Bounds on Amplitudes and EFTs

Brando Bellazzini

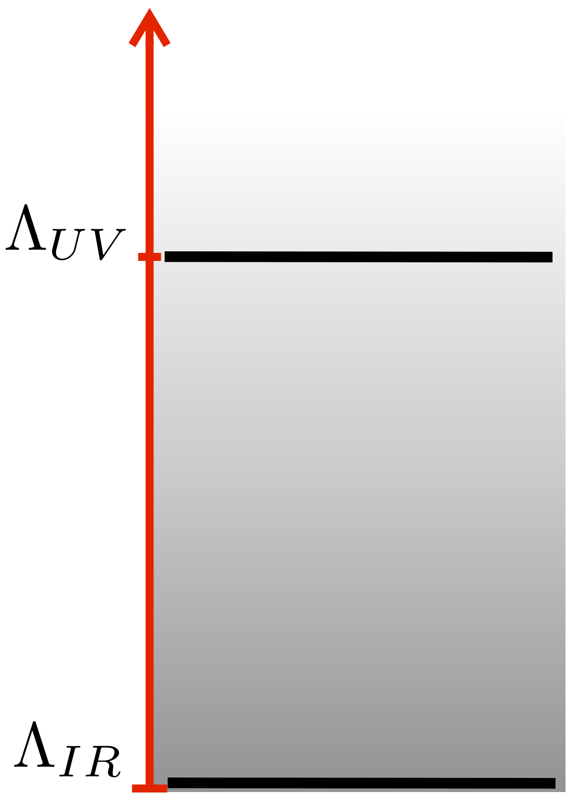
IPhT - CEA/Saclay

based on 1605.06111

CERN, 'Charting the unknown', August 9th 2016

Hierarchy of scales

a blessing…



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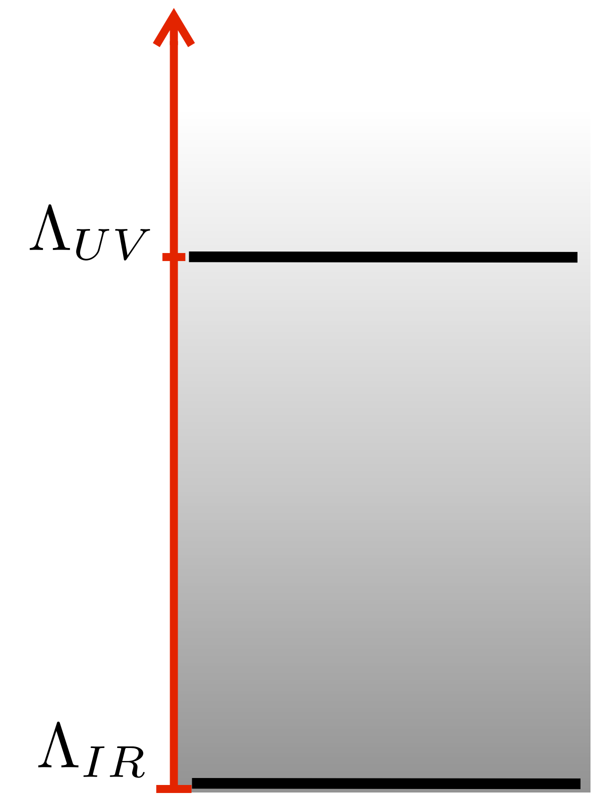
- small parameter E/Λ_{UV}
- emerging patterns
- suppress dangerous operators



$$\mathcal{L}_{\mathcal{IR}} = \mathcal{L}^{\Delta\leq 4} + \sum_{\mathcal{O}} \frac{\mathcal{O}(x)}{\Lambda_{UV}^{\Delta-4}}$$

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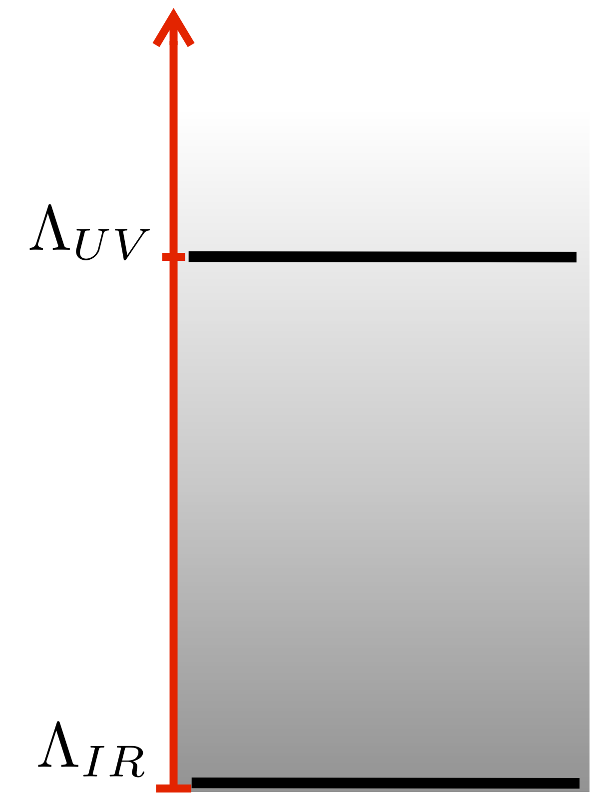

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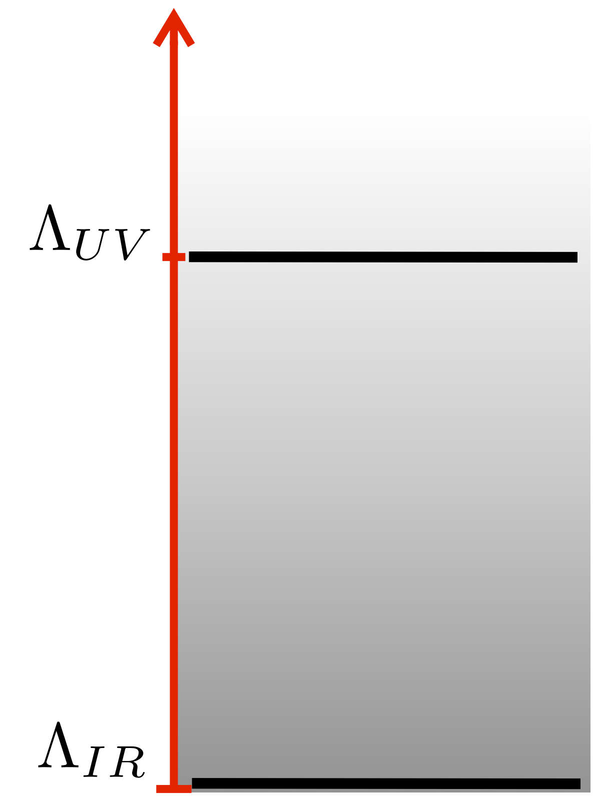

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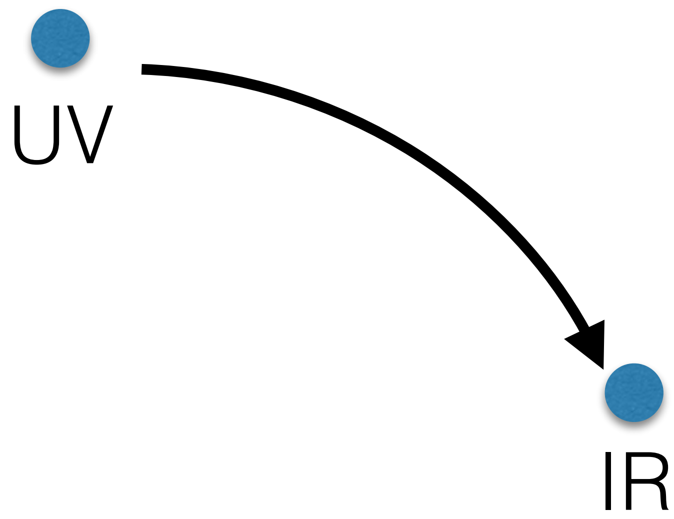
large couplings from a strong sector may help

e.g. in CHM: $\mathcal{L} = \frac{g_*^2}{m_*^2}(\partial H^2)^2$

The EFT paradigm



$$\mathcal{L}_{\mathcal{IR}} = \sum_i c_i \frac{\mathcal{O}_i(x)}{\Lambda_{UV}^{\Delta-4}}$$

EFT encodes UV-info via c_i

Finite set of C's is needed at any order in E/Λ_{UV}

Power counting = understanding = symmetries

Symmetry <—> Softness

Higher dim-operators may dominate the amplitude within EFT

just suppress relevant, marginal and less-irrelevant operators by symmetries

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1-to-1 amplitude dominated by a less-relevant operator

$$\mathcal{M}(1 \to 1) \sim \frac{1}{E} \qquad \epsilon \cdot m_* < E < m_*$$

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Example

(2) $\bar{\psi} i \partial \psi - g A_\mu \bar{\psi} \gamma^\mu \psi + \ldots$

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$$(2) \quad \bar{\psi} i \partial \psi - g A_\mu \bar{\psi} \gamma^\mu \psi + \dots \xrightarrow[g \ll 1]{} \bar{\psi} i \partial \psi - \epsilon \cdot g_* A_\mu \bar{\psi} \gamma^\mu \psi + \frac{g_*^2}{m_*^2} (\bar{\psi} \gamma^\mu \psi)^2 + \dots$$

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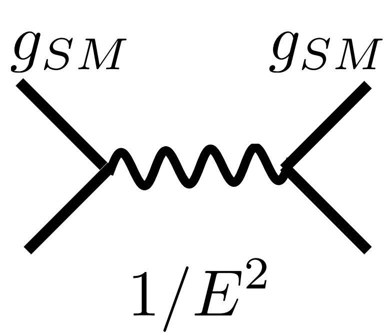
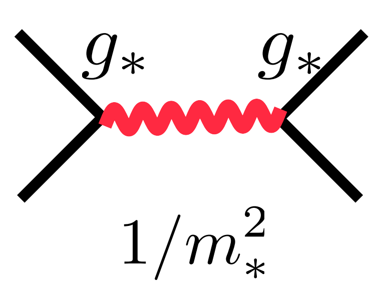
dominated by dim-6 at intermediate energy $\quad \epsilon \cdot m_* < E < m_*$





$$\mathcal{M}(2 \to 2) = \frac{g_{SM}^2}{E^2}\left(1 + \frac{1}{\epsilon^2}\frac{E^2}{m_*^2}\right)$$

