Classification of effective operators using conformal group

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why effective operators

- In the absence of any concrete signal of new particles, we need to discuss effective operators to *Chart the Unknown*, i.e.probe physics at higher energies or weaker couplings
    - precision Higgs
    - precision flavor
    - *B*, *L* violation
- similar to four-fermion operators in weak interactions

Effective Operators

- Surprisingly difficult question
- In the case of the Standard Model
  - Weinberg (1980) on D=6 $\not{B}$, D=5 $\not{L}$
  - Buchmüller-Wyler (1986) on D=6 ops
    - 80 operators for N_f=1, B, L conserving
  - Grzadkowski et al (2010) removed redundancies and discovered one missed
    - 59 operators for N_f=1, B, L conserving
  - Mahonar et al (2013) general N_f
  - Lehman-Martin (2014,15) D=7 for general N_f, D=8 for N_f=1 (incomplete)

Repeating this at order ϵ^6 we obtain the Hilbert series for dimension-six operators of the SM EFT:

$\not{B}, \not{L}$

$$
\begin{aligned}
\widehat{H}_6 \; = \; & H^3 H^{\dagger\, 3} + u^\dagger Q^\dagger H H^{\dagger\, 2} + 2Q^2 Q^{\dagger\, 2} + \boxed{Q^{\dagger\, 3} L^\dagger + Q^3 L} + 2QQ^\dagger L L^\dagger + L^2 L^{\dagger\, 2} + uQH^2 H^\dagger \\
& + 2uu^\dagger QQ^\dagger + uu^\dagger LL^\dagger + u^2 u^{\dagger\, 2} + \boxed{e^\dagger u^\dagger Q^2} + e^\dagger L^\dagger H^2 H^\dagger + 2e^\dagger u^\dagger Q^\dagger L^\dagger + eLHH^{\dagger\, 2} + \boxed{euQ^{\dagger\, 2}} \\
& + 2euQL + ee^\dagger QQ^\dagger + ee^\dagger LL^\dagger + ee^\dagger uu^\dagger + e^2 e^{\dagger\, 2} + d^\dagger Q^\dagger H^2 H^\dagger + 2d^\dagger u^\dagger Q^{\dagger\, 2} + \boxed{d^\dagger u^\dagger QL} \\
& + \boxed{d^\dagger e^\dagger u^{\dagger\, 2}} + d^\dagger e Q^\dagger L + dQHH^{\dagger\, 2} + 2duQ^2 + \boxed{duQ^\dagger L^\dagger} + de^\dagger QL^\dagger + \boxed{deu^2} + 2dd^\dagger QQ^\dagger + dd^\dagger LL^\dagger \\
& + 2dd^\dagger uu^\dagger + dd^\dagger ee^\dagger + d^2 d^{\dagger\, 2} + u^\dagger Q^\dagger H^\dagger G_R + d^\dagger Q^\dagger H G_R + HH^\dagger G_R^2 + G_R^3 + uQHG_L \\
& + dQH^\dagger G_L + HH^\dagger G_L^2 + G_L^3 + u^\dagger Q^\dagger H^\dagger W_R + e^\dagger L^\dagger H W_R + d^\dagger Q^\dagger H W_R + HH^\dagger W_R^2 + W_R^3 \\
& + uQHW_L + eLH^\dagger W_L + dQH^\dagger W_L + HH^\dagger W_L^2 + W_L^3 + u^\dagger Q^\dagger H^\dagger B_R + e^\dagger L^\dagger H B_R \\
& + d^\dagger Q^\dagger H B_R + HH^\dagger B_R W_R + HH^\dagger B_R^2 + uQHB_L + eLH^\dagger B_L + dQH^\dagger B_L + HH^\dagger B_L W_L \\
& + HH^\dagger B_L^2 + 2QQ^\dagger HH^\dagger \mathcal{D} + 2LL^\dagger HH^\dagger \mathcal{D} + uu^\dagger HH^\dagger \mathcal{D} + ee^\dagger HH^\dagger \mathcal{D} + d^\dagger u H^2 \mathcal{D} + du^\dagger H^{\dagger\, 2} \mathcal{D} \\
& + dd^\dagger HH^\dagger \mathcal{D} + 2H^2 H^{\dagger\, 2} \mathcal{D}^2 \, .
\end{aligned} \tag{3.16}
$$

Setting all of the spurions equal to unity gives $\widehat{H}_6 = 84$, the total number of independent local operators at dimension 6, but more information is contained in eq.(3.16). For instance, the counting can easily be further decomposed by baryon number violation, $76 + 8$. The perhaps more familiar '$59 + 4$' counting is one in which hermitian conjugates of fermionic operators are not counted separately (such counting can of course also be obtained from eq. (3.16)).

Repeating this at order ϵ^6 we obtain the Hilbert series for dimension-six operators of the SM EFT:

$$
\begin{aligned}
\widehat{H}_6 \; = \; & H^3 H^{\dagger\,3} + u^\dagger Q^\dagger H H^{\dagger\,2} + 2Q^2 Q^{\dagger\,2} + \text{[redacted]} + 2QQ^\dagger LL^\dagger + L^2 L^{\dagger\,2} + \boxed{uQH^2H^\dagger} \\
& + 2uu^\dagger QQ^\dagger + uu^\dagger LL^\dagger + u^2 u^{\dagger\,2} + \text{[redacted]} + e^\dagger L^\dagger H^2 H^\dagger + 2e^\dagger u^\dagger Q^\dagger L^\dagger + \boxed{eLHH^{\dagger\,2}} + \text{[redacted]} \\
& + \boxed{2euQL} + ee^\dagger QQ^\dagger + ee^\dagger LL^\dagger + ee^\dagger uu^\dagger + e^2 e^{\dagger\,2} + d^\dagger Q^\dagger H^2 H^\dagger + 2d^\dagger u^\dagger Q^{\dagger\,2} + \text{[redacted]} \\
& + \text{[redacted]} + d^\dagger e Q^\dagger L + \boxed{dQHH^{\dagger\,2} + 2duQ^2} + \text{[redacted]} + \boxed{de^\dagger QL^\dagger} + \text{[redacted]} + 2dd^\dagger QQ^\dagger + dd^\dagger LL^\dagger \\
& + 2dd^\dagger uu^\dagger + dd^\dagger ee^\dagger + d^2 d^{\dagger\,2} + u^\dagger Q^\dagger H^\dagger G_R + d^\dagger Q^\dagger H G_R + HH^\dagger G_R^2 + G_R^3 + \boxed{uQHG_L} \\
& + \boxed{dQH^\dagger G_L} + HH^\dagger G_L^2 + G_L^3 + u^\dagger Q^\dagger H^\dagger W_R + e^\dagger L^\dagger H W_R + d^\dagger Q^\dagger H W_R + HH^\dagger W_R^2 + W_R^3 \\
& + \boxed{uQHW_L + eLH^\dagger W_L + dQH^\dagger W_L} + HH^\dagger W_L^2 + W_L^3 + u^\dagger Q^\dagger H^\dagger B_R + e^\dagger L^\dagger H B_R \\
& + d^\dagger Q^\dagger H B_R + HH^\dagger B_R W_R + HH^\dagger B_R^2 + \boxed{uQHB_L + eLH^\dagger B_L + dQH^\dagger B_L} + HH^\dagger B_L W_L \\
& + HH^\dagger B_L^2 + 2QQ^\dagger HH^\dagger \mathcal{D} + 2LL^\dagger HH^\dagger \mathcal{D} + uu^\dagger HH^\dagger \mathcal{D} + ee^\dagger HH^\dagger \mathcal{D} + d^\dagger u H^2 \mathcal{D} + \boxed{du^\dagger H^{\dagger\,2}\mathcal{D}} \\
& + dd^\dagger HH^\dagger \mathcal{D} + 2H^2 H^{\dagger\,2} \mathcal{D}^2 \,.
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Hermitian conjugates

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59 operators

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& + 2uu^\dagger QQ^\dagger + uu^\dagger LL^\dagger + u^2 u^{\dagger\,2} + \text{[redacted]} + e^\dagger L^\dagger H^2 H^\dagger + 2e^\dagger u^\dagger Q^\dagger L^\dagger + \text{[redacted]} + \text{[redacted]} \\
& + \text{[redacted]} + ee^\dagger QQ^\dagger + ee^\dagger LL^\dagger + ee^\dagger uu^\dagger + e^2 e^{\dagger\,2} + d^\dagger Q^\dagger H^2 H^\dagger + 2d^\dagger u^\dagger Q^{\dagger\,2} + \text{[redacted]} \\
& + \text{[redacted]} + d^\dagger e Q^\dagger L + \text{[redacted]} + \text{[redacted]} + \text{[redacted]} + \text{[redacted]} + 2dd^\dagger QQ^\dagger + dd^\dagger LL^\dagger \\
& + 2dd^\dagger uu^\dagger + dd^\dagger ee^\dagger + d^2 d^{\dagger\,2} + u^\dagger Q^\dagger H^\dagger G_R + d^\dagger Q^\dagger H G_R + HH^\dagger G_R^2 + G_R^3 + \text{[redacted]} \\
& + \text{[redacted]} + HH^\dagger G_L^2 + G_L^3 + u^\dagger Q^\dagger H^\dagger W_R + e^\dagger L^\dagger H W_R + d^\dagger Q^\dagger H W_R + HH^\dagger W_R^2 + W_R^3 \\
& + \text{[redacted]} + HH^\dagger W_L^2 + W_L^3 + u^\dagger Q^\dagger H^\dagger B_R + e^\dagger L^\dagger H B_R \\
& + d^\dagger Q^\dagger H B_R + HH^\dagger B_R W_R + HH^\dagger B_R^2 + \text{[redacted]} + HH^\dagger B_L W_L \\
& + HH^\dagger B_L^2 + 2QQ^\dagger HH^\dagger \mathcal{D} + 2LL^\dagger HH^\dagger \mathcal{D} + uu^\dagger HH^\dagger \mathcal{D} + ee^\dagger HH^\dagger \mathcal{D} + d^\dagger u H^2 \mathcal{D} + \text{[redacted]} \\
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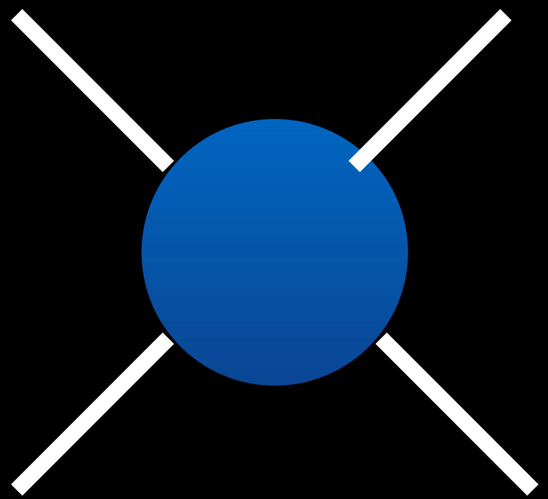
redundancies

- effective operators are invariants under the gauge group, Lorentz group, etc
- their classifications go back to Hilbert, Weyl
- applied to superpotentials, Standard Model
- but so far no general discussions on operators with derivatives
- two sources of redundancies
  - equation of motion (EOM)
  - integration by parts (IBP)

Simple Example



- scalars four-point at $O(\partial^2)$: 4(4+1)/2=10

  $(\partial_\mu \partial_\mu \varphi_i)\varphi_j \varphi_k \varphi_l$ $\quad$ $(\partial_\mu \varphi_i)(\partial_\mu \varphi_j)\varphi_k \varphi_l$
- $\partial^2 \varphi_i = m_i^2$ removes the first class: 4
- We know only 2 out of 6 are independent
  - $s, t, u, s+t+u=m_1^2+m_2^2+m_3^2+m_4^2$

$$(\partial_\mu \varphi_i)(\partial_\mu \varphi_j)\varphi_k \varphi_l - \varphi_i \varphi_j (\partial_\mu \varphi_k)(\partial_\mu \varphi_l) = \frac{1}{2}\partial^2(\varphi_i \varphi_j)(\varphi_k \varphi_l) - \frac{1}{2}(\varphi_i \varphi_j)\partial^2(\varphi_k \varphi_l) \approx 0$$

$$\partial_\mu \varphi_i \partial_\mu \varphi_j \varphi_k \varphi_l + \partial_\mu \varphi_i \varphi_j \partial_\mu \varphi_k \varphi_l + \partial_\mu \varphi_i \varphi_j \varphi_k \partial_\mu \varphi_l = \partial_\mu \varphi_i \partial_\mu (\varphi_j \varphi_k \varphi_l) \approx 0$$

- In addition, there are only *d* linearly independent momenta in *d*-dimensions

Main idea

- Take kinetic terms as the zeroth order Lagrangian $(\partial\phi)^2, \ \bar{\psi} i \partial\!\!\!/ \psi, \ (F_{\mu\nu})^2$
- Classically, it is conformally invariant under SO(4,2)≃SO(6,$\mathbb{C}$)
- Operator-State correspondence tells us that operators fall into representations of the conformal group
    - equation of motion: short multiplets
    - remove total derivatives: primary states

Master formula

- Define a multi-variate Hilbert series

$$H(p,\phi_1,\cdots,\phi_n) = \int d\mu_{\text{conformal}} d\mu_{\text{gauge}} \sum_{n=1}^{\infty} p^n \chi^*_{[n;0]} \prod_i PE[\phi_i \chi_i(q,\alpha,\beta)]$$

- PE are (anti-)symmetric products of characters for each field ϕ_i of dimension d_i
- integration over the gauge groups pick up gauge invariants
- integration over the conformal group picks only the primary states and Lorentz scalars
- expand it in power series in ϕ_i and p to find operators at given order in them

*There are corrections for operators d≤4 due to lack of orthonormality among characters for short multiplets

Hilbert series

- ring freely generated by φ:
  - 1, φ, φ^2, φ^3, φ^4, …

$$H(\varphi) = \frac{1}{1-\varphi}$$

- mod out by ideal, e.g. $\varphi^2=0$

$$H(\varphi) = \frac{1-\varphi^2}{1-\varphi} = 1+\varphi$$

- convenient way to encode all possible operators in a given theory
- basically a "generating function"

characters

- character $\chi(x_1, x_2, \ldots, x_r)=\mathrm{Tr}_R\, g$
- e.g., SU(2)

$$e^{i\theta T_3} = \mathrm{diag}(e^{ij\theta}, e^{i(j-1)\theta}, \cdots, e^{i(-j)\theta}) = (y^{2j}, y^{2j-2}, \cdots y^{-2j})$$

$$y = e^{i\theta/2}$$

$$\chi = y^{2j} + y^{2j-2} + \cdots + y^{-2j} = y^{2j}\frac{1-y^{-4j-2}}{1-y^{-2}} = \frac{y^{2j+1} - y^{-2j-1}}{y - y^{-1}}$$

