

$B \rightarrow PP$ form factors in B -meson semileptonic decays

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Introducing the form factors

- $B \rightarrow 2P$ form factors (FFs), general definition: $(P = \pi, K)$

$$\langle P(k_1)P(k_2)|\bar{q}\Gamma^a b|\bar{B}(p)\rangle = \sum_i L_i^a(k_1, k_2, q) F_i(q^2, k^2, \zeta)$$

- various flavour combinations: $q = u, d, s$, $\bar{B} = \{\bar{B}_{u,d,s}\}$
- kinematics: $k = k_1 + k_2$, $q = p - k$, $\bar{k} = k_1 - k_2$
- generic notation: L_i^a -Lorentz structure, $\Gamma^a = \{\gamma^\mu, \gamma^\mu\gamma_5, \sigma^{\mu\nu}\}$
- the “angular” variable:

$$q \cdot \bar{k} = \frac{1}{2}(2\zeta - 1)\lambda^{1/2}(p^2, q^2, k^2),$$

$(2\zeta - 1) \sim \cos\theta_P$ in dimeson c.m.

- partial wave expansion

$$F_i(q^2, k^2, \zeta) = \sum_\ell F_i^{(\ell)}(q^2, k^2) \mathcal{P}_\ell(\cos\theta_P)$$

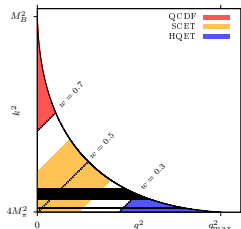
\mathcal{P}_ℓ expressed via the (associated) Legendre polynomials

Introducing the form factors

- $B \rightarrow 2\pi$:
 - used in $B \rightarrow \pi\pi\ell\nu_\ell$ for $|V_{ub}|$ determination
 - enter the rich set of observables in $B \rightarrow \pi\pi\ell\nu_\ell$,
[S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van Dyk, (2013)]
 - serve as factorizable parts of $B \rightarrow 3\pi$ nonleptonic amplitudes
- $B \rightarrow K\pi$ FFs determine the $B \rightarrow K\pi\ell^+\ell^-$ observables
- open questions :
 - S-waves vs P-waves in $B \rightarrow PP$
 - role of higher partial waves, e.g. , $\ell = 2, 4, \dots$ vs $\ell = 0,$
 $\ell = 3, 5, \dots$ vs $\ell = 1$
 - lowest resonance dominance: ρ, ϕ, K^* in P wave, f_0, \dots in S wave
- no lattice QCD calculation of $B \rightarrow PP$ FFs yet
- applying QCD light-cone sum rules (LCSRs) to $B \rightarrow 2P$ FFs

An exploratory study, first results

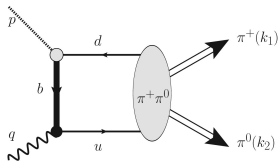
- so far only the form factors in $\bar{B}^0 \rightarrow \pi^+ \pi^0 \ell^- \nu_\ell$,
isospin 1, $\ell = 1, 3, \dots$
- accessible kinematical region
dipion with a small invariant mass
and large recoil:
 $k^2 \lesssim 1 \text{ GeV}^2$, $0 \leq q^2 \leq 12\text{-}14 \text{ GeV}^2$.



- method 1: LCSRs with dipion distribution amplitudes
[Ch. Hambrock, AK, 1511.02509 [hep-ph]]
- method 2: LCSRs with B meson distribution amplitudes
[S. Cheng, AK, J. Virto, work in progress]

LCSRs with dipion distribution amplitudes

- the method: similar to the LCSRs for $B \rightarrow \pi$ form factors,
- vacuum \rightarrow on-shell dipion hadronic matrix elements of nonlocal $\bar{u}(x)d(0)$ operators



- The correlation function:

$$\Pi_\mu(q, k_1, k_2) = i \int d^4x e^{iqx} \langle \pi^+(k_1) \pi^0(k_2) | T \{ \bar{u}(x) \gamma_\mu (1 - \gamma_5) b(x), i m_b \bar{b}(0) \gamma_5 d(0) \} | 0 \rangle$$

