

# Next-to-next-to-leading logarithmic resummation for transverse thrust

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Based on T. Becher and XGT, JHEP **1506** (2015) 071; T. Becher, XGT and J. Piclum, arXiv:1512.00022 [hep-ph].

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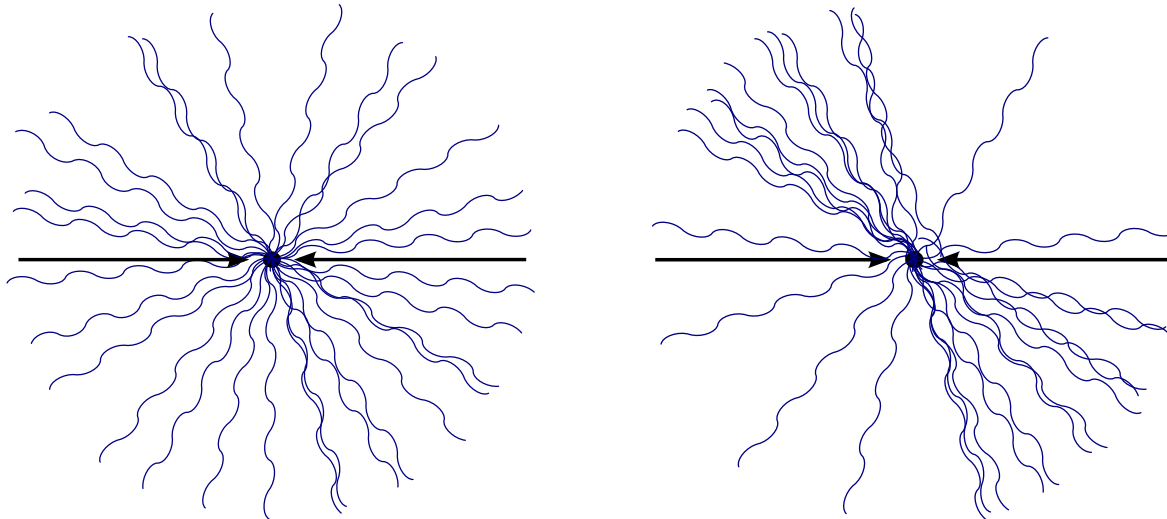
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**UNIVERSITÄT  
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FOR FUNDAMENTAL PHYSICS

# Event shapes $e$

**Event shapes:** measure energy flow in collisions. Important tool to characterize QCD effects at colliders

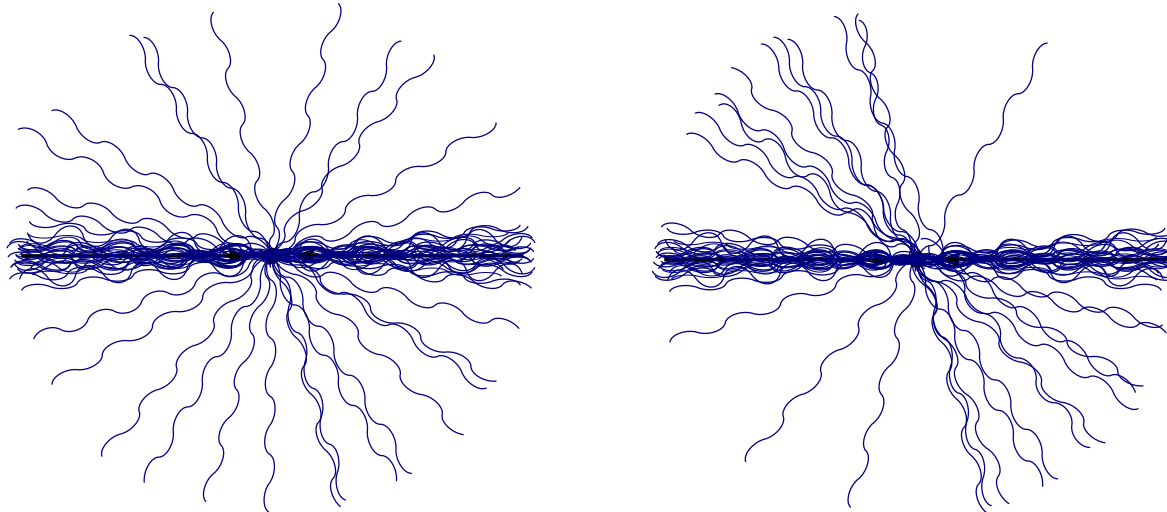


Inclusive nature: computed perturbatively, mild sensitivity to hadronisation  
Can be used to test QCD, and to discriminate new physics against SM  
e.g.: precision determinations of  $\alpha_s$