Amplitude runs fast within the validity of EFT

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Examples

(3) $(\partial\pi)^2 - m_*^2\pi^2 + g_*^2\pi^4 + \dots$

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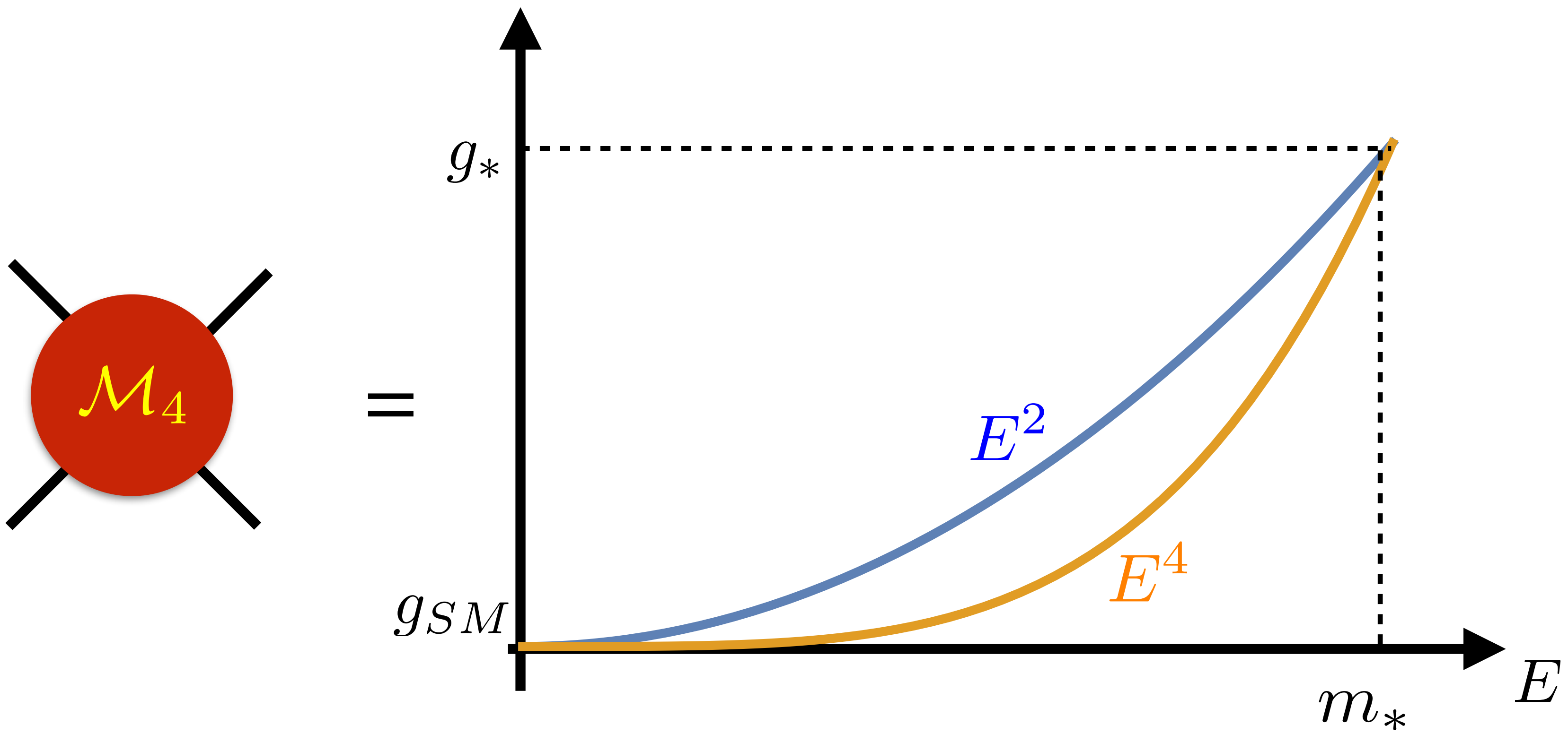
Amplitude runs fast within the validity of EFT: $\mathcal{M}(2 \to 2) = g_*^2\epsilon^4\left(1 + \frac{E^4}{m_*^4\epsilon^4}\right)$

$$\epsilon \cdot m_* < E < m_*$$

Running Coupling



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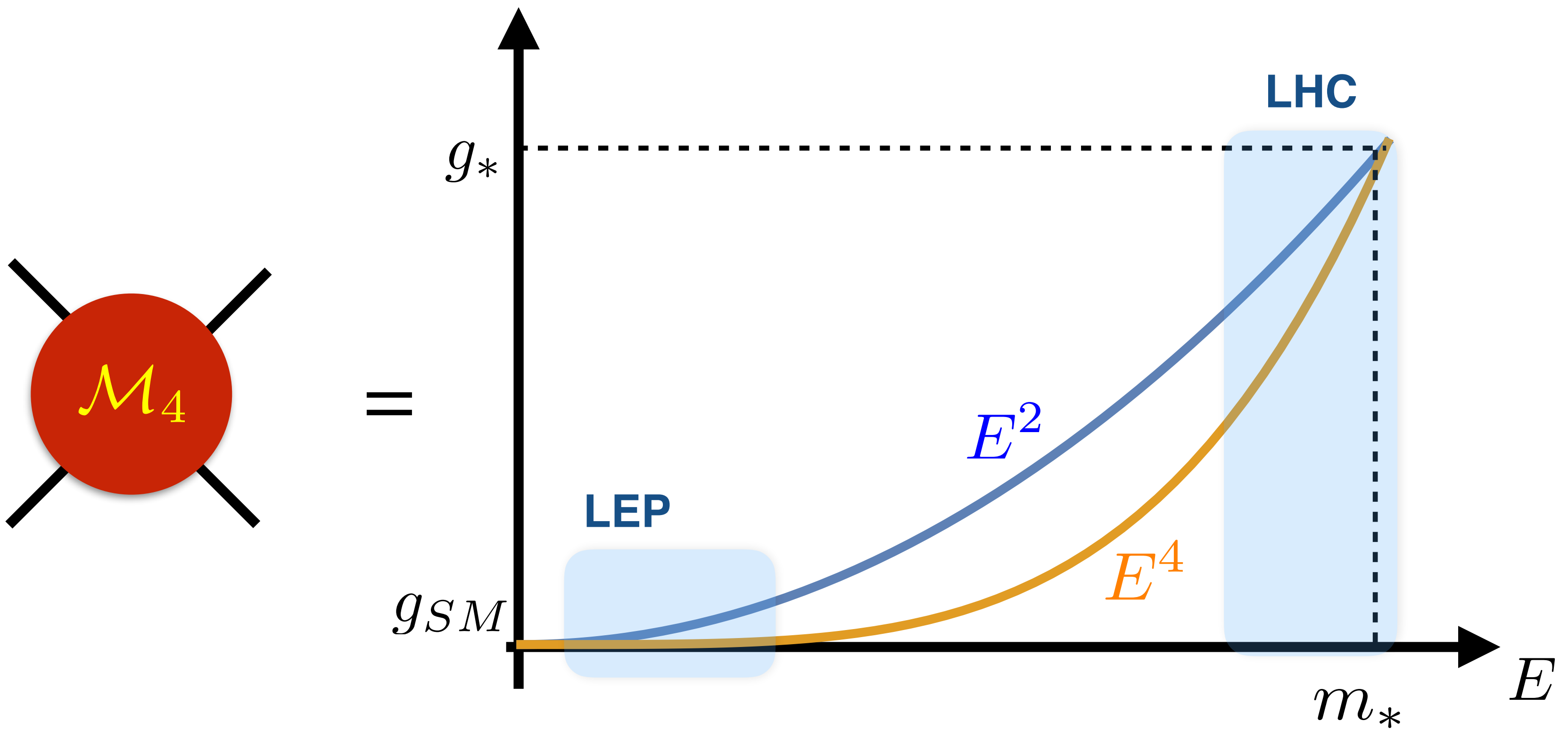

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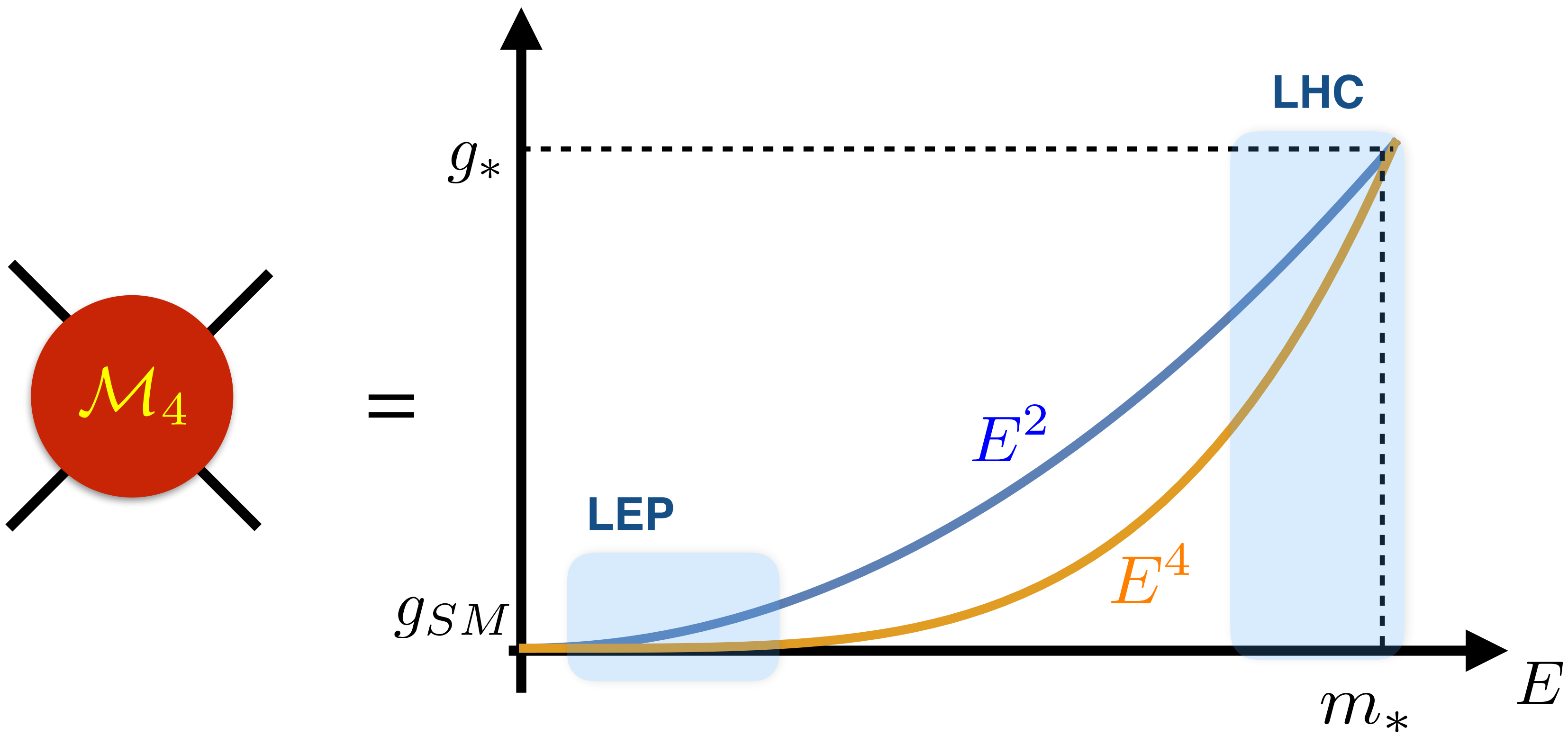
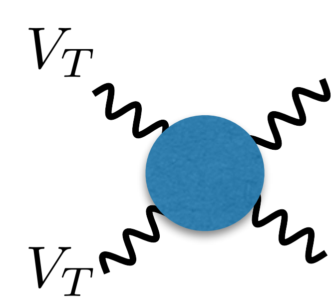


'remedios': strongly int. transv. vectors

1603.03064 Liu, Pomarol, Rattazzi, Riva

$$\frac{g_*^2}{m_*^4} W_{\mu\nu}^4 \longrightarrow$$



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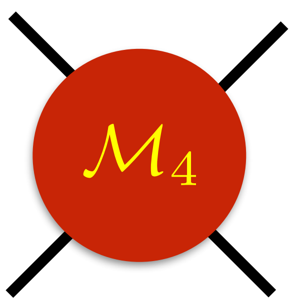