- OPE valid at $q^2 \ll m_b^2$ (b -quark virtual),
 $k^2 \ll m_b^2$ (2-pion system produced near the LC)
- nonperturbative input: dipion distribution amplitudes (DAs)
with ρ -resonances "embedded"
- only LO, twist-2 approximation for dipion DAs available

Dipion light-cone DAs

- introduced and developed for $\gamma^* \gamma \rightarrow 2\pi$ processes

[M. Diehl, T. Gousset, B. Pire and O. Teryaev, (1998)

D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes and J. Horejsi, (1994)

M. V. Polyakov, (1999)]

- twist-2 DAs:

$$\langle \pi^+(k_1) \pi^0(k_2) | \bar{u}(x) \gamma_\mu [x, 0] d(0) | 0 \rangle = -\sqrt{2} k_\mu \int_0^1 du e^{iu(k \cdot x)} \Phi_{\parallel}^{l=1}(u, \zeta, k^2),$$

$$\langle \pi^+(k_1) \pi^0(k_2) | \bar{u}(x) \sigma_{\mu\nu} [x, 0] d(0) | 0 \rangle = 2\sqrt{2} i \frac{k_{1\mu} k_{2\nu}^0 - k_{2\mu} k_{1\nu}}{2\zeta - 1} \int_0^1 du e^{iu(k \cdot x)} \Phi_{\perp}^{l=1}(u, \zeta, k^2)$$

- the “angular” variable: $\zeta = k_1^+ / k^+$, $1 - \zeta = k_2^+ / k^+$, $\zeta(1 - \zeta) \geq \frac{m_\pi^2}{k^2}$.

$$q \cdot \bar{k} = \frac{1}{2} (2\zeta - 1) \lambda^{1/2} (p^2, q^2, k^2), \quad \text{in dipion c.m. } (2\zeta - 1) = (1 - 4m_\pi^2/k^2)^{1/2} \cos\theta_\pi,$$

- normalization conditions \rightarrow pion timelike form factors,

$$\int_0^1 du \begin{cases} \Phi_{\parallel}^{l=1}(u, \zeta, k^2) \\ \Phi_{\perp}^{l=1}(u, \zeta, k^2) \end{cases} = (2\zeta - 1) \begin{cases} F_\pi^{em}(k^2) \\ F_\pi^t(k^2) \end{cases} \quad \begin{array}{l} \text{pion e.m. form factor} \\ \text{pion “tensor” form factor} \end{array}$$

- $\Phi_{\perp, \parallel}(u, \zeta, k^2)$ at $k^2 > 4m_\pi^2$ contain Im part
- $F_\pi^{em}(0) = 1$, \bullet “tensor” charge of the pion $F_\pi^t(0) = 1/f_{2\pi}^\perp$

Hadronic dispersion relation

- the ground B -meson state contribution:

$$\Pi_\mu(q, k_1, k_2) = \frac{\langle \pi^+(k_1)\pi^0(k_2) | \bar{u}\gamma_\mu(1 - \gamma_5)b | \bar{B}^0(p) \rangle f_B m_B^2}{m_B^2 - p^2} + \dots,$$

- expansion of $B \rightarrow \pi\pi$ matrix element in form factors:

$$\begin{aligned} i\langle \pi^+(k_1)\pi^0(k_2) | \bar{u}\gamma^\mu(1 - \gamma_5)b | \bar{B}^0(p) \rangle = & -F_\perp(q^2, k^2, \zeta) \frac{4}{\sqrt{k^2\lambda_B}} i\epsilon^{\mu\alpha\beta\gamma} q_\alpha k_{1\beta} k_{2\gamma} \\ & + F_t(q^2, k^2, \zeta) \frac{q^\mu}{\sqrt{q^2}} + F_0(q^2, k^2, \zeta) \frac{2\sqrt{q^2}}{\sqrt{\lambda_B}} \left(k^\mu - \frac{k \cdot q}{q^2} q^\mu \right) \\ & + F_\parallel(q^2, k^2, \zeta) \frac{1}{\sqrt{k^2}} \left(\bar{k}^\mu - \frac{4(q \cdot k)(q \cdot \bar{k})}{\lambda_B} k^\mu + \frac{4k^2(q \cdot \bar{k})}{\lambda_B} q^\mu \right), \end{aligned}$$

