

# Numerical Investigation of External Losses for SRF Resonators

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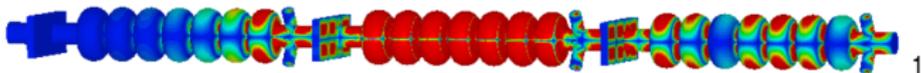
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## Investigation of accelerating structures

- For the thorough investigation and optimization of accelerating structures, we need to solve some variation of the equation:

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}, t) - \epsilon\mu \frac{\partial^2}{\partial t^2} \mathbf{E}(\mathbf{r}, t) = -\mu \frac{\partial}{\partial t} \mathbf{J}(\mathbf{r}, t)$$

- Computation of **all** eigenmodes in the frequency domain of interest is inevitable



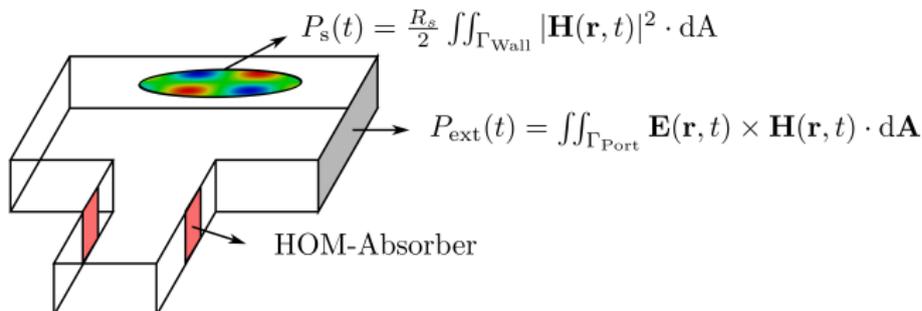
- Usually employed on HPC-Clusters or supercomputers<sup>2</sup>
- Of special interest are the losses of certain modes in the cavity

<sup>1</sup> F. Marhauser, Investigations on Critical Higher Order Modes in CEBAF Upgrade Cavities, Accelerator Seminar, JLAB, 27.08.2009.

<sup>2</sup> L.-Q. Lee, Omega3P: A Parallel Finite-Element Eigenmode Analysis Code for Accelerator Cavities, SLAC-PUB-13529, February 2009

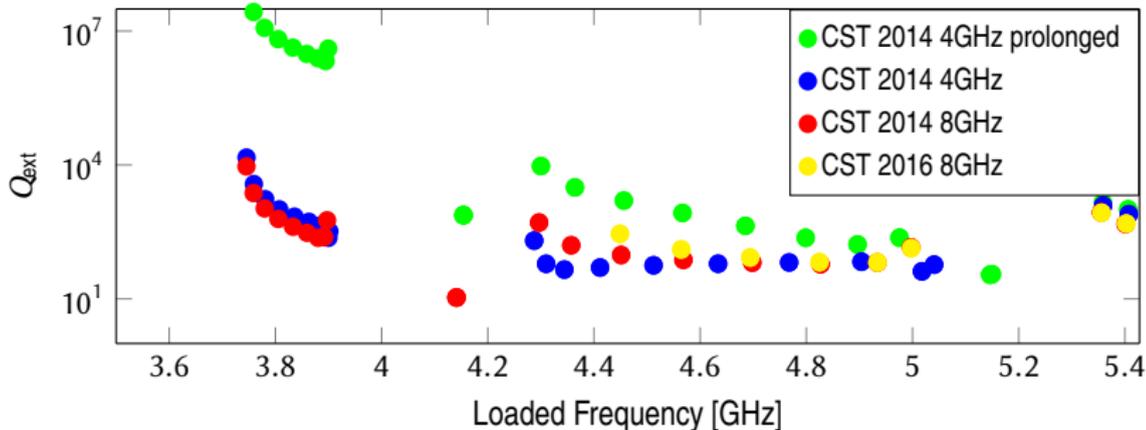
## Losses in Cavities

- Three different loss mechanisms are considered:
  - *Surface losses* play hardly any role in superconducting structures
  - *Dielectric and magnetic losses* are only important for beam pipe absorbers or residual gas in the cavity
  - **External losses** are determined by energy leaving the investigated structure, assuming a reflection-free port



## CST Microwave Studio ®

- Are we able to employ commercial software?
- $Q_{\text{ext}}$  of cavity, computed with CST MWS for:
  - different evaluation frequencies
  - different versions of CST MWS
  - different length of the attached beampipe



## External Losses

- For external losses we simply assume that an infinitely long beampipe is attached to any waveguide port
  - This holds if the termination impedance is equal to the wave impedance of the 2D port modes

$$Z_{\text{Wave}}^{\text{TE}}(s) = Z_0 \frac{s}{\sqrt{s^2 + \omega_{co}^2}}, \quad Z_{\text{Wave}}^{\text{TM}}(s) = Z_0 \frac{\sqrt{s^2 + \omega_{co}^2}}{s}$$

- Problem: the computation of the resulting fields leads to a **complex-valued, nonlinear eigenvalue problem** (NLEVP)

$$\underbrace{(\mathbf{K}_1 - \mathbf{K}_2(\lambda) - \lambda \mathbf{K}_3)}_{\mathbf{T}(\lambda)} \mathbf{x}(\lambda) = \mathbf{0}.$$