- orthonormality on Haar measure

$$\delta_{R_i,R_j} = \int d\mu_{SU(2)}\chi^*_{R_i}\chi_{R_j} = \oint_{|y|=1}\frac{dy}{2\pi i}\frac{(1-y^2)(1-y^{-2})}{y}\chi^*_{R_i}\chi_{R_j}$$

conformal characters

- Primary field characterized by its spin $s=(j_1,j_2)$ and conformal weight Δ

$$\chi_{[\Delta,s]}(q,\alpha,\beta) = q^{\Delta} P(q;\alpha,\beta)\chi_s(\alpha,\beta)$$

$$P(q;\alpha,\beta) = \frac{1}{(1-q\alpha\beta)(1-q\alpha\beta^{-1})(1-q\alpha^{-1}\beta)(1-q\alpha^{-1}\beta^{-1})}$$

- $\Delta=1+j_1+j_2$ ($j_1 j_2=0$) saturates the unitarity bound, there are "short multiplets" for EoM

$$\chi_0(\alpha,\beta) = 1-q^2 \qquad \phi$$

$$\chi_{(\frac{1}{2},0)}(\alpha,\beta) = \alpha+\alpha^{-1}-q(\beta+\beta^{-1}) = \chi_{(0,\frac{1}{2})}(\beta,\alpha) \qquad \psi_\alpha$$

$$F_{\mu\nu}$$

$$\chi_{(1,0)}(\alpha,\beta) = \alpha^2+1+\alpha^{-2}-q(\alpha+\alpha^{-1})(\beta+\beta^{-1})+q^2 = \chi_{(0,1)}(\beta,\alpha)$$

Plethystic Exponential

- symmetric tensor product R^n of R

$$PE[u\chi_R](x_1, x_2, \cdots, x_r) \equiv \frac{1}{\det_R(1-ug)}$$

$$= \sum_n u^n \chi_{R^n} = \exp\left[-\mathrm{Tr}_R \log(1-ug)\right]$$

$$= \exp\left[\sum_{n=1}^{\infty} \frac{u^n}{n} \chi_R(x_1^n, \cdots, x_r^n)\right]$$

$$PE[u\chi_{1/2}] = \frac{1}{\det\begin{pmatrix} 1-uy & 0 \\ 0 & 1-uy^{-1} \end{pmatrix}}$$

$$= \frac{1}{(1-uy)(1-uy^{-1})} = 1 + u(y+y^{-1}) + u^2(y^2+1+y^{-2}) + u^3(y^3+y+y^{-1}+y^{-3}) + \cdots$$

Plethystic Exponential

- anti-symmetric tensor product *R^n* of *R*

$$PE[u\chi_R](x_1, x_2, \cdots, x_r) \equiv \det{}_R(1+ug)$$

$$= \sum_n u^n \chi_{R^n} = \exp\left[\mathrm{Tr}_R \log(1+ug)\right]$$

$$= \exp\left[-\sum_{n=1}^{\infty} \frac{(-u)^n}{n} \chi_R(x_1^n, \cdots, x_r^n)\right]$$

$$PE[u\chi_{1/2}] = \det\begin{pmatrix} 1+uy & 0 \\ 0 & 1+uy^{-1} \end{pmatrix}$$

$$= (1+uy)(1+uy^{-1}) = 1 + u(y+y^{-1}) + u^2$$

```
χH[t_, α_, β_, x_, y_, z1_, z2_] := χscal[t, α, β] * u1[3, x] * su2f[y];
χHd[t_, α_, β_, x_, y_, z1_, z2_] := χscal[t, α, β] * u1[-3, x] * su2fb[y];
χQ[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[1, x] * su2f[y] * su3f[z1, z2];
χQd[t_, α_, β_, x_, y_, z1_, z2_] :=
  χfermR[t, α, β] * u1[-1, x] * su2fb[y] * su3fb[z1, z2];
χu[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[-4, x] * su3fb[z1, z2];
χud[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[4, x] * su3f[z1, z2];
χd[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[2, x] * su3fb[z1, z2];
χdd[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[-2, x] * su3f[z1, z2];
χL[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[-3, x] * su2f[y];
χLd[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[3, x] * su2fb[y];
χe[t_, α_, β_, x_, y_, z1_, z2_] := χfermL[t, α, β] * u1[6, x];
χed[t_, α_, β_, x_, y_, z1_, z2_] := χfermR[t, α, β] * u1[-6, x];
χBl[t_, α_, β_, x_, y_, z1_, z2_] := χfsL[t, α, β];
χBr[t_, α_, β_, x_, y_, z1_, z2_] := χfsR[t, α, β];
χWl[t_, α_, β_, x_, y_, z1_, z2_] := χfsL[t, α, β] * su2ad[y];
χWr[t_, α_, β_, x_, y_, z1_, z2_] := χfsR[t, α, β] * su2ad[y];
χGl[t_, α_, β_, x_, y_, z1_, z2_] := χfsL[t, α, β] * su3ad[z1, z2];
χGr[t_, α_, β_, x_, y_, z1_, z2_] := χfsR[t, α, β] * su3ad[z1, z2];
```

