Precision Physics at the LHC

V. Ravindran

Institute of Mathematical Sciences, Chennai

- Experiment and Theory
- QCD improved parton model
- Strong coupling constant
- Parton distribution function
- NLO, NNLO results
- Jet physics

Multi-leg, Multi-loop Processes at SINP, Kolkata 23-27 Feb 2016

Aspects of QCD at the LHC

- Experiment and Theory
- QCD improved parton model
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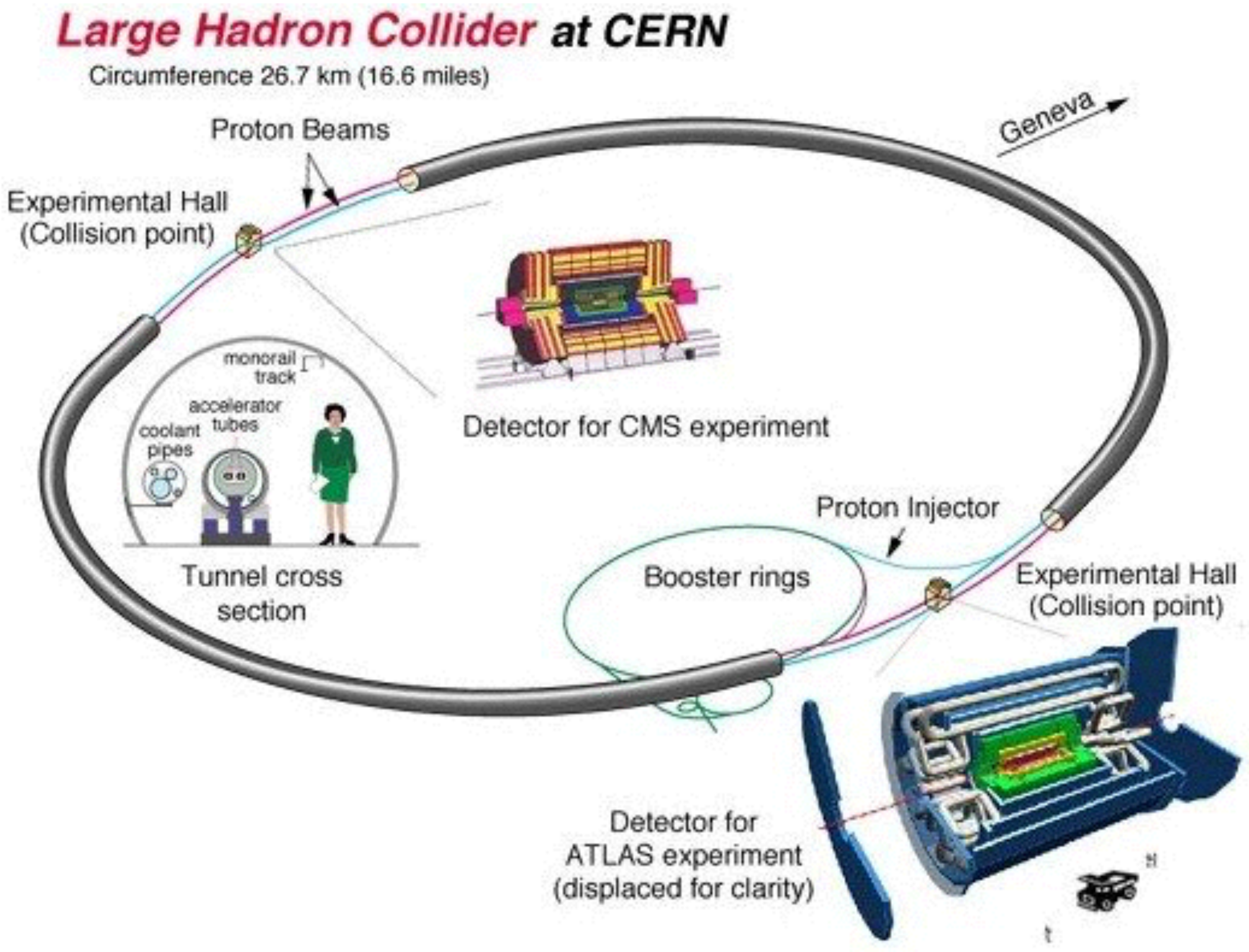
  - Higgs
  - Supersymmery
  - Extra-Dimensional models
  - Anything else



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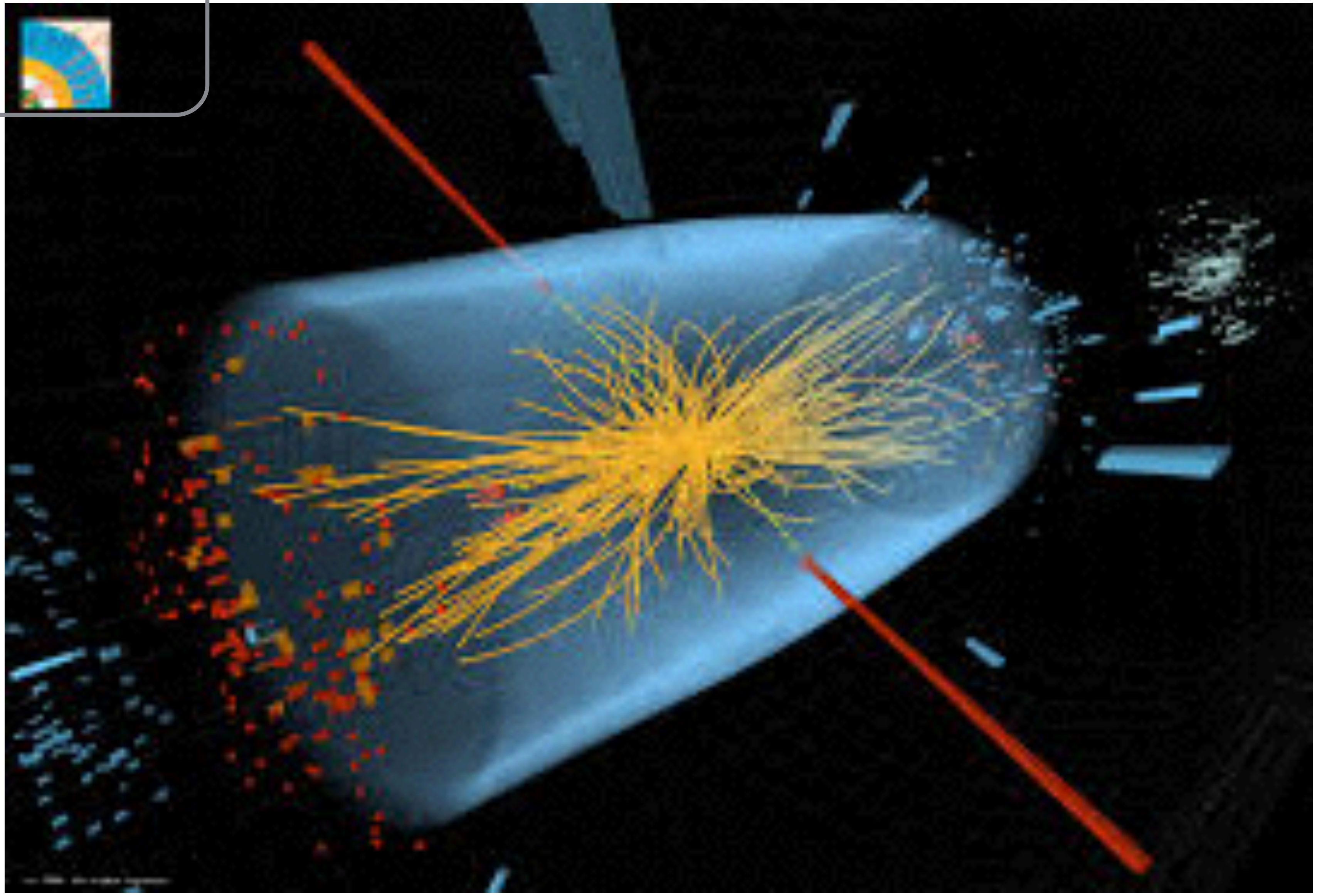

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  - Large number of $\gamma, l^{\pm}, Z, W^{\pm}$
  - Jets
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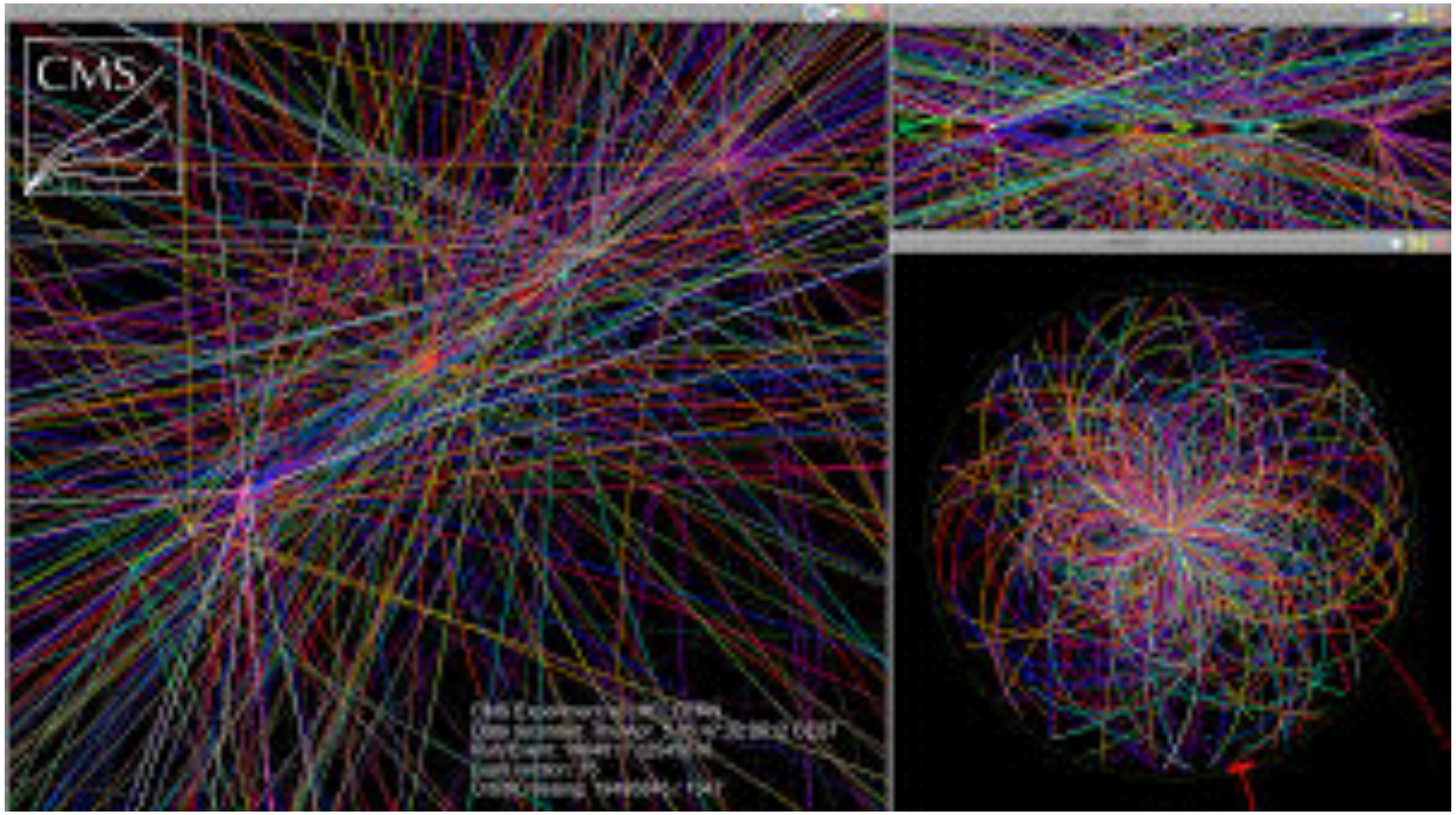

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  - Quantum Chromodynamics (QCD) effects
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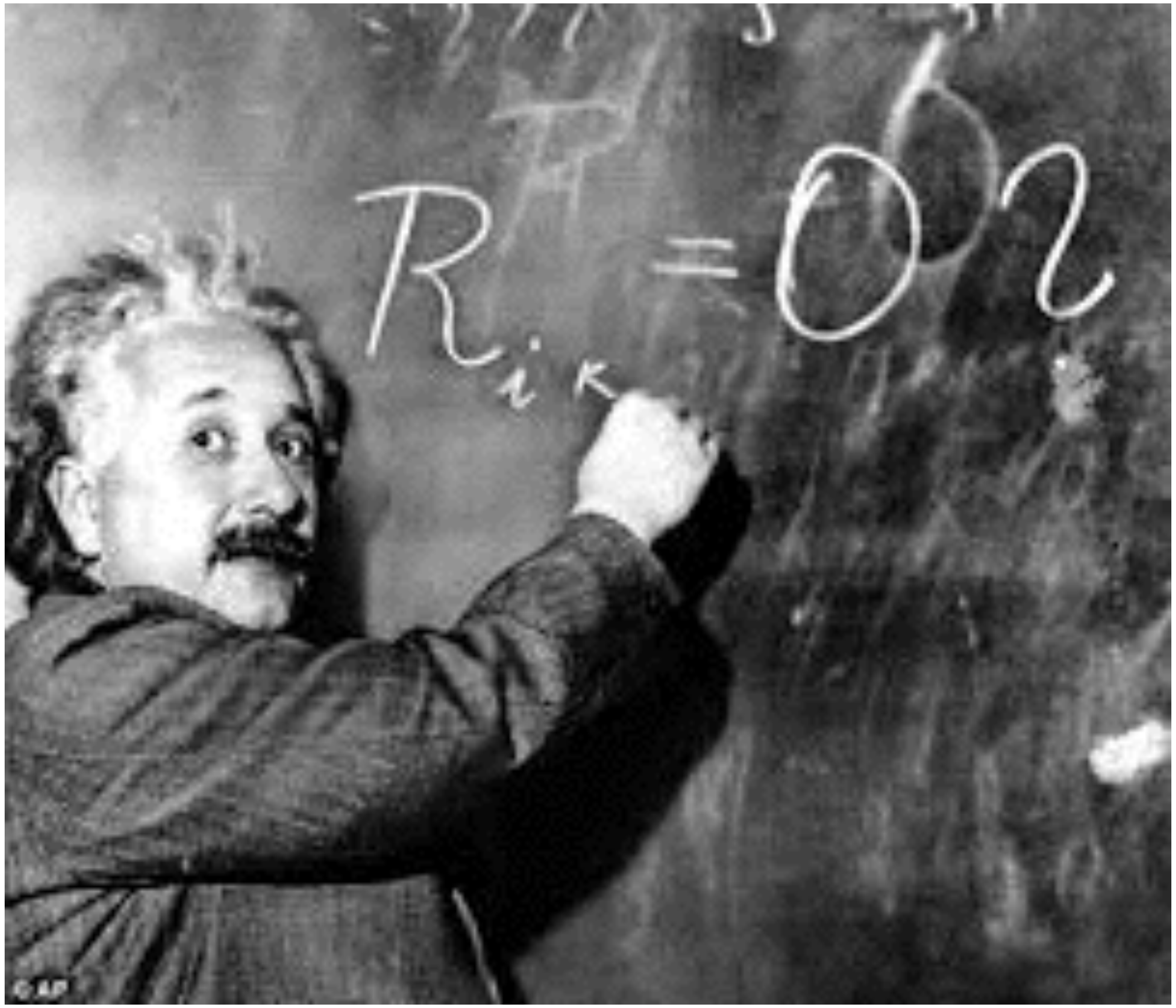

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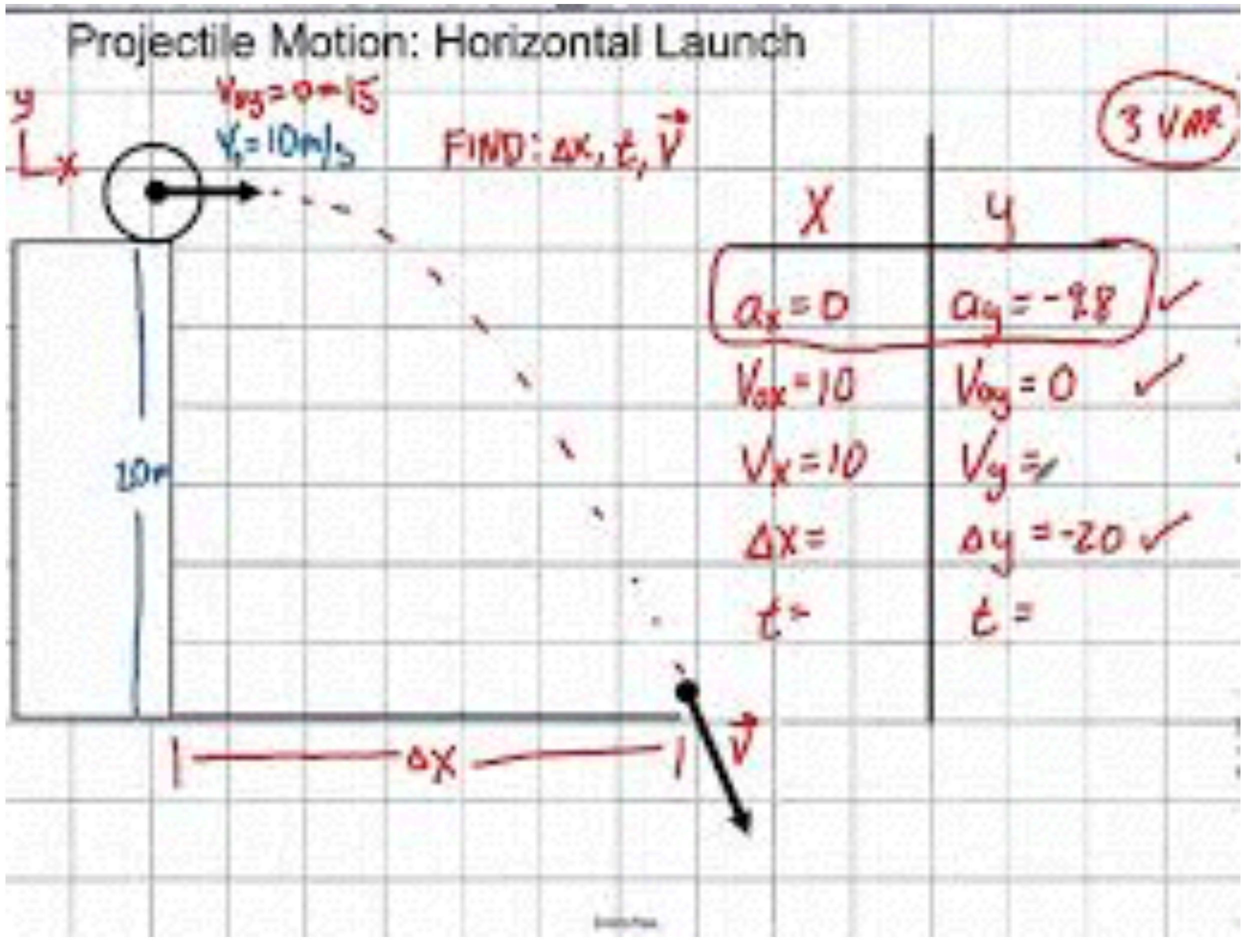
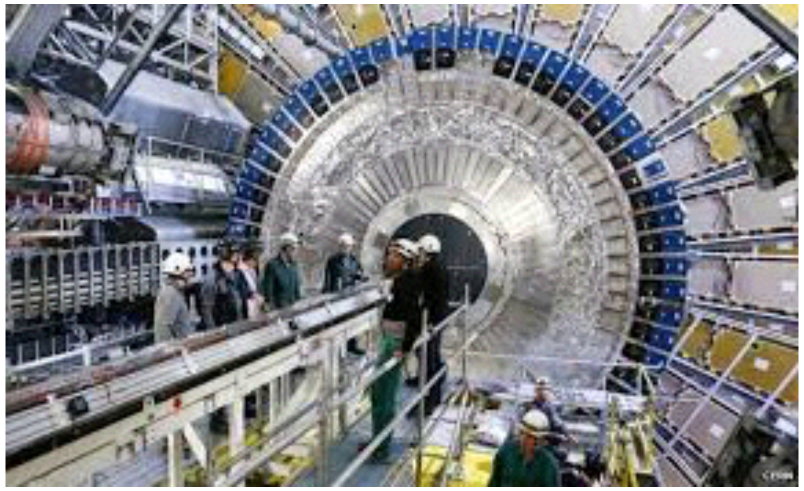
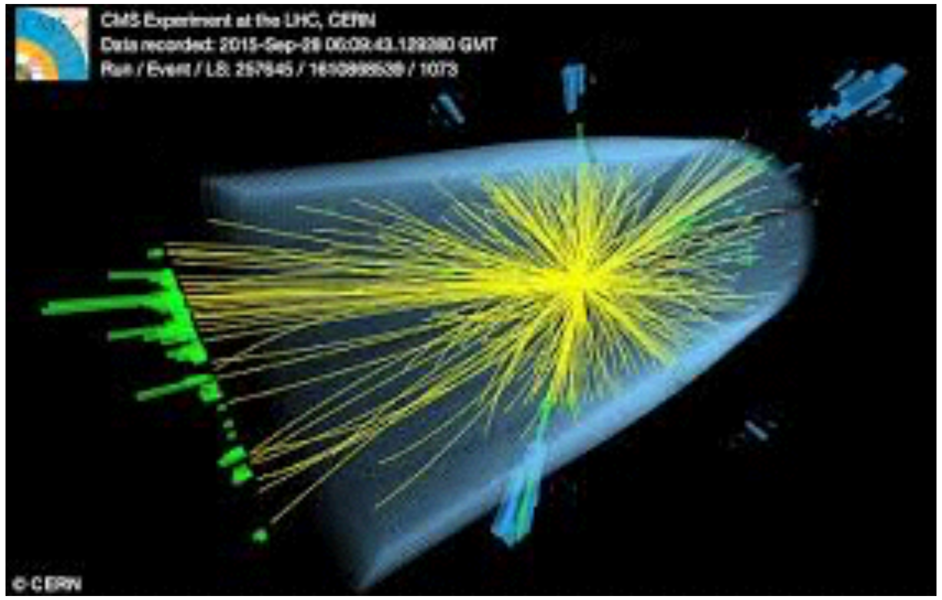


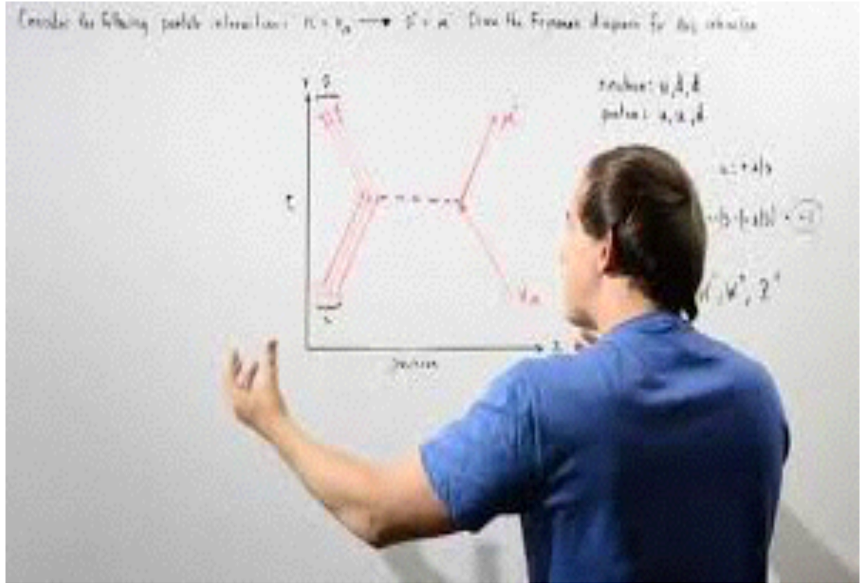
- Issues to be tackled:
  - Kinematics
  - Normalisation
  - Renormalisation and factorisation scale uncertainities
  - Parton Distribution Functions
  - Phase Space boundary effects and resummation of large logs

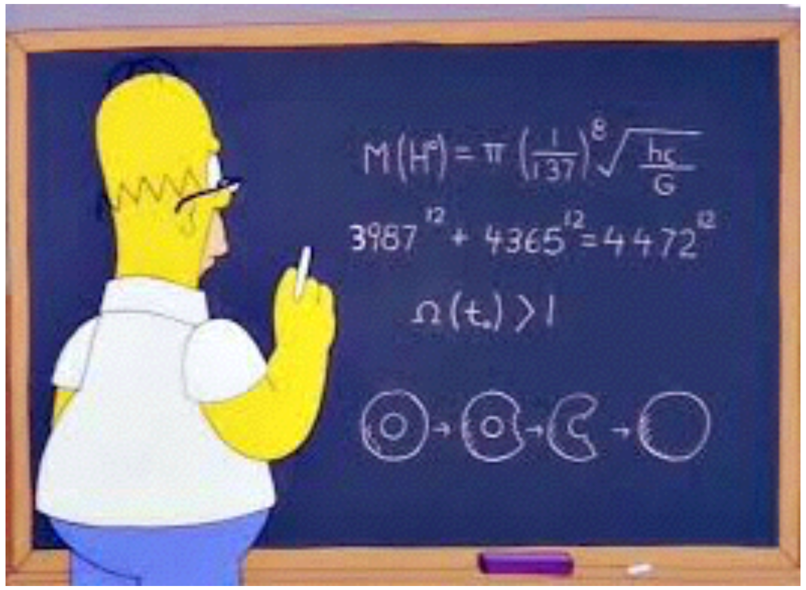
What experimentalists see

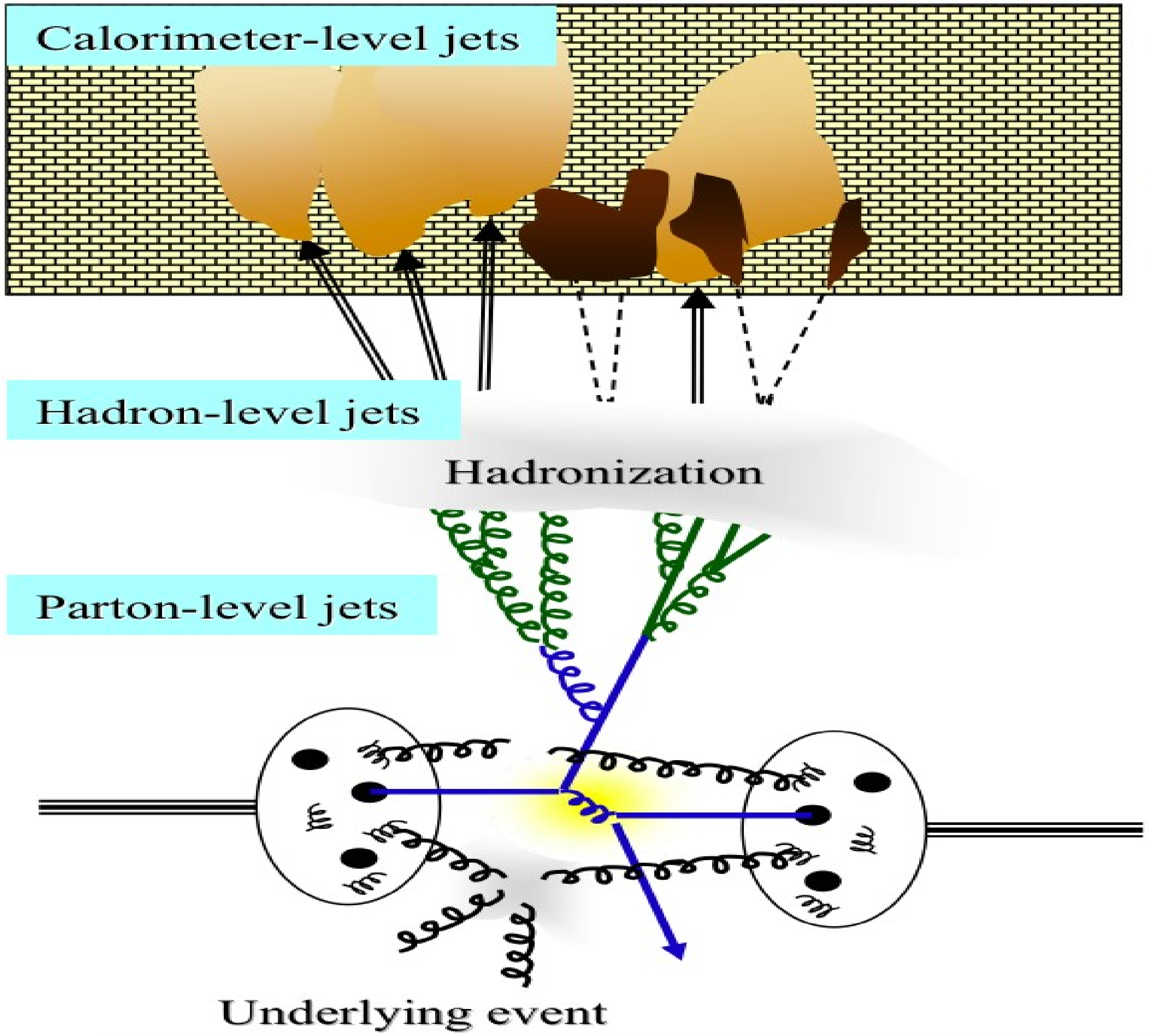




What really happens







Calorimeter-level jets

Hadron-level jets

Hadronization

Parton-level jets

Underlying event

- Large number of events of different kinds involving variety of particles at the production and detector levels
- The underlying theory, Quantum Chromodynamics provides a physical picture.
- Exact computation of such an observable is unrealistic.

Quantum Chromodynamics

- LHC being hadronic machine Quantum Chromodynamics(QCD), - the theory of Strong interaction physics plays an important role.

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- We can compute New physics signal and large SM background very precisely

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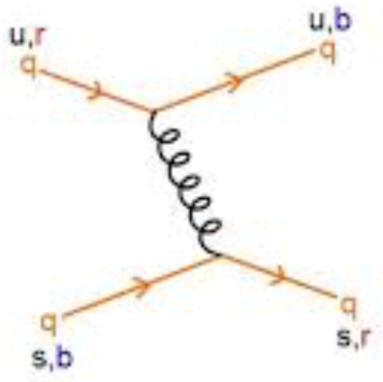
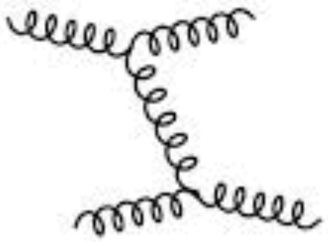
- Parameter of QCD is strong coupling constant g_s or Λ_{QCD}.