Traditionally mostly studied and used in leptonic collisions

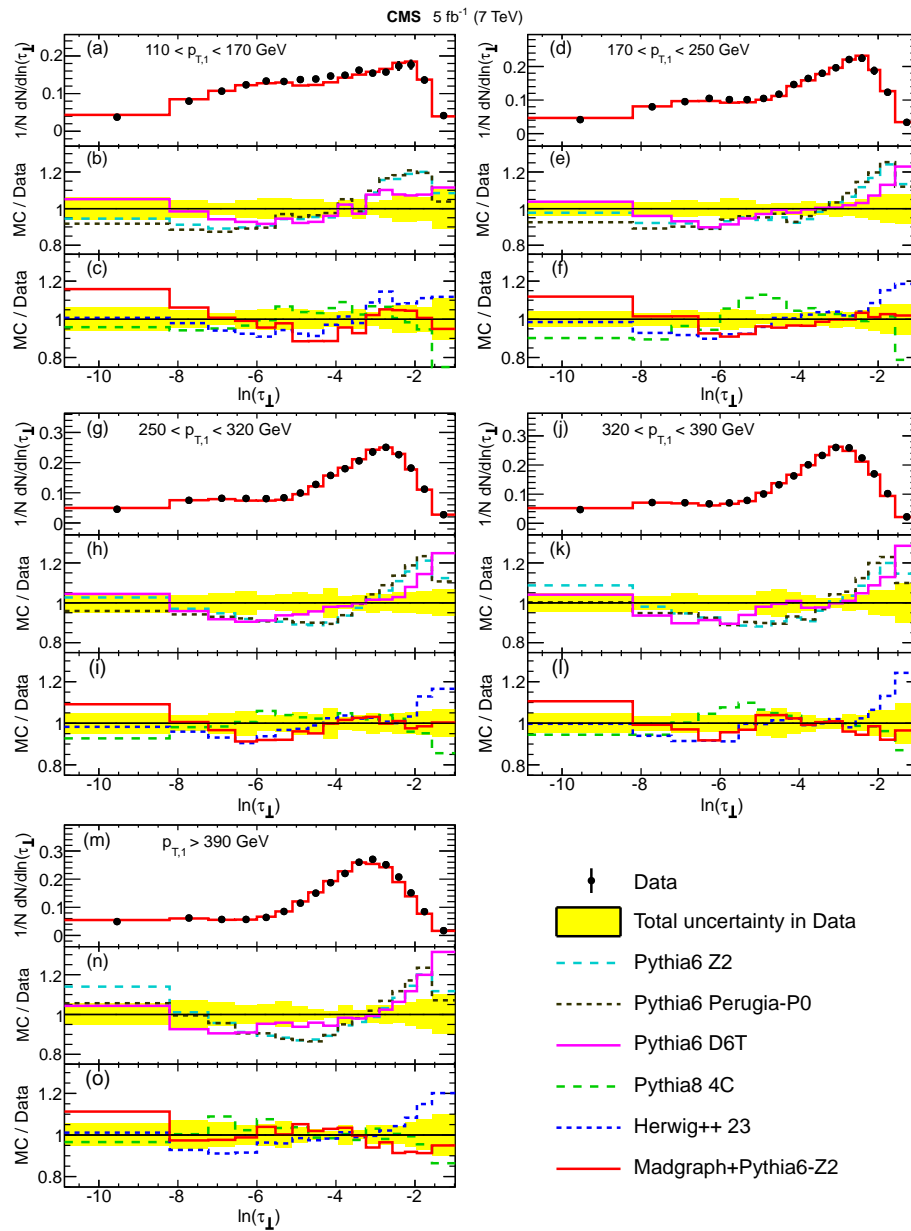
# Hadronic event shapes $e_{\perp}$

Also of great interest in hadronic collisions. e.g.: study jet substructure, improve knowledge of underlying-event effects, ...