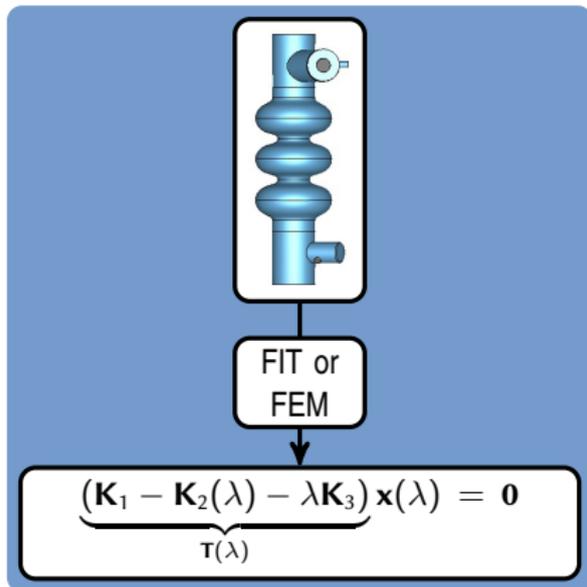
## Generally used simplifications

- **Linearization:** Wave impedance is evaluated at a certain frequency
  - Used by CST MWS

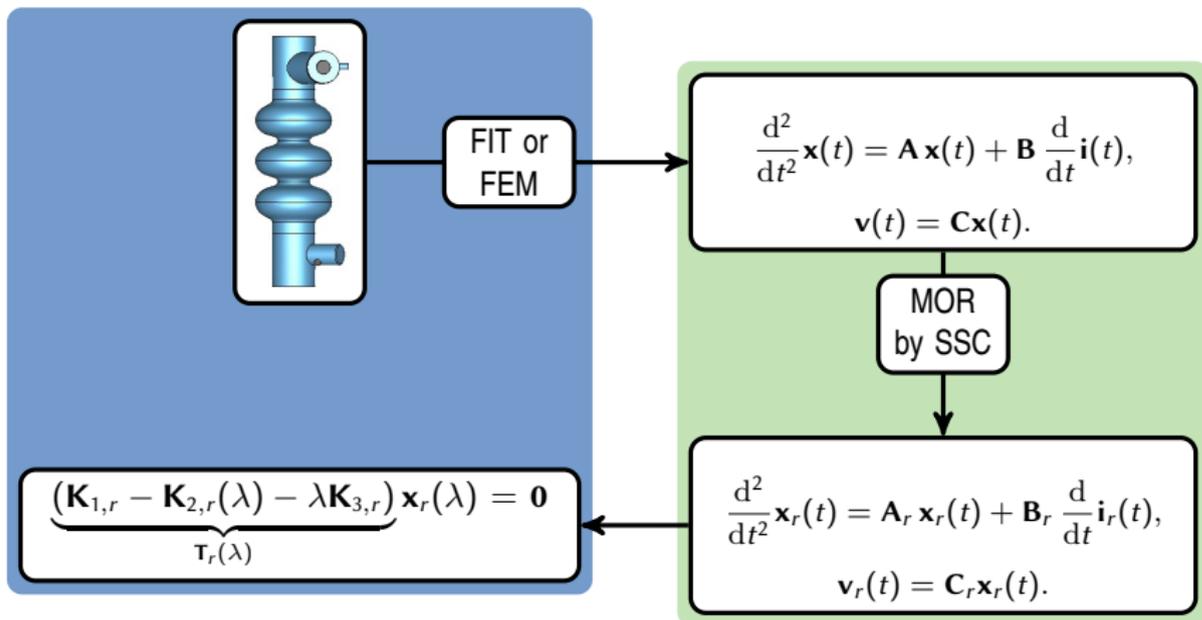
$$(\mathbf{K}_1 - |\mathbf{K}_2(f_{lin})| - \lambda \mathbf{K}_3) \mathbf{x}(\lambda) = \mathbf{0}$$

- **Only one 2D mode per port:** NLEVP can be converted to a quadratic, non-linear EVP with frequency-independent matrices
  - Used by Omega3P (SLAC), we tried this too with higher order polynomials
  - Generally ports have multiple modes (e.g. 5 to 15)
- **Solution of surrogate problem:**
  - Perfectly matched layers (PML)
  - Pole Fitting (later used for validation)

