The Information Paradox
Quantum Mechanics and Black Holes

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The quest for Quantum Gravity

Gravity as a QFT is non-renormalizable

String Theory
Why Quantum Gravity?

Space-time at short scales/scattering at $E \gtrsim 10^{19}$ GeV

Cosmology

Black holes
Classical black holes

\[ ds^2 = - \left( 1 - \frac{2GM}{r} \right) dt^2 + \left( 1 - \frac{2GM}{r} \right)^{-1} dr^2 + r^2 d\Omega^2 \]

Horizon: \( r = 2GM \)
Singularity: \( r = 0 \)
Hawking radiation

Hawking found that black holes emit thermal body radiation at temperature

\[ T = \frac{1}{8\pi GM} \]

They lose energy and after a long time they evaporate

Evaporation time

\[ t_{\text{evap}} \sim G^2 M^3 \]
Black hole entropy

From the Hawking temperature \( T = \frac{1}{8\pi GM} \), the radius of the horizon \( R = 2GM \) and thermodynamics we find the Bekenstein-Hawking entropy:

\[
S = \frac{A}{4G}
\]

Scales like area instead of volume.

\[
S_{\text{solarBH}} \approx 10^{77} \quad \text{while} \quad S_{\text{sun}} \approx 10^{57}
\]

In Statistical Mechanics

\[
S = \log N
\]

Number of states for solar-mass BH: \( N = e^{10^{77}} \), while in general relativity \( N = 1 \)

What are the “microstates” of a black hole?
The information paradox

\[ |\Psi(0)\rangle \rightarrow |\Psi(t)\rangle = e^{-iHt} |\Psi(0)\rangle \]

Inconsistent with **unitary** evolution in quantum mechanics

\[ \rho = \frac{e^{-\beta H}}{Z} \]
Unitarity from small corrections

\[ \rho = \rho_{\text{thermal}} + e^{-S} \rho_{\text{cor}} \]

Exponentially small correlations between the outgoing Hawking particles contain the information.

To preserve unitarity the Hawking particles must be entangled to each other.
Entanglement near the horizon

Hawking particles are produced in entangled pairs

This entanglement is necessary for the smoothness of spacetime near the horizon

Example: flat space, Unruh effect

\[ |0\rangle_M = \sum_{n=0}^{\infty} e^{-\pi \omega n} |n\rangle_L \otimes |n\rangle_R \]

\[ |\Psi\rangle = |0\rangle_L \otimes |0\rangle_R \rightarrow \langle T_{\mu\nu} \rangle \neq 0 \]
The firewall paradox

[Mathur], [Almheiri, Marolf, Polchinski, Sully]

For information to escape black hole: $B$ must be entangled with $A$.

For horizon to be smooth: $B$ must be entangled with $C$.

This violates the **monogamy of entanglement** for the particle $B$. 
Is the horizon smooth?

Breaking the B-C entanglement near the horizon creates a huge energy density creating a “firewall” on the horizon, which would burn up an infalling observer.

Fuzzball proposal: every black hole microstate corresponds to a different geometry behind the black hole horizon.
These proposals would be able to solve the information paradox, however they lead to massive violations of general relativity.

The curvature of spacetime near the black hole horizon is

$R_{ijkl}R^{ijkl} \sim \frac{1}{(GM)^4}$

For a large black hole this curvature is very low and we expect standard general relativity to hold.
General Relativity vs Quantum Mechanics

Proposed resolutions of information paradox

1. Information is lost

2. Remnants

3. Information is encoded in radiation but general relativity violated at horizon (firewall, fuzzball)
   
   but ideally we would like

4. Information is encoded in radiation and horizon is smooth, as predicted by GR
The AdS/CFT correspondence

A $d + 1$-dimensional theory of gravity with negative cosmological constant, is equivalent to a $d$-dimensional large $N$ $SU(N)$ gauge theory without gravity

Questions about quantum gravity can be translated in the QFT

QFT is strongly coupled

Emergence of extra AdS-dimension from QFT remains mysterious (related to RG-scale)
Black Holes in AdS/CFT

Quark gluon plasma

Black Hole in AdS

Understanding of black hole entropy, dynamical properties of QGP (viscosity, etc.)

