

# Understanding cosmic ray small-scale anisotropies

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# Outline

① Observations

② Large-scale anisotropies

③ Models for small-scale anisotropies

- Magnetic lenses etc.
- Non-uniform pitch-angle scattering
- Small-scale turbulence
- Heliospheric effects

# Outline

① Observations

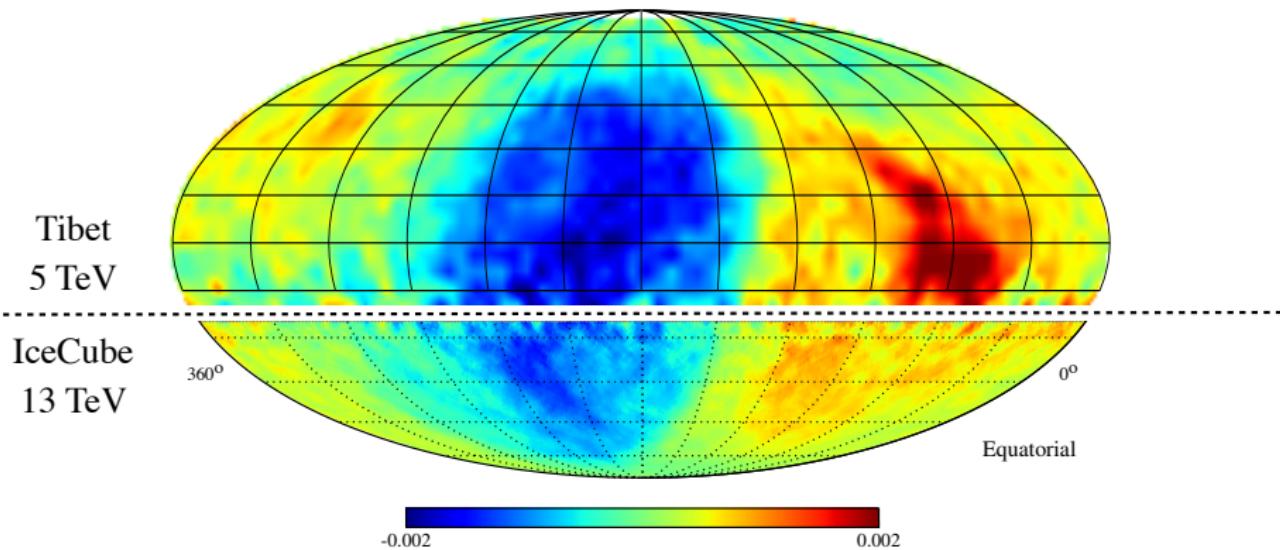
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# Cosmic ray anisotropies

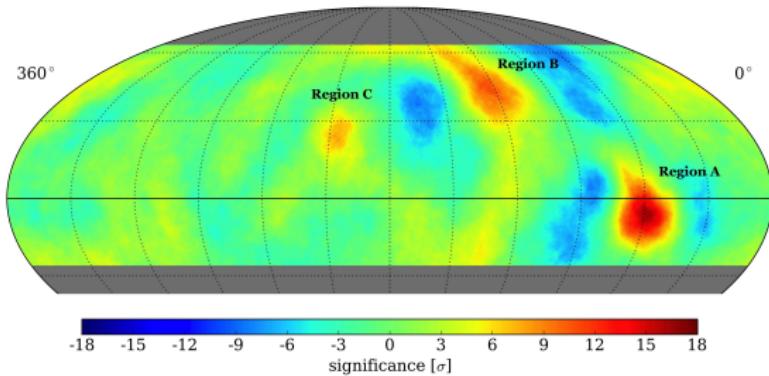
$$I(\mathbf{n}) \equiv \frac{\phi(\mathbf{n})}{\phi^{\text{iso}}} \equiv 1 + \delta I(\mathbf{n})$$



Amenomori *et al.*, ApJ 711 (2010) 119, Saito *et al.*, Proc. 32nd ICRC 1 (2011) 62

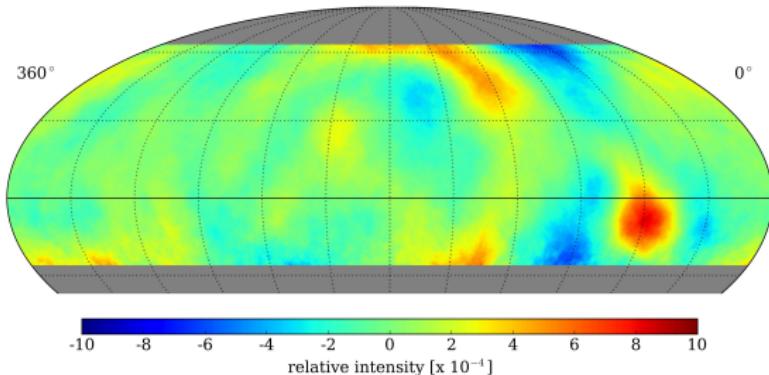
Aartsen *et al.*, ApJ 826 (2016) 220

# Small-scale anisotropies

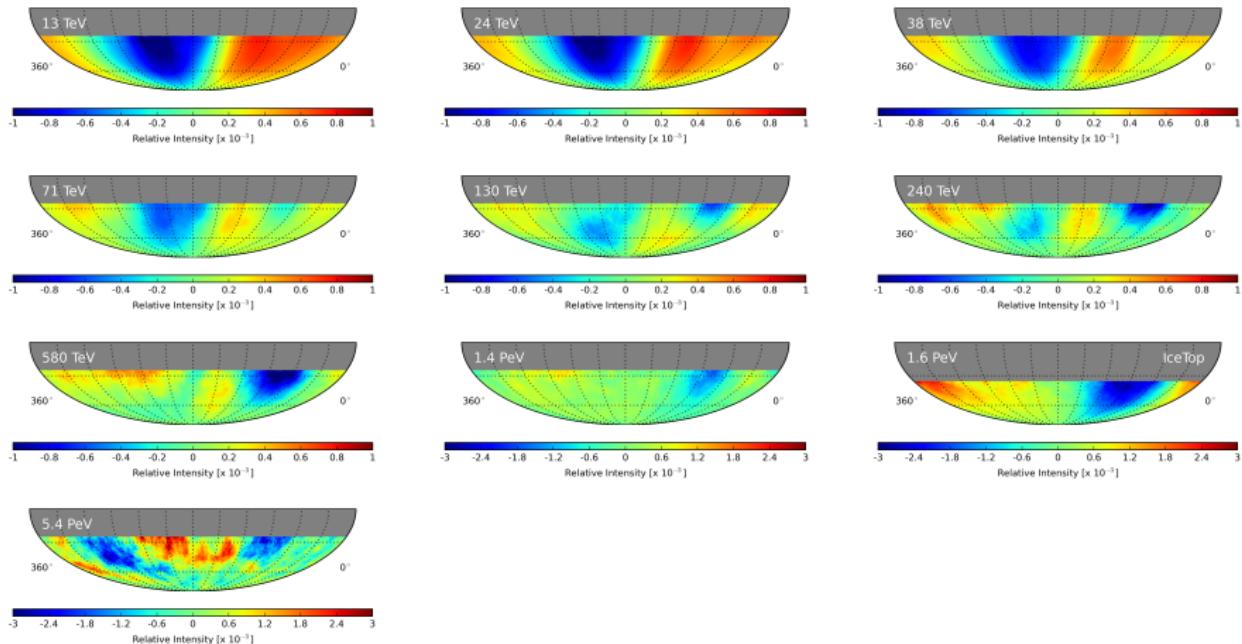


- subtract off dipole and quadrupole
- smooth with  $10^\circ$  disk
- small-scale features