Master formula

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Terminal — tcsh — ttys001

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Terminal — tcsh — ttys000

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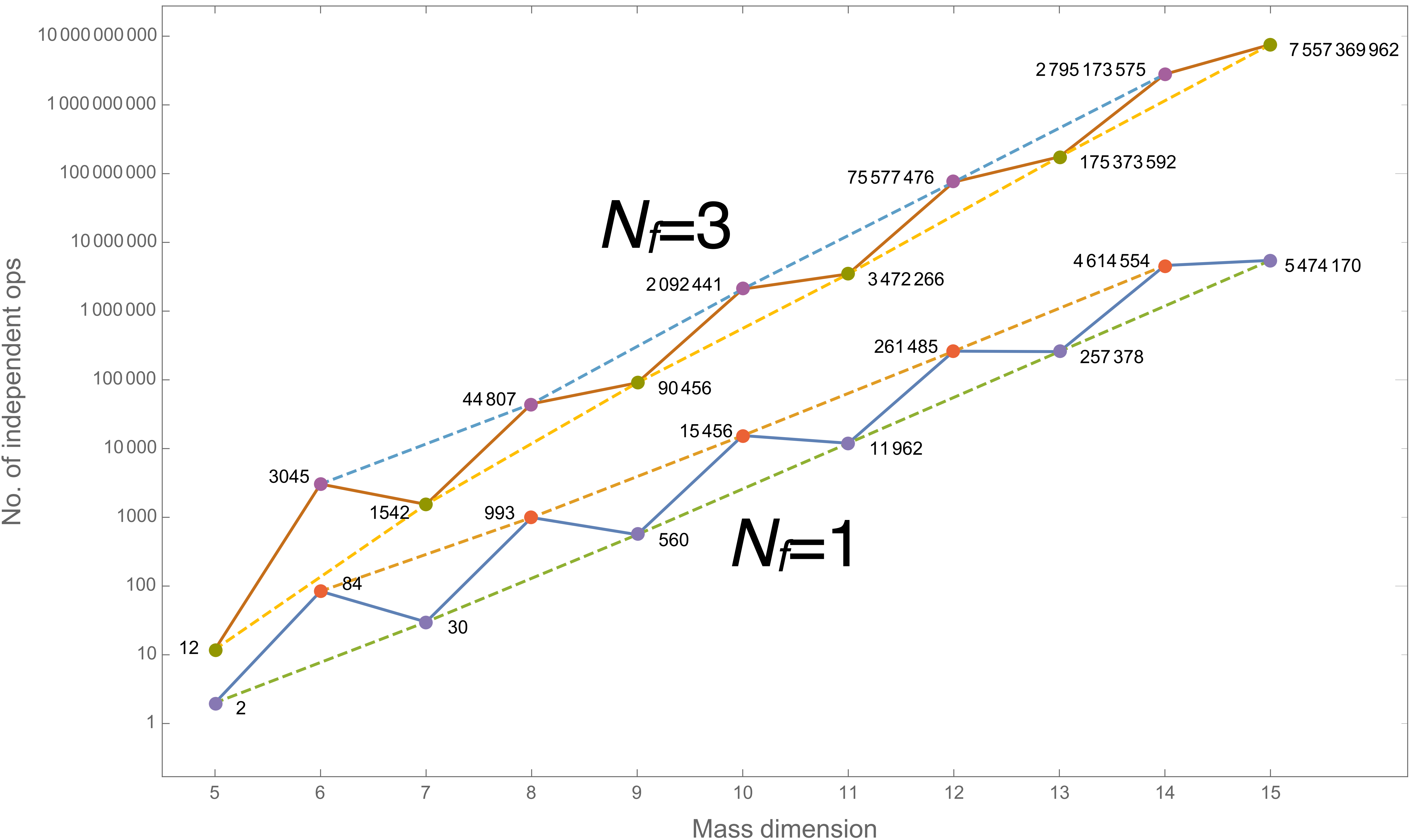
D=8 operators

```
f =
   2*L^2*Ld^2*t^2 + 2*ee*ed*L*Ld*t^2 + ee^2*ed^2*t^2 + 2*d*dd*L*Ld*t^2 + 2*
   d*dd*ee*ed*t^2 + 2*d^2*dd^2*t^2 + ud^2*dd*ed*t^2 + 2*u*ud*L*Ld*t^2 + 2*u
   *ud*ee*ed*t^2 + 4*u*ud*d*dd*t^2 + u^2*d*ee*t^2 + 2*u^2*ud^2*t^2 + 2*Qd*
   dd*ee*L*t^2 + 3*Qd*ud*ed*Ld*t^2 + 2*Qd*u*d*Ld*t^2 + 3*Qd^2*ud*dd*t^2 +
   Qd^2*u*ee*t^2 + Qd^3*Ld*t^2 + 2*Q*d*ed*Ld*t^2 + 2*Q*ud*dd*L*t^2 + 3*Q*u*
   ee*L*t^2 + 4*Q*Qd*L*Ld*t^2 + 2*Q*Qd*ee*ed*t^2 + 4*Q*Qd*d*dd*t^2 + 4*Q*Qd
   *u*ud*t^2 + Q^2*ud*ed*t^2 + 3*Q^2*u*d*t^2 + 4*Q^2*Qd^2*t^2 + Q^3*L*t^2
    + Wr*L^2*Ld^2 + Wr*ee*ed*L*Ld + Wr*d*dd*L*Ld + Wr*u*ud*L*Ld + Wr*Qd*dd*
   ee*L + 3*Wr*Qd*ud*ed*Ld + Wr*Qd*u*d*Ld + 3*Wr*Qd^2*ud*dd + Wr*Qd^2*u*ee
    + 2*Wr*Qd^3*Ld + Wr*Q*d*ed*Ld + Wr*Q*ud*dd*L + 3*Wr*Q*Qd*L*Ld + Wr*Q*Qd
   *ee*ed + 2*Wr*Q*Qd*d*dd + 2*Wr*Q*Qd*u*ud + 2*Wr*Q^2*Qd^2 + Wr^2*L*Ld*t
    + Wr^2*Q*Qd*t + 2*Wr^4 + Wl*L^2*Ld^2 + Wl*ee*ed*L*Ld + Wl*d*dd*L*Ld +
   Wl*u*ud*L*Ld + Wl*Qd*dd*ee*L + Wl*Qd*u*d*Ld + Wl*Q*d*ed*Ld + Wl*Q*ud*dd*
   L + 3*Wl*Q*u*ee*L + 3*Wl*Q*Qd*L*Ld + Wl*Q*Qd*ee*ed + 2*Wl*Q*Qd*d*dd + 2*
   Wl*Q*Qd*u*ud + Wl*Q^2*ud*ed + 3*Wl*Q^2*u*d + 2*Wl*Q^2*Qd^2 + 2*Wl*Q^3*L
    + 2*Wl*Wr*L*Ld*t + Wl*Wr*ee*ed*t + Wl*Wr*d*dd*t + Wl*Wr*u*ud*t + 2*Wl*
   Wr*Q*Qd*t + Wl^2*L*Ld*t + Wl^2*Q*Qd*t + 2*Wl^2*Wr^2 + 2*Wl^4 + Gr*d*dd*L
   *Ld + Gr*d*dd*ee*ed + Gr*d^2*dd^2 + 3*Gr*ud^2*dd*ed + Gr*u*ud*L*Ld + Gr*
   u*ud*ee*ed + 4*Gr*u*ud*d*dd + Gr*u^2*ud^2 + Gr*Qd*dd*ee*L + 3*Gr*Qd*ud*
   ed*Ld + 2*Gr*Qd*u*d*Ld + 6*Gr*Qd^2*ud*dd + Gr*Qd^2*u*ee + 2*Gr*Qd^3*Ld
    + Gr*Q*d*ed*Ld + 2*Gr*Q*ud*dd*L + 2*Gr*Q*Qd*L*Ld + Gr*Q*Qd*ee*ed + 4*Gr
   *Q*Qd*d*dd + 4*Gr*Q*Qd*u*ud + Gr*Q^2*ud*ed + 2*Gr*Q^2*Qd^2 + Gr*Wr*Q*Qd*
   t + Gr*Wl*Q*Qd*t + Gr^2*d*dd*t + Gr^2*u*ud*t + Gr^2*Q*Qd*t + 2*Gr^2*Wr^2
    + Gr^2*Wl^2 + 3*Gr^4 + Gl*d*dd*L*Ld + Gl*d*dd*ee*ed + Gl*d^2*dd^2 + Gl*
   u*ud*L*Ld + Gl*u*ud*ee*ed + 4*Gl*u*ud*d*dd + 3*Gl*u^2*d*ee + Gl*u^2*ud^2
    + Gl*Qd*dd*ee*L + 2*Gl*Qd*u*d*Ld + Gl*Qd^2*u*ee + Gl*Q*d*ed*Ld + 2*Gl*Q
   *ud*dd*L + 3*Gl*Q*u*ee*L + 2*Gl*Q*Qd*L*Ld + Gl*Q*Qd*ee*ed + 4*Gl*Q*Qd*d*
   dd + 4*Gl*Q*Qd*u*ud + Gl*Q^2*ud*ed + 6*Gl*Q^2*u*d + 2*Gl*Q^2*Qd^2 + 2*Gl
   *Q^3*L + Gl*Wr*Q*Qd*t + Gl*Wl*Q*Qd*t + Gl*Gr*L*Ld*t + Gl*Gr*ee*ed*t + 3*
   Gl*Gr*d*dd*t + 3*Gl*Gr*u*ud*t + 3*Gl*Gr*Q*Qd*t + Gl*Gr*Wl*Wr + Gl^2*d*dd
   *t + Gl^2*u*ud*t + Gl^2*Q*Qd*t + Gl^2*Wr^2 + 2*Gl^2*Wl^2 + 3*Gl^2*Gr^2
    + 3*Gl^4 + Br*ee*ed*L*Ld + Br*d*dd*L*Ld + Br*d*dd*ee*ed + 2*Br*ud^2*dd*
   ed + Br*u*ud*L*Ld + Br*u*ud*ee*ed + 2*Br*u*ud*d*dd + Br*Qd*dd*ee*L + 3*
   Br*Qd*ud*ed*Ld + Br*Qd*u*d*Ld + 3*Br*Qd^2*ud*dd + Br*Qd^3*Ld + Br*Q*d*ed
   *Ld + Br*Q*ud*dd*L + 2*Br*Q*Qd*L*Ld + Br*Q*Qd*ee*ed + 2*Br*Q*Qd*d*dd + 2
   *Br*Q*Qd*u*ud + Br*Q^2*ud*ed + Br*Wr*L*Ld*t + Br*Wr*Q*Qd*t + Br*Wl*L*Ld*
   t + Br*Wl*Q*Qd*t + Br*Gr*d*dd*t + Br*Gr*u*ud*t + Br*Gr*Q*Qd*t + Br*Gr^3
    + Br*Gl*d*dd*t + Br*Gl*u*ud*t + Br*Gl*Q*Qd*t + Br*Gl^2*Gr + 2*Br^2*Wr^2
    + Br^2*Wl^2 + 2*Br^2*Gr^2 + Br^2*Gl^2 + Br^4 + Bl*ee*ed*L*Ld + Bl*d*dd*
   L*Ld + Bl*d*dd*ee*ed + Bl*u*ud*L*Ld + Bl*u*ud*ee*ed + 2*Bl*u*ud*d*dd + 2
   *Bl*u^2*d*ee + Bl*Qd*dd*ee*L + Bl*Qd*u*d*Ld + Bl*Qd^2*u*ee + Bl*Q*d*ed*
   Ld + Bl*Q*ud*dd*L + 3*Bl*Q*u*ee*L + 2*Bl*Q*Qd*L*Ld + Bl*Q*Qd*ee*ed + 2*
   Bl*Q*Qd*d*dd + 2*Bl*Q*Qd*u*ud + 3*Bl*Q^2*u*d + Bl*Q^3*L + Bl*Wr*L*Ld*t
    + Bl*Wr*Q*Qd*t + Bl*Wl*L*Ld*t + Bl*Wl*Q*Qd*t + Bl*Gr*d*dd*t + Bl*Gr*u*
   ud*t + Bl*Gr*Q*Qd*t + Bl*Gl*d*dd*t + Bl*Gl*u*ud*t + Bl*Gl*Q*Qd*t + Bl*Gl
   *Gr^2 + Bl*Gl^3 + Bl*Br*L*Ld*t + Bl*Br*ee*ed*t + Bl*Br*d*dd*t + Bl*Br*u*
   ud*t + Bl*Br*Q*Qd*t + Bl*Br*Wl*Wr + Bl*Br*Gl*Gr + Bl^2*Wr^2 + 2*Bl^2*
   Wl^2 + Bl^2*Gr^2 + 2*Bl^2*Gl^2 + Bl^2*Br^2 + Bl^4 + 3*Hd*ee*L^2*Ld*t +
   Hd*ee^2*ed*L*t + 3*Hd*d*dd*ee*L*t + 3*Hd*ud*d*ed*Ld*t + 2*Hd*ud^2*dd*L*t
    + 2*Hd*u*d^2*Ld*t + 3*Hd*u*ud*ee*L*t + 6*Hd*Qd*ud*L*Ld*t + 3*Hd*Qd*ud*
   ee*ed*t + 6*Hd*Qd*ud*d*dd*t + 3*Hd*Qd*u*d*ee*t + 3*Hd*Qd*u*ud^2*t + 3*Hd
   *Qd^2*d*Ld*t + Hd*Qd^3*ee*t + 6*Hd*Q*d*L*Ld*t + 3*Hd*Q*d*ee*ed*t + 3*Hd*
   Q*d^2*dd*t + 2*Hd*Q*ud^2*ed*t + 6*Hd*Q*u*ud*d*t + 6*Hd*Q*Qd*ee*L*t + 6*
   Hd*Q*Qd^2*ud*t + 3*Hd*Q^2*ud*L*t + 6*Hd*Q^2*Qd*d*t + Hd*Wr*ee*L*t^2 + 2*
   Hd*Wr*Qd*ud*t^2 + Hd*Wr*Q*d*t^2 + Hd*Wr^2*ee*L + 2*Hd*Wr^2*Qd*ud + Hd*
   Wr^2*Q*d + 2*Hd*Wl*ee*L*t^2 + Hd*Wl*Qd*ud*t^2 + 2*Hd*Wl*Q*d*t^2 + 2*Hd*
   Wl^2*ee*L + Hd*Wl^2*Qd*ud + 2*Hd*Wl^2*Q*d + 2*Hd*Gr*Qd*ud*t^2 + Hd*Gr*Q*
   d*t^2 + 2*Hd*Gr*Wr*Qd*ud + Hd*Gr*Wr*Q*d + Hd*Gr^2*ee*L + 3*Hd*Gr^2*Qd*ud
    + 2*Hd*Gr^2*Q*d + Hd*Gl*Qd*ud*t^2 + 2*Hd*Gl*Q*d*t^2 + Hd*Gl*Wl*Qd*ud +
   2*Hd*Gl*Wl*Q*d + Hd*Gl^2*ee*L + 2*Hd*Gl^2*Qd*ud + 3*Hd*Gl^2*Q*d + Hd*Br*
   ee*L*t^2 + 2*Hd*Br*Qd*ud*t^2 + Hd*Br*Q*d*t^2 + Hd*Br*Wr*ee*L + 2*Hd*Br*
   Wr*Qd*ud + Hd*Br*Wr*Q*d + 2*Hd*Br*Gr*Qd*ud + Hd*Br*Gr*Q*d + Hd*Br^2*ee*L
    + Hd*Br^2*Qd*ud + Hd*Br^2*Q*d + 2*Hd*Bl*ee*L*t^2 + Hd*Bl*Qd*ud*t^2 + 2*
   Hd*Bl*Q*d*t^2 + 2*Hd*Bl*Wl*ee*L + Hd*Bl*Wl*Qd*ud + 2*Hd*Bl*Wl*Q*d + Hd*
   Bl*Gl*Qd*ud + 2*Hd*Bl*Gl*Q*d + Hd*Bl^2*ee*L + Hd*Bl^2*Qd*ud + Hd*Bl^2*Q*
   d + Hd^2*ee^2*L^2 + Hd^2*ud*d*t^3 + Hd^2*ud*d*L*Ld + Hd^2*Qd*ud*ee*L + 2
   *Hd^2*Qd^2*ud^2 + 2*Hd^2*Q*d*ee*L + 2*Hd^2*Q*Qd*ud*d + 2*Hd^2*Q^2*d^2 +
   Hd^2*Wr*ud*d*t + Hd^2*Wl*ud*d*t + Hd^2*Gr*ud*d*t + Hd^2*Gl*ud*d*t + Hd^2
   *Br*ud*d*t + Hd^2*Bl*ud*d*t + 3*H*ed*L*Ld^2*t + H*ee*ed^2*Ld*t + 3*H*d*
   dd*ed*Ld*t + 2*H*ud*dd^2*L*t + 3*H*u*dd*ee*L*t + 3*H*u*ud*ed*Ld*t + 2*H*
   u^2*d*Ld*t + 6*H*Qd*dd*L*Ld*t + 3*H*Qd*dd*ee*ed*t + 3*H*Qd*d*dd^2*t + 6*
   H*Qd*u*ud*dd*t + 2*H*Qd*u^2*ee*t + 3*H*Qd^2*u*Ld*t + 3*H*Q*ud*dd*ed*t +
   6*H*Q*u*L*Ld*t + 3*H*Q*u*ee*ed*t + 6*H*Q*u*d*dd*t + 3*H*Q*u^2*ud*t + 6*H
   *Q*Qd*ed*Ld*t + 6*H*Q*Qd^2*dd*t + 3*H*Q^2*dd*L*t + 6*H*Q^2*Qd*u*t + H*
   Q^3*ed*t + 2*H*Wr*ed*Ld*t^2 + 2*H*Wr*Qd*dd*t^2 + H*Wr*Q*u*t^2 + 2*H*Wr^2
   *ed*Ld + 2*H*Wr^2*Qd*dd + H*Wr^2*Q*u + H*Wl*ed*Ld*t^2 + H*Wl*Qd*dd*t^2
    + 2*H*Wl*Q*u*t^2 + H*Wl^2*ed*Ld + H*Wl^2*Qd*dd + 2*H*Wl^2*Q*u + 2*H*Gr*
   Qd*dd*t^2 + H*Gr*Q*u*t^2 + 2*H*Gr*Wr*Qd*dd + H*Gr*Wr*Q*u + H*Gr^2*ed*Ld
    + 3*H*Gr^2*Qd*dd + 2*H*Gr^2*Q*u + H*Gl*Qd*dd*t^2 + 2*H*Gl*Q*u*t^2 + H*
   Gl*Wl*Qd*dd + 2*H*Gl*Wl*Q*u + H*Gl^2*ed*Ld + 2*H*Gl^2*Qd*dd + 3*H*Gl^2*Q
   *u + 2*H*Br*ed*Ld*t^2 + 2*H*Br*Qd*dd*t^2 + H*Br*Q*u*t^2 + 2*H*Br*Wr*ed*
   Ld + 2*H*Br*Wr*Qd*dd + H*Br*Wr*Q*u + 2*H*Br*Gr*Qd*dd + H*Br*Gr*Q*u + H*
   Br^2*ed*Ld + H*Br^2*Qd*dd + H*Br^2*Q*u + H*Bl*ed*Ld*t^2 + H*Bl*Qd*dd*t^2
    + 2*H*Bl*Q*u*t^2 + H*Bl*Wl*ed*Ld + H*Bl*Wl*Qd*dd + 2*H*Bl*Wl*Q*u + H*Bl
   *Gl*Qd*dd + 2*H*Bl*Gl*Q*u + H*Bl^2*ed*Ld + H*Bl^2*Qd*dd + H*Bl^2*Q*u + 4
   *H*Hd*L*Ld*t^3 + 2*H*Hd*L^2*Ld^2 + 2*H*Hd*ee*ed*t^3 + 2*H*Hd*ee*ed*L*Ld
    + H*Hd*ee^2*ed^2 + 2*H*Hd*d*dd*t^3 + 2*H*Hd*d*dd*L*Ld + H*Hd*d*dd*ee*ed
    + H*Hd*d^2*dd^2 + H*Hd*ud^2*dd*ed + 2*H*Hd*u*ud*t^3 + 2*H*Hd*u*ud*L*Ld
    + H*Hd*u*ud*ee*ed + 2*H*Hd*u*ud*d*dd + H*Hd*u^2*d*ee + H*Hd*u^2*ud^2 +
   2*H*Hd*Qd*dd*ee*L + 4*H*Hd*Qd*ud*ed*Ld + 2*H*Hd*Qd*u*d*Ld + 4*H*Hd*Qd^2*
   ud*dd + H*Hd*Qd^2*u*ee + 2*H*Hd*Qd^3*Ld + 2*H*Hd*Q*d*ed*Ld + 2*H*Hd*Q*ud
   *dd*L + 4*H*Hd*Q*u*ee*L + 4*H*Hd*Q*Qd*t^3 + 5*H*Hd*Q*Qd*L*Ld + 2*H*Hd*Q*
   Qd*ee*ed + 4*H*Hd*Q*Qd*d*dd + 4*H*Hd*Q*Qd*u*ud + H*Hd*Q^2*ud*ed + 4*H*Hd
   *Q^2*u*d + 3*H*Hd*Q^2*Qd^2 + 2*H*Hd*Q^3*L + 6*H*Hd*Wr*L*Ld*t + 2*H*Hd*Wr
   *ee*ed*t + 2*H*Hd*Wr*d*dd*t + 2*H*Hd*Wr*u*ud*t + 6*H*Hd*Wr*Q*Qd*t + 2*H*
   Hd*Wr^2*t^2 + H*Hd*Wr^3 + 6*H*Hd*Wl*L*Ld*t + 2*H*Hd*Wl*ee*ed*t + 2*H*Hd*
   Wl*d*dd*t + 2*H*Hd*Wl*u*ud*t + 6*H*Hd*Wl*Q*Qd*t + 2*H*Hd*Wl*Wr*t^2 + 2*H
   *Hd*Wl^2*t^2 + H*Hd*Wl^3 + 2*H*Hd*Gr*d*dd*t + 2*H*Hd*Gr*u*ud*t + 4*H*Hd*
   Gr*Q*Qd*t + H*Hd*Gr^2*t^2 + H*Hd*Gr^3 + 2*H*Hd*Gl*d*dd*t + 2*H*Hd*Gl*u*
   ud*t + 4*H*Hd*Gl*Q*Qd*t + H*Hd*Gl*Gr*t^2 + H*Hd*Gl^2*t^2 + H*Hd*Gl^3 + 4
   *H*Hd*Br*L*Ld*t + 2*H*Hd*Br*ee*ed*t + 2*H*Hd*Br*d*dd*t + 2*H*Hd*Br*u*ud*
   t + 4*H*Hd*Br*Q*Qd*t + 2*H*Hd*Br*Wr*t^2 + H*Hd*Br*Wr^2 + H*Hd*Br*Wl*t^2
    + H*Hd*Br^2*t^2 + 4*H*Hd*Bl*L*Ld*t + 2*H*Hd*Bl*ee*ed*t + 2*H*Hd*Bl*d*dd
   *t + 2*H*Hd*Bl*u*ud*t + 4*H*Hd*Bl*Q*Qd*t + H*Hd*Bl*Wr*t^2 + 2*H*Hd*Bl*Wl
   *t^2 + H*Hd*Bl*Wl^2 + H*Hd*Bl*Br*t^2 + H*Hd*Bl^2*t^2 + 6*H*Hd^2*ee*L*t^2
    + 6*H*Hd^2*Qd*ud*t^2 + 6*H*Hd^2*Q*d*t^2 + 2*H*Hd^2*Wr*Qd*ud + 2*H*Hd^2*
   Wl*ee*L + 2*H*Hd^2*Wl*Q*d + H*Hd^2*Gr*Qd*ud + H*Hd^2*Gl*Q*d + H*Hd^2*Br*
   Qd*ud + H*Hd^2*Bl*ee*L + H*Hd^2*Bl*Q*d + H*Hd^3*ud*d*t + H^2*ed^2*Ld^2
    + H^2*u*dd*t^3 + H^2*u*dd*L*Ld + 2*H^2*Qd*dd*ed*Ld + 2*H^2*Qd^2*dd^2 +
   H^2*Q*u*ed*Ld + 2*H^2*Q*Qd*u*dd + 2*H^2*Q^2*u^2 + H^2*Wr*u*dd*t + H^2*Wl
   *u*dd*t + H^2*Gr*u*dd*t + H^2*Gl*u*dd*t + H^2*Br*u*dd*t + H^2*Bl*u*dd*t
    + 6*H^2*Hd*ed*Ld*t^2 + 6*H^2*Hd*Qd*dd*t^2 + 6*H^2*Hd*Q*u*t^2 + 2*H^2*Hd
   *Wr*ed*Ld + 2*H^2*Hd*Wr*Qd*dd + 2*H^2*Hd*Wl*Q*u + H^2*Hd*Gr*Qd*dd + H^2*
   Hd*Gl*Q*u + H^2*Hd*Br*ed*Ld + H^2*Hd*Br*Qd*dd + H^2*Hd*Bl*Q*u + 3*H^2*
   Hd^2*t^4 + 4*H^2*Hd^2*L*Ld*t + H^2*Hd^2*ee*ed*t + H^2*Hd^2*d*dd*t + H^2*
   Hd^2*u*ud*t + 4*H^2*Hd^2*Q*Qd*t + 2*H^2*Hd^2*Wr*t^2 + 2*H^2*Hd^2*Wr^2 +
   2*H^2*Hd^2*Wl*t^2 + 2*H^2*Hd^2*Wl^2 + H^2*Hd^2*Gr^2 + H^2*Hd^2*Gl^2 +
   H^2*Hd^2*Br*t^2 + H^2*Hd^2*Br*Wr + H^2*Hd^2*Br^2 + H^2*Hd^2*Bl*t^2 + H^2
   *Hd^2*Bl*Wl + H^2*Hd^2*Bl^2 + H^2*Hd^3*ee*L + H^2*Hd^3*Qd*ud + H^2*Hd^3*
   Q*d + H^3*Hd*u*dd*t + H^3*Hd^2*ed*Ld + H^3*Hd^2*Qd*dd + H^3*Hd^2*Q*u + 2
   *H^3*Hd^3*t^2 + H^4*Hd^4;
```

