

A Monodromy from London

Nemanja Kaloper

UC Davis

based on work with A. Lawrence, and A. Lawrence and L. Sorbo
arXiv:1607.06105, arXiv:1404.2912, arXiv:1101.0026,
arXiv:0811.1989, arXiv:0810.5346.

Outline

- Large field inflation: dangers... and appeals! of simplicity
- Models, models, models... keep it simple, silly: why **could** it work?...
- Of Londons, and Julia and Toulouse
- Summary

Large field inflation: bottom up (from the top down!)

- Instead of focusing on *how*, let's try to figure out *why*?
- Many different inflation candidates now thanks to some older and a lot of new interesting work.
- “Axions”: shift symmetry?
- A flood of theories: necessity a mother of invention, wisdom after the fact it's great assistant.
- A tempting conjecture: an “equivalence class” of inflatons: explicit models somewhat detail-dependent, sharing a hidden underlying dynamics which is the same???

Simplest Inflaton

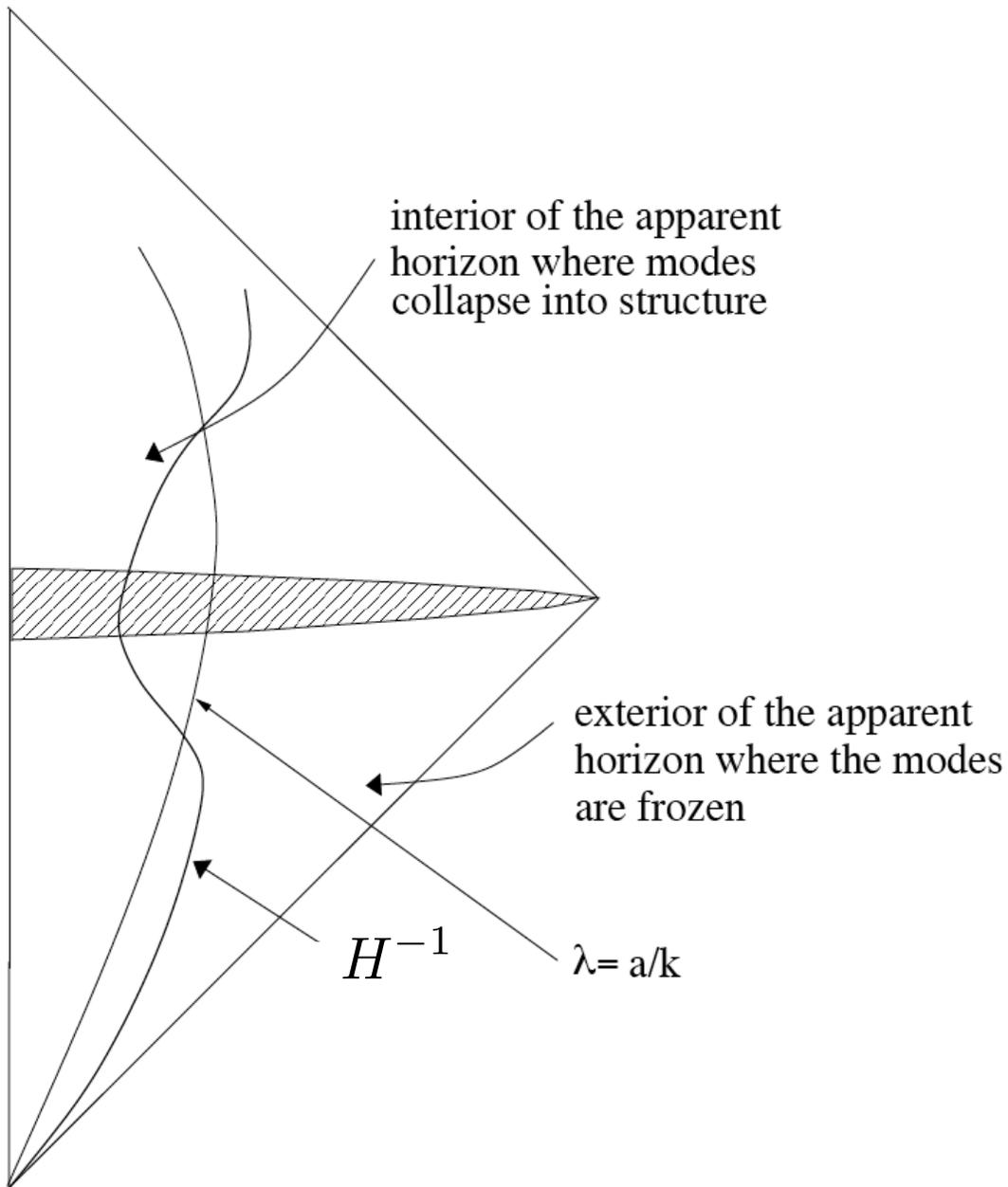
Linde: 1980's: take any polynomial; eg just a parabola, $m^2 \phi^2 / 2$