- quark-hadron duality in the B -channel, \Rightarrow effective threshold s_0 ,
Borel transformation, $p^2 \rightarrow M^2$

Final results: LCSRs for the form factors at twist-2 LO

- the two sum rules, only the chiral-odd twist-2 DA contributes:

$$\frac{F_{\perp}(q^2, k^2, \zeta)}{\sqrt{k^2} \sqrt{\lambda_B}} = \frac{m_b}{\sqrt{2} f_B m_B^2 (1 - 2\zeta)} \int_{u_0(s_0)}^1 \frac{du}{u} \Phi_{\perp}(u, \zeta, k^2) e^{\frac{m_B^2}{M^2} - \frac{m_b^2 - q^2 \bar{u} + k^2 u \bar{u}}{u M^2}},$$

$$\frac{F_{\parallel}(q^2, k^2, \zeta)}{\sqrt{k^2}} = \frac{m_b}{\sqrt{2} f_B m_B^2 (1 - 2\zeta)} \int_{u_0(s_0)}^1 \frac{du}{u^2} (m_b^2 - q^2 + k^2 u^2) \Phi_{\perp}(u, \zeta, k^2) e^{\frac{m_B^2}{M^2} - \frac{m_b^2 - q^2 \bar{u} + k^2 u \bar{u}}{u M^2}}$$

- an additional relation between the axial-current form factors:

$$\frac{1}{\sqrt{\lambda_B}} (m_B^2 - q^2 - k^2) F_0(q^2, k^2, \zeta) = F_t(q^2, k^2, \zeta) + 2 \frac{\sqrt{k^2} \sqrt{q^2} (2\zeta - 1)}{\sqrt{\lambda_B}} F_{\parallel}(q^2, k^2, \zeta).$$

- additional sum rule obtained from a modified correlator:

$$\frac{F_t(q^2, k^2, \zeta)}{\sqrt{q^2}} = \frac{m_b^2}{\sqrt{2} f_B m_B^2} \int_{u_0}^1 \frac{du}{u^2} (m_b^2 - q^2 + k^2 u^2) \Phi_{\parallel}(u, \zeta, k^2) e^{\frac{m_B^2}{M^2} - \frac{m_b^2 - q^2 \bar{u} + k^2 u \bar{u}}{u M^2}},$$

[AK, work in preparation]

What do we know about LCDAs

[M. V. Polyakov, Nucl. Phys. B 555 (1999) 231.]

- double expansion in Legendre and Gegenbauer polynomials:

$$\Phi_{\perp(\parallel)}(u, \zeta, k^2) = 6u(1-u)F_{\pi}^{t(em)}(0) \sum_{n=0,2,\dots}^{\infty} \sum_{\ell=1,3,\dots}^{n+1} B_{n\ell}^{\perp(\parallel)}(k^2) C_n^{3/2}(2u-1) \beta_{\pi} P_{\ell}^{(0)}\left(\frac{2\zeta-1}{\beta_{\pi}}\right)$$

- input: complex coefficient functions $B_{n\ell}^{\perp,\parallel}(k^2)$ - at $k^2 > 4m_{\pi}^2$
- instanton vacuum model: $n = 0, 2, 4$, valid at small $k^2 \sim 4m_{\pi}^2$

[M. V. Polyakov and C. Weiss, (1999)]

$$B_{01}^{\perp}(k^2) = 1 + \frac{k^2}{12M_0^2}, B_{21}^{\perp}(k^2) = \frac{7}{36} \left(1 - \frac{k^2}{30M_0^2}\right), B_{23}^{\perp}(k^2) = \frac{7}{36} \left(1 + \frac{k^2}{30M_0^2}\right),$$
$$B_{41}^{\perp}(k^2) = \frac{11}{225} \left(1 - \frac{5k^2}{168M_0^2}\right), B_{43}^{\perp}(k^2) = \frac{77}{675} \left(1 - \frac{k^2}{630M_0^2}\right), B_{45}^{\perp}(k^2) = \frac{11}{135} \left(1 + \frac{k^2}{56M_0^2}\right).$$

$f_{2\pi}^{\perp} = 4\pi^2 f_{\pi}^2 / 3M_0 \simeq 650$ MeV, where $f_{\pi} = 132$ MeV is the pion decay constant.