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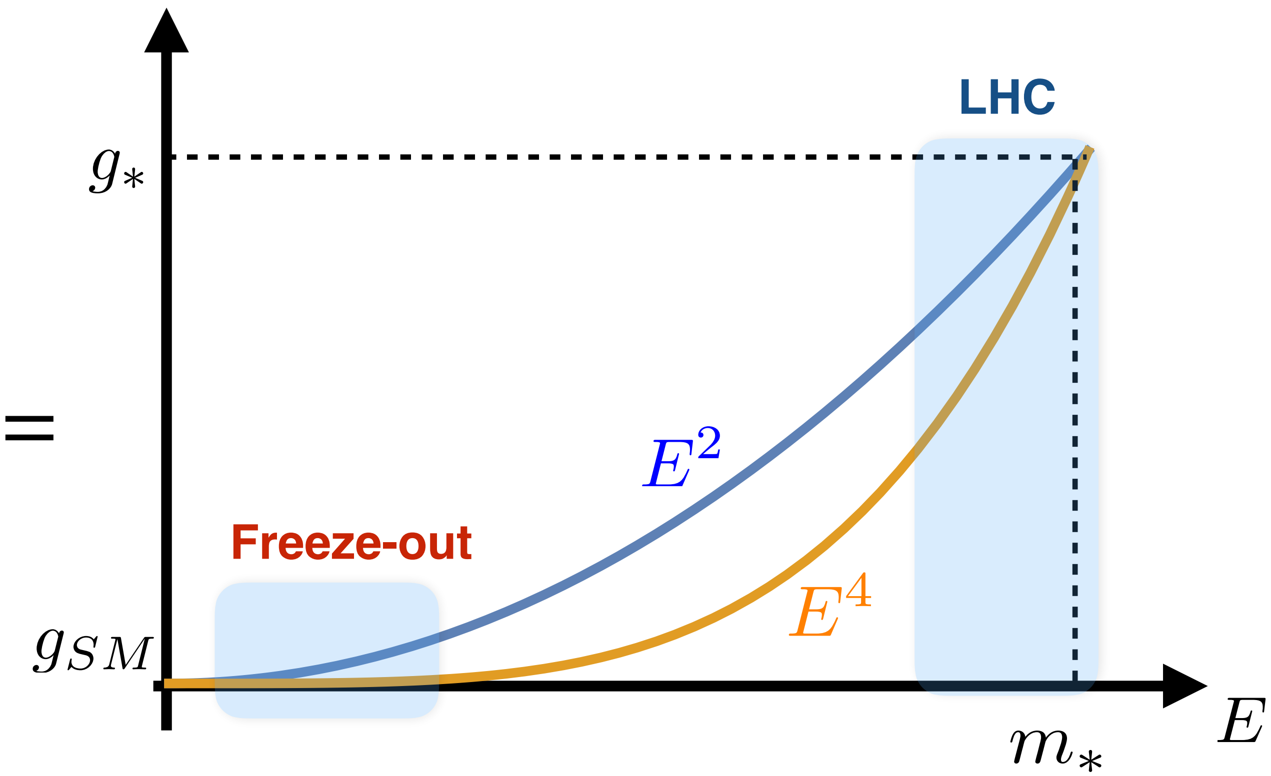

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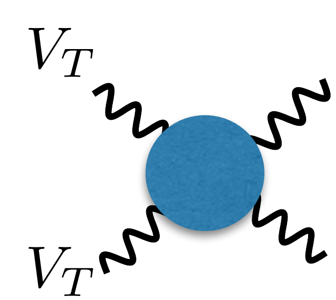

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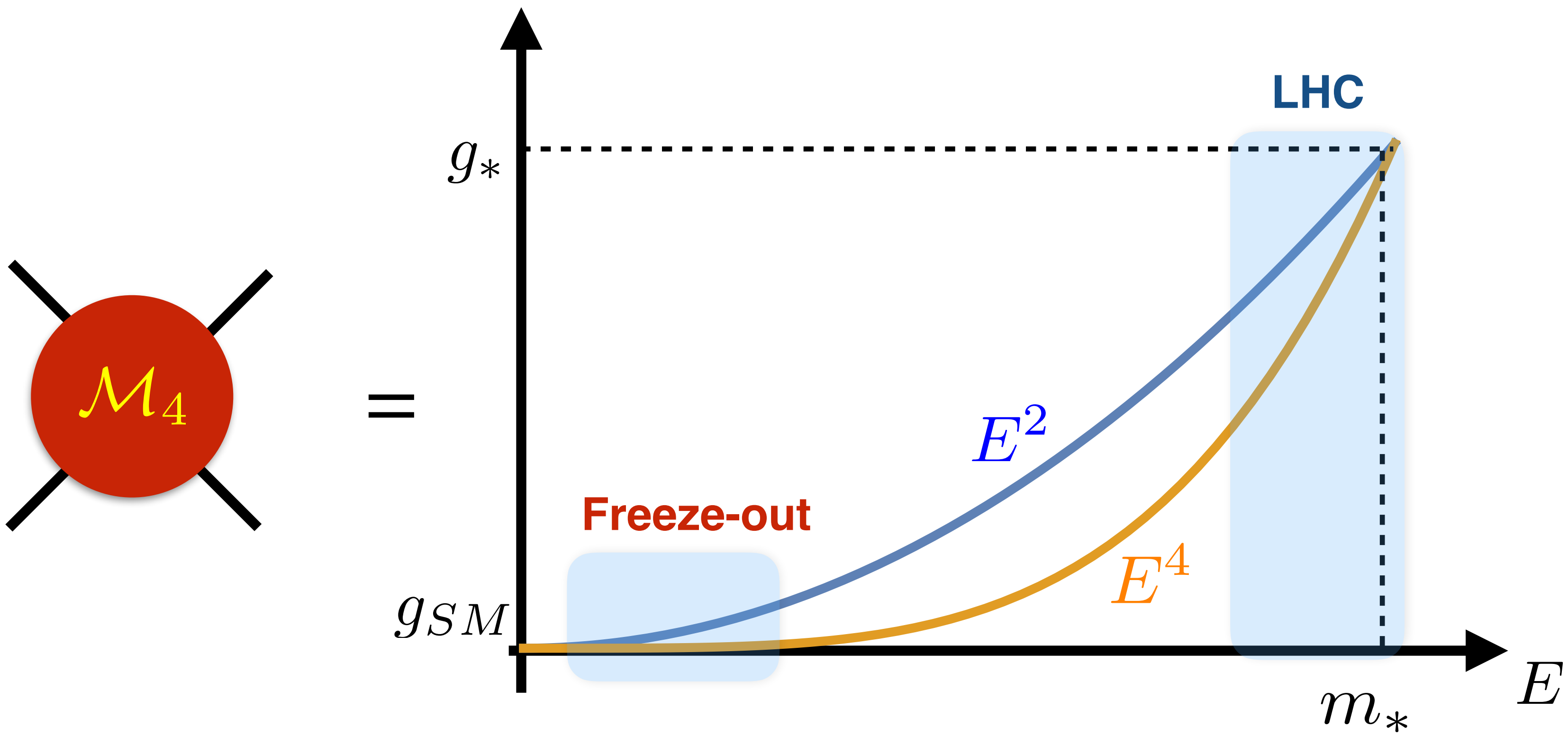
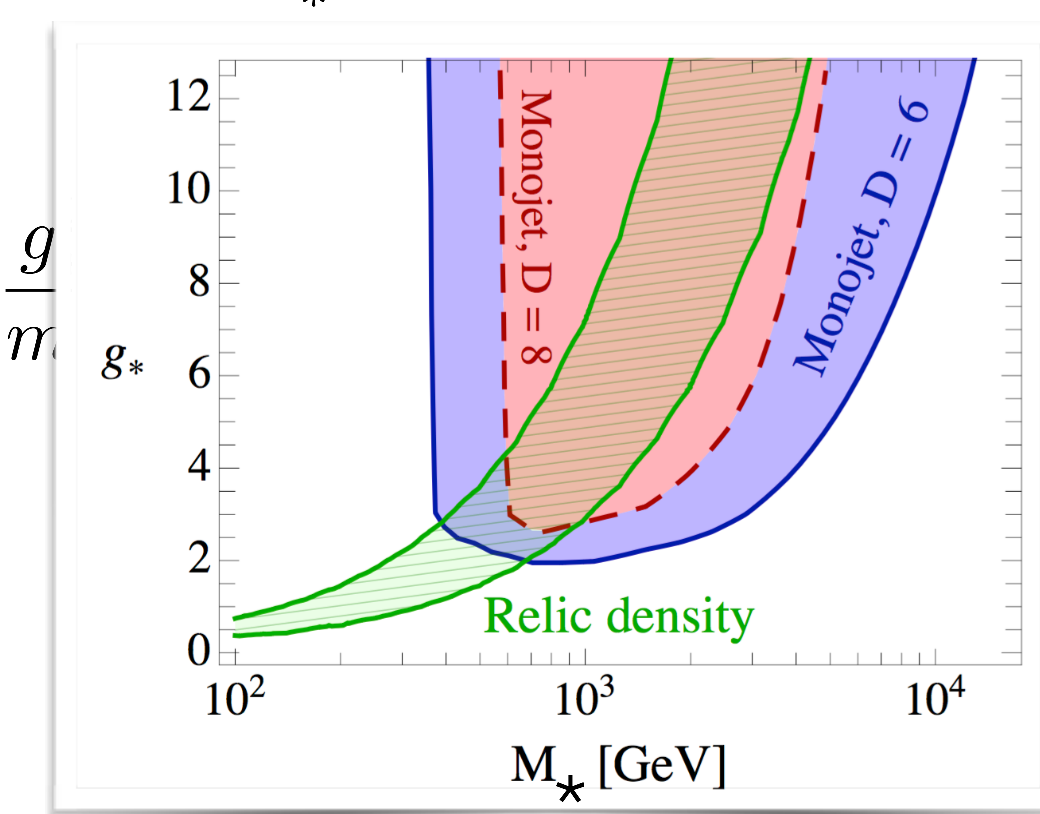


‘remedios’: strongly int. transv. vectors

1603.03064 Liu, Pomarol, Rattazzi, Riva

DM as light pseudo-goldstino

1607.02474 Brugisser, Riva, Urbano



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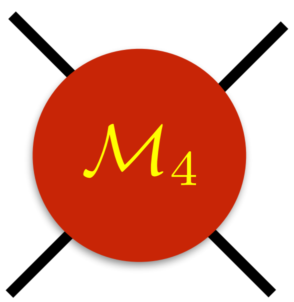

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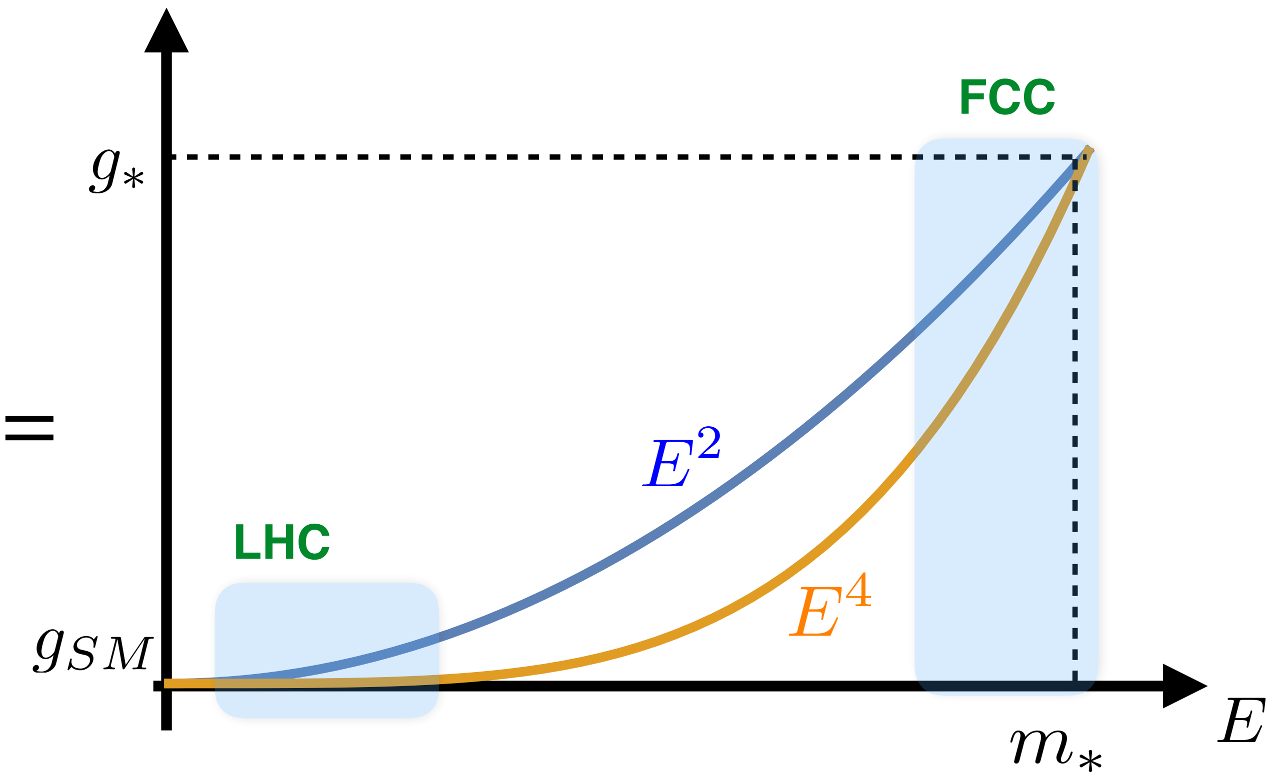

