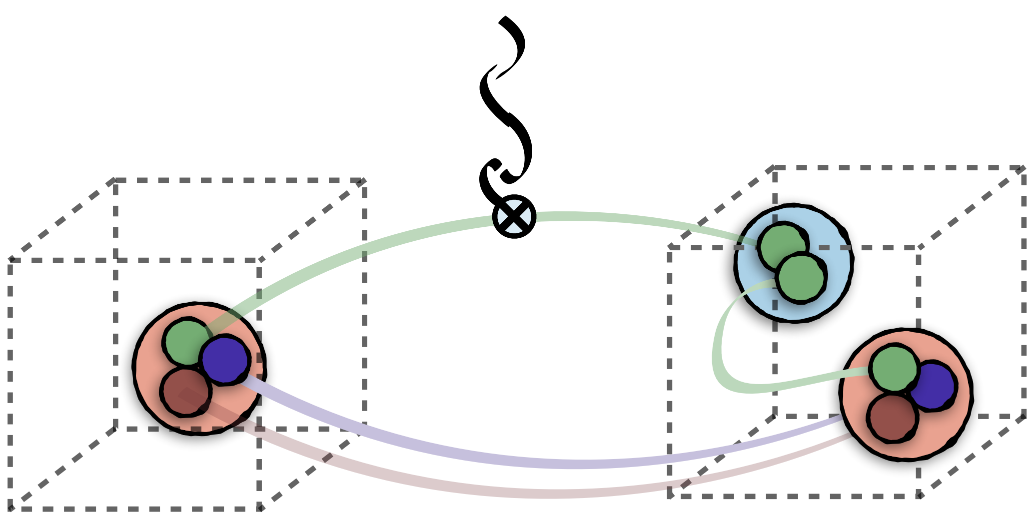
2) Interpret matrix elements:

$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\,\mathcal{R}\,\mathcal{H}$$

known finite volume function

$$\mathcal{R}\left(E_{\mathbf{2}}, L, \delta, \frac{\partial \delta}{\partial E_{\mathbf{2}}}\right)$$



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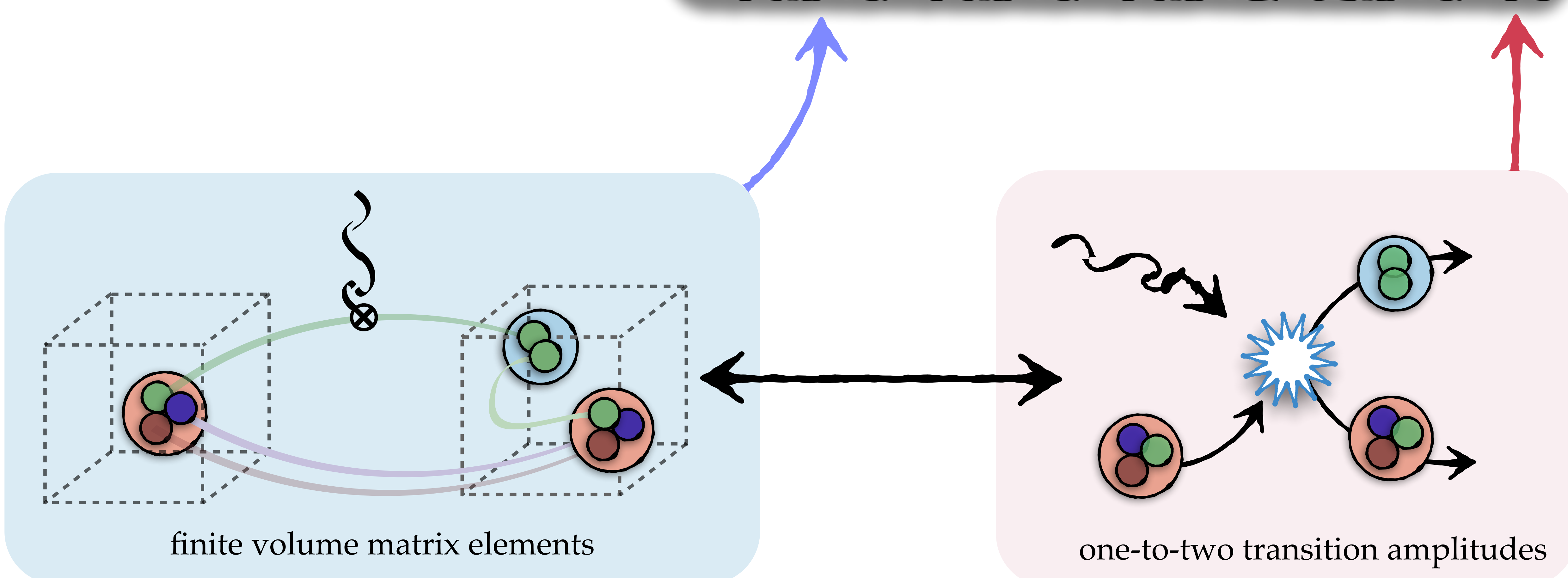

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2) Interpret matrix elements:

$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\ \mathcal{R}\ \mathcal{H}$$

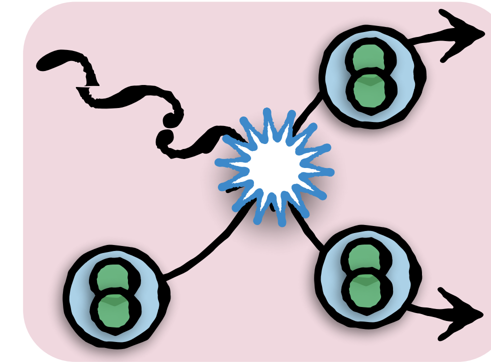
summarizes everything previously done and more!

Lellouch-Lüscher formalism

- Lellouch & Lüscher (2000) [K-to-ππ at rest]
- Christ, Kim & Yamazaki / Kim, Sachrajda & Sharpe (2005) [moving K-to-ππ]
- Meyer [Bγ-to-BB] (2011)
- Hansen & Sharpe [moving D-to-ππ/KK] (2012)
- Agadjanov, V. Bernard, Meissner & Rusetsky [Nγ-to-Nπ] (2013)

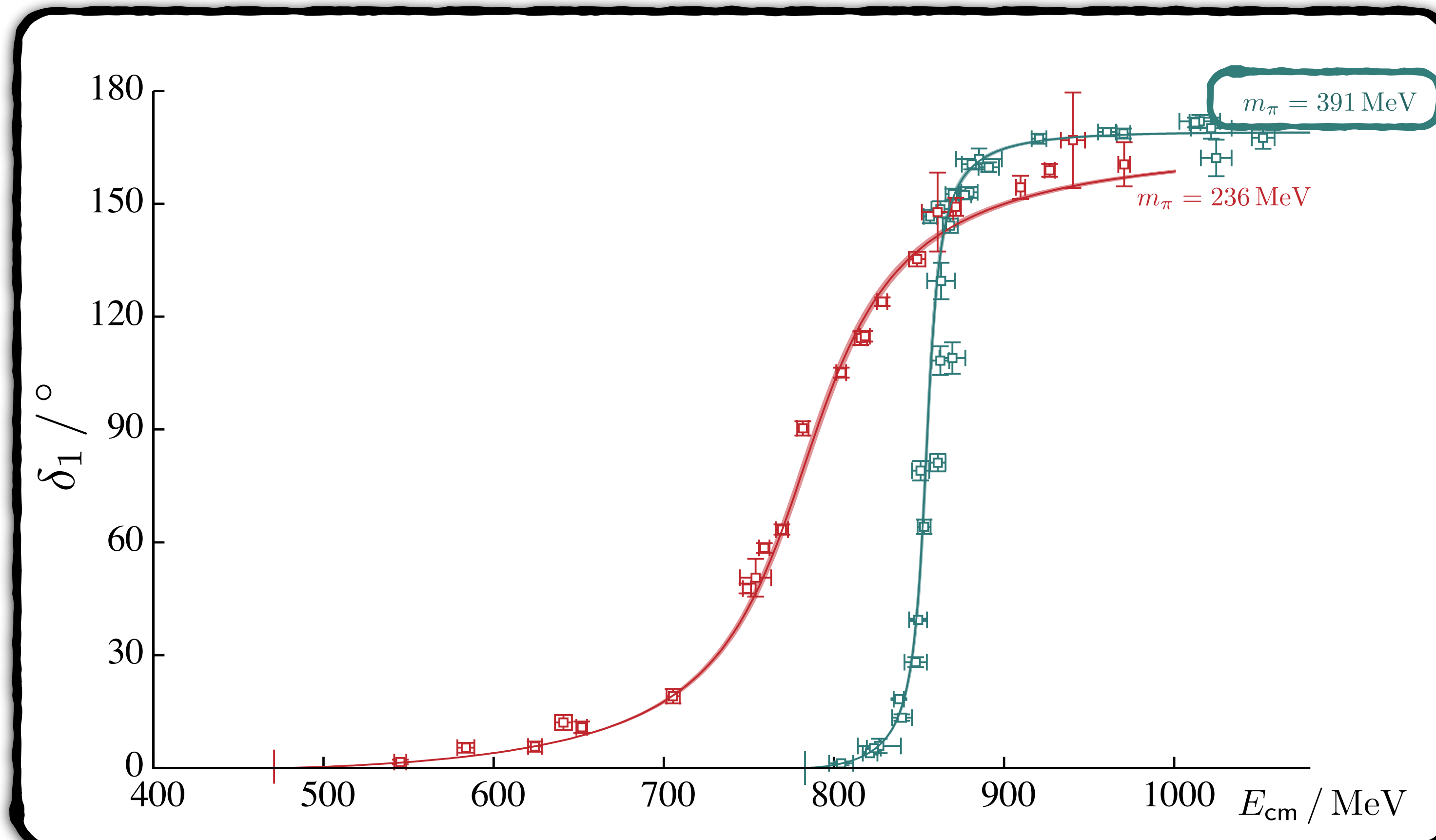
RB, Hansen - Phys.Rev. D92 (2015) no.7, 074509.
RB, Hansen, Walker-Loud - Phys.Rev. D91 (2015) no.3, 034501.

$\pi\gamma^*$-to-$\pi\pi$



Exploratory $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ρ calculation:

- $m_\pi = 391$ MeV



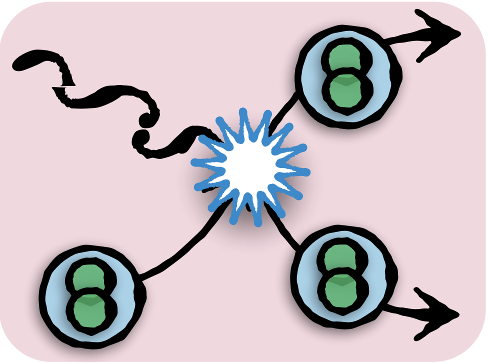
- Matrix element determined in **42** kinematic point: $(E_{\pi\pi}, Q^2)$

- Lorentz decomposition:

$$\mathcal{H}^{\mu}_{\pi\pi,\pi\gamma^\star} = \epsilon^{\mu\nu\alpha\beta}\, P_{\pi,\nu}\, P_{\pi\pi,\alpha}\, \epsilon_\beta(\lambda_{\pi\pi}, \mathbf{P}_{\pi\pi})\, \frac{2}{m_\pi} \mathcal{A}_{\pi\pi,\pi\gamma^\star}$$

$\pi\pi/\rho$ polarization *$\pi\pi/\rho$ helicity* *Lorentz scalar*

$\pi\gamma^*$-to-$\pi\pi$



1. ϱ-to-$\pi\gamma^*$ decay
2. chiral anomaly
3. Building block of $N\gamma^*$-to-$N\pi$
4. Hadronic light-by-light contribution to g_μ-2

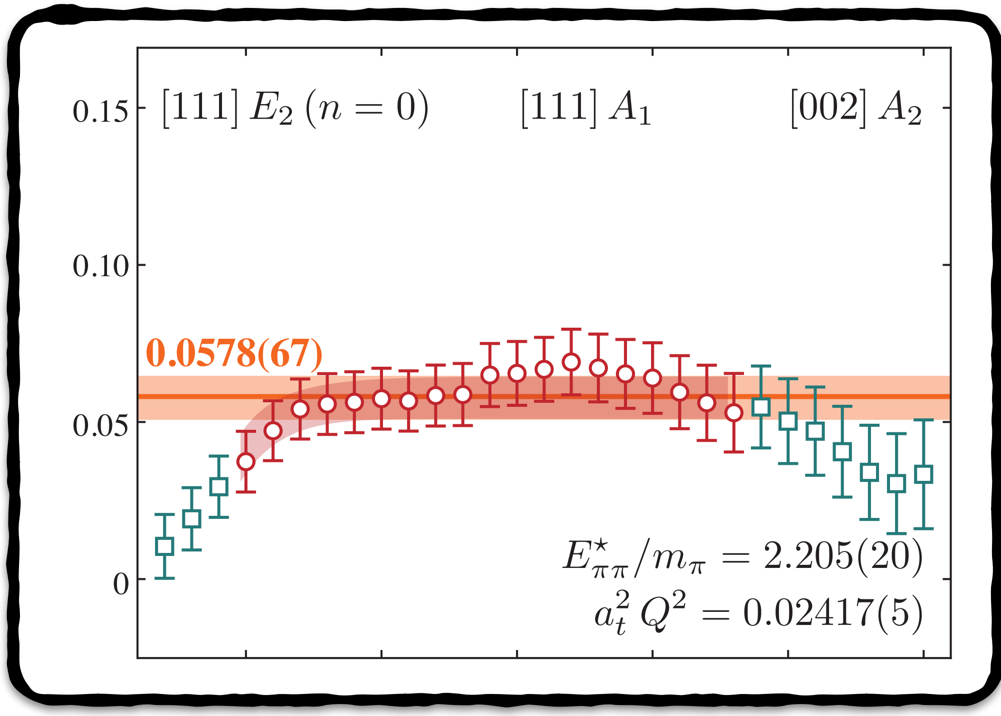
⋮

5a. First resonating 1-to-2 calculation

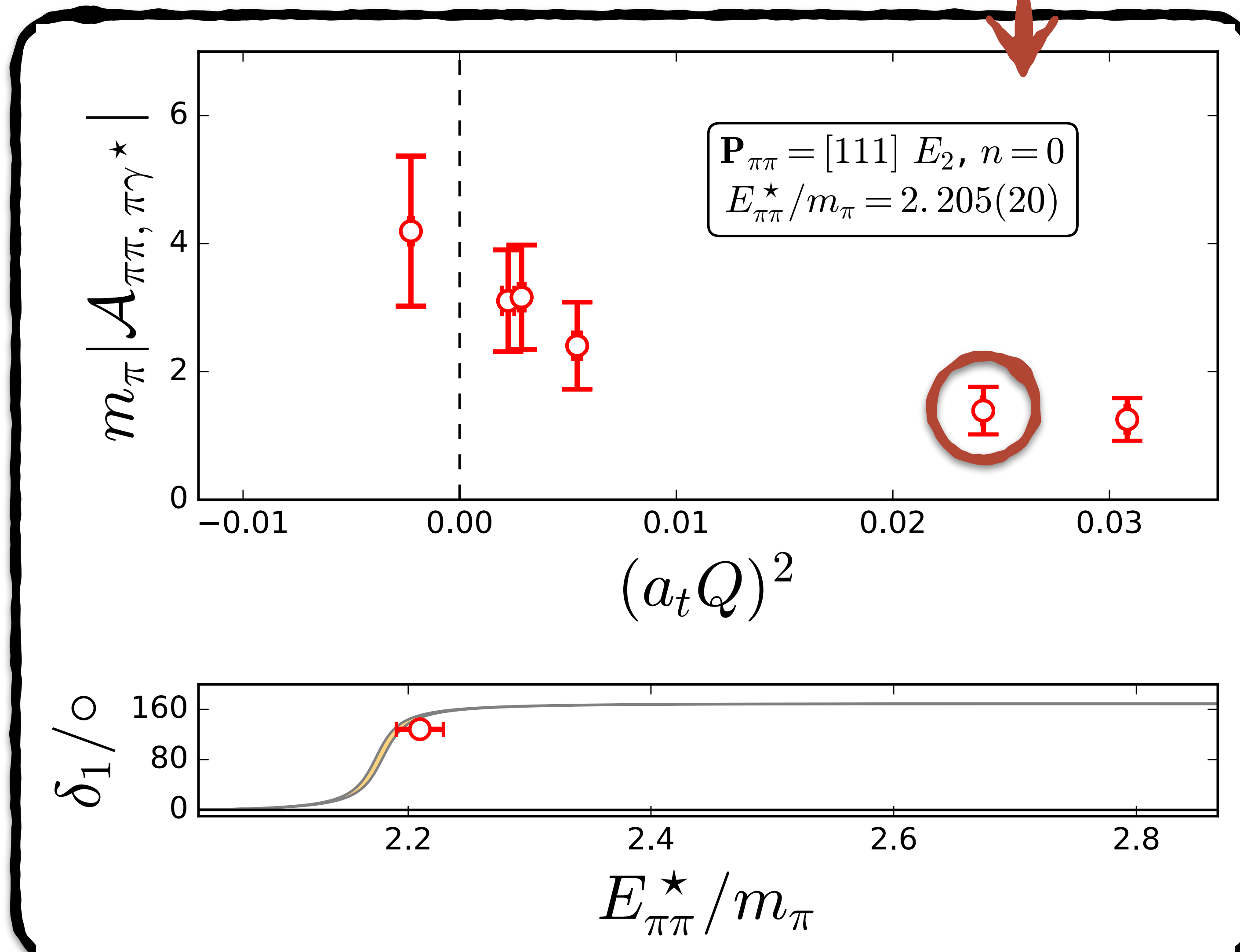
5b. First resonance form factor

5c. Testing ground for more challenging processes

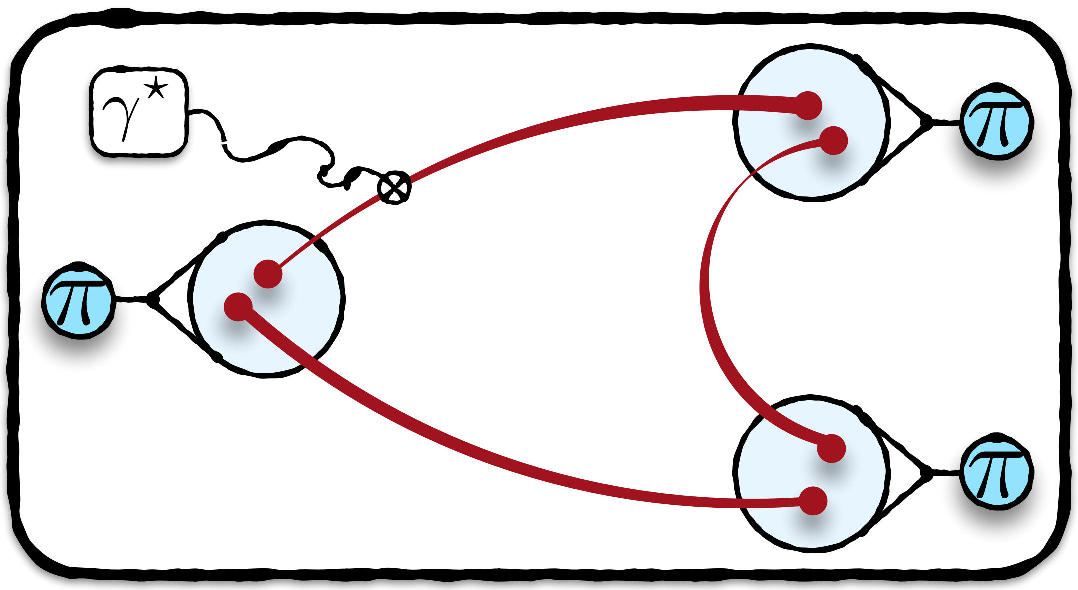
$\pi\gamma^*$-to-$\pi\pi$ amplitude



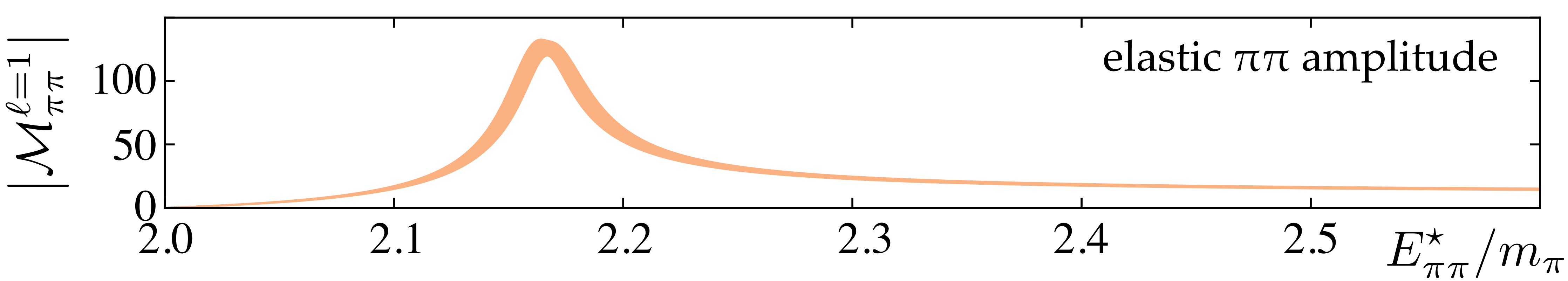
$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\,\mathcal{R}\,\mathcal{H}$$



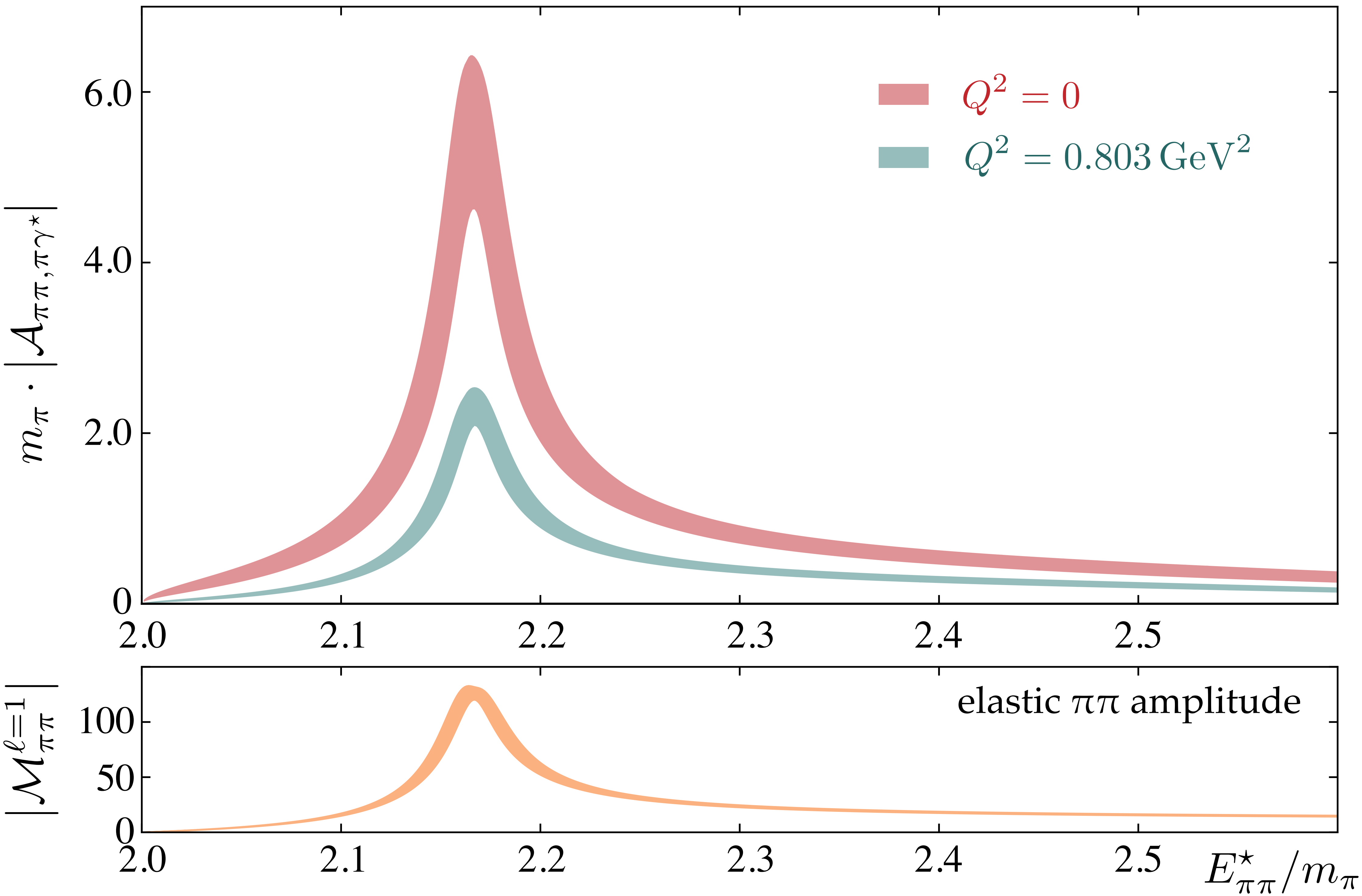
$\pi\gamma^*$-to-$\pi\pi$ amplitude



any guesses how the $\pi\gamma^*$-to-$\pi\pi$ amplitude should look like?