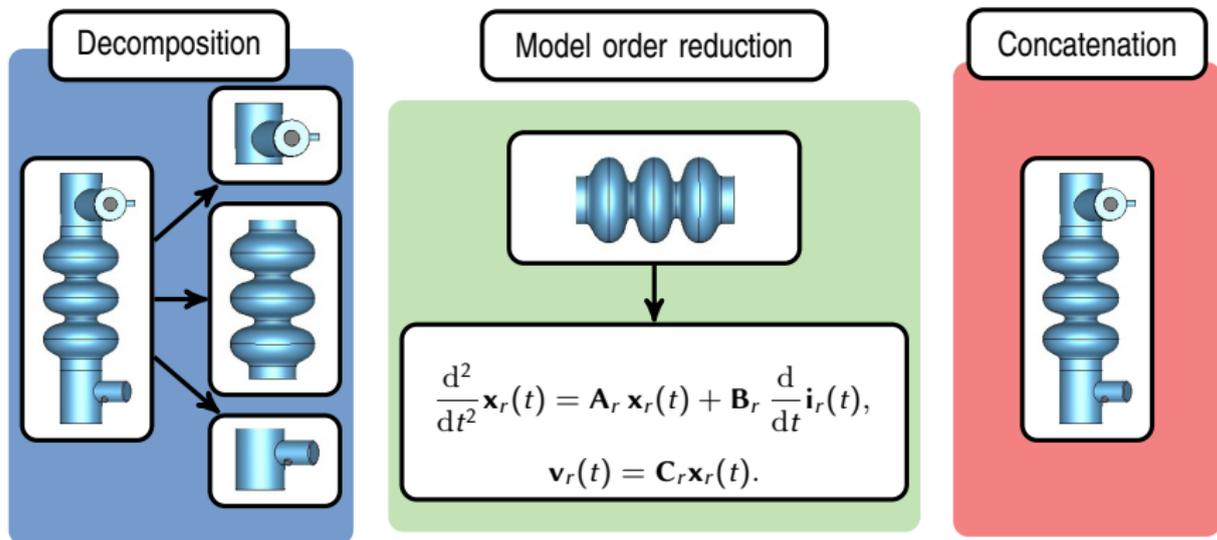
## General Approach



## Proposed Approach



## Reduction of State Matrices by SSC



T. Flisgen, H.-W. Glock, and U. van Rienen. Compact Time-Domain Models of Complex RF Structures Based on the Real Eigenmodes of Segments. IEEE

Transactions on Microwave Theory and Techniques, 61(6), June 2013.

## Solution Technique with SSC

- Perturbation approach for the reduced-order model: change termination impedance, such that the boundary gets non-reflective

$$G_{\text{Wave},n}(s) = Z_{\text{Wave},n}^{-1}(s) = -\frac{i_{r,n}(s)}{u_{r,n}(s)}$$

- Plugging this into our initial state-space system
- Nonlinear Eigenvalue Problem (NLEVP) that is partly determined by the **SS-system** of the closed system and the **termination condition**

$$\underbrace{(\mathbf{A}_r - \mathbf{B}_r \mathbf{G}_{\text{Wave}}(\lambda) \mathbf{B}_r^T - \lambda \mathbf{I})}_{\boldsymbol{\tau}(\lambda)} \mathbf{x}_r(\lambda) = \mathbf{0}$$

## Inverse \ Newton Iteration

- We try to find all eigenpairs  $\{\lambda_i, \mathbf{x}_i\}$  of the NLEVP, and therefore introduce normalization condition

$$\mathbf{P} \begin{bmatrix} \mathbf{x}_i \\ \lambda_i \end{bmatrix} = \begin{bmatrix} \mathbf{T}(\lambda_i)\mathbf{x}_i \\ \mathbf{v}^H \mathbf{x}_i - 1 \end{bmatrix} = \mathbf{0}$$

- We differentiate with  $\begin{bmatrix} \frac{\partial}{\partial \mathbf{x}_i} & \frac{\partial}{\partial \lambda_i} \end{bmatrix}$  and obtain the so-called Fréchet derivative

$$\mathbf{P}' \begin{bmatrix} \mathbf{x}_i \\ \lambda_i \end{bmatrix} = \begin{bmatrix} \mathbf{T}(\lambda_i) & \frac{\partial \mathbf{T}(\lambda_i)}{\partial \lambda_i} \mathbf{x}_i \\ \mathbf{v}^H & 0 \end{bmatrix}$$

- The derivative  $\frac{\partial \mathbf{T}(\lambda_i)}{\partial \lambda_i}$  is known analytically

## Newton Iteration

- Having the operator  $\mathbf{P}$  we can use Newton-Iteration<sup>3</sup>

$$\begin{bmatrix} \mathbf{T}(\lambda_{i,j}) & \frac{\partial \mathbf{T}(\lambda_{i,j})}{\partial \lambda_{i,j}} \mathbf{x}_{i,j} \\ \mathbf{v}^H & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_{i,j+1} - \mathbf{x}_{i,j} \\ \lambda_{i,j+1} - \lambda_{i,j} \end{bmatrix} = - \begin{bmatrix} \mathbf{T}(\lambda_{i,j}) \mathbf{x}_{i,j} \\ \mathbf{v}^H \mathbf{x}_{i,j} - 1 \end{bmatrix}$$

- This can be reformulated as

$$\mathbf{T}(\lambda_{i,j}) \mathbf{u}_{j+1} = \frac{\partial \mathbf{T}(\lambda_{i,j})}{\partial \lambda_{i,j}} \mathbf{x}_{i,j},$$

$$\lambda_{i,j+1} = \lambda_{i,j} - \frac{\mathbf{v}_i^H \mathbf{x}_{i,j}}{\mathbf{v}_i^H \mathbf{u}_{i,j+1}},$$

$$\mathbf{x}_{i,j+1} = \mathbf{C} \mathbf{u}_{i,j+1}.$$

<sup>3</sup>A. Ruhe, "Algorithms for the nonlinear eigenvalue problem." SIAM Journal on Numerical Analysis 10.4 (1973): 674-689.