Now 'just' take $\phi \gg M_{Pl}$ and keep $m \ll M_{Pl}$ and use the fact that far far away the potential looks flat! Indeed,

$$\frac{\partial_\phi V}{V/M_{Pl}} = \frac{M_{Pl}}{\phi}, \quad \frac{\partial_\phi^2 V}{V/M_{Pl}^2} = \frac{M_{Pl}^2}{\phi^2}$$

and done we are! Better yet: look at small fluctuations; our pendula (scalars +tensors), while overdamped, do NOT stay put like in classical physics. They tremble, due to the uncertainty principle. The amplitude evolution freezes when the wavelength hits horizon scale ("antitunneling" in time)

These frozen tremors seed galaxy formation long after the end of inflation



This did not seem to be very difficult at all; we (i.e., Linde) just needed to be clever a bit, and voila, it all came out on a silver platter...

The Specter of QM

Radiative corrections **could** deform the inflationary potential

A theory of inflation cannot ignore the rest of the world:
inflation must end, the universe must be repopulated: the field
driving inflation **MUST** couple to other stuff!!!



... LOOP DIVERGENCES!

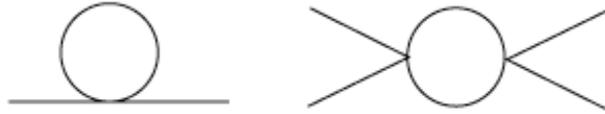
Due to quantum corrections these couplings are **NOT** inert: they
could change the potential dramatically and spoil inflation badly !!

This suggests sensitivity of inflation to unknown, new physics

(Really???)

Quantum dangers ?

1) Self-interacting scalars: no, even though the corrections individually look **terrible**



etc $\equiv (-1)^n \lambda^n \phi^4 \left(\frac{\phi}{M}\right)^{2n-4}$

They **alternate** and resum to log corrections: $\lambda \phi^4 \left(1 + c \ln\left(\frac{\phi}{M}\right)\right)$

2) Graviton loops: no, finite potential and Planck mass renormalizations that go like

$$\left(\frac{\partial_\phi^2 V}{M_{Pl}^2} + \frac{V}{M_{Pl}^4}\right) V \quad \partial_\phi^2 V R$$

which are small in the inflationary regime: the couplings \sim energies and momenta, and so are small when the energies \ll Planck scale!

Reason: (weakly broken) shift symmetry $\phi \rightarrow \phi + c$; radiative corrections proportional only to the breaking terms, going as some derivatives of $V'(\phi)$.

If all couplings to matter are derivative, the inflaton immune to loop corrections

NO PROBLEM EVER WITH LOOP CORRECTIONS

So what's all the bother?

UV completions - eg embeddings in... say string theory: the lore is QG breaks all global symmetries (such as shift symmetry) - and breaks them badly. This implies that there may be large corrections to low energy EFT beyond perturbation theory, and that the perturbative stability is unreliable since in full theory the protection mechanism - global shift symmetry - is no more.

