Reference:\\cern.ch\dfs\users\r\rudi\documents\MathCAD\definition-constants-units.xmcd

Reference:\\cern.ch\dfs\users\r\rudi\documents\MathCAD\definition-constants-materials.xmcdz

Bethe Bloch equation: energy loss of charge particles in material

Parameter of the projectile (ion)

http://diff2012-lhc.physi.uni-heidelberg.de/Talks/grafstrom.pdf

$TeV := 1000GeV$

Charge of the projectile for ions $Z_p := 82$ Atomic weight lead: $A_p := 208$

Kinetic energy of one nucleon in an ion assuming LHC operates with a magnetic field for a proton energy of $E_p := 6.37TeV$ proton equivalent field: $E_{kin\_1nucleon} := 6.37 \cdot TeV \cdot \left(\frac{Z_p}{A_p}\right)$

Energy of the ion: $E_{kin\_ion} := E_{kin\_1nucleon} \cdot A_p$

$E_{kin\_1nucleon} = 2.51 \cdot TeV$

$E_{kin\_ion} = 5.2234 \times 10^2 \cdot TeV$

Assuming that the ion beam is lost, and the power is given by: $P_{loss} := 100kW$

$$N_{loss\_ion} := \frac{P_{loss}}{E_{kin\_ion}}$$

Total number of ions lost per second: $N_{loss\_ion} = 1.1950 \times 10^9 \frac{1}{s}$

Total number of ions in the LHC (nominal): $N_{nominal\_ion} := 592 \cdot 7 \cdot 10^7$

Energy stored in the LHC ion beam: $E_{nominal\_ion} := N_{nominal\_ion} \cdot E_{kin\_ion}$

$E_{nominal\_ion} = 3.4677 \times 10^6 J$

$E_{ion} := Z_p \cdot 7TeV$

$E_{ion} = 5.7400 \times 10^{2} \cdot TeV$

Energy stored in an ion pulse: $E_{ion\_pulse} := E_{kin\_ion} \cdot N_{loss\_ion}$

$E_{ion\_pulse} = 1.0000 \times 10^{5} W$

$E_p := m_p \cdot c^2 + E_{kin\_ion}$

$$\gamma_p := \frac{E_p}{m_p \cdot c^2}$$

$$\beta_p := \sqrt{1 - \frac{1}{\gamma_p^{2}}}$$

$\beta_p = 1.000000$ $\gamma_p = 556659.52$

Beam sizes: $\sigma_x := 0.2mm$ and $\sigma_y := 0.01mm$

Beam spot size on material: $F_{beam} := 2 \cdot \sigma_x \cdot \sigma_x$

Energy per ion $E_p = 522.341 \cdot TeV$

material depth of : $l_{material} := 1mm$

Volume of material material:
$Volume := F_{beam} \cdot l_{material}$

Heating of matter by an ion beam via dEdx for C

Electron density of the material material $n_t = \frac{Z_t \cdot \rho}{A_t \cdot M_u}$ **for Al**

Atomic mass unit: $M_{unit} := 1.660 \cdot 10^{-27} kg$

$$\rho_C = 2.26 \cdot \frac{gm}{cm^3}$$

$$n_{t\_C} := \frac{Z_C \cdot \rho_C}{A_C \cdot M_{unit}}$$

Electron density in material: $n_{t\_C} = 6.8072 \times 10^{23} \cdot \frac{1}{cm^3}$

Mean excitation potential: $I_C := 70eV \cdot Z_C$

dEdx for one ion and impact on a C material

$$dEdx_C := \frac{4\pi}{m_e \cdot c^2} \cdot n_{t\_C} \cdot Z_p^2 \cdot \left(\frac{e_0^2}{4 \cdot \pi \cdot \varepsilon_0}\right)^2 \cdot \frac{1}{\beta_p^2} \cdot \left[\ln\left[\frac{2 \cdot m_e \cdot c^2 \cdot \beta_p^2}{I_C \cdot \left(1 - \beta_p^2\right)}\right] - \beta_p^2\right]$$

$$\frac{4\pi}{m_e \cdot c^2} \cdot \frac{e_0^2}{4 \cdot \pi \cdot \varepsilon_0} = 3.5404 \times 10^{-14} m$$

$$\ln\left[\frac{2 \cdot m_e \cdot c^2 \cdot \beta_p^2}{I_C \cdot \left(1 - \beta_p^2\right)}\right] - \beta_p^2 = 3.3257 \times 10^1$$

$$dEdx_C = 7.7596 \cdot \frac{GeV}{mm}$$

Parameter for C: $c_{C\_spec} = 711 \cdot \frac{J}{kg \cdot K}$ $\rho_C = 2.26 \cdot \frac{gm}{cm^3}$

Energy into material : $E_{loss\_C} := N_{loss\_ion} l_{material} \cdot dEdx_C$

$E_{loss\_C} = 1.486\,W$

Energy density in Material: $E_{dvol\_C} := \frac{E_{loss\_C}}{F_{beam} \cdot l_{material}}$ Mass of material material: $Mass_C := F_{beam} \cdot l_{material} \cdot \rho_C$

$Mass_C = 1.81 \times 10^{-7}\,kg$

$E_{dvol\_C} = 1.857 \times 10^{4} \frac{1}{s} \cdot \frac{J}{cm^3}$ $E_{dvol\_C} = 1.159 \times 10^{14} \frac{1}{s} \cdot \frac{GeV}{cm^3}$ $\frac{E_{dvol\_C}}{\rho_{Al}} = 6.8775 \times 10^{3} \frac{1}{s} \cdot \frac{J}{gm}$

Temperature increase: $dT_C := \frac{E_{loss\_C}}{c_{Al\_spec} \cdot Mass_C}$

$dT_C = 9.143 \times 10^{3} \frac{K}{s}$

Results for ion impact on Carbon

Comments: No cooling is taken into account

Beam area: $F_{beam} = 0.08 \cdot mm^2$

Number of ions $N_{loss\_ion} = 1.20 \times 10^9 \frac{1}{s}$

Power loss in this part of the collimator: $E_{loss\_C} = 1.49\,W$

Total energy of the nuclei: $E_{ion\_pulse} = 1.00 \times 10^5 \frac{1}{s} \cdot J$

Temperature increase when beam enters into the material: $dT_C = 9.143 \times 10^3 \frac{K}{s}$

Energy deposition of one ion when entering matter: $dEdx_C = 7.76 \cdot \frac{GeV}{mm}$