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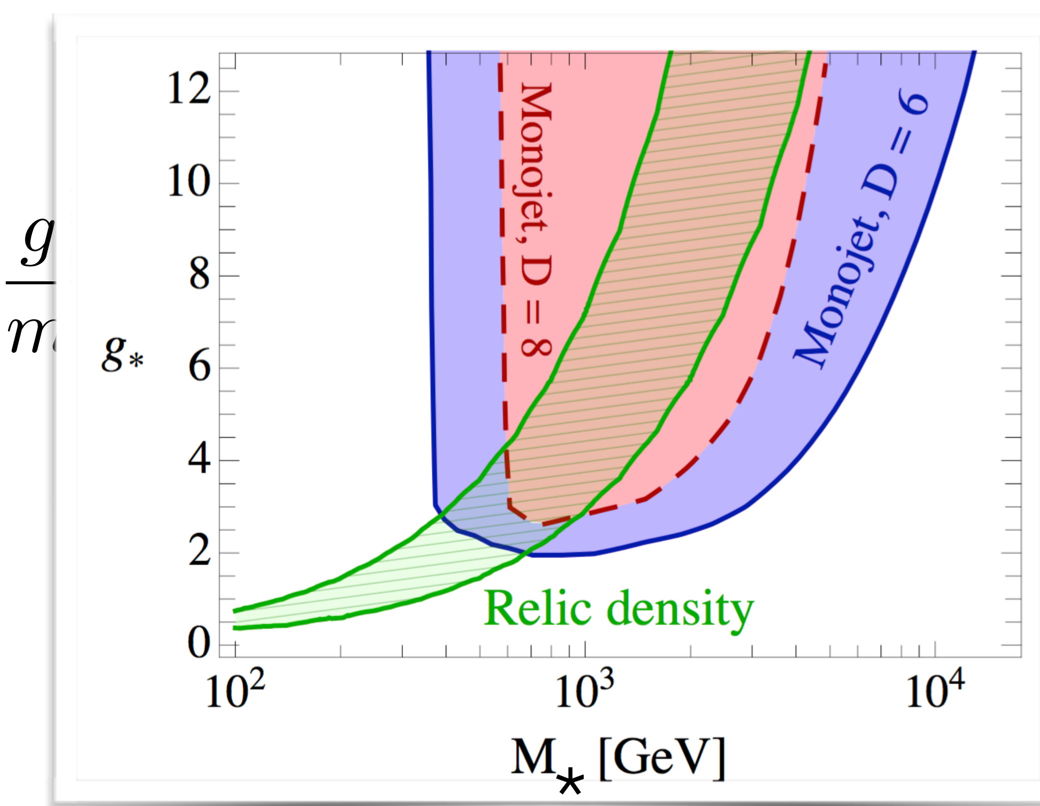

How fast?

Goldstones	$(\partial\pi)^2\pi^2$	
4-Fermions	$(\bar{\psi}\gamma^\mu\psi)^2$	$\Bigg\} \sim E^2$
	$\ldots$	

can amplitudes be softer than E^4 ?

(within an EFT)

dilaton	$(\partial\sigma)^4$	
Goldstino	$\bar{\psi}^2\Box\psi^2$	$\Bigg\} \sim E^4$
remedios	$F_{\mu\nu}^4$	
	$\ldots$	

UV-IR connection

(well known)

For spin-0 particles the answer is: No!

e.g. hep-th/0602178
Adams, Arkani-Hamed,
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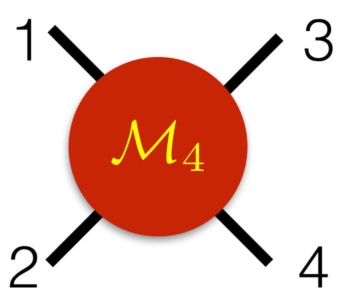

Analyticity, Crossing, and Unitarity

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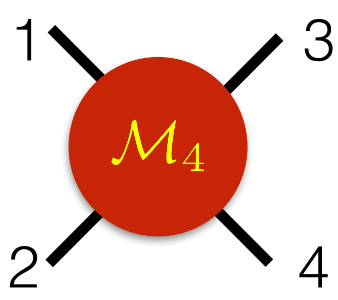

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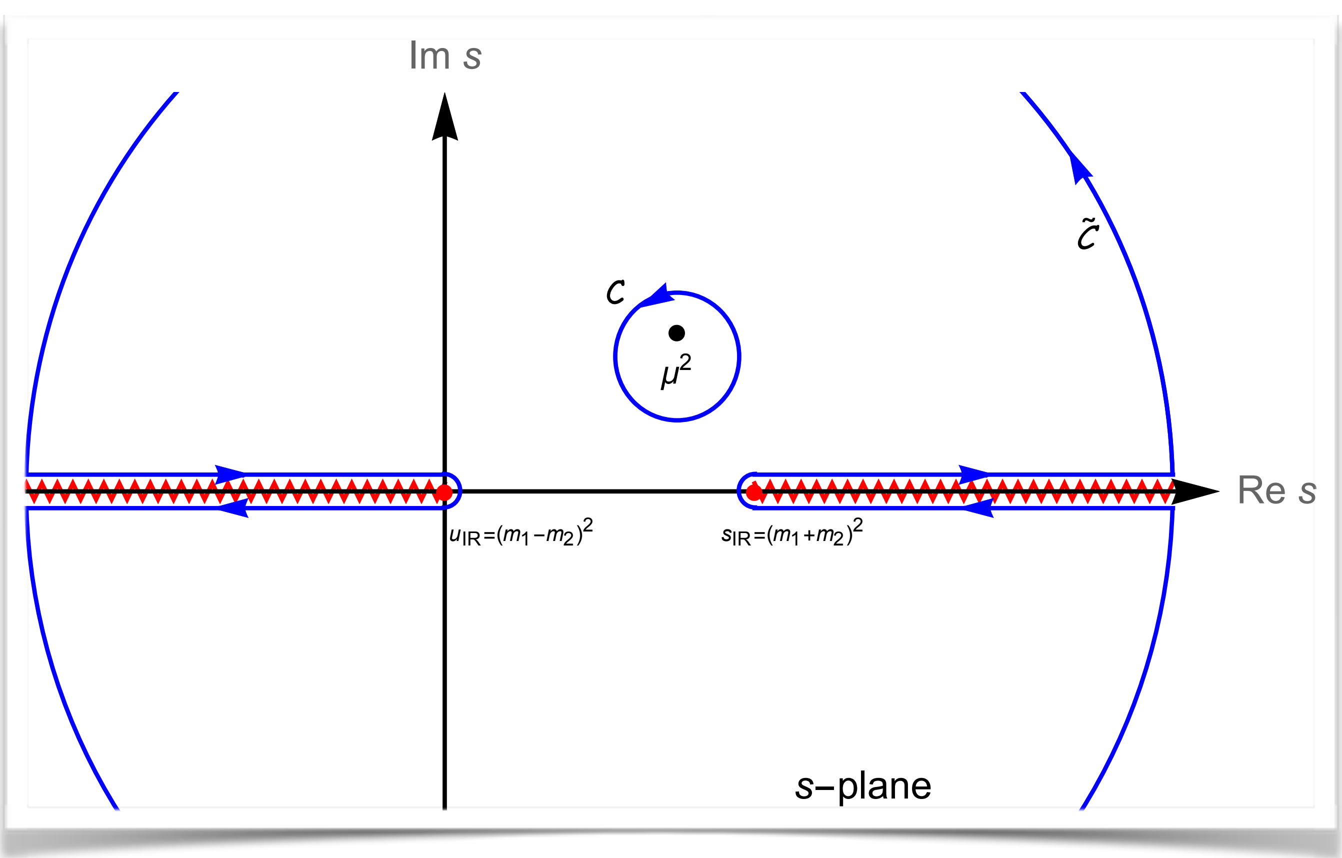

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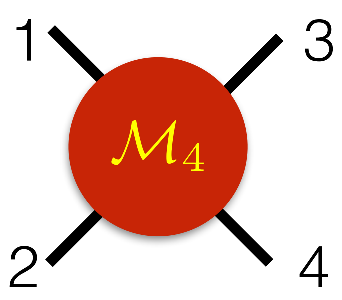

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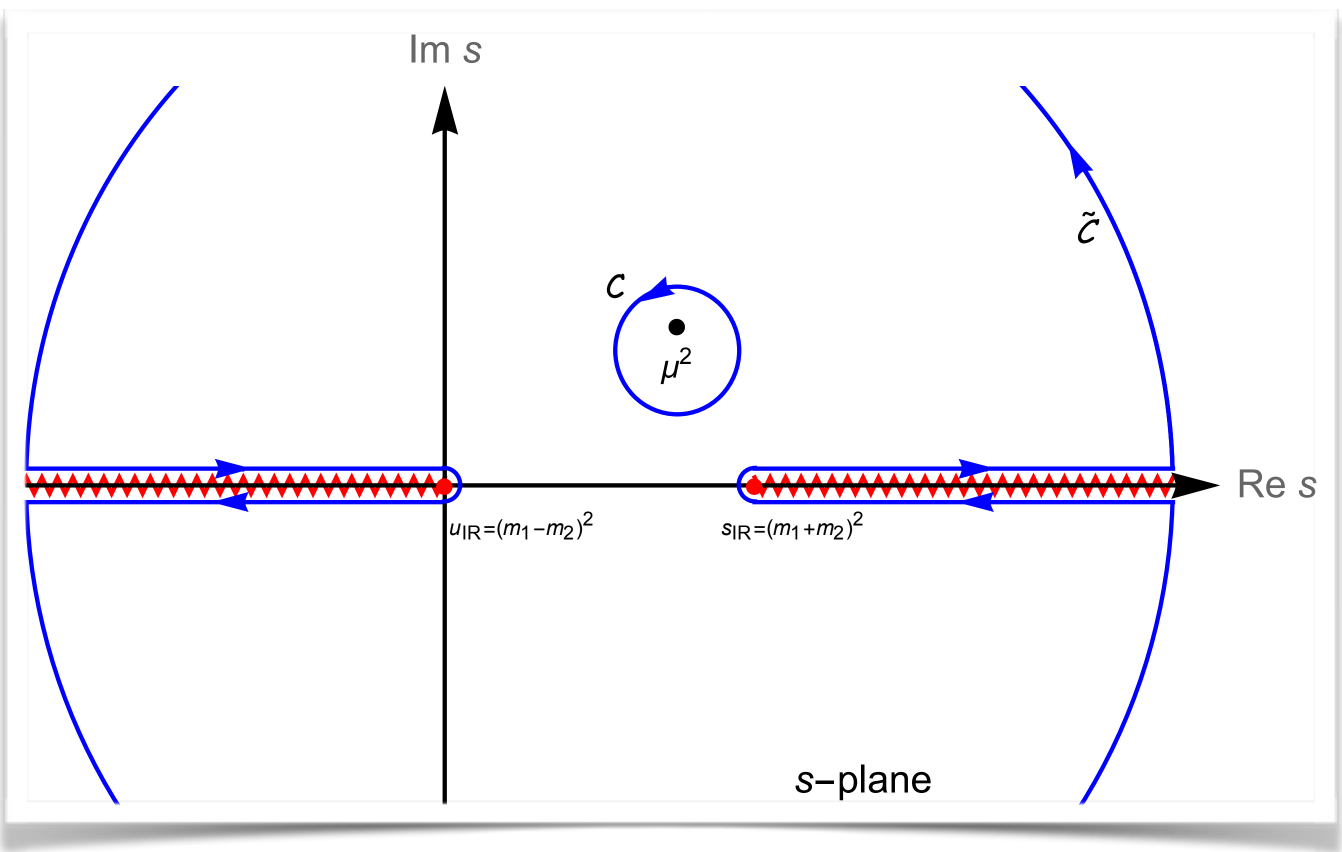

