$B \to \pi \ell^+ \ell^-$ and $B \to K \ell^+ \ell^-$ at large recoil and $|V_{td}/V_{ts}|$ determination

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Introduction

- $B \to \pi \ell^+ \ell^-$ and $B \to K \ell^+ \ell^-$ decays are induced by Flavour Changing Neutral Current (FCNC) $b \to d \ell^+ \ell^-$ and $b \to s \ell^+ \ell^-$, respectively
- These decays are a possible source of New physics
- Both were measured by LHCb collaboration:
  $B \to K \ell^+ \ell^-$: arXiv:1403.8044 [hep-ex]
  $B \to \pi \ell^+ \ell^-$: arXiv:1509.00414 [hep-ex]

Effective Hamiltonian

$$H_{\rm eff}^{b\to q} = \frac{4G_F}{\sqrt{2}}\left(\lambda_u^{(q)}\sum_{i=1}^{2} C_i\,\mathcal{O}_i^u + \lambda_c^{(q)}\sum_{i=1}^{2} C_i\,\mathcal{O}_i^c - \lambda_t^{(q)}\sum_{i=3}^{10} C_i\,\mathcal{O}_i\right) + h.c.$$

$$\lambda_p^{(q)} = V_{pb}V_{pq}^*, \quad p = u, c, t, \quad q = d, s$$

- $B \to K\ell^+\ell^-$: $\quad \lambda_t^{(s)} \approx -\lambda_c^{(s)} \sim \lambda^2 \gg \lambda_u^{(s)} \sim \lambda^4$
- $B \to \pi\ell^+\ell^-$: $\quad \lambda_t^{(d)} \sim \lambda_c^{(d)} \sim \lambda_u^{(d)} \sim \lambda^3$

Amplitude

$$A(B \to P\ell^+\ell^-) =$$

$$= \frac{G_F}{\sqrt{2}} \frac{\alpha_{\rm em}}{\pi} \lambda_t^{(q)} f_{BP}^+(q^2) \Bigg[(\bar\ell \gamma^\mu \ell)\, p_\mu \left(C_9 + \frac{2(m_b+m_q)}{m_B+m_P} C_7^{\rm eff} \frac{f_{BP}^T(q^2)}{f_{BP}^+(q^2)} \right)$$

$$+ (\bar\ell \gamma^\mu \gamma_5 \ell)\, p_\mu C_{10} - (\bar\ell\gamma^\mu\ell) p_\mu \frac{16\pi^2}{f_{BP}^+(q^2)} \left(\frac{\lambda_u^{(q)}}{\lambda_t^{(q)}} \mathcal{H}_P^{(u)}(q^2) + \frac{\lambda_c^{(q)}}{\lambda_t^{(q)}} \mathcal{H}_P^{(c)}(q^2) \right) \Bigg]$$

$f_{BP}^+(q^2), f_{BP}^T(q^2)$ — $B \to P$ transitions form factors
$\mathcal{H}_P^{(u,c)}(q^2)$ — nonlocal hadronic amplitudes

- $B \to K\ell^+\ell^-$: only one amplitude $\mathcal{H}_K^{(c)}(q^2)$
- $B \to \pi\ell^+\ell^-$: two amplitudes $\mathcal{H}_\pi^{(u)}(q^2)$ and $\mathcal{H}_\pi^{(c)}(q^2)$

Hadronic input

- Form Factors

$$\langle P(p)|\bar{q}\gamma^\mu b|B(p+q)\rangle = f_{BP}^{+}(q^2)\,(2p^\mu + q^\mu) + \left(f_{BP}^{+}(q^2) - f_{BP}^{0}(q^2)\right)\frac{m_B^2 - m_P^2}{q^2}q^\mu$$

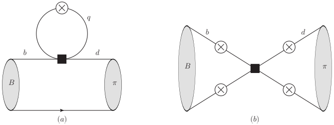
$$\langle P(p)|\bar{q}\sigma^{\mu\nu}q_\nu b|B(p+q)\rangle = \frac{i f_{BP}^{T}(q^2)}{m_B + m_P}\left[2q^2 p^\mu + \left(q^2 - \left(m_B^2 - m_P^2\right)\right)q^\mu\right]$$