Examples: wormholes, WGC, and also direct constructions

Eg: imagine a simple axion with $V = \Lambda^4 \cos(\frac{\phi}{f}) + \dots$

From inflationary slow roll, $m_{pl} < \phi < f$

But then higher order corrections are unsuppressed, $c_n \simeq e^{-n \frac{M}{f}} \rightarrow 1$

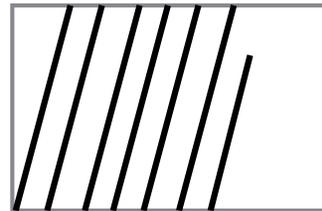
Higher harmonics will generically decohere the potential and make it very bumpy so slow roll will be lost

Enter monodromy

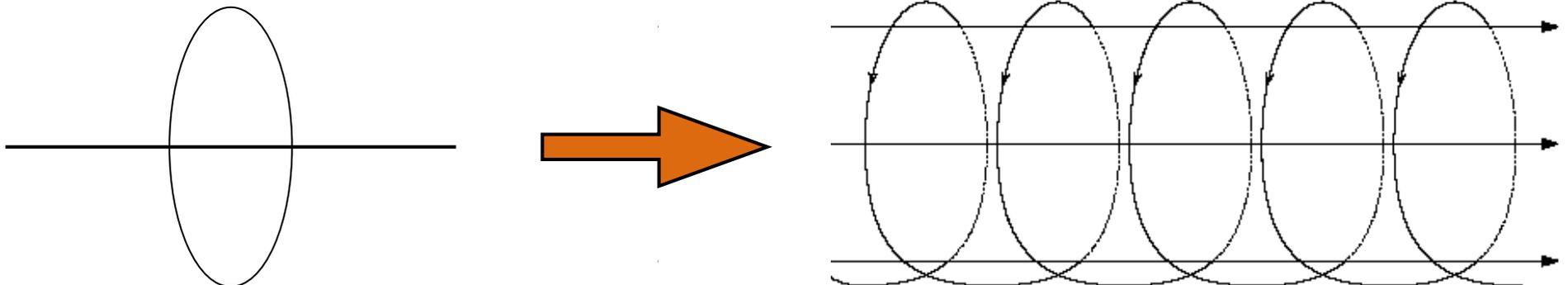
Silverstein & Westphal; NK & Sorbo, 2008

Linguistically: monodromy = “a single road” (“running ‘round singly’”, according to wikipedia; but ask the Greeks...)

Idea: try to get large field ranges in small compact spaces which do not violate the reliability of low energy EFT - so you want to move a lot with colliding with your path in phase space