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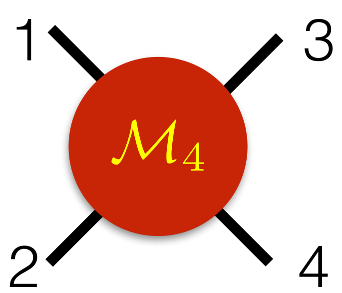

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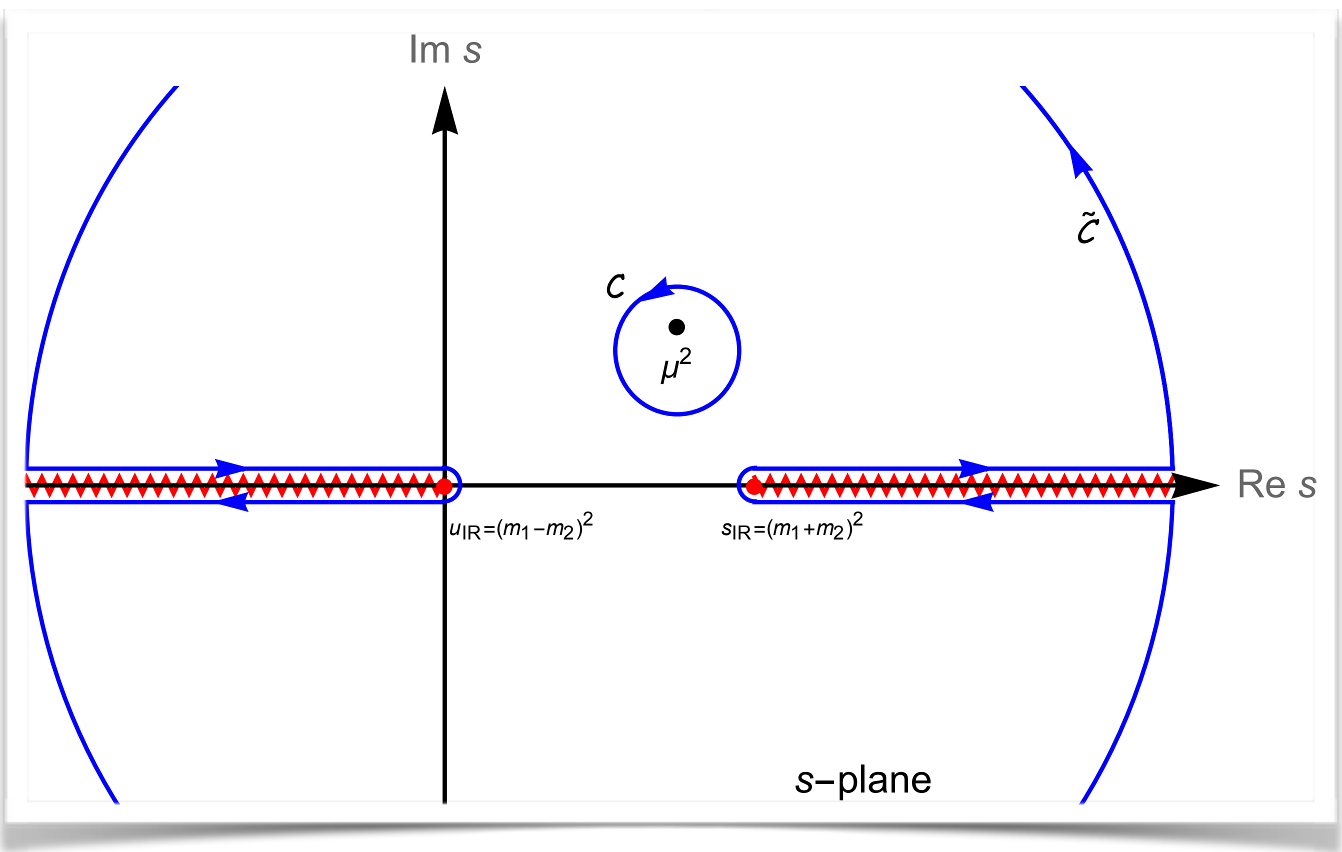
$$s \leftrightarrow u$$
$$Disc_s = Disc_u$$



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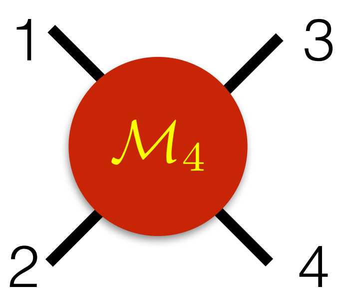

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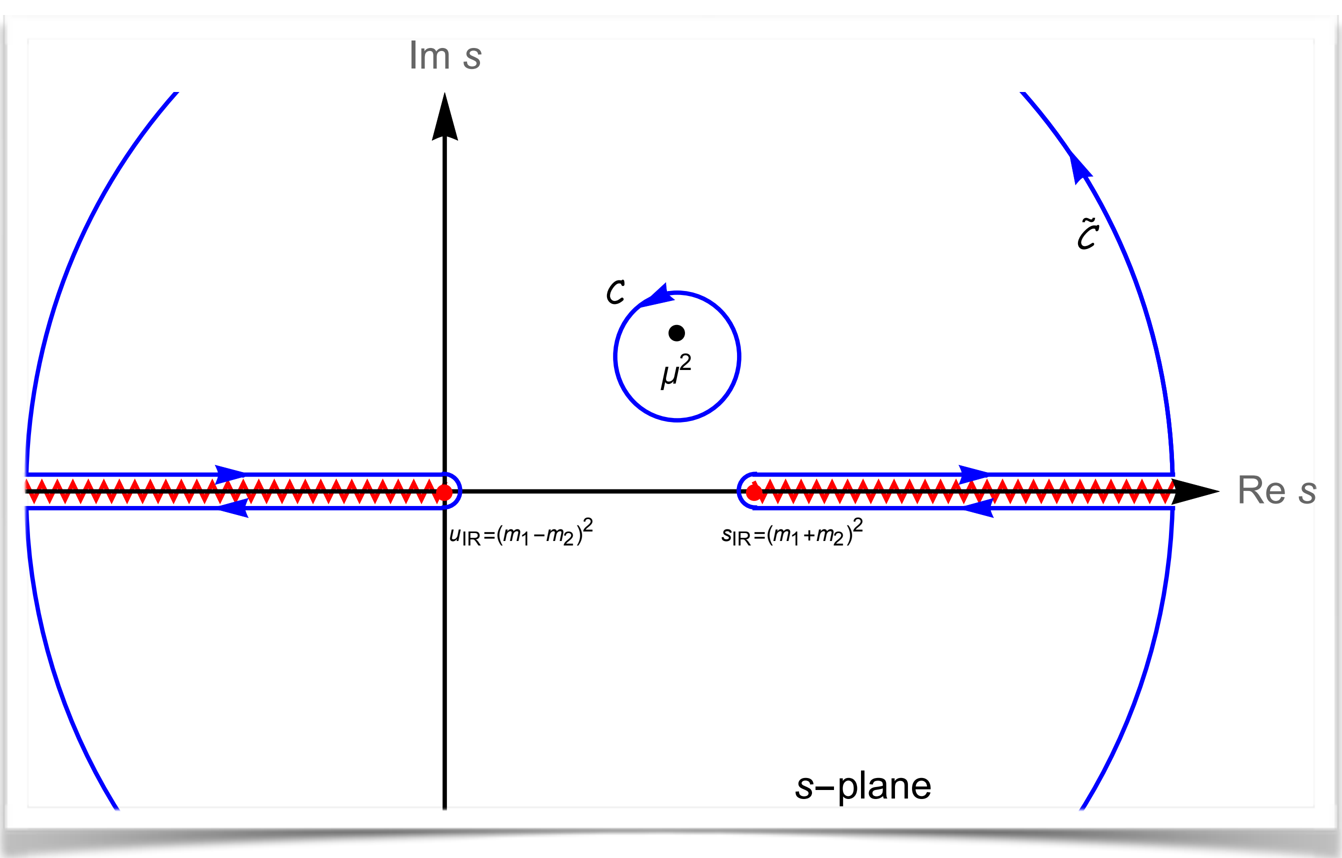
$Disc \sim s\sigma_{Tot}(s) > 0$

t=0



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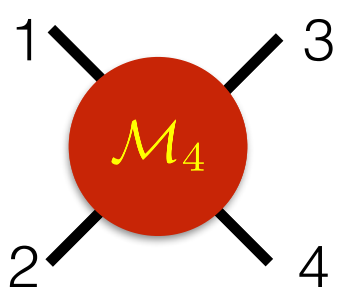

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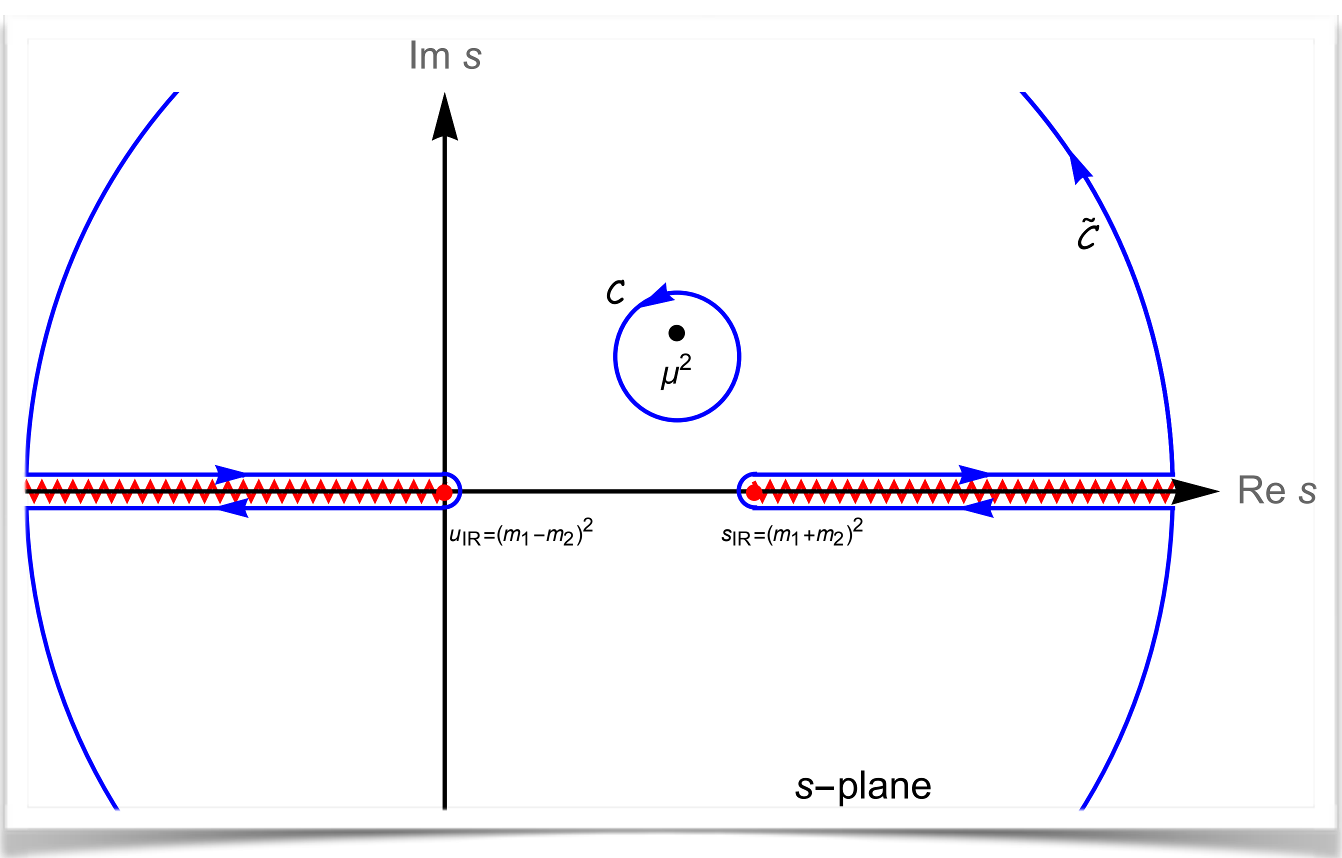

$$\mathcal{M}''(2 \to 2)\Big|_{IR} = \int_0^\infty \frac{ds}{s^3}\sigma_{12\to\text{anything}}(s) > 0$$