Large class of event shapes involving only momentum components transverse to the beam  $e_{\perp}$  Banfi, Salam, Zanderighi '04'10. Archetypal example is transverse thrust  $T_{\perp}$

$$T_{\perp} = \max_{\vec{n}_{\perp}} \frac{\sum_i |\vec{p}_{i\perp} \cdot \vec{n}_{\perp}|}{\sum_i |\vec{p}_{i\perp}|} \quad ; \quad \tau_{\perp} = 1 - T_{\perp}$$



In the dijet limit  $\tau_{\perp} \rightarrow 0$ , enhanced effects need to be resummed to obtain reliable perturbative predictions

$$\sigma(\tau_{\perp}) \sim 1 + \alpha_s (\ln^2 \tau_{\perp} + \ln \tau_{\perp} + \dots) + \alpha_s^2 \dots$$

Resummation was performed at NLL accuracy, using the automated framework provided by CAESAR, and combined with matching to fixed order

Banfi, Salam, Zanderighi '10

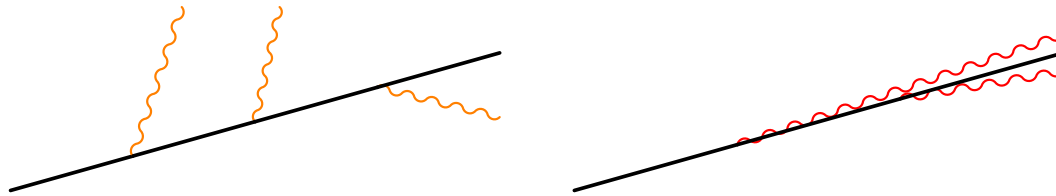
$\tau_{\perp}$  can be seen as ratio of disparate energy scales

$$\tau_{\perp} \sim \frac{\text{jet mass}}{\text{hard-collision energy}}$$

Effective Field theory techniques exploit hierarchy of energy scales to simplify the problem. Systematically factorize effects from different scales, resum enhanced terms

# Soft Collinear Effective Theory

Collider processes receive enhancements from **soft** (low energy) and **collinear** (small angle) emission of radiation



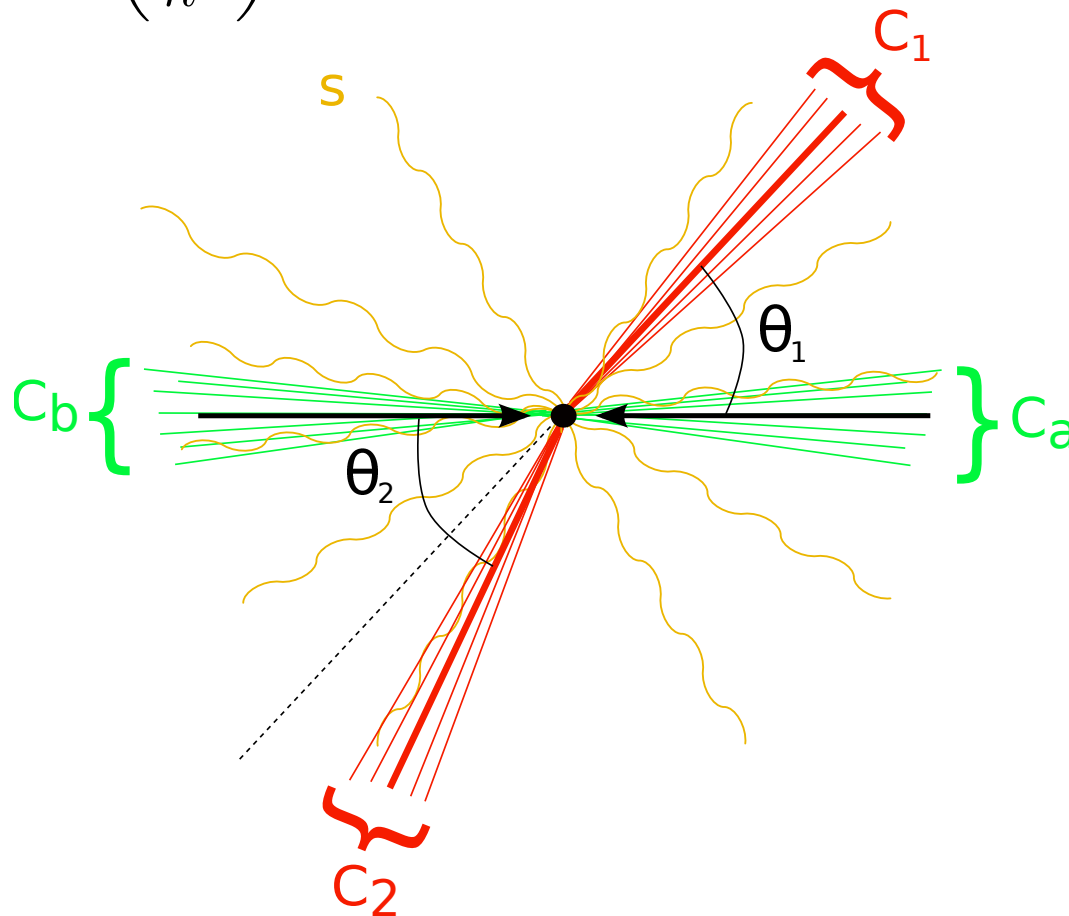
We study the transverse-thrust distribution within the effective theory framework of SCET.