993 of them



No. of independent ops
Mass dimension
N_f=3
N_f=1
10 000 000 000
1 000 000 000
100 000 000
10 000 000
1 000 000
100 000
10 000
1000
100
10
1
5
6
7
8
9
10
11
12
13
14
15
12
2
3045
84
1542
30
44 807
993
90 456
560
2 092 441
15 456
3 472 266
11 962
75 577 476
261 485
175 373 592
257 378
2 795 173 575
4 614 554
7 557 369 962
5 474 170

Conclusions

- Nailed the question of classifying effective operators in a given Lorentz-inv theory
- Connections to amplitudes?
- perturbation around non-free theories?
- EFT important in many other contexts
  - condensed matter physics
  - nuclear physics
  - cosmological density fluctuations

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UTIAS 東京大学国際高等研究所 THE UNIVERSITY OF TOKYO INSTITUTES FOR ADVANCED STUDY

KAVLI IPMU INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

Dark Pions as Dark Matter

with Yonit Hochberg and Eric Kuflik

CERN Theory Institute, Aug 4, 2016

arXiv:1411.3727 w/ Tomer Volansky Jay Wacker,
arXiv:1512.07917, 1609.xxxxx

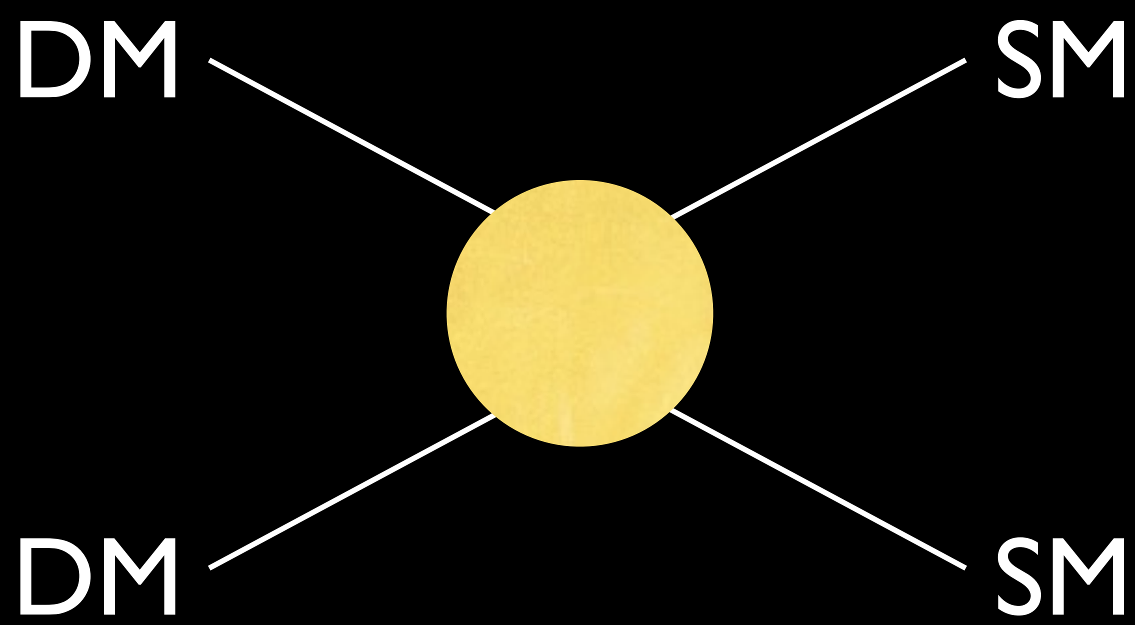
BERKELEY CENTER FOR THEORETICAL PHYSICS

BERKELEY LAB

THE UNIVERSITY OF CALIFORNIA 1868 LET THERE BE LIGHT

$$\frac{n_{\rm DM}}{s} = 4.4 \times 10^{-10} \frac{\rm GeV}{m_{\rm DM}}$$

Miracles

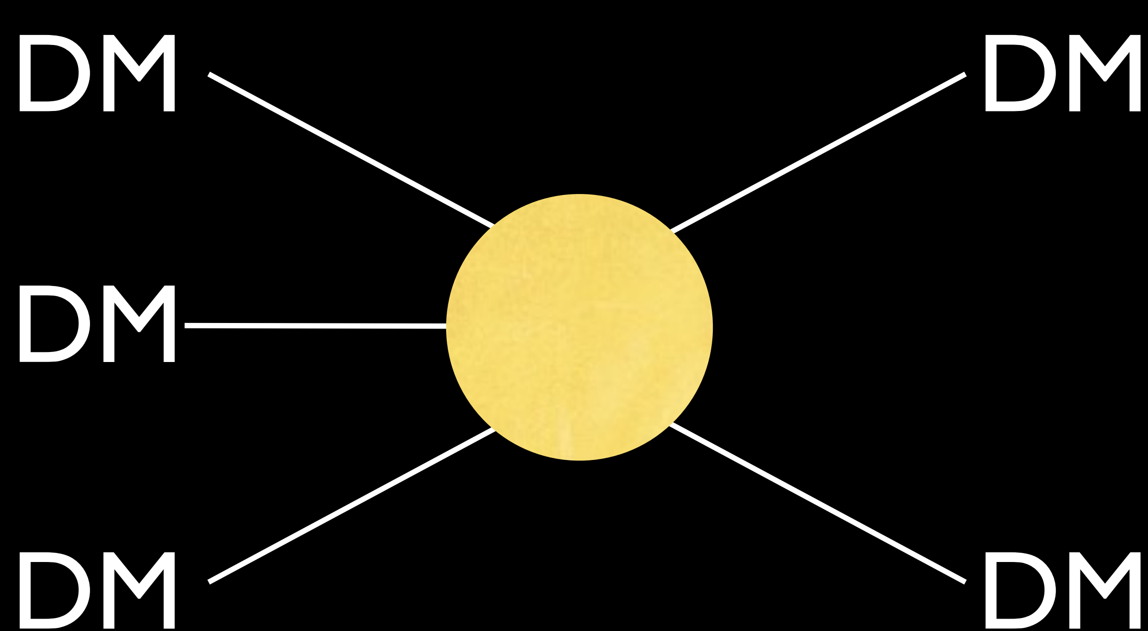


$$\langle \sigma_{2\to 2} v \rangle \approx \frac{\alpha^2}{m^2}$$

$$\alpha \approx 10^{-2}$$

$$m \approx 300 \text{ GeV}$$

WIMP miracle!



$$\langle \sigma_{3\to 2} v^2 \rangle \approx \frac{\alpha^3}{m^5}$$

$$\alpha \approx 4\pi$$

$$m \approx 300 \text{MeV}$$

SIMP miracle!

Hochberg, Kuflik, Volansky, Wacker arXiv:1402.5143

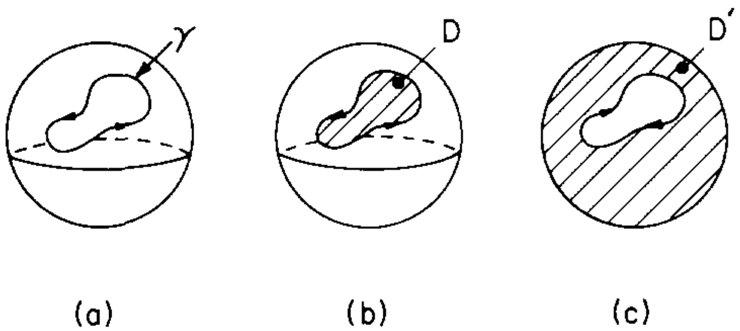
LAGRANGIANS

Quark theory

$$\mathcal{L}_{\text{quark}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \bar{q}_i i \not{D} q_i \; -\frac{1}{2} m_Q \; J^{ij} q_i q_j + h.c.$$

Sigma theory

$$\mathcal{L}_{\text{Sigma}} = \frac{f_\pi^2}{16} \text{Tr} \partial_\mu \Sigma \; \partial^\mu \Sigma^\dagger - \frac{1}{2} m_Q \mu^3 \text{Tr} J \Sigma + h.c. - \frac{i N_c}{240 \pi^2} \int \text{Tr}(\Sigma^\dagger d\Sigma)^5$$



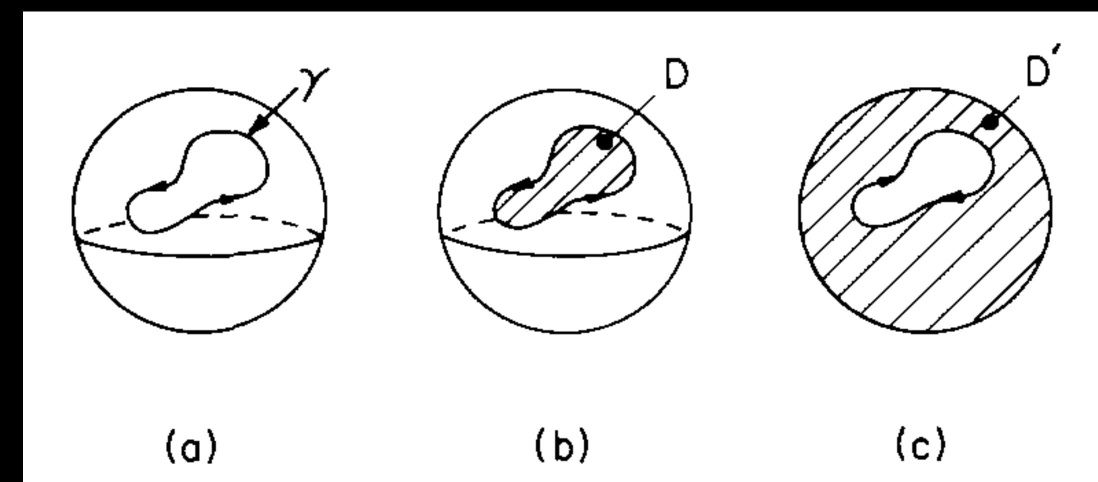
Pion theory

$$\mathcal{L}_{\text{pion}} = \frac{1}{4} \text{Tr} \partial_\mu \pi \partial^\mu \pi - \frac{m_\pi^2}{4} \text{Tr} \pi^2 + \frac{m_\pi^2}{12 f_\pi^2} \text{Tr} \pi^4 - \frac{1}{6 f_\pi^2} \text{Tr} \left(\pi^2 \partial^\mu \pi \partial_\mu \pi - \pi \partial^\mu \pi \pi \partial_\mu \pi \right)$$

$$+ \frac{2 N_c}{15 \pi^2 f_\pi^5} \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left[\pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi \right] + \mathcal{O}(\pi^6)$$

Wess-Zumino term

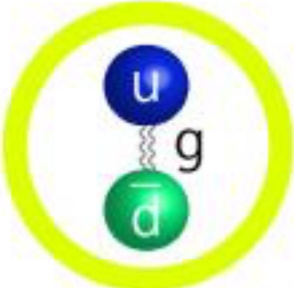
- SU(N_c) gauge theory
  - $\pi_5(SU(N_f))=\mathbb{Z}$ ($N_f \geq 3$)
- Sp(N_c) gauge theory
  - $\pi_5(SU(2N_f)/Sp(N_f))=\mathbb{Z}$ ($N_f \geq 2$)
- SO(N_c) gauge theory
  - $\pi_5(SU(N_f)/SO(N_f))=\mathbb{Z}$ ($N_f \geq 3$)



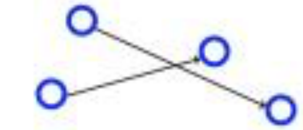
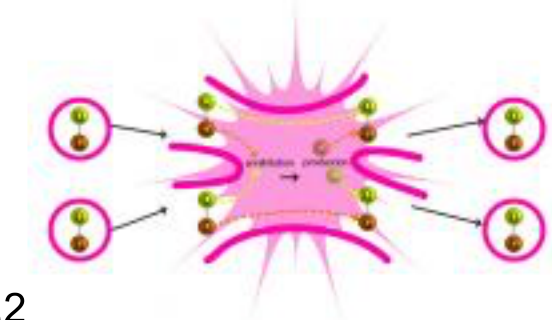
Witten