## Solution pairs $\{\lambda_i, \mathbf{x}_{r,i}\}$

- From the eigenvalues  $\lambda_n$  we can compute external quality factors

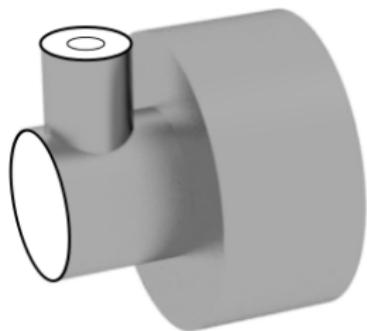
$$Q_{\text{ext},n} = -\frac{|\text{Im}\{\lambda_n\}|}{2 \text{Re}\{\lambda_n\}} \quad f_n = \frac{\text{Im}\{\lambda_n\}}{2\pi}$$

- From the eigenvectors we can compute the fields and derive the interaction with a particle beam

$$(r/Q)_n = \frac{\left| \int_0^L E_n(x_0, y_0, z) e^{j\omega \frac{z}{c_0}} dz \right|^2}{\omega_n W_{\text{stored},n}}$$

## Minimalistic Resonator

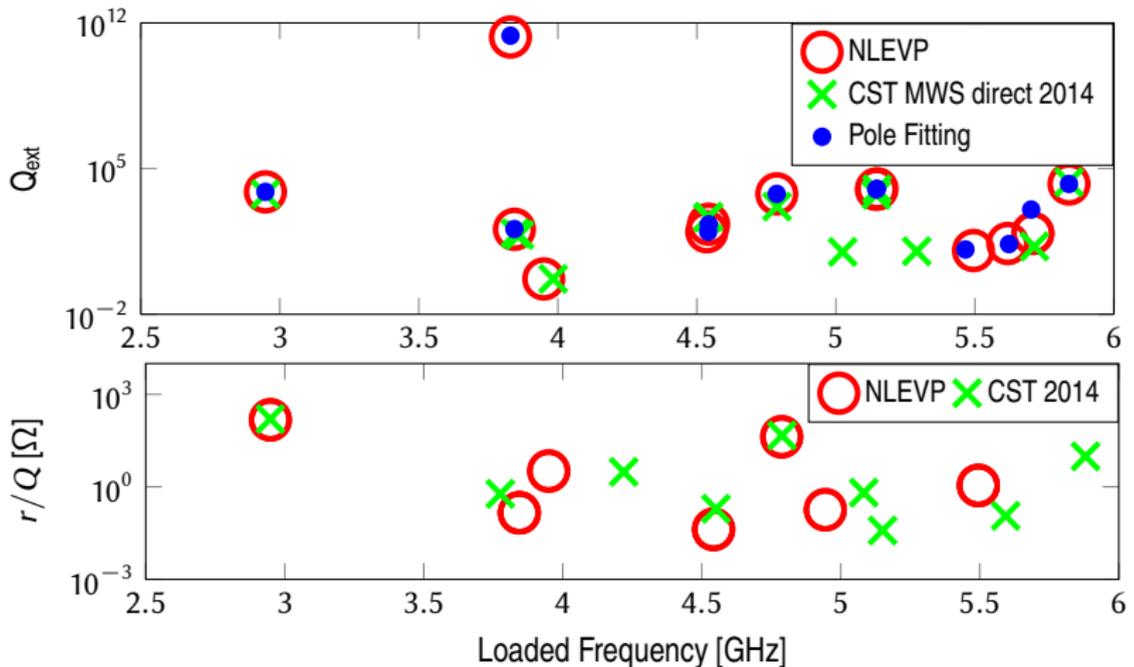
- First we test a very minimalistic resonator with 50k DOFs for 1 – 6 GHz
  - This structure has no practical relevance, just serves as example



Port	Mode	$f_{co}$ [GHz]
1	TEM	0
1	TE <sub>11</sub> pol. 1	5.902
1	TE <sub>11</sub> pol. 2	5.902
2	TE <sub>11</sub> pol. 1	4.389
2	TE <sub>11</sub> pol. 2	4.389
2	TM <sub>01</sub>	5.732
2	TE <sub>21</sub> pol. 1	7.275
2	TE <sub>21</sub> pol. 2	7.275

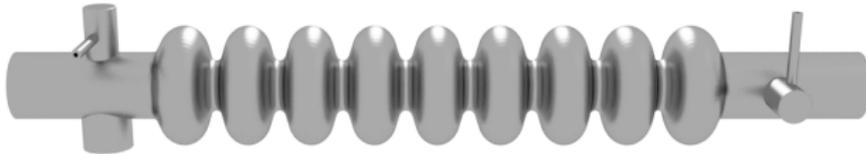
- This example is then compared to CST MWS and Pole-Fitting
  - We compare  $Q_{ext}$  and  $r/Q$