Factorization formula for a generic event shape  $e_{\perp}$  ( $ab \rightarrow ij$ )

$$\frac{d\sigma}{de_{\perp}} = \sum_{a,b,i,j} P_{IJ}^{ab \rightarrow ij} \otimes S_{JI}^{ab \rightarrow ij} \otimes J_i \otimes J_j \otimes B_a \otimes B_b$$

$$\tilde{t}(\kappa) \sim H_{IJ}^{ab \rightarrow ij} \left( \frac{Q^2}{\kappa^2} \right)^{-F^{ab \rightarrow ij}(\kappa)} \tilde{S}_{JI}^{ab \rightarrow ij}(\kappa) \tilde{B}_a(\kappa) \tilde{B}_b(\kappa) \tilde{J}_i(\kappa) \tilde{J}_j(\kappa)$$

$$\tilde{t}(\kappa) \sim H_{IJ}^{ab \rightarrow ij} \left( \frac{Q^2}{\kappa^2} \right)^{-F^{ab \rightarrow ij}(\kappa)} \tilde{S}_{JI}^{ab \rightarrow ij}(\kappa) \tilde{B}_a(\kappa) \tilde{B}_b(\kappa) \tilde{J}_i(\kappa) \tilde{J}_j(\kappa)$$



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Target is N<sup>2</sup>LL accuracy; need **2-loop**  $\gamma_f$ , **2-loop anomaly exponent**, 1-loop functions (and 1-loop  $H$ , 3-loop  $\gamma_{\text{cusp}}$ )

$$\frac{d}{d \ln \mu} \tilde{f}(L, \mu) = [-C_f \gamma_{\text{cusp}} L + \gamma_f] \tilde{f}(L, \mu)$$



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$$\frac{d}{d \ln \mu} \tilde{f}(L, \mu) = [-C_f \gamma_{\text{cusp}} L + \gamma_f] \tilde{f}(L, \mu)$$

Consider different processes:

$$ab \rightarrow ij: \quad H_{IJ}^{ab \rightarrow ij} \left( \frac{Q^2}{\kappa^2} \right)^{-F^{ab \rightarrow ij}(\kappa)} \tilde{S}_{JI}^{ab \rightarrow ij}(\kappa) \tilde{B}_a(\kappa) \tilde{B}_b(\kappa) \tilde{J}_i(\kappa) \tilde{J}_j(\kappa)$$

$$e^+ e^- \rightarrow ij: \quad H^{ij} \left( \frac{Q^2}{\kappa^2} \right)^{-F^{ij}(\kappa)} \tilde{S}^{ij}(\kappa) \tilde{J}_i(\kappa) \tilde{J}_j(\kappa),$$

$$ab \rightarrow e^+ e^-: \quad H^{ab} \left( \frac{Q^2}{\kappa^2} \right)^{-F^{ab}(\kappa)} \tilde{S}^{ab}(\kappa) \tilde{B}_a(\kappa) \tilde{B}_b(\kappa),$$