IR-side UV-side

E^4 -terms are strictly positive

Example

$$\pi \to \pi + \text{const}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \pi)^2 + \frac{c}{\Lambda^4}(\partial_\mu \pi)^4 + \ldots$$

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$\pi\pi \to \pi\pi$

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This interacting theory can't be softer than E^4

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$$s, t, u$$
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(not amplitudes squared)

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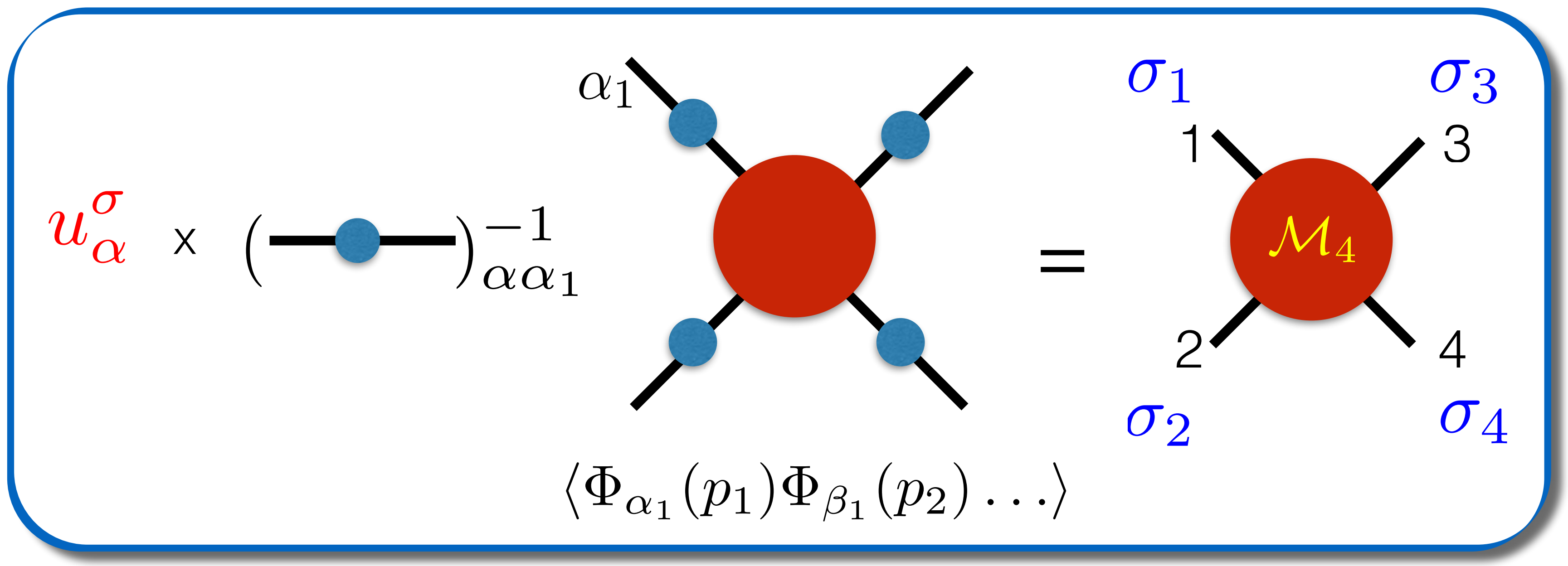

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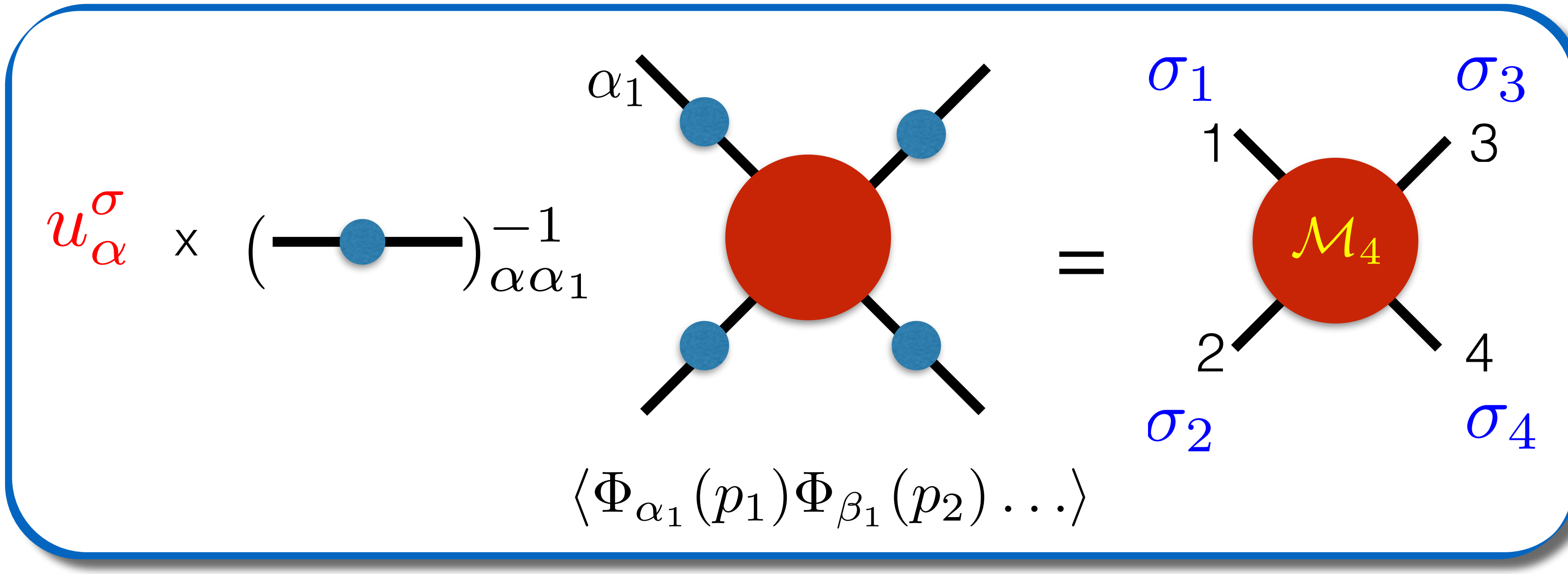

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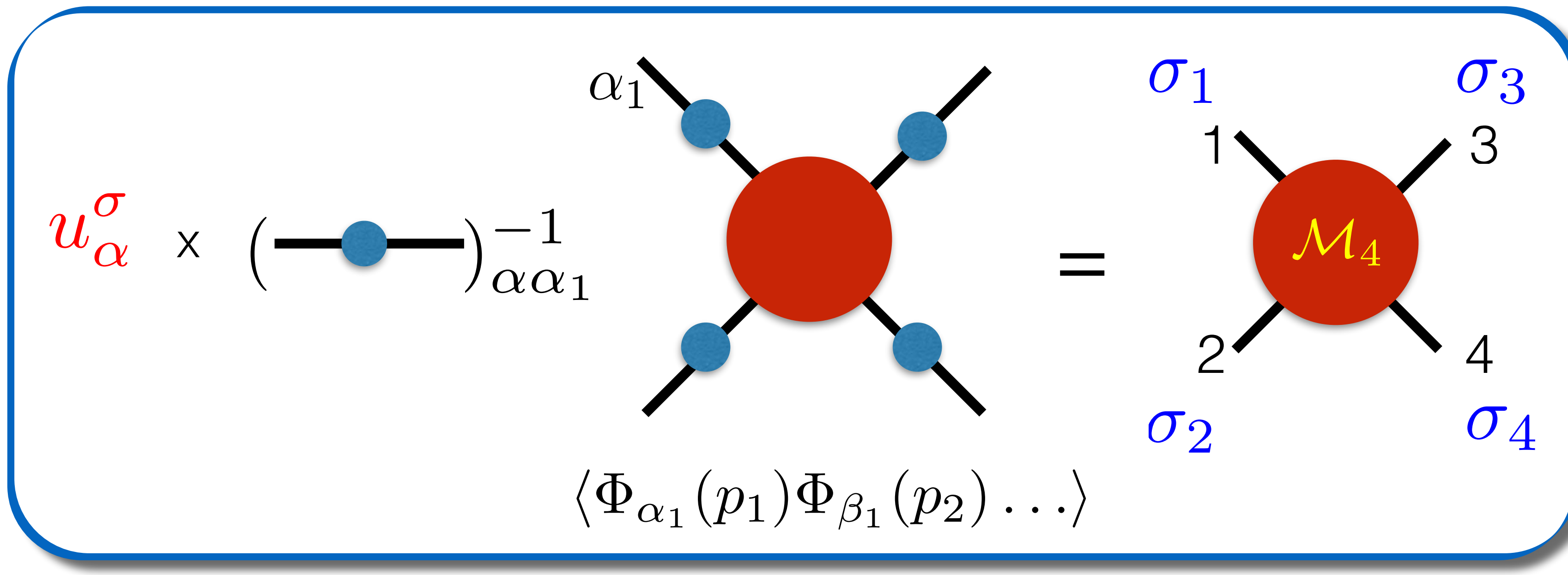

(2) polarizations carry non-analyticities $\begin{cases} \epsilon^L_\mu(\mathbf{p}) \sim (p_z, 0, 0, \sqrt{p_z^2+m^2})^T \\ u(\mathbf{p}) \sim \sqrt{p_\mu \sigma^\mu} \\ \dots \end{cases}$

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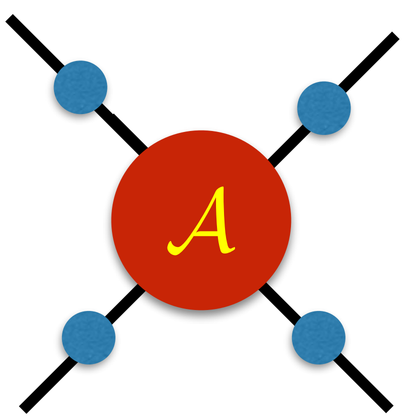

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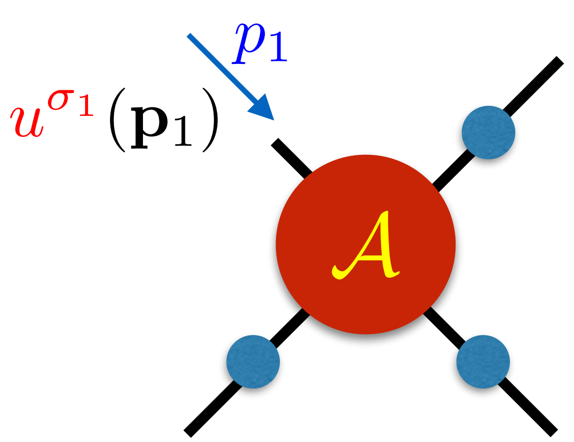