Abeysekara et al., ApJ 796 (2014) 108



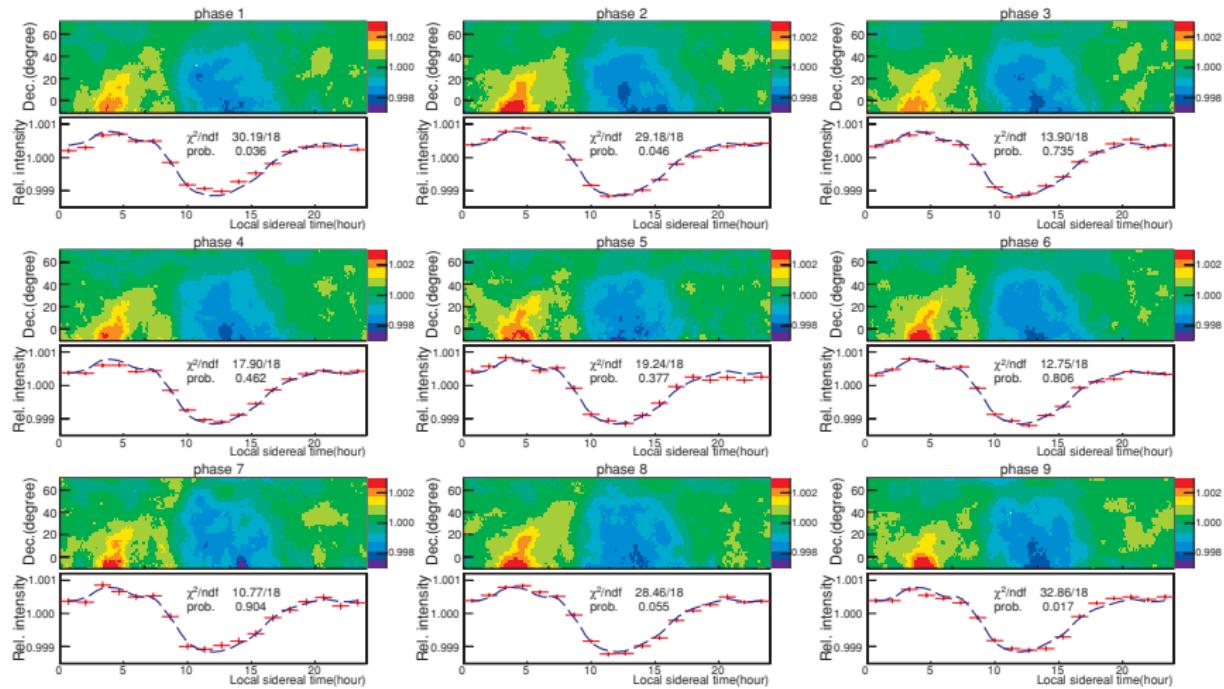
# Energy dependence



Aartsen et al., ApJ 826 (2016) 220

Flip of direction around 100 TeV

# Time dependence



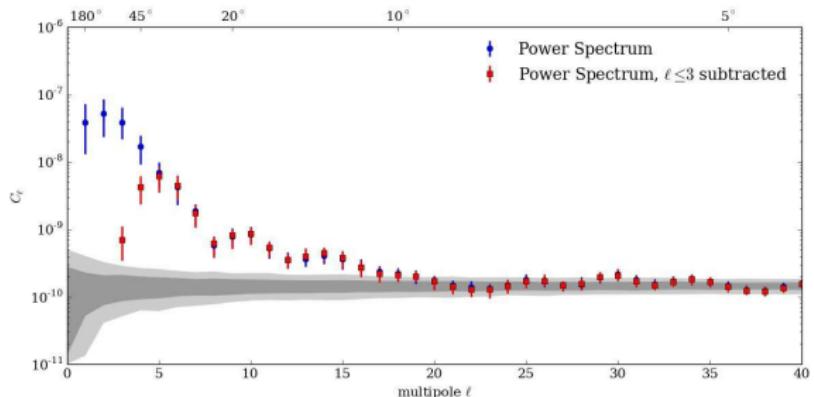
Amenomori et al., ApJ 711 (2010) 119

No significant time-dependence over 9 years.

# Angular power spectrum

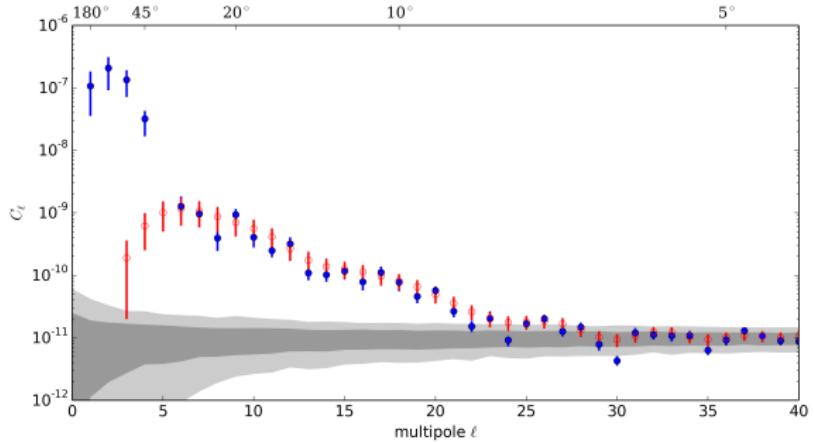
HAWC

Abeysekara *et al.*,  
ApJ 796 (2014) 108



IceCube

Aartsen *et al.*, ApJ  
826 (2016) 220



# Score sheet

## Properties

- large-scale anisotropy of the order  $10^{-3} \dots 10^{-4}$  at TeV ... PeV energies
- small-scale anisotropy of similar size
- directional pattern also changes with energy
- no time-dependence

## Limitation

Relative intensity in declination bands not fixed by reconstruction  
→ insensitive to anisotropies that align with Earth's rotation axis

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- Heliospheric effects

# Standard diffusion

e.g. Jokipii, Rev. Geophys. 9 (1971) 27

- Liouville's theorem:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \dot{\vec{x}} \cdot \nabla_{\vec{x}} f + \dot{\vec{p}} \cdot \nabla_{\vec{p}} f = 0$$

- in a regular and turbulent magnetic field  $\vec{B}(\vec{r}) = \vec{B}_0 + \delta\vec{B}(\vec{r}) = p_0/e (\vec{\Omega} + \vec{\omega})$   
the Lorentz force is:

$$\dot{\vec{p}} = \vec{p} \times (\vec{\Omega} + \vec{\omega}(\vec{r}))$$

- rotation operator  $\vec{L} \equiv -i\vec{p} \times \vec{\nabla}_{\vec{p}}$ :

$$\dot{\vec{p}} \cdot \nabla_{\vec{p}} f = \vec{p} \times (\vec{\Omega} + \vec{\omega}(\vec{r})) \cdot \nabla_{\vec{p}} f = -i(\vec{\Omega} + \vec{\omega}(\vec{r})) \cdot \vec{L} f$$