QCD-a toolkit for discovering NEW PHYSICS at LHC



gluon exchange by 2 quarks



gluon-gluon scattering

QCD improved Parton Model

$$P_1 + P_2 \to higgs + X$$

$$d\sigma^{P_1 P_2} = \sum_{ab} \int dx_1 \int dx_2 f_{\frac{a}{P_1}}\left(x_1, \mu_F^2\right) f_{\frac{b}{P_2}}\left(x_2, \mu_F^2\right) d\hat{\sigma}^{ab}\left(x_1, x_2, \{p_i\}, \mu_F^2\right),$$

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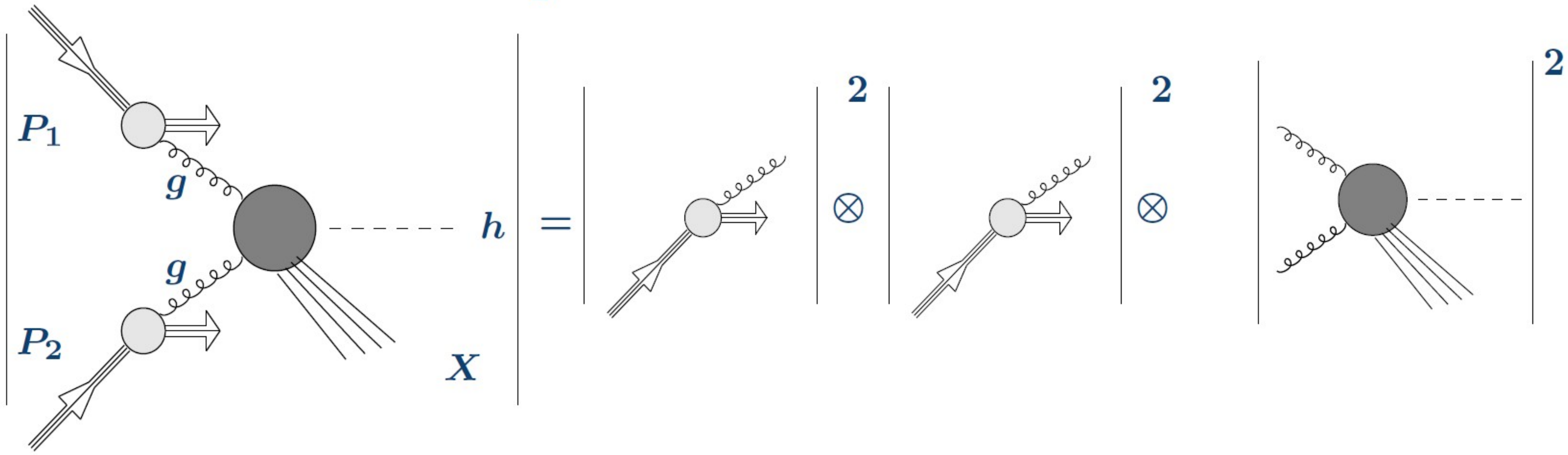

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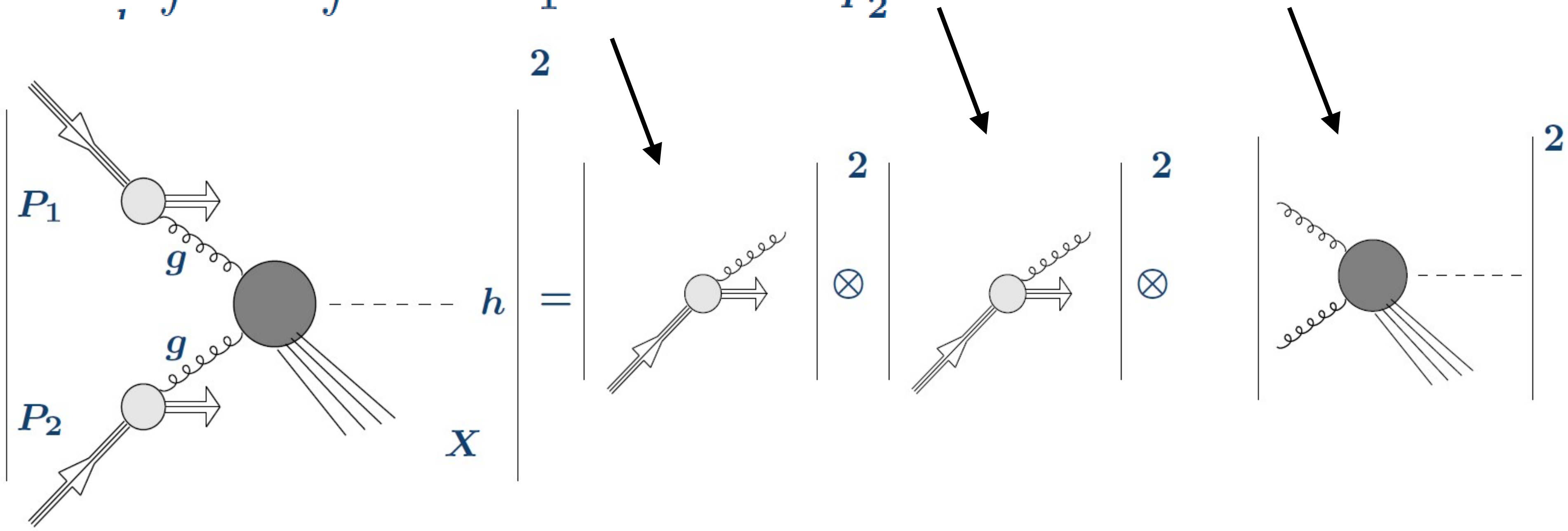

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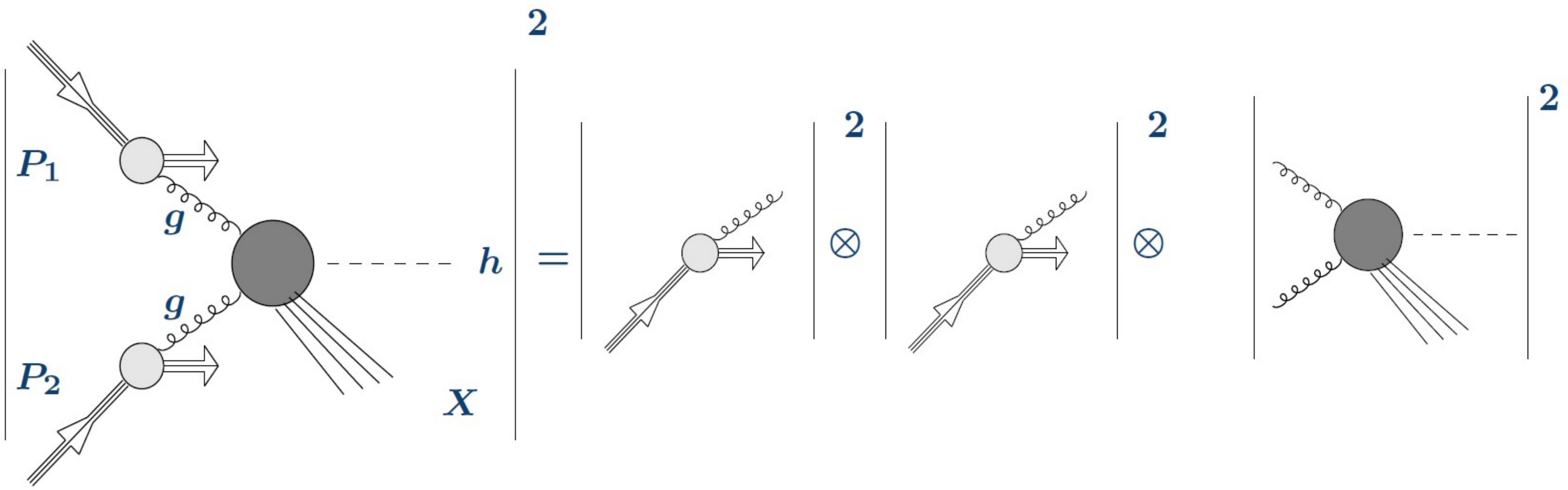

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- $\hat{\sigma}_{ab}(x_i, \{p_i\}, \mu_F^2)$ are the partonic cross sections.
- Perturbatively calculable.

Factorisation Theorem

Hadronic cross section in terms of partonic cross sections convoluted with appropriate PDF:

$$2S\, d\sigma^{P_1 P_2}\left(\tau, m_h^2\right) = \sum_{ab} \int_\tau^1 \frac{dx}{x} \Phi_{ab}\left(x, \mu_F\right) 2\hat{s}\, d\hat{\sigma}^{ab}\left(\frac{\tau}{x}, m_h^2, \mu_F\right)$$

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- The perturbatively calculable partonic cross section:

$$d\hat{\sigma}^{ab}\left(z, m_h^2, \mu_F\right) = \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu_R)}{4\pi}\right)^i d\hat{\sigma}^{ab,(i)}\left(z, m_h^2, \mu_F, \mu_R\right)$$

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- The Renormalisation group invariance:

$$\boxed{\frac{d}{d\mu}\sigma^{P_1 P_2}(\tau, m_h^2) = 0, \qquad \mu = \mu_F, \mu_R}$$

LO is a crude approximation!

Higgs Production through gluon fusion:

$$2S\, d\sigma^{PP}(x, m_H) = \int_x^1 \frac{dz}{z} \Phi_{gg}^{(0)}(z, \mu_F)\, 2\hat{s}\, d\hat{\sigma}_{gg}^{(0)}\left(\frac{x}{z}, m_H^2, \mu_R\right) + \cdots$$

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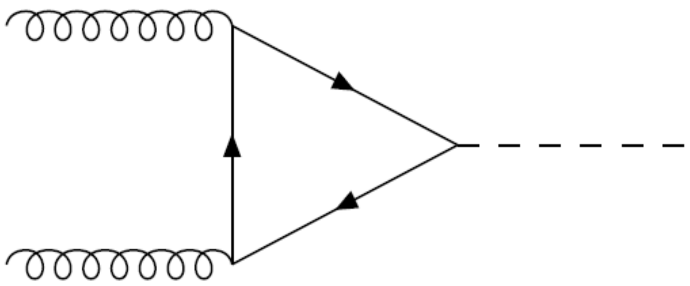

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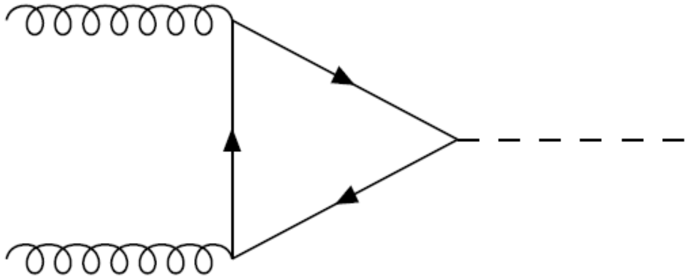

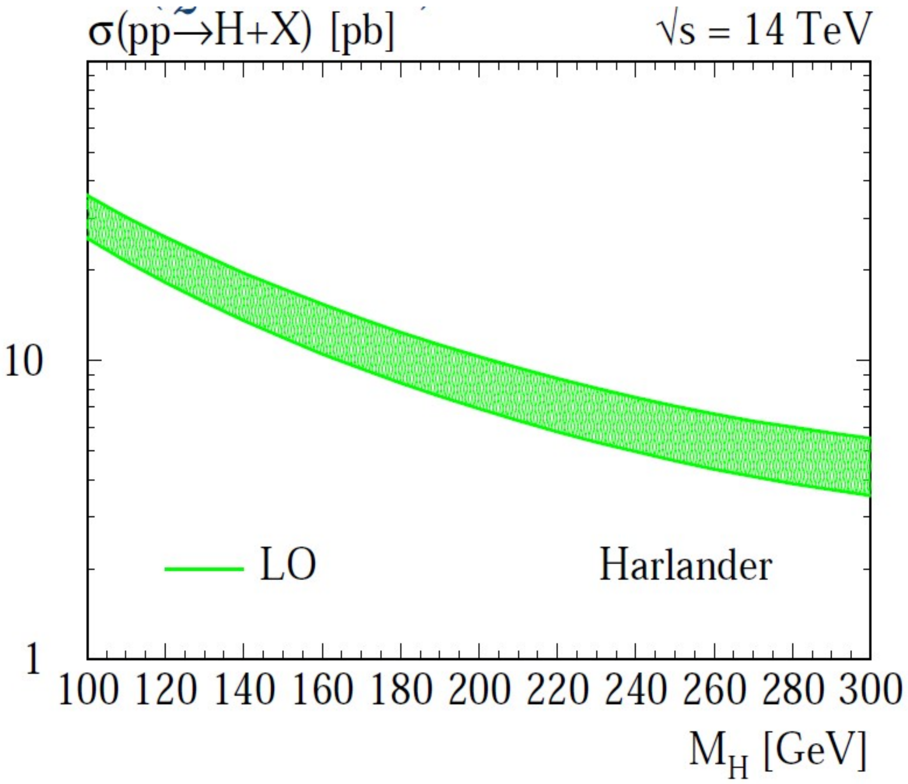

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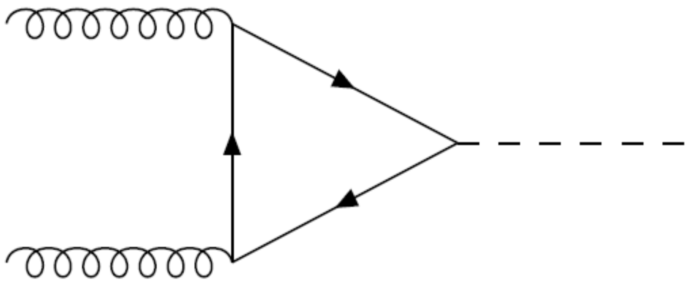

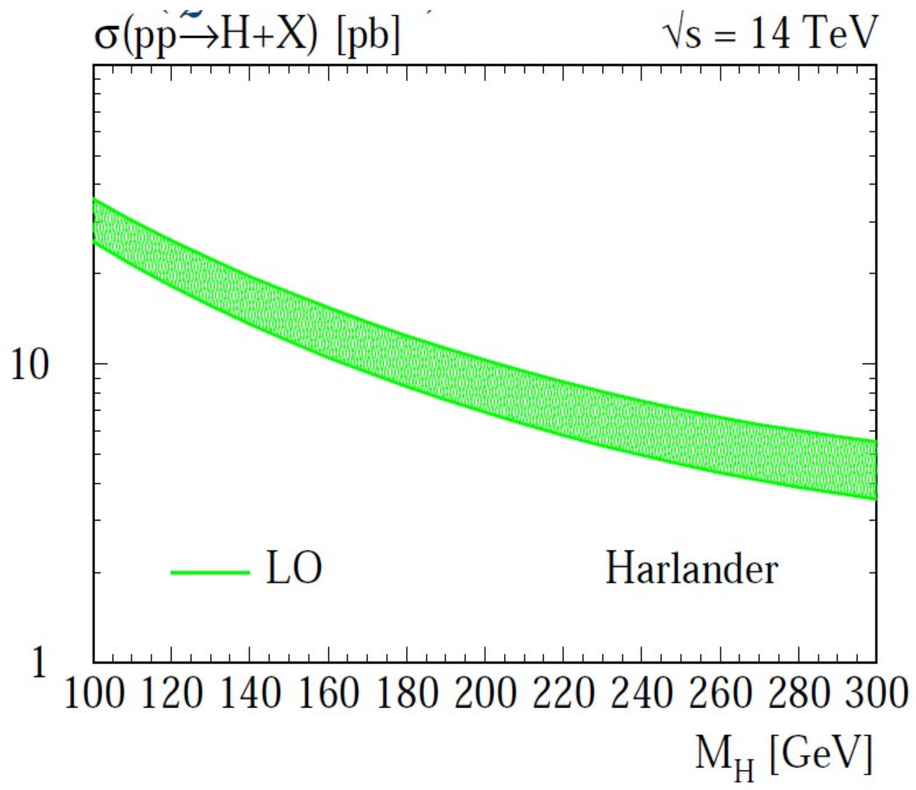

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$$2\hat{s}\, \hat{\sigma}^{(0)}_{gg}(\hat{s}, \mu_R) \sim \alpha_s^2(\mu_R)\, G_F \left[\frac{4m_t^2}{m_H^2} F\left(\frac{4m_t^2}{m_H^2}\right)\right], \qquad \frac{m_H}{2} < \mu_R = \mu_F < 2m_H$$

LO prediction is Unreliable due $100 - 200\%$ scale uncertainity

Inputs that can affect the predictions

- Renormalisation scale due to UV divergences

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- Parton Distribution Functions PDF extracted from experiments

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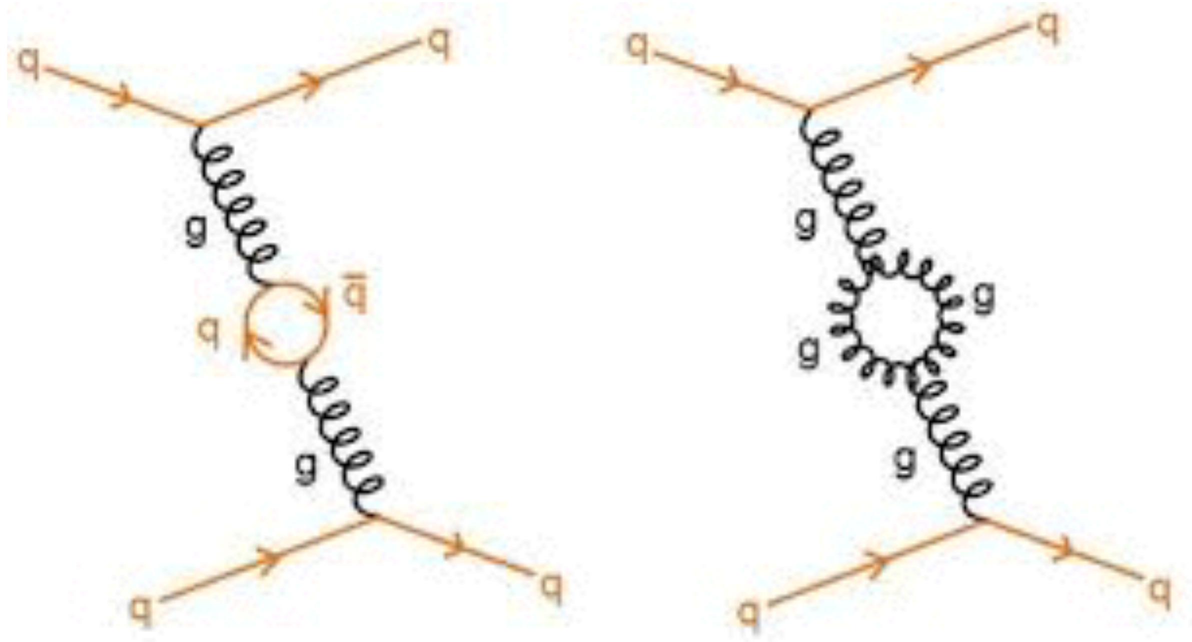
- Any "Fixed order" perturbative result is bound to depend on μ_R and μ_F and type of PDF sets.
- Observables are "free" of μ_R and μ_F.

$$\mu \frac{d}{d\mu} \sigma^{P_1 P_2} = 0, \qquad \mu = \mu_F, \mu_R, PDF$$

Strong Coupling Constant

$$\alpha_s(\mu_R)$$



Renormalisation Group Equation α_s

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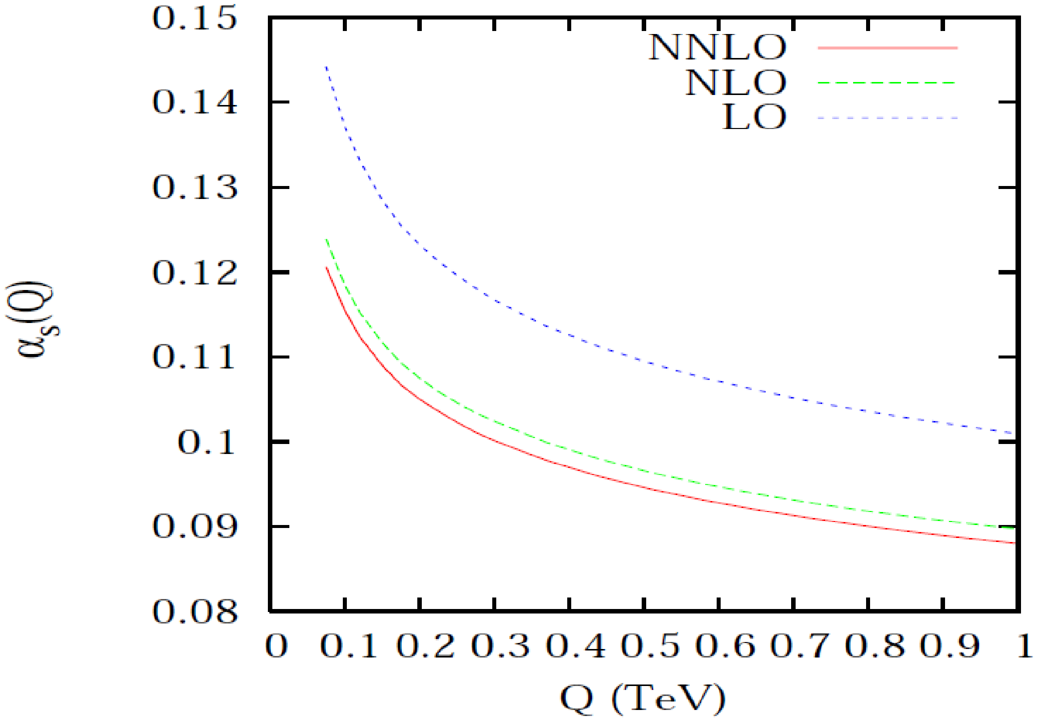

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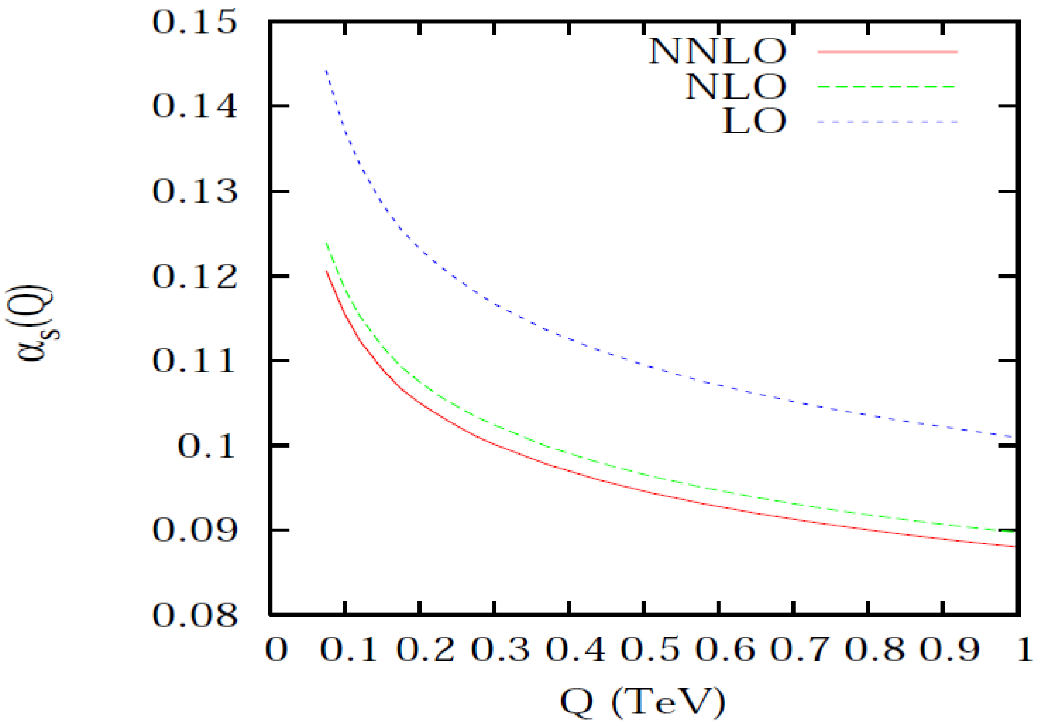
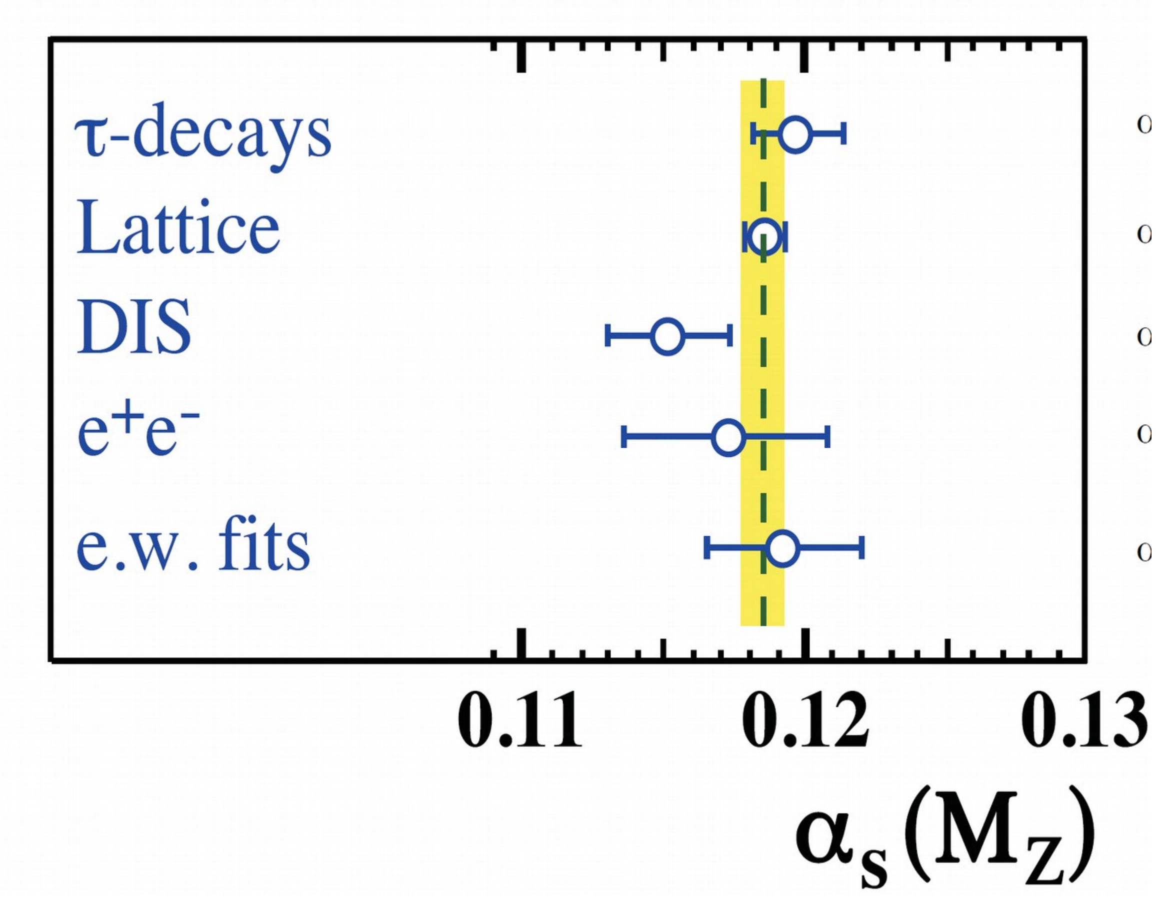


Measured from :

- Tau decays,
- lattice,
- heavy quarkonia decays,
- non-single structure functions,
- Jets from HERA,
- event shape variables from LEP

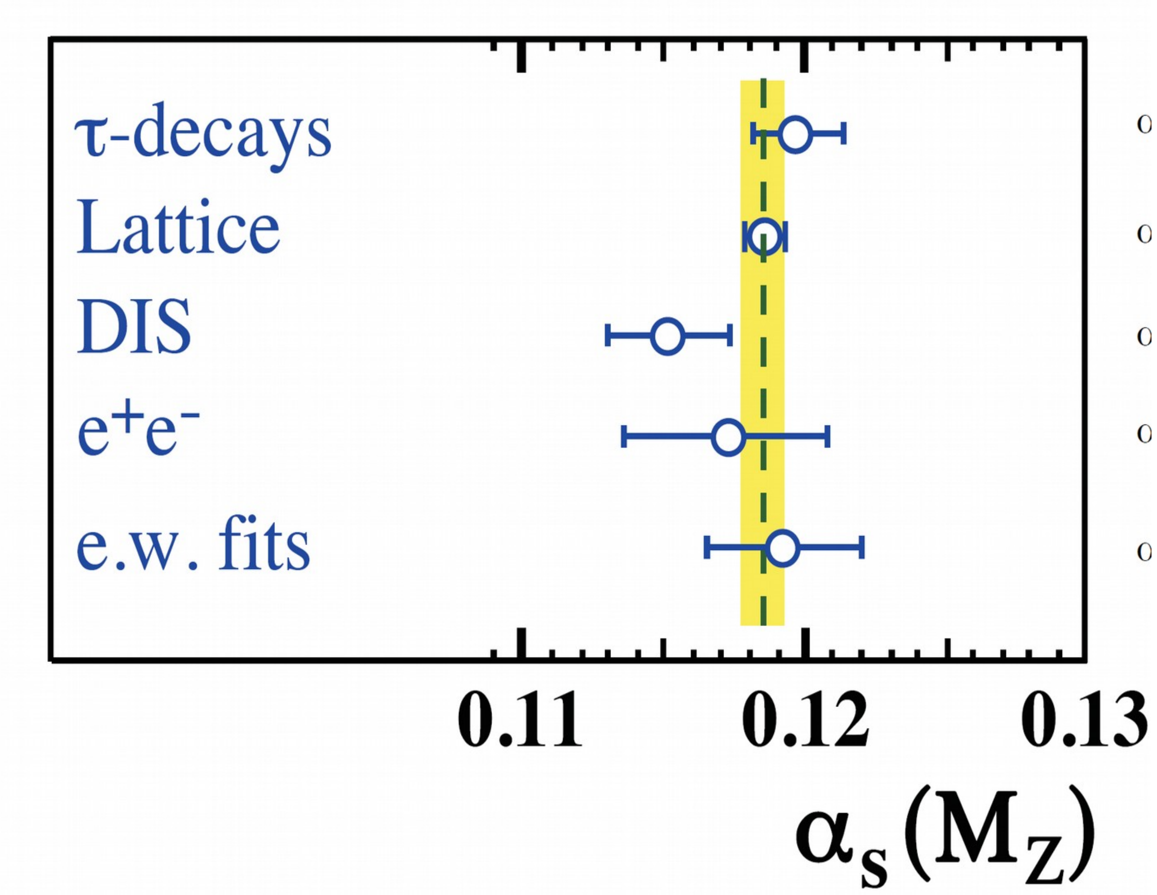
World Summary of α_s 2012:

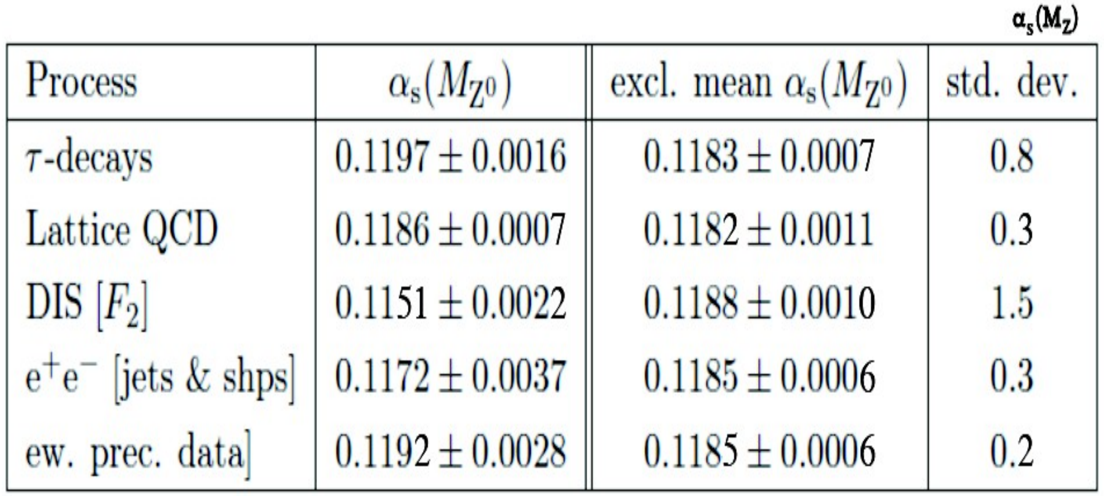
S. Bethke



World Summary of α_s 2012:

S. Bethke




$\alpha_s(M_Z)$

Process	$\alpha_s(M_{Z^0})$	excl. mean $\alpha_s(M_{Z^0})$	std. dev.
τ-decays	0.1197 ± 0.0016	0.1183 ± 0.0007	0.8
Lattice QCD	0.1186 ± 0.0007	0.1182 ± 0.0011	0.3
DIS [F_2]	0.1151 ± 0.0022	0.1188 ± 0.0010	1.5
e^+e^- [jets & shps]	0.1172 ± 0.0037	0.1185 ± 0.0006	0.3
ew. prec. data]	0.1192 ± 0.0028	0.1185 ± 0.0006	0.2

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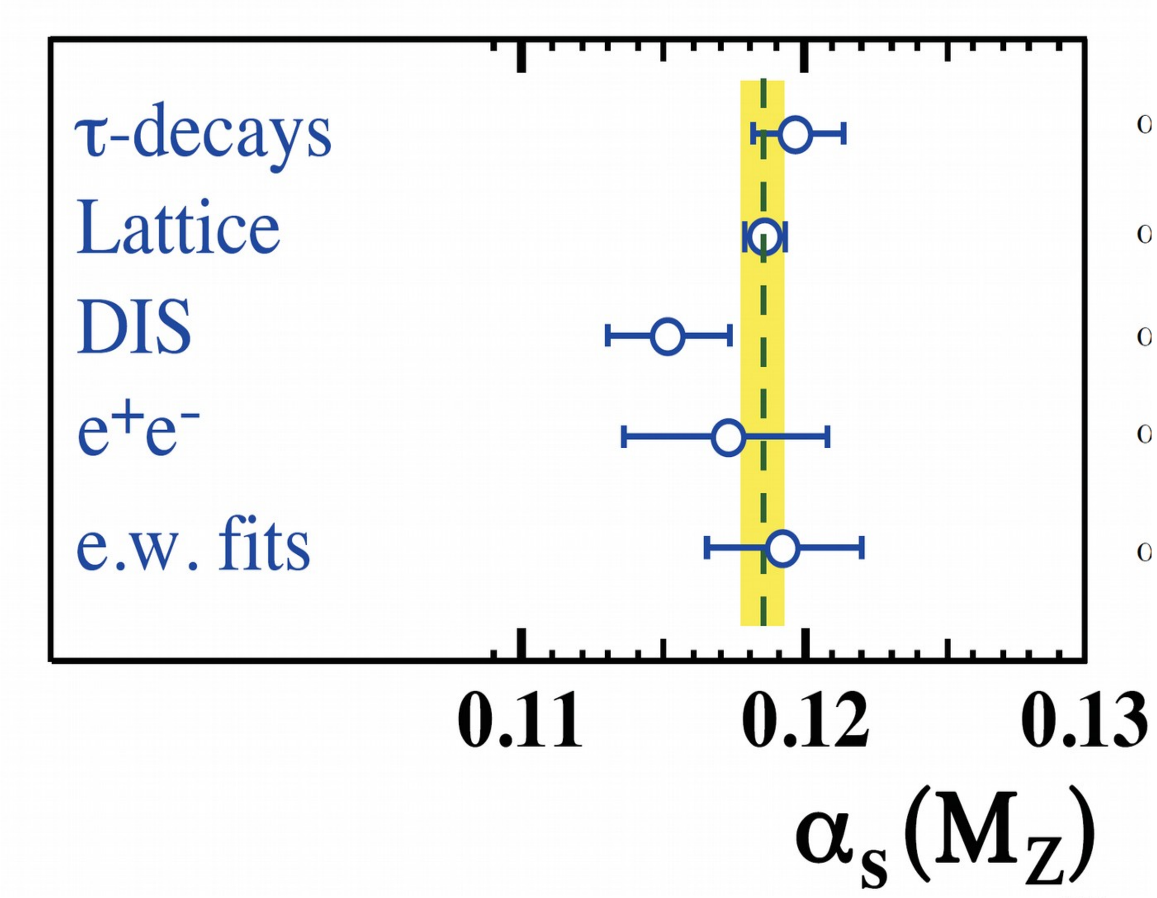

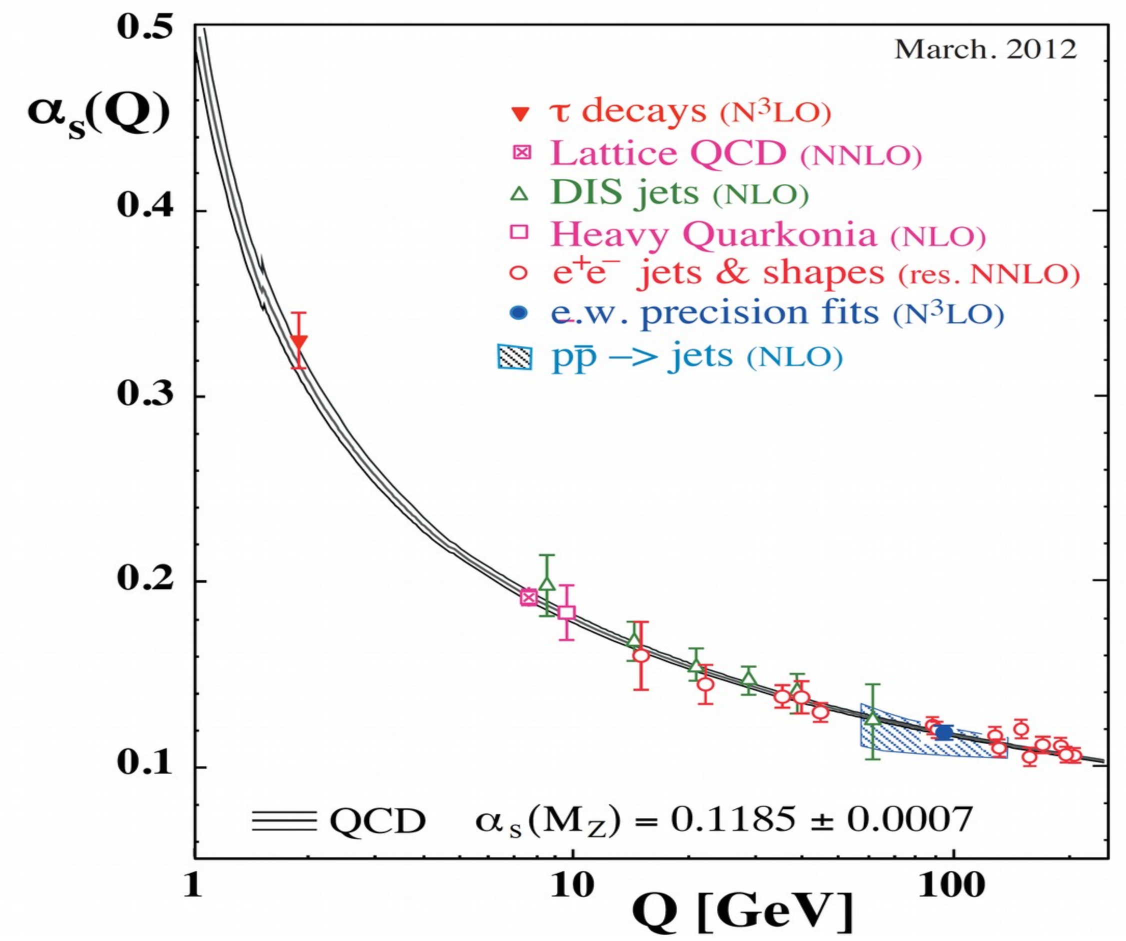

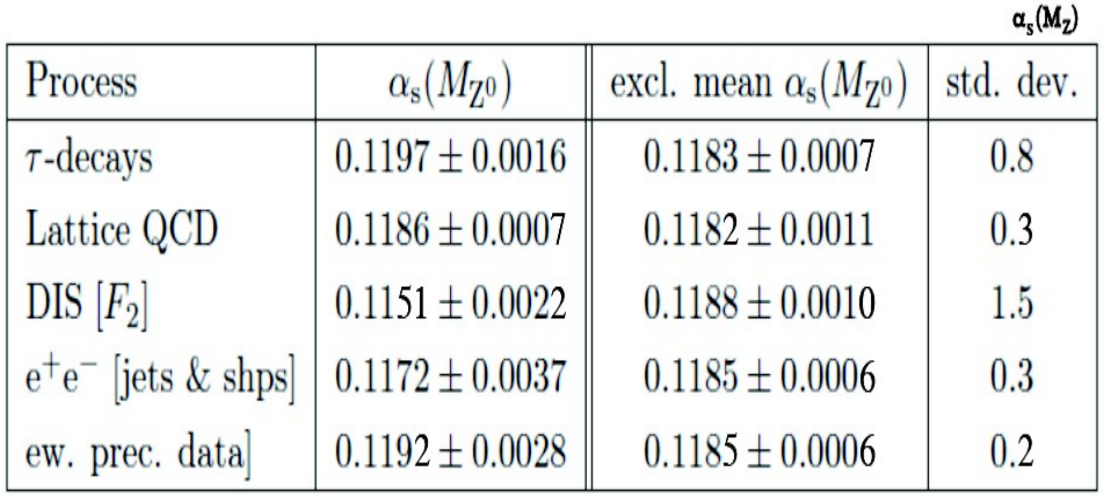

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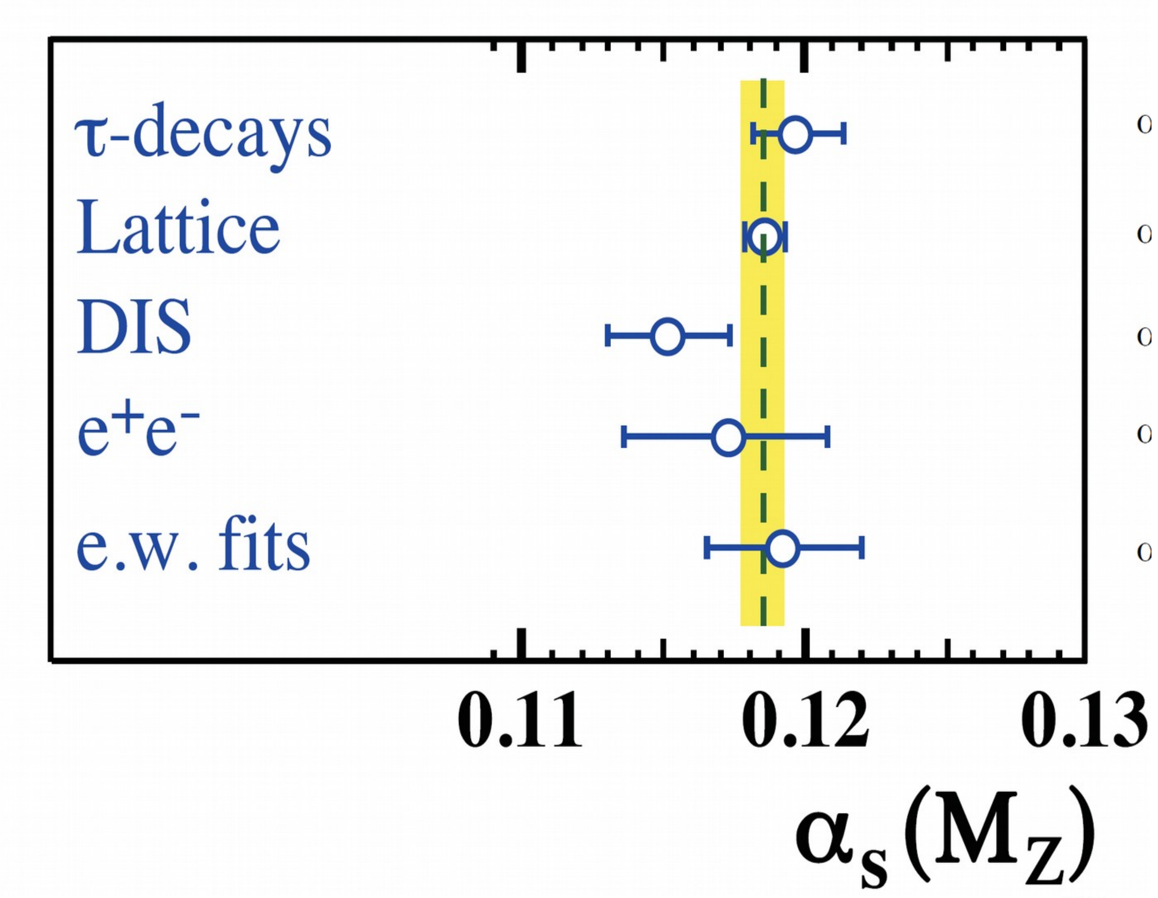

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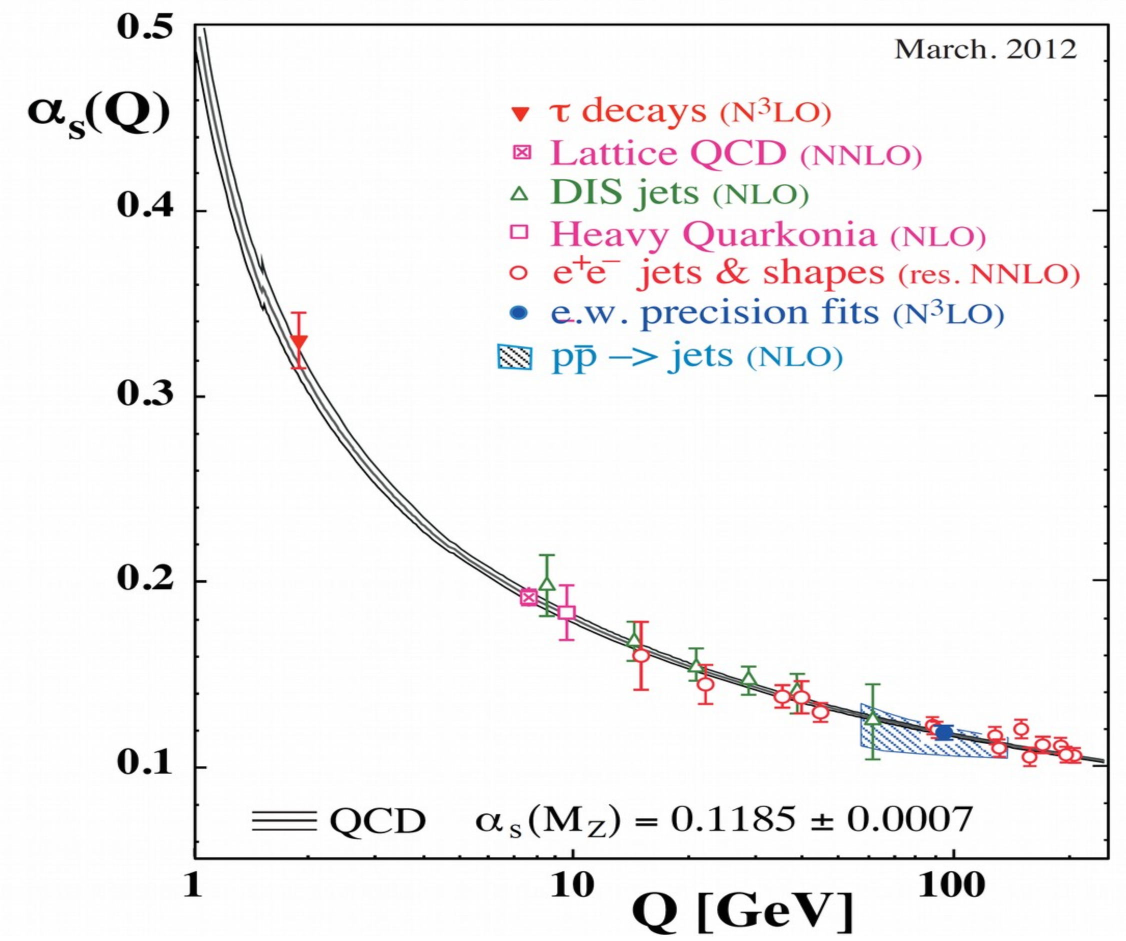
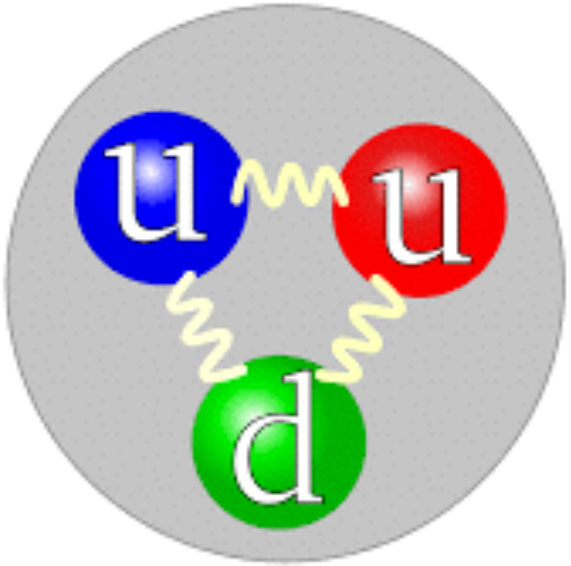


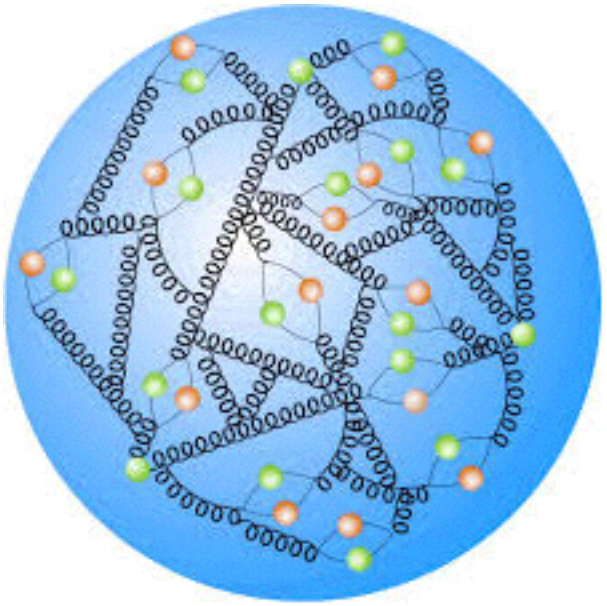
$$\alpha_s(M_Z) = 0.1185 \pm 0.0007$$

$$\Lambda_{\overline{MS}}^{(5)} = (214 \pm 9)\ \mathrm{MeV}$$
$$\Lambda_{\overline{MS}}^{(4)} = (297 \pm 11)\ \mathrm{MeV}$$

Parton Distribution Function

$$f_a\left(z, \mu_F\right)$$

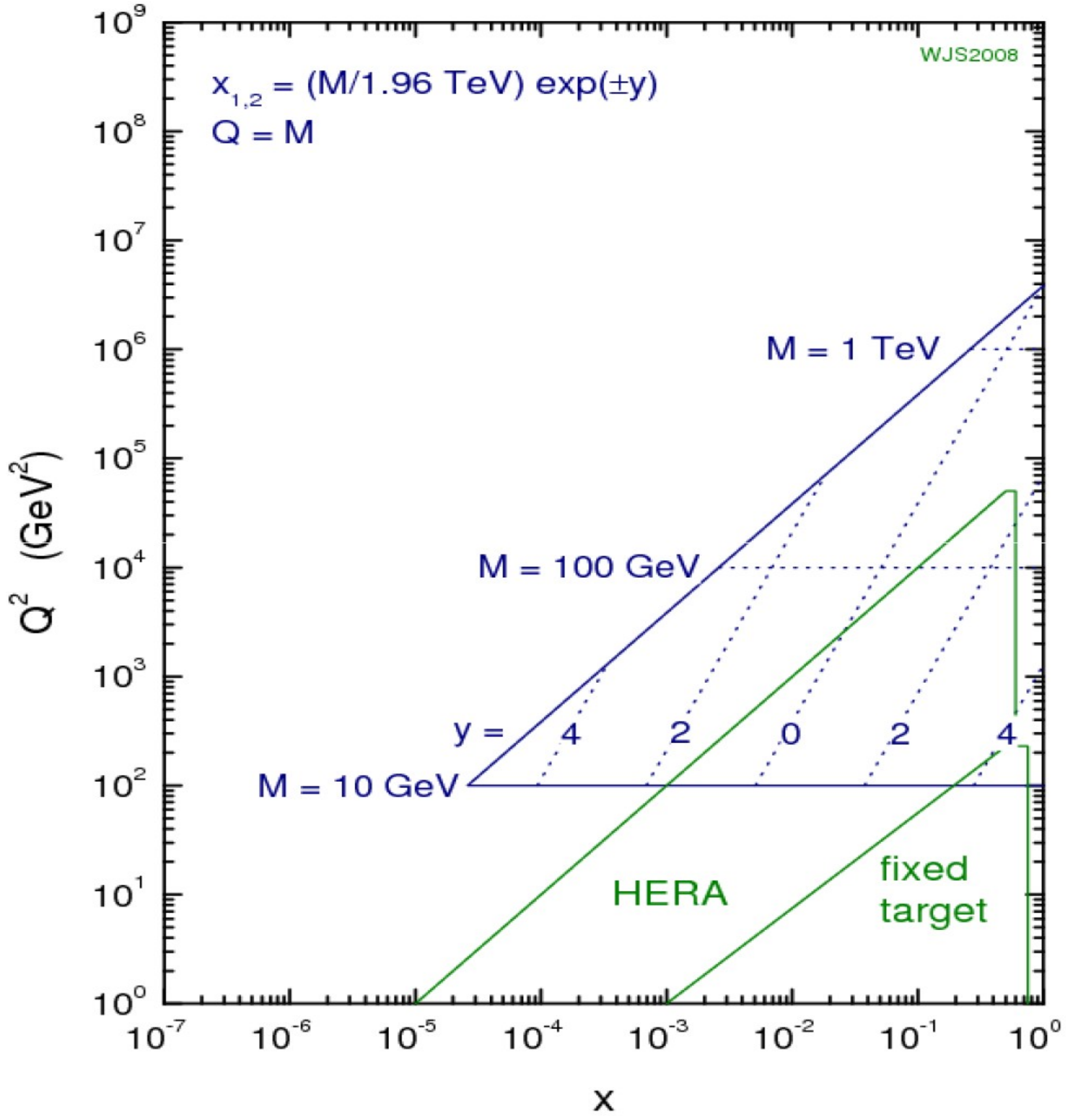




LHC-testing ground

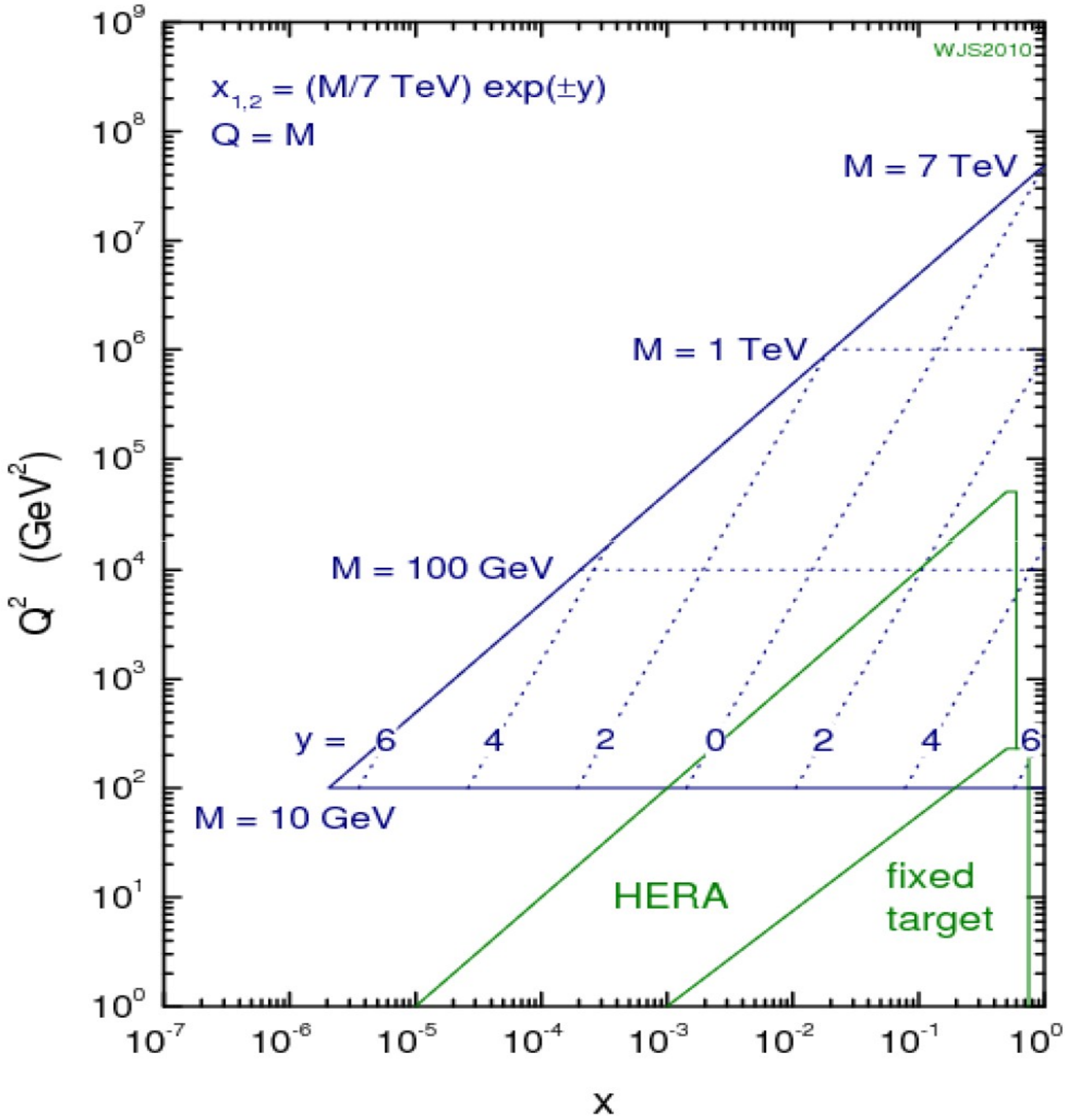
J. Stirling

Tevatron parton kinematics



0

7 TeV LHC parton kinematics



PDF and DGLAP evolution equation

Renormalised parton density:

$$f_a(z, \mu_F) = \Gamma_{ab}\left(z, \mu_F, \frac{1}{\varepsilon_{\mathrm{IR}}}\right) \otimes \; f_a^B(z)$$

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Dakshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) Evolution equation:

$$\mu_F \frac{d}{d\mu_F} f_a(x,\mu_F) = \int_x^1 \frac{dz}{z} P_{ab}(z,\mu_F) f_b\left(\frac{x}{z},\mu_F\right), \qquad P \equiv \Gamma^{-1}\left(\mu_F \frac{d}{d\mu_F}\right)\Gamma$$

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Perturbatively Calculable:

$$\begin{aligned} P_{ab}(z,\mu_F) \quad &= \quad \left(\frac{\alpha_s(\mu_F)}{4\pi}\right) P^{(0)}(z) && \text{one loop} \quad (LO) \\ &+\left(\frac{\alpha_s(\mu_F)}{4\pi}\right)^2 P^{(1)}(z) && \text{two loop} \quad (NLO) \\ &+\left(\frac{\alpha_s(\mu_F)}{4\pi}\right)^3 P^{(2)}(z) && \text{three loop} \quad (NNLO) \end{aligned}$$

NNLO is already known (summer **2004**)

Scale Variation of Flux at the LHC

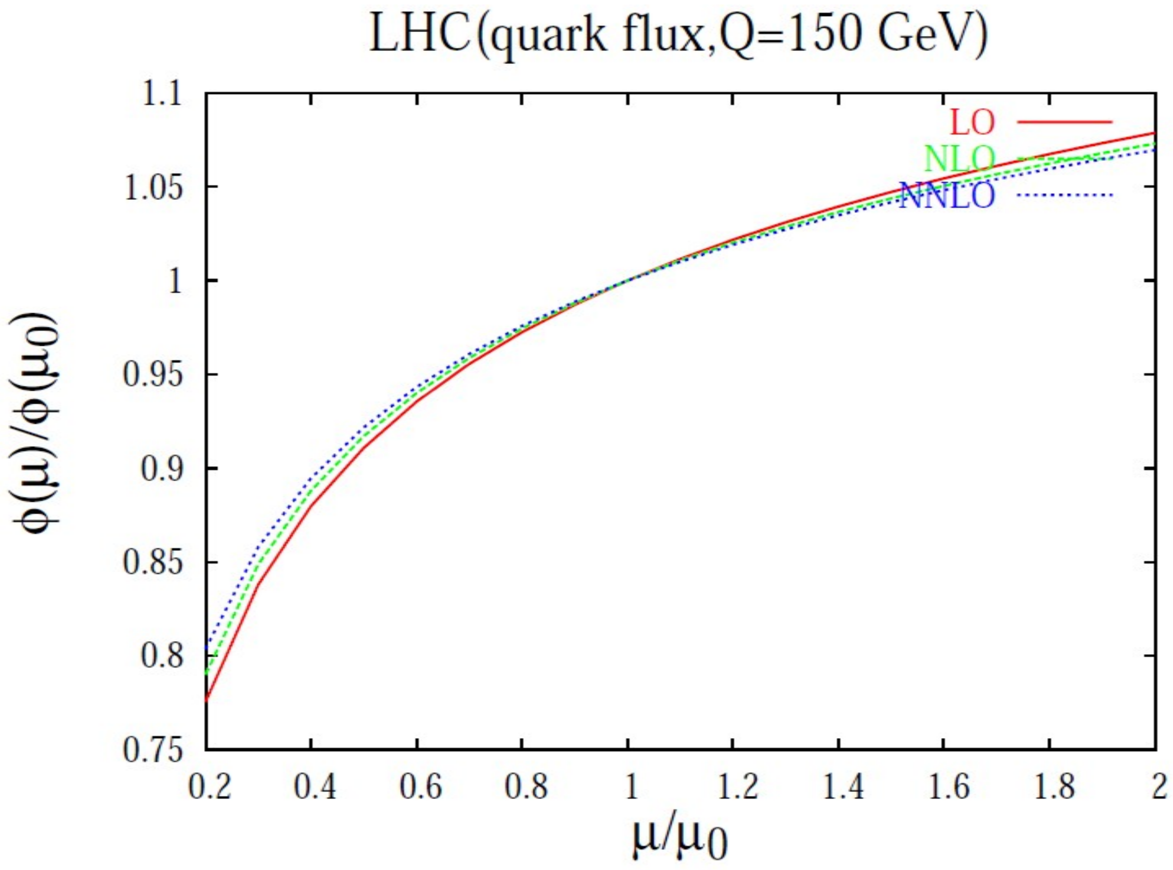
$$\Phi^{I}_{ab}(x,\mu_F) \quad = \quad \int_x^1 \quad \frac{dz}{z} \quad f^I_a\,(z,\mu_F) \quad f^I_b\left(\frac{x}{z},\mu_F\right) \qquad\qquad I = LO, NLO, NNLO$$