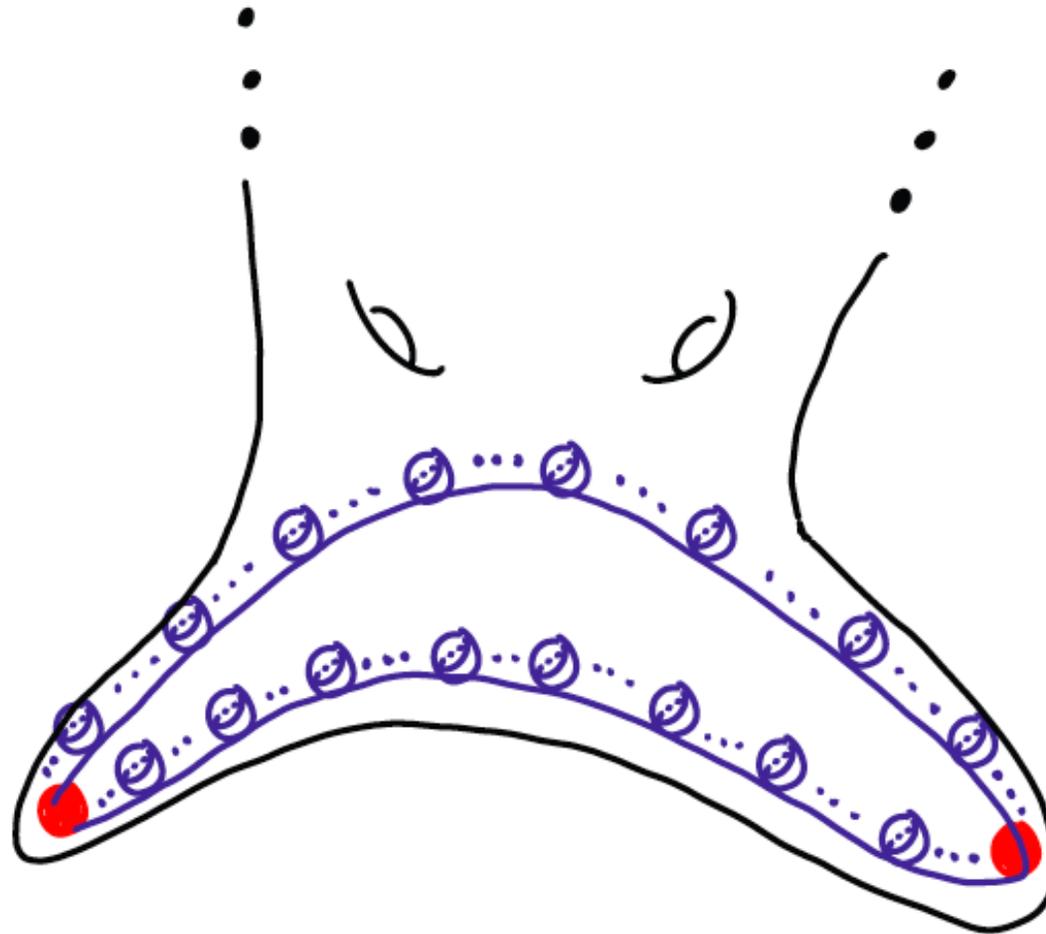


A simple physical example: a particle in a magnetic field, with a general velocity vector (in QM a very complex system, many many Landau levels...)



Constructions

McAllister, Westphal, Silverstein



Constructions by pedestrians

NK, Lawrence, Sorbo

$$\mathcal{S}_{bulk} = \int d^4x \sqrt{g} \left(\frac{M_{Pl}^2}{2} R - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{48} F_{\mu\nu\lambda\sigma}^2 + \frac{\mu\phi}{24} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} F_{\mu\nu\lambda\sigma} + \dots \right)$$

Di Vecchia and Veneziano; Quevedo and Trugenberger; Dvali and Vilenkin; NK & Sorbo; NK, Lawrence & Sorbo.

Action invariant under (perturbative!) shift symmetry!

under $\phi \rightarrow \phi + c$, $\mathcal{L} \rightarrow \mathcal{L} + c \mu \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu\rho\lambda} / 24$

Dynamics: the scalar propagator, 4-form propagator and scalar-form vertex:

Diagrammatic representation of the propagators and vertex:

- A solid horizontal line representing the scalar propagator, labeled $\frac{1}{p^2}$.
- A dashed horizontal line representing the 4-form propagator, labeled 1 .
- A solid horizontal line with a red dot in the middle, connected to a dashed horizontal line, representing the scalar-form vertex, labeled μ .

So the physical scalar propagates as a massive field! ... with a radiatively stable mass!

Diagrammatic equation showing the resummation of scalar propagators:

Left side: A sum of diagrams representing the expansion of a massive propagator. It starts with a solid line, followed by a solid line with a red dot connected to a dashed line, then a solid line with two red dots connected to two dashed lines, and so on.

Right side: A single solid line with an 'x' mark, representing the massive propagator $\frac{1}{p^2 - \mu^2}$.

$$\frac{1}{p^2} + \frac{1}{p^2} \mu^2 \frac{1}{p^2} + \frac{1}{p^2} \mu^2 \frac{1}{p^2} \mu^2 \frac{1}{p^2} + \dots = \frac{1}{p^2 - \mu^2}$$

Making symmetry manifest

- First order formalism: enforce $F = dA$ with a constraint

$$\mathcal{S}_q = \int d^4x \frac{q}{24} \epsilon^{\mu\nu\lambda\sigma} (F_{\mu\nu\lambda\sigma} - 4\partial_\mu A_{\nu\lambda\sigma})$$

NK, 1994

- Then change variables

$$\tilde{F}_{\mu\nu\lambda\sigma} = F_{\mu\nu\lambda\sigma} - \sqrt{g}\epsilon_{\mu\nu\lambda\sigma}(q + \mu\phi)$$

- This completes the square; integrate F out. What remains:

$$\mathcal{S}_{eff} = \int d^4x \sqrt{g} \left(\frac{M_{Pl}^2}{2} R - \frac{1}{2} (\nabla\phi)^2 - \frac{1}{2} (q + \mu\phi)^2 + \frac{1}{6} \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}} A_{\nu\lambda\sigma} \partial_\mu q \right)$$