## Minimalistic Resonator



## SRF Cavity with couplers

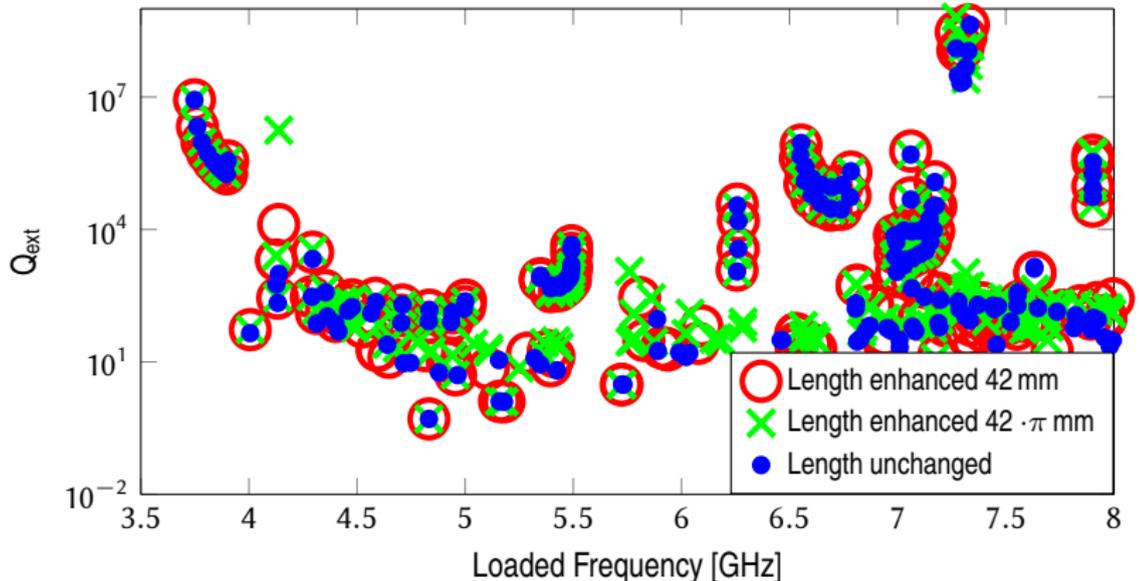
- Validation on a real life example with  $2.2 \cdot 10^6$  DOFs



- FLASH Third Harmonic Cavity with successively extended beampipe
  - $Q_{ext}$ -values should not change when the beampipes are extended
- Model uses a total of 25 port-modes
- MOR takes roughly 5 hours
- Solution takes roughly 7 minutes

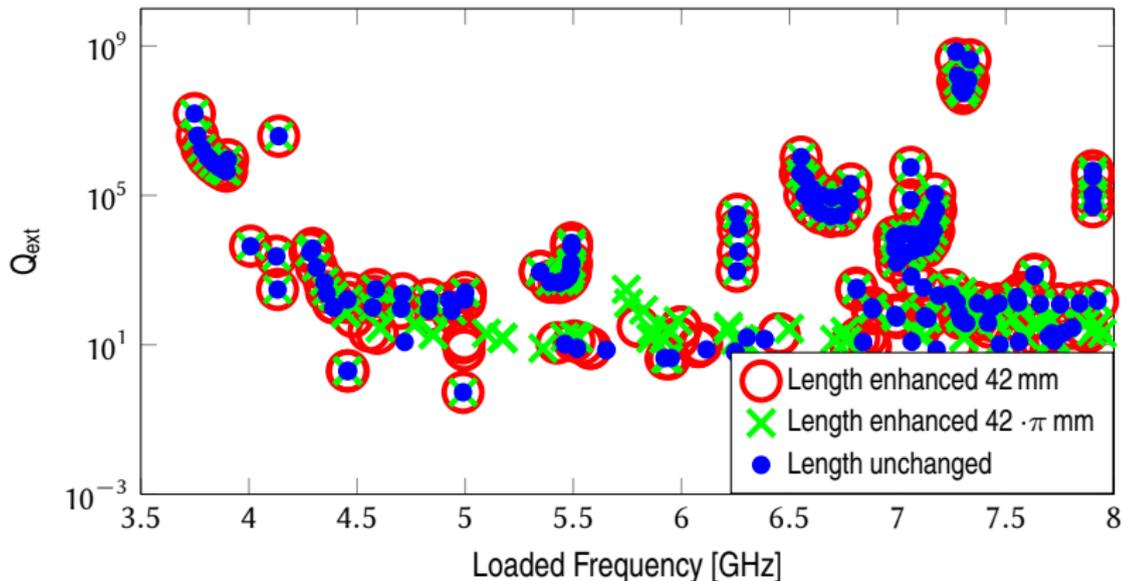
## SRF Cavity with couplers with CST MWS

- CST MWS: Cavity with successively extended beampipe



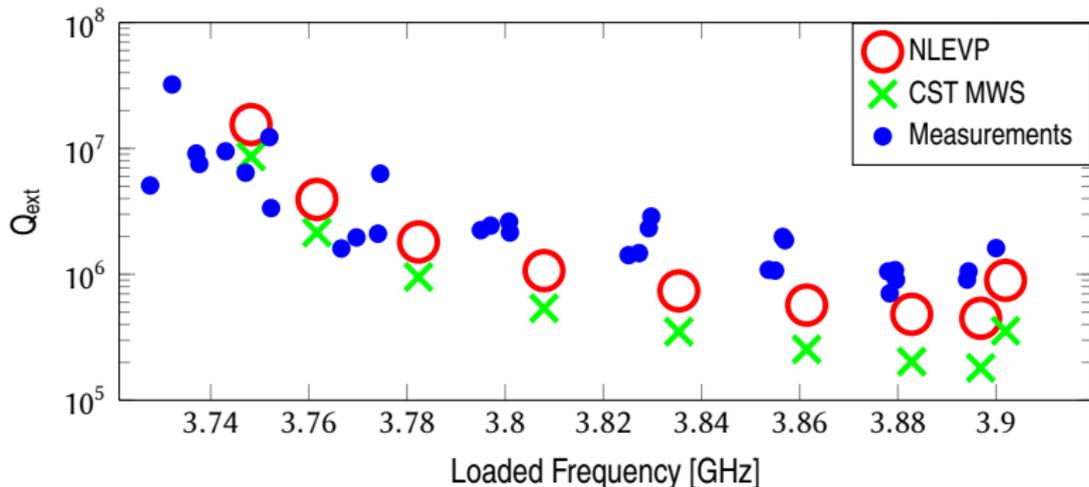
## SRF Cavity with couplers with NLEVP

- NLEVP: The attached beampipe is successively extended and we compute  $Q_{\text{ext}}$