- Nonlocal effects via correlation functions

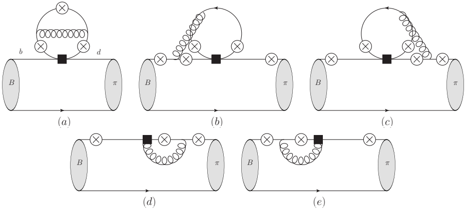
$$\mathcal{H}_{P,\mu}^{(p)} = i\int d^4x e^{iqx}\langle P(p)|\mathrm{T}\Big\{j_\mu^{\mathrm{em}}(x), \Big[C_1\mathcal{O}_1^p(0) + C_2\mathcal{O}_2^p(0) + \sum_{k=3-6,8g} C_k\mathcal{O}_k(0)\Big]\Big\}|B(p+q)\rangle = \left[(p\cdot q)q_\mu - q^2 p_\mu\right]\mathcal{H}_P^{(p)}(q^2)$$

Contributions to $\mathcal{H}_P(q^2)$

- LO, factorizable and weak annihilation (QCD factorization)
  [M. Beneke, Th. Feldmann, D. Seidel (2001)]

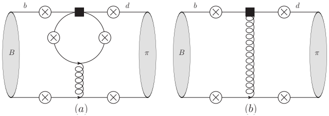


- NLO, factorizable
  [H.H.Asatryan, H.M. Asatrian, C. Greub, M. Walker (2002);
  H.M.Asatrian, K. Bieri, C. Greub, M. Walker (2004)]

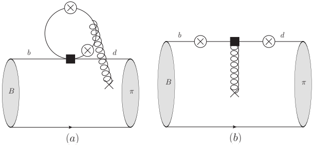


Contributions to $\mathcal{H}_P(q^2)$

- NLO, nonfactorizable (hard gluons)
  [M. Beneke, Th. Feldmann, D. Seidel (2001)]



- Soft gluons, nonfactorizable
  [A. Khodjamirian, Th. Mannel, A.A. Pivovarov, Y.-M. Wang (2010)]
  [A. Khodjamirian, Th. Mannel, Y.-M. Wang (2013)]



Hadronic amplitudes

Use the results for invariant hadronic amplitudes:

- $\mathcal{H}_K^{(c)}(q^2)$ for $B \to K\ell^+\ell^-$:
  [A. Khodjamirian, Th. Mannel, Y.M. Wang (2013)]

- $\mathcal{H}_\pi^{(c)}(q^2)$ and $\mathcal{H}_\pi^{(u)}(q^2)$ for $B \to \pi\ell^+\ell^-$:
  [Ch. Hambrock, A. Khodjamirian, A. Rusov (2015)]

- Both use the same framework and similar input

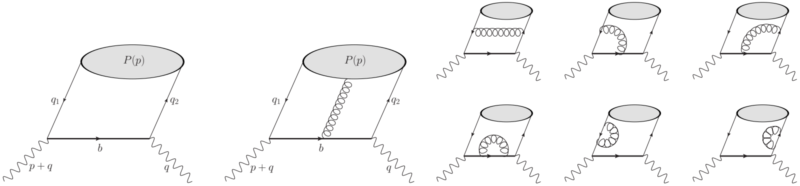
Form Factors from Light-Cone Sum Rules

Underlying correlation functions:

$$F^{\mu}_{BP}(p,q) = i\int d^4x e^{iqx}\langle P(p)|T\{\bar{q}_1(x)\Gamma^{\mu}b(x), m_b\bar{b}(0)i\gamma_5 q_2(0)\}|0\rangle$$

$$= \begin{cases} F_{BP}(q^2,(p+q)^2)p^{\mu} + \tilde{F}_{BP}(q^2,(p+q)^2)q^{\mu}, & \Gamma^{\mu} = \gamma^{\mu} \\ F^{T}_{BP}(q^2,(p+q)^2)\left[q^2p^{\mu} - (q\cdot p)q^{\mu}\right], & \Gamma^{\mu} = -i\sigma^{\mu\nu}q_{\nu} \end{cases}$$