- The membrane term enforces jump on q (ie $*F$):

$$\Delta q|_{\vec{n}} = e$$

Now properly quantize this system!

$$S_{class} = \int d^4x \sqrt{g} \left(m_{pl}^2 R - \frac{1}{48} F^2 - \frac{1}{2} (\partial\varphi)^2 + \frac{\mu}{24} \varphi^* F \right)$$

$$F_{\mu\nu\lambda\rho} = \partial_{[\mu} A_{\nu\lambda\rho]} \quad \delta A_{\mu\nu\lambda} = \partial_{[\mu} \Lambda_{\nu\lambda]} \quad \varphi \equiv \varphi + 2\pi f_\varphi$$

U(1) gauge symmetry +
discrete gauge symmetry

$$H_{tree} = \frac{1}{2} p_\phi^2 + \frac{1}{2} (p_A + \mu\phi)^2 + grav. \quad p_A = ne^2$$

Compactness of U(1)

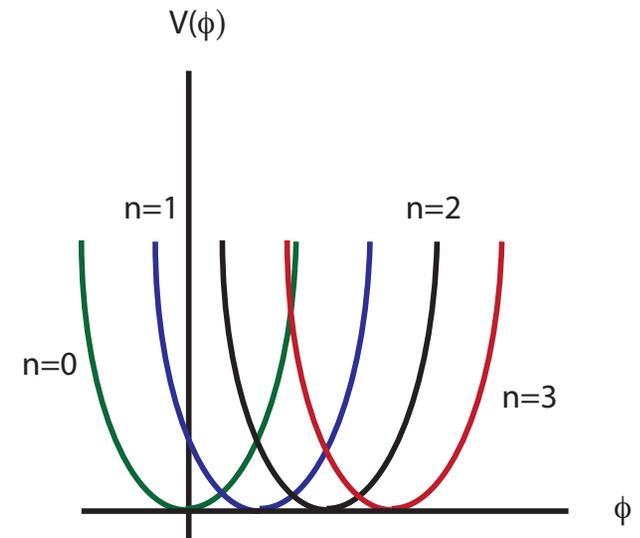
Dim. red. to 0+1:
charged particle
in B-field on torus.
k = magnetic flux quantum
= LLL degeneracy



$$\mu f_\phi = ke^2$$

$$\varphi \rightarrow \varphi + 2\pi f_\varphi ; n \rightarrow n - k$$

discrete gauge symmetry in phase space



Déjà vu! It really **IS** just a particle in a magnetic field!

The theory has **TWO** sets of gauge symmetries: the nonlinearly realized U(1) of the massive 4-form and the discrete gauge symmetry of the scalar!

Higher order shift-symmetry invariant terms

Gauge symmetry invariant terms are the only ones allowed!

After dimensional reduction and stabilization of moduli:

$$S_{\phi F} = \int d^4x \sqrt{g} \left(-\tilde{F}^2 + (\partial\tilde{\phi})^2 + \mu\tilde{\phi}\epsilon\tilde{F} + \dots + \frac{\tilde{F}^{2n}}{(V_d M^{4+d})^{n-1}} + \dots \right)$$

So the higher corrections are suppressed by $V^d M^{4+d} = \Lambda_F^4 \sim M_{GUT}^4$

$$V_{eff} = \frac{\mu^2 \phi^2}{2} \left(1 + \sum c_n \frac{\mu^2 \phi^2}{\Lambda_F^4} \right) !!!$$

There are also higher derivative corrections which we are ignoring here;
... stay tuned (in preparation, D'Amico, NK & Lawrence)

Er... Why is this???

What is *really* going on here?

What do we see: a 4-form gauge field + a ‘random’ massless (pseudo)scalar who mix up and yield... a massive theory???

SSB! But... there is no “Higgs” - i.e. no “radial” mode - just a “phase”

Rings a bell???

Well, a massive U(1) gauge theory - eg BCS superconductor???

Electrons in a material: a free gas - sometimes interactions with the lattice generate a state with long range correlations breaking QM phase shift invariance. This symmetry is gauged - it’s U(1) charge. The resulting Goldstone eaten by the U(1) field which becomes massive.