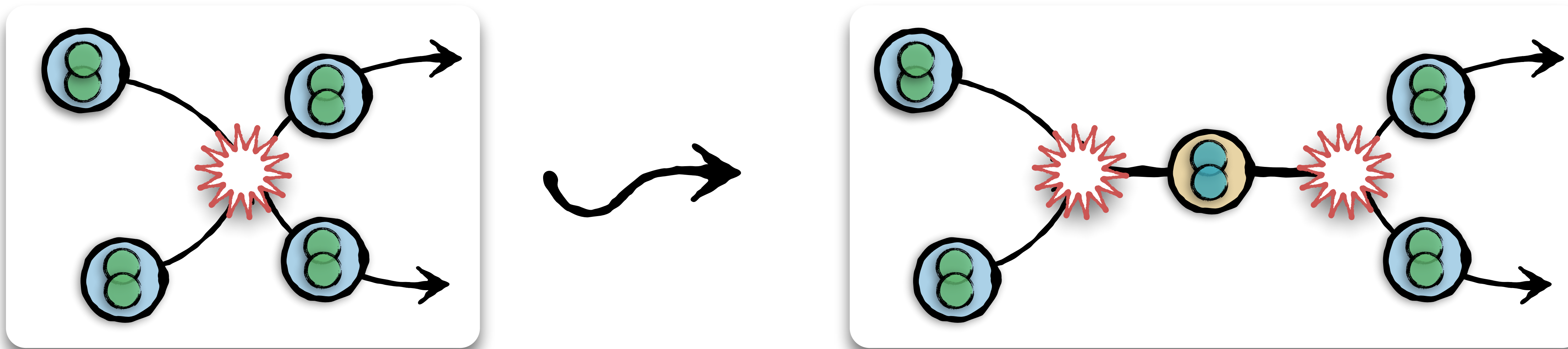


$\pi\gamma^{*}$-to-$\pi\pi$ amplitude

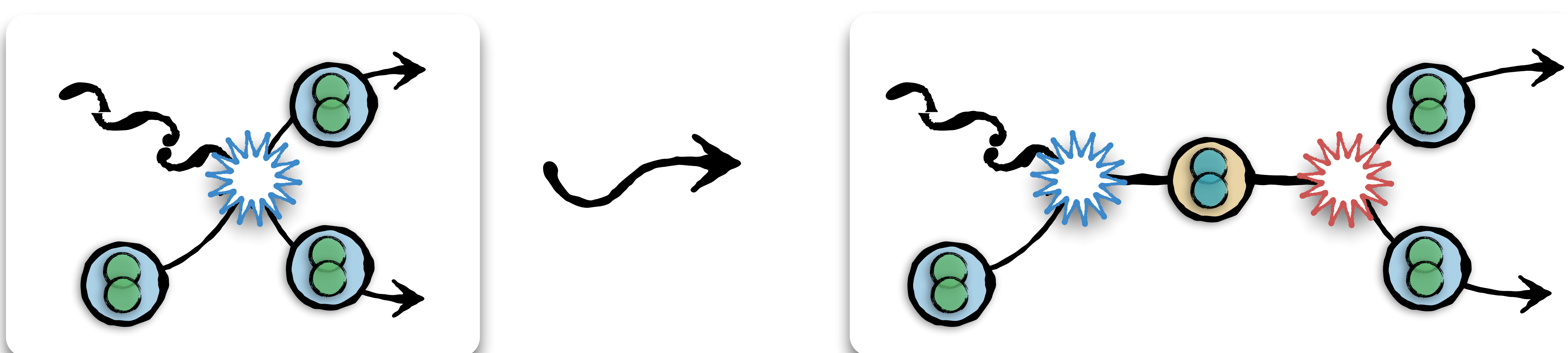


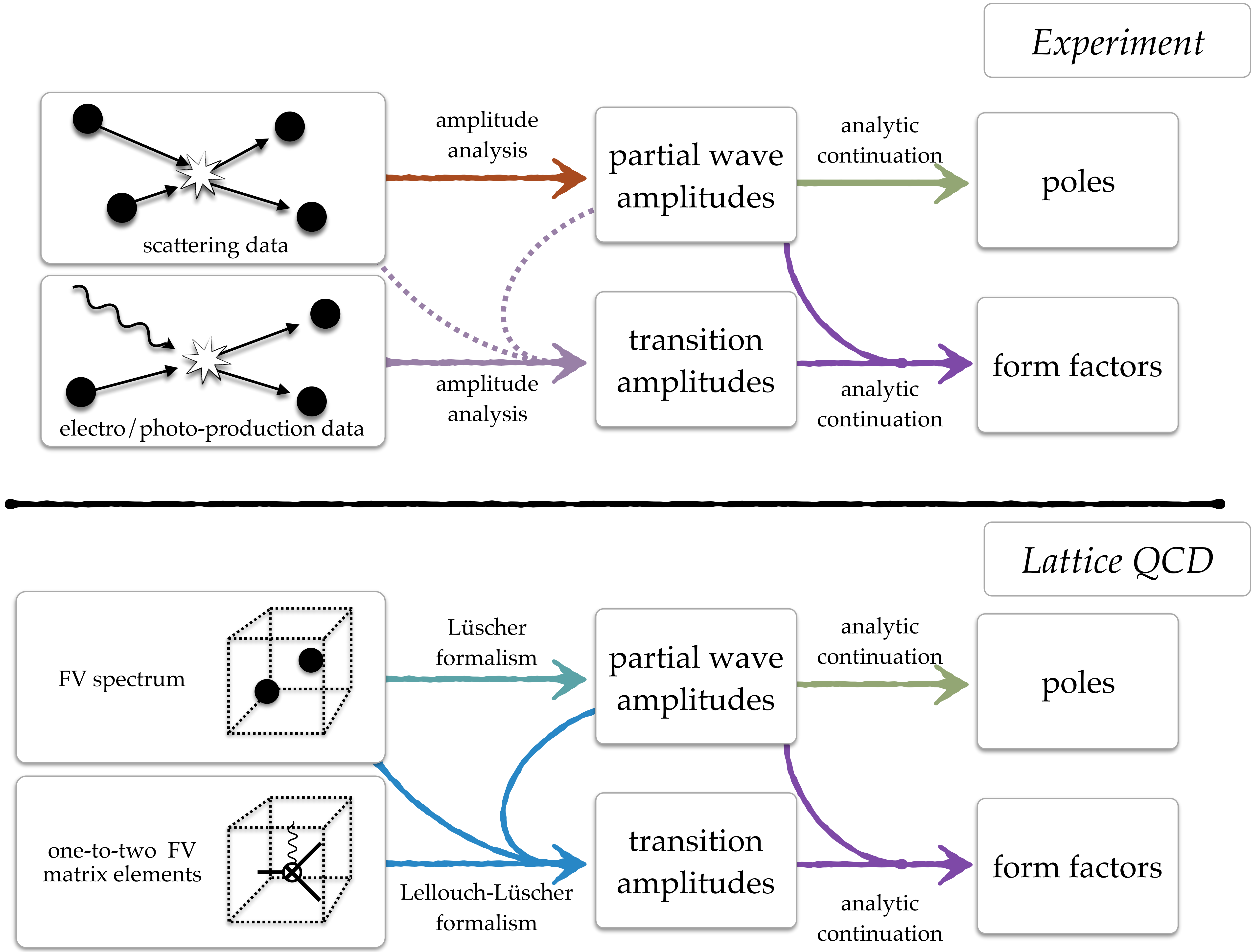
Intuitive explanation

- the elastic $\pi\pi$ amplitude is dynamically enhanced by the presence of the ϱ-meson



- Similarly, the $\pi\gamma^*$-to-$\pi\pi$ amplitude is enhanced by the ϱ-meson

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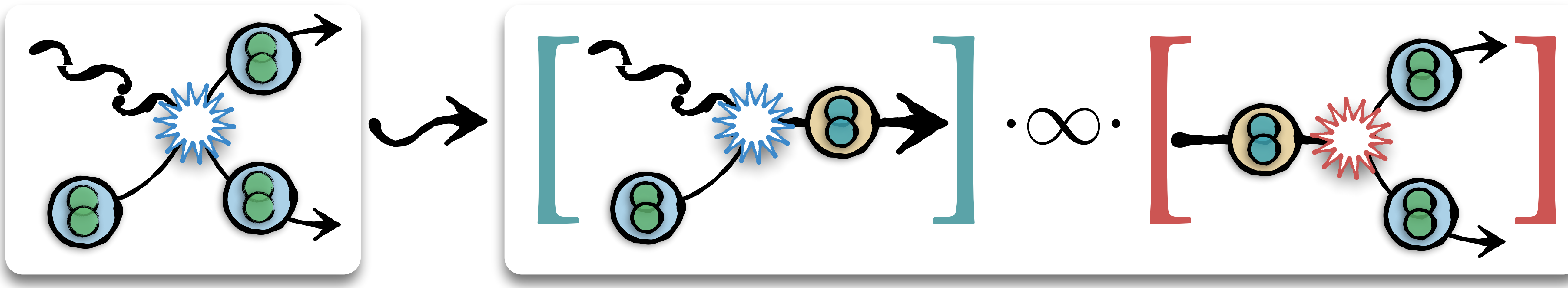
Form factor at ϱ pole

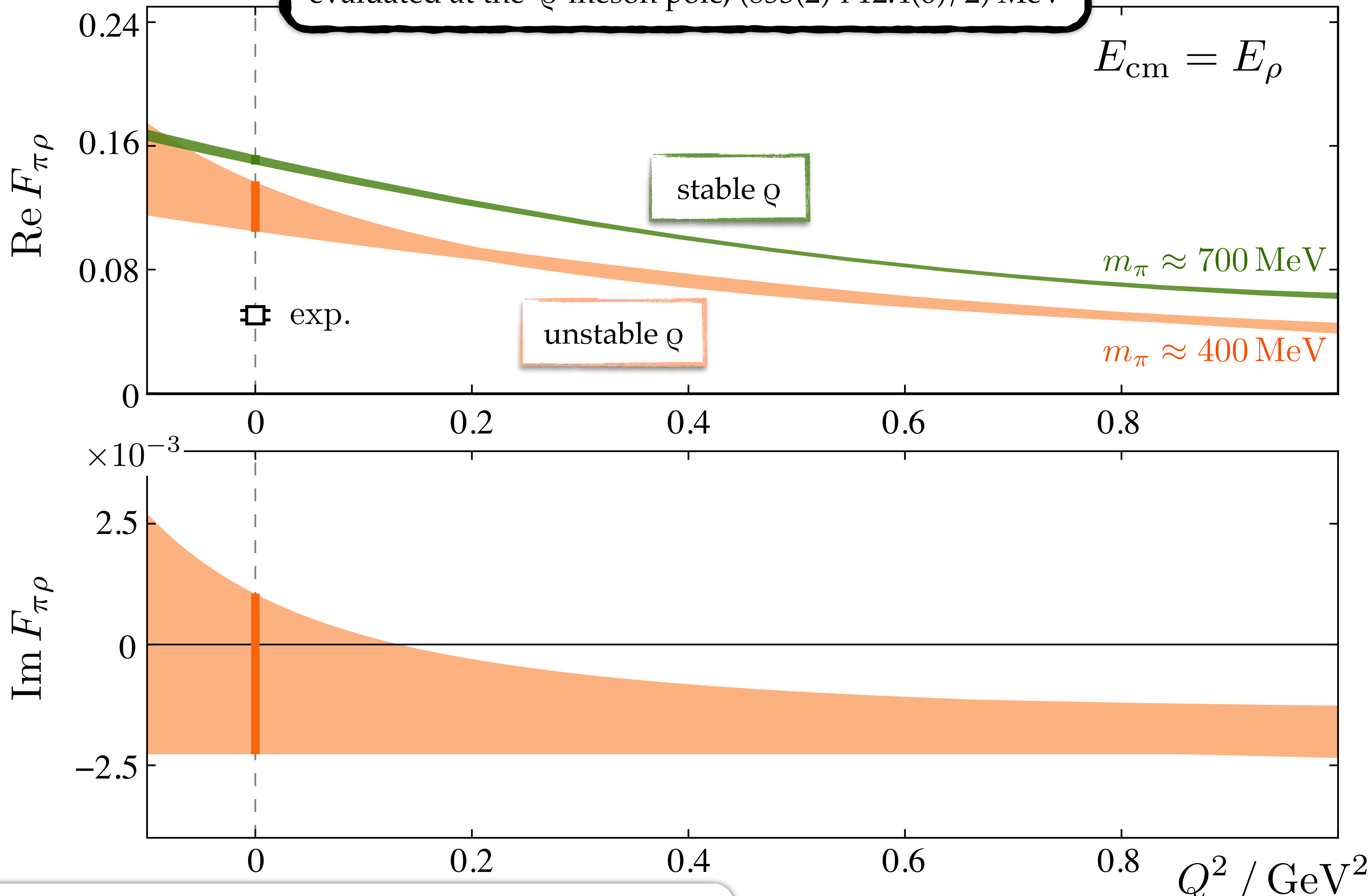
- The residue encodes the $\pi\gamma^*$-to-ϱ form factor



$$\mathcal{A}_{\pi\pi,\pi\gamma^\star}(E_{\pi\pi}, Q^2) = F(E_{\pi\pi}, Q^2) \times \left[\frac{1}{\cot\delta_1(E_{\pi\pi}) - i} \right] \times \sqrt{\frac{16\pi}{q_{\pi\pi}\,\Gamma(E_{\pi\pi})}}$$

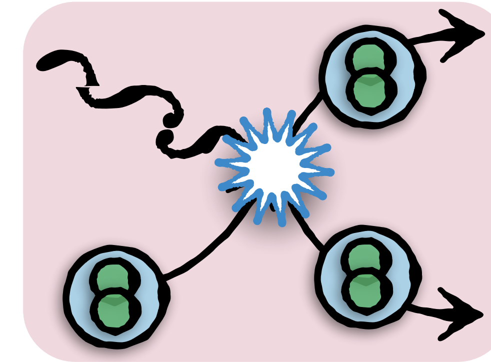
Form factor at ϱ pole

evaluated at the ϱ-meson pole, (853(2)-i 12.4(6)/2) MeV



Shultz, Dudek, & Edwards (2014)
RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)

$\pi\gamma^*$-to-$\pi\pi$



1. ρ-to-$\pi\gamma^*$ decay ✓
2. chiral anomaly → Under way!
3. Building block of $N\gamma^*$-to-$N\pi$
4. Hadronic light-by-light contribution to g_μ-2

⋮

5a. First resonating 1-to-2 calculation

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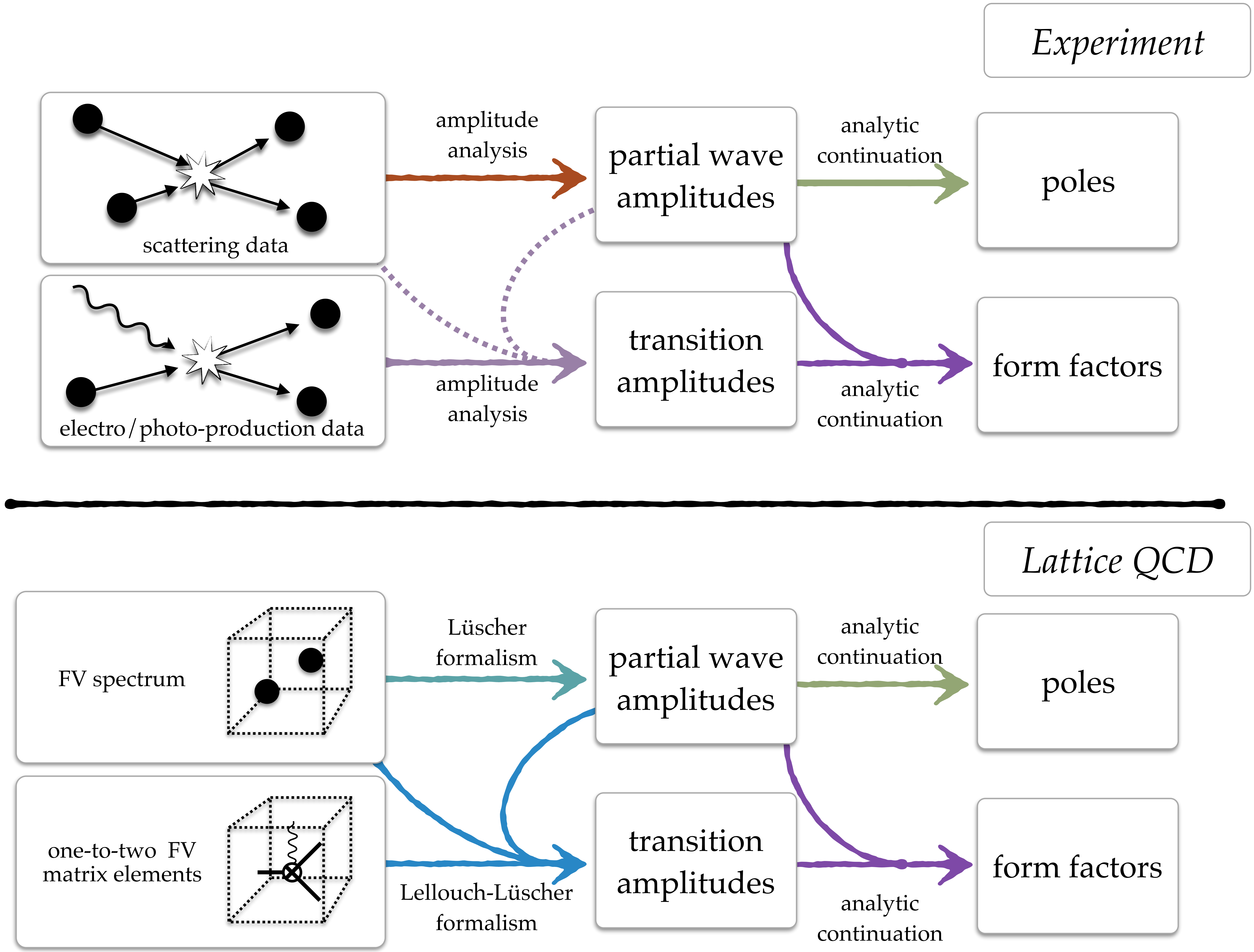
Marco Carrillo
UNAM



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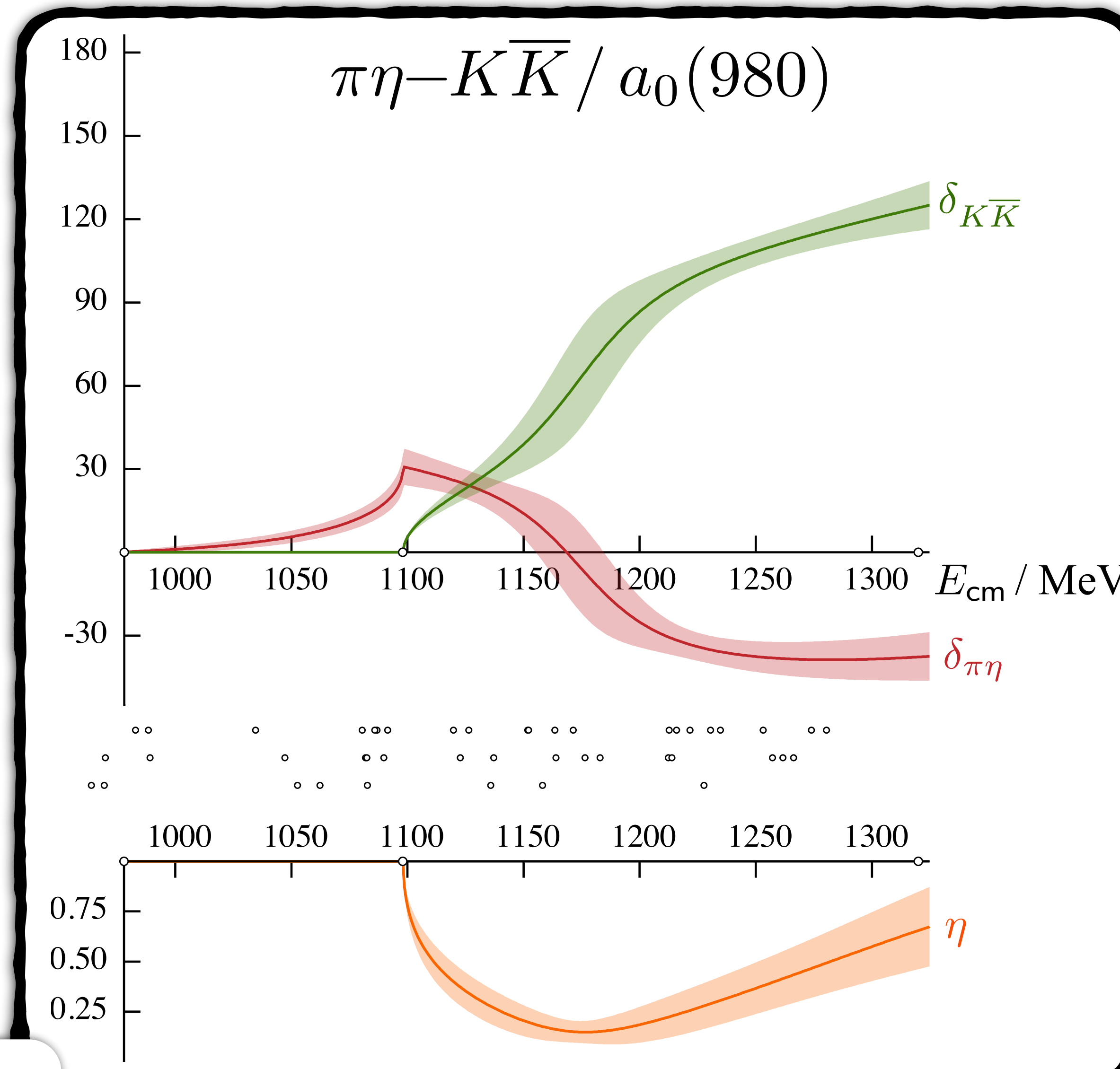
The future of spectroscopy