DGLAP evolution:

$$\mu_F \frac{d}{d\mu_F} f_a(x,\mu_F) = \int_x^1 \frac{dz}{z} P_{ab}(z,\mu_F) f_b\left(\frac{x}{z},\mu_F\right) \qquad\qquad \mu_F = \mu, \quad \mu_0 = 150\,GeV$$

Scale Variation of Flux at the LHC

$$\Phi^I_{ab}(x,\mu_F) \;=\; \int_x^1 \frac{dz}{z}\; f^I_a(z,\mu_F)\;\; f^I_b\left(\frac{x}{z},\mu_F\right) \qquad I = LO, NLO, NNLO$$



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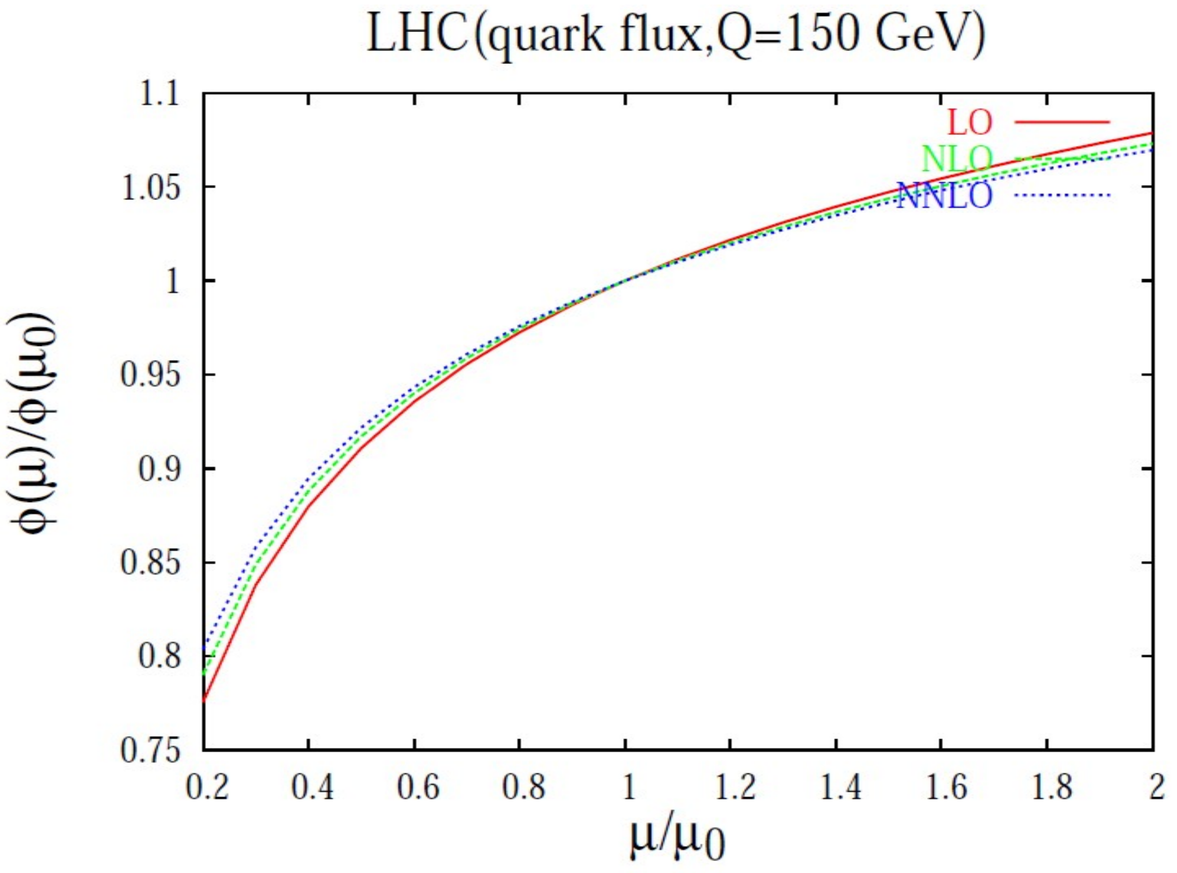
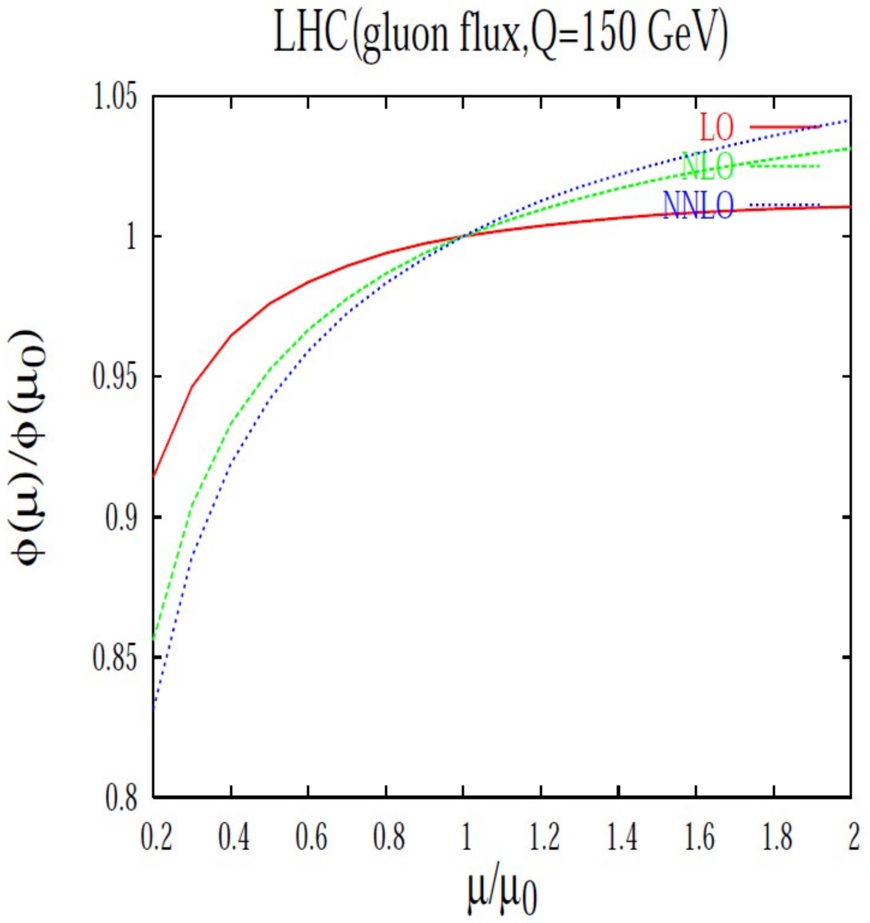

LHC(quark flux,Q=150 GeV)


LHC(gluon flux,Q=150 GeV)

DGLAP evolution:

$$\mu_F\frac{d}{d\mu_F}f_a(x,\mu_F) = \int_x^1 \frac{dz}{z}P_{ab}(z,\mu_F)f_b\left(\frac{x}{z},\mu_F\right) \qquad\qquad \mu_F = \mu, \quad \mu_0 = 150\,GeV$$

PDF sets

Different Groups:

MSTW, CTEQ, ABKM, ABM, NNPD,HERAPDF, GJR, ...

PDF sets

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MSTW, CTEQ, ABKM, ABM, NNPD,HERAPDF, GJR, ...

Exterimental inputs:

Deep Inelastic Scattering,
Drell-Yan,
Tevatron jets, Tevatron W,Z , ...

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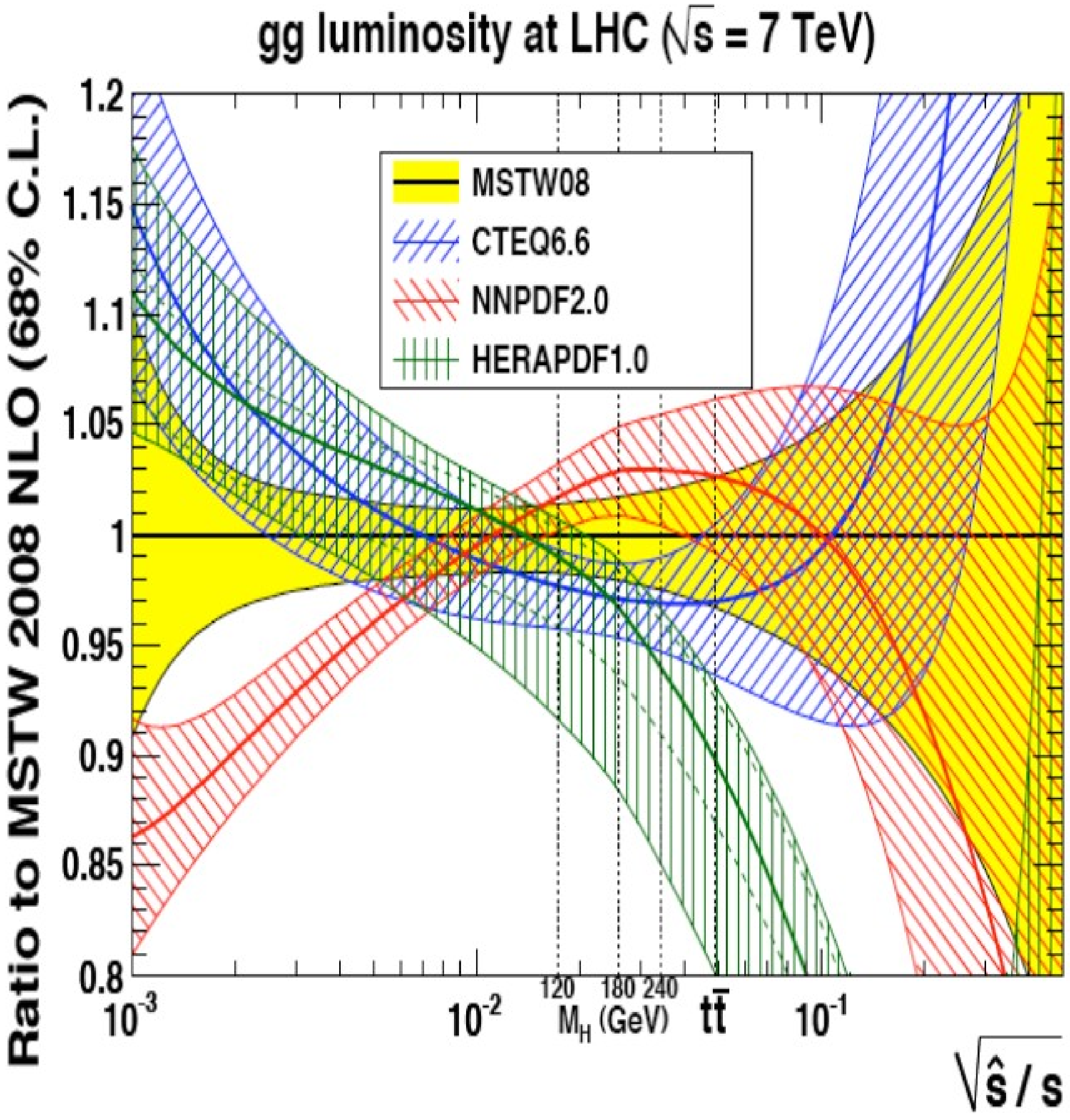
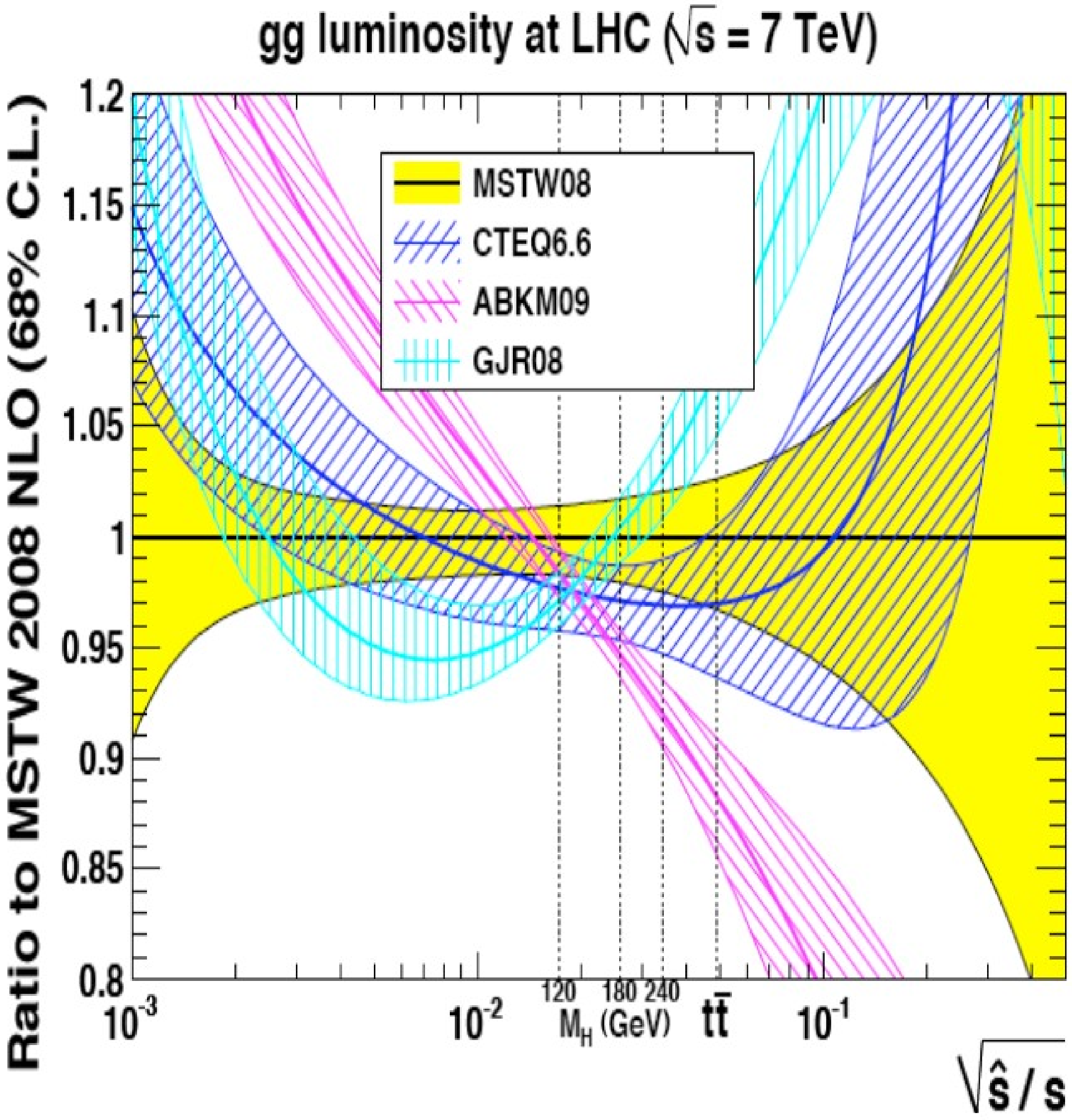
PDF uncertainity:

Choice of data sets
Treatment of heavy quarks
Treatment of errors
Order of perturbation theory
Parametrisation of densities
Flavour symmetries
Asymptotic behavious of pdfs

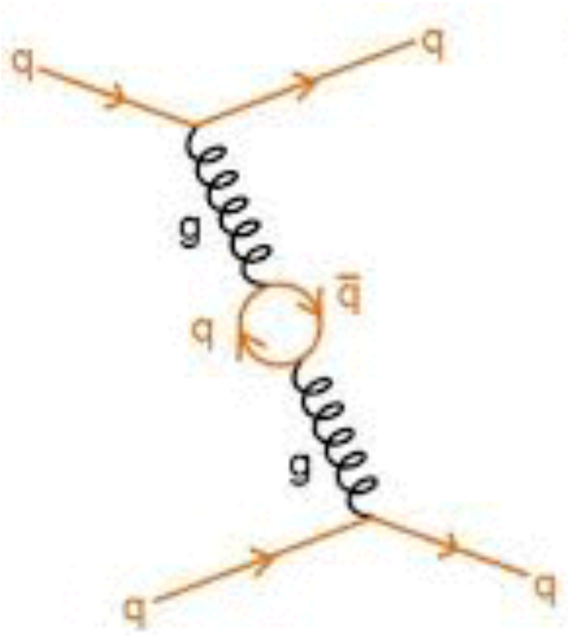
Gluon Luminosity

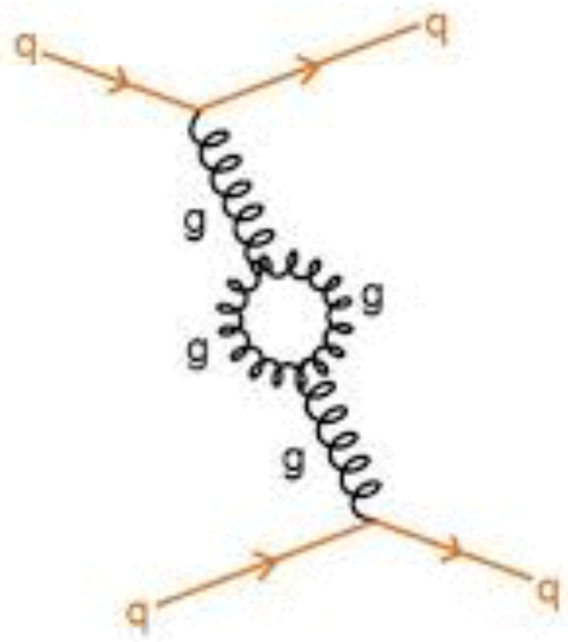




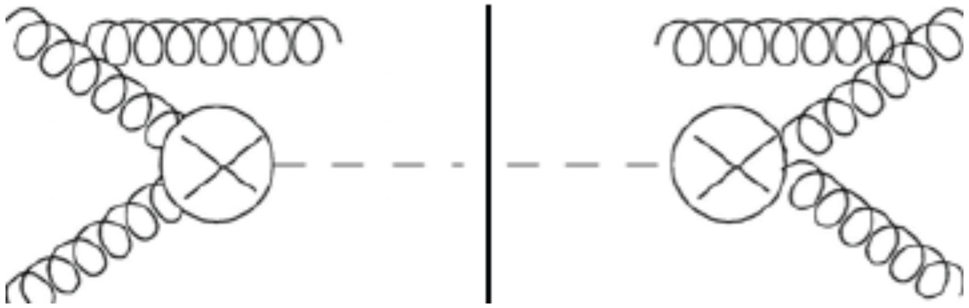
- Data sets: Electroproduction, hadron production (fixed target and collider)
- Fits procedure: Hessian and Monte Carlo
- Treatment: α_s, m_b and m_c

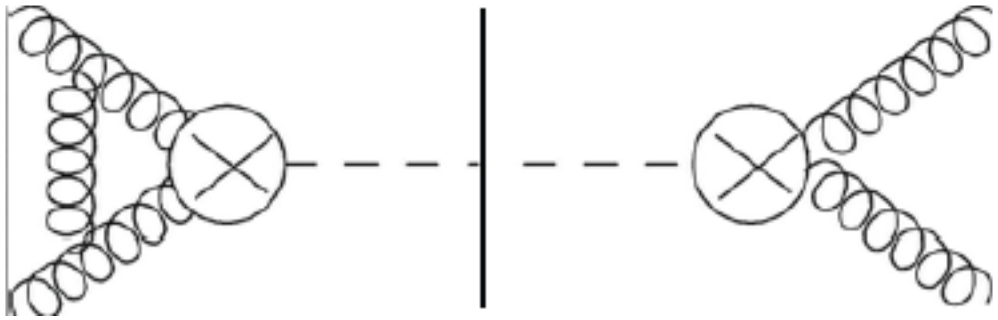
J. Stirling



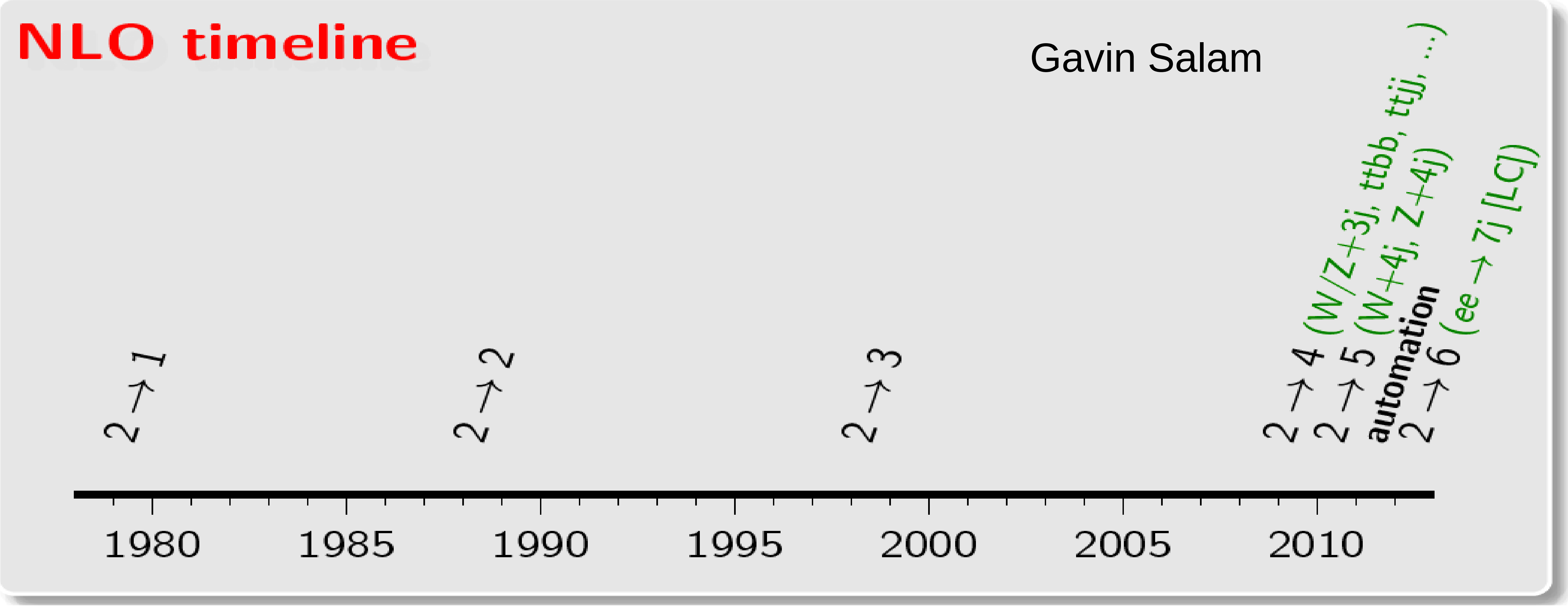


NLO



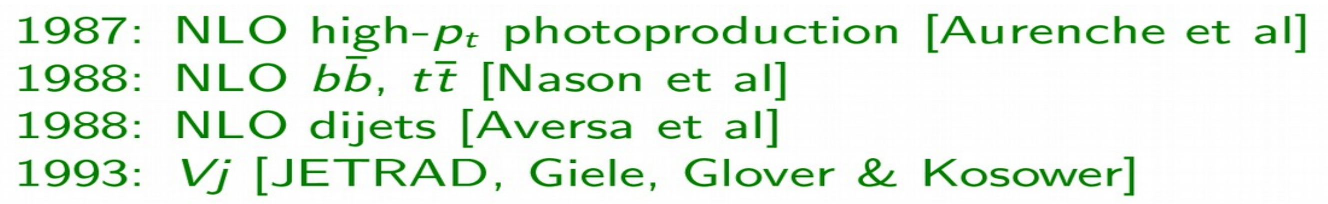


NLO revolution


NLO timeline

Gavin Salam


1979: NLO Drell-Yan [Altarelli, Ellis & Martinelli]
1991: NLO $gg \to$ Higgs [Dawson; Djouadi, Spira & Zerwas]


1987: NLO high-p_t photoproduction [Aurenche et al]
1988: NLO $b\bar{b}$, $t\bar{t}$ [Nason et al]
1988: NLO dijets [Aversa et al]
1993: Vj [JETRAD, Giele, Glover & Kosower]


1998: NLO $Wb\bar{b}$ [MCFM: Ellis & Veseli]
2000: NLO $Zb\bar{b}$ [MCFM: Campbell & Ellis]
2001: NLO $3j$ [NLOJet++: Nagy]
...
2007: NLO $t\bar{t}j$ [Dittmaier, Uwer & Weinzierl '07]
...

Advances at NLO

Analytical Methods

- Faster way of generating Feynman diagrams:

  QGRAF

- Sympolic manupulation:

  FORM, Mathematica

- On-shell methods
- Recursion techniques

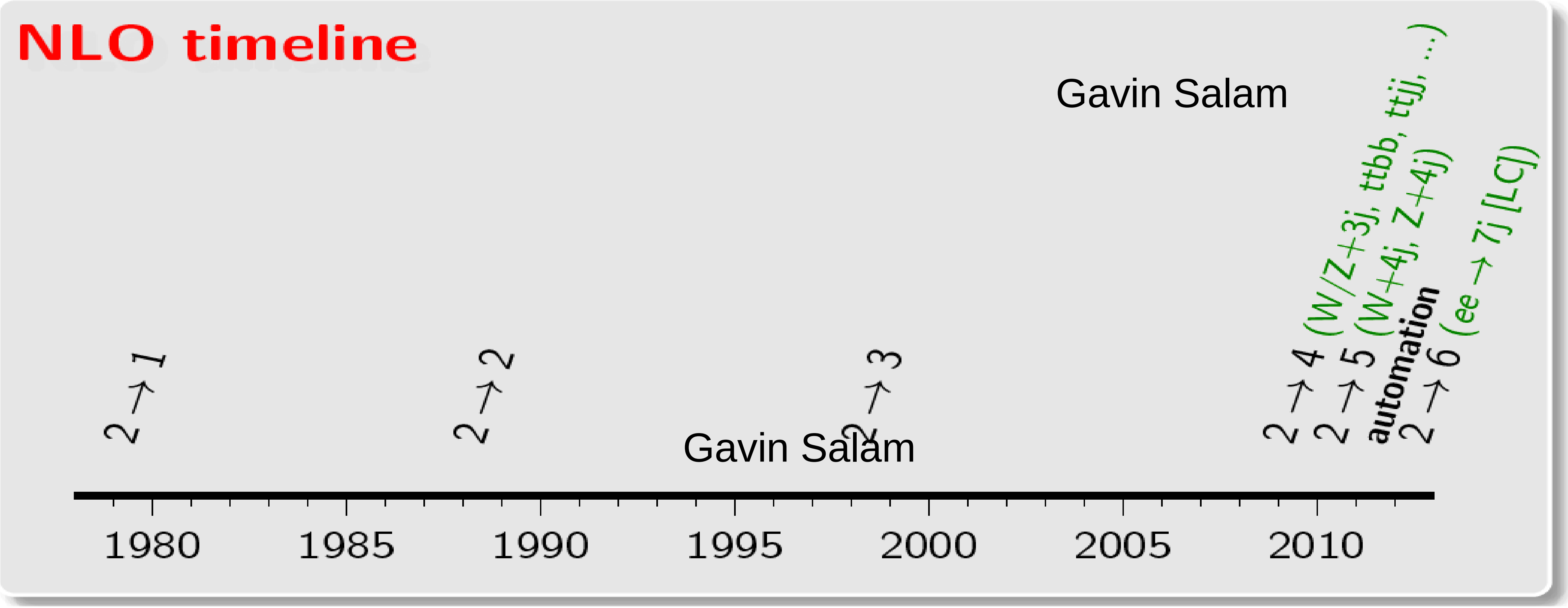
Merging NLO with Parton Showers:

- MC@NLO
- POWEG
- SHERPA
- VINCIA
- GENeVa
- aMC@NLO
- KRKMC

Semi-numerical methods

- Helac-NLO
- CutTools
- BlackHat
- Rocket
- SAMURAI
- MadLoop
- GoSam
- Ngluon

NLO revolution

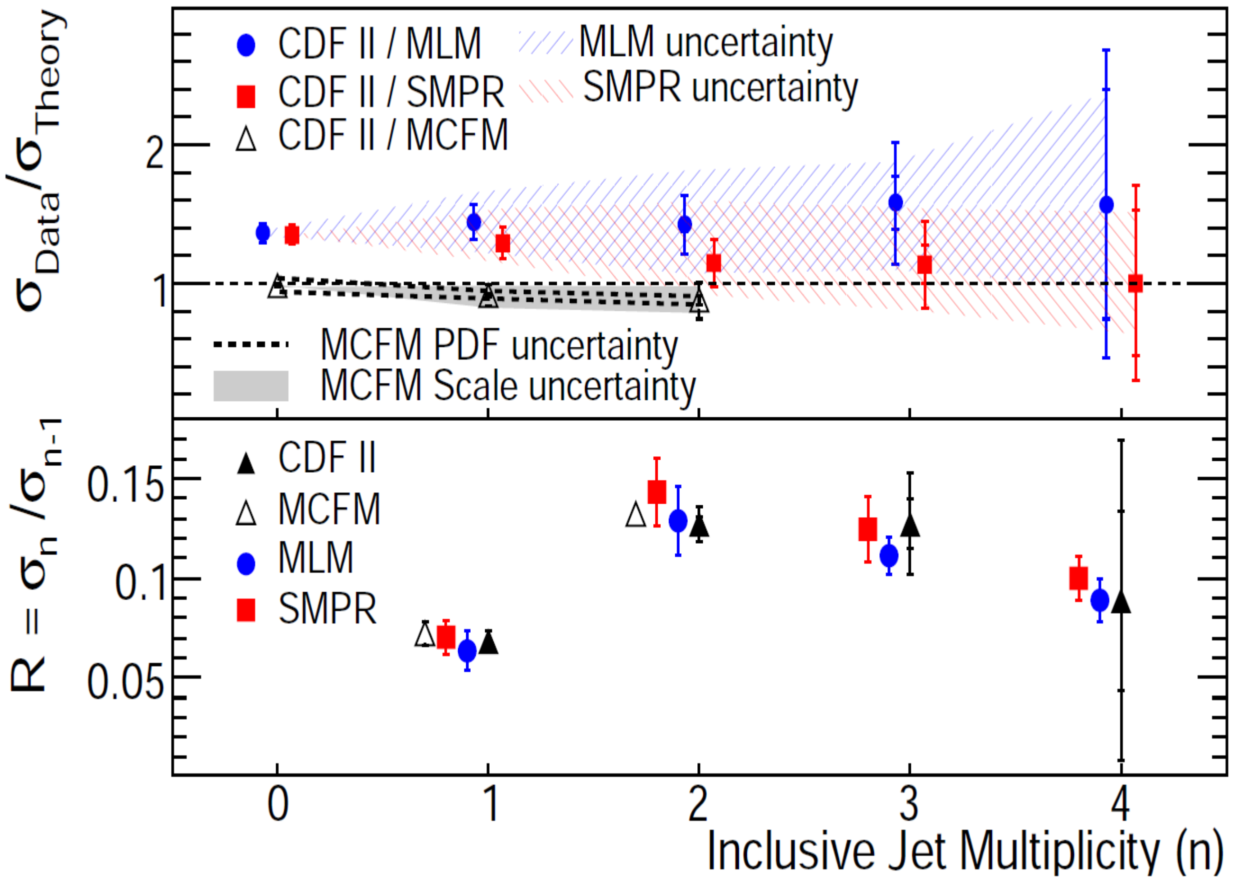


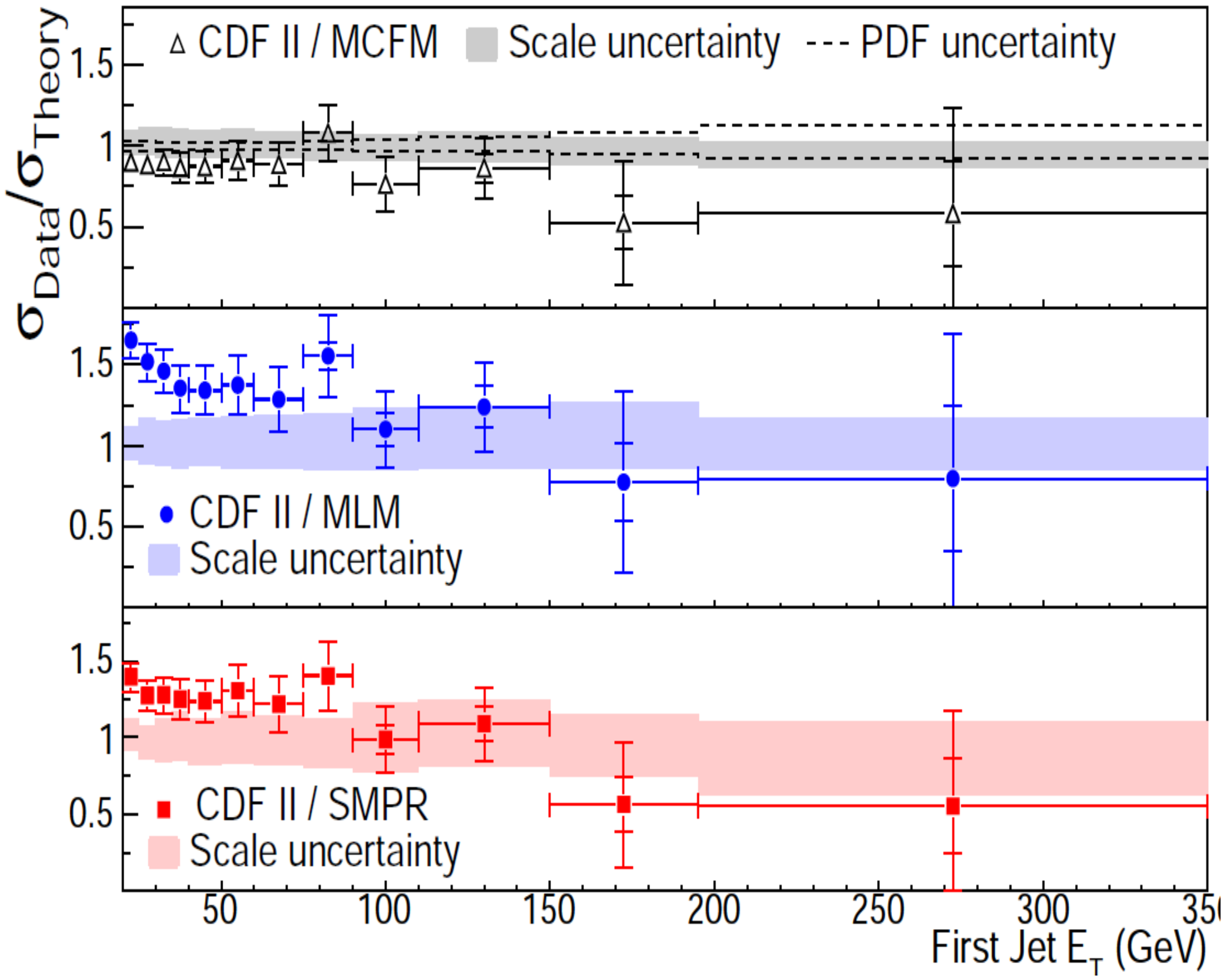
2009: NLO $W+3j$ [Rocket: Ellis, Melnikov & Zanderighi]
2009: NLO $W+3j$ [BlackHat+Sherpa: Berger et al]
2009: NLO $t\bar{t}b\bar{b}$ [Bredenstein et al]
2009: NLO $t\bar{t}b\bar{b}$ [HELAC-NLO: Bevilacqua et al]
2009: NLO $q\bar{q} \to b\bar{b}b\bar{b}$ [Golem: Binoth et al]
2010: NLO $t\bar{t}jj$ [HELAC-NLO: Bevilacqua et al]
2010: NLO $Z+3j$ [BlackHat+Sherpa: Berger et al]
...

Role of NLO corrections

W + n-jet cross section

- SMPR: Madgraph, Pythia, Jetglu
- MLM: ALPGEN, Herwig, Jetglu
- MCFM: NLO QCD + showering

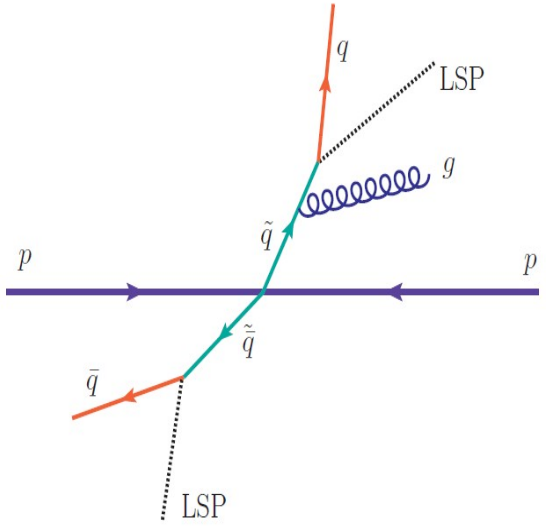


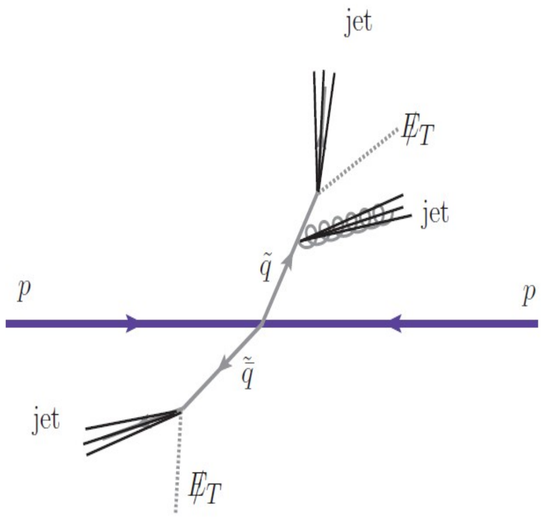


MCFM: NLO QCD works better

Z background to SUSY searches

- Susy searches require estimate on the Z background
- Hard to measure Z background
- Photon rates are 6 times larger easy to measure.
- Use theory to get the ratio $R_{Z/\gamma}$





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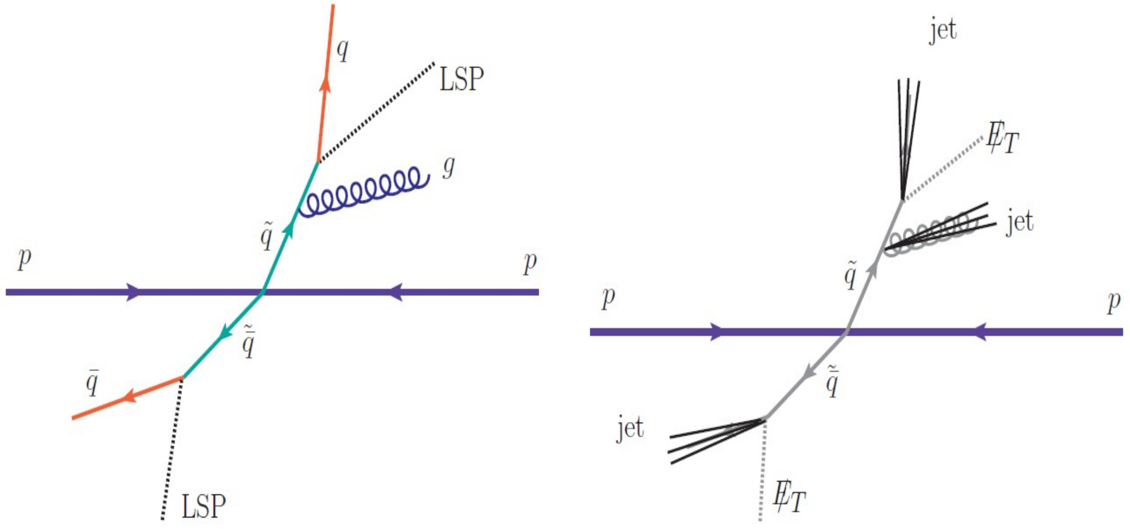
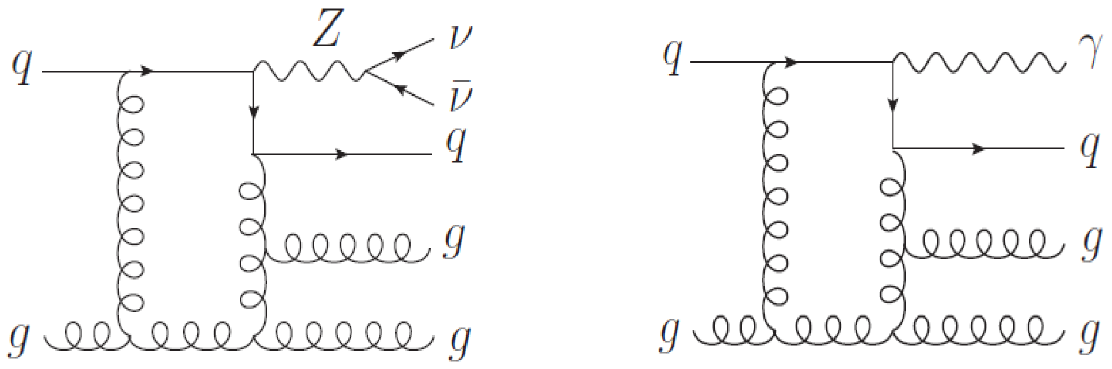
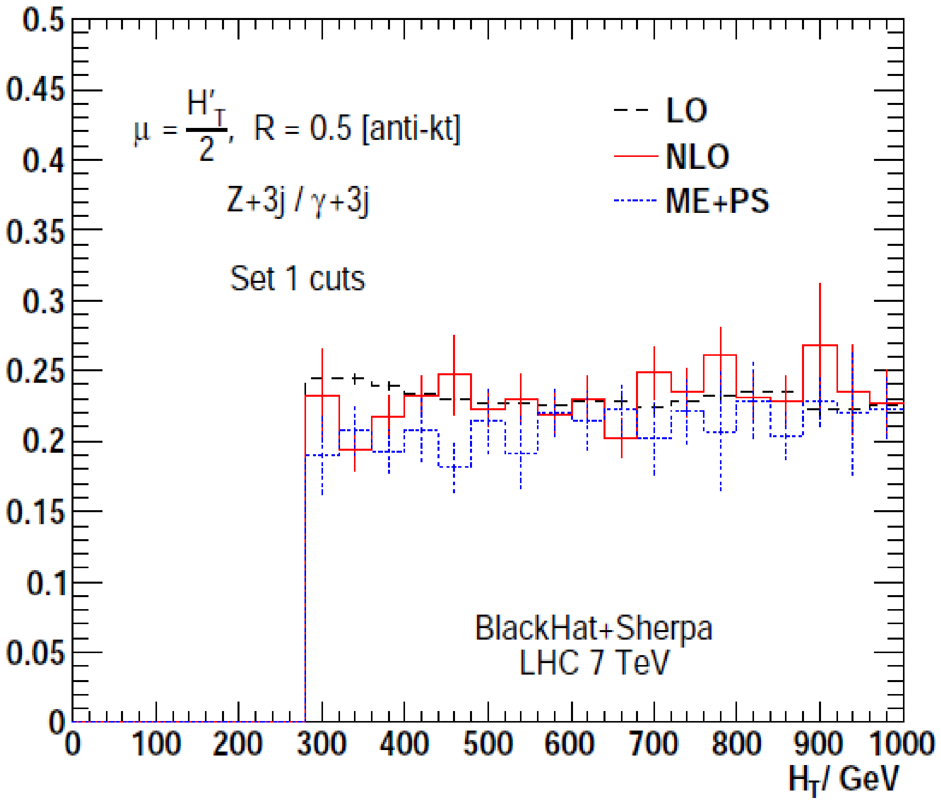


$$\sigma(pp \to Z(\to \nu\bar{\nu}) + \text{jets}) = \sigma(pp \to \gamma + \text{jets}) \times R_{Z/\gamma}$$

Background measured theory



Theory predictions

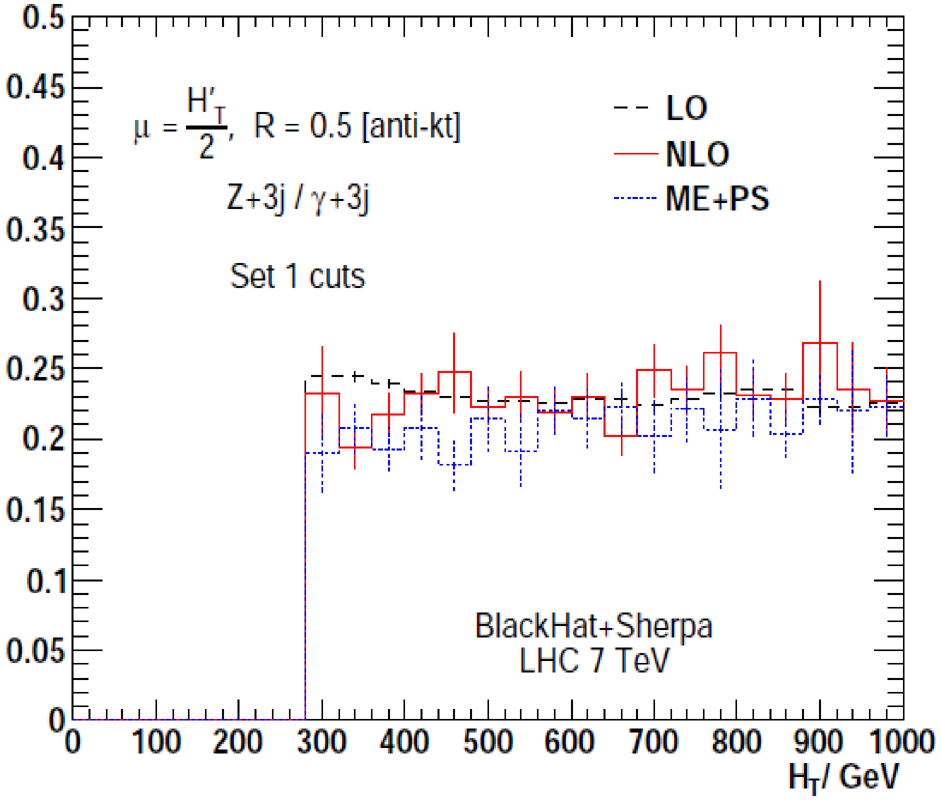


BlackHat

Virtual: On-shell and Unitarity cut techniques
Real : SHERPA

CMS and ALTAS use this
to estimate METZJ background
for SUSY searches

Theory predictions



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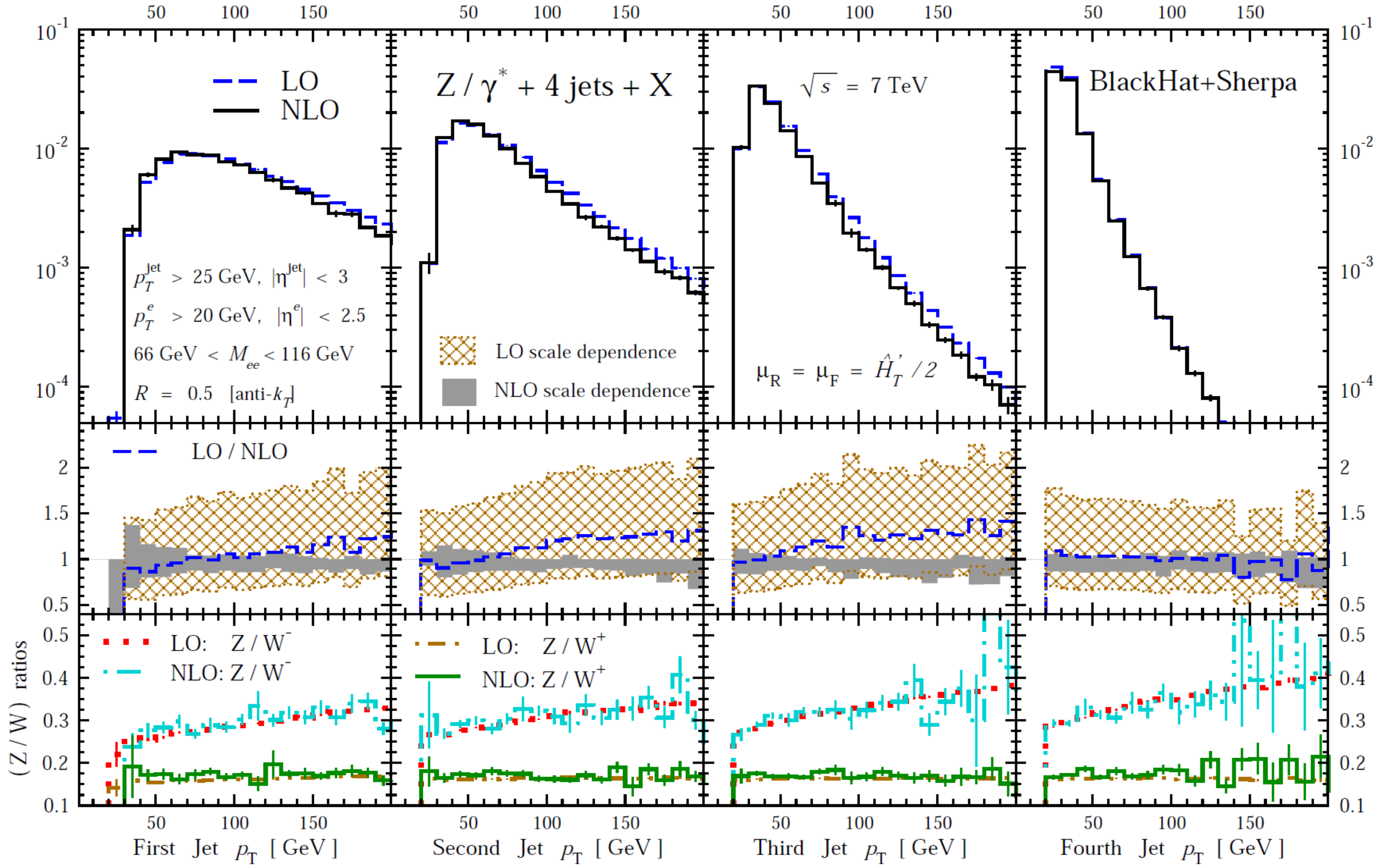
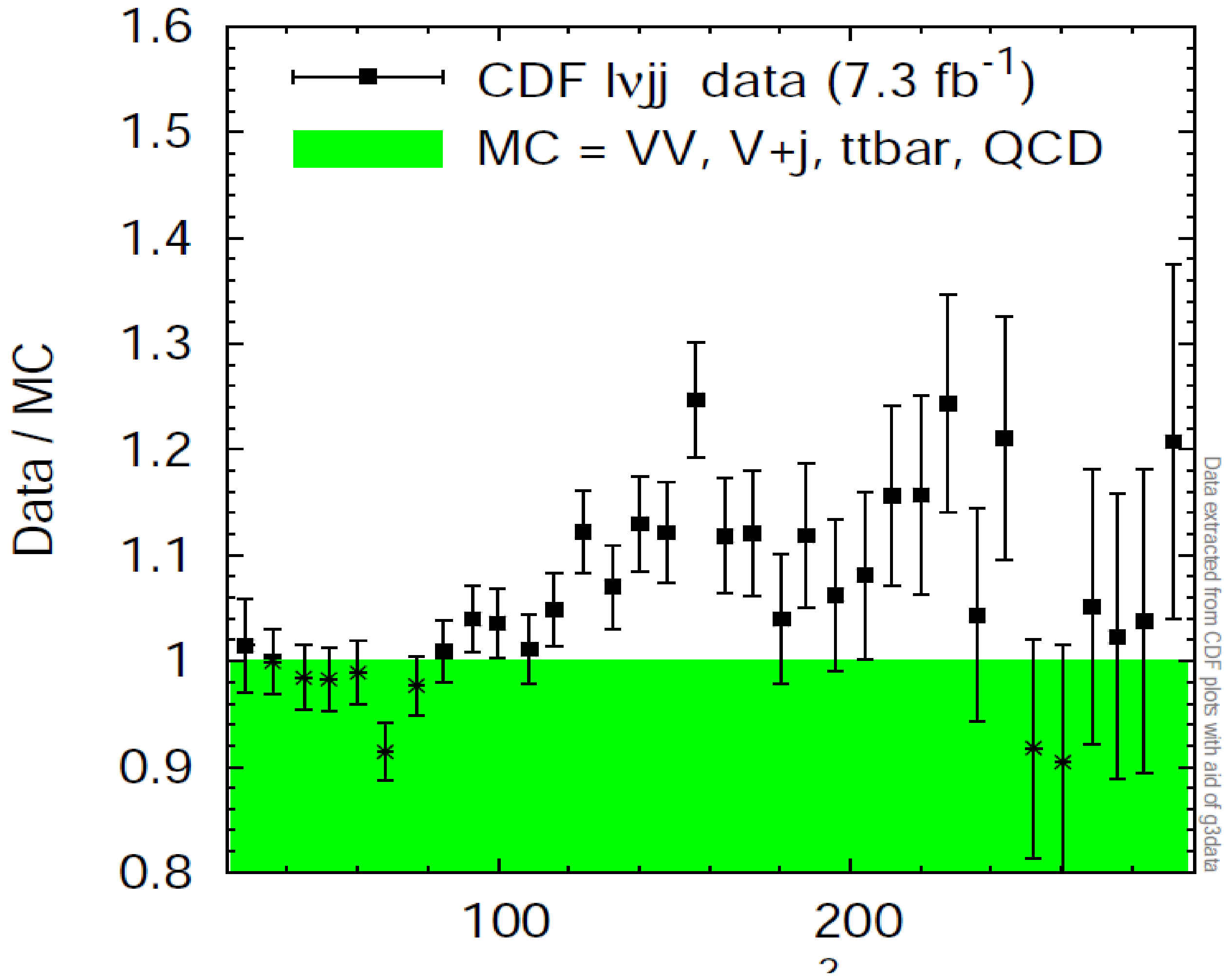


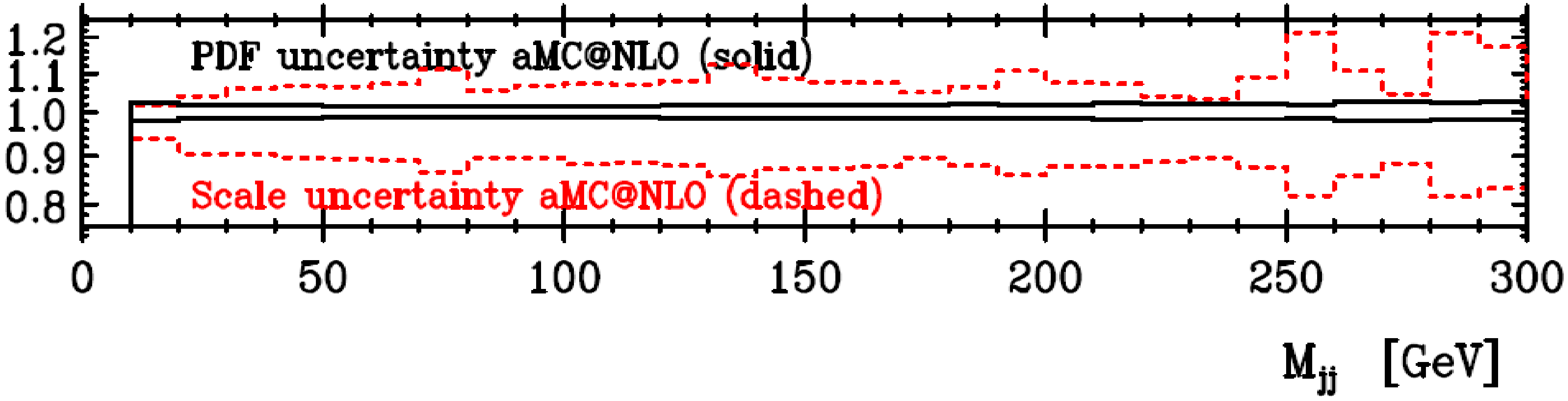
CMS and ALTAS use this
to estimate METZJ background
for SUSY searches

W+2 jet anomaly at CDF – NLO effect?



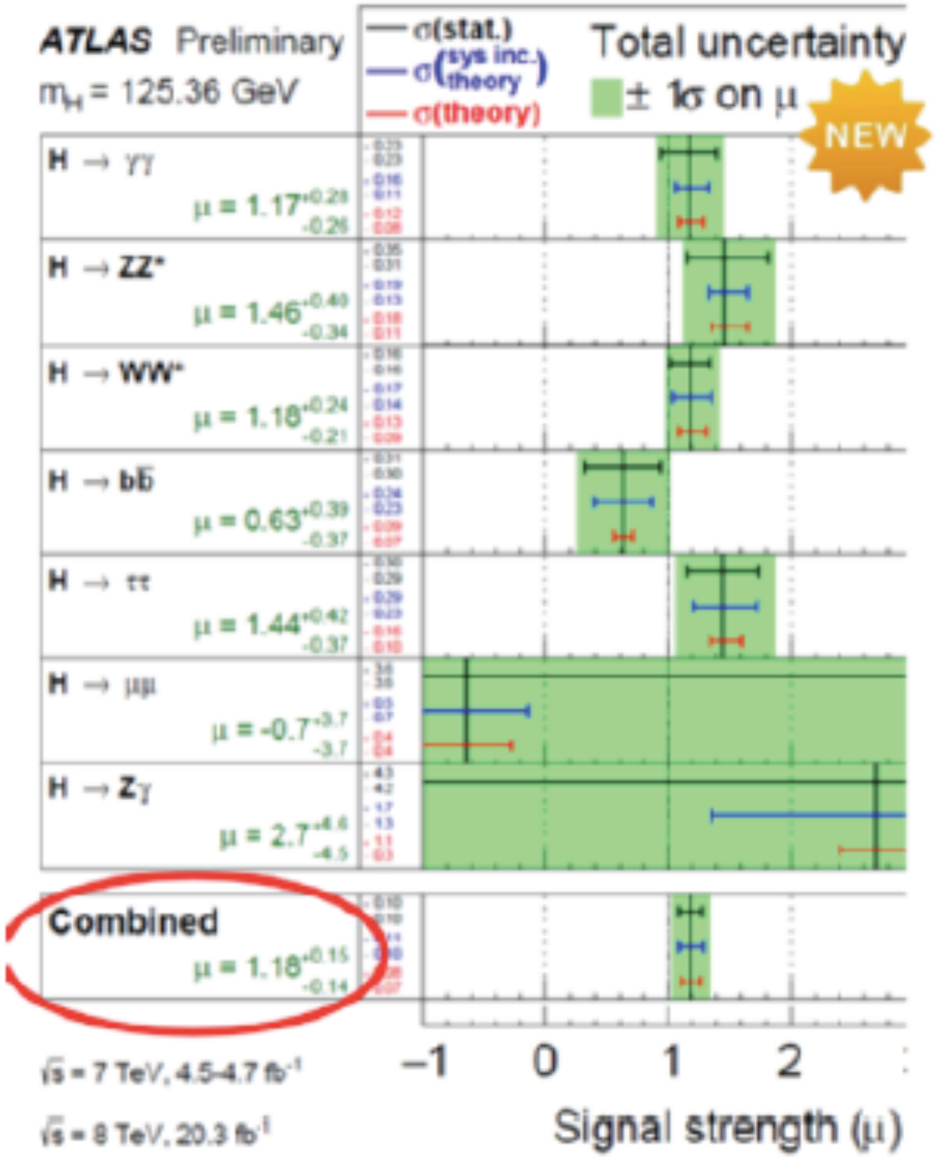
- CDF and D0 use ALPGEN and Parton showering for their analysis
- AMC@NLO Gives the similar prediction
- Theory uncertainty is comparable to Anomaly!