- the ensemble average  $\langle f \rangle$  evolves as

$$\frac{\partial \langle f \rangle}{\partial t} + \hat{p} \cdot \vec{\nabla}_{\vec{x}} \langle f \rangle - i\vec{\Omega} \cdot \vec{L} \langle f \rangle = \left\langle i\vec{\omega} \cdot \vec{L} \delta f \right\rangle$$

# Evaluating the scattering term

## (1) Quasi-linear theory

$$\partial_t \delta f + \left( \hat{p} \cdot \vec{\nabla}_{\vec{x}} - i \vec{\Omega} \cdot \vec{L} \right) \delta f \simeq i \vec{\omega} \cdot \vec{L} \langle f \rangle$$

$$\delta f(t, \mathbf{r}, \mathbf{p}) \simeq \delta f(t - T, \mathbf{r}(t - T), \mathbf{p}(t - T)) + \int_{t-T}^t dt' \left[ i \vec{\omega} \cdot \vec{L} \langle f \rangle \right]_{P(t')}$$

## (2) BGK ansatz

$$\frac{\partial \langle f \rangle}{\partial t} + \hat{p} \cdot \vec{\nabla}_{\vec{x}} \langle f \rangle - i \vec{\Omega} \cdot \vec{L} \langle f \rangle = \left\langle i \vec{\omega} \cdot \vec{L} \delta f \right\rangle \rightarrow -\nu \left( \langle f \rangle - \frac{n}{4\pi} \right)$$

Bhatnagar, Gross, Krook *Phys. Rev.* **94** (1954) 511

## (3) Pitch-angle diffusion ( $\mu = \vec{p} \cdot \vec{B} / (pB)$ )

$$\left\langle i \vec{\omega} \cdot \vec{L} \delta f \right\rangle \simeq \frac{\partial}{\partial \mu} D_{\mu\mu} \frac{\partial}{\partial \mu} \langle f \rangle \quad \Rightarrow \quad \partial_t \langle f \rangle + \nu \mu \frac{\partial}{\partial z} \langle f \rangle = \frac{\partial}{\partial \mu} D_{\mu\mu} \frac{\partial}{\partial \mu} \langle f \rangle$$

# Two sources of large-scale anisotropy

Relative intensity

$$I = 4\pi \frac{f(t_{\oplus}, \mathbf{r}_{\oplus}, -\mathbf{p})}{\phi(t_{\oplus}, \mathbf{r}_{\oplus}, p)} = 1 - \hat{\mathbf{p}} \cdot \underbrace{\frac{3\Phi(t_{\oplus}, \mathbf{r}_{\oplus}, p)}{\phi(t_{\oplus}, \mathbf{r}_{\oplus}, p)}}_{\equiv -\vec{\delta}} + \mathcal{O}(\{a_{\ell m}\}_{\ell \geq 2})$$

## Diffusive anisotropy

if source distribution wrt observer is asymmetric  $\Rightarrow \nabla n_{\text{CR}} \neq 0$

$$\boldsymbol{\delta}^* = 3\mathbf{K} \cdot \nabla \ln n_{\text{CR}}$$

## Compton-Getting effect

For a power-law CR spectrum  $\propto p^{-2-\Gamma}$

$$\boldsymbol{\delta}_{\text{CG}} \simeq (2 + \Gamma)\boldsymbol{\beta}$$

Compton & Getting, Phys. Rev. 47 (1935) 817; Gleeson & Axford, Astrophys. Space Sci. 2 (1968) 431

Total: vectorial sum  $\boldsymbol{\delta} \simeq \boldsymbol{\delta}^* + (2 + \Gamma)\boldsymbol{\beta}$

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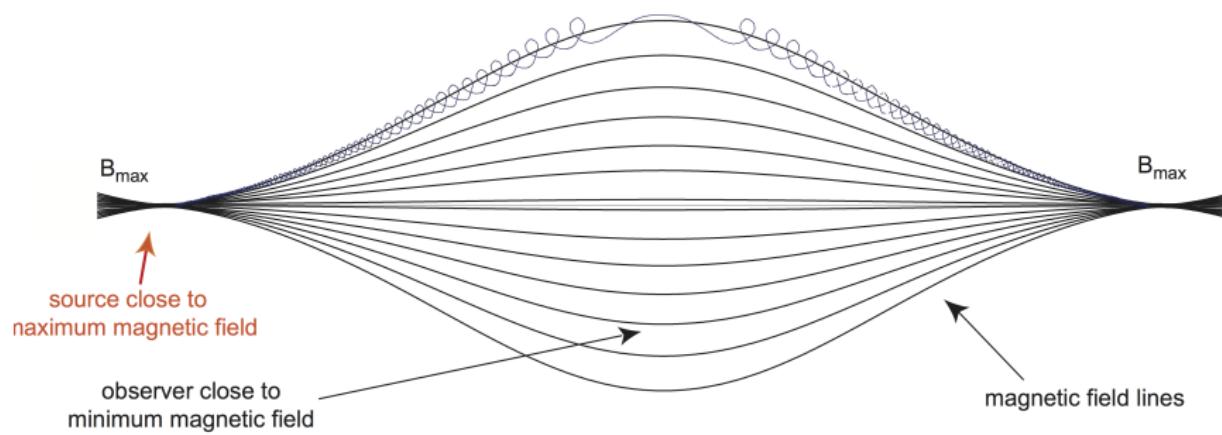
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# Focussing CRs