- *Coupled channels*

few implementations to date by HadSpec



formalism understood:

Hansen & Sharpe / RB & Davoudi (2012)
RB (2014) / RB & Hansen (2015)

HadSpec Collaboration

Dudek, Edwards & Wilson (2016)

~~RB~~

The future of spectroscopy

- *Coupled channels*
- *Baryons*

no implementation to date!

formalism understood:

RB (2014) / RB & Hansen (2015)

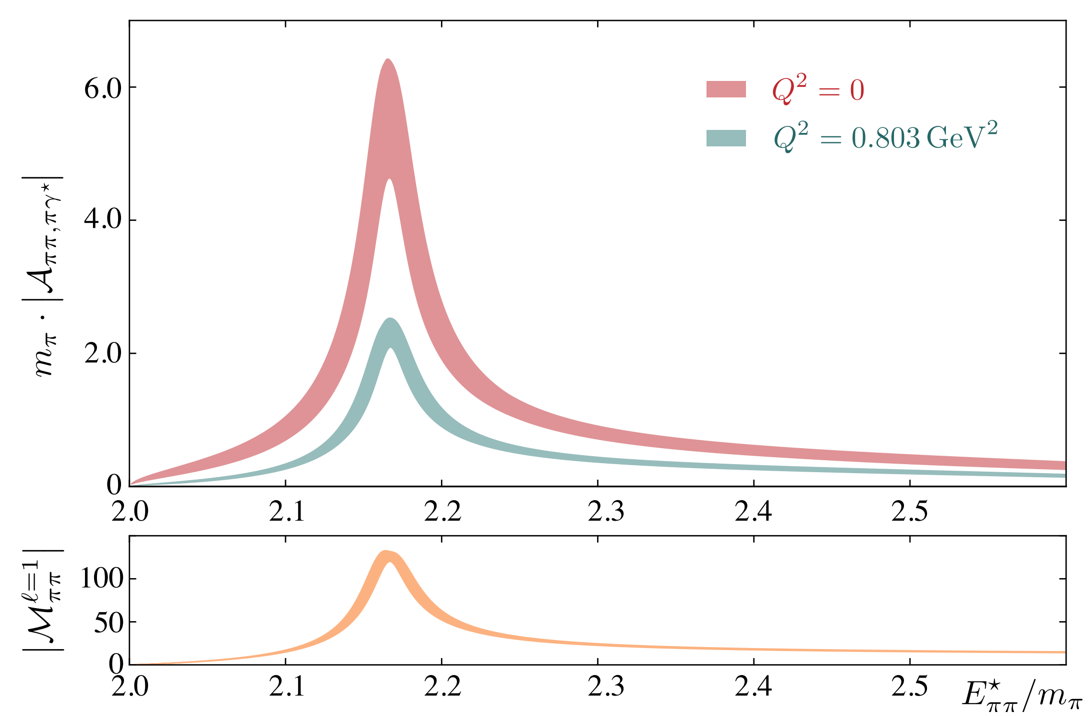
The future of spectroscopy

- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*

formalism understood:

RB, Hansen (2016)
RB, Hansen (2015)
RB, Hansen, Walker-Loud (2015)

first implementation: $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ρ



RB, Dudek, Edwards, Thomas, Shultz, Wilson (2015, 2016)
RB, Dudek, Edwards, Thomas, Shultz, Wilson (2015, 2016)

The future of spectroscopy

- *Coupled channels*
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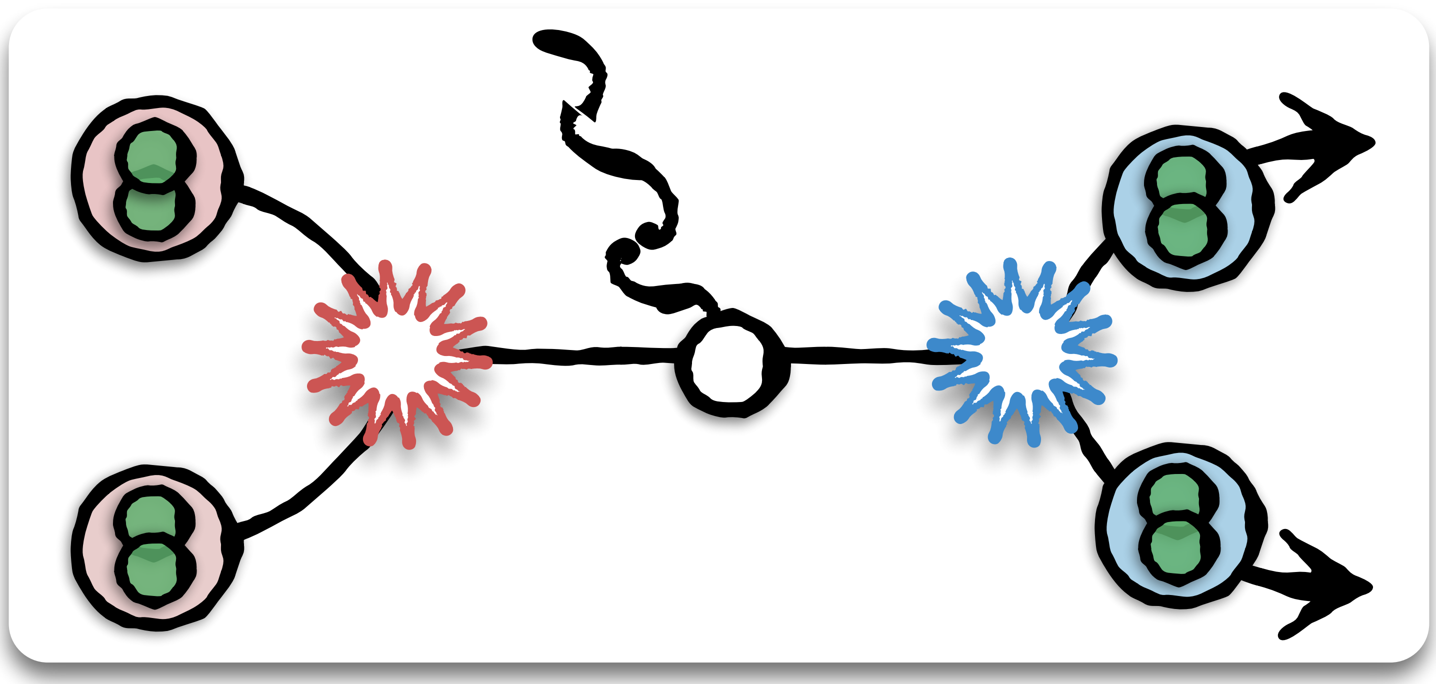
formalism understood:

RB, Hansen (2016)
RB, Hansen (2015)
RB, Hansen, Walker-Loud (2015)



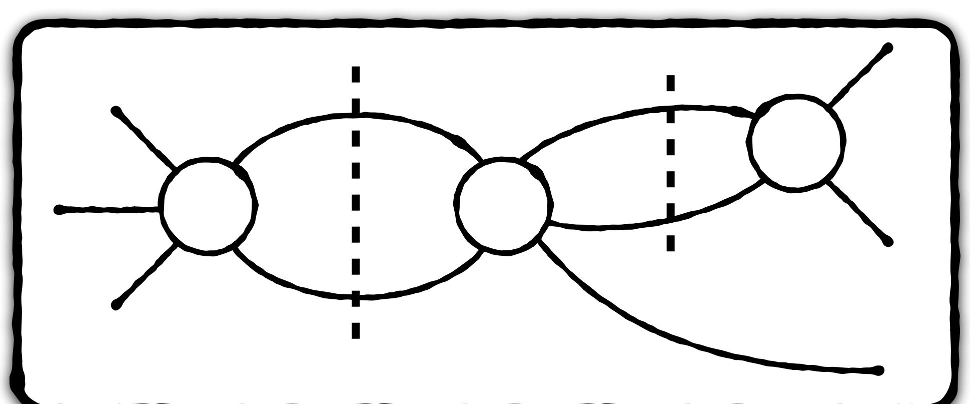
Can study elastic form factors !

The future of spectroscopy

- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems* [crucial for physical point calculations of interesting channels]

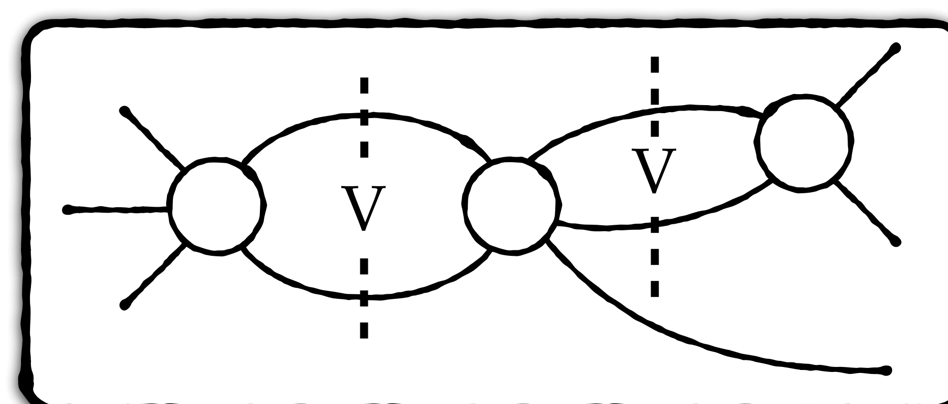
Challenges are not unlike those present in experiment

Experiment



three-particle unitarity is hard to satisfy,
e.g. simple Brett-Wigner & K-matrices violate it

Lattice QCD



understanding how to satisfy this exactly plus some finite-volume tricks amounts to solving this problem…

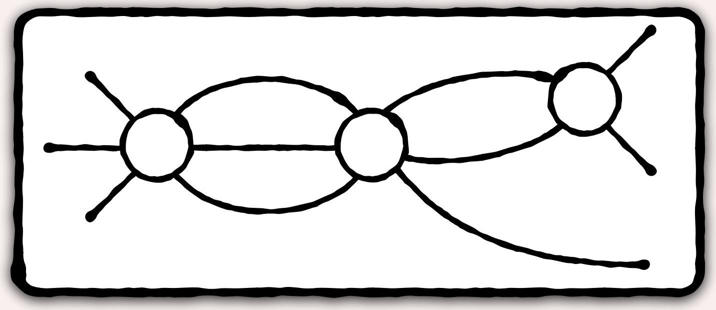
The future of spectroscopy

- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems* [crucial for physical point calculations of interesting channels]

formalism under construction:

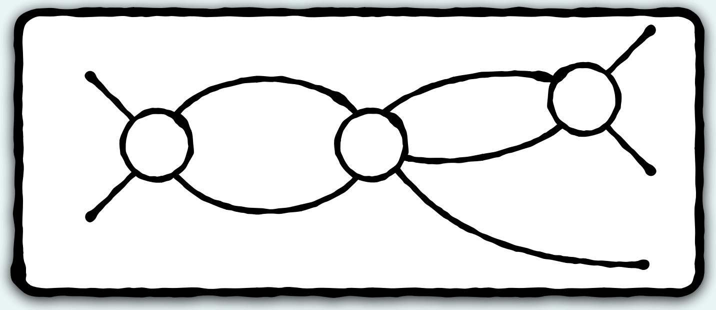
$$\det\left[1 + F_3 \mathcal{K}_{\mathrm{df},3}\right] = 0$$

Hansen & Sharpe (2014)



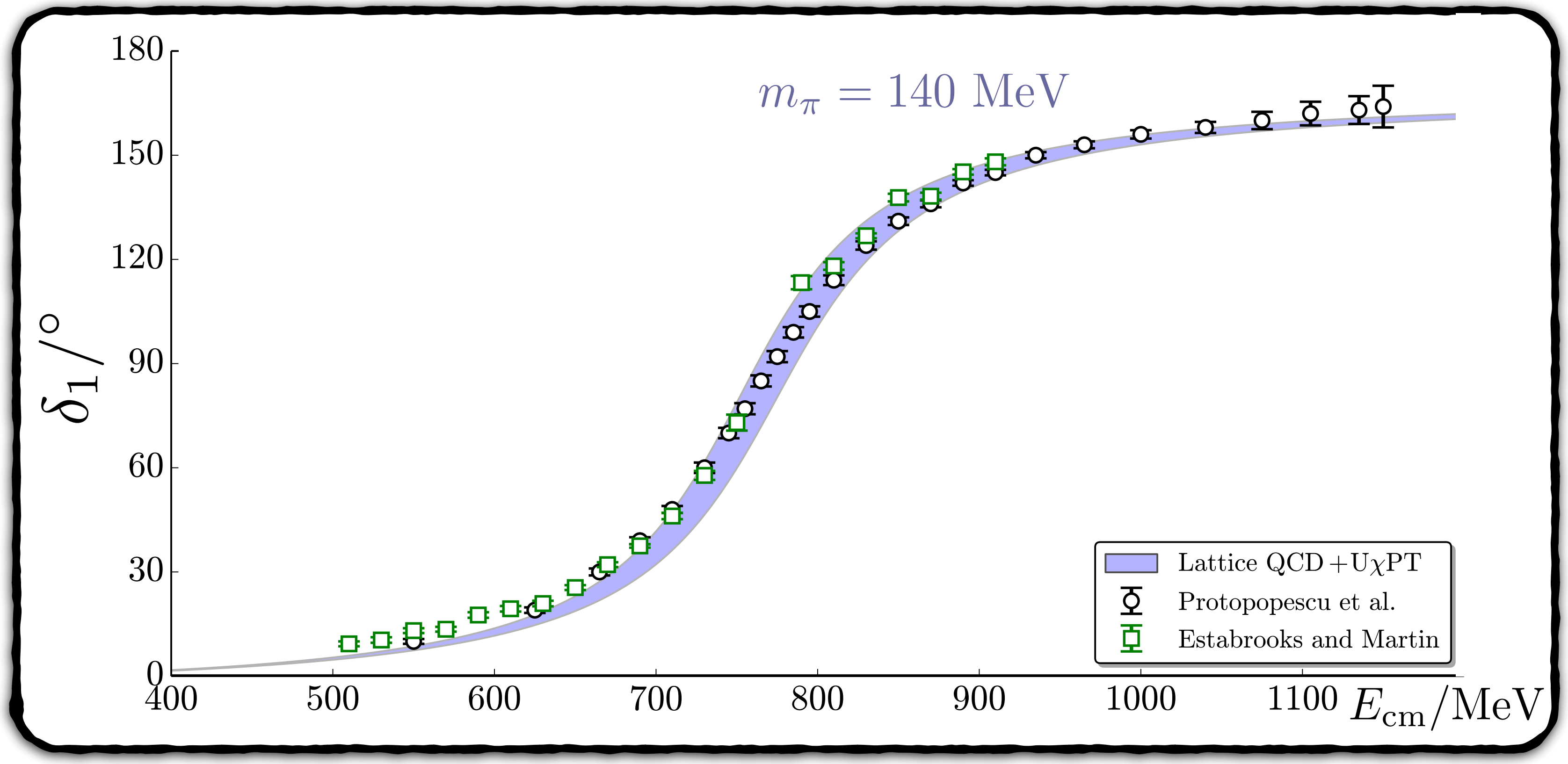
$$\det\left[1 + \begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix}\begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\mathrm{df},3} \end{pmatrix}\right] = 0$$

RB, Hansen & Sharpe **[in preparation]**



The future of spectroscopy

- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems*
- *Physical point, chiral extrapolation?*



Bolton, RB & Wilson Phys.Lett. B757 (2016) 50-56.

The future of spectroscopy

- *Coupled channels*
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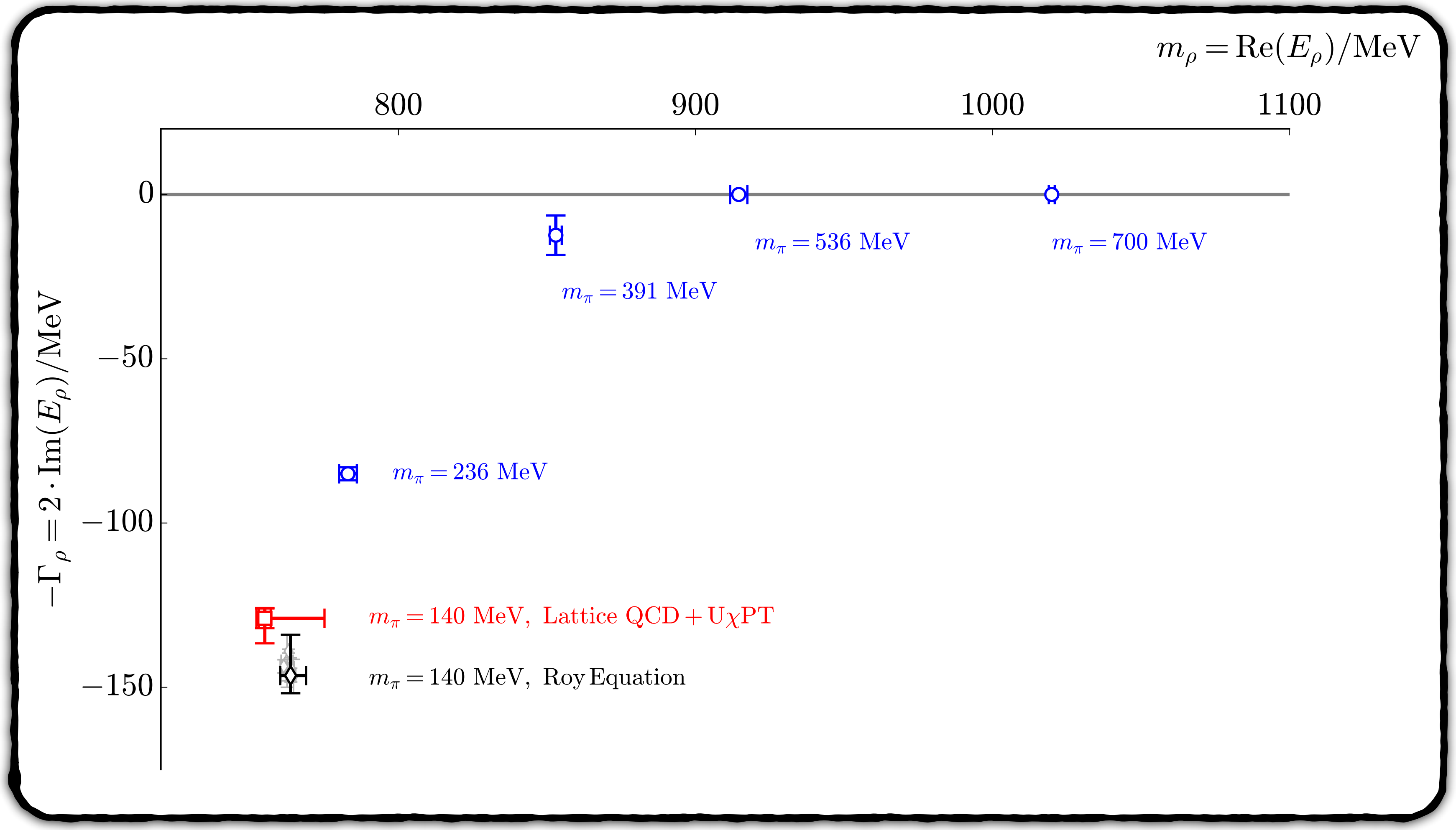
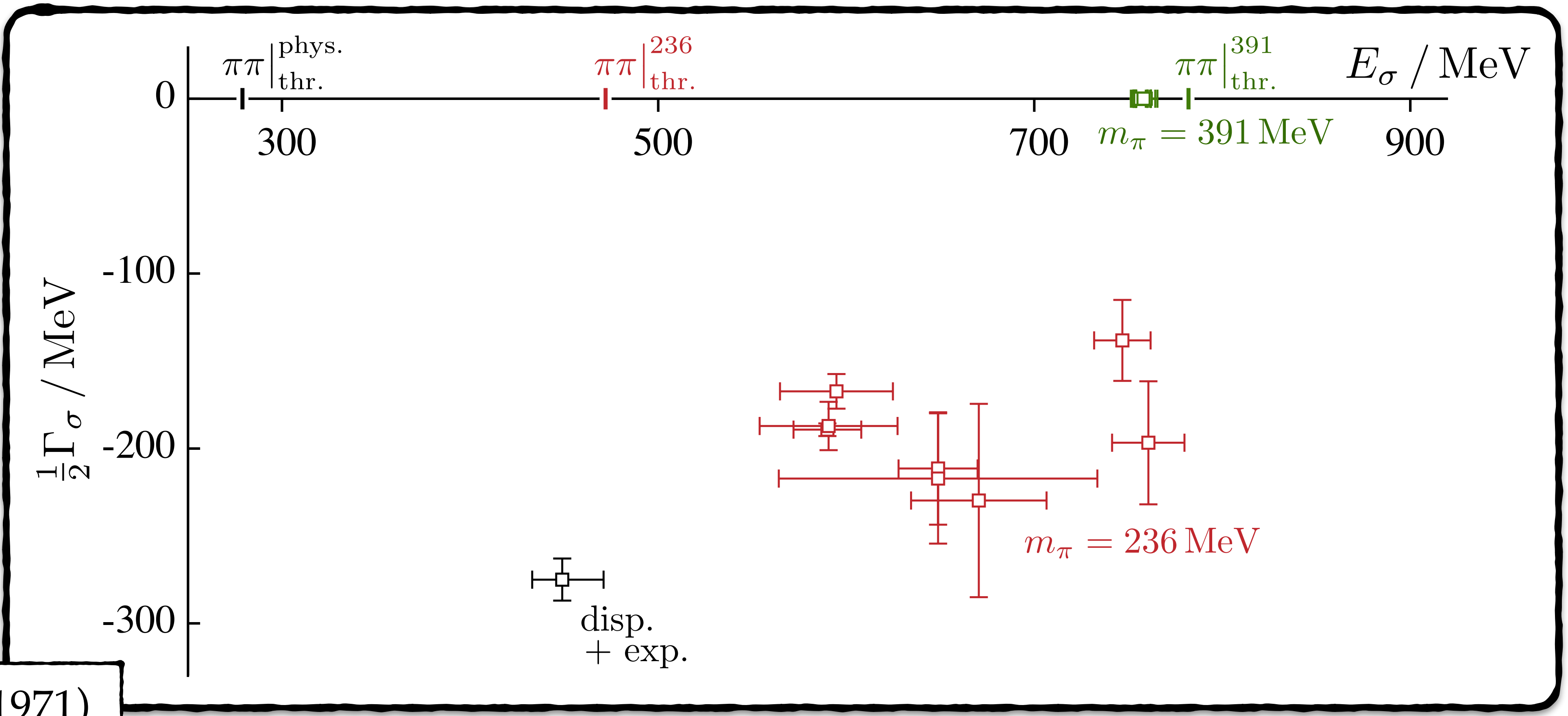
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems*
- *Physical point, chiral extrapolation?*
- *pole tracking*