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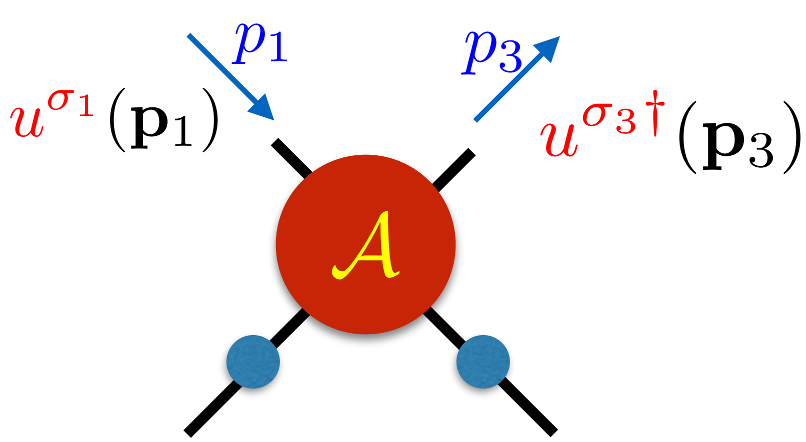

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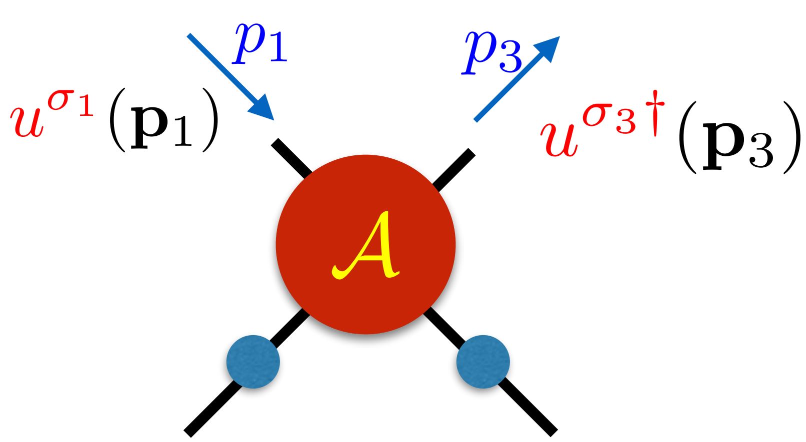

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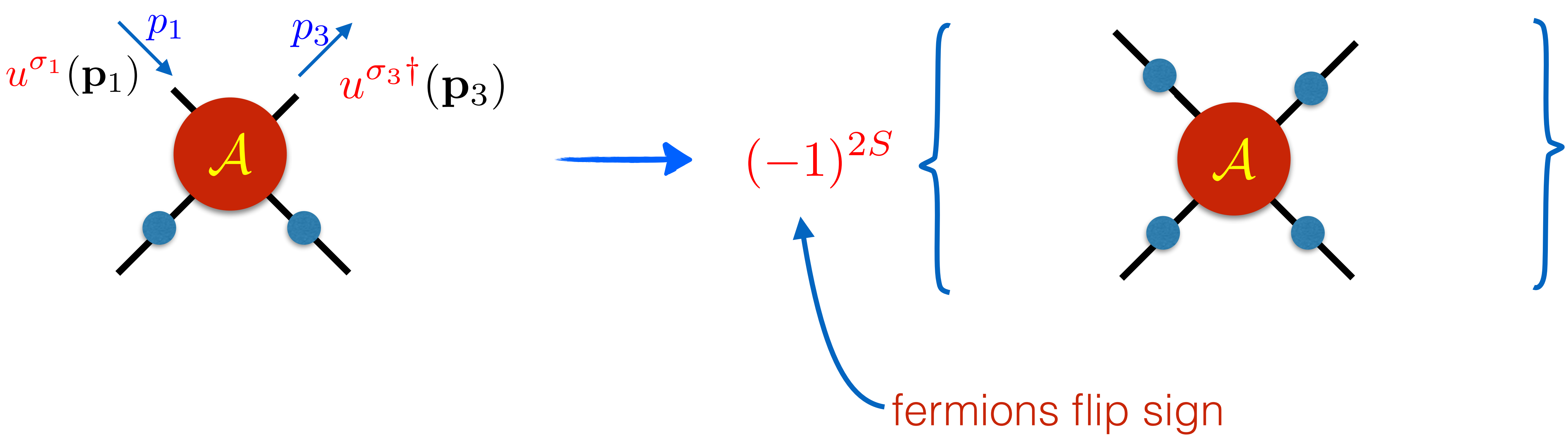

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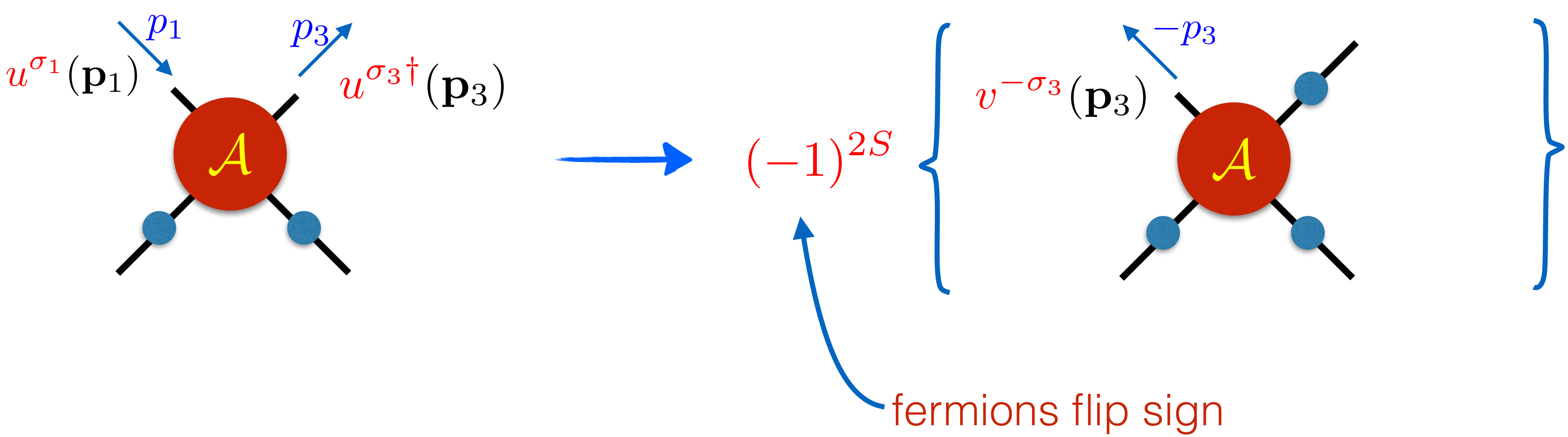

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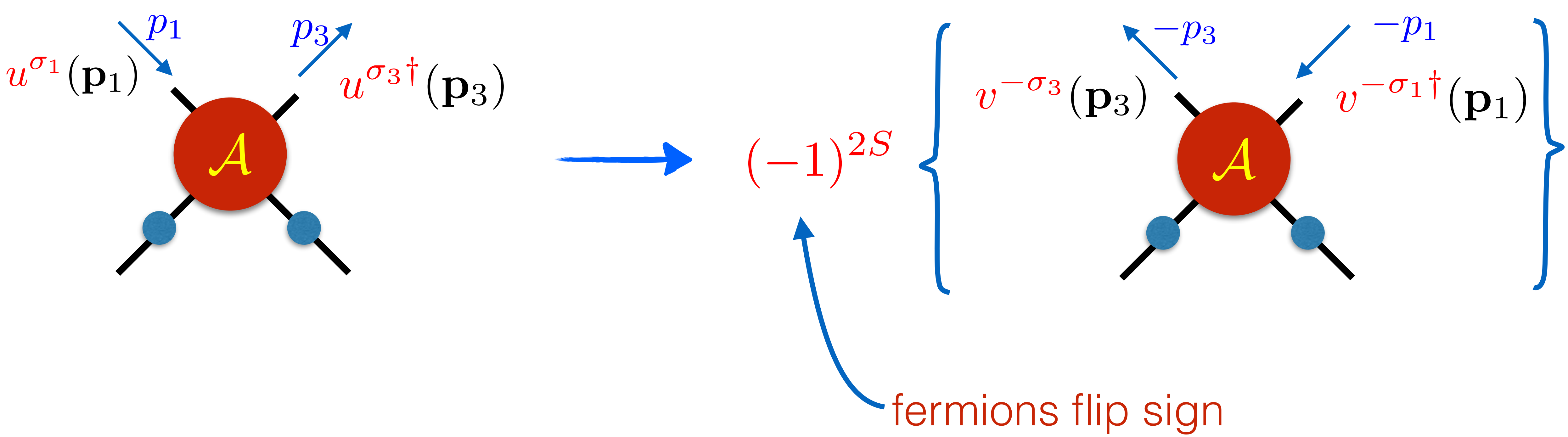

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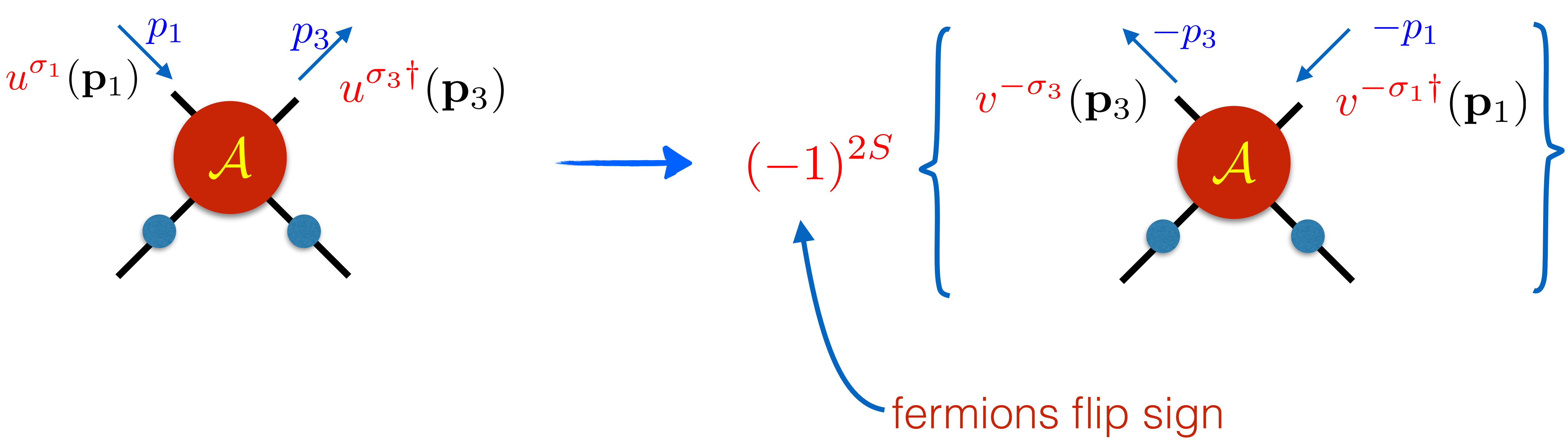

$$p_1 \leftrightarrow -p_3 \qquad\qquad u^{\sigma}(\mathbf{p}_1) \leftrightarrow v^{-\sigma}(\mathbf{p}_3)$$

Forward elastic scattering is special!