- form factors calculated at $k^2 \sim k_{min}^2 \simeq 4m_{\pi}^2$

Sum rules for partial waves

- form factors expanded in partial waves:

$$F_{\perp,\parallel}(q^2, k^2, \zeta) = \sum_{\ell=1}^{\infty} \sqrt{2\ell+1} F_{\perp,\parallel}^{(\ell)}(q^2, k^2) \frac{P_{\ell}^{(1)}(\cos \theta_{\pi})}{\sin \theta_{\pi}},$$

$\zeta \sim \cos \theta$, $P_{\ell}^{(m)}$ - the (associated) Legendre polynomials

- sum rules for separate partial waves

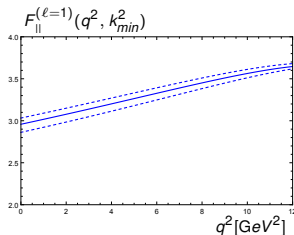
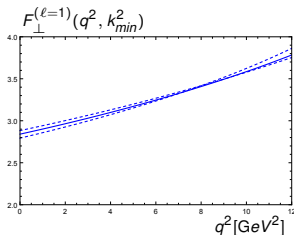
$$F_{\perp}^{(\ell)}(q^2, k^2) = \frac{\sqrt{k^2}}{\sqrt{2}f_{\perp}^{2\pi}} \frac{\sqrt{\lambda_B} m_b}{m_B^2 f_B} e^{m_B^2/M^2} \sum_{n=0,2,\dots} \sum_{\ell'=1,3,\dots}^{n+1} I_{\ell\ell'} B_{n\ell'}^{\perp}(k^2) J_n^{\perp}(q^2, k^2, M^2, s_0^B),$$

$$F_{\parallel}^{(\ell)}(q^2, k^2) = \frac{\sqrt{k^2}}{\sqrt{2}f_{\perp}^{2\pi}} \frac{m_b^3}{m_B^2 f_B} e^{m_B^2/M^2} \sum_{n=0,2,4,\dots} \sum_{\ell'=1,3,\dots}^{n+1} I_{\ell\ell'} B_{n\ell'}^{\perp}(k^2) J_n^{\parallel}(q^2, k^2, M^2, s_0^B),$$

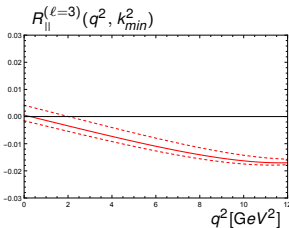
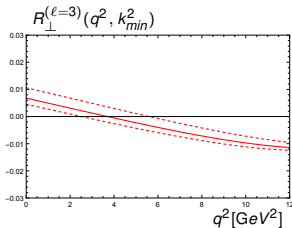
- $I_{\ell\ell'}$ - integrals over Legendre polynomials,
- $J_n^{\perp,\parallel}$ - the Borel-weighted integrals over $C_n^{3/2}(2u-1)$
- in the limit of asymptotic DA, ($B_{01} \neq 0$, $B_{n>0,\ell} = 0$),
only P -wave form factors are $\neq 0$

Numerical results

- *P*-wave form factors: (only twist-2)



- *P*-wave dominance: ratios of *F*- and *P*-wave form factors



--- uncertainties from the variation of M^2 .

How much $B \rightarrow \rho$ contributes to the $B \rightarrow 2\pi$?