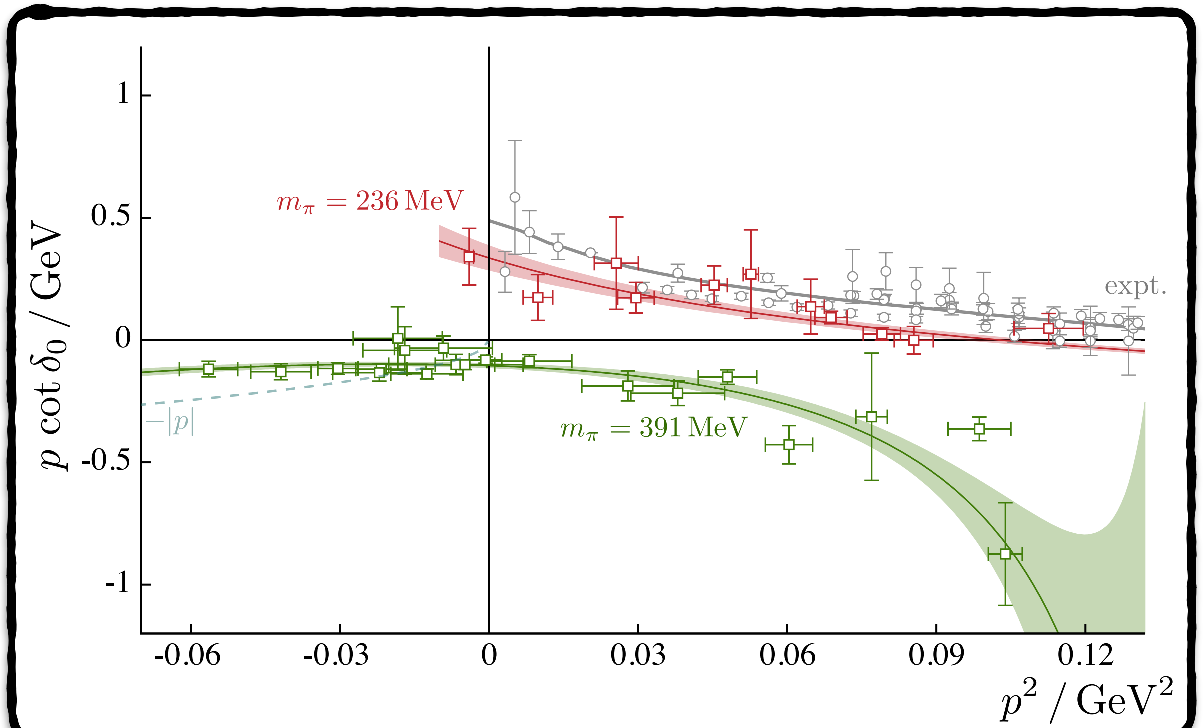
The future of spectroscopy

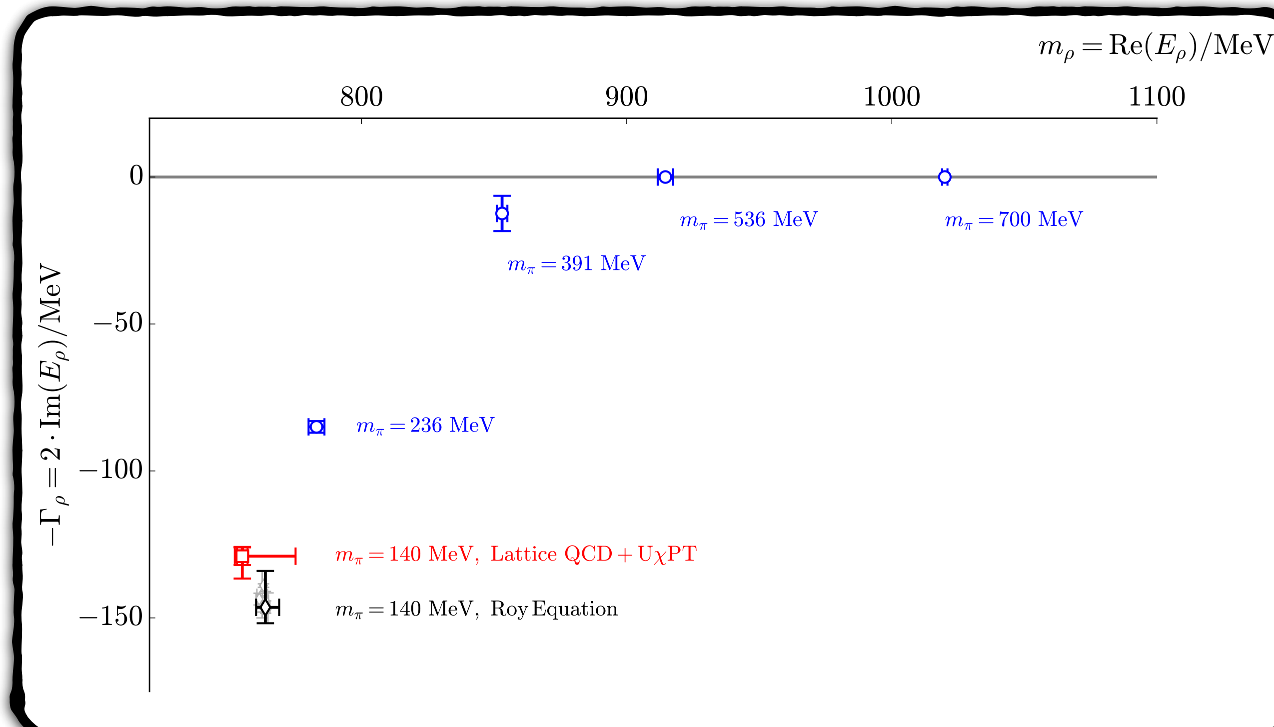
- *Coupled channels*
- *Baryons*
- *Electroweak form factors / structure - tetraquarks, molecules, etc.*
- *Three-particle systems*
- *Physical point, chiral extrapolation?*
- *pole tracking*
- *dispersive analysis*

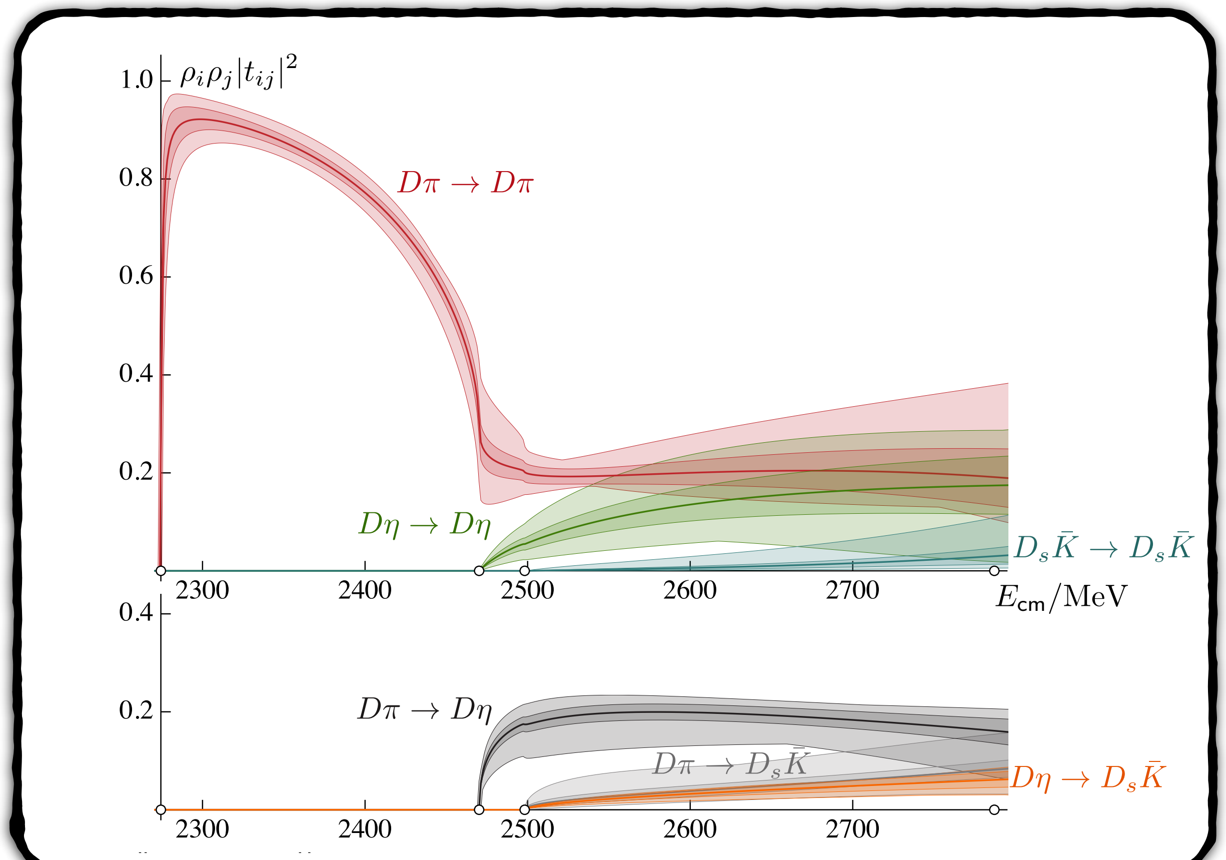


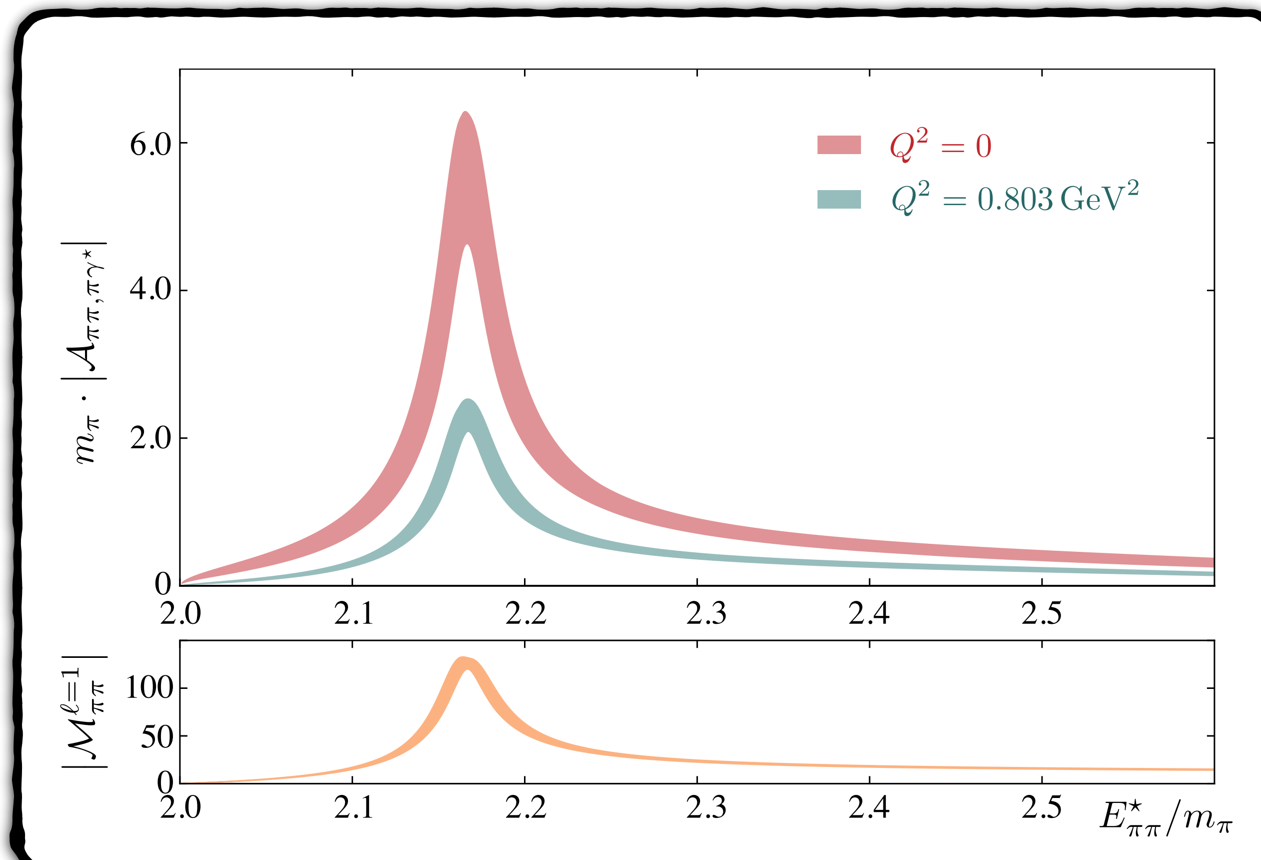
Roy (1971)

The big picture!









Collaborators & references

formalism


Hansen


Walker-Loud


Sharpe

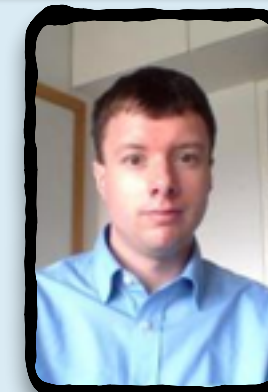
numerical

HadSpec Collaboration


Wilson


Moir


Shultz


Thomas


Dudek


Edwards


Ryan


Peardon

RB, Hansen - arXiv:1509.08507 [hep-lat]
RB, Hansen - Phys.Rev. D92 (2015) no.7, 074509.
RB, Hansen, Walker-Loud - Phys.Rev. D91 (2015) no.3, 034501.
RB - Phys.Rev. D89 (2014) no.7, 074507.

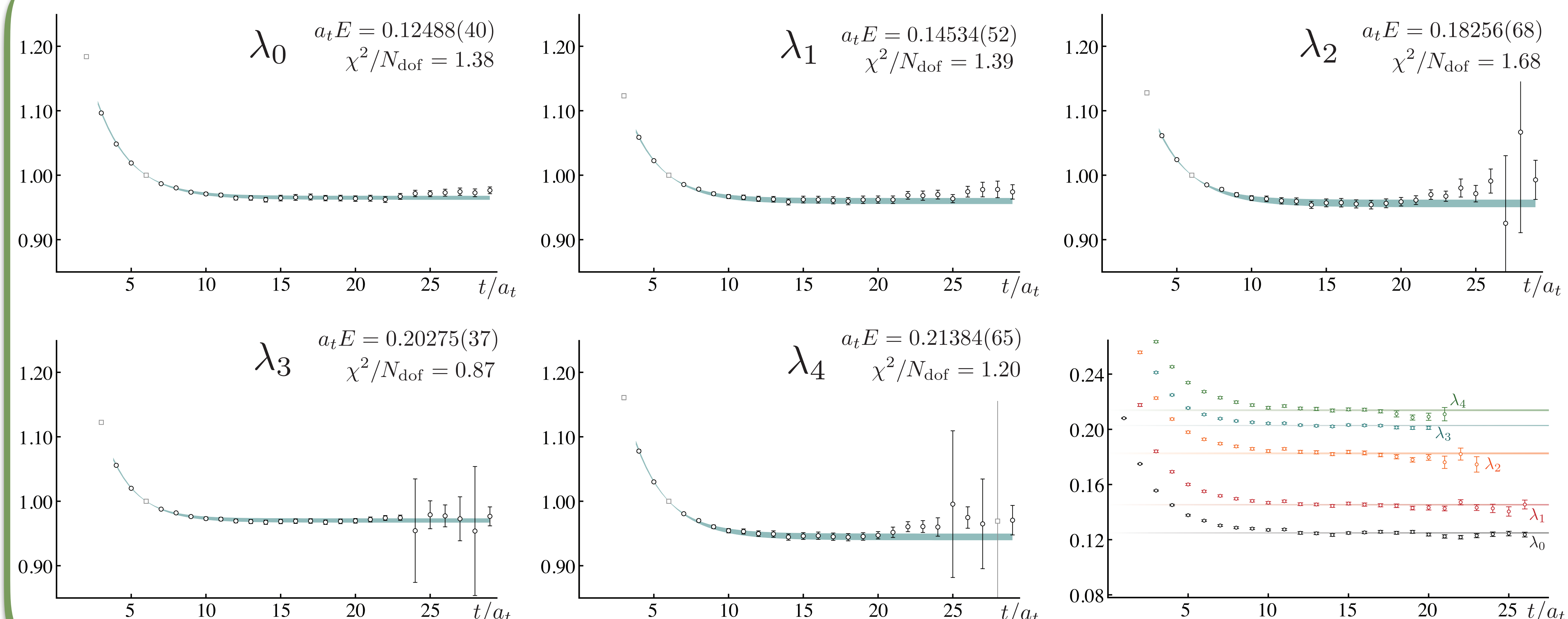
RB, Dudek, Edwards, Wilson - arXiv:1607.05900 [hep-ph].
Moir, Peardon, Ryan, Thomas, Wilson - arXiv:1607.07093 [hep-lat].
RB, Dudek, Edwards, Thomas, Shultz, Wilson - Phys.Rev. D93 (2016) 114508.
RB, Dudek, Edwards, Thomas, Shultz, Wilson - Phys.Rev.Lett. 115 (2015) 242001
Wilson, RB, Dudek, Edwards, Thomas - Phys.Rev. D92 (2015) no.9, 094502

Back-up slides

Determining spectrum

$$C(t)v_n(t) = \lambda_n(t)C(t_0)v_n(t),$$
$$\lambda_n(t) \sim e^{-E_n(t-t_0)}$$

$[000]\ T_1^-$



Parametrization

$$t(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\,\Gamma(s)}{m_R^2 - s - i\sqrt{s}\,\Gamma(s)},$$

$$\Gamma(s) = \frac{g_R^2}{6\pi} \frac{k^3}{s}$$

$$t_{ij}^{-1}(s) = \frac{1}{(2k_i)^\ell} K_{ij}^{-1}(s) \frac{1}{(2k_j)^\ell} + I_{ij}(s)\,,$$

$$\mathrm{Im}\, I_{ij}(s) = -\delta_{ij}\, \rho_i(s)$$

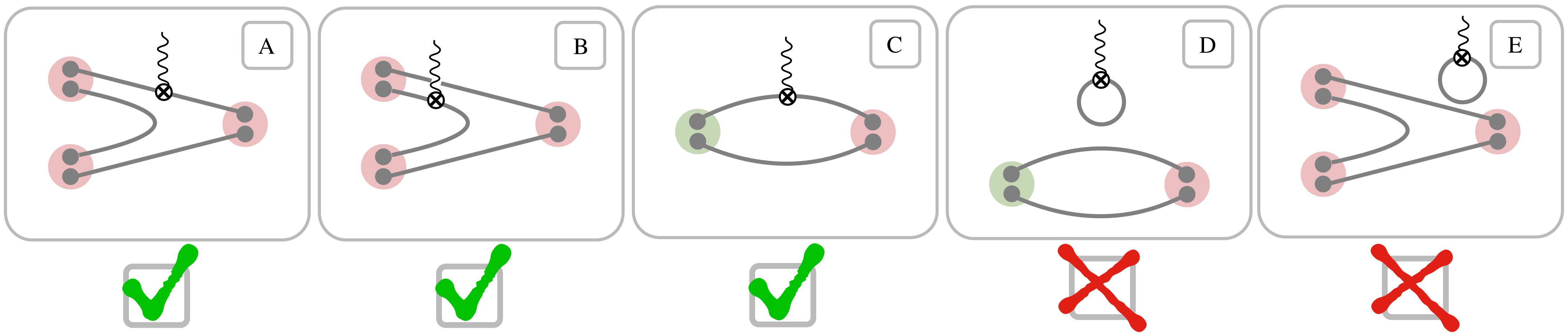
$$K_{ij}(s) = \frac{g_i\, g_j}{m^2 - s} + \sum_{n=0}^{N} \gamma_{ij}^{(n)} \left(\frac{s}{s_0}\right)^n,$$

$$K_{ij}^{-1} = \sum_{m=0}^{M} c_{ij}^{(m)} s^m,$$

$\pi\gamma^*$-to-$\pi\pi$ amplitude

Correlation functions

Contractions:



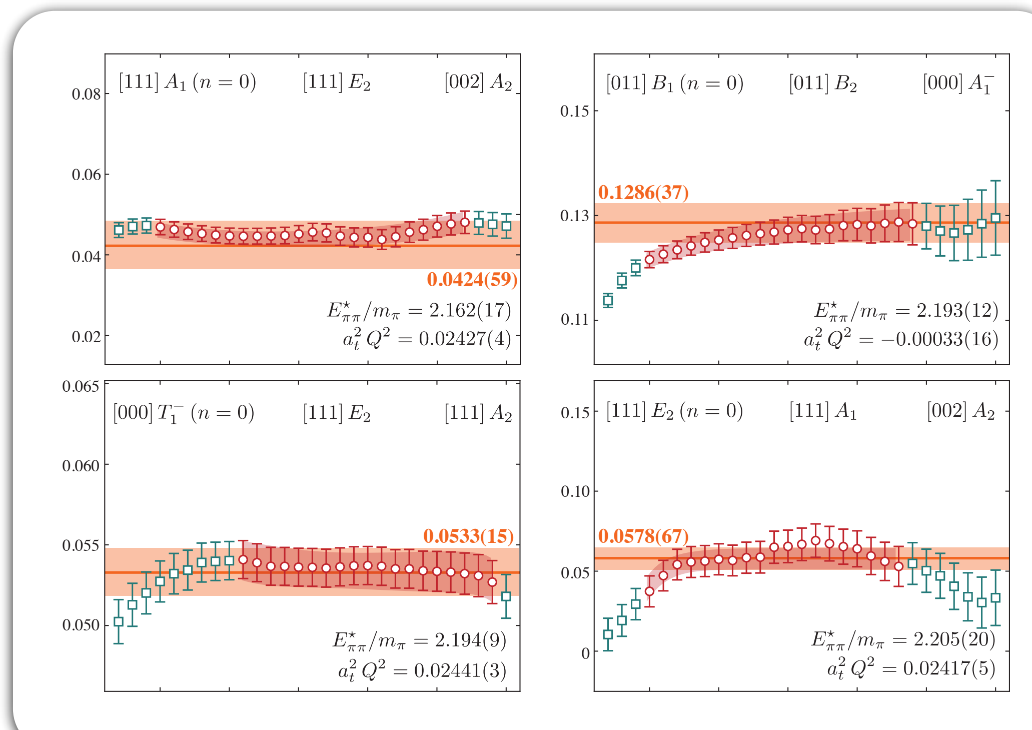
Operators and matrix elements:

$$C^{(3)}_{\pi\pi_n,\mu,\pi}(\mathbf{P}_\pi, \mathbf{P}_{\pi\pi}; \Delta t, t) = \langle 0 | \Omega_\pi(\Delta t, \mathbf{P}_\pi)\, \widetilde{\mathcal{J}}_\mu(t, \mathbf{P}_\pi - \mathbf{P}_{\pi\pi})\, \Omega^\dagger_{\pi\pi}(0, \mathbf{P}_{\pi\pi}) | 0 \rangle$$
$$= e^{-(E_{\pi\pi}-E_\pi)t}\, e^{-E_\pi \Delta t}\, \langle \pi; L | \widetilde{\mathcal{J}}_\mu | \pi\pi; L \rangle + \ldots$$