## SRF Cavity with couplers, Measurements

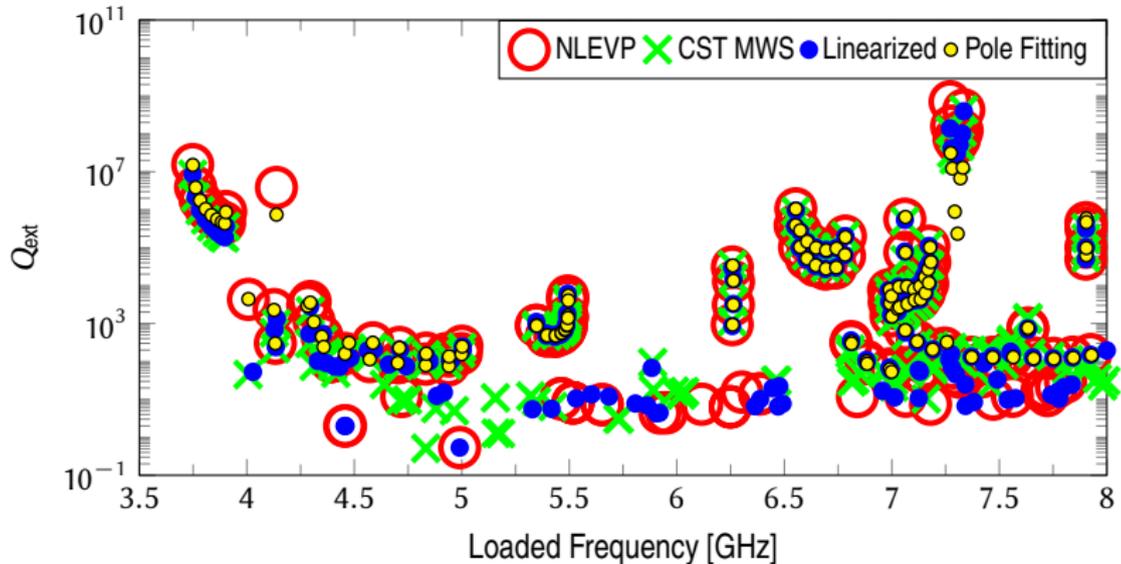
- For the first monopole band, there exist some measurements<sup>4</sup>



<sup>4</sup>T. Flisgen, et al., "Scattering parameters of the 3.9 GHz accelerating module in a free-electron laser linac: A rigorous comparison between simulations and measurements," Physical Review Special Topics-Accelerators and Beams 17:2: 022003, (2014).

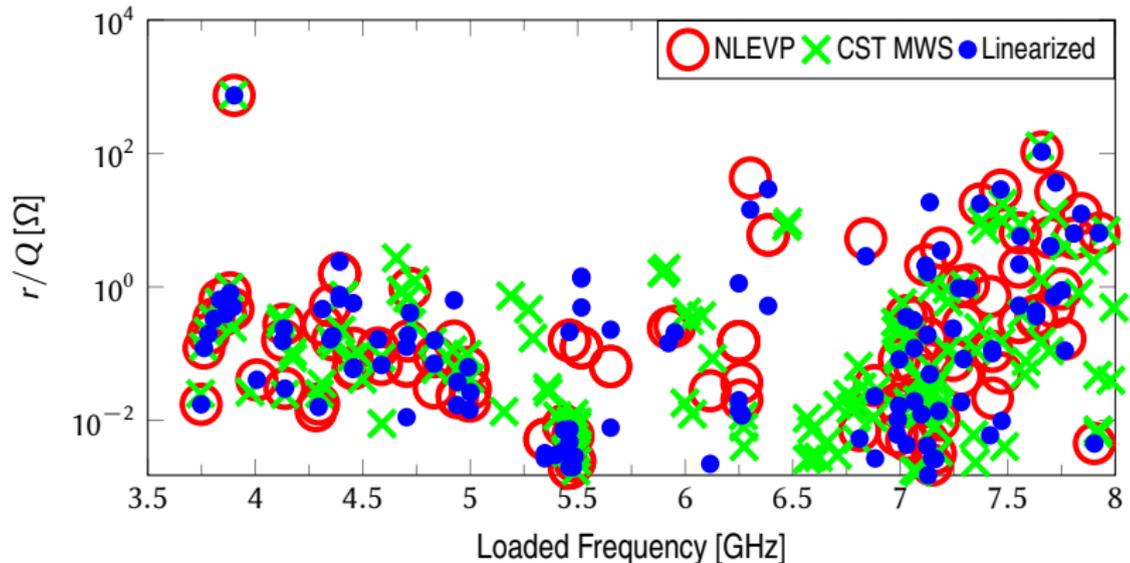
## SRF Cavity with couplers

- All approaches compared regarding  $Q_{ext}$



## SRF Cavity with couplers

- Interestingly the termination condition also influences the electromagnetic fields