$$\frac{p_{\perp}^2}{2B} = \text{const.}$$

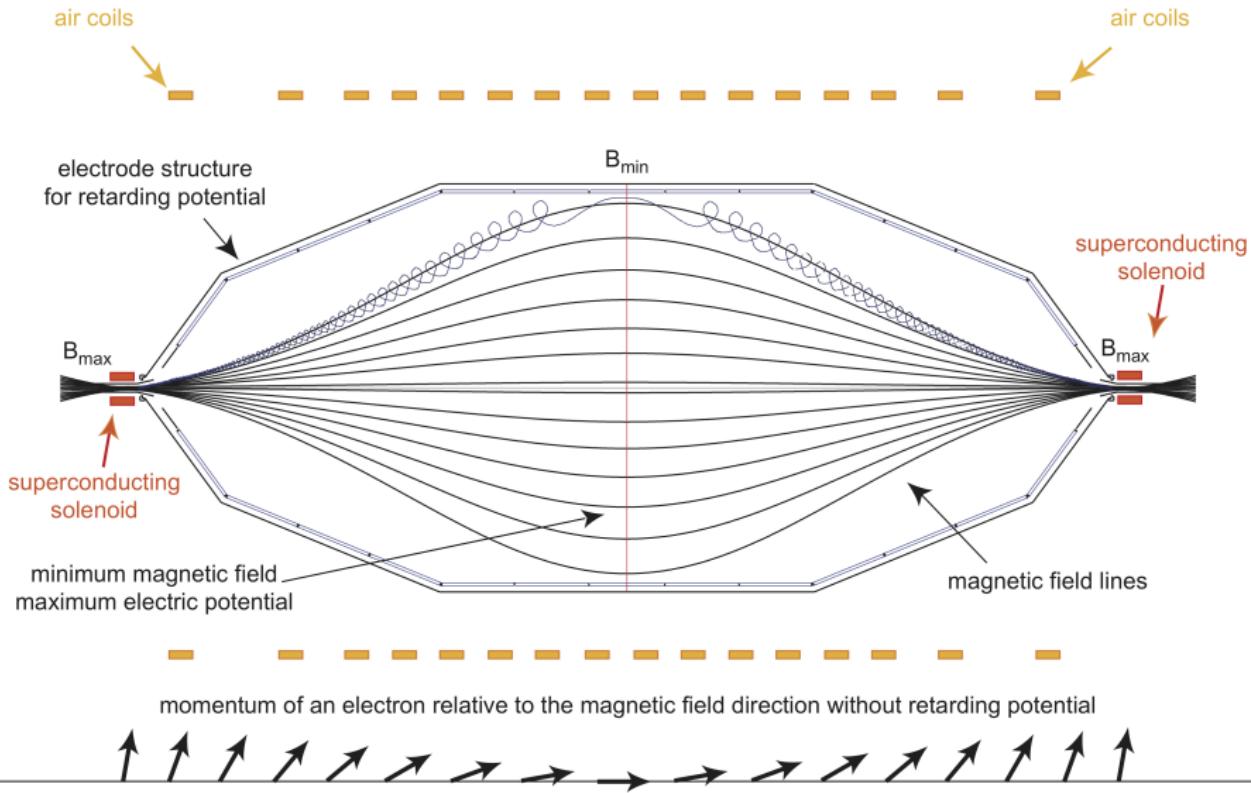


momentum of a CR particle relative to the magnetic field direction



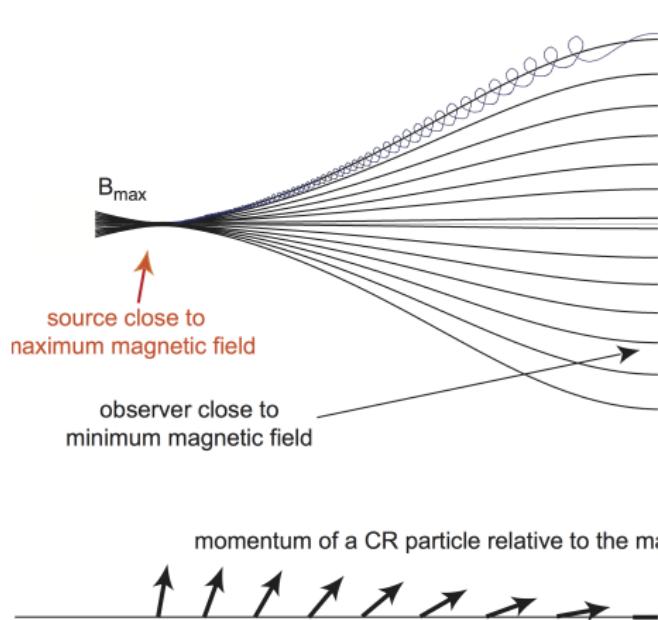
## Focussing CRs

$$\frac{p_\perp^2}{2B} = \text{const.}$$



Beck *et al.*, JINST 9 (2014) P11020

# Focussing CRs



- beam width

$$\delta\theta \simeq \sqrt{\frac{B_{\min}}{B_{\max}}} \\ \simeq 5^\circ \left( \frac{B_{\max}/B_{\min}}{100} \right)^{-1/2}$$

- beam can be subdominant

- source needs to be close to maximum  $\rightarrow$  unnatural?
- small-scale turbulence will broaden beam  $\rightarrow$  source needs to be closer than scattering length  $\mathcal{O}(10)$  pc at 1 PeV

# Non-uniform pitch-angle scattering

Solve Fokker-Planck equation but with  $D_{\mu\mu} \neq D_0(1 - \mu^2)$

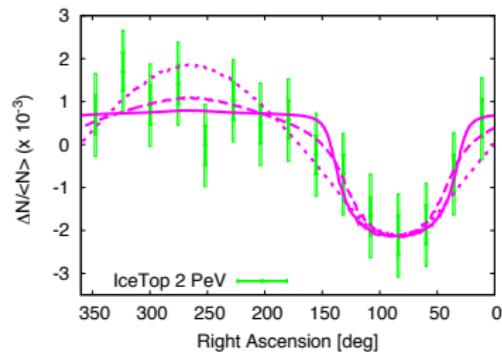
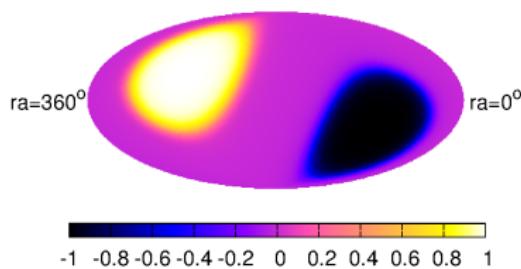
- ① Goldreich-Sridhar turbulence  $\rightarrow$  narrow peak in  $D_{\mu\mu}$   $\rightarrow$  narrow beam in CRs

Malkov et al., ApJ 721 (2010) 750

- ② modification of the large-scale anisotropy:

- ▶ compute  $D_{\mu\mu}$  in quasi-linear theory in various turbulence models
- ▶ can have peak close to  $\mu = 0$
- ▶ consider higher-order terms in series in  $\mu$
- ▶ large-scale anisotropy modified

Giacinti & Kirk, ApJ 835 (2017) 258



# Small-scale turbulence and ensemble averaging

- in standard diffusion, compute  $C_\ell$  from  $\langle f \rangle$ :

$$C_\ell^{\text{std}} = \frac{1}{4\pi} \int d\hat{\mathbf{p}}_1 \int d\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle f(\hat{\mathbf{p}}_1) \rangle \langle f(\hat{\mathbf{p}}_2) \rangle$$

- however, in an individual realisation of  $\delta B$ ,  $\delta f = f - \langle f \rangle \neq 0$

$$\langle C_\ell \rangle = \frac{1}{4\pi} \int d\hat{\mathbf{p}}_1 \int d\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \hat{\mathbf{p}}_2) \langle f(\hat{\mathbf{p}}_1) f(\hat{\mathbf{p}}_2) \rangle$$

- if  $f(\hat{\mathbf{p}}_1)$  and  $f(\hat{\mathbf{p}}_2)$  are correlated,

$$\langle f(\hat{\mathbf{p}}_1) f(\hat{\mathbf{p}}_2) \rangle \geq \langle f(\hat{\mathbf{p}}_1) \rangle \langle f(\hat{\mathbf{p}}_2) \rangle \quad \Rightarrow \quad \langle C_\ell \rangle \geq C_\ell^{\text{std}}$$

Source of the small scale anisotropies?