- dispersion relations for the $B \rightarrow \pi\pi$ *P-wave* ($\ell = 1$) form factors:

$$\frac{\sqrt{3}F_{\perp}^{(\ell=1)}(q^2, k^2)}{\sqrt{k^2}\sqrt{\lambda_B}} = \frac{g_{\rho\pi\pi}}{m_{\rho}^2 - k^2 - im_{\rho}\Gamma_{\rho}(k^2)} \frac{V^{B \rightarrow \rho}(q^2)}{m_B + m_{\rho}} + \dots$$

and

$$\frac{\sqrt{3}F_{\parallel}^{(\ell=1)}(q^2, k^2)}{\sqrt{k^2}} = \frac{g_{\rho\pi\pi}}{m_{\rho}^2 - k^2 - im_{\rho}\Gamma_{\rho}(k^2)} (m_B + m_{\rho})A_1^{B \rightarrow \rho}(q^2) + \dots$$

$$\Gamma_{\rho}(k^2) = \frac{m_{\rho}^2}{k^2} \left(\frac{k^2 - 4m_{\pi}^2}{m_{\rho}^2 - 4m_{\pi}^2} \right)^{3/2} \theta(k^2 - 4m_{\pi}^2) \Gamma_{\rho}^{\text{tot}},$$

- +... - the contributions of ρ', ρ'', \dots
- using the definition of $B \rightarrow \rho$ FFs:

$$\begin{aligned} \langle \rho^+(k) | \bar{u}\gamma_{\mu}(1 - \gamma_5)b | \bar{B}^0(p) \rangle &= \epsilon_{\mu\alpha\beta\gamma} \epsilon_{\alpha}^{*(\rho)} p^{\beta} k^{\gamma} \frac{2V^{B \rightarrow \rho}(q^2)}{m_B + m_{\rho}} \\ &\quad - i\epsilon_{\mu}^{*(\rho)} (m_B + m_{\rho}) A_1^{B \rightarrow \rho}(q^2) + \dots \end{aligned}$$

LCSRs for $B \rightarrow \rho$ form factors

e.g., [P. Ball and V. M. Braun, Phys. Rev. D **55** (1997) 5561]

- LCSRs for $B \rightarrow \rho$ form factors ($\Gamma_\rho = 0$)
in terms of the ρ -meson DAs in the twist-2 approximation:

$$V^{B \rightarrow \rho}(q^2) = \frac{(m_B + m_\rho)m_b}{2m_B^2 f_B} f_\rho^\perp e^{\frac{m_B^2}{M^2}} \int_{u_0}^1 \frac{du}{u} \phi_\perp^{(\rho)}(u) e^{-\frac{m_b^2 - q^2 \bar{u} + m_\rho^2 u \bar{u}}{uM^2}},$$

$$A_1^{B \rightarrow \rho}(q^2) = \frac{m_b^3}{2(m_B + m_\rho)m_B^2 f_B} f_\rho^\perp e^{\frac{m_B^2}{M^2}} \int_{u_0}^1 \frac{du}{u^2} \phi_\perp^{(\rho)}(u) \left(1 - \frac{q^2 - m_\rho^2 u^2}{m_b^2}\right) e^{-\frac{m_b^2 - q^2 \bar{u} + m_\rho^2 u \bar{u}}{uM^2}}.$$

- both sum rules determined by the chiral-odd ρ -meson DA:

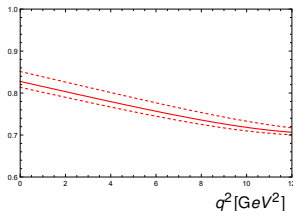
$$\langle \rho^+(k) | \bar{u}(x) \sigma_{\mu\nu} [x, 0] d(0) | 0 \rangle = -if_\rho^\perp (\epsilon_\mu^{*(\rho)} k_\nu - k_\mu \epsilon_\nu^{*(\rho)}) \int_0^1 du e^{iuk \cdot x} \phi_\perp^{(\rho)}(u),$$

- the Gegenbauer polynomial expansion:

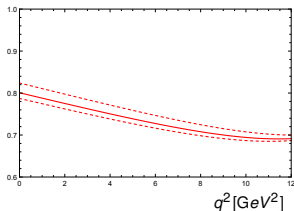
$$\phi_\perp^{(\rho)}(u) = 6u(1-u) \left(1 + \sum_{n=2,4,\dots} a_n^{(\rho)\perp} C_n^{3/2}(2u-1) \right),$$