$$B^+ \to K^+ :\ q_1 = u,\ q_2 = s, \qquad B^+ \to \pi^+ :\ q_1 = u,\ q_2 = d$$



The OPE result

$$\{F^{(T)}_{B\pi}(q^2), F^{(T)}_{BK}(q^2)\} \Rightarrow \mathrm{OPE} \Rightarrow \{f^{+,T}_{B\pi}(q^2), f^{+,T}_{BK}(q^2)\}$$

$$\mathrm{OPE} = \left(T_0^{(2)} + (\alpha_s/\pi)T_1^{(2)} + (\alpha_s/\pi)^2 T_2^{(2)}\right) \otimes \varphi_P^{(2)}$$

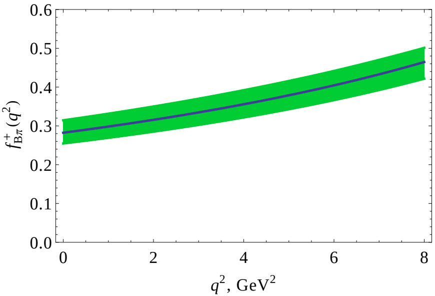
$$+\frac{\mu_P}{m_b}\left(T_0^{(3)} + (\alpha_s/\pi)T_1^{(3)}\right) \otimes \varphi_P^{(3)} + \frac{\delta_\pi^2}{m_b\chi} T_0^{(4)} \otimes \varphi_P^{(4)}$$

$$+\langle\bar{q}q\rangle\left(T_0^{(5)} \otimes \varphi_P^{(2)} + T_0^{(6)} \otimes \varphi_P^{(3)}\right)$$

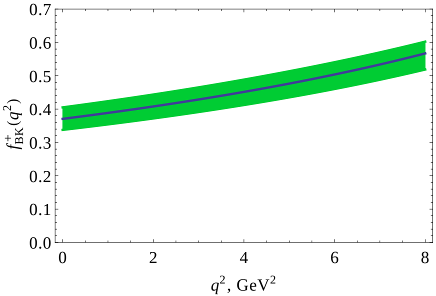
- LO twist 2, 3, 4 $q\bar{q}$ and $\bar{q}qG$ terms
  [V.Belyaev, A.Khodjamirian, R.Rückl (1993);
  V.Braun, V.Belyaev, A.Khodjamirian, R.Rückl (1996)]
- NLO $O(\alpha_s)$ twist 2 (collinear factorization)
  [A.Khodjamirian, R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997);]
- NLO $O(\alpha_s)$ twist 3 (coll. factorization for asympt. DA)
  [P. Ball, R. Zwicky (2001); G.Duplancic, A.Khodjamirian, B.Melic, Th.Mannel, N.Offen (2007)]
- LO twist 5 and twist 6 in factorization approximation
  [A. Rusov (paper in preparation)]
- Part of NNLO $O(\alpha_s^2\beta_0)$ twist 2 [A. Bharucha (2012)]

$B \to \pi, K$ form factors from LCSR: results

$B \to \pi$



$B \to K$



Recalculating both form factors with the same input as in [Imsong et al. (2015)] and for the kaon DA using the same parameters as in [A. Khodjamirian, Th. Mannel, Y.M. Wang (2013)]

Preliminary!

$B \to P\ell^+\ell^-$ phenomenology

- Differential branching fraction of the $B \to P\ell^+\ell^-$ decay

$$\frac{1}{\tau_{B^-}} \frac{dB(B^- \to P^-\ell^+\ell^-)}{dq^2} = \frac{G_F^2 \alpha_{\rm em}^2 |\lambda_t^{(q)}|^2}{1536\pi^5 m_B^3} |f_{BP}^+(q^2)|^2 \lambda^{3/2}(m_B^2, m_P^2, q^2)$$

$$\times \left\{ \left| C_9 + \Delta C_9^{(BP)}(q^2) + \frac{2(m_b + m_q)}{m_B + m_P} C_7 \frac{f_{BP}^T(q^2)}{f_{BP}^+(q^2)} \right|^2 + |C_{10}|^2 \right\}$$

$$\Delta C_9^{(BP)}(q^2) = \frac{16\pi^2}{f_{BP}^+(q^2)} \left[\mathcal{H}_P^{(c)}(q^2) + \frac{\lambda_u^{(q)}}{\lambda_t^{(q)}} \left(\mathcal{H}_P^{(c)}(q^2) - \mathcal{H}_P^{(u)}(q^2) \right) \right]$$