## Renormalization group and factorization constraints require

$$\gamma_{H^{ab \rightarrow ij}} + \gamma_{S^{ab \rightarrow ij}} + \gamma_{B_a} + \gamma_{B_b} + \gamma_{J_i} + \gamma_{J_j} = 0$$

$$\gamma_{H^{ij}} + \gamma_{S^{ij}} + \gamma_{J_i} + \gamma_{J_j} = 0$$

$$\gamma_{H^{ab}} + \gamma_{S^{ab}} + \gamma_{B_a} + \gamma_{B_b} = 0$$

$$F^{ab \rightarrow ij} = F^{ab} + F^{ij} = \frac{C_a + C_b}{2} F_{\perp} + \frac{C_i + C_j}{2} F'_{\perp}$$

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$$F^{ab \rightarrow ij} = F^{ab} + F^{ij} = \frac{C_a + C_b}{2} F_{\perp} + \frac{C_i + C_j}{2} F'_{\perp}$$

From  $pp \rightarrow e^+e^-/\gamma\gamma$  get  $\gamma_B$  at two loops

From  $e^+e^- \rightarrow q\bar{q}$  and using fixed-order code get  $\gamma_{J_q}$  and  $\gamma_{S^{qq}}$

$\gamma_{S^{qq}}$  and  $\gamma_{S^{gg}}$  at two loops are related by Casimir scaling, from  $e^+e^- \rightarrow gg$  get  $\gamma_{J_g}$

$\gamma_S$  in any other channel is fixed by RG invariance

To get  $F_{\perp} (d_2^{\perp})$ : define new observable, with known  $d_2$ , that starts differing from  $e_{\perp}$  when there are at least two emissions. Compute difference of observables

$\tau_{\perp}$  in  $pp \rightarrow e^+e^-$ :

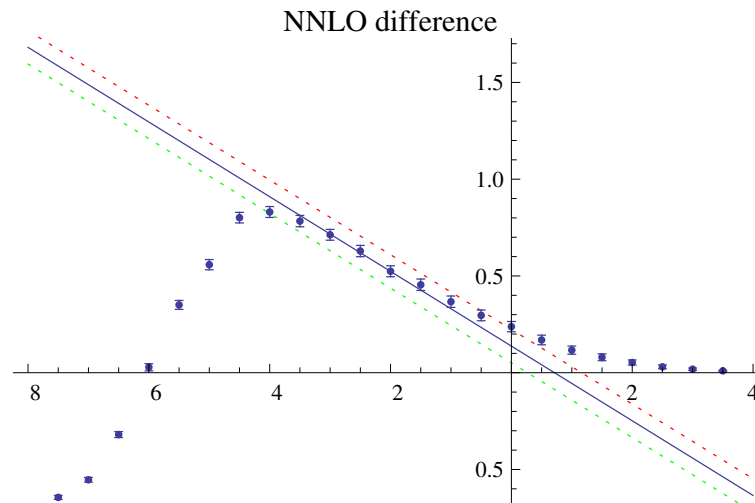
$$\mathcal{S}_{\perp} := |\vec{q}_{\perp}| - |\vec{q}_{\perp} \cdot \vec{n}_{\perp}| = \left| \sum_m \vec{p}_{m\perp} \right| - \left| \sum_m \vec{p}_{m\perp} \cdot \vec{n}_{\perp} \right|$$

$$\mathcal{T}_{\perp} := Q_{\perp} \tau_{\perp} = \sum_m (|\vec{p}_{m\perp}| - |\vec{p}_{m\perp} \cdot \vec{n}_{\perp}|)$$

Two-loop result for  $\mathcal{S}_{\perp}$  from known Drell-Yan results. Difference involves  $d_2^{DY} - d_2^{\perp}$ ; this term is determined by rapidity divergences of soft function. Numerically evaluate coefficient of rapidity divergence in tree-level two-emission soft amplitude squared