Numerical estimates

$$\frac{[F_{\perp}^{(\ell=1)}(q^2, k_{min}^2)](\rho)}{[F_{\perp}^{(\ell=1)}(q^2, k_{min}^2)](LCSR)}$$



$$\frac{[F_{\parallel}^{(\ell=1)}(q^2, k_{min}^2)](\rho)}{[F_{\parallel}^{(\ell=1)}(q^2, k_{min}^2)](LCSR)}$$



Relative contribution of ρ -meson to the $B \rightarrow \pi^+ \pi^0$ P-wave form factors $F_{\perp}^{(\ell=1)}(q^2, k_{min}^2)$ (left panel) and $F_{\parallel}^{(\ell=1)}(q^2, k_{min}^2)$ (right panel) from LCSRs.

Dashed lines - the uncertainty due to the variation of the Borel parameter.

Further development and applications

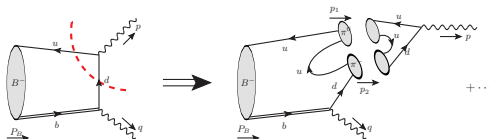
- ansatz for coefficient functions $B_{nl}^{\perp,\parallel}(k^2)$ at $k^2 \lesssim 1 \text{ GeV}^2$
from LCSRs for pion FFs at $k^2 < 0 \oplus$ dispersion representations at $k^2 > 4m_\pi^2$
- including twist-3,4 and $\bar{q}qG$ components of OPE,
identifying twist-3,4 DAs and their double expansions,
the methods used for vector meson DAs [V.Braun et al.]
- NLO gluon radiative corrections
- $B \rightarrow \pi^+\pi^-, \pi^0\pi^0$ channels, including dipions in S, D, \dots -waves
($\ell = 0, 2, \dots$), scalar f_0 and tensor f_2 dominance?,...
need more inputs for corresponding DAs
- $B \rightarrow K\pi(K^*)$ form factors,
SU(3)-violating asymmetry of Gegenbauer moments
- LCSR for S -wave $K\pi$ state in $B \rightarrow K\pi$
[U. G. Meißner, W. Wang, (2014)]
no ζ dependence, asymptotic DA, timelike form factor from ChPT

Alternative method to access $B \rightarrow \pi\pi$ FFs

[S.Cheng, AK, J.Virto, work in progress]

- LCSRs with B -meson DA and $\bar{u}\gamma_\mu d$ interpolating current
- the method introduced to calculate $B \rightarrow P, V$ form factors,

[A.K., N. Offen, Th. Mannel (2006)], "SCET sum rules", [F. De Fazio, Th. Feldmann, T.Hurth (2006)]
 NLO corrections to $B \rightarrow \pi$ [Y-M. Wang, Y-L.Shen (2015)]



- one of the new LCSRs: (preliminary)

$$\int_0^{s_0^2} ds e^{-s/M^2} \frac{[\beta_\pi]^3 \sqrt{s}}{8\sqrt{3}\pi\sqrt{\lambda}} F_\pi^*(s) F_\perp^{(\ell=1)}(s, q^2) = f_B m_B \left\{ \int_0^{\sigma_0^2} d\sigma e^{-s/M^2} \frac{\phi_+^B(\sigma m_B)}{1-\sigma} + \dots \right\},$$

- insert a dispersion relation for $B \rightarrow 2\pi$ form factors and a (dispersion rel. \oplus experiment) parametrization for F_π
- not a direct calculation, given the shape of the $B \rightarrow 2\pi$ form factors, these sum rules can provide normalization

Conclusions

- $B \rightarrow PP$ ($P = \pi, K$) form factors are calculable from LCSRs with dipion DAs at small dipion mass and large recoil
- first exploratory study: all $B \rightarrow \pi^+ \pi^0$ form factors at LO, twist-2
- provide quantitative estimates for P -wave dominance, ρ -meson dominance in P -wave, etc.
- need dedicated studies on dipion DAs \rightarrow coefficient functions
- more accurate data on pion and kaon timelike form factors needed
- LCSRs with B -meson DAs will provide additional constraints on $B \rightarrow PP$ form factors
- providing experiment with viable models for dimeson spectra in $B \rightarrow \pi\pi\nu_\ell$ and $B \rightarrow K\pi\ell\ell$