## Full FLASH Third Harmonic Module

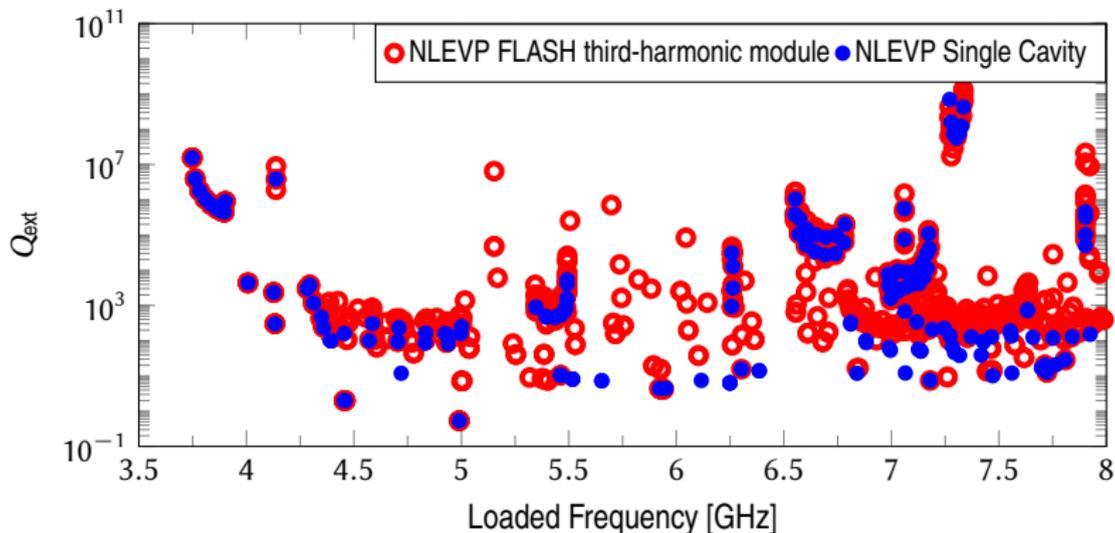
- Finally, a large scale example:  $10.2 \cdot 10^6$  DOFs



- FLASH Third Harmonic Module is operated at DESY in Hamburg
- This has roughly 800 eigenmodes
- Full solution takes roughly a day (on a workstation computer)
- Even the linearised problem is actually beyond the scope of a workstation computer

## Full FLASH Third Harmonic Module

- Interestingly the behavior of the single cavity differs significantly from the concatenation of several cavities

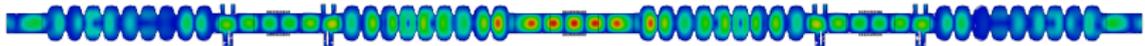


## Full FLASH Third Harmonic Module

- Some interesting fields computed by solving the NLEVP
  - The  $\pi$ -mode remains the same



- Some dipole-modes are distributed along the entire structure



- By investigating the entire structure instead of single cavities, additional modes can be found