Microscopics challenging - but effective theory straightforward: **Londons’ eq!**



Fritz

$$\vec{A} = \frac{\vec{J}}{m^2}$$

Heinz



Some massive vector U(1) gauge theory

- Gauge symmetry is nonlinearly realized

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2(A_\mu - \partial_\mu\phi)^2 + (A_\mu - \partial_\mu\phi)J^\mu - \frac{1}{2\alpha}(\partial \cdot A - \alpha m^2\phi)^2$$

- Gauge symmetry protects the theory - it is renormalizable! The key:

$$\Delta_{\mu\nu} = \frac{i}{p^2 + m^2} \left(\eta_{\mu\nu} + (1 - \alpha) \frac{p_\mu p_\nu}{p^2 + \alpha m^2} \right), \quad \Delta_\phi = -\frac{i}{p^2 + \alpha m^2}$$

- Can send m to zero without messing the theory up! (the scalar is free in QFT and is “modded out” by BCs; but it seems to gravitate (Deser et al).
- Even though it seems to have a global symmetry... this is vacuous since the scalar period $\sim m/e$ so only fluctuations mix with gravity. Yet this is “vacuous”: *only* renormalizes gravity
- This limit is dual to a 2-form gauge theory but with ZERO charges - and NO sources - this is consistent with the WGC; if a string is introduced it becomes light when m is zero, implying that the dual U(1) must be completed at zero mass - introducing massless charges

Phases of a massive U(1): an insight of Julia and Toulouse

- Conventional UV embedding is by way of a Higgs mechanism
- But for U(1) there are other options...!!! JULIA & TOULOUSE
- EXAMPLE: consider a gauge theory with defects - eg a U(1) theory with vortices (= strings), go to the phase containing MANY defects and let them couple strongly. LET THEM CONDENSE!
- ASK: fluctuate this system. What is the EFT of fluctuations???
- Wisdom after the fact: superfluids! In the presence of many vortices, the superfluid velocity is not irrotational; superfluid current: $\vec{j} \simeq \vec{v}$
- Julia-Toulouse: take that current and DECLARE IT to be a gauge field potential - and add a gauge field strength for it to boot!

Table II. — Hydrodynamic variables for He₄.

p	Number of components		No vortex	Many vortices
	Space only	Space + time		
0	1	1	Variable φ	
1	3	4	Superfluid velocity $v, dx = d\varphi$	Variables A_μ



Julia - Toulouse cont.

- Take a compact scalar - a phase. Consider its variation around the moving vortices

$$\delta\phi \sim \vec{v} \cdot \delta\vec{x}$$

- Since the fluid is NOT irrotational, $\vec{v} \neq \vec{\nabla}\phi$; instead $\vec{j} \simeq \vec{v}$ contains nontrivial physical information; JT declare it to be the new variable and use it to describe the EFT of the new phase. This is London's eq and adding kinetic terms implies that the new gauge theory is a $U(1)$ massive vector.
- Since we are dealing with a system of vortices (strings) it is natural to start with a 3-form. As the system is not perturbatively connected to the usual 3-form vacuum, it is also natural to assume that the 3-form is massive. So we start with

$$\mathcal{L}_H = -\frac{1}{12}H_{\mu\nu\lambda}^2 - \frac{m^2}{4}\left(B_{\mu\nu} - \frac{1}{m}\tilde{F}_{\mu\nu}\right)^2 + \dots$$

- We then attempt to link it with the JT proposal

Julia - Toulouse cont.

- Consider **massive U(1) dualities!** start with a massive 2-form

$$\mathcal{L}_H = -\frac{1}{12}H_{\mu\nu\lambda}^2 - \frac{m^2}{4}\left(B_{\mu\nu} - \frac{1}{m}\tilde{F}_{\mu\nu}\right)^2 + \dots$$

- Now dualize! write

$$\mathcal{L}_{BF} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{12}H_{\mu\nu\lambda}^2 - \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}B^{\mu\nu}F^{\lambda\sigma} + \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\tilde{A}_\mu\partial_\nu F_{\lambda\sigma} + \dots$$