All previous issues cancel against each other out

Elastic and Forward

(1) **Lorentz Invariance**

Elastic and Forward

(1) **Lorentz Invariance** forward elastic amp. is invariant

$$\mathcal{M} = \mathcal{M}(s)$$

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$\longrightarrow$

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Elastic and Forward

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$$\overbrace{\quad\quad\quad\quad}^{\rho^{\sigma}(k_1)}$$ =density matrices

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$\rho(-k_1)^{\sigma_1}$?

if so, crossing in elastic scattering **t=0** would be become again **s ⟷ u=-s**

$$\mathcal{M}_{\text{particles}}(s) = \mathcal{M}_{\text{antiparticles}}(u=-s)$$

Elastic and Forward

(4) **Locality**

analytically continue density matrices off-shell

$$u^{\sigma}(\mathbf{k})u^{\sigma\,\dagger}(\mathbf{k}) \equiv \rho(\mathbf{k}) \quad \longrightarrow \quad \rho^{\sigma}(\mathbf{k}) \to \rho^{\sigma}(k)$$

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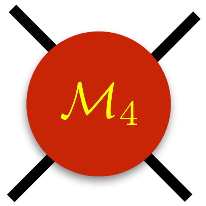
$$\langle T\psi(x_1)\psi^\dagger(x_2)\rangle = \int \frac{d^4k}{(2\pi)^4} e^{-ikx_{12}} \frac{\rho^\sigma(k)}{k^2 - i\epsilon}$$

propagator's numerator:
its parity fixed by Spin-Statistics

Bound on Softness



can amplitudes be **softer** than E^4 ?

(within an EFT)

Bound on Softness

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Universal statement irrespectively spins

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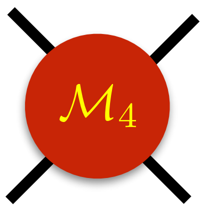

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For Spinning particles:

Analyticity, Crossing, and Unitarity act like for spin-0 in the forward elastic scattering

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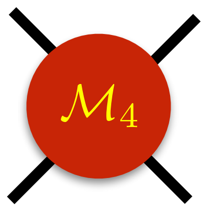

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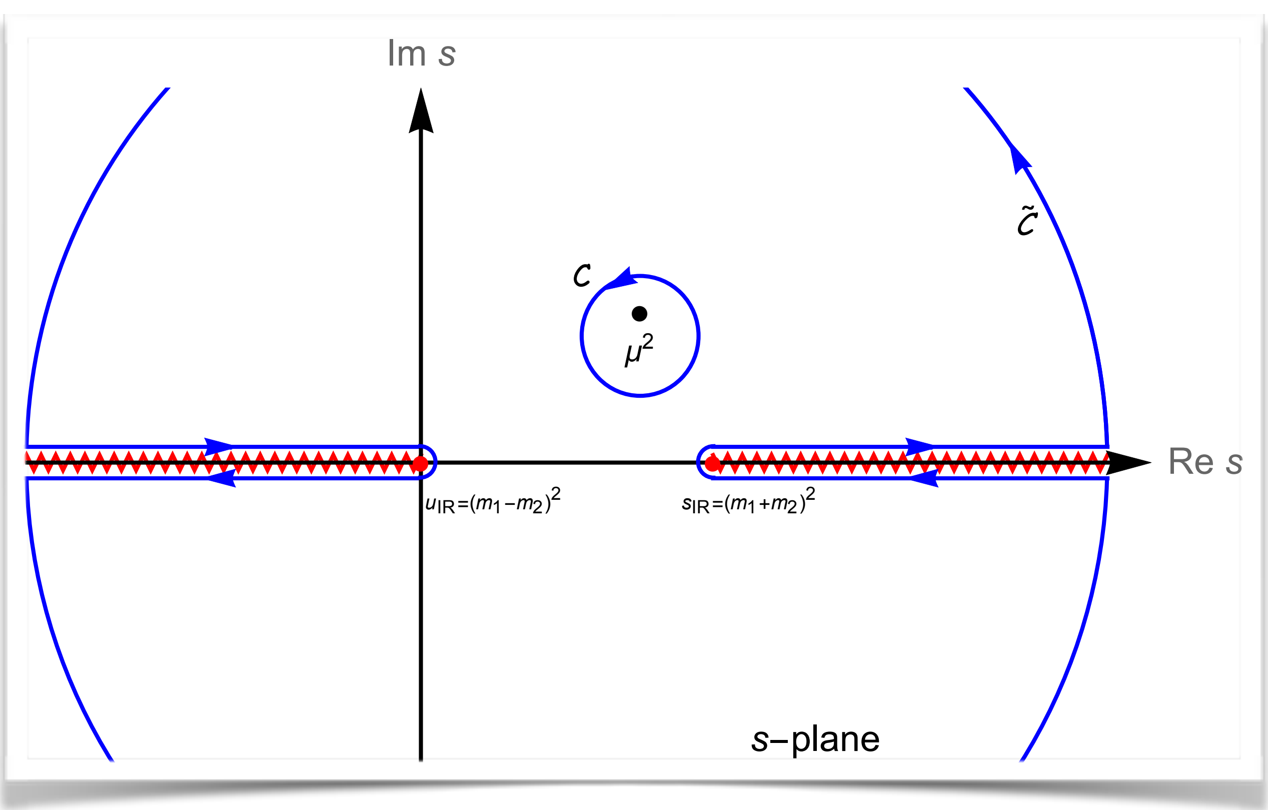

$$\mathcal{M}''(2\to 2)\big|_{IR} = \int_0^\infty \frac{ds}{s^3}\sigma_{12\to\text{anything}}(s) > 0$$

IR-side UV-side

E^4 -terms are strictly positive

Examples

(1) HD partial compositeness

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doesn't admit a local unitary UV completion

Examples

(2) Goldstino $$\psi(x) \to \psi'(x') = \psi(x) + \xi$$

$$x \to x' = x - i\xi\sigma\theta + i\theta^\dagger\sigma\xi$$

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up to field red. →

$$\mathcal{L}_{eff} = -\frac{1}{4F^2} G^{\dagger\,2} \Box G^2 + \dots$$

$$\mathcal{M}(GG \to GG)(s, t = 0) = \frac{s^2}{F^2}$$

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+light fields

$$\begin{cases} \mathcal{L} = -\dfrac{a_\psi}{F^2}(G^\dagger\psi^\dagger)\Box(G\psi) + \dfrac{\widetilde{a}_\psi}{F^2}(\partial_\nu G^\dagger \bar{\sigma}^\mu \partial^\nu G)(\psi^\dagger \bar{\sigma}^\mu \psi) \\ +\dfrac{i a_\pi}{4F^2} \partial_\mu \pi \partial^\nu \pi (G^\dagger \bar{\sigma}^\mu \partial_\nu G) + h.c. \\ -\dfrac{i a_A}{2F^2} (G^\dagger \bar{\sigma}^\mu \partial_\nu G) F_{\mu\rho} F^{\nu\rho} + h.c. \end{cases}$$

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→ $a_\psi > 0 \qquad a_A > 0 \qquad a_\pi > 0$

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(3) Goldstino and R-axion

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$$\mathcal{L} = \int d^4\theta \left(X_{NL}^\dagger X_{NL} + f_a^2 R_{NL}^\dagger R_{NL} \right) + \left(\int d^2\theta \, F X_{NL} + w_R \, R_{NL}^2 + h.c. \right)$$

$$X_{NL}^2 = 0 \qquad X_{NL}(A_{NL} - A_{NL}^\dagger) = 0 \qquad R_{NL} = e^{iA_{NL}} \qquad A_{NL} \to A_{NL} + \epsilon$$

constrained superfields Komargodski Seiberg 0907.2441

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bound on VEV superpotential
Komargodski Festuccia Dine 0910.2527

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bound on VEV superpotential
Komargodski Festuccia Dine 0910.2527

$$w_R < 2 f_a F \longrightarrow \Gamma(a \to GG) < \frac{1}{32\pi} \left(\frac{m_a^5}{F^2} \right)$$

Conclusions

- *Universal bound on softness and positivity of 2-to-2 amplitudes, for arbitrary spins*
- *Unitarity crossing, and analyticity work as usual only in the forward elastic scattering*
- *2-to-2 Amplitudes can't run arbitrarily fast*

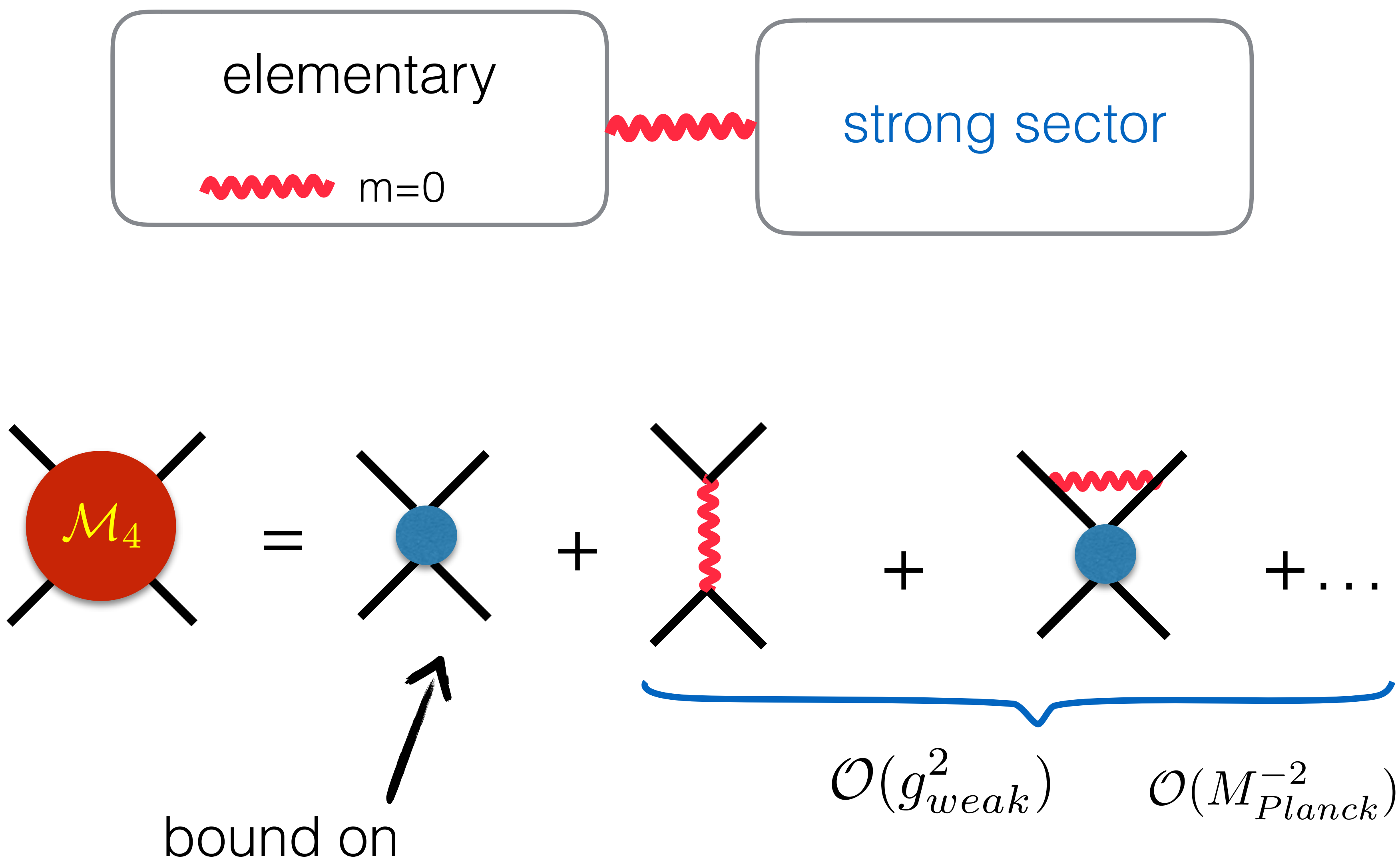
  (low-energy constraints can't be made arbitrarily irrelevant).
- *Non-trivial constraints on EFTs beyond symmetries*

  (R-axion, Goldstino, fermionic shift sym…)

thank you!

backup slides

MASSLESS HIGHER-SPIN STATES



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