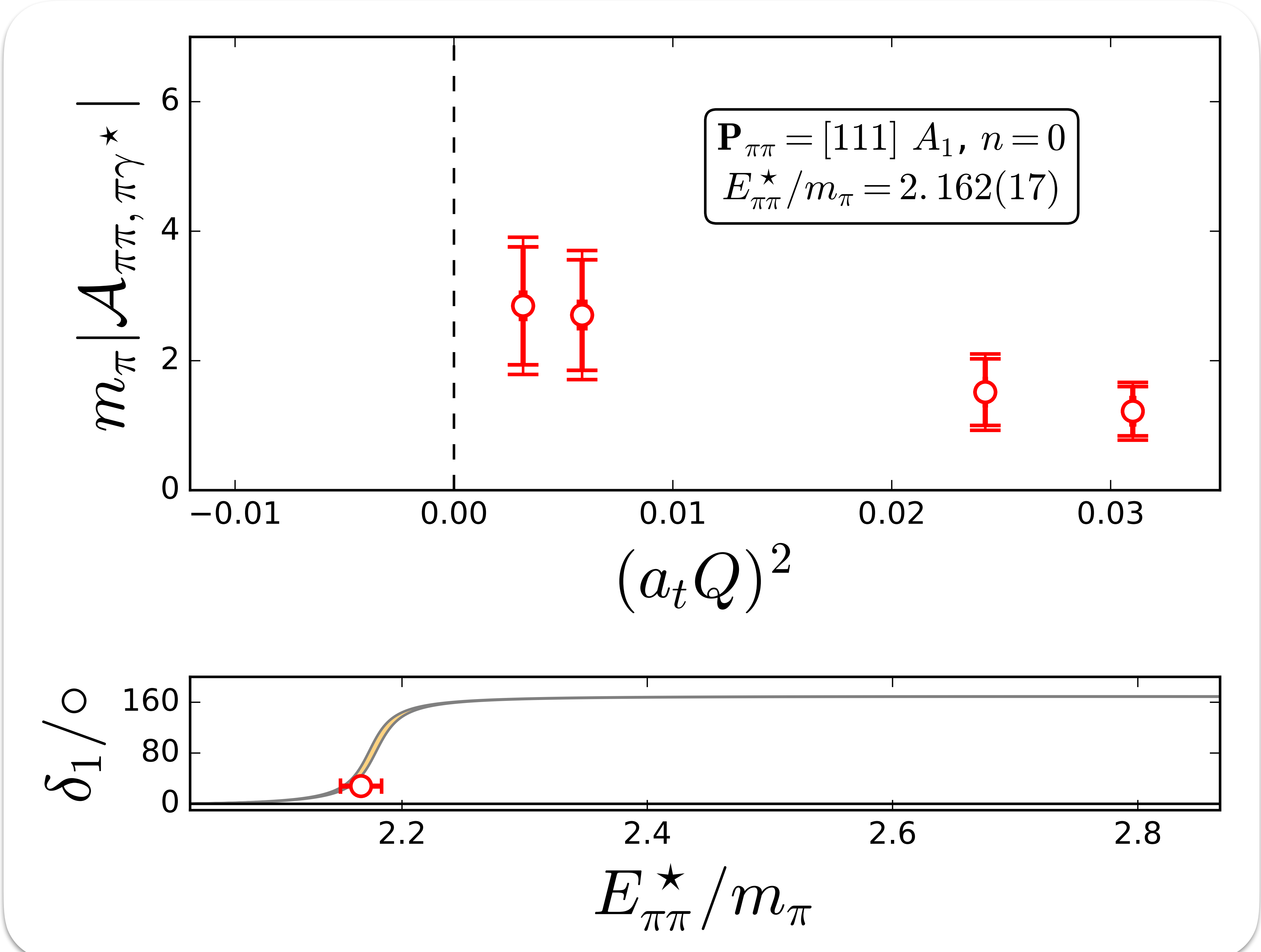
Ω_π =optimized 'π' operator,
linear combo. of ~ 10 ops.

$\Omega_{\pi\pi}$ =optimized '$\pi\pi$' operator,
linear combo. of ~ 20-30 ops.

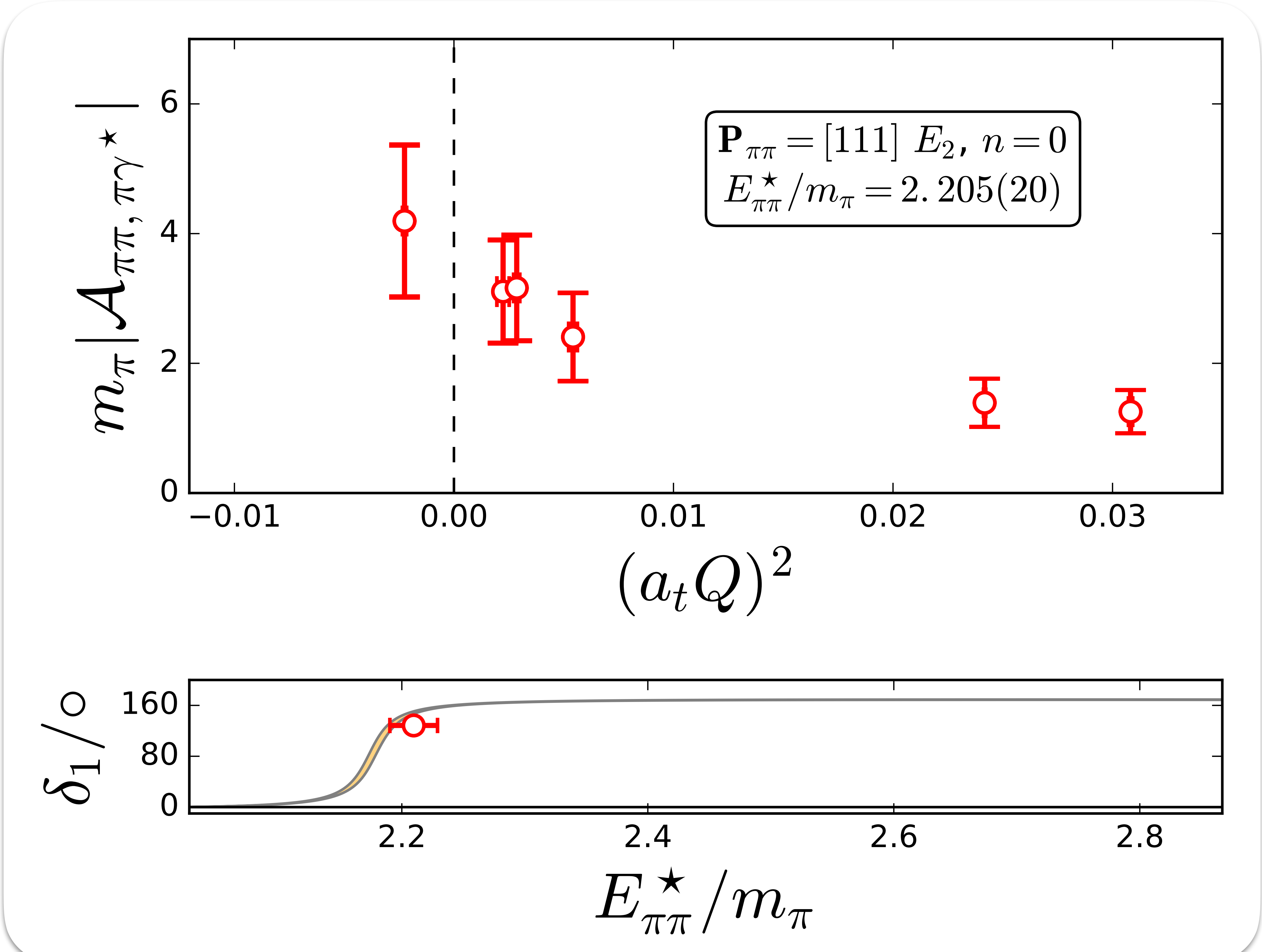
$\widetilde{\mathcal{J}}_\mu$ =electromagnetic current