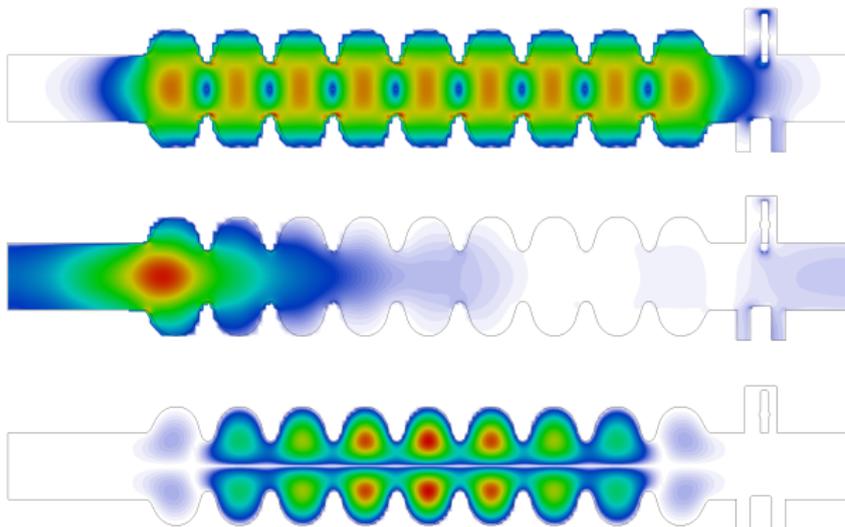


## Conclusion

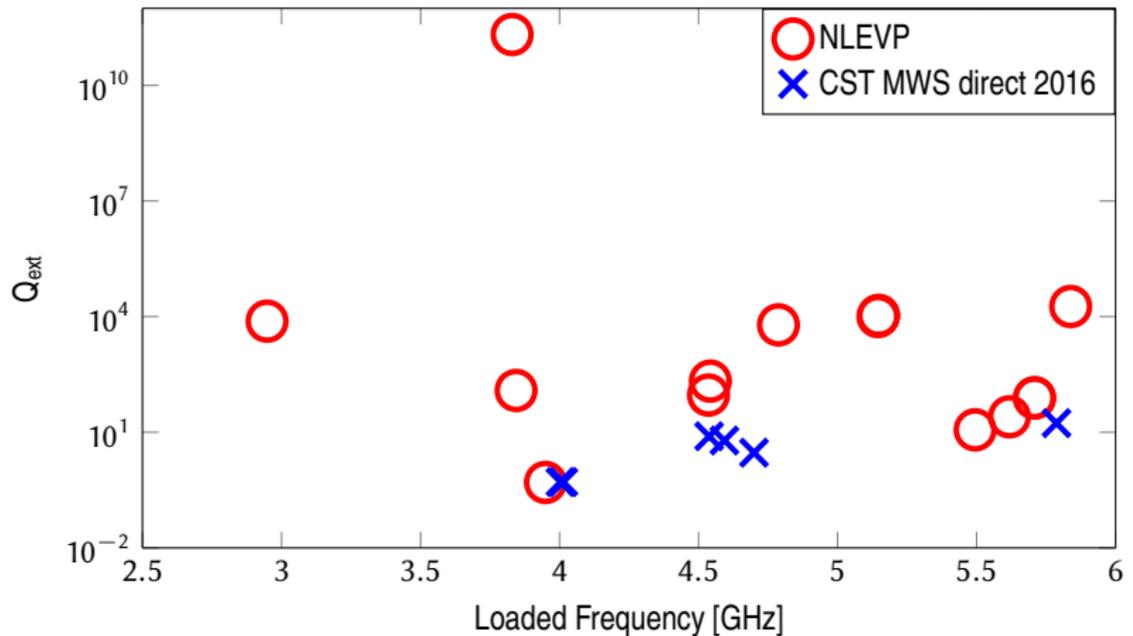
- The proposed technique (though relatively simple) allows for the fast solution of the NLEVP
- Approach works for relatively large structures (10 Mio DOFs), with better deflation scheme we could go to  $> 20$  Mio DOFs
- Deflation scheme could be improved (major bottleneck)
- Higher order Fréchet derivatives, and preconditioning might give better performance
- Further reading:
  - J. Heller, T. Flisgen, T. Galek and U. van Rienen, *Numerical Investigation of External Losses in Superconducting Radio-Frequency Cavities*, Phys. Rev. ST Accel. Beams, submitted 20.06.2016.

## Cavity with coupler

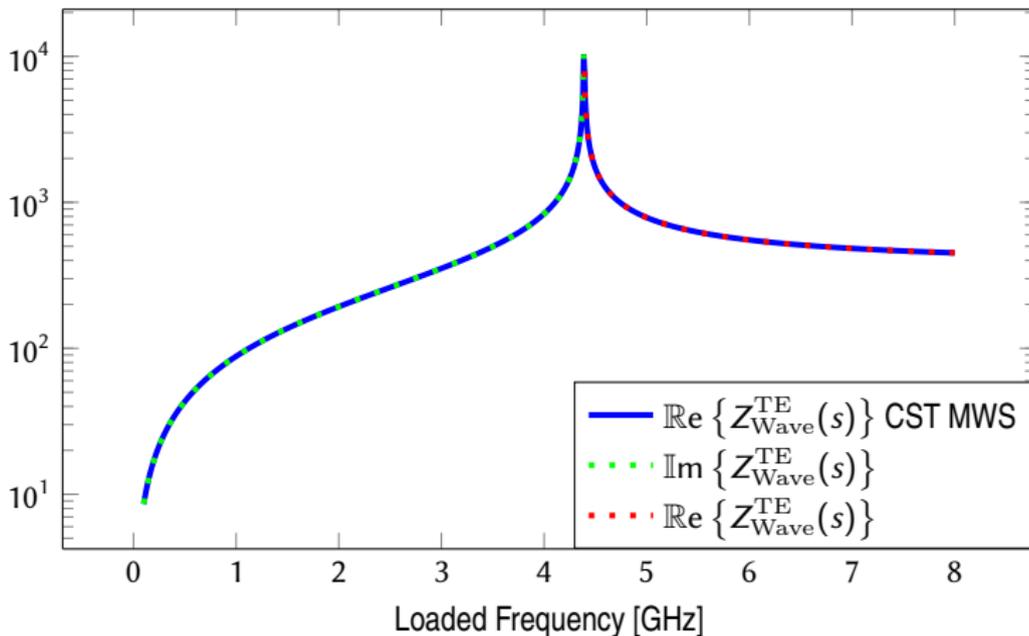
- Some interesting fields originating from the NLEVP



## CST Studio 2016?



## Wave impedance CST Studio



## General remarks to applications

- All computations were performed on a Intel(R) Xeon(R) CPU E5-2687W @3.4 GHz with 256 GB of RAM
- In all shown cases all eigenmodes (that were found by the algorithm) in the given frequency interval were computed with a relative residual smaller than  $10^{-6}$

$$r = \|\mathbf{T}_j(\lambda_{i,j}) \mathbf{x}_{i,j}\| / \|\mathbf{x}_{i,j}\|$$

- The implementation was done in Matlab ®(in future we would rather do it in [SLEPc](#))
- Field visualizations done in [Blender](#) and [Paraview](#)

## SRF Cavity with couplers

- For the optimization of the coupler, often the product  $r/Q \cdot Q_{\text{ext}}$  is considered