- Integrating out F gives massive 2-form; integrating out A gives

$$\mathcal{L}_{BF} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{12}H_{\mu\nu\lambda}^2 - \frac{m}{4}\epsilon_{\mu\nu\lambda\sigma}B^{\mu\nu}F^{\lambda\sigma} + \dots$$

- Integrate BF term by parts, add $\phi\epsilon^{\mu\nu\lambda\sigma}\partial_\mu H_{\nu\lambda\sigma}$ and integrate out H

$$\mathcal{L}_{JT} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{m^2}{2}(A_\mu - \partial_\mu\phi)^2 + \dots$$

- De facto JT ansatz:

$$A_\mu = \frac{1}{6m}\epsilon_{\mu\nu\lambda\sigma}H^{\nu\lambda\sigma}$$

The Scoop on Julia-Toulouse

- Dualization of the gauge theory where the field variables of the initial setup are traded for field momenta of the dual
- The idea: take a theory with defects, crank up the coupling, let the defects be light and let them condense. Trade fields and momenta; map the theory just below the strong coupling to its dual just above the strong coupling.
- Generalizations to other form fields - Quevedo & Trnkeberger
- **The new Lore:**
- TAKE A GAUGE THEORY WITH DEFECTS, LET THEM MULTIPLY, INTERACT AND CONDENSE; WHAT IS THE NEW EFT OF THE SYSTEM?
- The rule: TAKE THE ***DUAL*** OF THE OLD GAUGE FIELD STRENGTH - which in the presence of the defects obeys $*F \sim J$ and so is NOT closed any more - AND DECLARE IT TO BE THE GAUGE POTENTIAL for the new phase
- THE DUALITY TRANSFORMATIONS **"KNOW"** THIS!
- (look up the magnetic field **scalar!** potential in Jackson's book! (in the section on magnetostatics))

4-form massive gauge theory

- Follow the known rules

$$\mathcal{L} = -\frac{1}{48} F_{\mu\nu\lambda\rho} F^{\mu\nu\lambda\rho} - \frac{m^2}{12} (A_{\mu\nu\lambda} - h_{\mu\nu\lambda})^2 - \frac{1}{2\xi} \left(\partial^\mu A_{\mu\nu\lambda} - \frac{\xi m^2}{2} b_{\nu\lambda} \right)^2$$

- Londons' eq: $A_{\mu\nu\lambda} \sim \frac{J_{\mu\nu\lambda}}{m^2}$: current of membranes!
- At high momenta:

$$\langle A_{\mu\nu\lambda}(p) A_{\mu'\nu'\lambda'}(-p) \rangle = \epsilon_{\mu\nu\lambda\rho} \epsilon_{\mu'\nu'\lambda'\rho'} \left(\frac{\frac{\xi}{2} \eta^{\rho\rho'}}{p^2 - \frac{\xi m^2}{2}} + \frac{(1 - \frac{\xi}{2}) p^\rho p^{\rho'}}{(p^2 - m^2)(p^2 - \frac{\xi m^2}{2})} \right)$$

- The mass term is innocuous as momenta get big so the theory is well behaved in the UV; gauge symmetry protects it. Further it has ONLY one propagating DOF - a scalar!
- To see this dualize! Ignore the GF and use 1st order formulation;

$$\mathcal{L} = -\frac{1}{48} F_{\mu\nu\lambda\sigma}^2 - \frac{m^2}{12} (A_{\mu\nu\lambda} - h_{\mu\nu\lambda})^2 + \frac{m}{6} \phi \epsilon^{\mu\nu\lambda\rho} \partial_\mu h_{\nu\lambda\rho}$$

- The constraint enforces $h = db$; integrate out h

More massive 4-form:

- The resulting action is, with $F = dA$,

$$\mathcal{L}_{\phi,A} = -\frac{1}{48} F_{\mu\nu\lambda\sigma}^2 - \frac{1}{2} (\partial_\mu \phi)^2 + \frac{m}{24} \phi \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu\lambda\rho}$$

- Now we can integrate out F ; adding $\frac{Q}{24} \epsilon_{\mu\nu\lambda\sigma} (F^{\mu\nu\lambda\sigma} - 4\partial^\mu A^{\nu\lambda\sigma})$

$$\mathcal{L}_{\phi,A} = -\frac{1}{2} (\partial_\mu \phi)^2 - \frac{m^2}{2} \left(\phi + \frac{Q}{m}\right)^2 + \frac{1}{6} \epsilon_{\mu\nu\lambda\sigma} Q \partial^\mu A^{\nu\lambda\sigma}$$