- Ratio of the differential branching fractions:

$$R(q^2) \equiv \frac{dB}{dq^2}(B^\pm \to \pi^\pm \ell^+\ell^-) / \frac{dB}{dq^2}(B^\pm \to K^\pm \ell^+\ell^-)$$

$B \to P\ell^+\ell^-$ phenomenology

$$R(q^2) = \left(\frac{\lambda(m_B^2, m_\pi^2, q^2)}{\lambda(m_B^2, m_K^2, q^2)}\right)^{3/2} \left|\frac{V_{td}}{V_{ts}}\right|^2 \left(\frac{f_{B\pi}^+(q^2)}{f_{BK}^+(q^2)}\right)^2 \times$$

$$\times \quad \frac{\left|A^{(\pi)}(q^2) + B^{(\pi,c)}(q^2) + \alpha_V B^{(\pi,c-u)}(q^2)\right|^2 + |C_{10}|^2}{\left|A^{(K)}(q^2) + B^{(K,c)}(q^2)\right|^2 + |C_{10}|^2}$$

- $A^{(P)}(q^2) = C_9 + \frac{2(m_b + m_q)}{m_B + m_\pi} C_7 \frac{f_{BP}^T(q^2)}{f_{BP}^+(q^2)}$
- $B^{(P,c)}(q^2) = \frac{16\pi^2 \mathcal{H}_P^{(c)}(q^2)}{f_{BP}^+(q^2)}, \; B^{(\pi,c-u)}(q^2) = \frac{16\pi^2 \left[\mathcal{H}_P^{(c)}(q^2) - \mathcal{H}_P^{(u)}(q^2)\right]}{f_{BP}^+(q^2)}$
- $\alpha_V \equiv \frac{\lambda_u^{(d)}}{\lambda_t^{(d)}} = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}$

Experimental data on $B^+ \to K^+\ell^+\ell^-$ and $B^+ \to \pi^+\ell^+\ell^-$

q^2 bin (GeV^2)	$B^K[q_1^2, q_2^2](10^{-9}\mathrm{GeV}^{-2})$
0.1 – 0.98	$33.2 \pm 1.8 \pm 1.7$
1.1 – 2.0	$23.3 \pm 1.5 \pm 1.2$
2.0 – 3.0	$28.2 \pm 1.6 \pm 1.4$
3.0 – 4.0	$25.4 \pm 1.5 \pm 1.3$
4.0 – 5.0	$22.1 \pm 1.4 \pm 1.1$
5.0 – 6.0	$23.1 \pm 1.4 \pm 1.2$
6.0 – 7.0	$24.5 \pm 1.4 \pm 1.2$
7.0 – 8.0	$23.1 \pm 1.4 \pm 1.2$
1.1 – 6.0	$24.2 \pm 0.7 \pm 1.2$

[arXiv:1403.8044 [hep-ex]]

q^2 bin (GeV^2)	$B^\pi[q_1^2, q_2^2](10^{-9}\mathrm{GeV}^{-2})$
0.1 – 2.0	$1.89\,^{+0.47}_{-0.41} \pm 0.06$
2.0 – 4.0	$0.62\,^{+0.39}_{-0.33} \pm 0.02$
4.0 – 6.0	$0.85\,^{+0.32}_{-0.27} \pm 0.02$
6.0 – 8.0	$0.66\,^{+0.30}_{-0.25} \pm 0.02$
1.0 – 6.0	$0.91\,^{+0.21}_{-0.20} \pm 0.03$