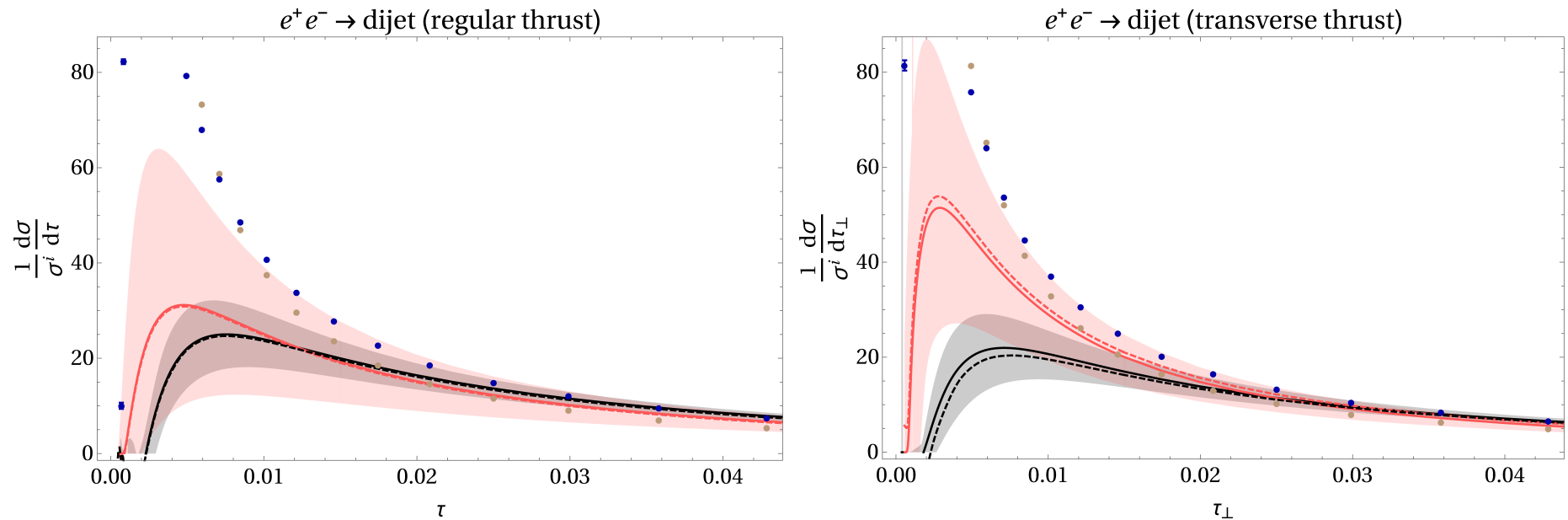
$$d_2^\perp = (182.3 \pm 0.1) C_A + (-51.881 \pm 0.006) T_F n_f$$

Cross-check with fixed order code (DYNNLO Grazzini)