AdS/CFT settles that information is not lost, no remnants
Spacetime behind the horizon

\[ \mathcal{O} = \text{Tr}[F_{\mu\nu}F^{\mu\nu}] \]

\[ \phi(x) = \int dy K(y; x) \mathcal{O}(y) \]

What are the operators \( \tilde{\mathcal{O}} \)?
Spacetime behind the horizon

- Until recently, understanding of black hole interior in AdS/CFT was limited

- In last few years we developed a proposal for the holographic description of the BH interior [K.P. and S. Raju, also with S. Banerjee and J.W. Bryan]
  

- This has provided some new insights for the modern version of the information paradox

- It is important to make further checks and to expand into a complete mathematical framework

- Concretely: we identified CFT operators $\widetilde{O}$ relevant for describing the black hole interior
Quantum fields outside the black hole are in an entangled state. We diagonalize the entanglement by going to a “Schmidt basis”

$$|\Psi\rangle = \sum_a d_a |\hat{\Psi}_a^{\text{out}}\rangle \otimes |\hat{\psi}_a^{\text{BH}}\rangle$$

For every operator outside the black hole

$$\mathcal{O} = \sum_{ij} \mathcal{O}_{ij} |\hat{\Psi}_i^{\text{out}}\rangle \langle \hat{\Psi}_j^{\text{out}}|$$

the entanglement naturally selects a “mirror operator” inside the black hole

$$\tilde{\mathcal{O}} = \sum_{ij} \mathcal{O}_{ij}^* |\hat{\psi}_i^{\text{BH}}\rangle \langle \hat{\psi}_j^{\text{BH}}|$$
Define antilinear map
\[ SA|\Psi\rangle = A^\dagger|\Psi\rangle \]
and
\[ \Delta = S^\dagger S \quad J = S\Delta^{-1/2} \]
Then the operators
\[ \tilde{O} = JOJ \]
i) commute with \( O \)
ii) are correctly entangled with \( O \)
These are the operators that we need for the Black Hole interior.
In the large \( N \) gauge theory for a Quark-Gluon-Plasma state in equilibrium, we find using the KMS condition
\[ \Delta = e^{-\beta(H-E)} \]
Spacetime behind the horizon

Black hole interior described by gauge theory operators $\tilde{O}$ defined by

$$\tilde{O}|\Psi\rangle = e^{-\frac{\beta H}{2}} O e^{\frac{\beta H}{2}} |\Psi\rangle$$

$$\tilde{O}O...O|\Psi\rangle = O...O\tilde{O}|\Psi\rangle$$
Reconstructing the black hole interior

Using these operators in AdS/CFT we showed that the horizon is a smooth region of space-time as predicted by general relativity.

Quantum field inside the black hole

$$\phi(t, r, \Omega) = \int_0^\infty d\omega \left[ \mathcal{O}_\omega f_\omega(t, \Omega, r) + \tilde{\mathcal{O}}_\omega g_\omega(t, \Omega, r) + h.c. \right]$$

Correlation functions of these operators

$$\langle \Psi | \phi(t_1, r_1, \Omega_1) \ldots \phi(t_n, r_n, \Omega_n) | \Psi \rangle$$

reproduce those of effective field theory in the exterior/interior of the black hole, without any indication for a firewall or fuzzball.

At the same time, the entire framework is unitary.

How have we been able to avoid the previous paradox?
Non-locality in Quantum Gravity

\[ [\mathcal{O}, \tilde{\mathcal{O}}] \approx 0 \text{ in simple correlators, not as exact operator equation} \]

\[ [\phi(P), \phi(Q)] = O(e^{-S}) \]

Hilbert space of Quantum Gravity does not factorize as

\[ \mathcal{H} \neq \mathcal{H}_{\text{inside}} \otimes \mathcal{H}_{\text{outside}} \]

Solves problem of Monogamy of Entanglement

Concrete realization of “Black Hole Complementarity”. We showed it is consistent with approximate locality in effective field theory
State-dependence of operators

Interior operators defined by

$$\tilde{O}|\Psi\rangle = e^{-\frac{\beta H}{2}} \mathcal{O} e^{\frac{\beta H}{2}} |\Psi\rangle \quad \tilde{O} \mathcal{O} \ldots \mathcal{O} |\Psi\rangle = \mathcal{O} \ldots \mathcal{O} \tilde{O} |\Psi\rangle$$

We notice the specific black hole microstate $|\Psi\rangle$ entering the equation

Operators depend on the state, they are defined in “patches” on the Hilbert space

Unusual in Quantum Mechanics, needs further study
Connection to ER = EPR

Entanglement & Wormholes [Maldacena, Susskind, Raamsdonk]

\[ H = H_L + H_R \]

\[ |\text{TFD}\rangle = \sum_E \frac{e^{-\beta E/2}}{\sqrt{Z}} |E\rangle_L \otimes |E\rangle_R \]
Ryu-Takayanagi proposal ⇒ Entanglement determines geometry of spacetime

Einstein equations from dynamics of entanglement

ER/EPR correspondence
Summary

1. The information paradox, and its recent reformulation as the firewall paradox is a fundamental conflict between general relativity and quantum mechanics.

2. Understanding how to resolve it may lead us towards the fundamental principles of quantum gravity.

3. I presented a proposal for its resolution, which relies on non-locality in quantum gravity and on state-dependence of observables.

4. More generally entanglement and quantum information seem to be increasingly important for understanding the quantum nature of spacetime.
Thank you