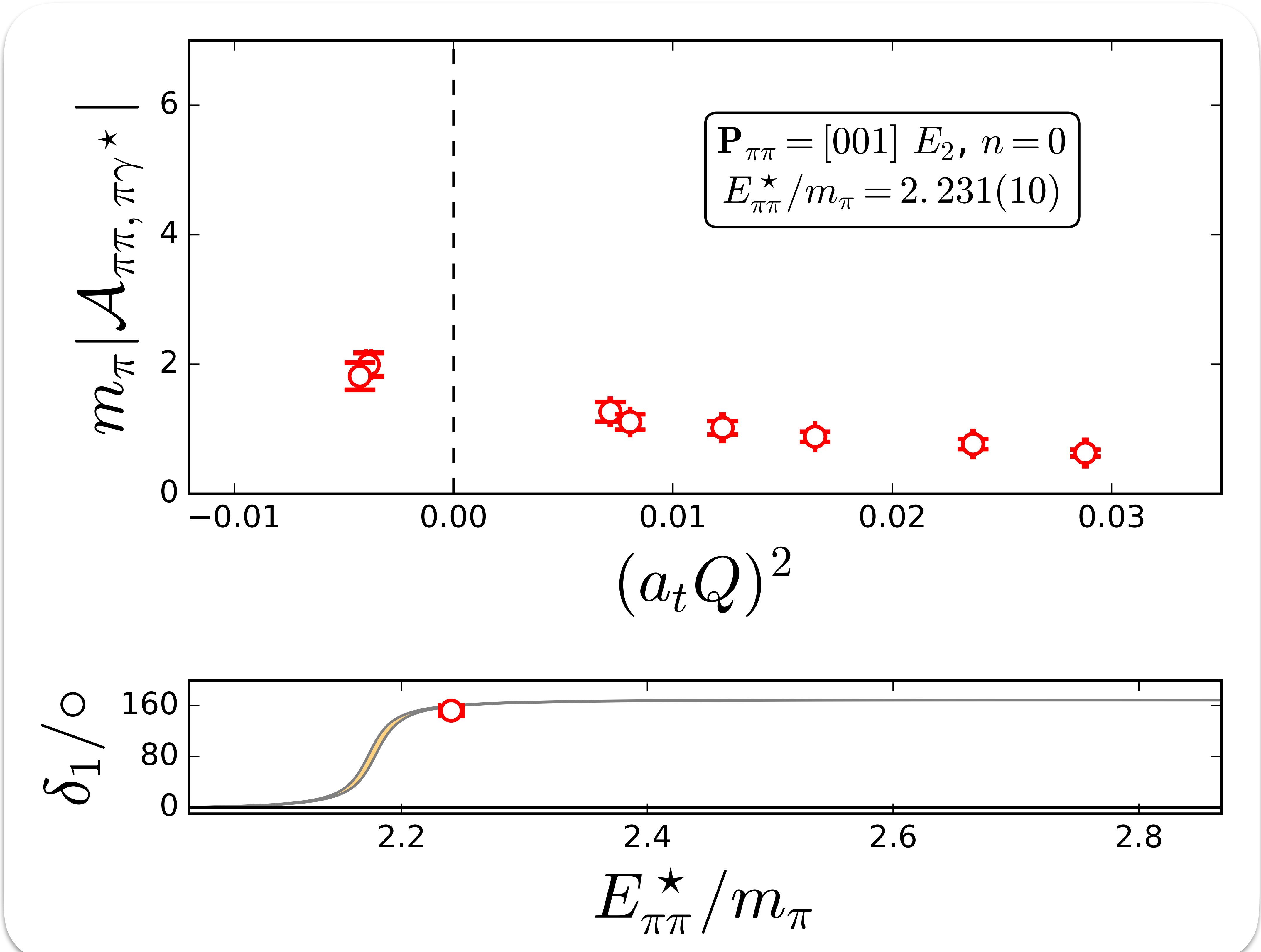
$\pi\gamma^*$-to-$\pi\pi$ amplitude



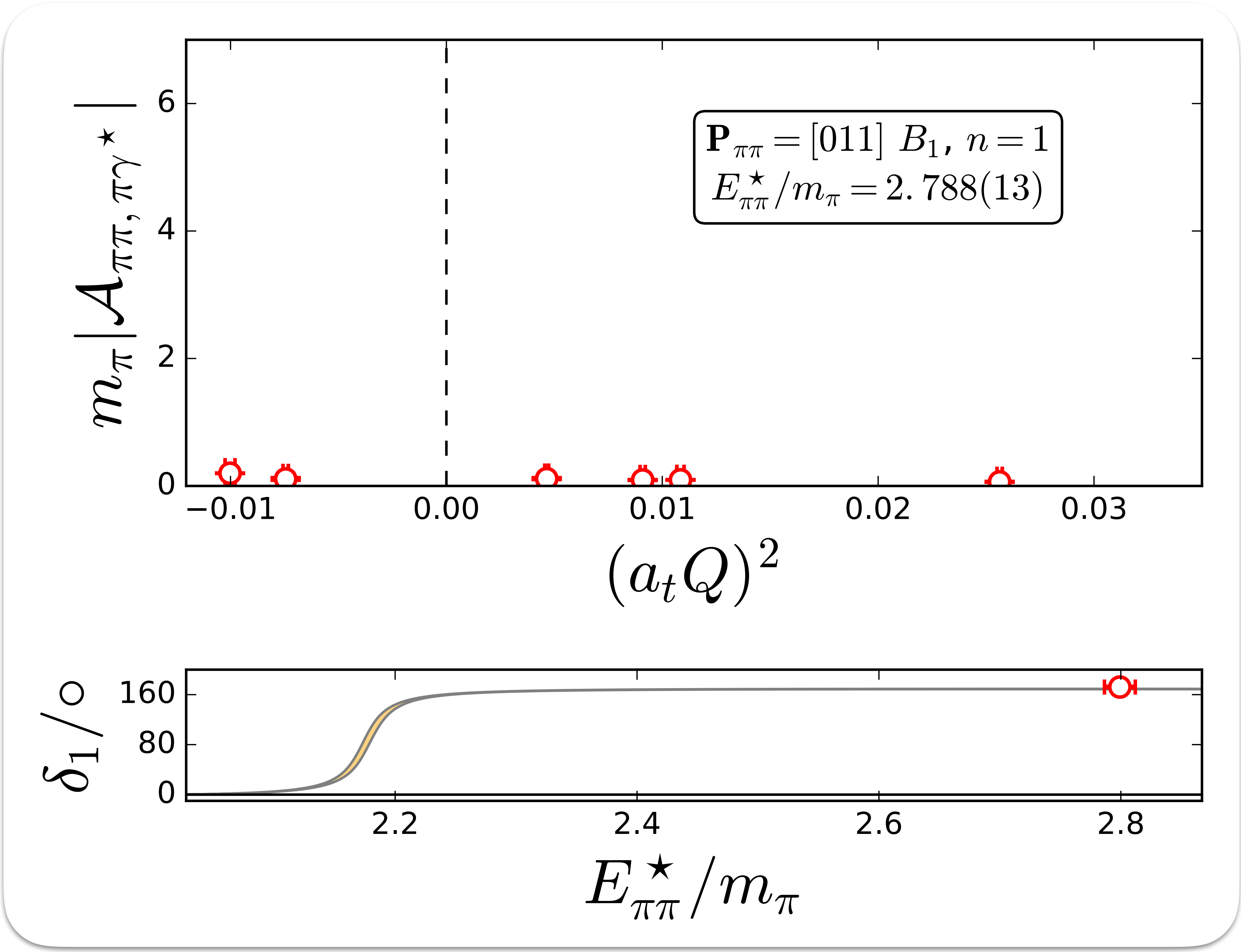
$\pi\gamma^*$-to-$\pi\pi$ amplitude



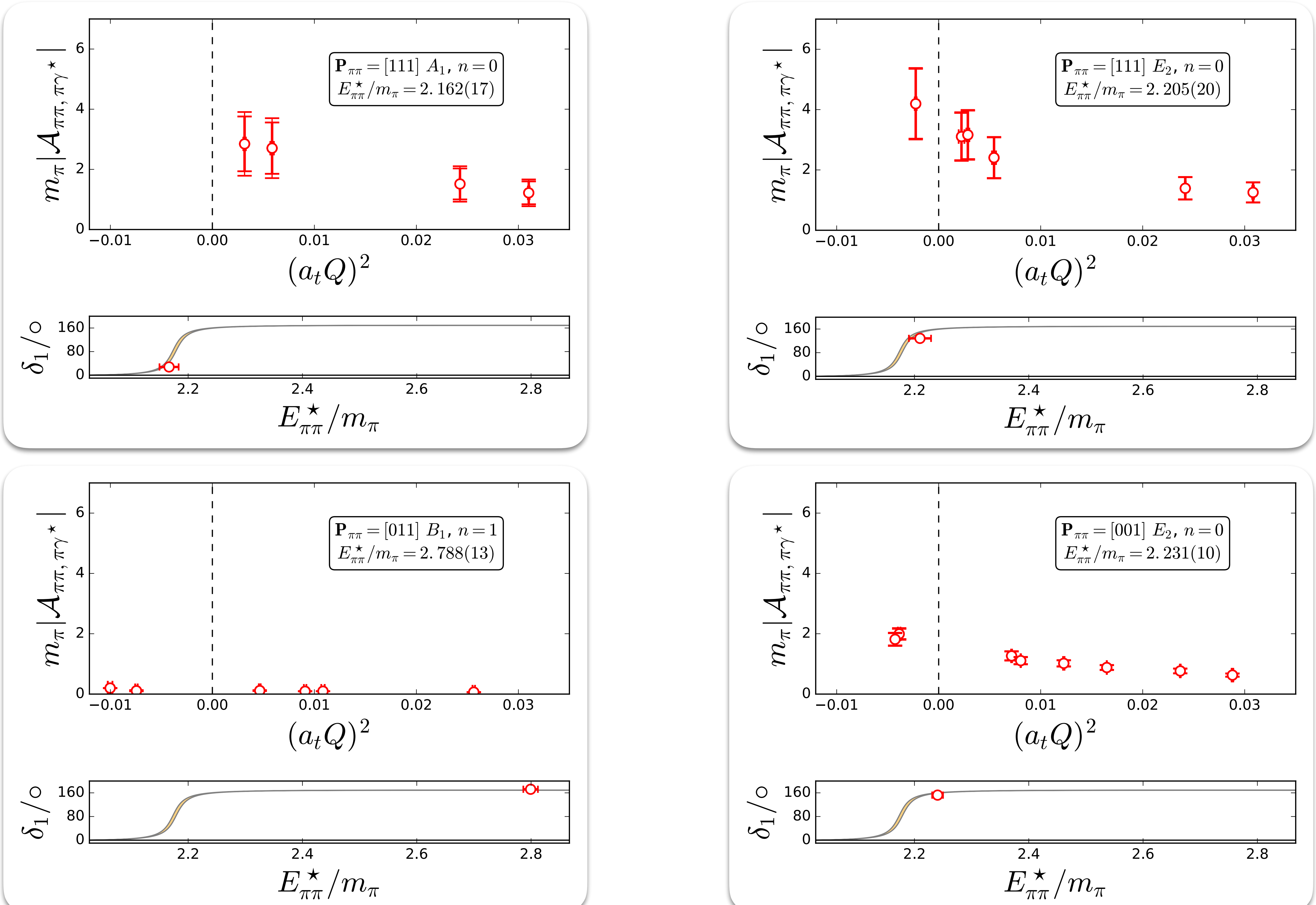
$\pi\gamma^*$-to-$\pi\pi$ amplitude



$\pi\gamma^*$-to-$\pi\pi$ amplitude

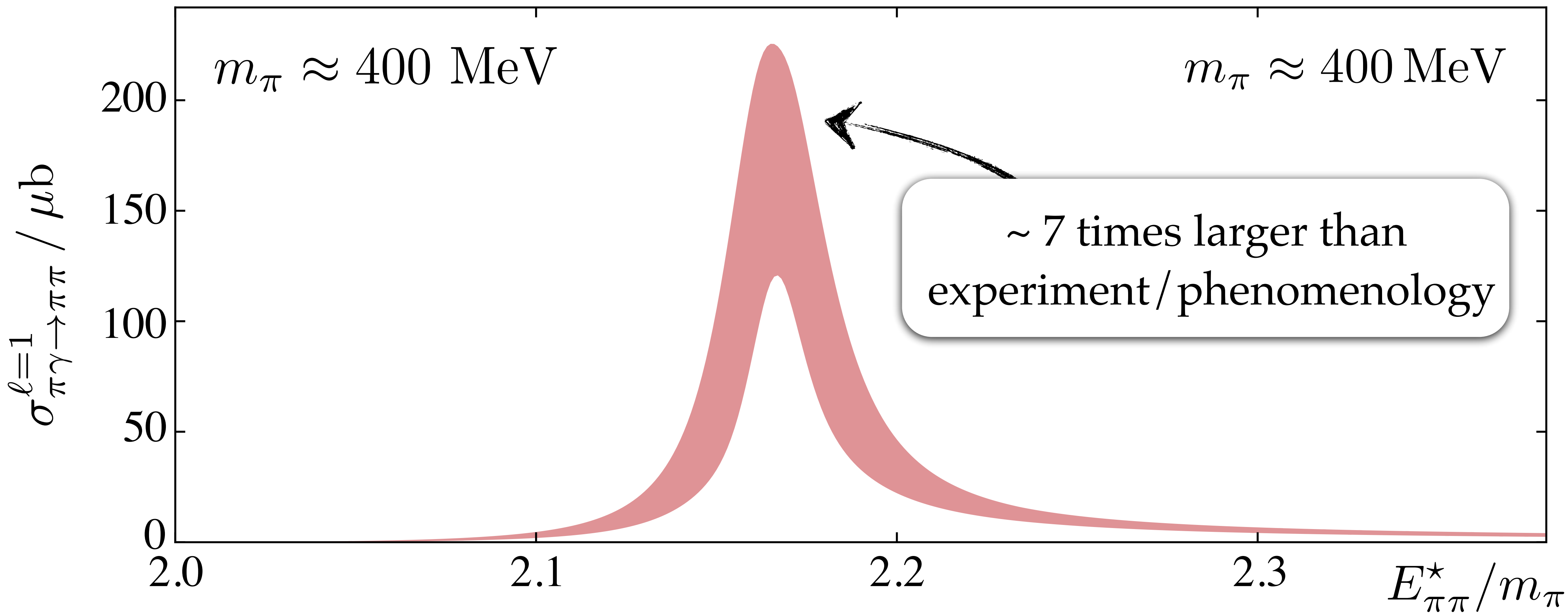


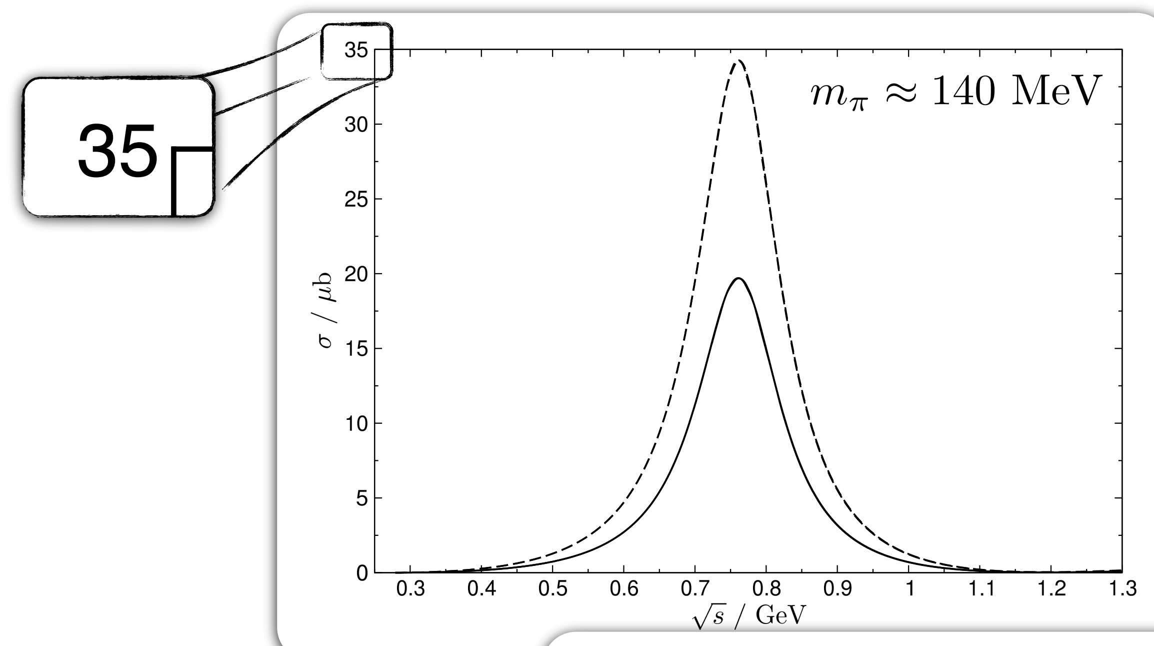
$\pi\gamma^*$-to-$\pi\pi$ amplitude



Comparison with phenomenology

$\pi\gamma$-to-$\pi\pi$ cross section

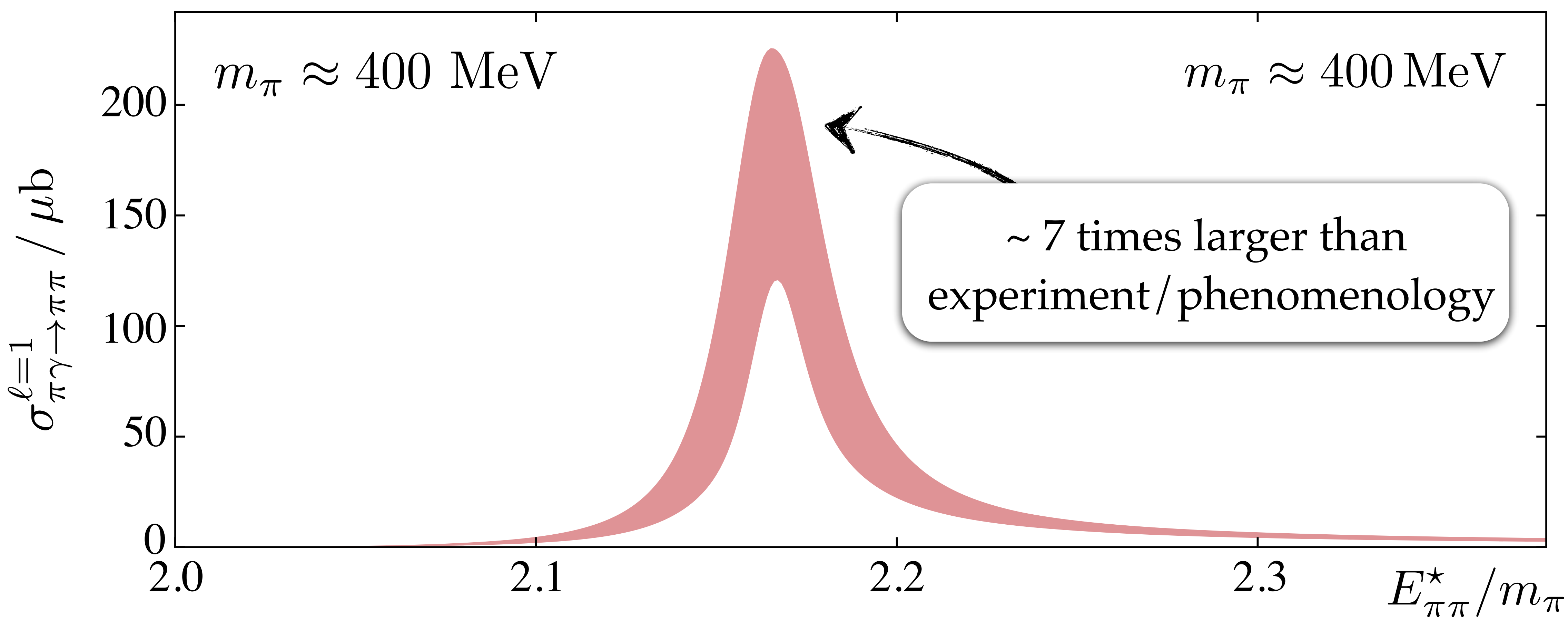




non trivial quark-mass dependence!

Hoferichter, Kubis, & Sakkas (2012)

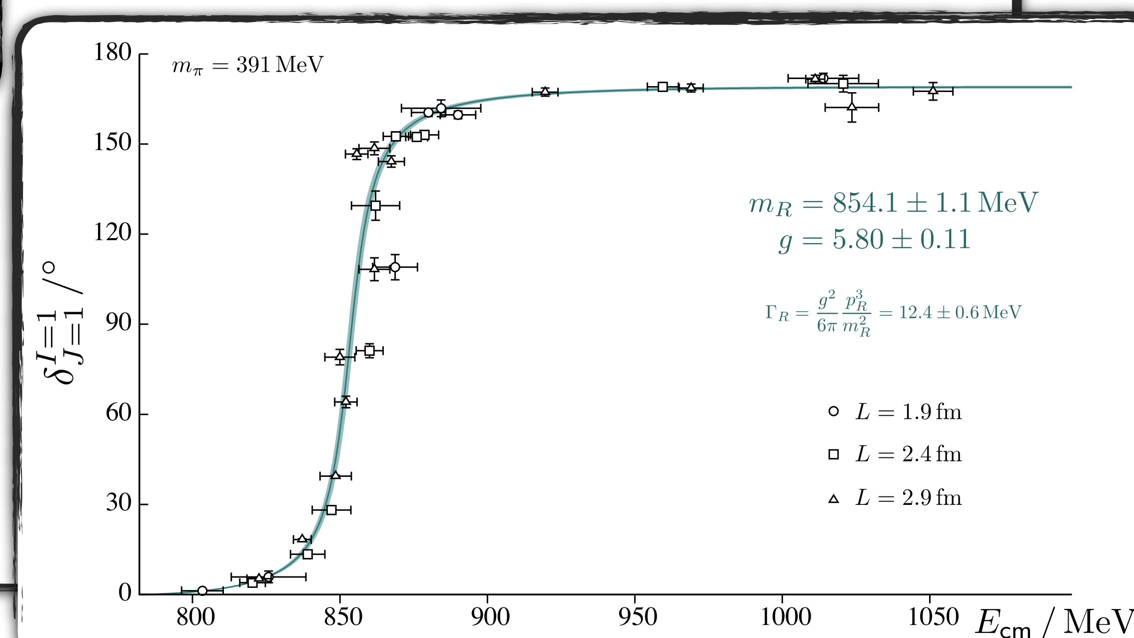
$\pi\gamma$-to-$\pi\pi$ cross section



$$\lim_{E^\star_{\pi\pi}\to m_\rho} \sigma(\pi^+\gamma \to \pi^+\pi^0) \propto \frac{q^\star_{\pi\gamma}\, F^2_{\pi\rho}(m_\rho, 0)}{m_\pi^2} \times \frac{1}{\Gamma_1(m_\rho)}$$

0.60 x (physical)

12 x (physical)



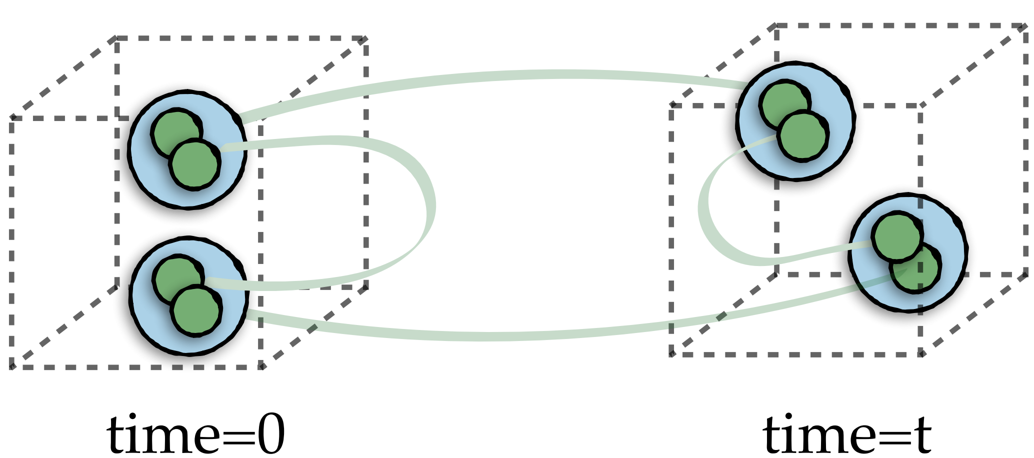
On determining correlation function using small basis of operators

Extracting the spectrum

Two-point correlation functions:

$$
\begin{aligned}
C_{ab}^{2pt.}(t,\mathbf{P}) &\equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n \langle 0|\mathcal{O}_b(t,\mathbf{P})|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n \langle 0|e^{t\hat{H}_{QCD}}\mathcal{O}_b(0,\mathbf{P})e^{-t\hat{H}_{QCD}}|n,L\rangle\langle n,L|\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle \\
&= \sum_n Z_{b,n}Z_{a,n}^\dagger e^{-E_n t}
\end{aligned}
$$

spectrum

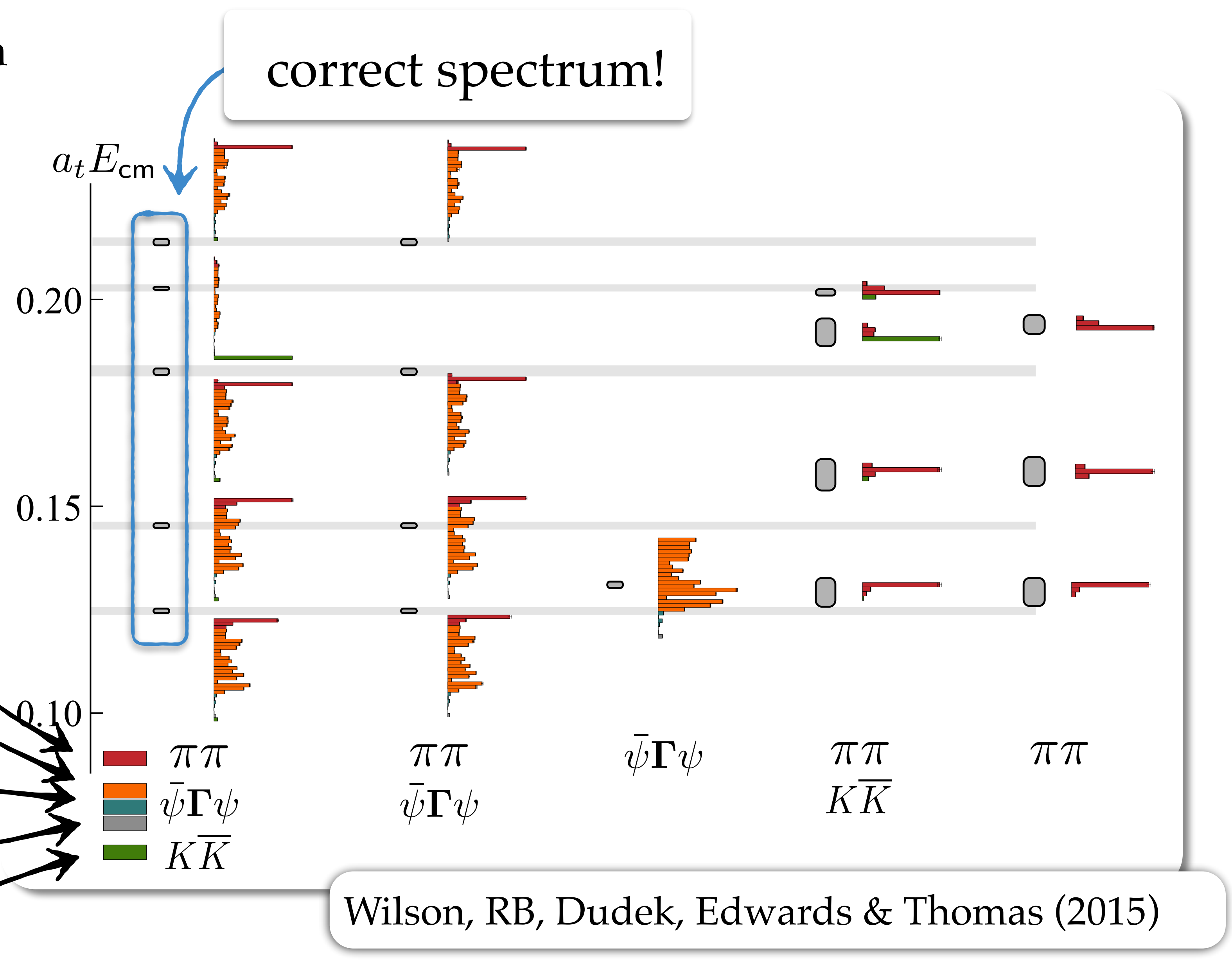


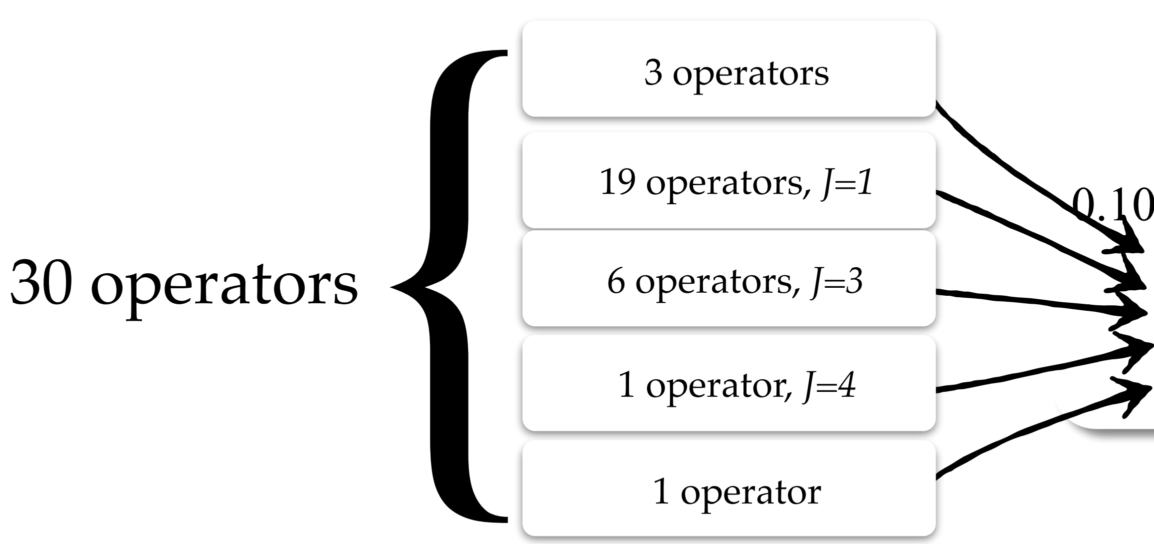
Extracting the spectrum

Two-point correlation functions:

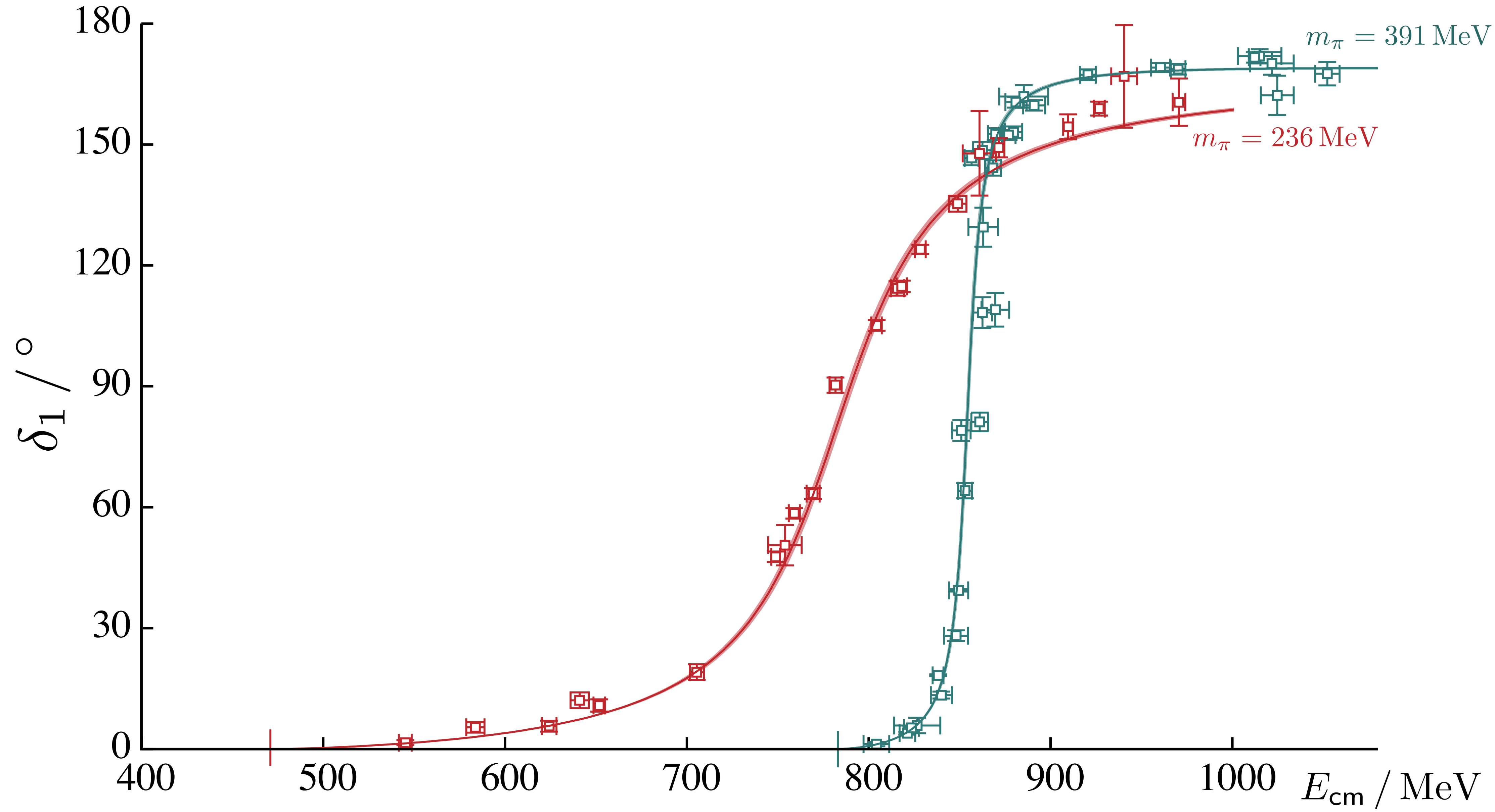
$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

- Use a large basis of operators with the same quantum numbers
- Diagonalize correlation function