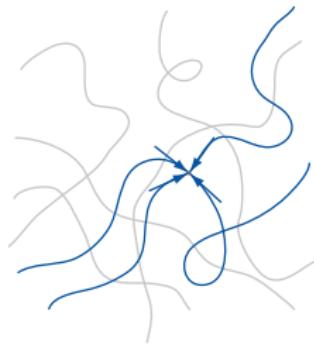
Giacinti & Sigl, PRL 109 (2012) 071101

Ahlers, PRL 112 (2014) 021101, Ahlers & Mertsch, ApJL 815 (2015) L2, Pohl & Rettig, Proc. 36th ICRC (2016) 451, López-Barquero *et al.*, ApJ 830 (2016) 19, López-Barquero *et al.*, ApJ 842 (2017) 54

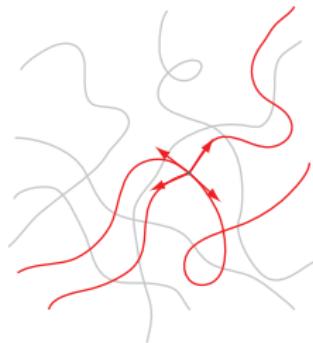
# Backtracking



# Backtracking



# Backtracking



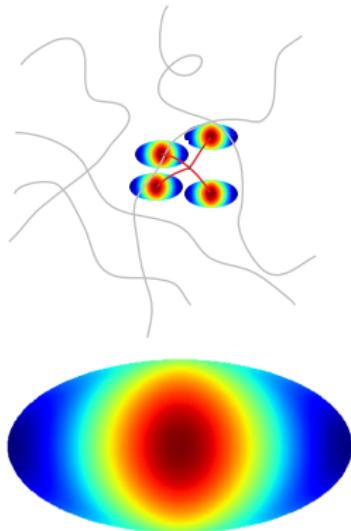
- Set of trajectories  $\{(\vec{x}_i(t), \vec{p}_i(t))\}$  with  $\vec{x}_i(0) = \vec{x}_\oplus = 0$  and  $\vec{p}_i(0) = \vec{p}_{i0}$
- Assume (time-independent) initial state  $f_{\text{ini}}(\vec{x}, \vec{p})$
- Exploit Liouville's theorem

$$df = 0 \quad \Rightarrow \quad f(\vec{x}_\oplus, \vec{p}_i, 0) = f_{\text{ini}}(\vec{x}_i(-T), \vec{p}_i(-T))$$

## Gedankenexperiment

$$df = 0 \quad \Rightarrow \quad f(\vec{x}_\oplus, \vec{p}_i, 0) = f_{\text{ini}}(\vec{x}_i(-T), \vec{p}_i(-T))$$

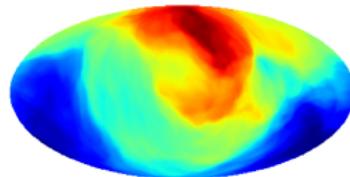
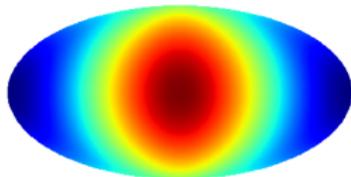
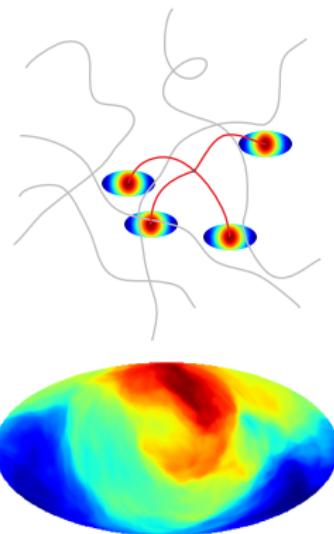
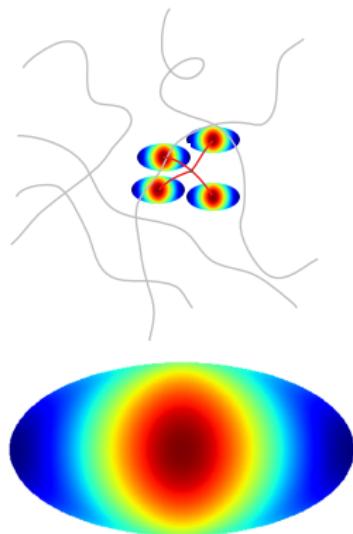
homogeneous, but anisotropic (dipole) initial state



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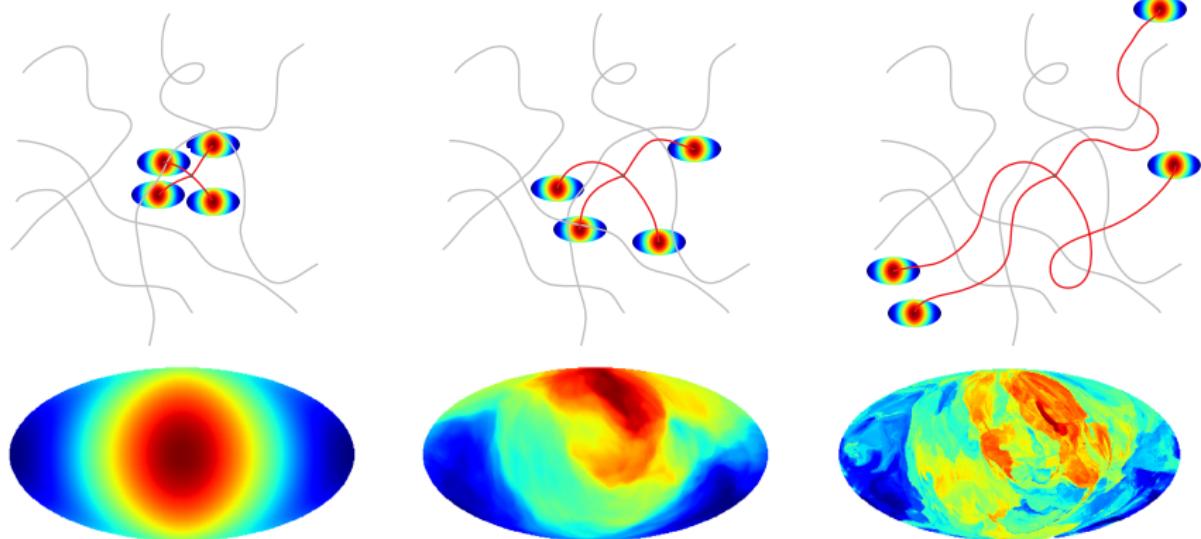
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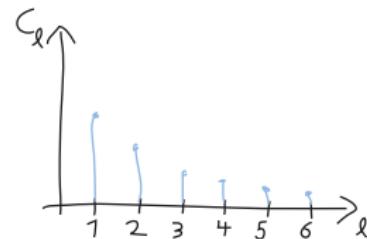
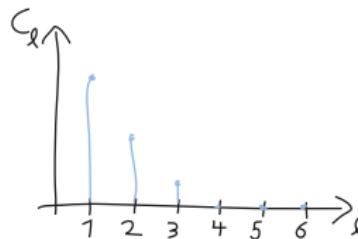
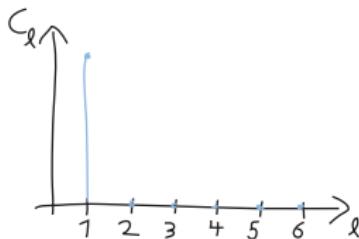
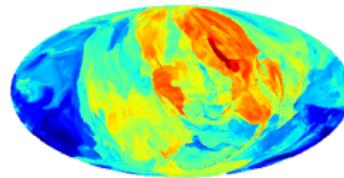
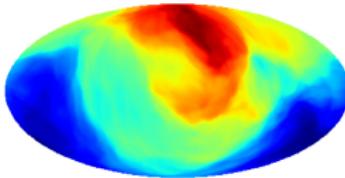
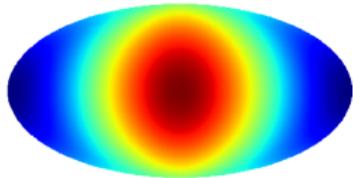
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$$df = 0 \quad \Rightarrow \quad f(\vec{x}_\oplus, \vec{p}_i, 0) = f_{\text{ini}}(\vec{x}_i(-T), \vec{p}_i(-T))$$