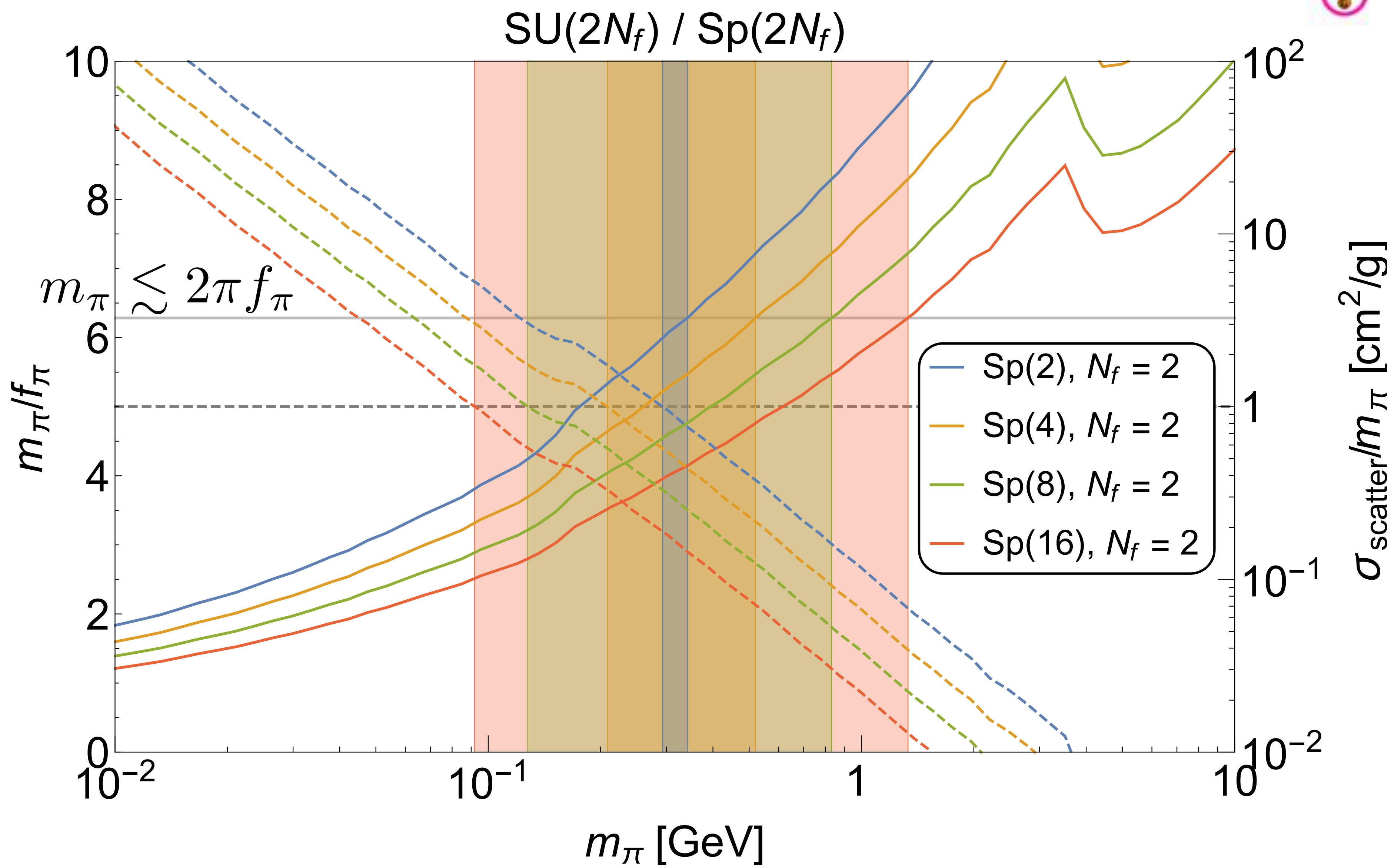
SIMPlest Miracle

- SU(2) gauge theory with four doublets
- SU(4)=SO(6) flavor symmetry
- $\langle q^i q^j \rangle \neq 0$ breaks it to Sp(2)=SO(5)
- coset space SO(6)/SO(5)=S^5
- $\pi_5(S^5)=\mathbb{Z} \Rightarrow$ Wess-Zumino term
- $\mathcal{L}_{WZ}=\epsilon_{abcde}\,\epsilon^{\mu\nu\rho\sigma}\,\pi^a\partial_\mu\pi^b\partial_\nu\pi^c\partial_\rho\pi^d\partial_\sigma\pi^e$



The Results

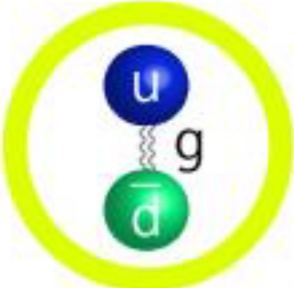


Solid curves: solution to Boltzmann eq.

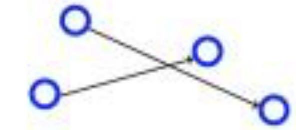
Dashed curves: along that solution

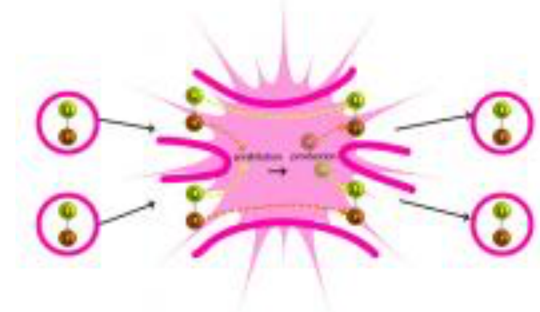
$$\frac{m_\pi}{f_\pi} \propto m_\pi^{3/10}$$

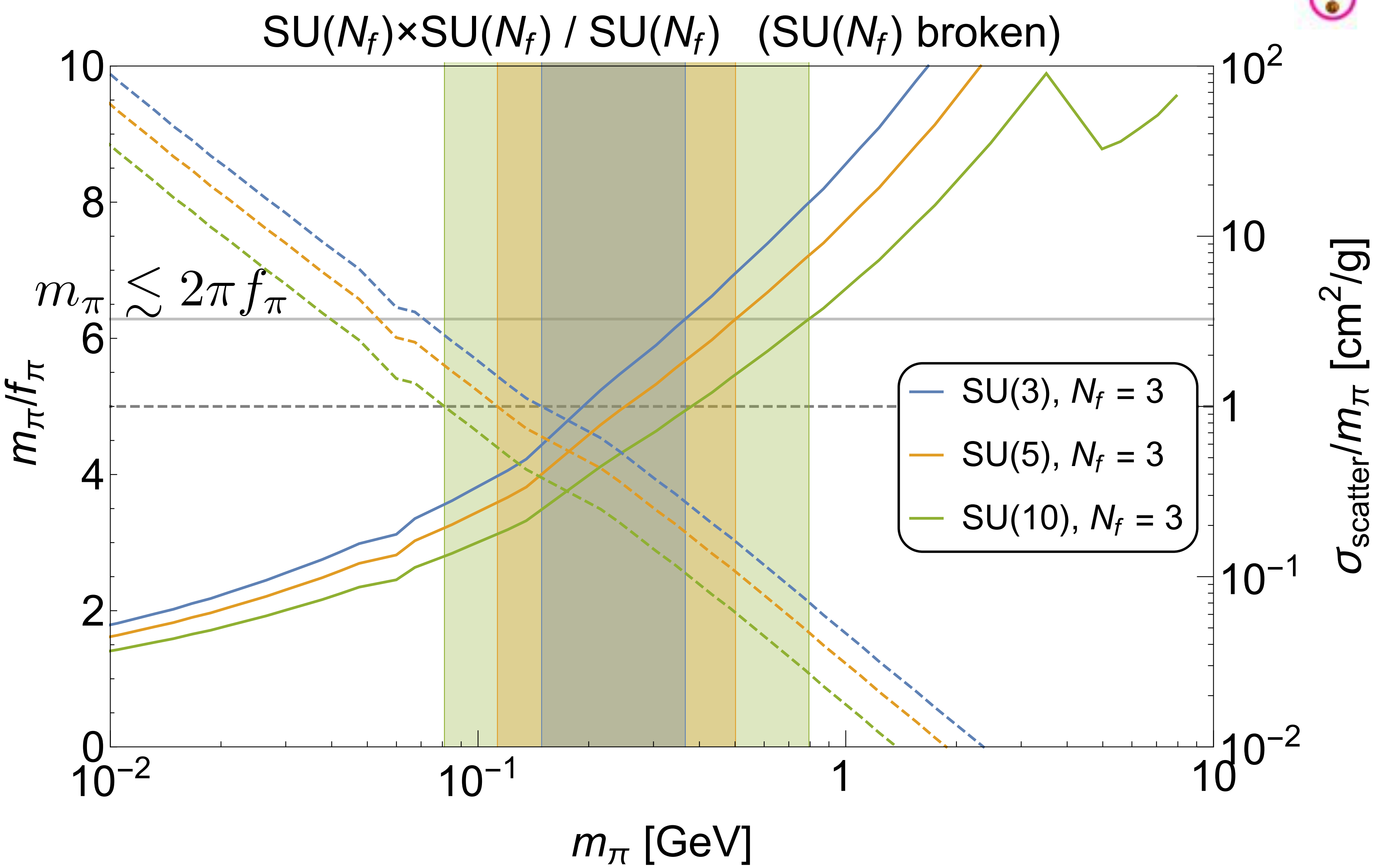
$$\frac{\sigma_{\mathrm{scatter}}}{m_\pi} \propto m_\pi^{-9/5}$$



The Results





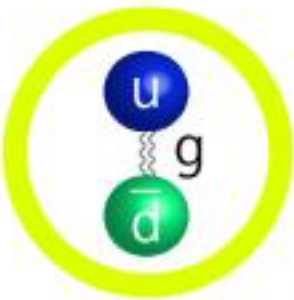


Solid curves: solution to Boltzmann eq.

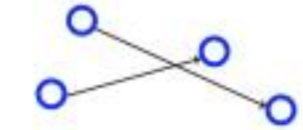
Dashed curves: along that solution

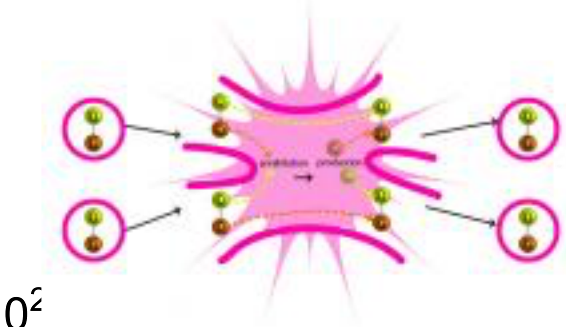
$$\frac{m_\pi}{f_\pi} \propto m_\pi^{3/10}$$

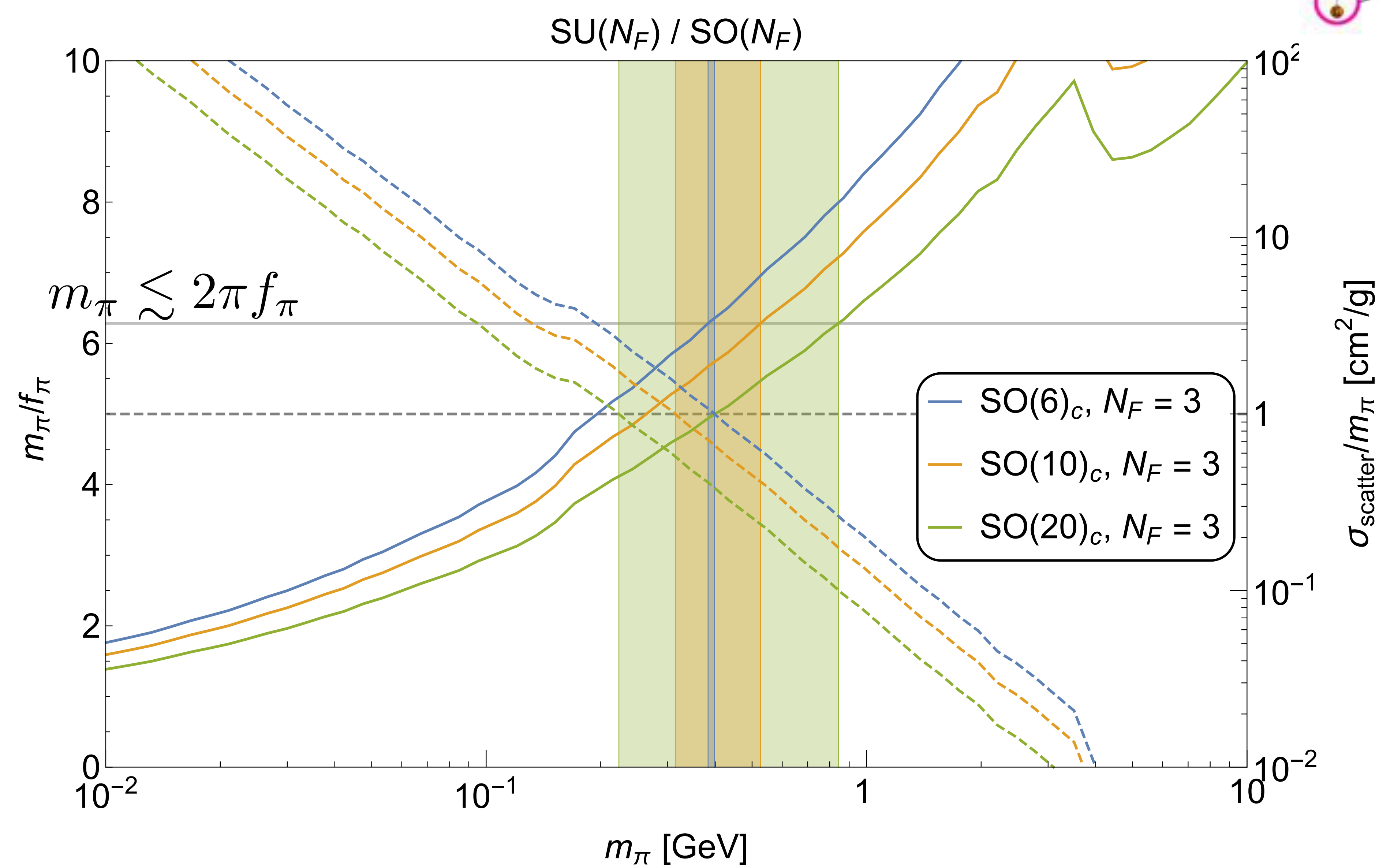
$$\frac{\sigma_{\rm scatter}}{m_\pi} \propto m_\pi^{-9/5}$$



The Results







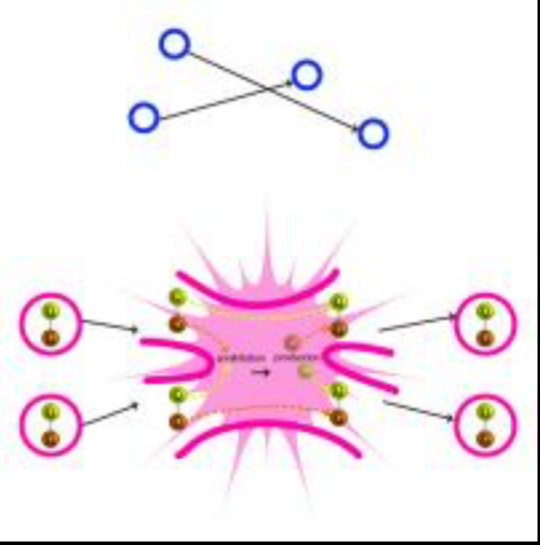
Solid curves: solution to Boltzmann eq.