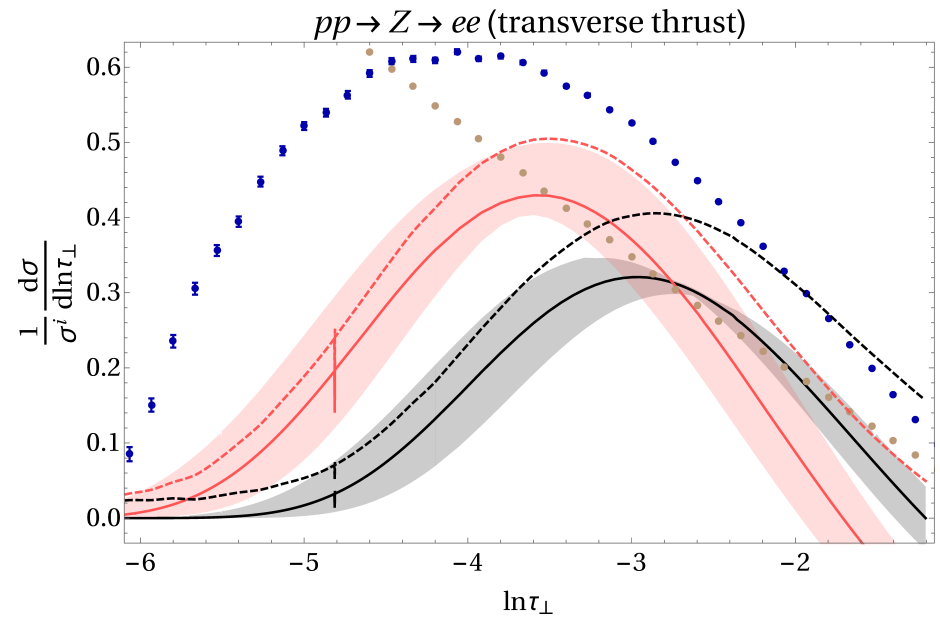
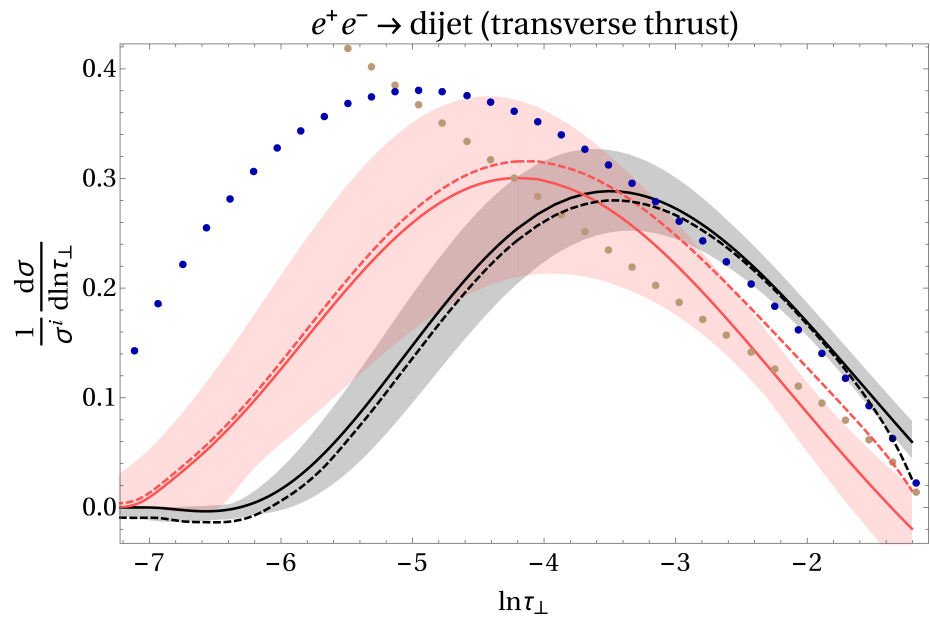
We find in general much larger coefficients than in lepton-collider event shapes

# Results



NLL   
  $N^2LL$    
 $\mathcal{O}(\alpha_s)$  fixed-order results   
 $\mathcal{O}(\alpha_s^2)$  fixed-order results

# Results



NLL     $N^2LL$      $\mathcal{O}(\alpha_s)$  fixed-order results     $\mathcal{O}(\alpha_s^2)$  fixed-order results

## Conclusions and outlook

Obtained unprecedented N<sup>2</sup>LL accuracy for hadronic  $e_{\perp}$  event shapes, by exploiting separation of scales from EFT

Procedure valid for generic  $e_{\perp}$

*Outlook:*

Perform comprehensive phenomenological study of relevant processes:

$pp \rightarrow \text{dijet}$ ,  $pp \rightarrow Z + \text{jet}$ ,  $pp \rightarrow ZZ$

- ◆ Need to combine with fixed order results, available or in progress from Zurich group

All ingredients are being implemented in a numerical code, to be made publicly available

Combined with numerical evaluation of one-loop jet, soft, and beam functions, obtain automated EFT-based framework for N<sup>2</sup>LL resummation

Study Glauber-gluon effects issues/Relation to UE effects

Novel  $\alpha_s$  extraction at much higher energies than with leptonic event shapes