[arXiv:1509.00414 [hep-ex]]

$$\mathcal{B}^P[q_1^2, q_2^2] \equiv \frac{1}{q_2^2 - q_1^2} \int_{q_1^2}^{q_2^2} dq^2 \frac{dB(B \to P\ell^+\ell^-)}{dq^2}$$

Ratio $|V_{ts}/V_{td}|$

- PDG 2014 (from the mixing of $B_d^0 \leftrightarrow \bar{B}_d^0$ and $B_s^0 \leftrightarrow \bar{B}_s^0$):

$$|V_{td}/V_{ts}| = 0.216 \pm 0.011$$

- LHCb 2015 (exp. data on $B^+ \to (K^+, \pi^+)\mu^+\mu^-$)

$$|V_{td}/V_{ts}| = 0.24^{+0.05}_{-0.04}$$

- Fermilab Lattice and MILC (using Lattice $B \to P$ form factors):

$$|V_{td}/V_{ts}|_{\mathrm{low}-q^2} = 0.25 \pm 0.04, \quad |V_{td}/V_{ts}|_{\mathrm{high}-q^2} = 0.19 \pm 0.02$$

$$|V_{td}/V_{ts}| = 0.20 \pm 0.02$$

- Very preliminary results:

3 bins $[2-4], [4-6], [6-8]$ — 1 bin $[1-6]$

$$|V_{td}/V_{ts}| = 0.22 \pm 0.03 \qquad |V_{td}/V_{ts}| = 0.24 \pm 0.04$$

α_V can not be extracted reliably (taken from global CKM fit)

Conclusion

- $B \to (\pi, K)\ell^+\ell^-$ decays analysed at large hadronic recoil
  - * LCSR for $B \to \pi, K$ form factors revisited
  - * Nonlocal effects accounted
  - * The ratio of the differential branching fractions of the $B \to (\pi, K)\ell^+\ell^-$ decays predicted
  - * The ratio of $|V_{td}/V_{ts}|$ extracted
- $B_s \to K$ form factors and $B_s \to K\ell\nu_\ell$ in progress

Backup

Transition to $B \to P$ form factors

$$f_{BP}^{+}(q^2) = \frac{e^{m_B^2/M^2}}{2m_B^2 f_B} F_{BP}(q^2, s_0^B, M^2)$$

$$f_{BP}^{0}(q^2) = \frac{e^{m_B^2/M^2}}{2m_B^2 f_B} \Bigg[F_{BP}(q^2, s_0^B, M^2) + \frac{q^2}{m_B^2 - m_P^2} \left[2\tilde{F}_{BP}(q^2, s_0^B, M^2) - F_{BP}(q^2, s_0^B, M^2) \right] \Bigg]$$

$$f_{BP}^{T}(q^2) = \frac{(m_B + m_P) e^{m_B^2/M^2}}{2m_B^2 f_B} F_{BP}^{T}(q^2, s_0^B, M^2)$$

s_0^B — continuum threshold, M^2 — Borel parameter

$$F(q^2, s_0^B, M^2) = F_0(q^2, s_0^B, M^2) + \frac{\alpha_s C_F}{4\pi} F_1(q^2, s_0^B, M^2)$$

Fitting

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{R_i^{exp} - R_i^{th}\{\alpha_k\}}{\sigma_i} \right)^2$$

- $\{\alpha_k\}$ - fitted parameters, N - number of bins
- Binned differential branching fraction:

$$\mathcal{B}^P[q_1^2, q_2^2] \equiv \int_{q_1^2}^{q_2^2} dq^2 \, \frac{dB(B \to P\ell^+\ell^-)}{dq^2}$$

- Bins of the ratio $R(q^2)$:

$$R^{exp}[q_1^2, q_2^2] = \frac{\mathcal{B}^{\pi,exp}[q_1^2, q_2^2]}{\mathcal{B}^{K,exp}[q_1^2, q_2^2]}, \quad R^{th}[q_1^2, q_2^2] = \frac{\mathcal{B}^{\pi,th}[q_1^2, q_2^2]}{\mathcal{B}^{K,th}[q_1^2, q_2^2]}$$