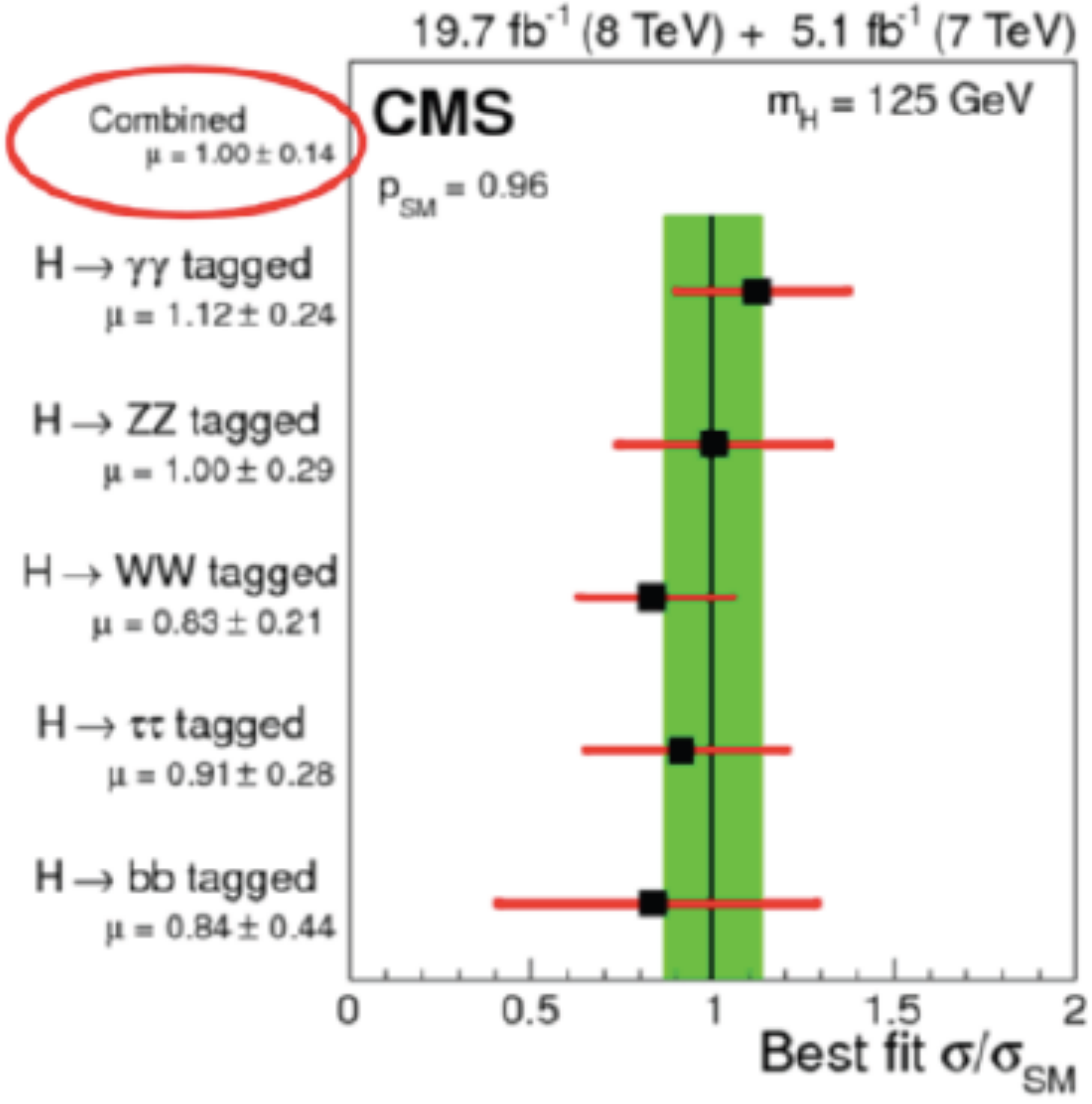
AMC@NLO



(N)NNLO

Higgs Results





$$\mu_{ATLAS} = 1.18^{+0.15}_{-0.14}$$

$$\mu_{CMS} = 1.00 \pm 0.14$$

$$\text{stat.} = {}^{+0.10}_{-0.10}$$

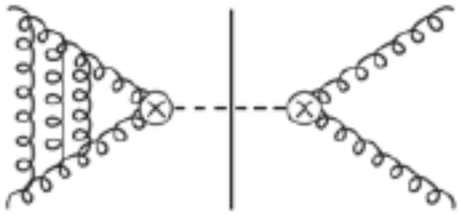
$$\text{theory} = {}^{+0.08}_{-0.07}$$

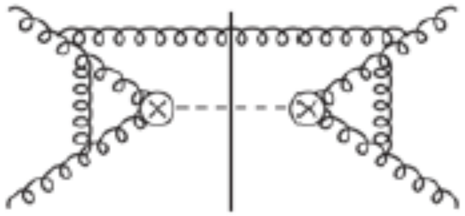
$$\text{sys. (inc. theo.)} = {}^{+0.11}_{-0.10}$$

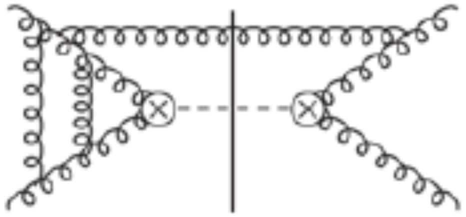
[M. Dührssen @ Moriond EW 2015]

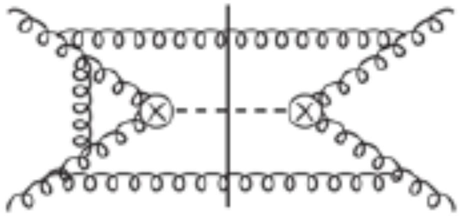
QCD Processes for Higgs Production

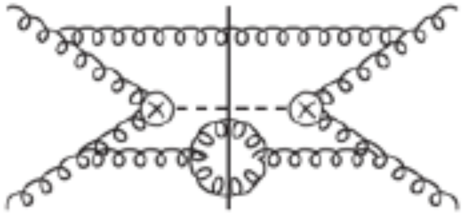
Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger


Triple virtual


Real-virtual squared


Double virtual real


Double real virtual


Triple real

	Integrals
NNLO	50 000
N3LO	517 531 178

100 000 diagrams

Integration By Parts Identities

[Tkachov, Chetyrkin]

- Generalization of **Gauss's theorem** in d dimension.
- Within dimensional regularization, all integrals in d dimension are well-defined and convergent.

$$\Downarrow$$

the integrand must be zero at boundary
(*necessary condition for convergence*)

- to make it free from Lorentz index

$$\int \prod_{i=1}^{l} \mathcal{D}^d k_l \frac{\partial}{\partial k_j^\mu} \left(\frac{v^\mu}{D_1^{n_1} \dots D_m^{n_m}} \right) = 0 \qquad \Big|_{v \equiv k_i, p_i}$$

Lorentz Invariance Identities

[Gehrmann, Remiddi]

- Under Lorentz transformation of external momenta

$$p_i^\mu \to p_i^\mu + \delta p_i^\mu = p_i^\mu + \omega^\mu_\nu p_i^\nu \quad \text{with } \omega^\mu_\nu = -\omega^\nu_\mu$$

  the integrals are invariant i.e.

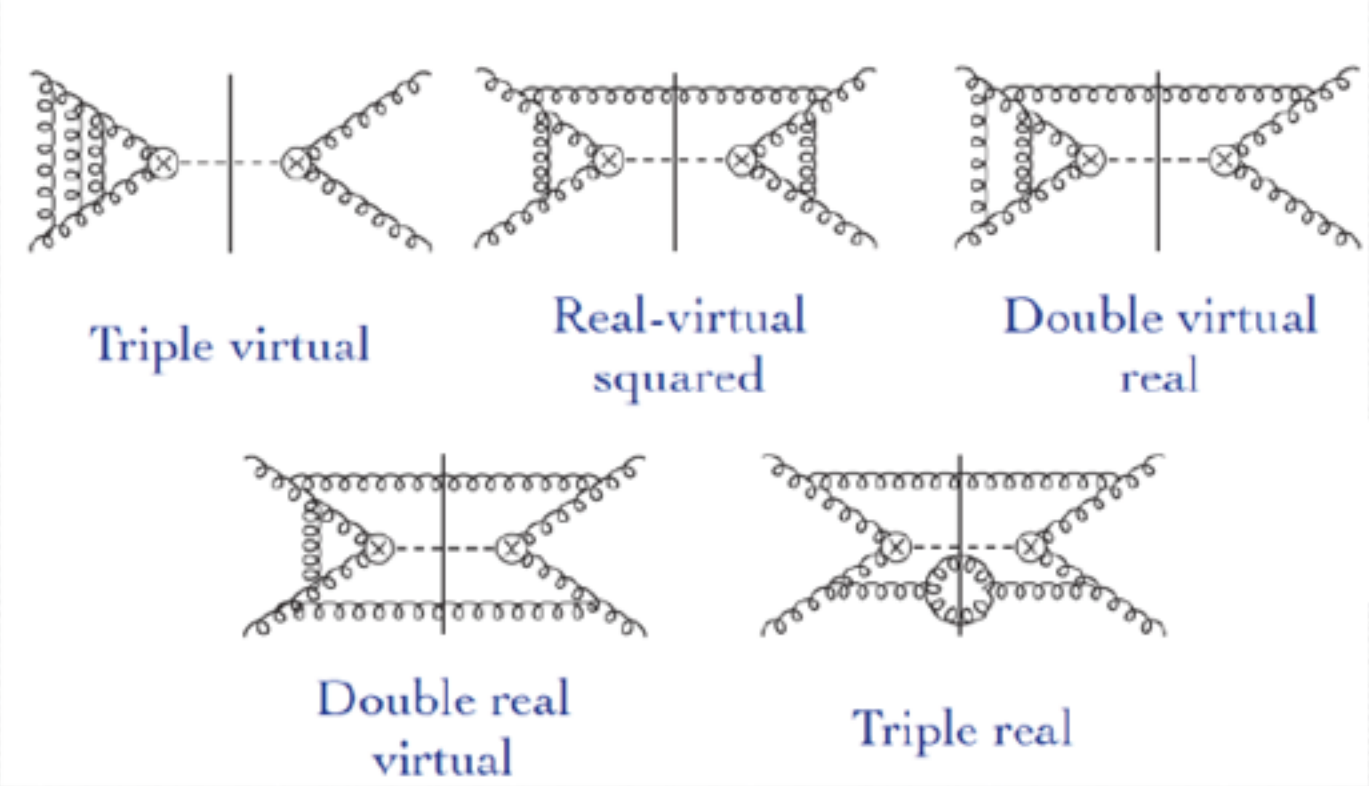
$$\mathcal{I}(p_i) = \mathcal{I}(p_i + \delta p_i) = \mathcal{I}(p_i) + \omega^\nu_\mu \sum_j p_j^\mu \frac{\partial}{\partial p_j^\nu} \mathcal{I}(p_i)$$

- from which the identity can be derived

$$\sum_j \left(p_{j,\mu} \frac{\partial}{\partial p_j^\nu} - p_{j,\nu} \frac{\partial}{\partial p_j^\mu} \right) \mathcal{I}(p_i) = 0$$

Master Integrals

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger



100 000 diagrams	Integrals	Master Integrals
NNLO	50 000	27
N3LO	517 531 178	1028

Methods

↓

Integration By Parts

$$\int \frac{d^dk_1}{(2\pi)^d}\cdots\int\frac{d^dk_3}{(2\pi)^d}\frac{\partial}{\partial k_i}\cdot\left(v_j\frac{1}{\prod_l D_l^{n_l}}\right)=0$$

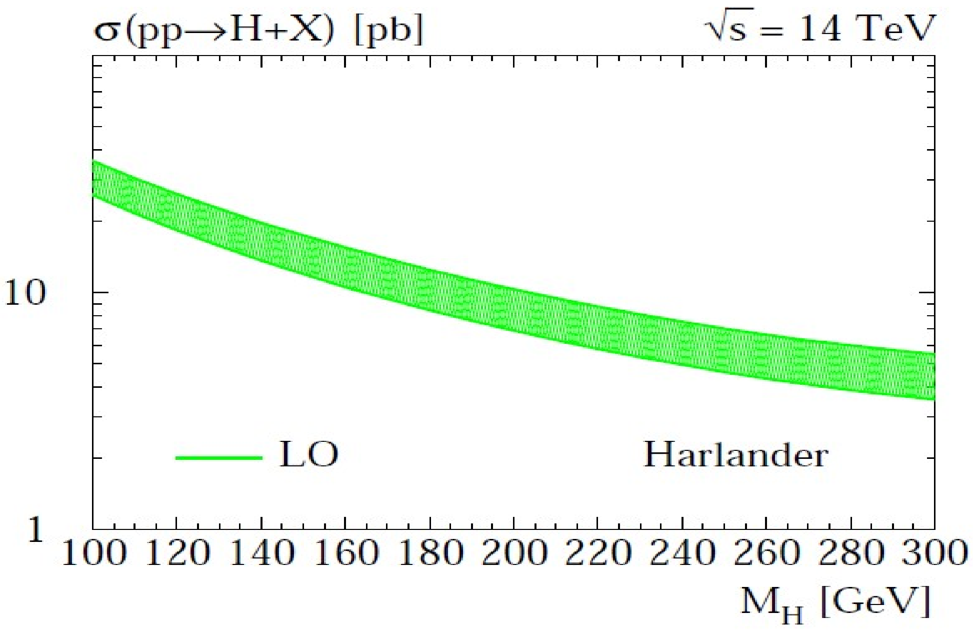
Lorentz Invariance

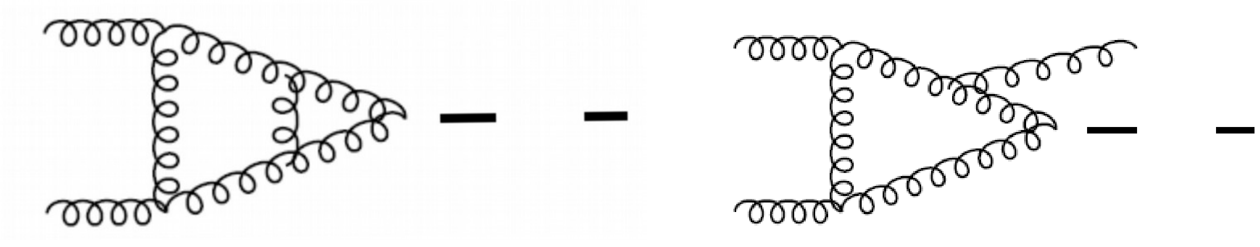
$$p_i^\mu p_j^\nu\left(\sum_k p_{k[\nu}\frac{\partial}{\partial p_k^{\mu]}}\right)J(\vec{n})=0.$$

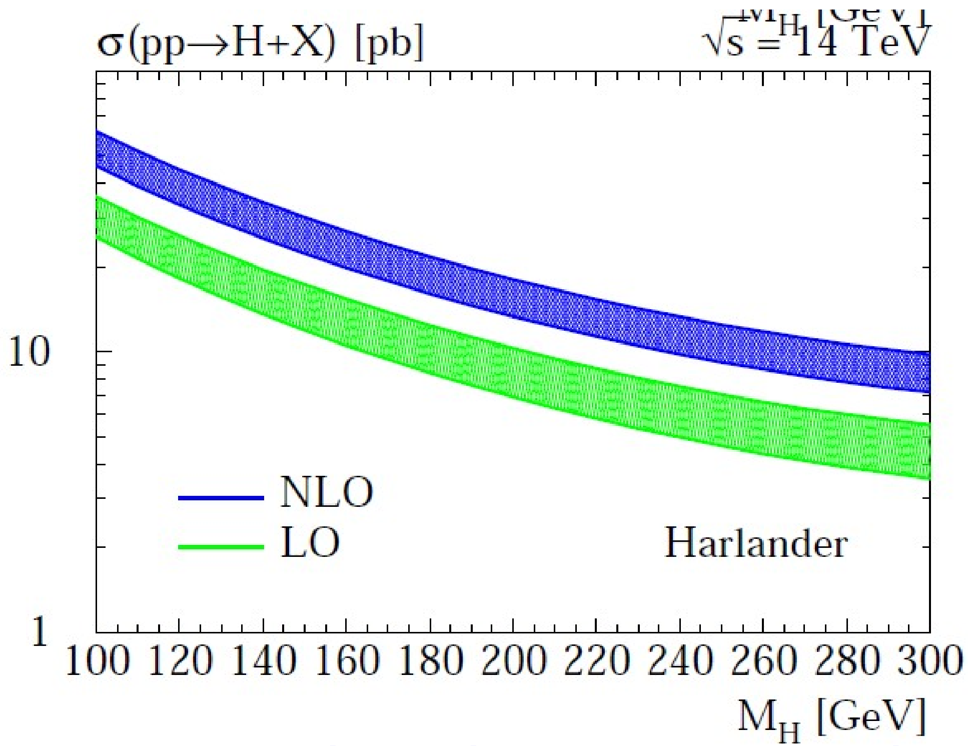
↓

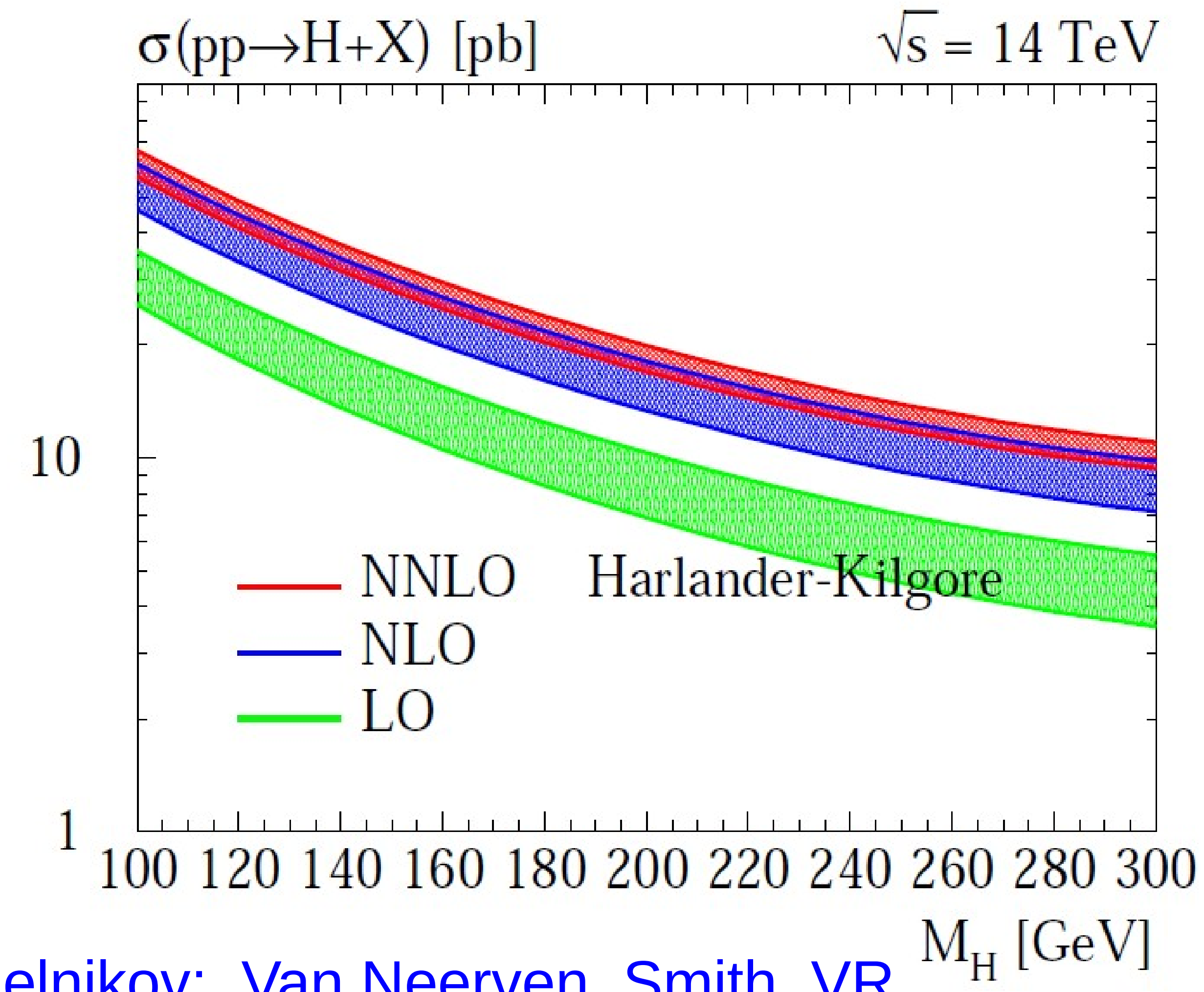
Master Integrals

Higgs cross section at NNLO





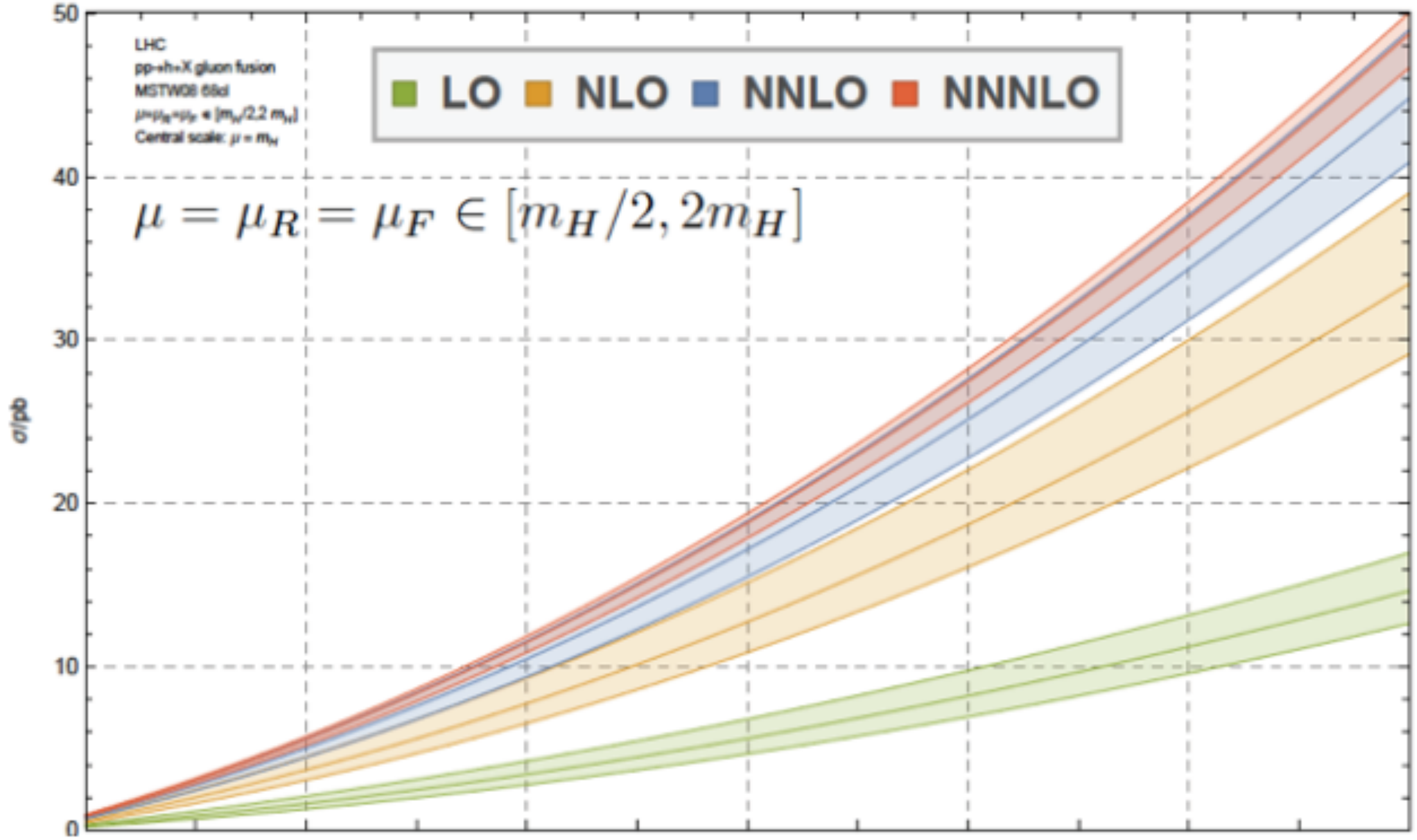


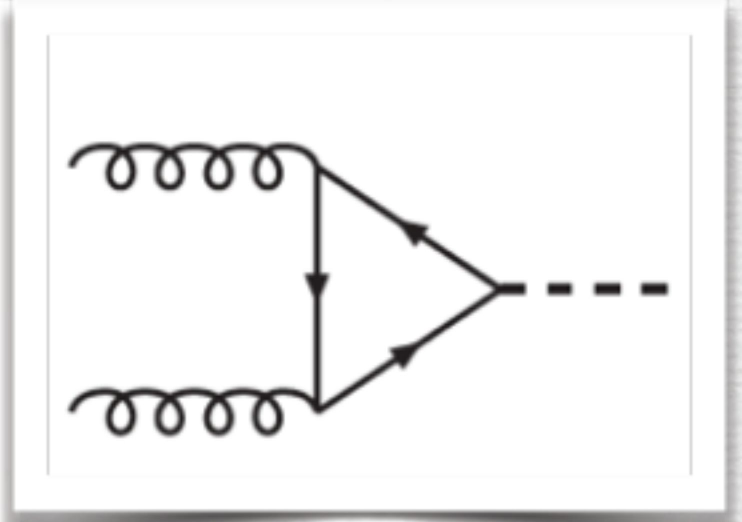


Harlander, Kilgore; Anastasiou, Melnikov; Van Neerven, Smith, VR

N3LO QCD results for Higgs

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

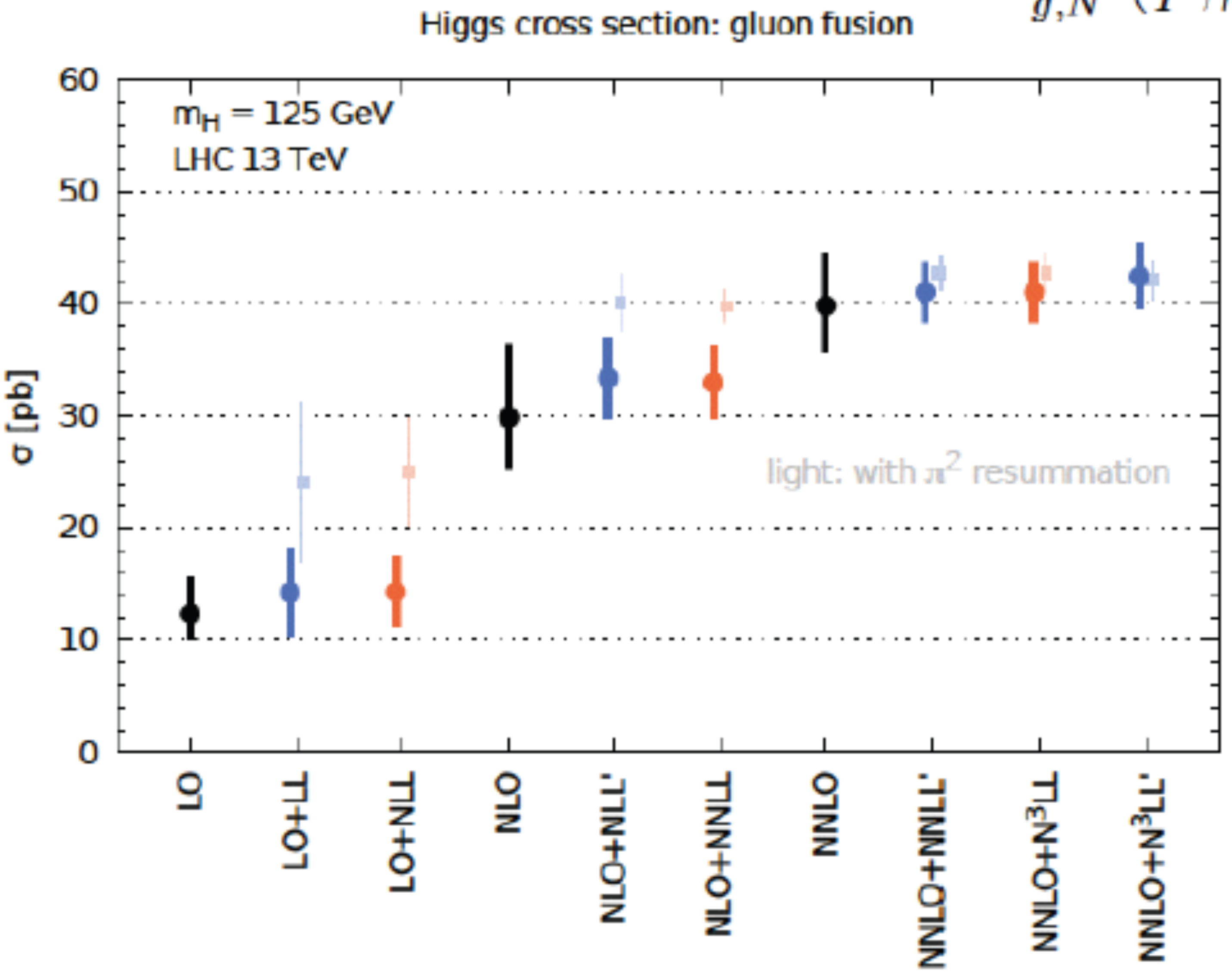




σ/pb	2 TeV	7 TeV	8 TeV	13 TeV	14 TeV
$\mu = \frac{m_H}{2}$	$0.99^{+0.43\%}_{-4.65\%}$	$15.31^{+0.31\%}_{-3.08\%}$	$19.47^{+0.32\%}_{-2.99\%}$	$44.31^{+0.31\%}_{-2.64\%}$	$49.87^{+0.32\%}_{-2.61\%}$
$\mu = m_H$	$0.94^{+4.87\%}_{-7.35\%}$	$14.84^{+3.18\%}_{-5.27\%}$	$18.90^{+3.08\%}_{-5.02\%}$	$43.14^{+2.71\%}_{-4.45\%}$	$48.57^{+2.68\%}_{-4.24\%}$

N3LL resummed results for Higgs

$$\Delta^{A,\mathrm{res}}_{g,N}(q^2,\mu_R^2,\mu_F^2) = C_g^{A,\mathrm{th}}(q^2,\mu_R^2,\mu_F^2)\,\Delta_{g,N}(q^2)$$

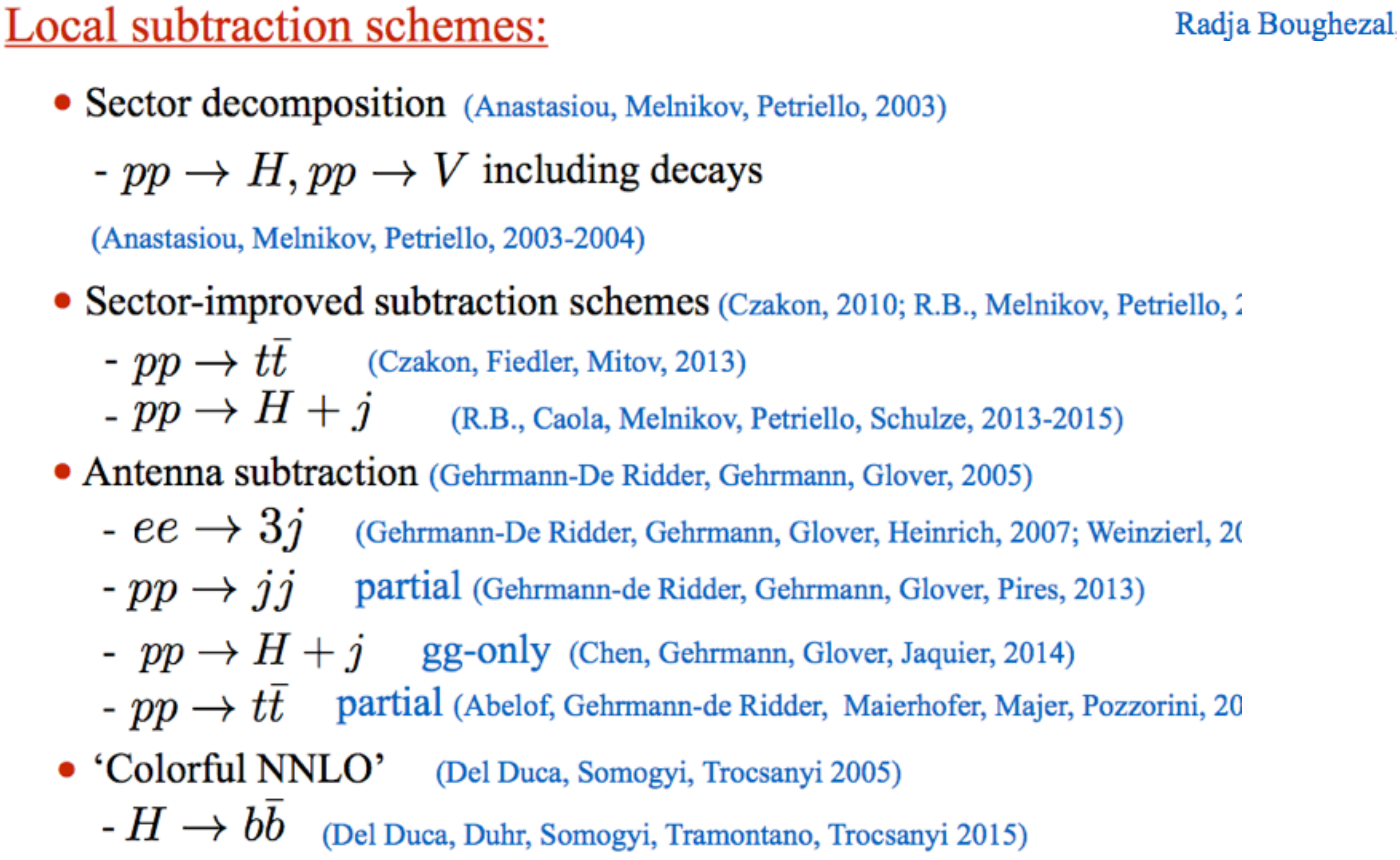


N3LL

Resummation

$$\Delta_{g,N} = \exp\left[\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\left\{2\int_{q^2}^{q^2(1-z)^2}\frac{d\lambda^2}{\lambda^2}A_g\left(a_s(\lambda^2)\right)+D_g\left(a_s(q^2(1-z)^2)\right)\right\}\right]$$