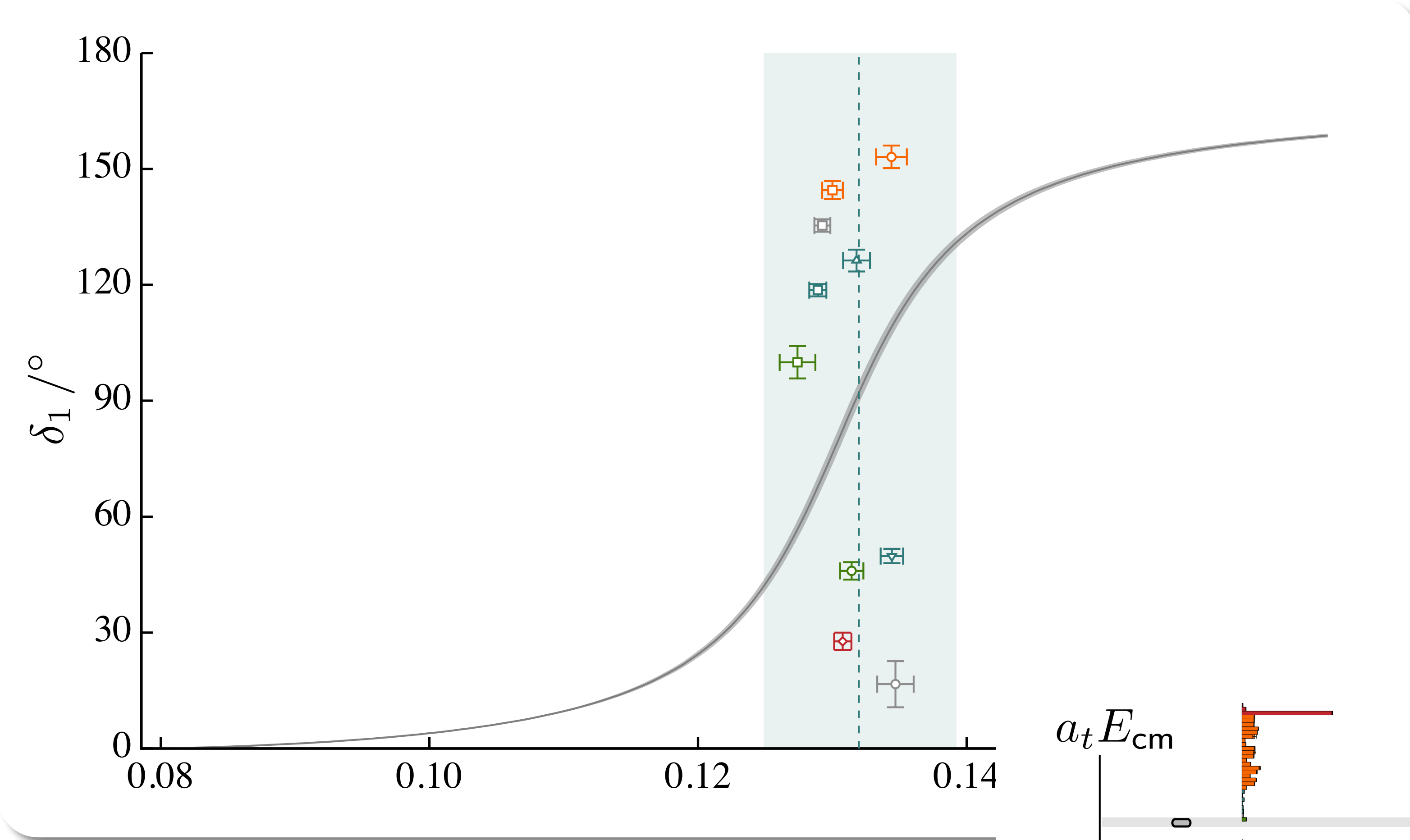
Isovector $\pi\pi$ scattering

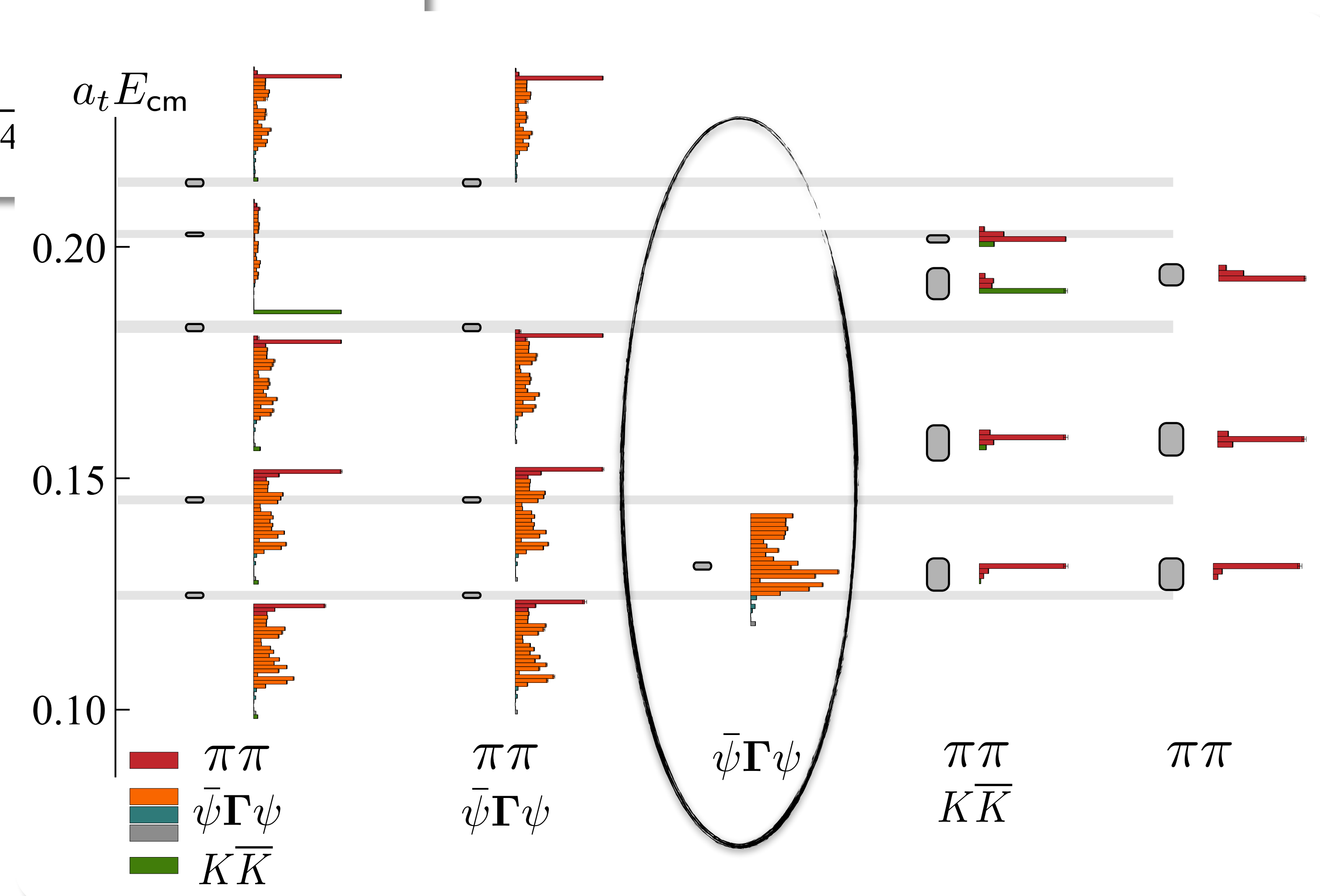


HadSpec Collaboration

Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

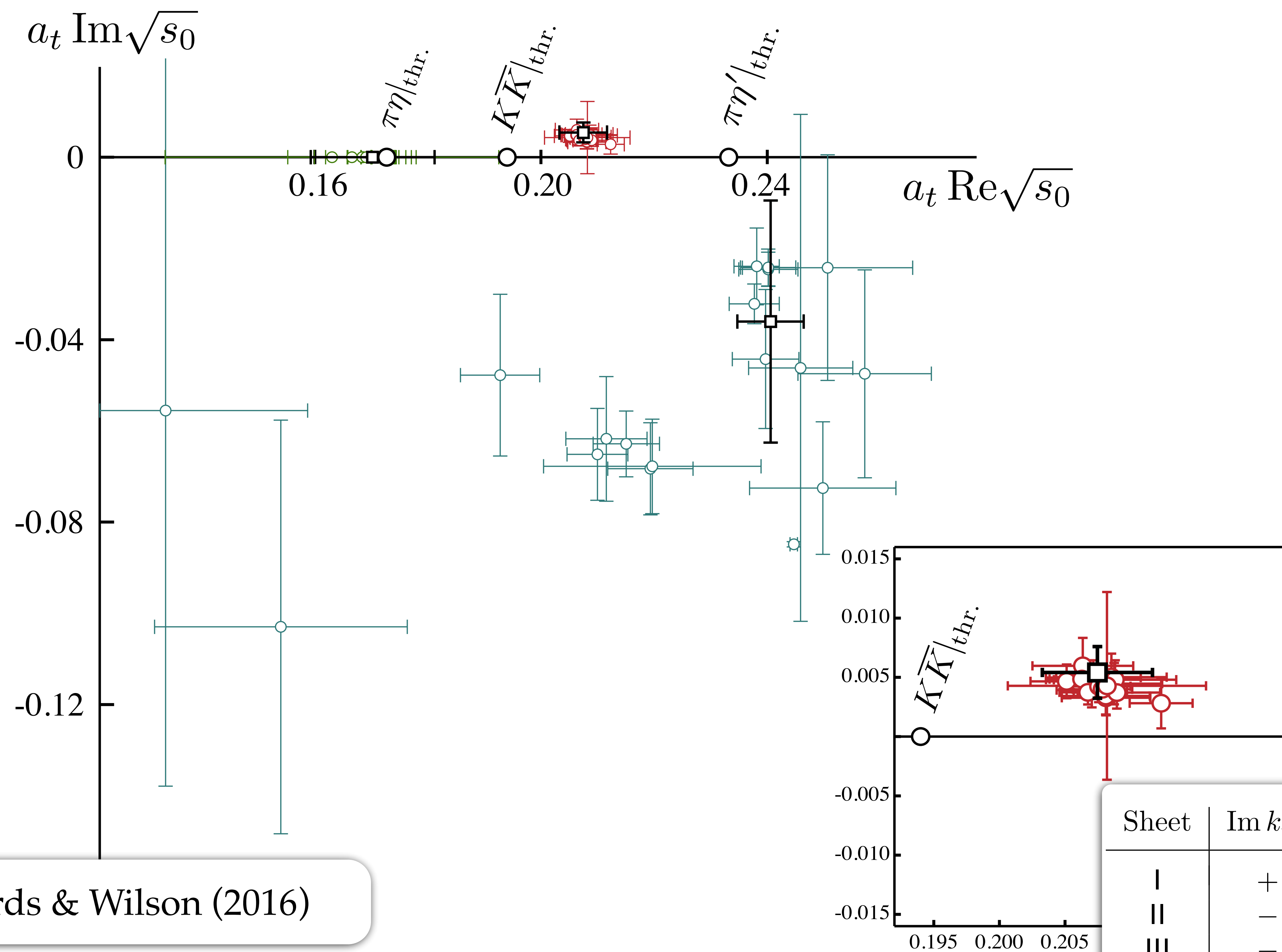
The incorrect answer





$a_0(980)$ poles

πη-KK-πη′ in I=1, m_π=391MeV



Sheet	Im $k_{\pi\eta}$	Im $k_{K\overline{K}}$	
I	+	+	
II	−	+	
III	−	−	[blue]
IV	+	−	[red]

Dudek, Edwards & Wilson (2016)

RB

Unitarized χPT

$$\mathcal{M}_{\mathrm{U}\chi\mathrm{PT}} = \mathcal{M}_{\mathrm{LO}} \frac{1}{\mathcal{M}_{\mathrm{LO}} - \mathcal{M}_{\mathrm{NLO}}} \mathcal{M}_{\mathrm{LO}}$$

$$\begin{aligned} S &= 1 + 2i\sigma\mathcal{M} \\ \mathcal{M} &= (\mathrm{Re}(\mathcal{M}^{-1}) - i\sigma)^{-1} \\ \mathcal{M}^{-1} &= \mathcal{M}_{\mathrm{LO}}^{-1} \frac{1}{1 + \mathcal{M}_{\mathrm{LO}}^{-1}\mathcal{M}_{\mathrm{NLO}} + \ldots} = \mathcal{M}_{\mathrm{LO}}^{-1}\left(1 - \mathcal{M}_{\mathrm{LO}}^{-1}\mathcal{M}_{\mathrm{NLO}} + \ldots\right) \\ \mathrm{Re}(\mathcal{M}^{-1}) &= \mathcal{M}_{\mathrm{LO}}^{-1}\left(1 - \mathcal{M}_{\mathrm{LO}}^{-1}\mathrm{Re}(\mathcal{M}_{\mathrm{NLO}}) + \ldots\right) \end{aligned}$$

Dobado and Pelaez (1997)

Oller, Oset, and Pelaez (1998)

Oller, Oset, and Pelaez (1999)

LL-factor

Relationship between amplitude and "form factor":

$$\mathcal{A}_{\pi\pi,\pi\gamma^\star}(E^\star_{\pi\pi},Q^2) = \left(\frac{F(E^\star_{\pi\pi},Q^2)}{\cot\delta_1(E^\star_{\pi\pi}) - i}\right)\sqrt{\frac{16\pi}{q^\star_{\pi\pi}\,\Gamma(E^\star_{\pi\pi})}}$$

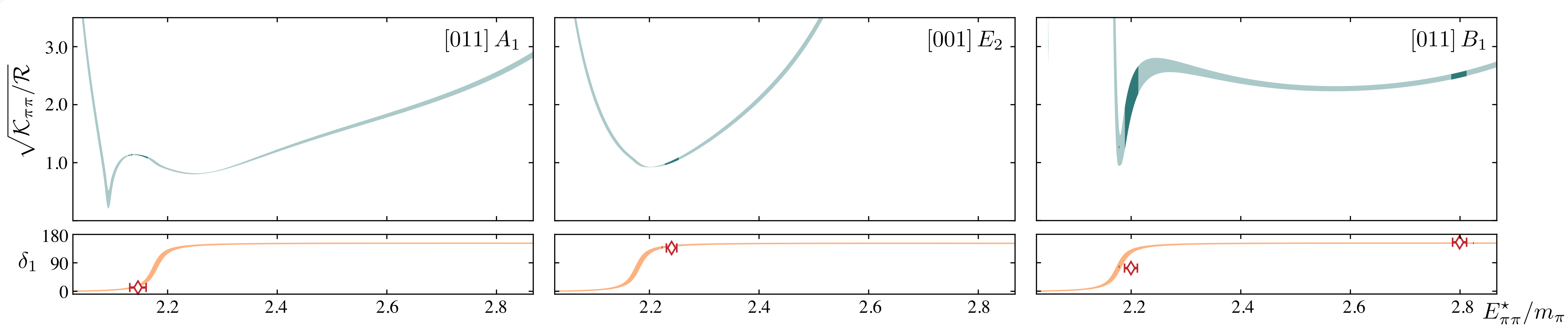
$$F(E^\star_{\pi\pi},Q^2) = \tilde{\mathcal{A}}(E^\star_{\pi\pi},Q^2;L)\sqrt{\frac{\mathcal{K}_{\pi\pi}}{\mathcal{R}}},$$

$$\frac{1}{\sqrt{2E^\star_{\pi\pi}\,\mathcal{K}_{\pi\pi}(E^\star_{\pi\pi})}} = \sin\delta_1(E^\star_{\pi\pi})\sqrt{\frac{16\pi}{q^\star_{\pi\pi}\,\Gamma(E^\star_{\pi\pi})}}$$

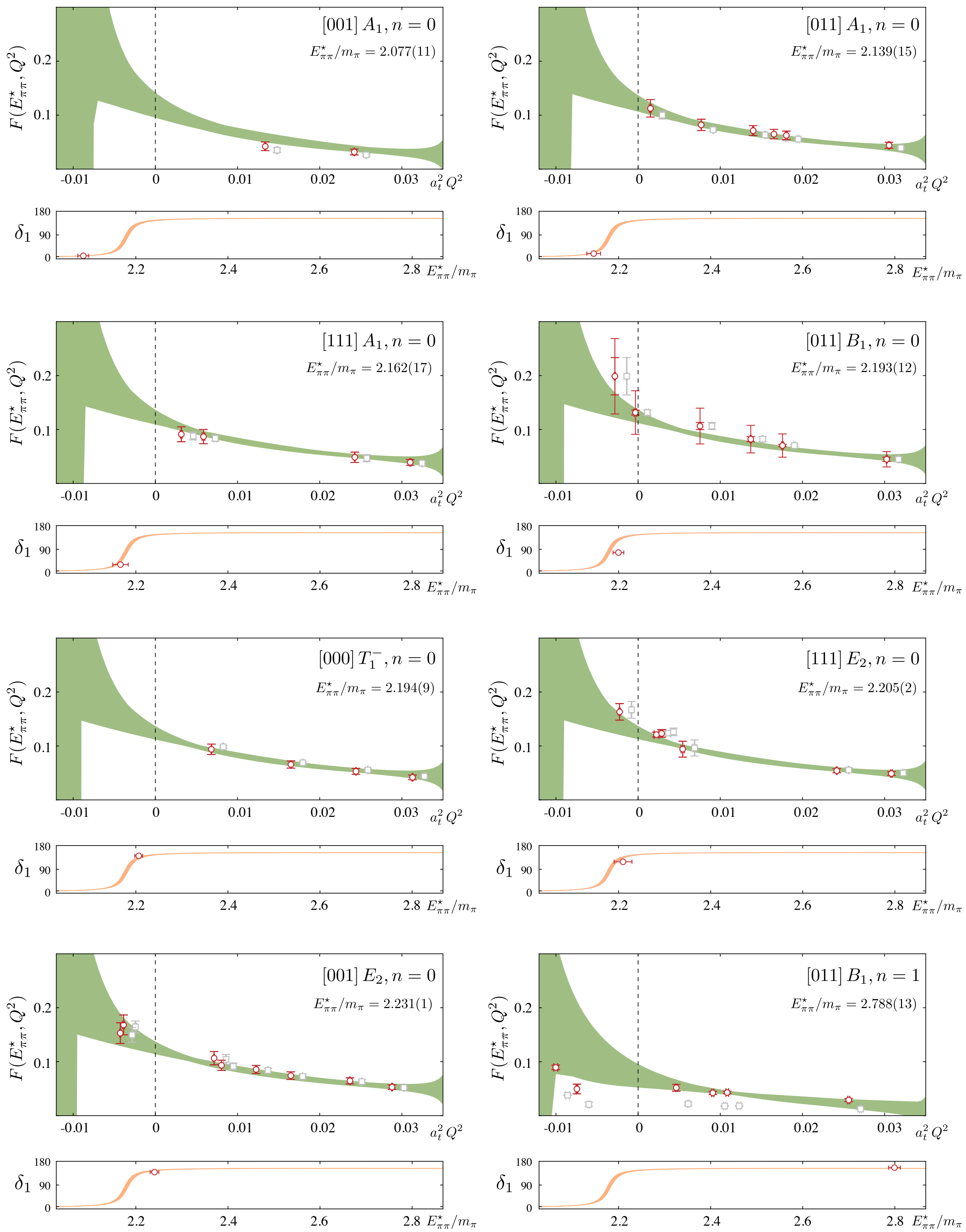
LL factor:

$$\frac{2E_\pi}{\mathcal{R}} = 32\pi\,\frac{E_\pi E_{\pi\pi}}{q^\star_{\pi\pi}}\,\cos^2\delta_1\,\frac{\partial}{\partial P^\star_{0,\pi\pi}}\left(\tan\delta_1 + \tan\phi^{\mathbf{P}_{\pi\pi},\Lambda_{\pi\pi}}\right)\Bigg|_{P^\star_{0,\pi\pi}=E^\star_{\pi\pi}}$$

$$= 32\pi\,\frac{E_\pi E_{\pi\pi}}{q^\star_{\pi\pi}}\left(\delta_1' + r\phi'\right),$$



“Form factor”

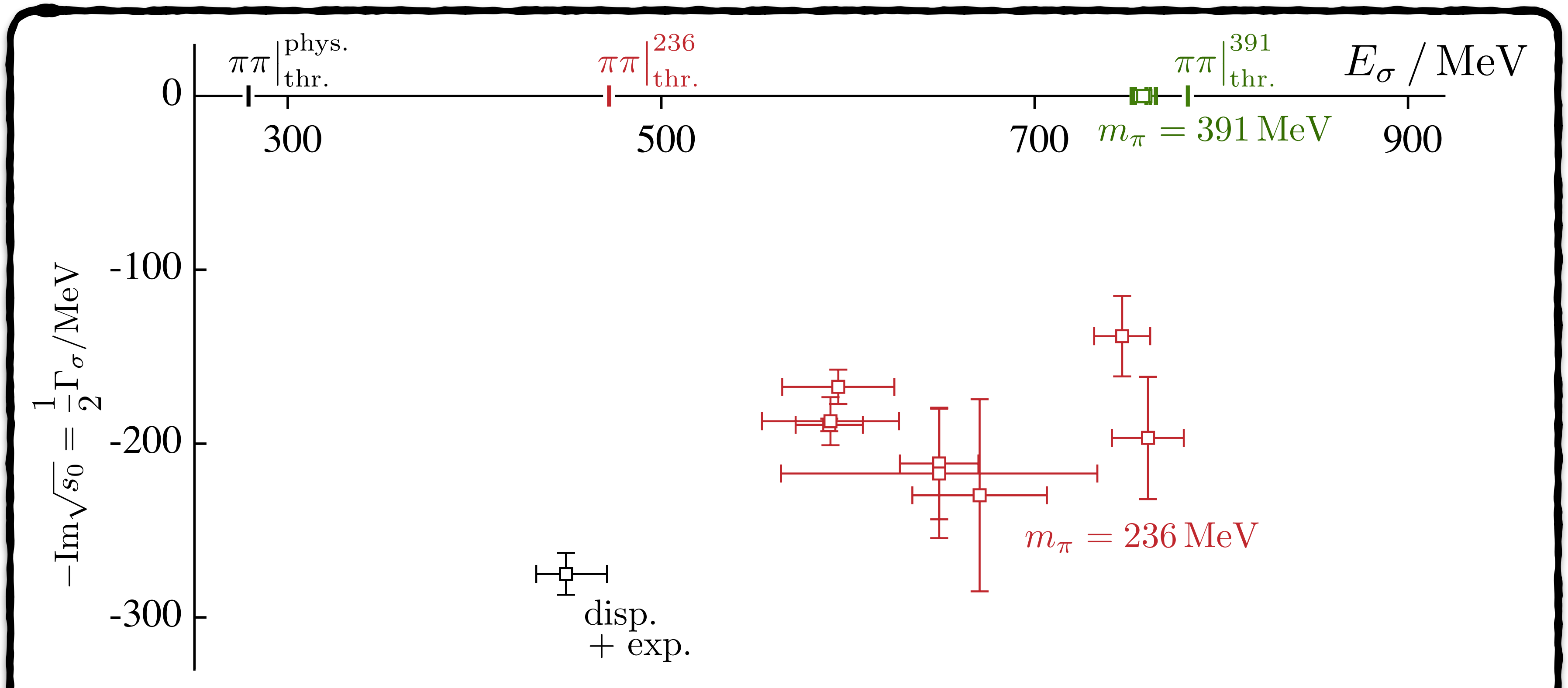


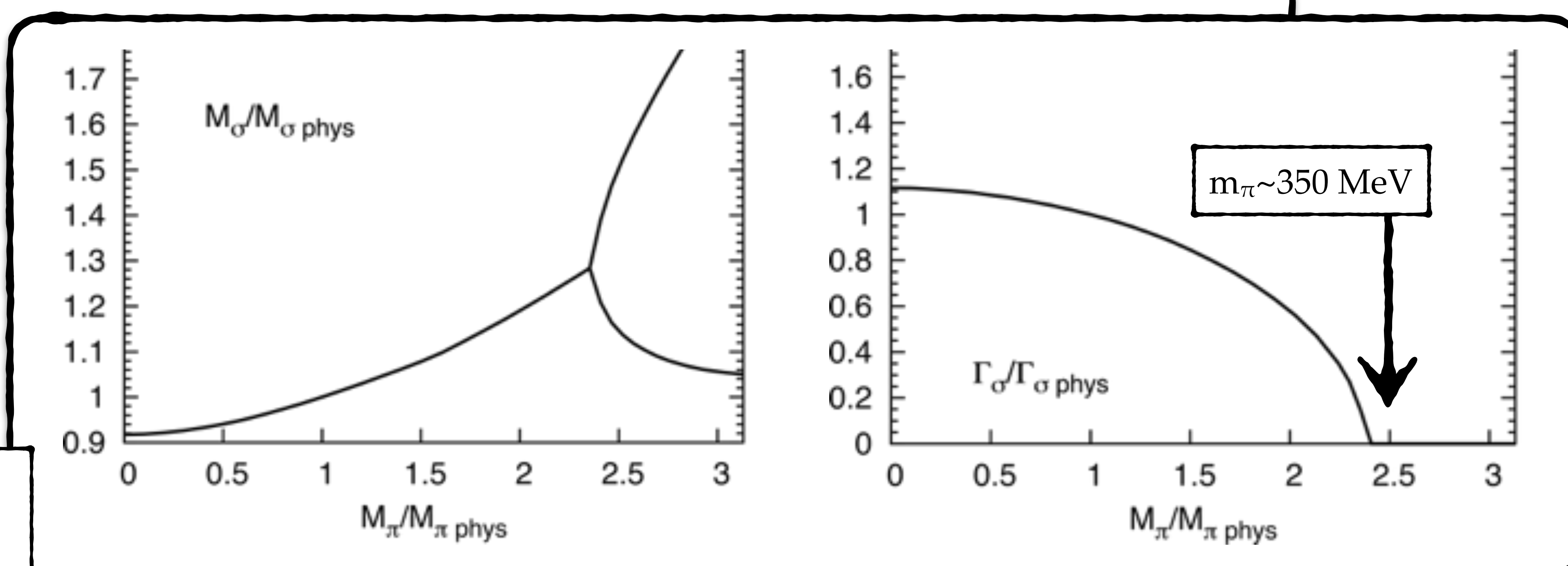
Fit parametrization:

$$h^{[\{\alpha,\beta\}]}(E^\star_{\pi\pi}, Q^2) = \frac{\alpha_1}{1+\alpha_2 Q^2 + \beta_1(E^{\star 2}_{\pi\pi} - m_0^2)} + \alpha_3 Q^2 + \alpha_4 Q^4 + \alpha_5 \exp\left[-\alpha_6 Q^2 - \beta_2(E^{\star 2}_{\pi\pi} - m_0^2)\right] + \beta_3(E^{\star 2}_{\pi\pi} - m_0^2) + \beta_4(E^{\star 4}_{\pi\pi} - m_0^4),$$