- NOTE: the dualizations show $\phi = -\frac{24}{m} \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu\lambda\sigma}$
- LET'S READ THIS RIGHT:
- we started with a magnetic dual of F , the Q . This is a 0-form FIELD STRENGTH! In a condensate of membranes, this will realize a DISCRETE GAUGE SYMMETRY $Q \rightarrow Q + ne^2$ when we complete the theory with a gauge field ϕ which transforms as $\phi \rightarrow \phi - ne^2/m$
- The gauge field potential is $(\partial\phi)^2$ and the gauging of Q is $(m\phi + Q)^2$
- All along we have $dQ = 0$ locally

Gauge theory of inflation

- Gauge symmetry is nonlinearly realized, and there is $U(1)$ symmetry of the 4-form and the discrete gauge symmetry of the dual
- THEY CANNOT BE BROKEN BY GRAVITY, OR ANYTHING ELSE... ON THEIR OWN - THEY ARE REDUNDANCIES BUILT INTO THE THEORY
- Corrections must come in the form that preserves these symmetries

$$\delta\mathcal{L}_1 = c_n \frac{F^{2n}}{M^{4n-4}}$$

$$\delta\mathcal{L}_2 = d_n \frac{m^{2n}}{M^{4n-4}} A^{2n}$$

- The m^{2n} in the latter term follows from Goldstone boson Equivalence Thm!
- We can also have cross terms
- After dualizing:

$$F_{\mu\nu\lambda\sigma} \sim (m\phi + Q)\epsilon_{\mu\nu\lambda\sigma}$$

$$m^2 A^2 \sim (\partial\phi)^2$$

Inflaton is the gauge flux!

- Note:

$$F_{\mu\nu\lambda\sigma} \sim (m\phi + Q)\epsilon_{\mu\nu\lambda\sigma}$$

- Define $m\varphi = m\phi + Q$; this is the physical inflaton

$$F_{\mu\nu\lambda\sigma} = m\varphi\epsilon_{\mu\nu\lambda\sigma}$$

- Large when F is large - or, when Q is large. Further, m can be dialed by hand since it is radiatively stable (as in massive $U(1)$ vector gauge theory). It makes the effective scalar super-Planckian even when the total energy is safely sub-Planckian
- Gauge symmetries prohibit large corrections which violate this structure. Gravity corrections are small as long as energy densities are sub-Planckian
- What sets the scale of energy density is the flux of F - it can be huge as long as its energy density is below the Planck scale

WGC

- In our setup WGC bounds reduce to the statement that the membranes cannot be too heavy for a given charge

$$T_{p,D}^e \lesssim \sqrt{g_D^2 m_{pl,D}^{D-2}} \qquad T_{D-p-3,D}^m \lesssim \sqrt{\frac{m_{pl,D}^{D-2}}{g_D^2}}$$

- An issue is that one needs to stabilize the monodromy structure of the system of potentials against flux discharge due to membrane emission

$$P \sim \exp \left[-\frac{27\pi^2}{2} \frac{\sigma^4}{V^3} \left(\frac{\mu}{2\Delta\mu} \right)^3 \right]$$

- If the tension is too small then the potential can be discharged quickly. But the numerics is favorable - one can easily get $\gg 100$ e-folds without coming even close to violating WGC
- For the electric dual, the scalar field itself plays the role of the discharging mechanism - it rolls down the potential neutralizing F

Summary

- Inflation is a simple way of explaining the origin of the universe, its dynamics fully consistent with local QFT.
- There is an issue of UV sensitivity - naturalness - and its paradigmatic implications. Monodromy addresses it well.
- Hidden gauge symmetries: a key controlling mechanism behind monodromy. They protect EFT from gravity.
- Gauge symmetries also explain why the large field vevs are ***fine***: they are dual gauge field strengths which count the sources! Large field = many sources.
- The ideas are experimentally predictive: so falsifiable... What we are really doing here is testing NATURALNESS in the sky. Soon we should know more. Patience!