homogeneous, but anisotropic (dipole) initial state



# Gedankenexperiment



- Total angular power conserved:  $\sum_\ell (2\ell + 1)C_\ell = \text{const.}$
- conservation equation

$$\partial_t C_\ell = M_{\ell\ell'} C_{\ell'}$$

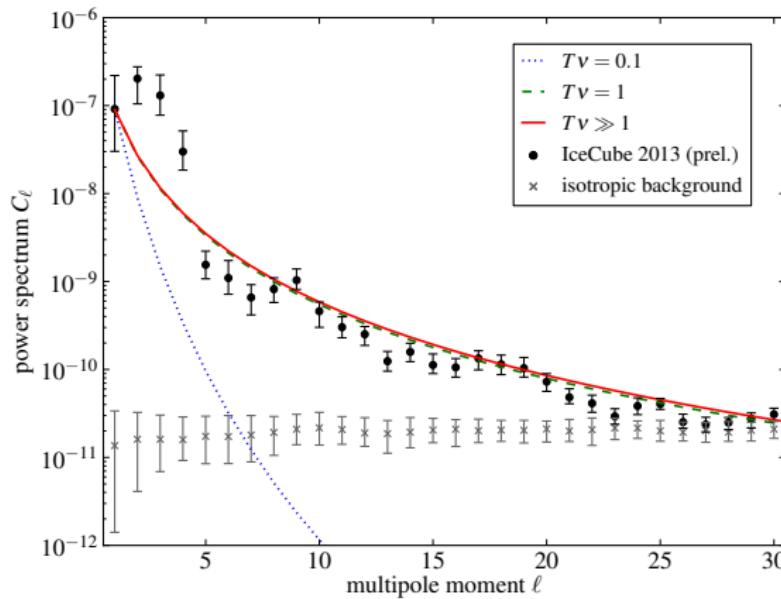
with properties

- ▶ initial condition:  $C_\ell(0) \propto \delta_{\ell 1}$  for  $\ell > 0$
- ▶ triangularity:  $M_{\ell\ell'} > 0$  for  $\ell > \ell'$  only
- ▶ decay of power:  $M_{\ell\ell} = -\nu_\ell = -\ell(\ell + 1)\nu$

# Gedankenexperiment

Ahlers, PRL 112 (2014) 021101

all  $C_\ell$  decrease with time, but  $C_\ell/C_0 \rightarrow \text{const.}$



# Relative diffusion

- quasi-stationary distribution:

$$4\pi \langle f \rangle \simeq n + \vec{r} \nabla n - 3\hat{\mathbf{p}} \mathbf{K} \nabla n$$

- the phase space density  $f$  at time  $t = 0$  depends on positions  $\mathbf{r}$  and velocities  $\hat{\mathbf{p}}$  at earlier time  $t = -T$

$$4\pi f \simeq 4\pi \delta f(-T) + n + (\mathbf{r}(-T) - 3\hat{\mathbf{p}}(-T)\mathbf{K}) \nabla n$$

- as before

$$\begin{aligned} \frac{1}{4\pi} \langle C_\ell \rangle &= \int d\hat{\mathbf{p}}_1 \int d\hat{\mathbf{p}}_2 P_\ell(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) \langle f(\hat{\mathbf{p}}_1) f(\hat{\mathbf{p}}_2) \rangle \\ &\simeq \int \frac{d\hat{\mathbf{p}}_1}{4\pi} \int \frac{d\hat{\mathbf{p}}_2}{4\pi} P_\ell(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) \lim_{T \rightarrow \infty} \langle r_{1i}(-T) r_{2j}(-T) \rangle \frac{\partial_i n \partial_j n}{n^2} \end{aligned}$$

# Relative diffusion

- as before:

$$\frac{1}{4\pi} \langle C_\ell \rangle \simeq \int \frac{d\hat{\mathbf{p}}_1}{4\pi} \int \frac{d\hat{\mathbf{p}}_2}{4\pi} P_\ell(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2) \lim_{T \rightarrow \infty} \langle r_{1i}(-T) r_{2j}(-T) \rangle \frac{\partial_i n \partial_j n}{n^2}$$

- variance from standard diffusion coefficient:

$$\frac{1}{4\pi} \sum_{\ell \geq 0} (2\ell + 1) \langle C_\ell \rangle \simeq \langle r_i(-T) r_j(-T) \rangle \frac{\partial_i n \partial_j n}{n^2} \simeq 2T K_{ij}^s \frac{\partial_i n \partial_j n}{n^2}$$

- monopole from difference of standard and *relative* diffusion coefficients:

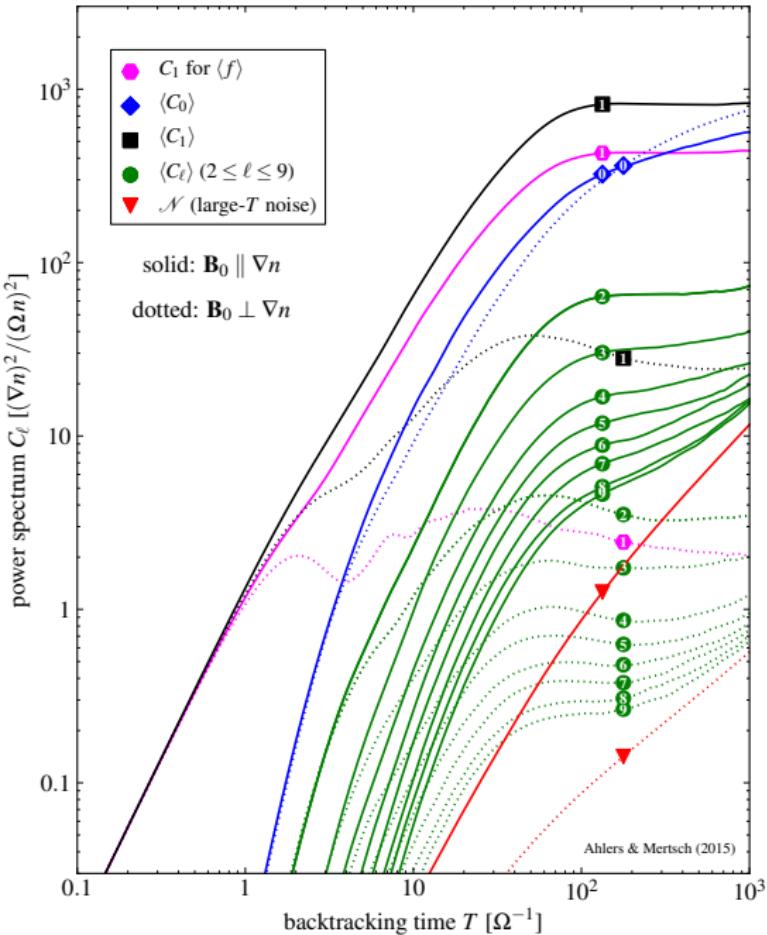
$$\frac{1}{4\pi} \langle C_0 \rangle \simeq 2T \left( K_{ij}^s - \tilde{K}_{ij}^s \right) \frac{\partial_i n \partial_j n}{n^2}$$