The σ / $f_0(500)$ vs m_π

$$s_0 = (E_\sigma - \tfrac{i}{2}\Gamma_\sigma)^2, \qquad g^2_{\sigma\pi\pi} = \lim_{s\to s_0} (s_0 - s)\, t(s)$$

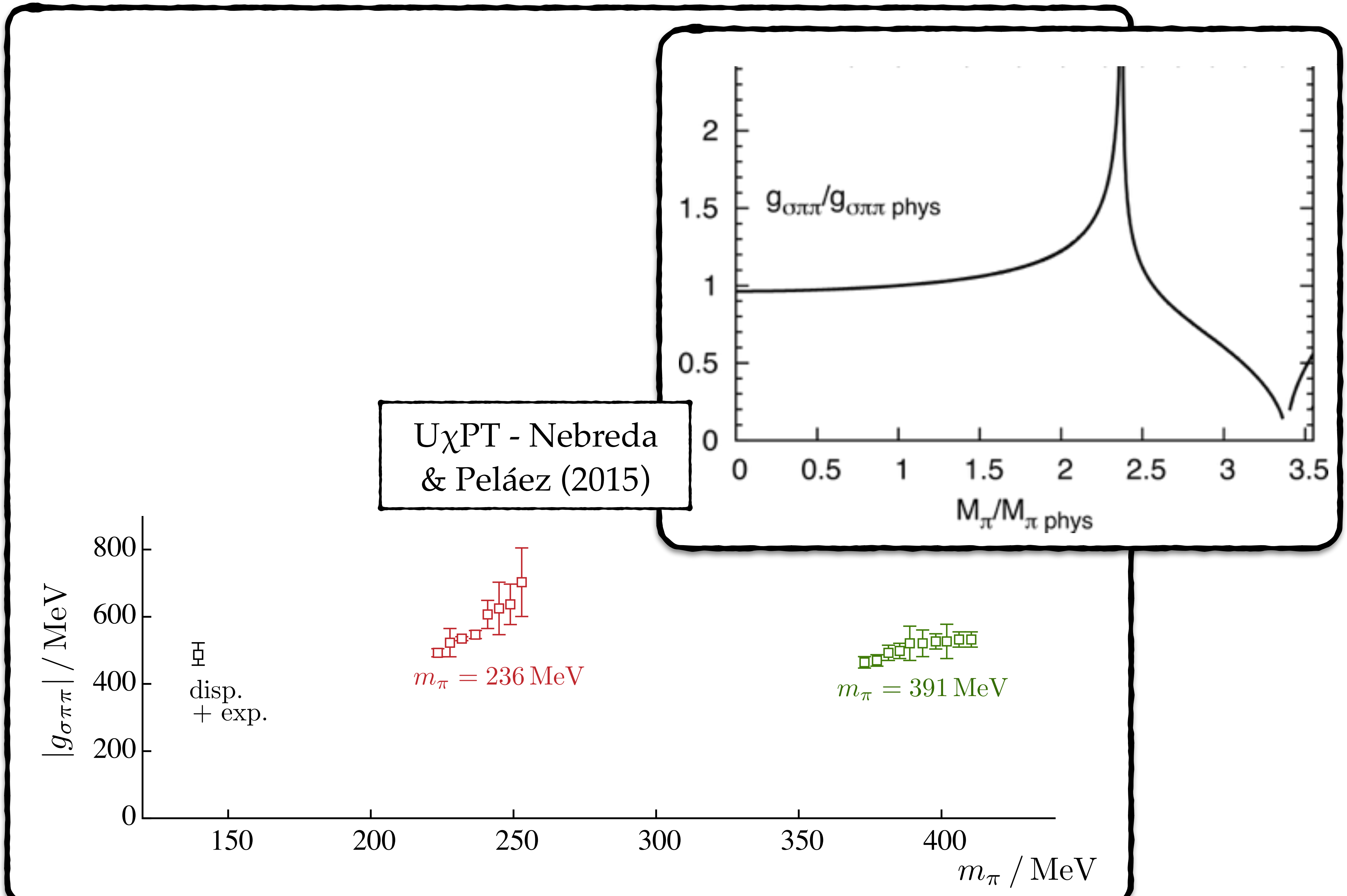




UχPT - Nebreda & Peláez (2015)

The σ / $f_0(500)$ vs m_π

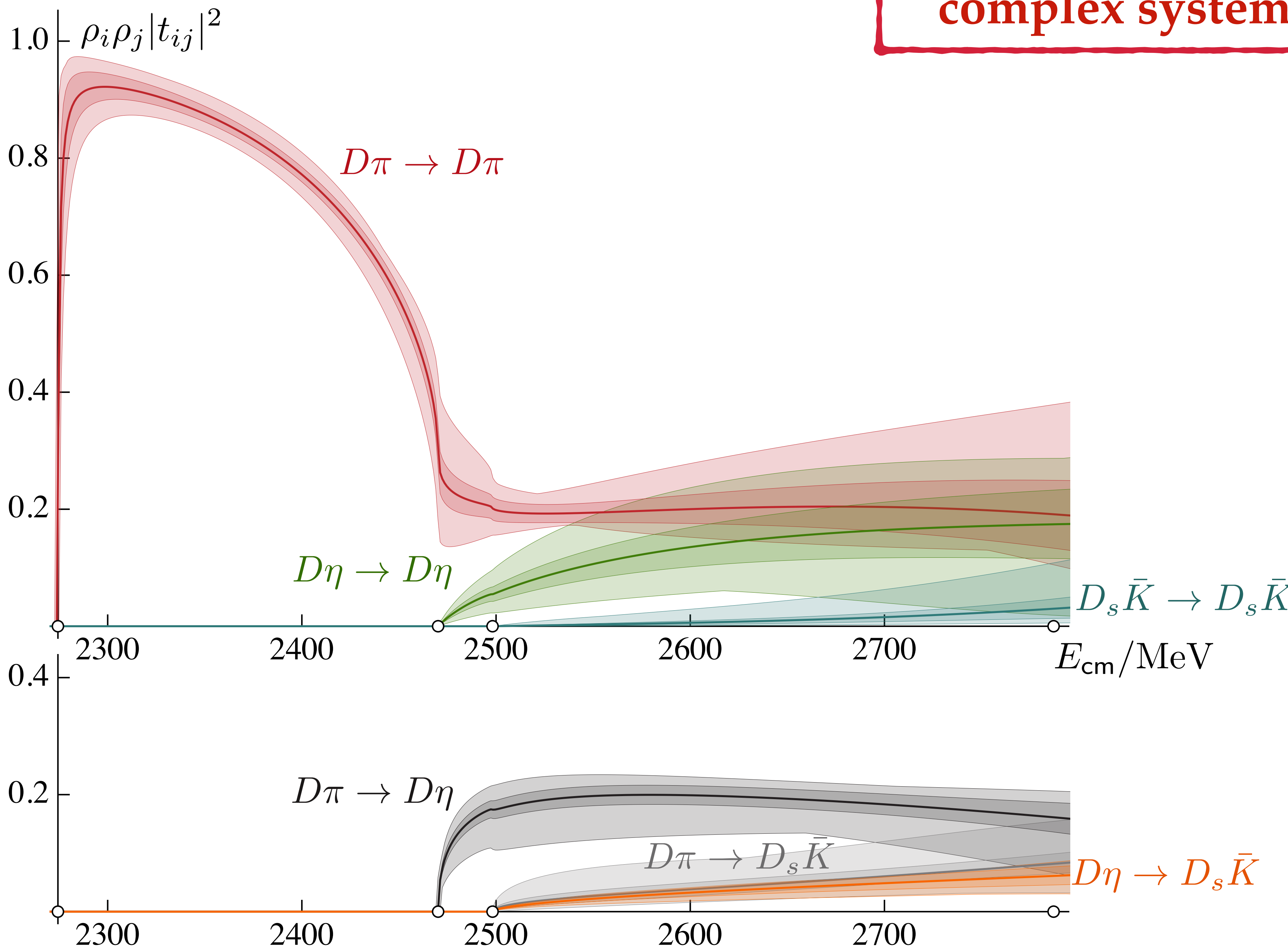
$$s_0 = (E_\sigma - \tfrac{i}{2}\Gamma_\sigma)^2, \qquad g^2_{\sigma\pi\pi} = \lim_{s\to s_0} (s_0 - s)\, t(s)$$



$D\pi$–$D\eta$–$D_s\bar{K}$ scattering

(I=1/2 channel)

Increasingly complex systems

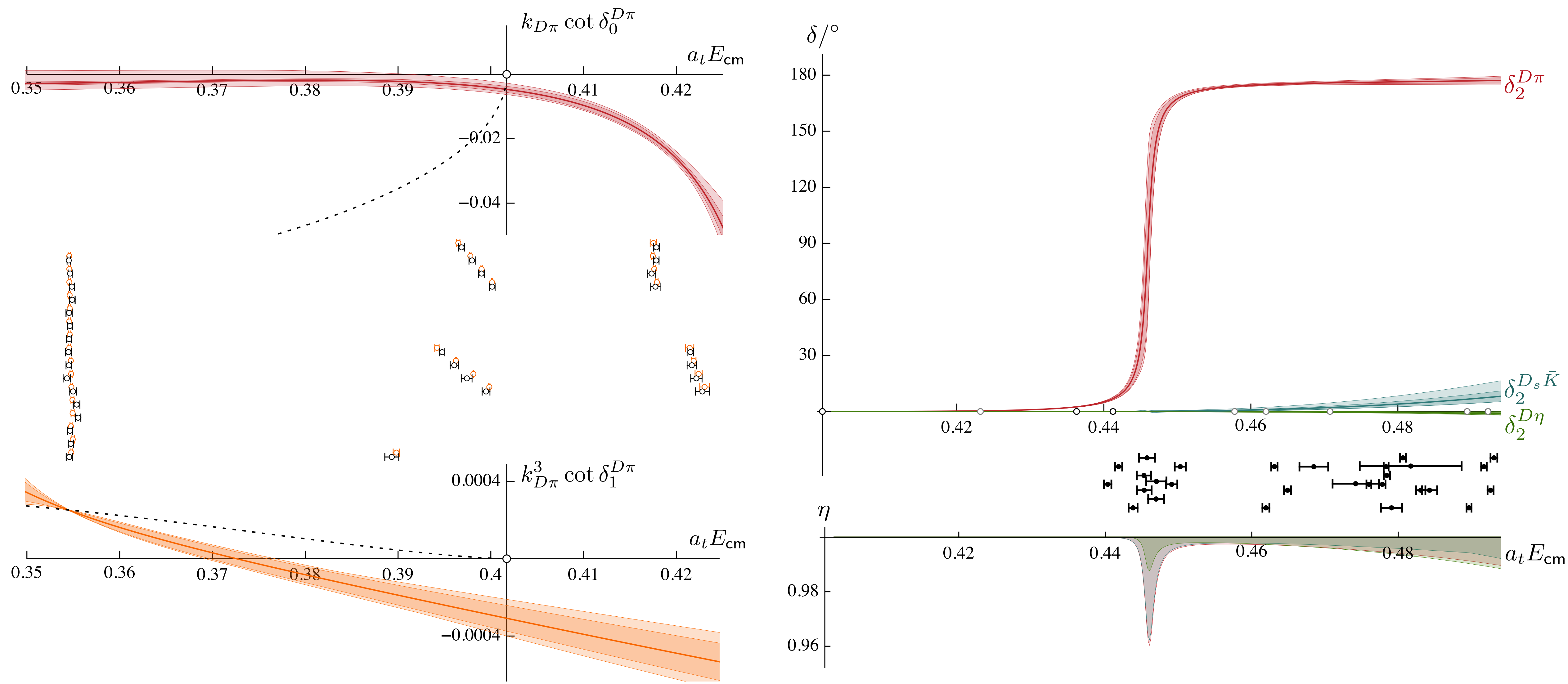


RB

Moir, Peardon, Ryan, Thomas, Wilson

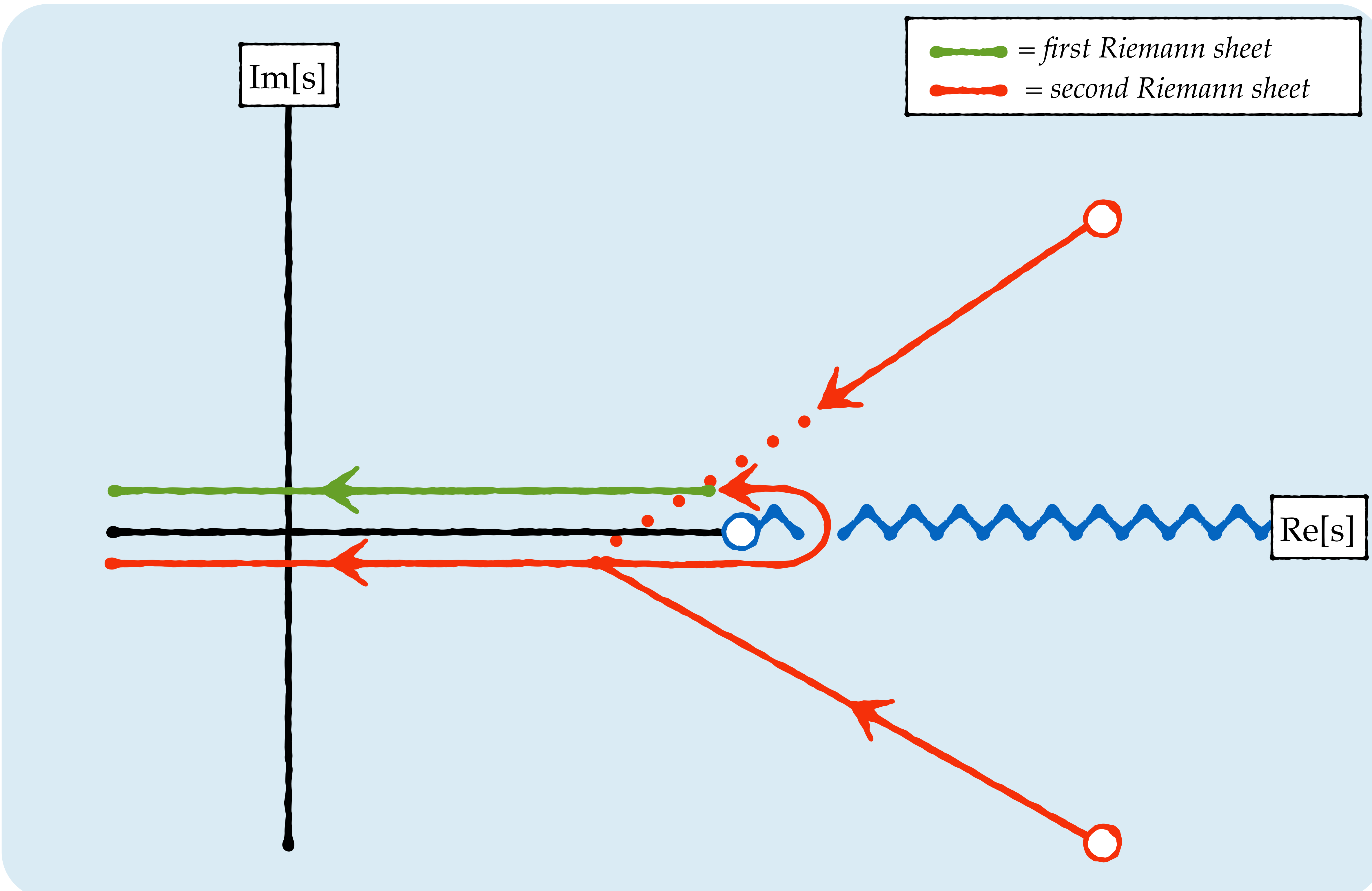
$D\pi$–$D\eta$–$D_s\overline{K}$ scattering

(I=1/2 channel)



UχPT expectation for σ / $f_0(500)$

σ / $f_0(500)$ vs m_π

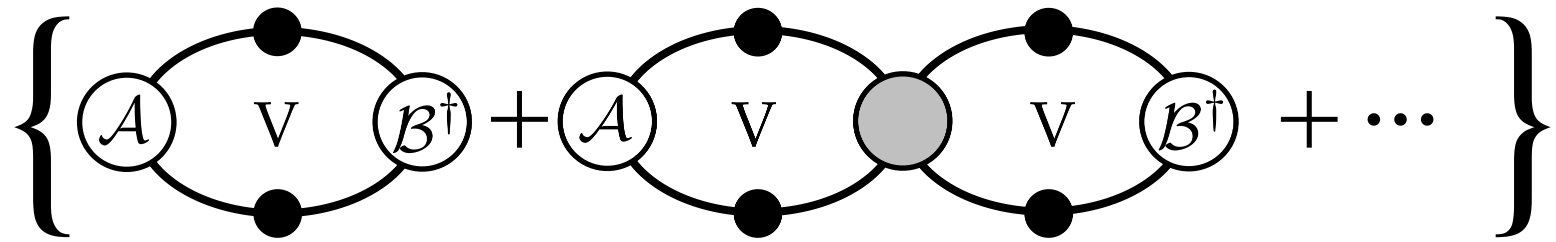


Sketch of Lüscher

Two particles in a box

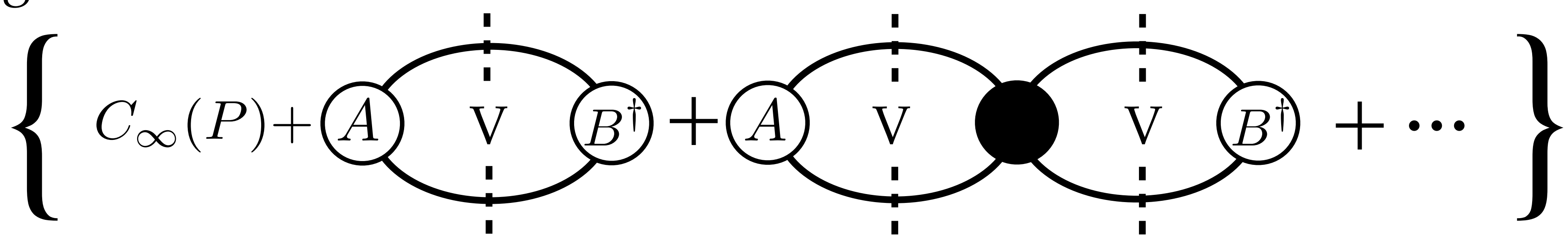
Onto two particles:

$$L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t}$$



After some massaging…

$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t}$$



$$= L^3 \int \frac{dP_0}{2\pi} e^{iP_0 t} \left\{ C_\infty(P) - A(P) \frac{1}{F^{-1}(P,L) + \mathcal{M}(P)} B^\dagger(P) \right\}$$

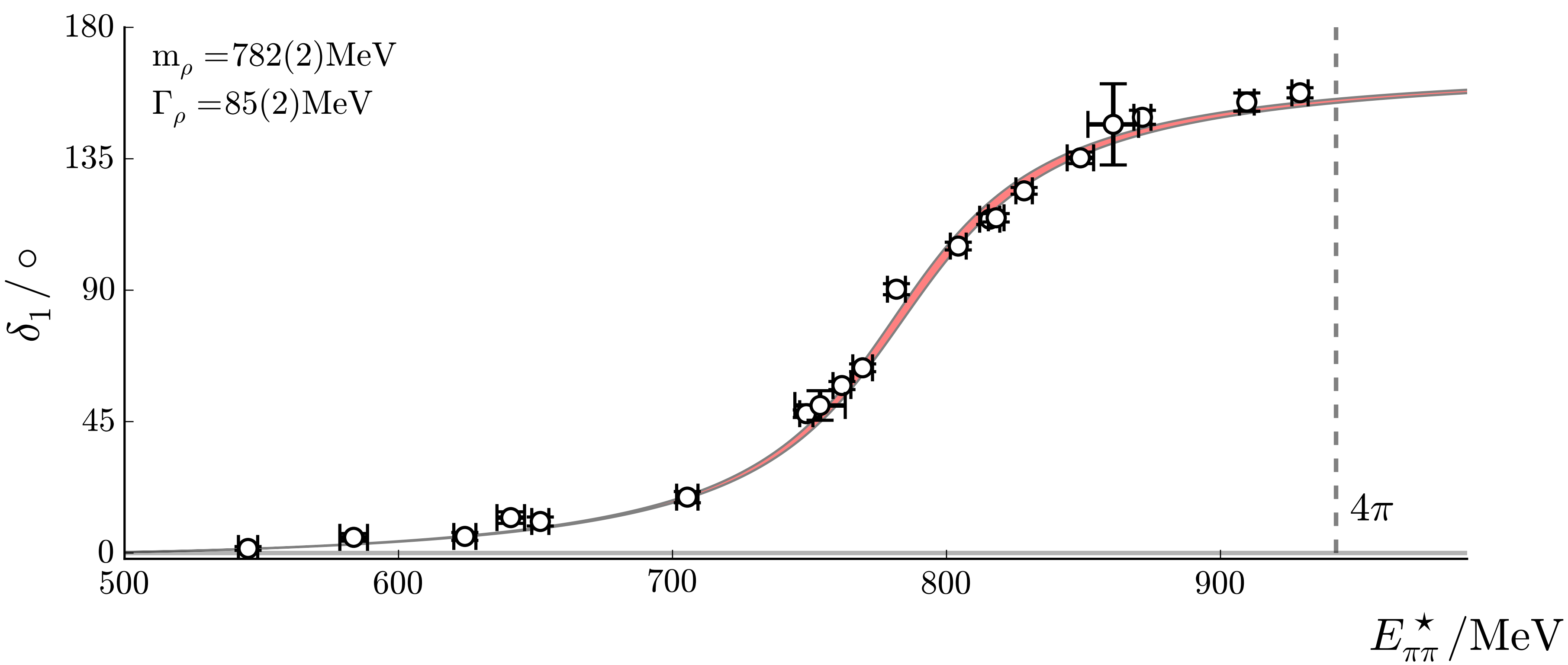
poles satisfy: $\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$

Chiral fits

Chiral fit

$$\alpha_1 \equiv -2\ell_1^r + \ell_2^r,\ \alpha_2 \equiv \ell_4^r$$

$$\alpha_1(770\ \text{MeV}) = 14.7(4)(2)(1) \times 10^{-3}$$
$$\alpha_2(770\ \text{MeV}) = -28(6)(3)\binom{01}{11} \times 10^{-3}$$



previos results:

$$\alpha_1(770\ \text{MeV}) \in [9, 13] \times 10^{-3}$$
$$\alpha_2(770\ \text{MeV}) \in [1, 12] \times 10^{-3}$$

m_π dependence