Subtraction Methods at NNLO



Local subtraction schemes:

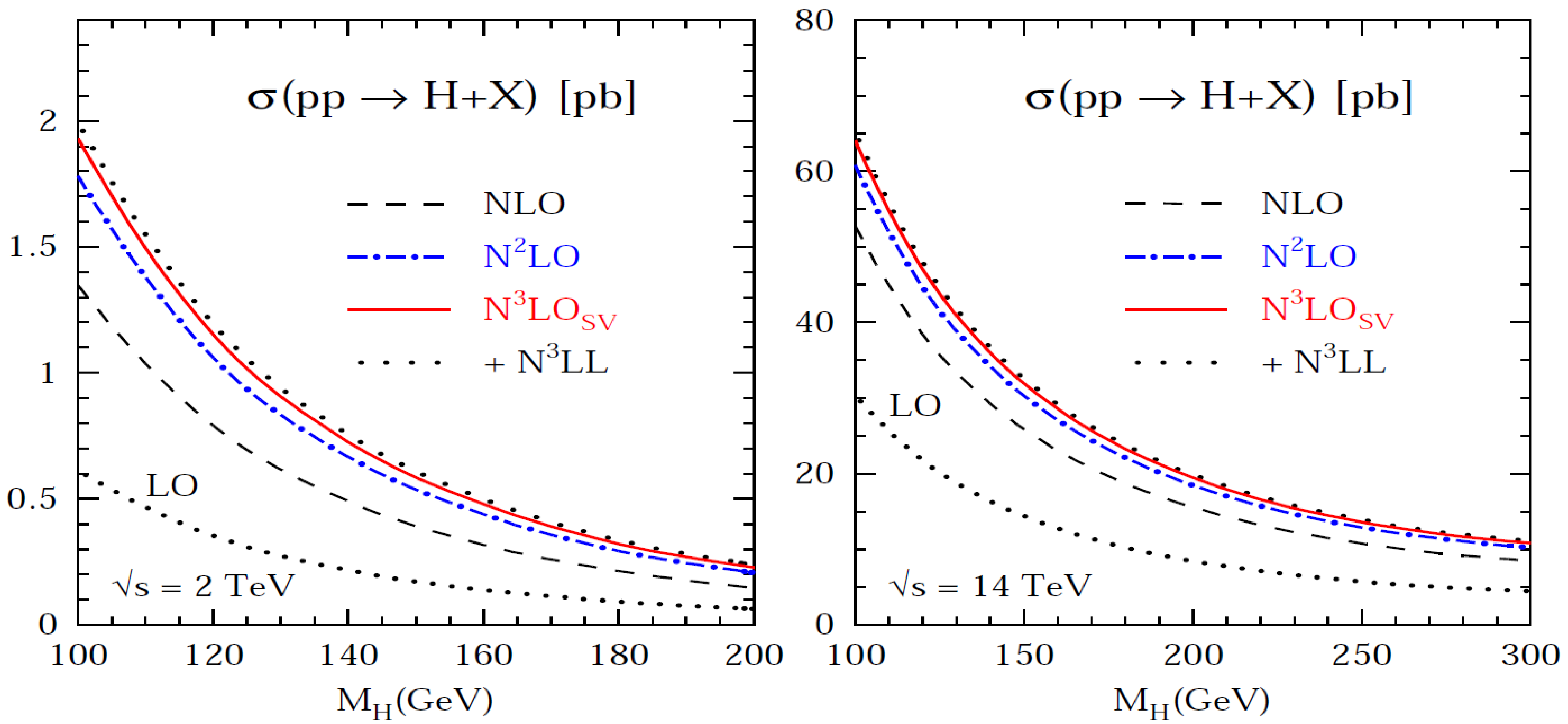
Radja Boughezal

- Sector decomposition (Anastasiou, Melnikov, Petriello, 2003)
  - $pp \to H, pp \to V$ including decays

  (Anastasiou, Melnikov, Petriello, 2003-2004)
- Sector-improved subtraction schemes (Czakon, 2010; R.B., Melnikov, Petriello,
  - $pp \to t\bar{t}$ (Czakon, Fiedler, Mitov, 2013)
  - $pp \to H + j$ (R.B., Caola, Melnikov, Petriello, Schulze, 2013-2015)
- Antenna subtraction (Gehrmann-De Ridder, Gehrmann, Glover, 2005)
  - $ee \to 3j$ (Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, 2007; Weinzierl, 2
  - $pp \to jj$ partial (Gehrmann-de Ridder, Gehrmann, Glover, Pires, 2013)
  - $pp \to H + j$ gg-only (Chen, Gehrmann, Glover, Jaquier, 2014)
  - $pp \to t\bar{t}$ partial (Abelof, Gehrmann-de Ridder, Maierhofer, Majer, Pozzorini, 20
- ‘Colorful NNLO’ (Del Duca, Somogyi, Trocsanyi 2005)
  - $H \to b\bar{b}$ (Del Duca, Duhr, Somogyi, Tramontano, Trocsanyi 2015)

Resummed Higgs cross section

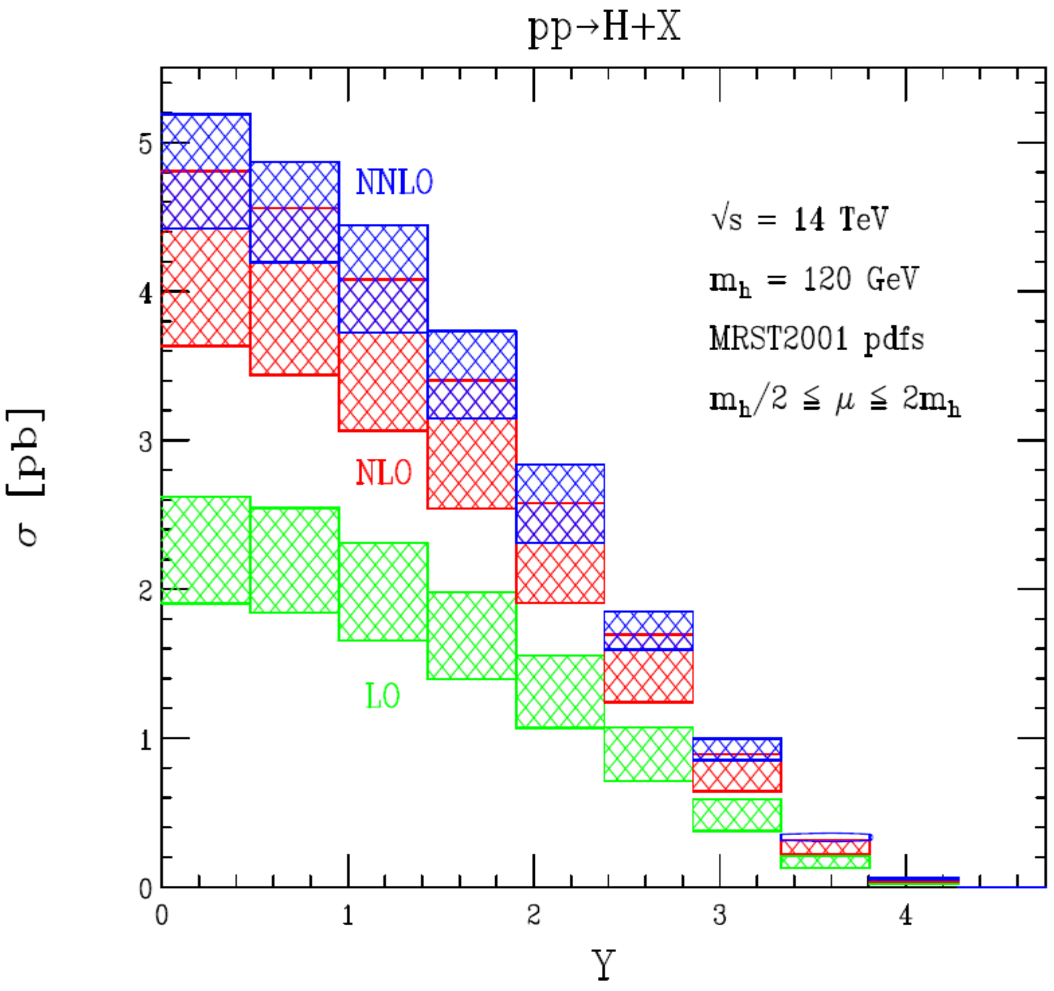
Catani and Grazzini; Vogt and Moch

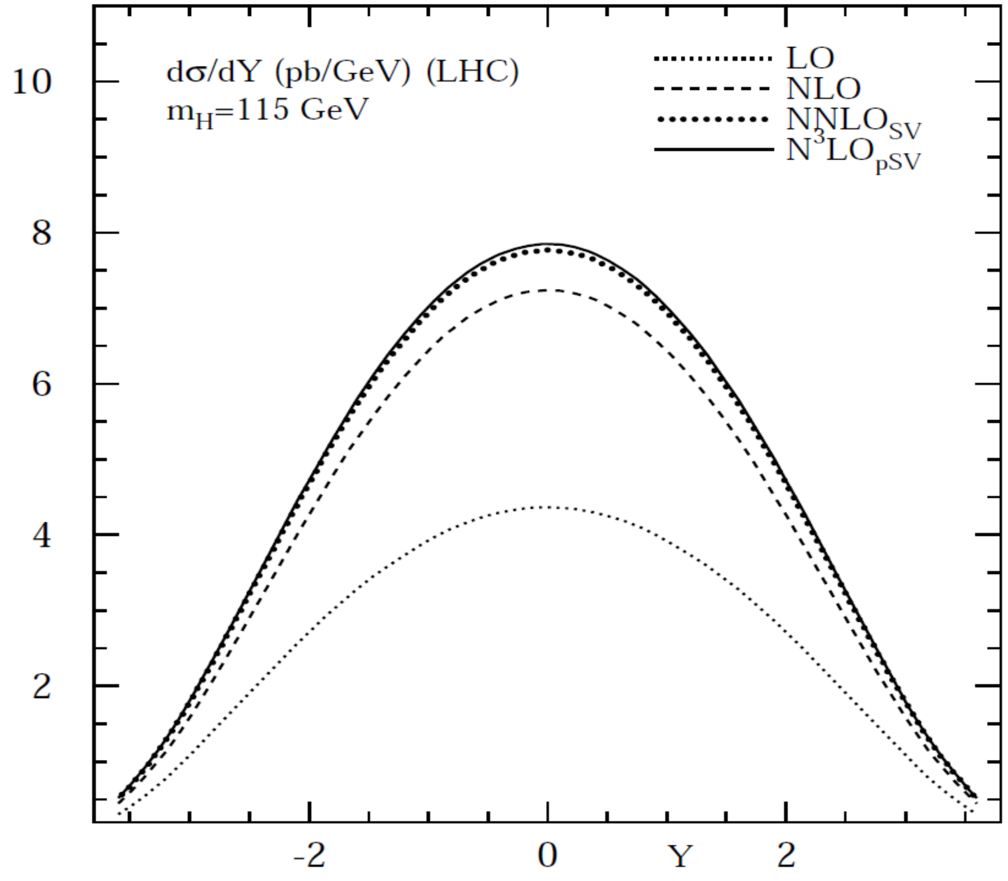


- N^3LL resummation exponents are available now.
- N^3LL resummation does not change the picture much. Fixed order N^3LO_{pSV} is very close to the N^3LL resummed result.

Rapidity of Higgs and its scale dependence at $NNLO, N^3LO$

Anastaiou et al



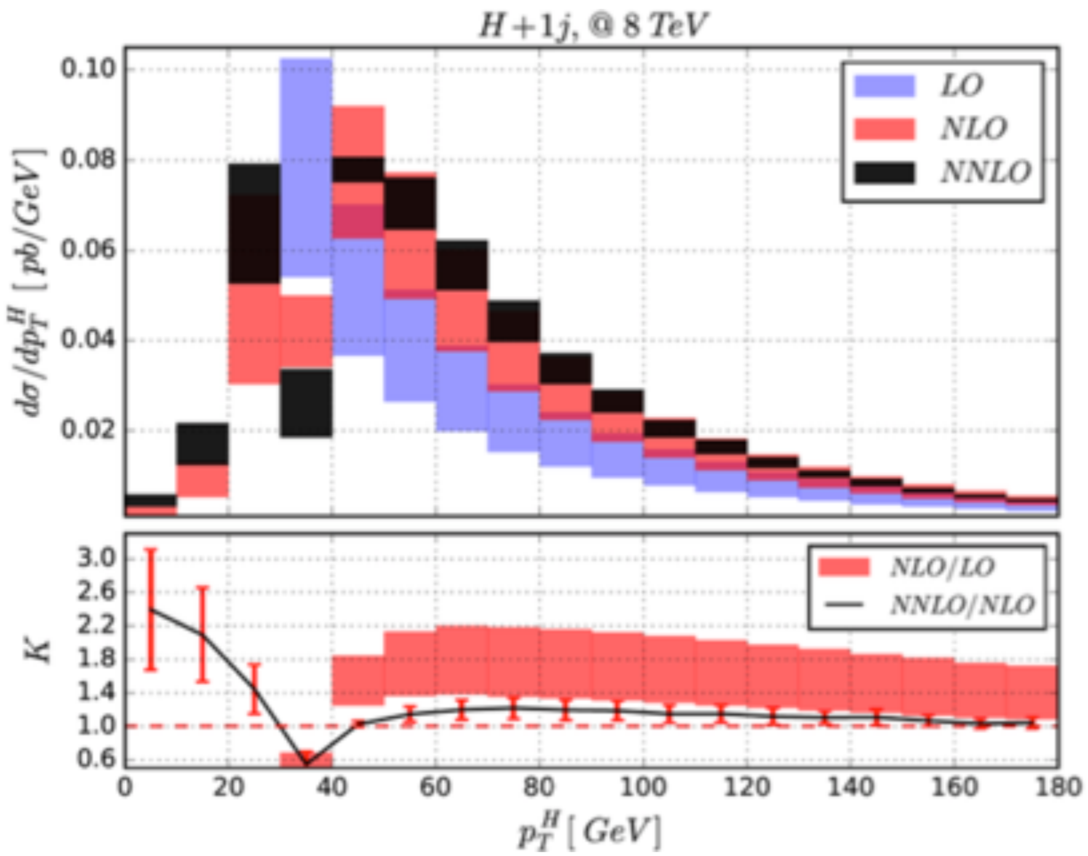


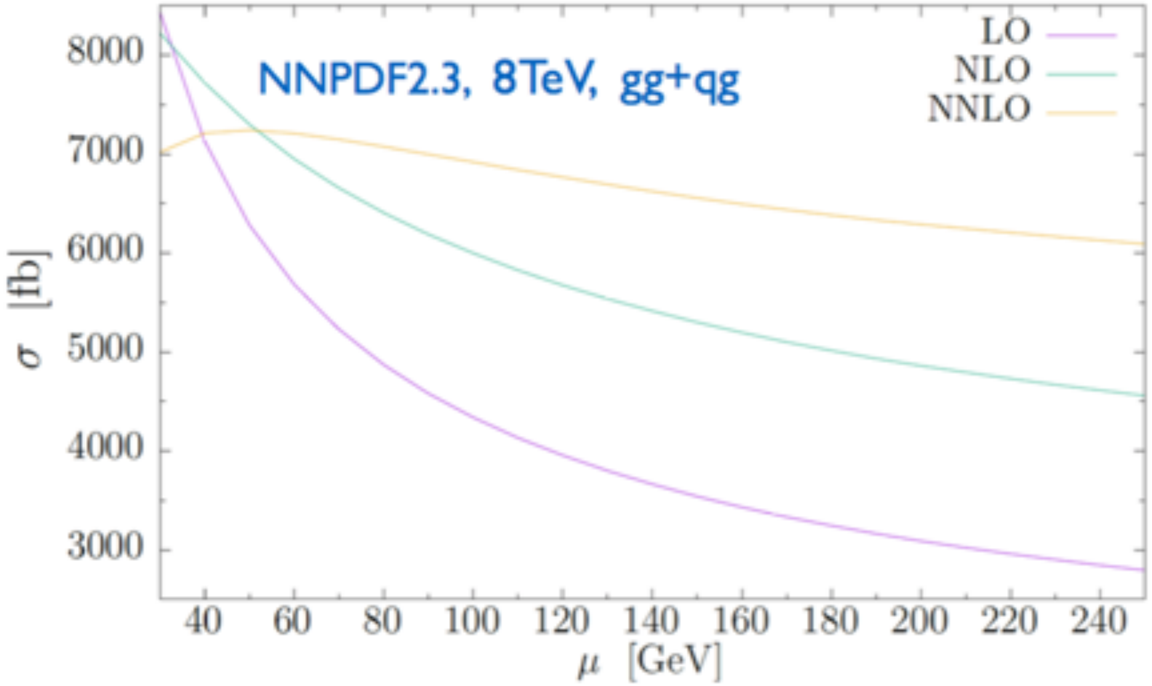
- NNLO exact in the large top limit reduces the scale uncertainity significantly

- One of the most difficult computations in QCD. Is it the end?

Higgs+jet at NNLO

R.B., Caola, Melnikov, Petriello, Schulze, 2015





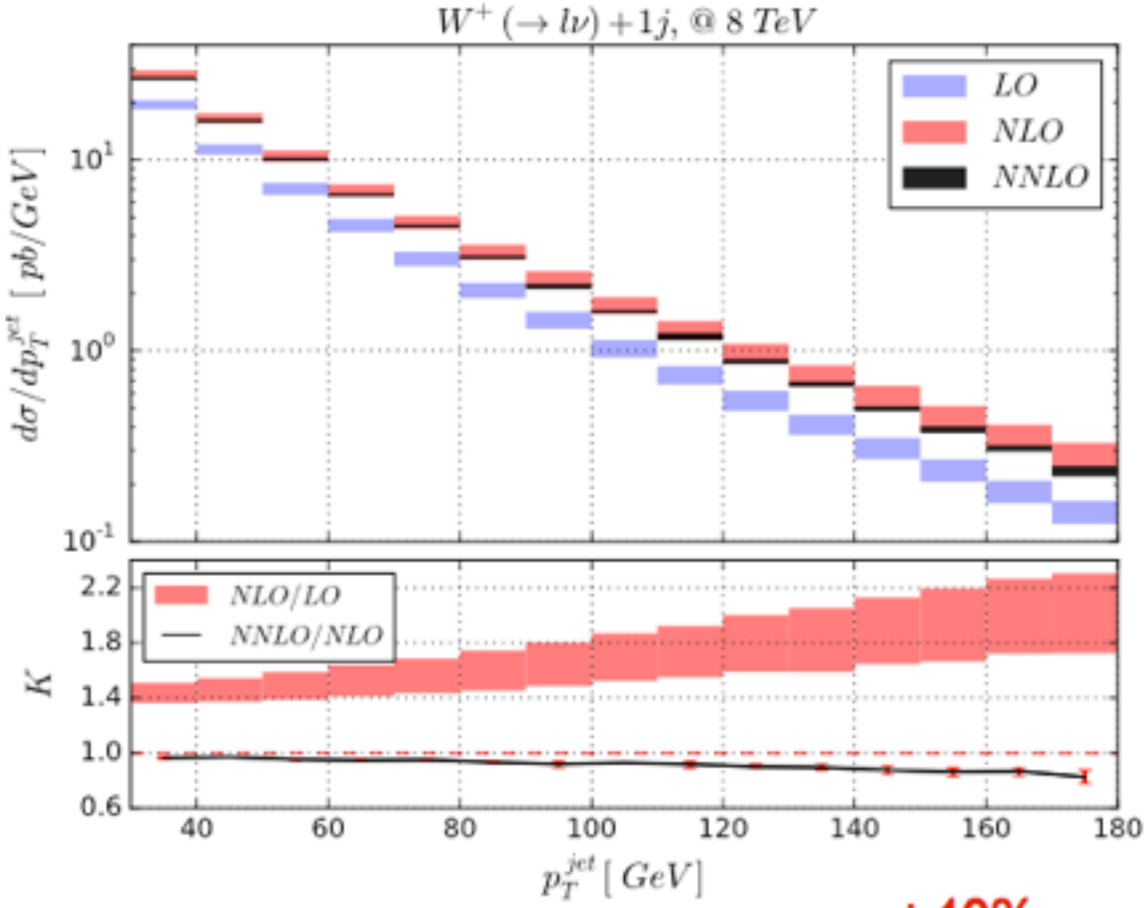
Corrections: $\text{LO} \xrightarrow{+44\%} \text{NLO} \xrightarrow{+20\%} \text{NNLO}$ for $\mu = m_H$

$\text{LO} \xrightarrow{+23\%} \text{NLO} \xrightarrow{+4\%} \text{NNLO}$ for $\mu = m_H/2$

Scale uncertainties: 36% LO 21% NLO 8% NNLO

W+jet at NNLO

Radja Boughezal



$p_T^{jet} > 30$ GeV, $\lvert\eta_{jet}\rvert < 2.4$	
Leading order:	533^{+39}_{-38} pb
Next-to-leading order:	797^{+63}_{-49} pb
Next-to-next-to-leading order:	791^{+0}_{-6} pb

CT10 PDFs, anti-K_T with $R = 0.5$

- K_{NNLO} is independent from τ_{cut} in each bin.

Corrections: $\text{LO} \xrightarrow{+40\%} \text{NLO} \xrightarrow{-1\%} \text{NNLO}$ for $\mu = m_W$

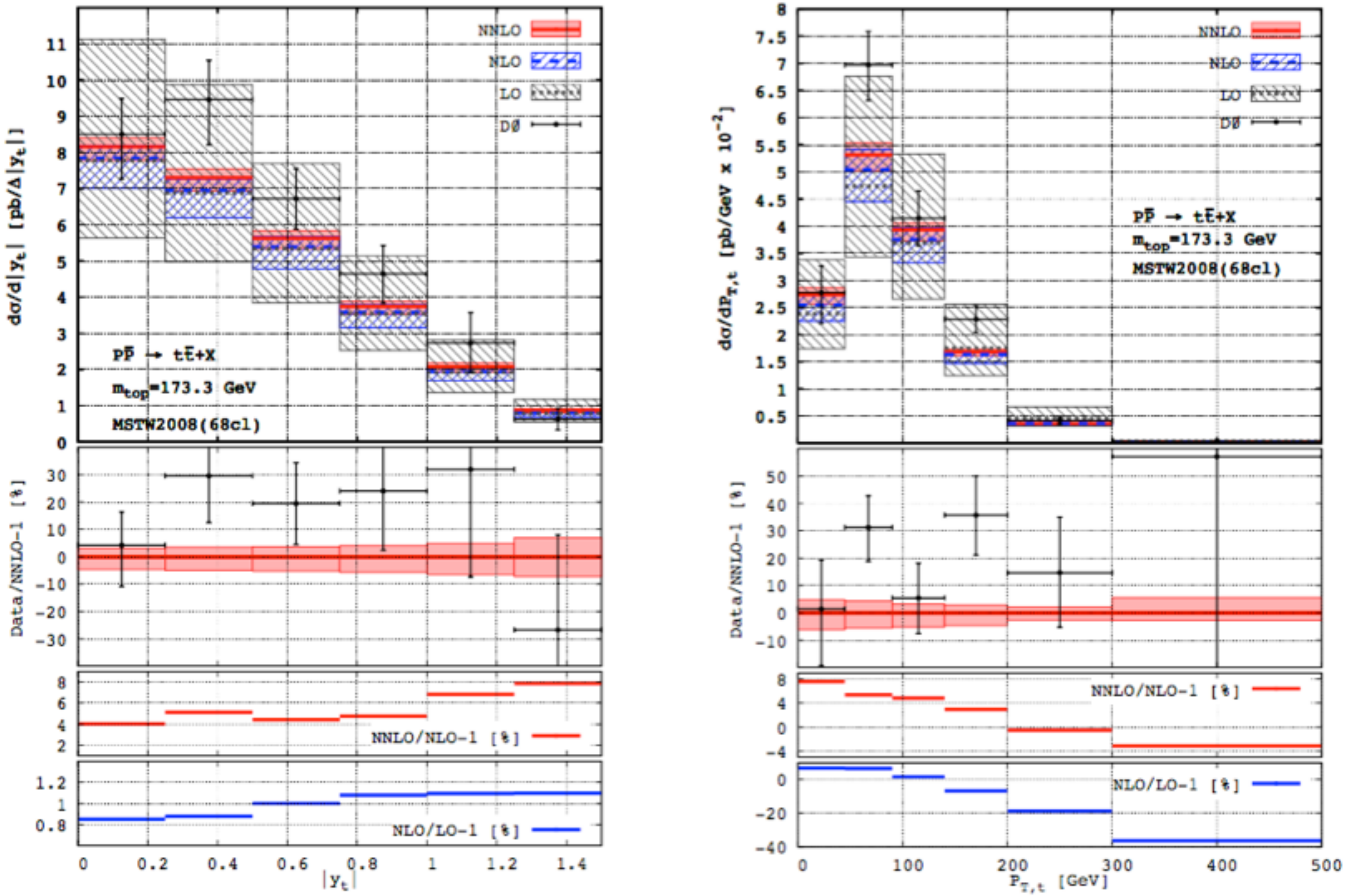
Scale uncertainties: 7% LO 7% NLO ~<1% NNLO

Very mild shift from NLO to NNLO and almost flat dependence on pTj

Top pair at NNLO

MC, Fiedler, Mitov, preliminary

M. Czakon

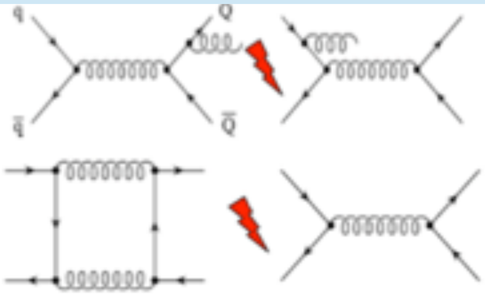


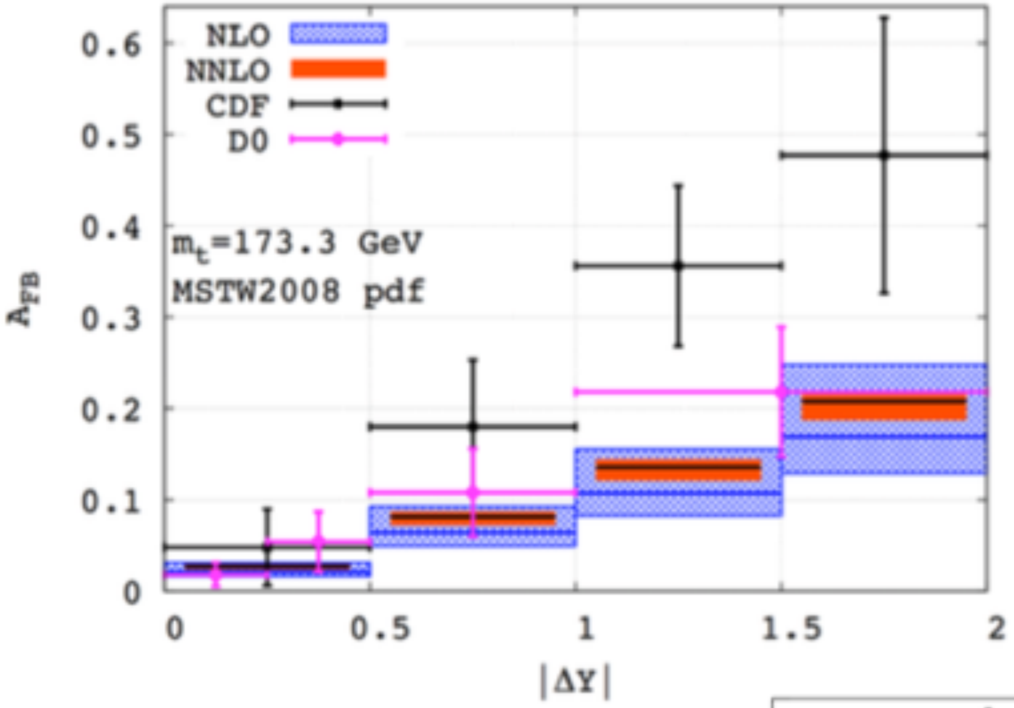
Top A_{FB} at NNLO

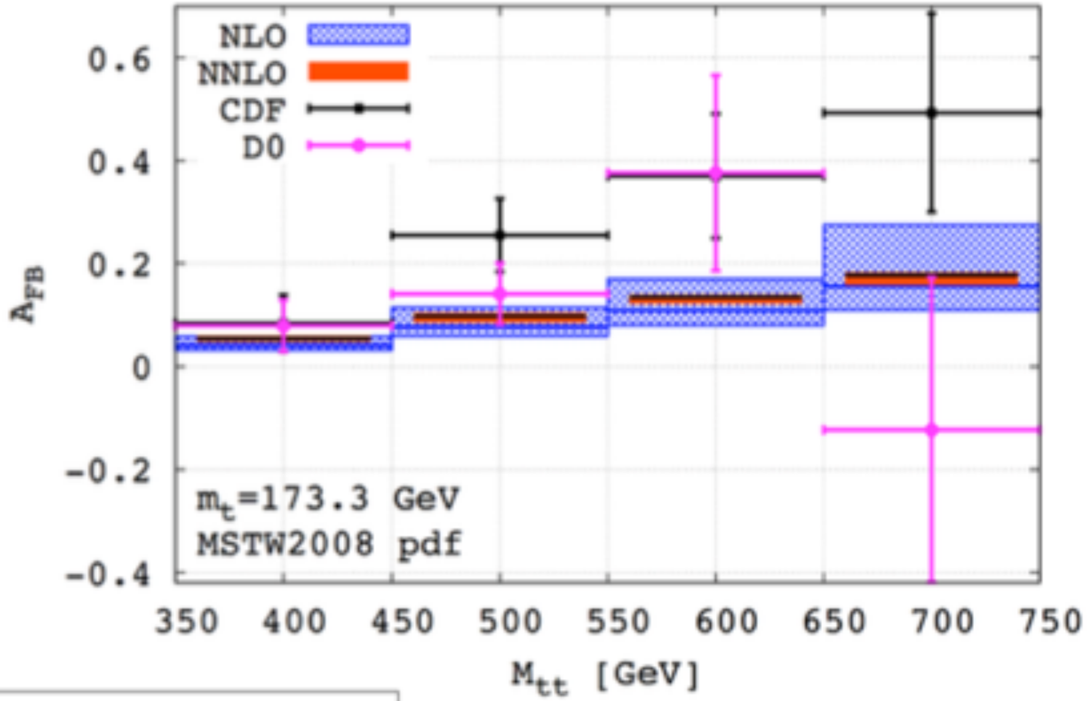
M. Czakon

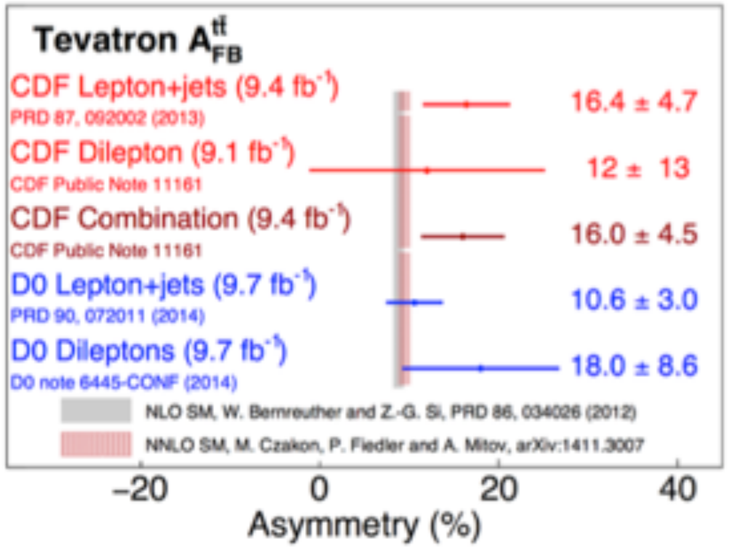
$$A^{t\bar{t}} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)}$$