Dashed curves: along that solution

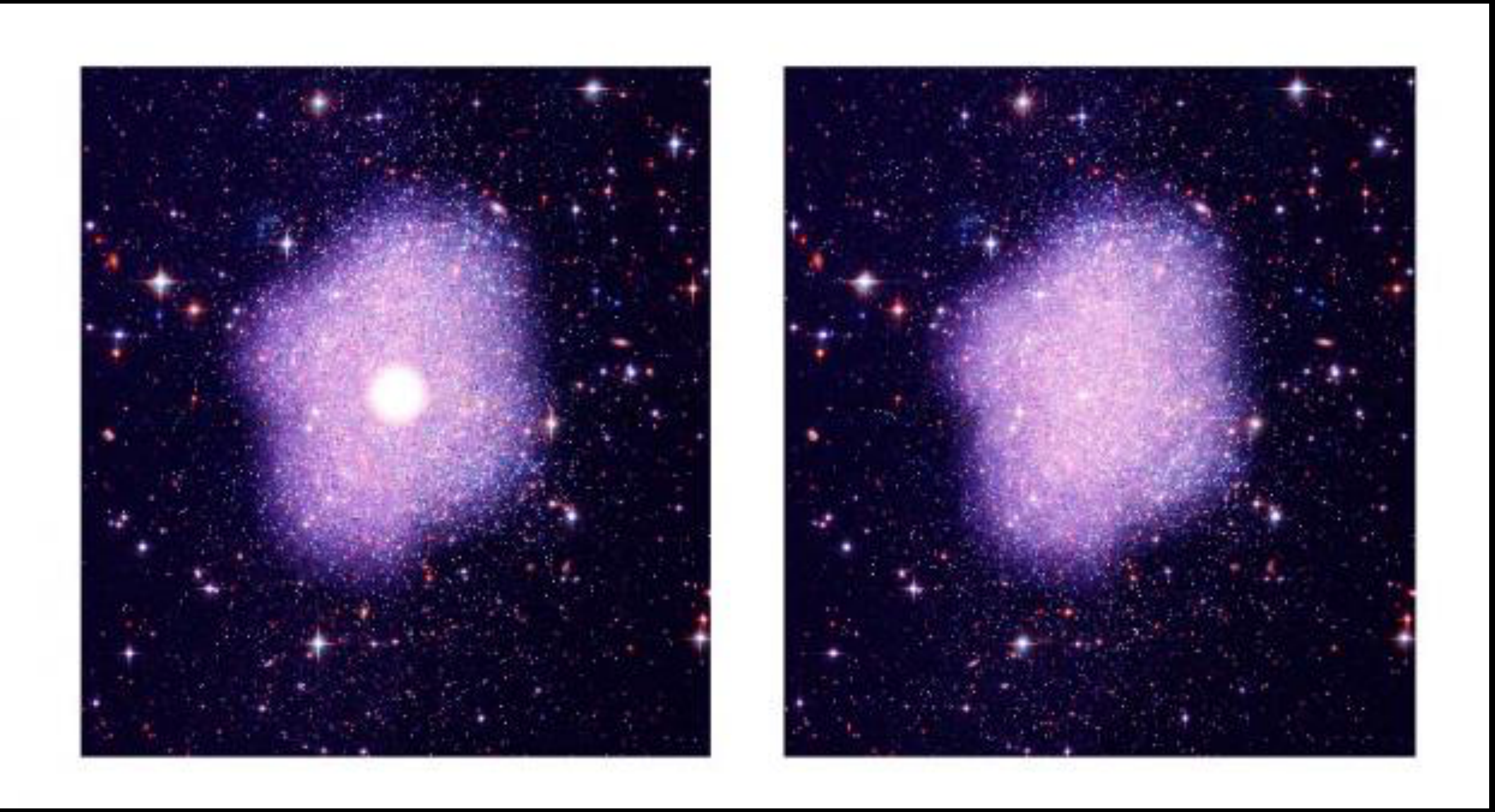
$$\frac{m_\pi}{f_\pi} \propto m_\pi^{3/10}$$

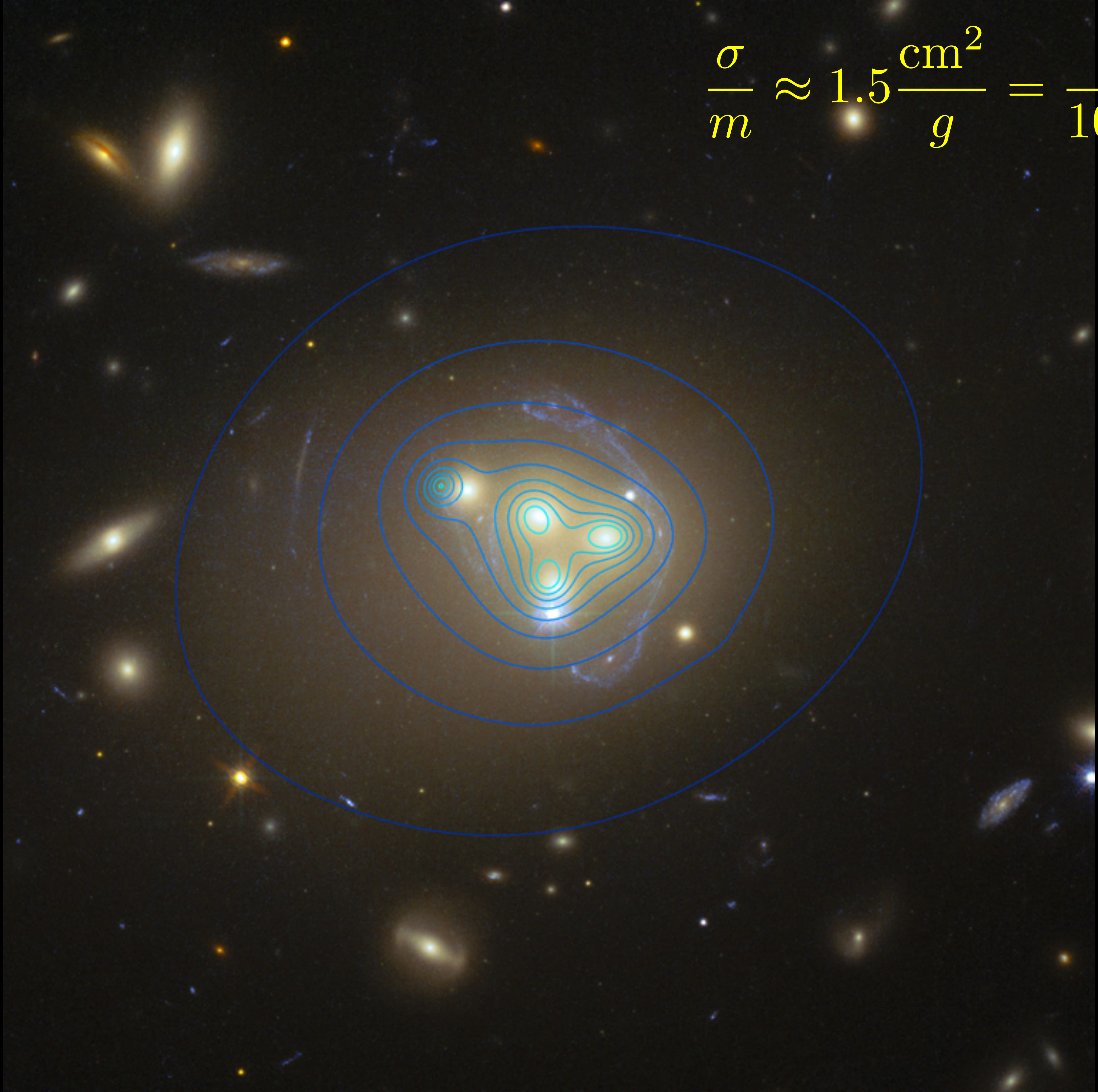
$$\frac{\sigma_{\rm scatter}}{m_\pi} \propto m_\pi^{-9/5}$$

self interaction



- self interaction of $\sigma/m \sim 10^{-24}\text{cm}^2/\text{GeV}$
- flattens the cusps in NFW profile
- actually desirable for dwarf galaxies?

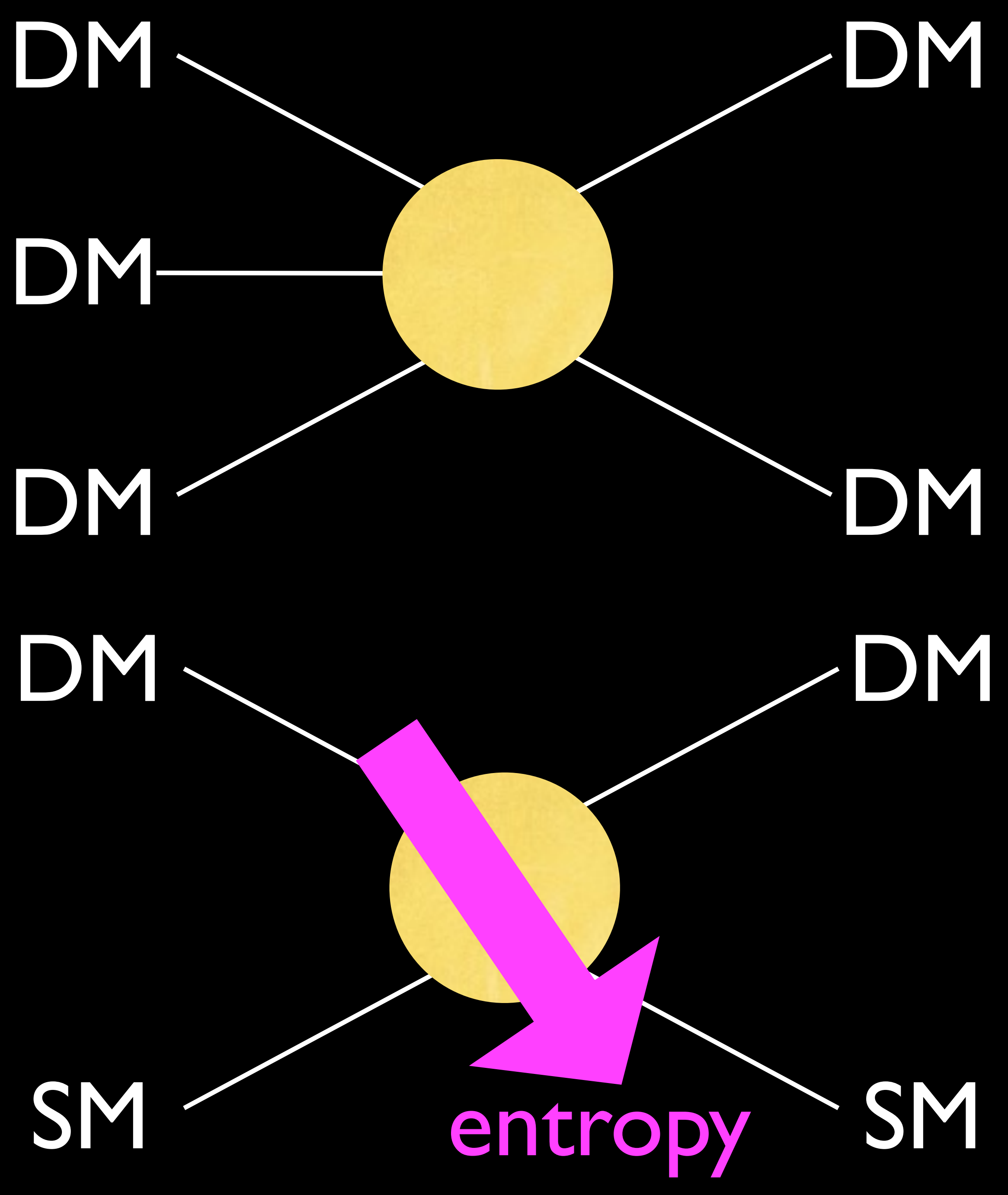




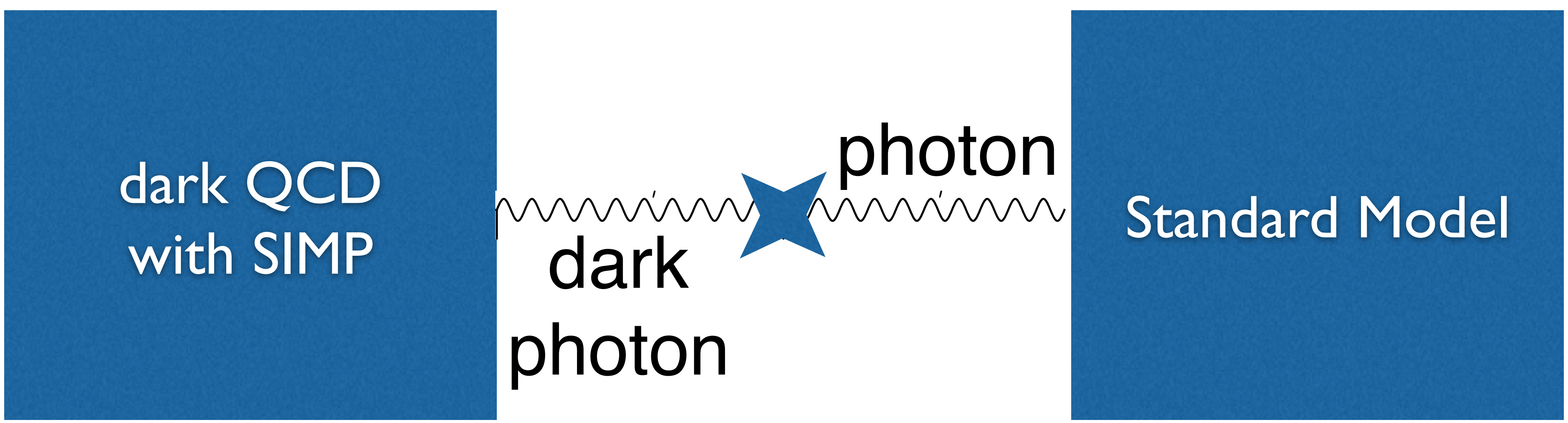
$$\frac{\sigma}{m} \approx 1.5 \frac{\text{cm}^2}{g} = \frac{0.27\text{b}}{100\text{MeV}}$$

communication

- 3 to 2 annihilation
- excess entropy *must* be transferred to $e^{\pm}$, γ
- need communication at some level
- leads to experimental signal



vector portal



$$\frac{\epsilon_\gamma}{2c_W} B_{\mu\nu} F_D^{\mu\nu}$$

Kinetically mixed U(1)