where  $\tilde{K}_{ij}^s = \int \frac{d\hat{\mathbf{p}}_1}{4\pi} \int \frac{d\hat{\mathbf{p}}_2}{4\pi} \lim_{T \rightarrow \infty} \frac{1}{4T} \langle \{r_{1i} - r_{2i}\} \{r_{1j} - r_{2j}\} \rangle$

- hence, all angular power for  $\ell \geq 1$  must be due to relative diffusion:

$$\frac{1}{4\pi} \sum_{\ell \geq 1} (2\ell + 1) \langle C_\ell \rangle(T) \simeq 2T \tilde{K}_{ij}^s \frac{\partial_i n \partial_j n}{n^2}$$

$$B_0^2 = \langle \delta B^2 \rangle, r_L/L_c = 0.1, \lambda_{\min}/L_c = 0.01, \lambda_{\max}/L_c = 100$$



## angular power spectrum of mean-subtracted map

- at early times, all moments increase; dipole  $\propto T^2$
  - later: asymptotic values
  - finite number of trajectories  
→ **shot noise**
  - variance =  $\sum_\ell (2\ell + 1) C_\ell \propto T$
  - relative difference between  $\vec{B}_0 \parallel \nabla n$  and  $\vec{B}_0 \perp \nabla n$
  - standard dipole  $C_1 < \langle C_1 \rangle$
  - non-vanishing monopole  $\langle C_0 \rangle$

# Generalised BGK–ansatz

- want to write down *local* ODE for  $C_\ell$ , so need

$$\partial_t \langle f_1 f_2 \rangle = \langle f_1 \left( -\hat{\mathbf{p}}_1 \cdot \nabla_{\mathbf{r}} + i\vec{\omega}_1 \cdot \vec{L} + i\vec{\Omega}_0 \cdot \vec{L} \right) f_2 \rangle + (1 \leftrightarrow 2)$$

- BGK–ansatz; drive  $\langle f \rangle$  to isotropic distribution  $n$ :

$$\left\langle i\vec{\omega} \cdot \vec{L} \delta f \right\rangle \rightarrow -\nu \left( \langle f \rangle - \frac{n}{4\pi} \right)$$

- diffusion on the sphere where Laplacian is  $\nabla^2 \sim -\vec{L}^2$

$$\left\langle i\vec{\omega} \cdot \vec{L} \delta f \right\rangle \rightarrow -(\nu/2)\vec{L}^2$$

- we therefore make the ansatz

$$\langle (i\omega_1 \mathbf{L}_1 + i\omega_2 \mathbf{L}_2) f_1 f_2 \rangle \simeq - \left[ \nu_r(x) \frac{\mathbf{L}_1^2 + \mathbf{L}_2^2}{2} + \nu_c(x) \mathbf{J}^2 \right] \langle f_1 f_2 \rangle$$

with  $x = \hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2$  and  $\mathbf{J} = \mathbf{L}_1 + \mathbf{L}_2$

# Generalised BGK–ansatz

- gradient term:

$$\langle f_1 \hat{\mathbf{p}}_2 \cdot \nabla f_2 \rangle \simeq -3/(4\pi)^2 (\hat{\mathbf{p}}_1 \cdot \nabla n) (\hat{\mathbf{p}}_2 \cdot \mathbf{K} \cdot \vec{\nabla} n)$$

- steady-state solution:

$$K_{ij} \frac{\partial_i n \partial_j n}{6\pi} \delta_{\ell 1} = \sum_k \langle C_k \rangle k(k+1) \frac{2k+1}{2} \int dx \nu_r(x) P_\ell(x) P_k(x)$$

- depends on relative scattering rate  $\nu_r$  only:

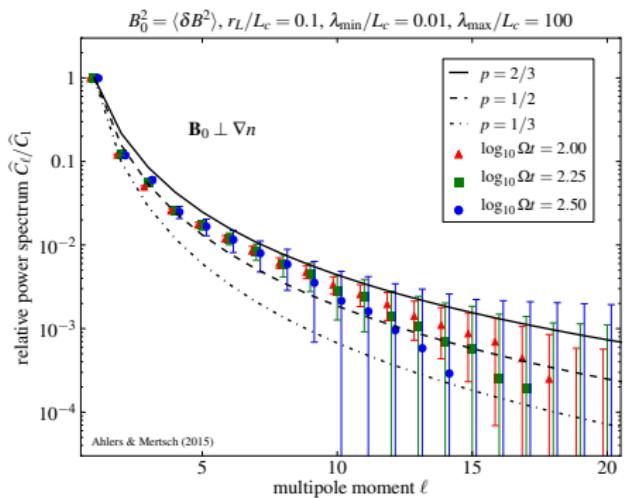
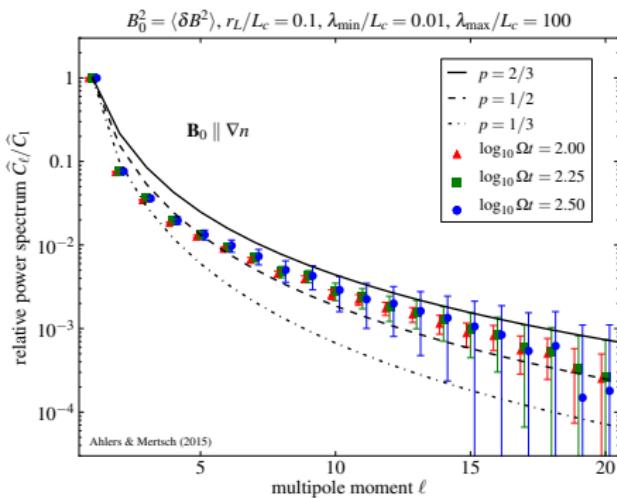
$$\langle C_\ell \rangle = \frac{3}{2} \frac{K_{ij} \partial_i n \partial_j n}{\ell(\ell+1)} \int_{-1}^1 dx \frac{x P_\ell(x)}{\nu_r(x)}$$

# Generalised BGK–ansatz

$$\langle C_\ell \rangle = \frac{3}{2} \frac{Q_1}{\ell(\ell+1)} \int_{-1}^1 dx \frac{x P_\ell(x)}{\nu_r(x)}$$

- ansatz for  $x$ –dependence:

$$\nu_r(x) \propto (1-x)^p$$



# Are heliospheric effects strong enough?

- Solar modulation in force field approximation with  $\mathcal{O}(100)$  MeV potentials
- Can this effect TeV-PeV cosmic rays?