$$\Delta y = y_t - y_{\bar{t}}$$



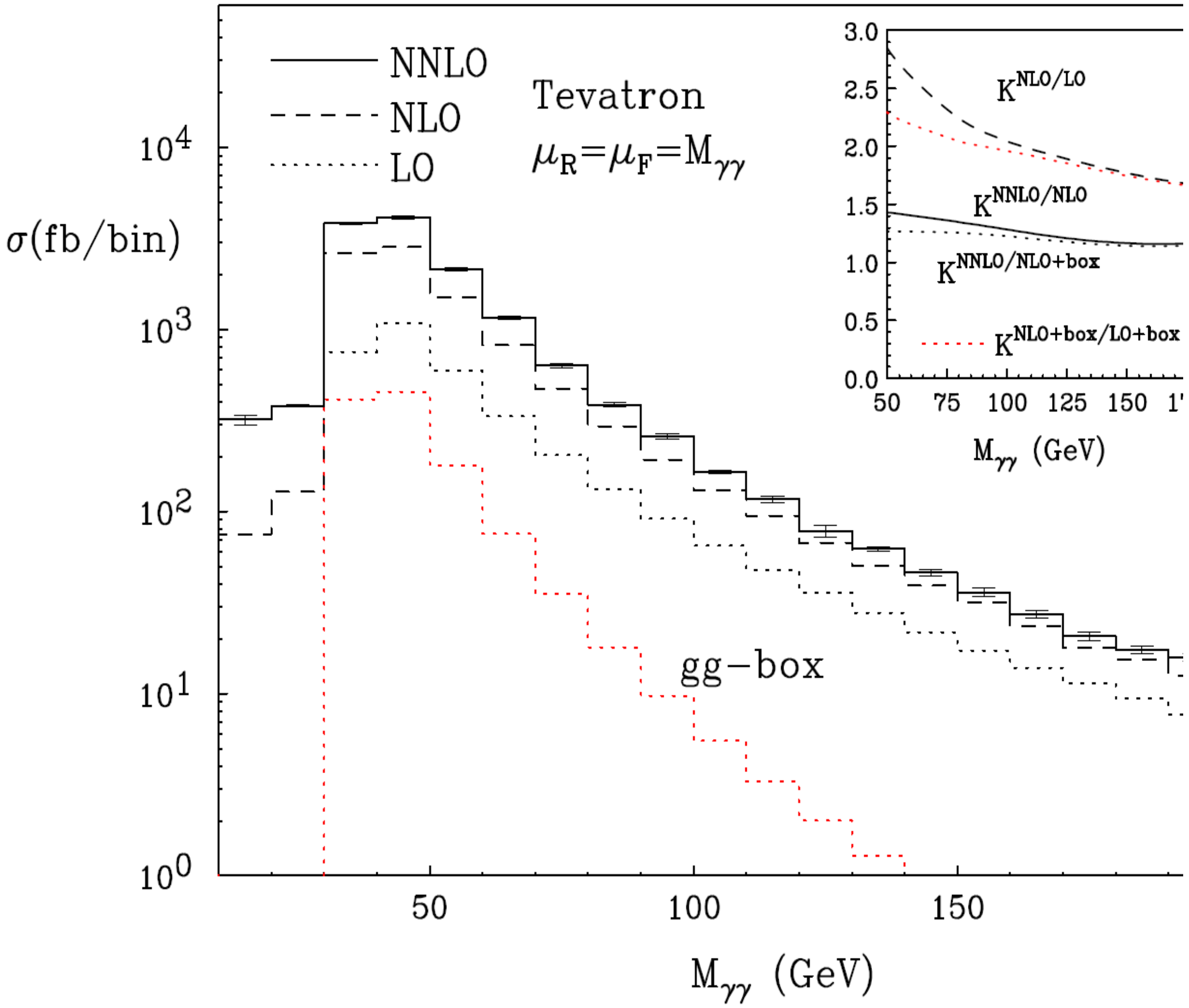






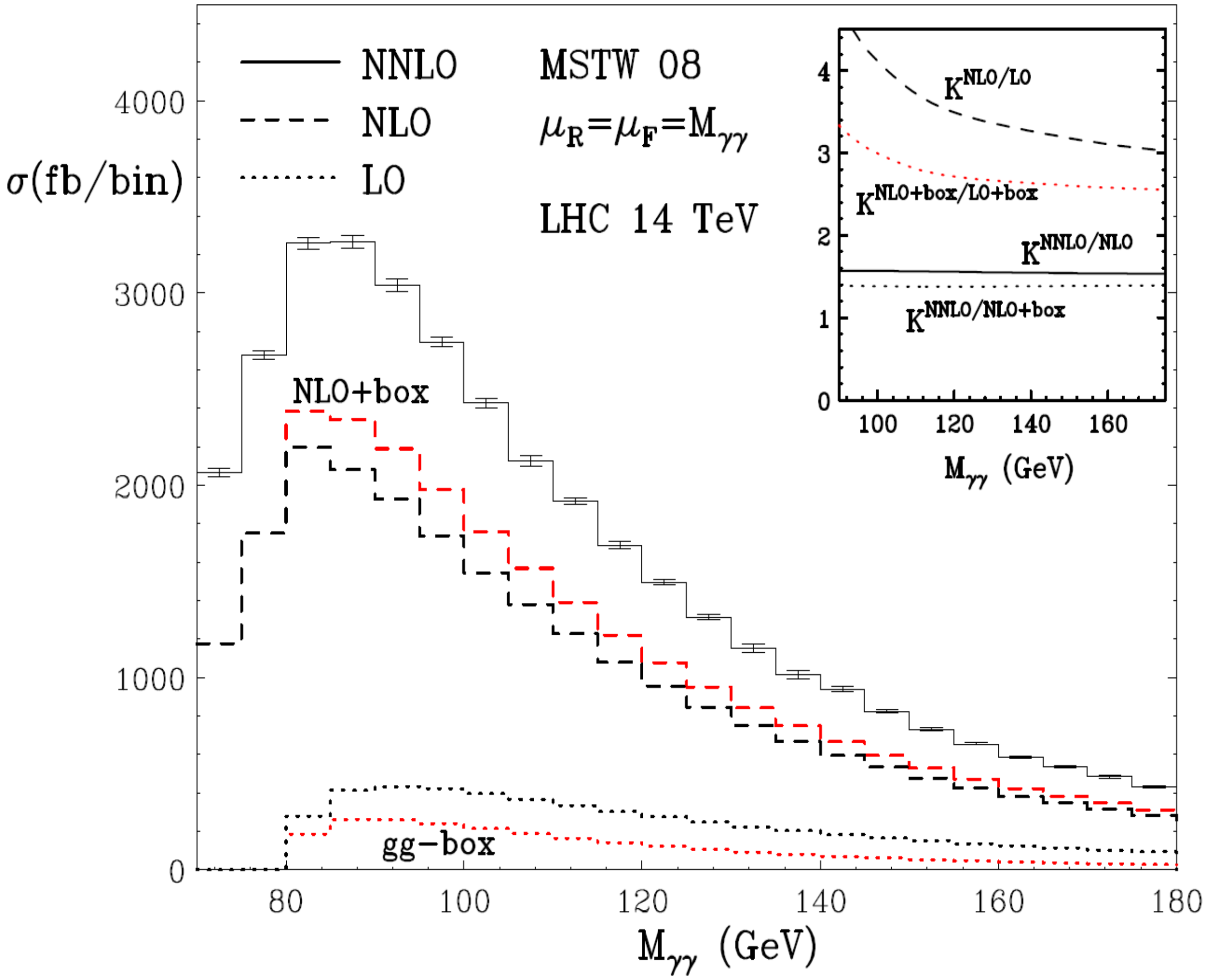
Di-photon at NNLO

Tevatron



LHC

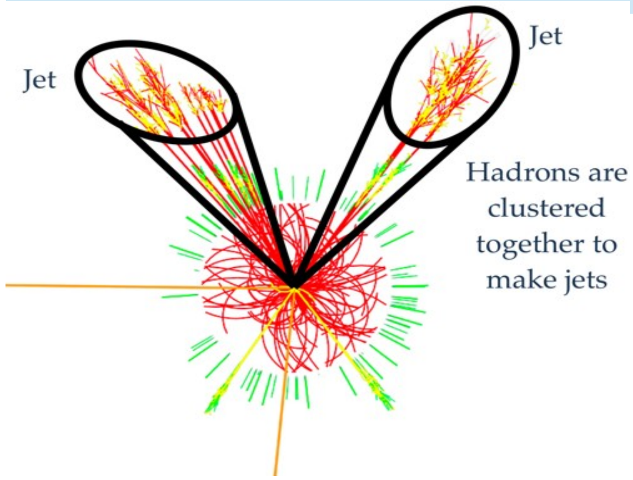
Catani et al.



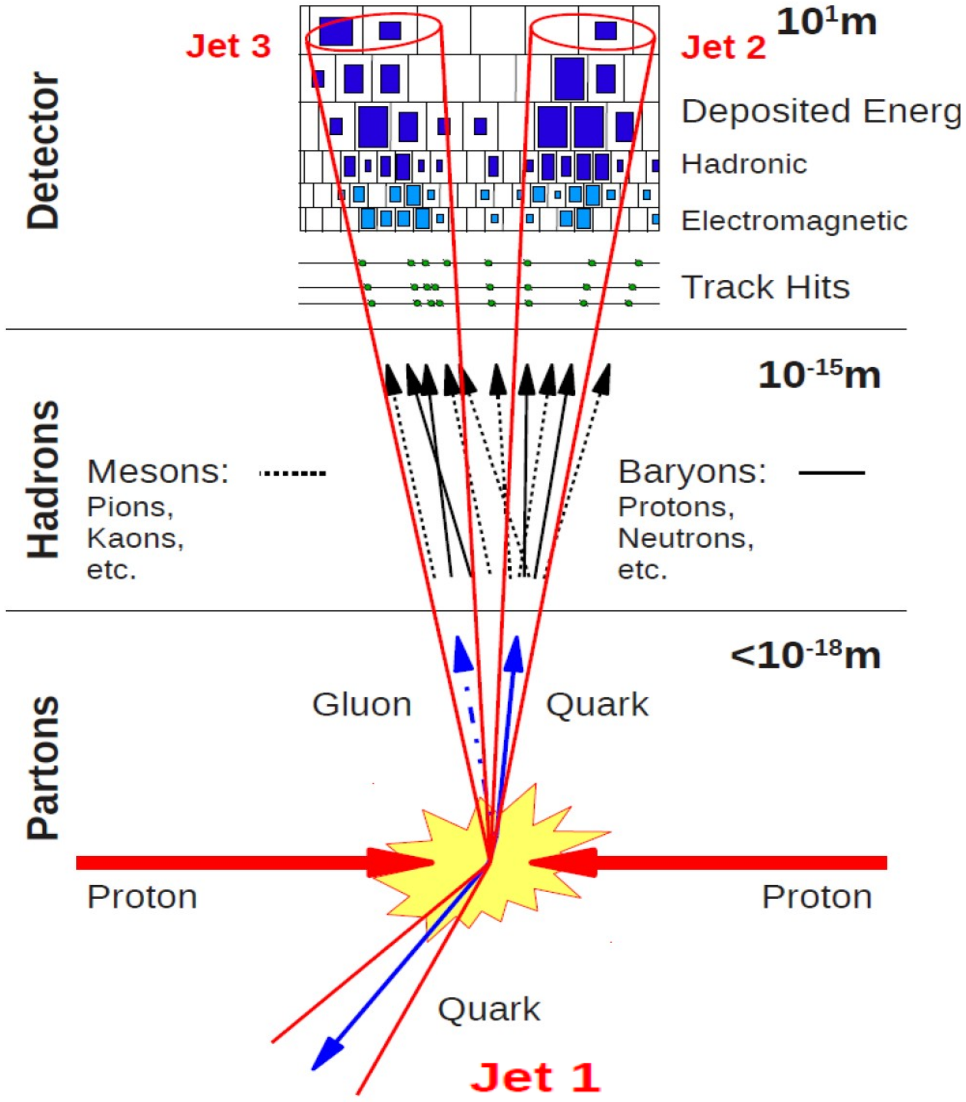
Cross section increases by 30-40%

Jets

Infra-red safe observables



- We do not see quarks and gluons, we see only
- hadrons/bunch photons, weak



- Infra-red Safe observables are the only meas
- How to construct infra-red safe quatities in Q
- Example: What is a Jet

Infra-red safe definition of a Jet

Algorithm

Collection of partons → Collection of hadrons (Jets)

Jet Agorithms

- k_t Algorithm
- Cambride/Aachen algorithm
- Anti k_t algorithm

SIS Cone
ATLAS Cone
CMS Iterative Cone
GetJet
....
....

Successively Recombine the nearby partons

$$d_{ij} = \min(k_{t,i}^{2p}, k_{t,j}^{2p})(\Delta y_{ij}^2 + \Delta\phi_{ij}^2)$$

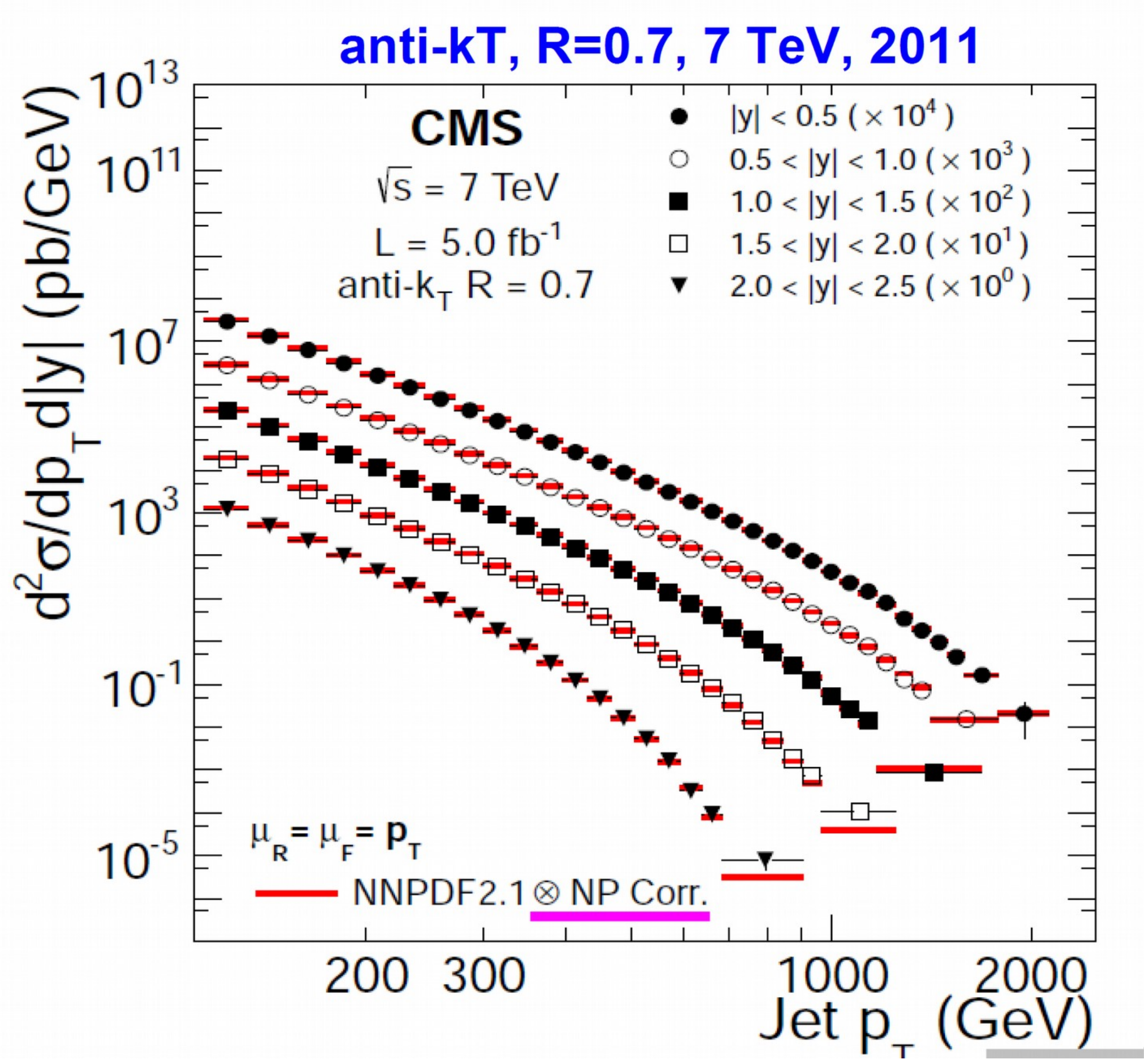
$p = 1$: k_t algorithm [Catani, Dokshitzer, Seymour, Webber, 93]

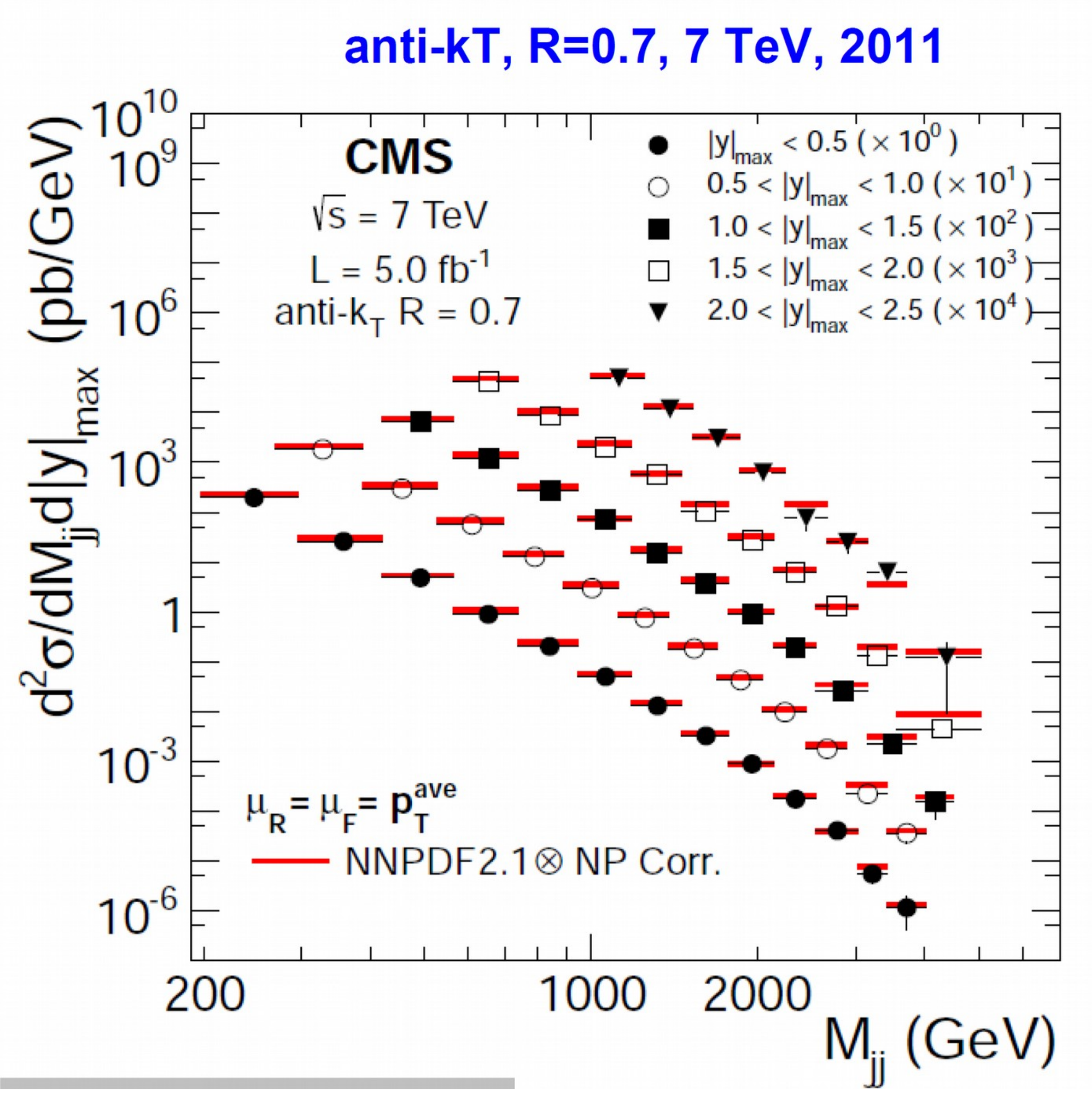
$p = 0$: Cambridge/Aachen (C/A) algorithm [Dokshitzer, Leder, Moretti, Webber, 93]

$p = -1$: anti-k_t algorithm [Cacciari, Salam, GS, 08]

Cone: $\approx$ flow of energy in a cone (of fixed R) centred on the cone centre: SISCone [Salam, GS, 07]

High Pt and invariant mass distributions of jets


anti-kT, R=0.7, 7 TeV, 2011


anti-kT, R=0.7, 7 TeV, 2011

Excellent agreement with NLO QCD predictions

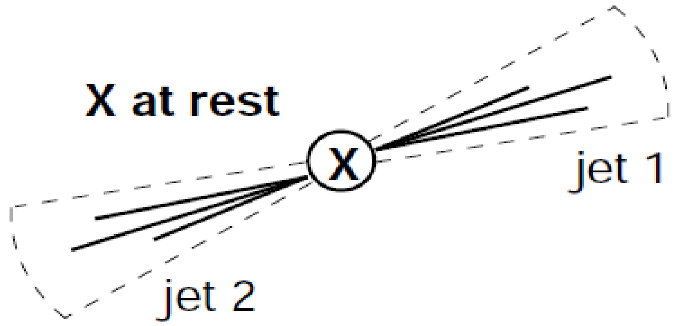
Fine Jets and Boosted Jets

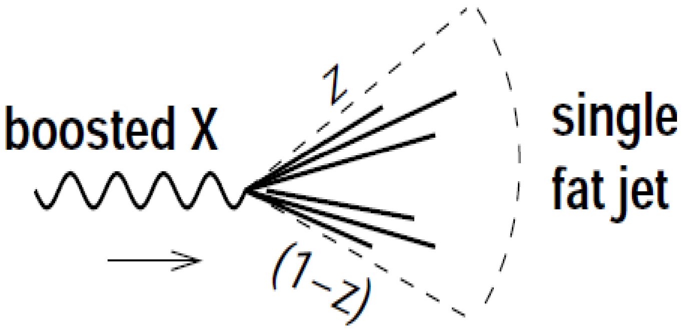
Filtering: undo the last recombination, keep the subjets

Trimming: remove low energetic deposits near a jet

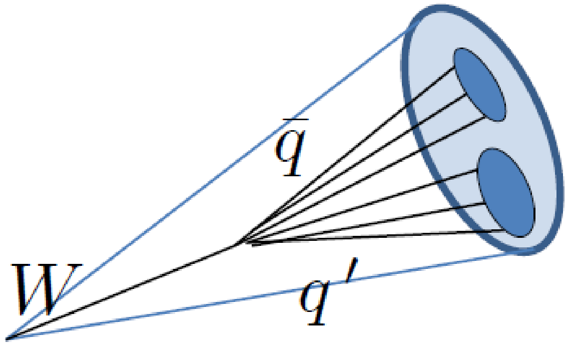
Pruning: recluster each jet in way wide angle recombinatio are absent

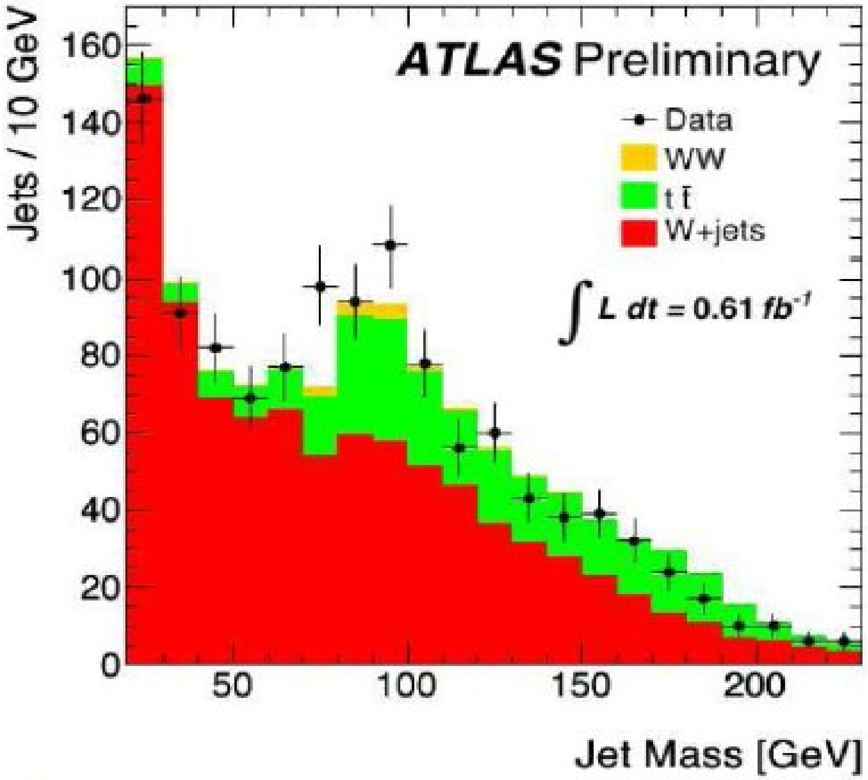
Boosted jets can probe Heavy states: new physics



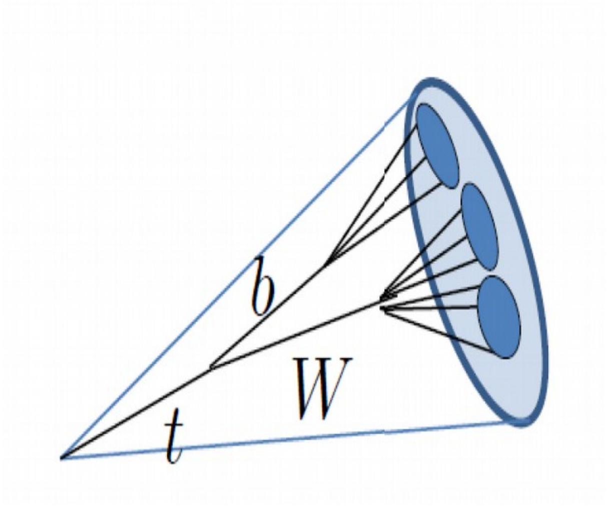


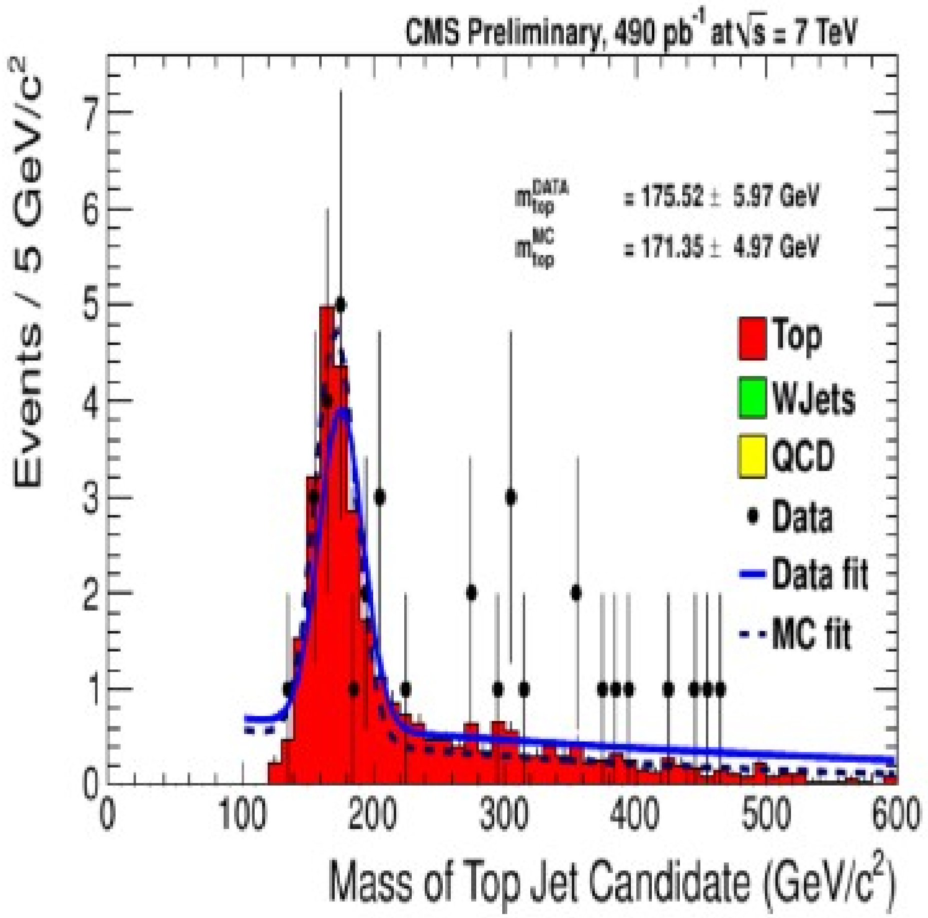
Boosted Jet from W Boson





Boosted Jet from top quark





Conclusions

- QCD is a tool kit at Hadron Colliders
- Factorisation plays an important role for predictions
- Strong coupling constant and PDFs are under control
- Many NLO and few NNLO results are available to test SM and new physics
- Jet physics provides alternate ground for probing new physics.