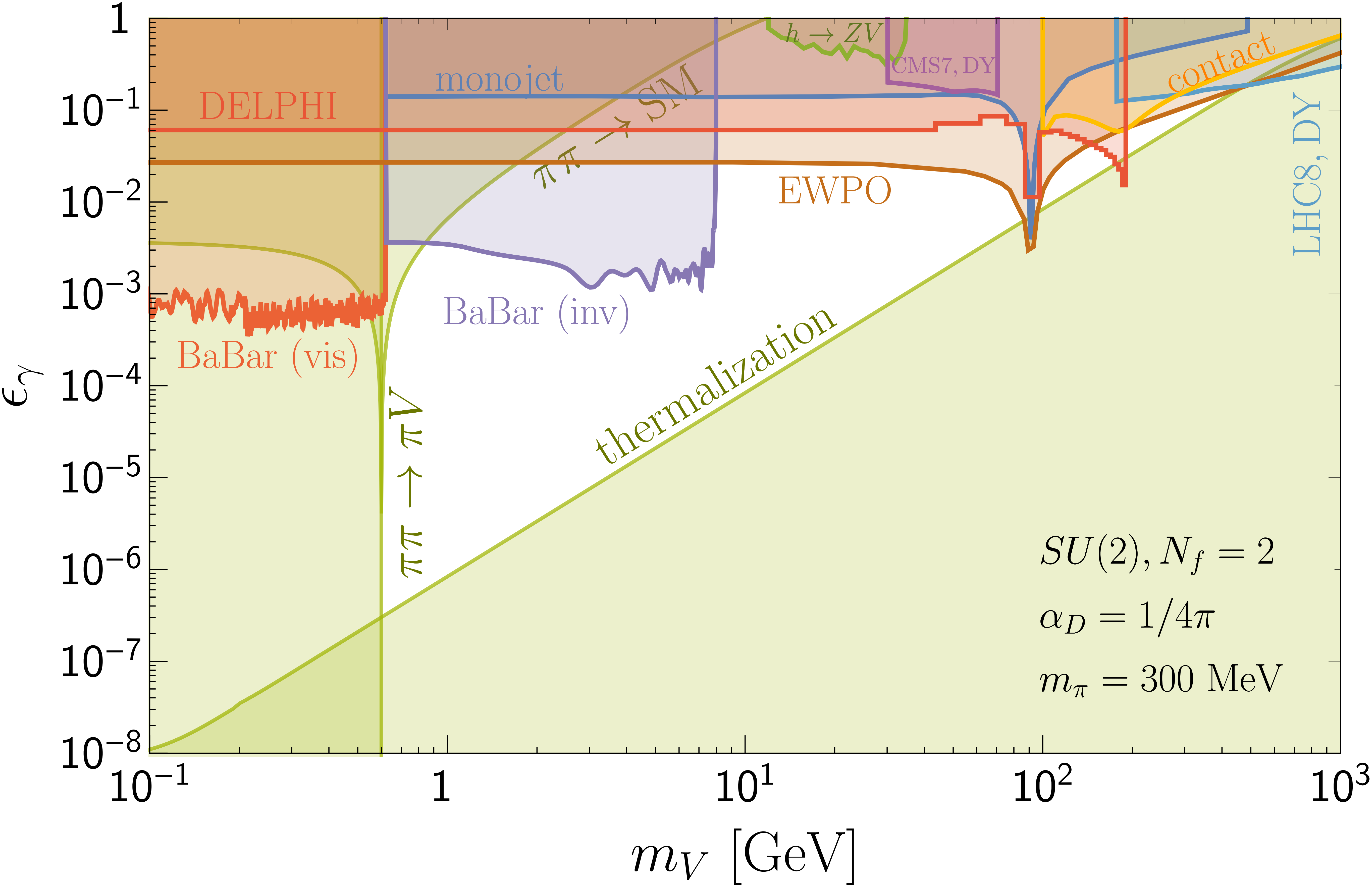
- e.g., the SIMPlest model SU(2) gauge group with N_f=2 (4 doublets)
- gauge U(1)=SO(2) ⊂ SO(2)×SO(3) ⊂ SO(5)=Sp(4)
- maintains degeneracy of quarks
- near degeneracy of pions for co-annihilation
- preserves SO(2)×SO(3) *s.t.* all pions are stable

$$SU(4)/Sp(4) = S^5$$

$$(q^+, q^+, q^-, q^-)$$

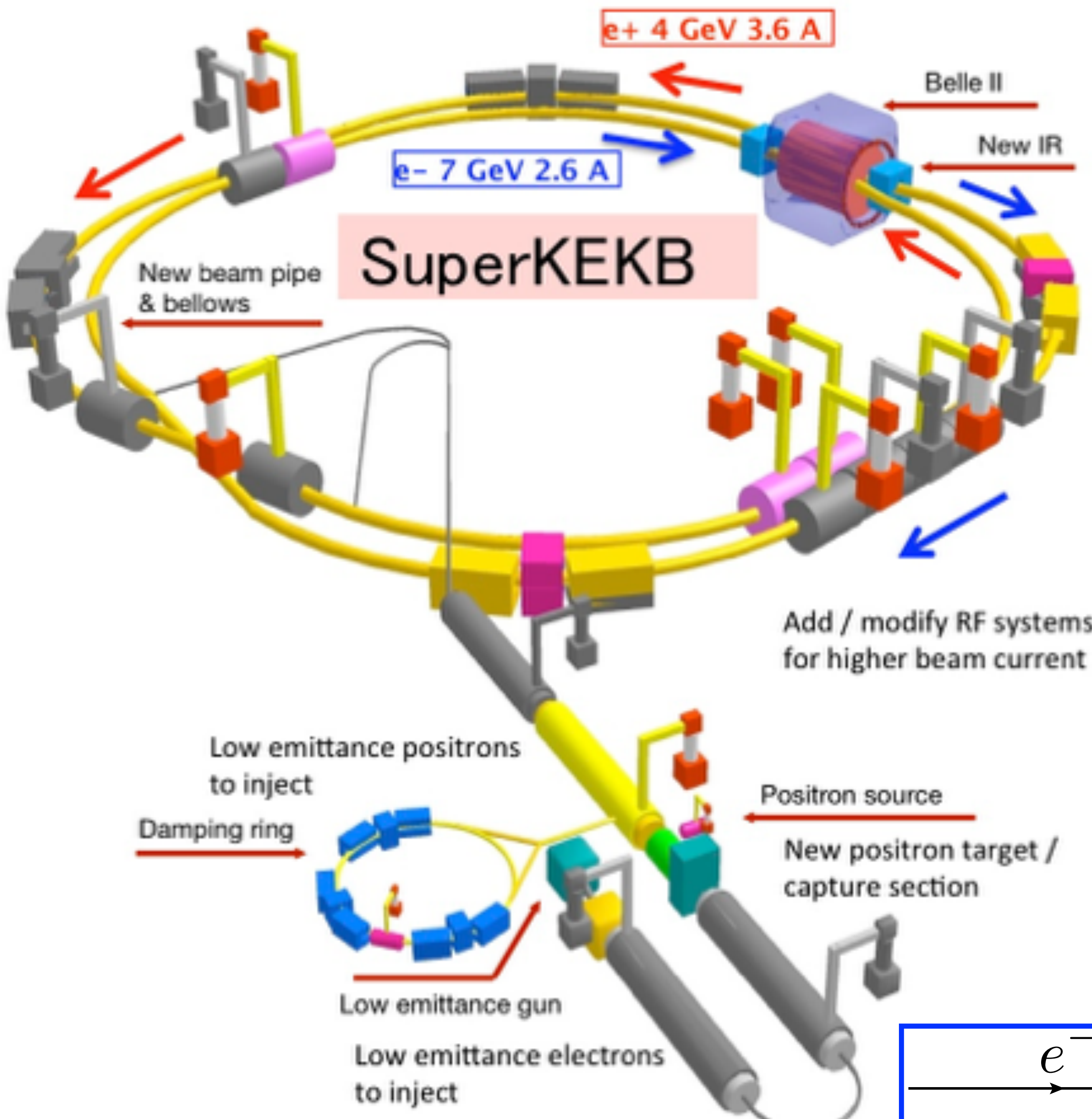
$$(\pi^{++}, \pi^{--}, \pi^0_x, \pi^0_y, \pi^0_z)$$

$$\frac{\epsilon_\gamma}{2c_W} B_{\mu\nu} F_D^{\mu\nu}$$

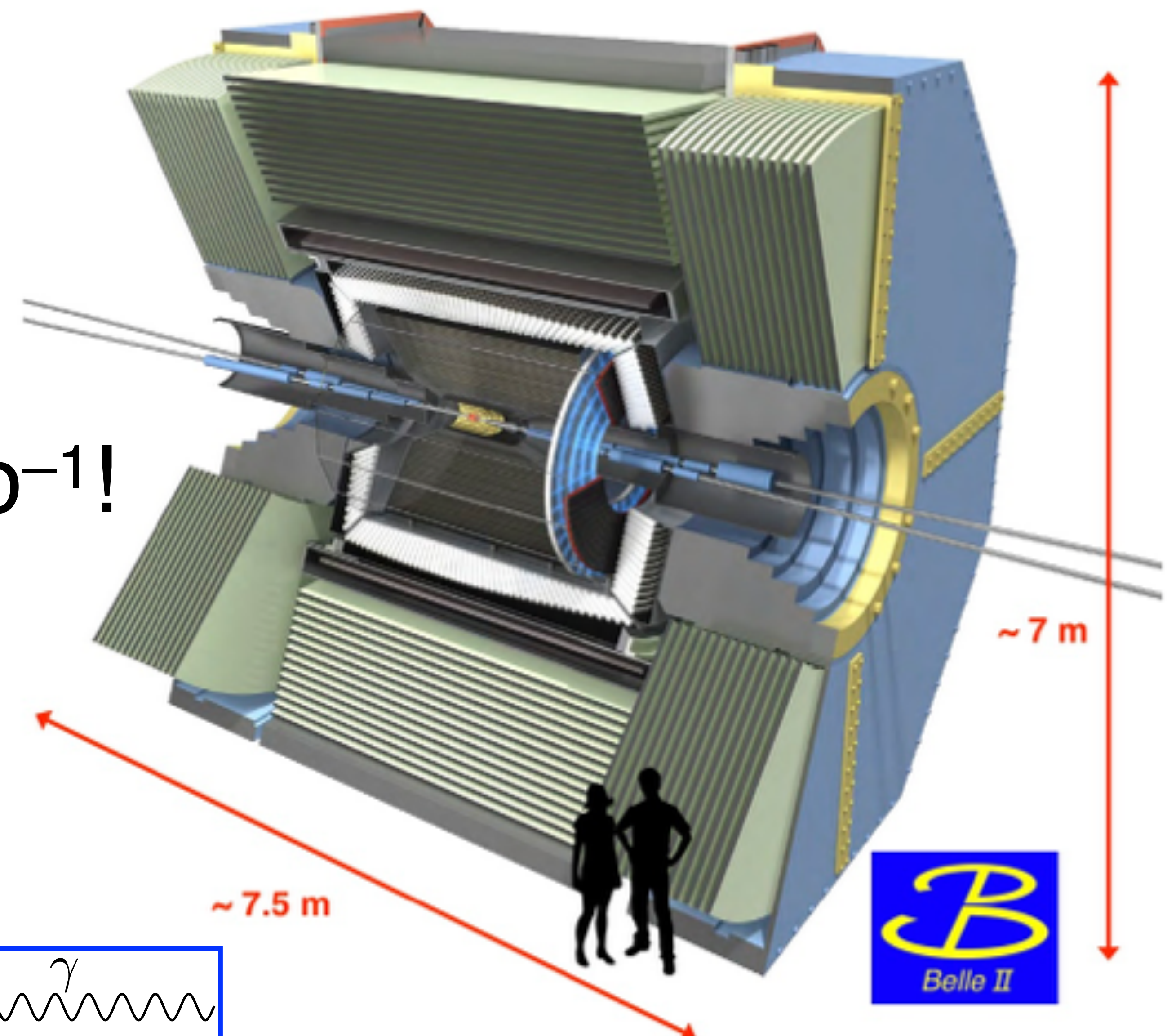


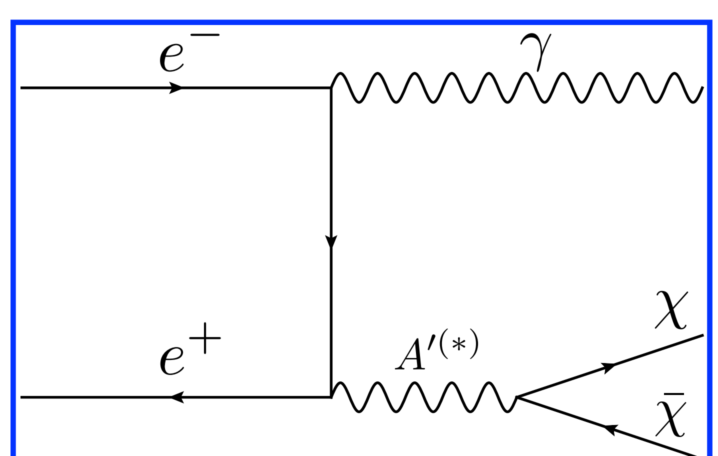
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
ϵ_γ
10^{-1}
1
10^{1}
10^{2}
10^{3}
m_V [GeV]
DELPHI
monojet
$\pi\pi \to SM$
$h \to ZV$
CMS7, DY
contact
LHC8, DY
EWPO
BaBar (inv)
BaBar (vis)
$\pi\pi \to \pi V$
thermalization
$SU(2), N_f = 2$
$\alpha_D = 1/4\pi$
$m_\pi = 300$ MeV