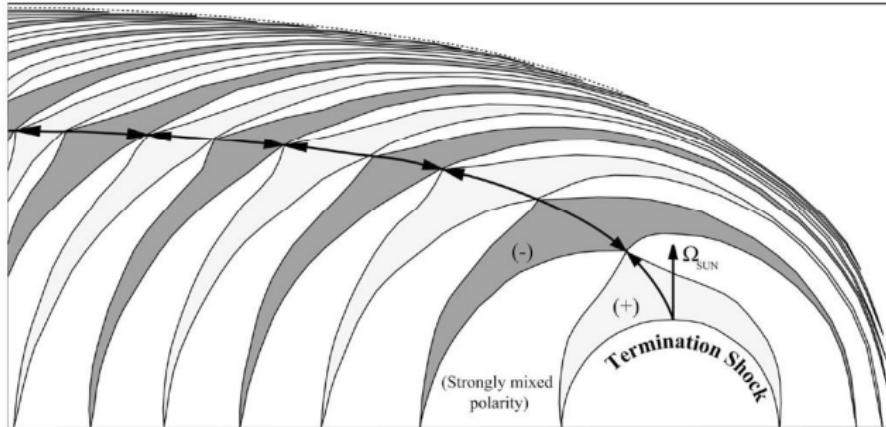
- Alignment of excess region with heliotail
- $r_g \simeq 200 (R/\text{TV})(B/\mu\text{G})^{-1}$  AU is  $\lesssim$  size of heliosphere
- Need not modify isotropic flux, but only arrival directions:

Drury (2013)

- Electric field due to relative bulk speed of ISM CRs in heliosphere:  $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$
- $v = 10 \text{ km/s}, B = 10 \mu\text{G} \rightarrow 1.5 \text{ MV/AU}$
- If field coherent over 100 AU  $\rightarrow 150 \text{ MV}$
- $10^{-4}$  effect for TeV particles

# Explaining the excess in the heliotail

Lazarian and Desiati, ApJ 722 (2010) 188, Desiati and Lazarian, ApJ 762 (2013) 44



Nerney, Suess and Schmahl, JGR 100 (1995) 3463

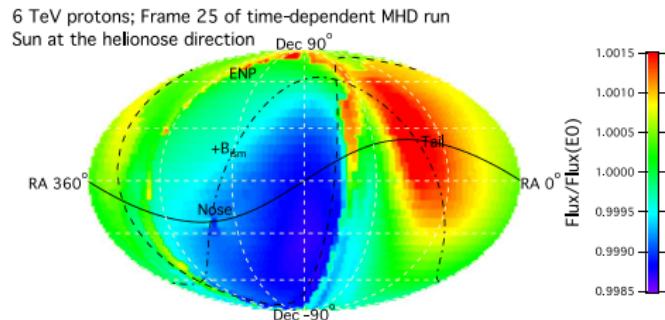
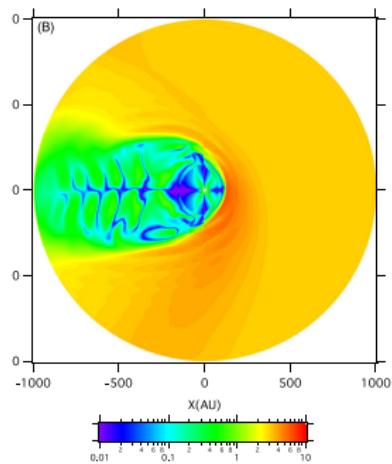
- Reconnection in the heliotail → harder spectrum in excess region
- Super-Alfvénic turbulence with  $\lambda_{\text{mfp}} \sim r_g$  → excess in the heliotail
- Misalignment of ISM flow and  $B$  direction → non-dipolar anisotropies
- Reconstruction errors of large-scale (angular) gradient → small-scale structure

# Detailed numerical model

Zhang, Zuo and Pogorelov, ApJ 790 (2014) 5

- state-of-the-art MHD model of heliosphere
- backtrack from initial distribution with  $\nabla_{\perp} \ln n$ , dipole and quadrupole

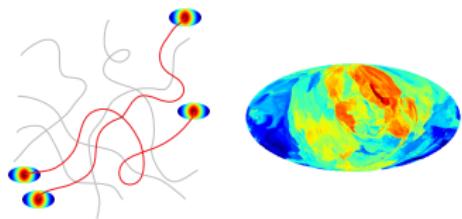
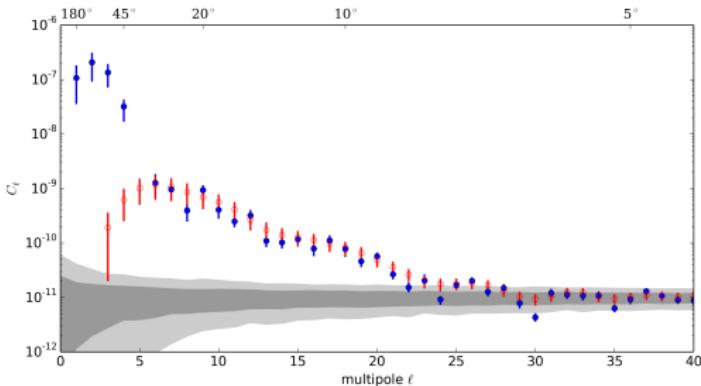
- ① acceleration in electric fields
- ② non-uniform pitch-angle scattering along the regular magnetic field
- ③ drift diffusion perpendicular to the field ("B-cross-gradient" forces)



# Summary

## Observations

- anisotropies down to  $\sim 5^\circ$
- power law in  $\ell$  for  $\ell > 5$
- no time-dependence



## Interpretations

- magnetic lenses: *ad hoc?* turbulence?
- non-uniform pitch-angle scattering
- heliospheric
- small-scale turbulence:  
guaranteed, predictive, test turbulence

# A shameless plug . . .



## Progress in Particle and Nuclear Physics

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Review

### Origin of small-scale anisotropies in Galactic cosmic rays

Markus Ahlers<sup>a</sup>, Philipp Mertsch<sup>b</sup>,

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<https://doi.org/10.1016/j.ppnp.2017.01.004>

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#### Abstract

The arrival directions of Galactic cosmic rays are highly isotropic. This is expected from the presence of turbulent magnetic fields in our Galactic environment that repeatedly scatter charged cosmic rays during propagation. However, various cosmic ray observatories have identified weak anisotropies of various angular sizes and with relative intensities of up to a level of 1 part in 1000. Whereas large-scale anisotropies are generally predicted by standard diffusion models, the appearance of small-scale anisotropies down to an angular size of 10° is surprising. In this review, we summarize the current experimental situation for both the large-scale and small-scale anisotropies.

# Harmonic decomposition

$$I(\alpha, \delta) = 1 + \sum_{\ell \geq 1} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\pi/2 - \delta, \alpha)$$

dipole amplitude and phase:  $(A, \alpha) = (\sqrt{\frac{3}{4\pi}} |a_{11}|^2, \arg a_{11})$