Super KEK B & Belle II

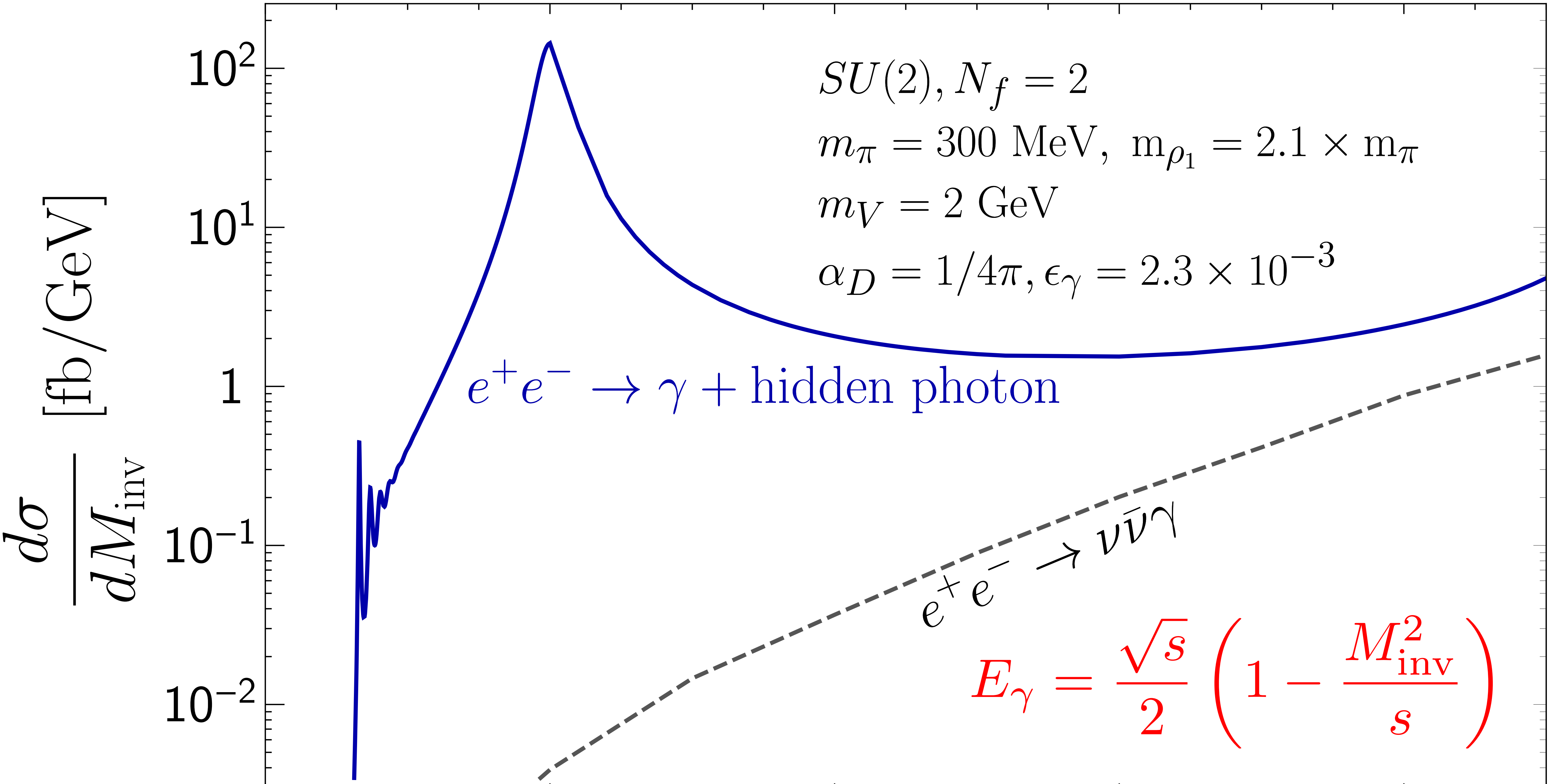


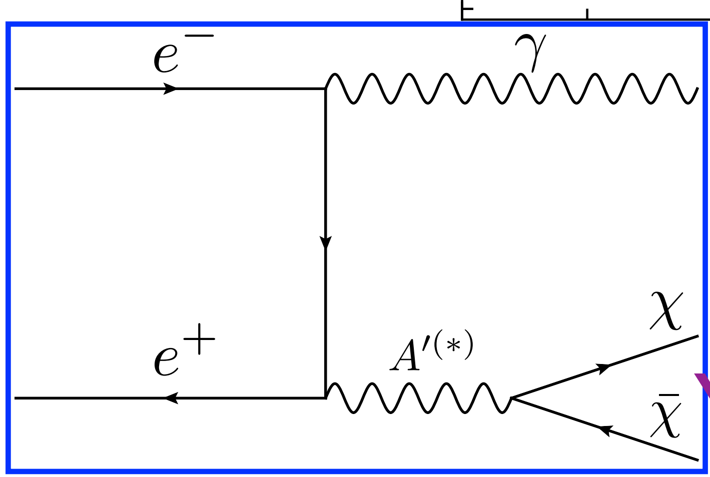
50 ab^{-1}!



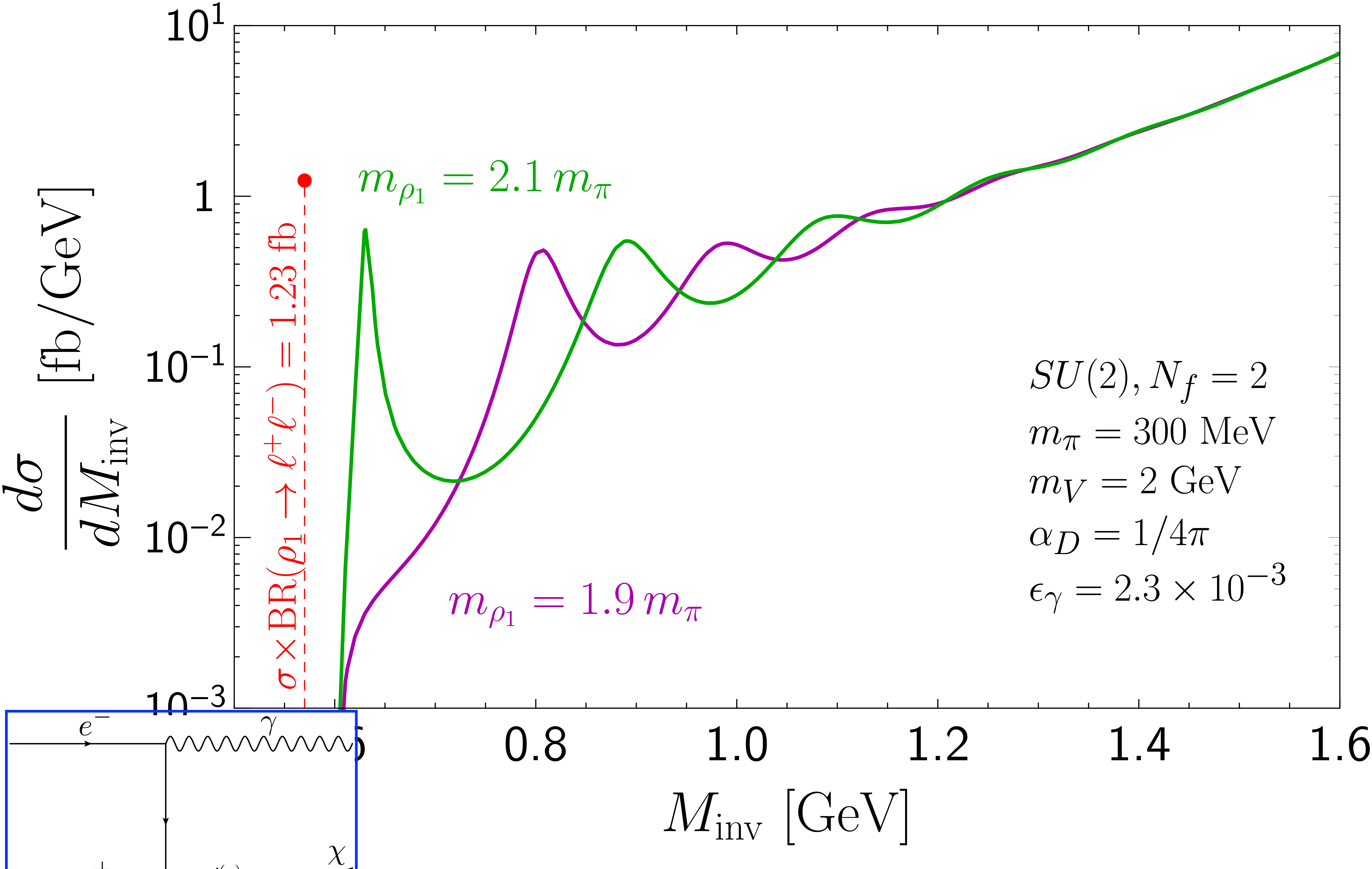


$$E_\gamma = \frac{\sqrt{s}}{2}\left(1 - \frac{M_{\text{inv}}^2}{s}\right)$$

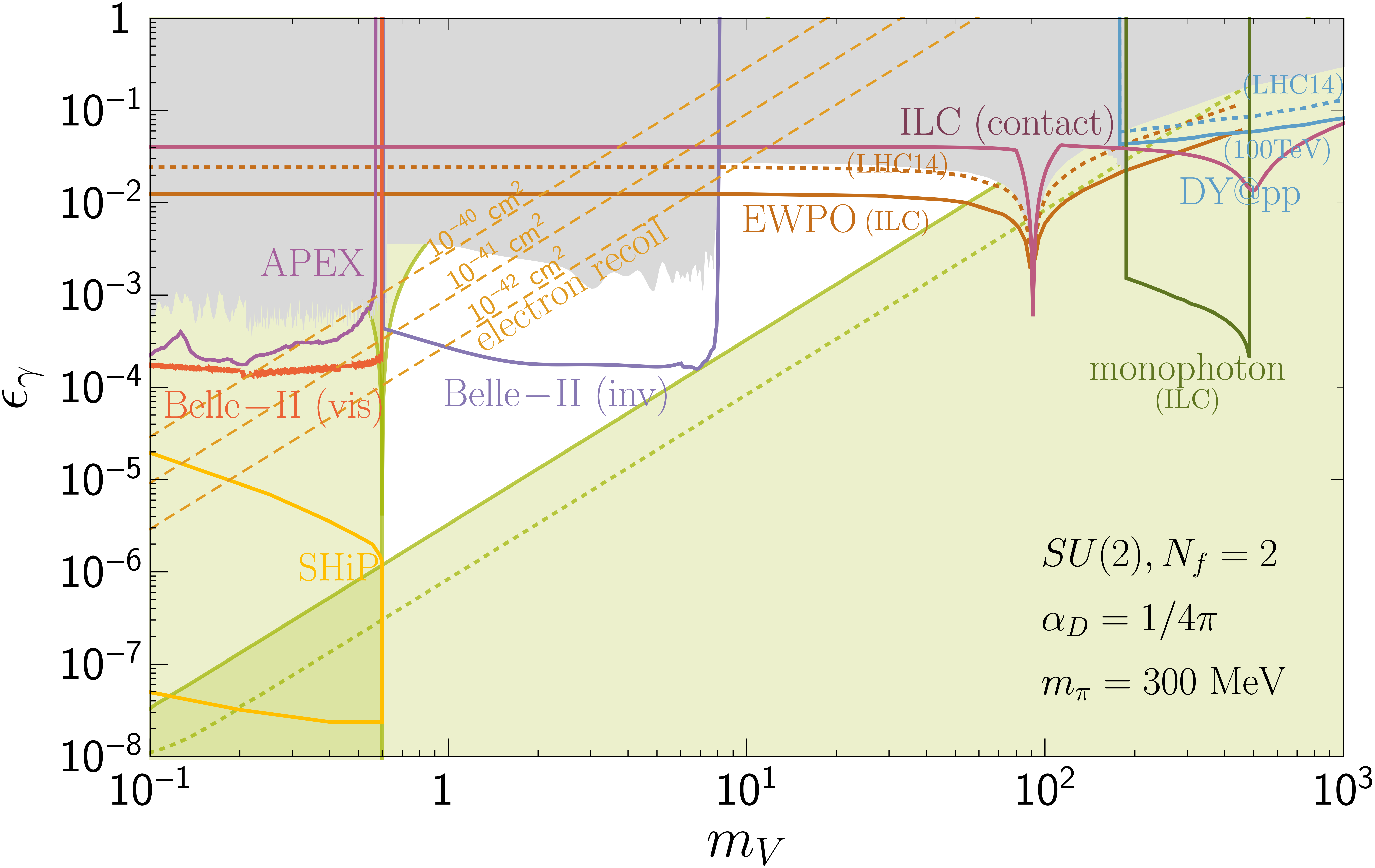




Yonit Hochberg, Eric Kuflik, HM



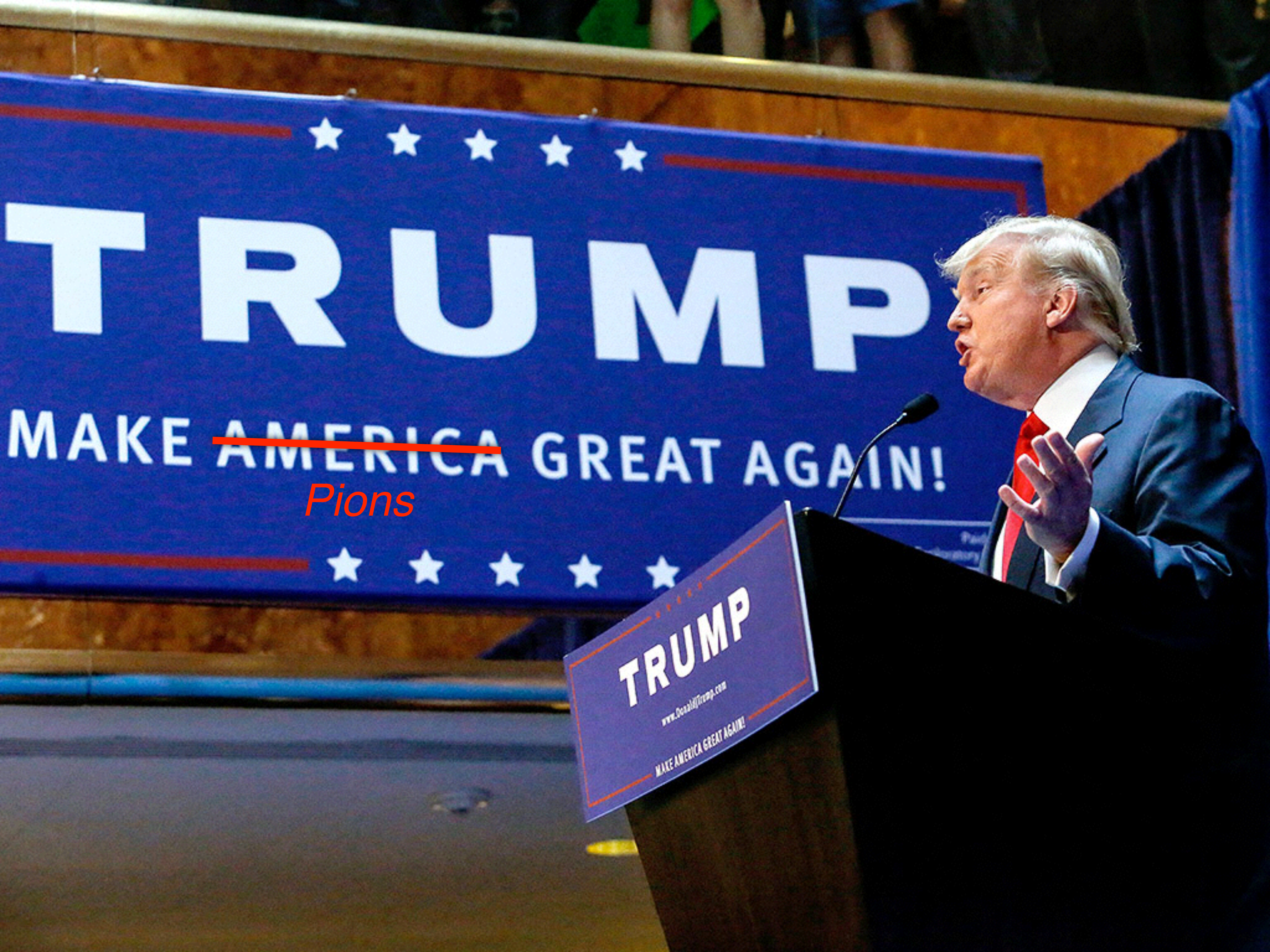
Yonit Hochberg, Eric Kuflik, HM



1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
ϵ_γ
10^{-1}
1
10^{1}
10^{2}
10^{3}
m_V
ILC (contact)
(LHC14)
(LHC14)
(100TeV)
DY@pp
EWPO (ILC)
10^{-40} cm^2
10^{-41} cm^2
10^{-42} cm^2
electron recoil
APEX
Belle−II (vis)
Belle−II (inv)
monophoton
(ILC)
SHiP
$SU(2), N_f = 2$
$\alpha_D = 1/4\pi$
$m_\pi = 300$ MeV

Conclusion

- surprising an *old* theory for dark matter
- SIMP Miracle^3
  - mass ~ QCD
  - coupling ~ QCD
  - theory ~ QCD
- can solve problem with DM profile
- very rich phenomenology
- Exciting *dark spectroscopy*!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