Measurements and combination of $\sin^2 \theta_{\text{eff}}$ at the Tevatron and extraction of the $W$ mass

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On behalf of the CDF and DØ Collaborations
Introduction
- Drell-Yan dilepton production and the weak mixing angle
- Forward-Backward asymmetry
- Measurement strategy

CDF Measurements

D0 Measurements

Tevatron Combination of $\sin^2 \theta_{eff}^l$
- Standardization of results
- BLUE combination

Inferring $\sin^2 \theta_W$ and $M_W$

Summary
Drell-Yan Production

Drell-Yan lepton pairs are produced at the Tevatron through

\[ p\bar{p} \rightarrow Z/\gamma^* \rightarrow l^+l^- \]

The weak mixing angle can be measured from the forward-backward asymmetry of the polar angle distribution of these Drell-Yan pairs

\[ q\bar{q} \rightarrow \gamma^* \rightarrow l^+l^- \]

Born level couplings

\[ g_V^f = Q_f \]

\[ g_A^f = 0 \]

\[ \langle f | (g_V + g_A \gamma^5) \gamma^\mu | f \rangle \]

Weak couplings altered by radiative corrections

- Multiplicative factor of a few %
- Gives effective \( \sin^2 \theta_W \) coupling \( \rightarrow \sin^2 \theta_{\text{eff}} \)
Forward-Backward Asymmetry

Measure $l^- l^*$ angular distribution in the Collins-Soper rest frame of the boson. Polar angle, $\theta^*$, of the $l^-$ is defined relative to the direction of the incoming quark.

- Forward: $\cos \theta^* > 0$, Backward: $\cos \theta^* < 0$

\[ dN/d\Omega \propto 1 + \cos^2 \theta^* + A_4 \cos \theta^* \]

- All coefficients but $A_4$ vanish as $P_T \to 0$

- $A_4 \cos \theta^*$: parity violating, from interference of vector and axial vector currents

  Sensitive to $\sin^2 \theta_W$ through $Z$ self-interference in $Z_{VV} \otimes Z_{AA}$:

  \[ I_{3l}(1 - 4|Q_l|\sin^2 \theta_W)I_{3q}(1 - 4|Q_q|\sin^2 \theta_W)I_{3q}I_{3l} \]

\[ A_{FB} = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} = \frac{3}{8} A_4 \]

\[ \text{NLO QCD: } dN/d\Omega = 1 + \cos^2 \theta^* + A_0 \left( 1 - 3 \cos^2 \theta^* \right)/2 + A_1 \sin 2 \theta^* \cos \phi + A_2 \left( \sin^2 \theta^* \cos 2 \phi \right)/2 + A_3 \sin \theta^* \cos \phi + A_4 \cos \theta^* + A_5 \sin^2 \theta^* \sin 2 \phi + A_6 \sin 2 \theta^* \sin \phi + A_7 \sin \theta^* \sin \phi \]
Measurement Strategy

- Measure $A_{FB}$ in bins of lepton pair invariant mass
- Produce Monte Carlo $A_{FB}(M, sin^2 \theta_W)$ templates
- Perform full corrections to data and simulation
- Extract $sin^2 \theta_W$ by a $\chi^2$ comparison between data and MC generated at different values of $sin^2 \theta_W$

ZZ interference
- Sensitive to $sin^2 \theta_W$
- Near Z-pole: Best statistics/precision, minimal $\gamma Z$ interference

$\gamma Z$ interference
- Independent of $sin^2 \theta_W$
- Zero at Z-pole
- Dominates away from Z-pole and sensitive to PDFs

Measure at the Z pole!
The Detectors and Data

**Dimuons: 9.4 fb⁻¹**
- \( p_T > 20 \text{ GeV/c} \)
- \( |\eta| < 1.1 \) (277K events)

**Dielectrons: 9.4 fb⁻¹**
- Central (C): 0.05<|\eta|<1.05
- End plug (P): 1.2<|\eta|<2.8
- CC: \( E_T > 25/15 \text{ GeV} \) (227K events)
- CP: \( E_T > 20 \text{ GeV} \) (258K events)
- PP: \( E_T > 25 \text{ GeV} \) (80K evts cal only)

**Dimuons: 8.6 fb⁻¹**
- \( p_T > 15 \text{ GeV/c} \)
- \( |\eta|_1 \leq 1.6, |\eta|_2 \leq 1.8 \) (481K events)

**Dielectrons: 9.7 fb⁻¹**
- Central (CC): \( |\eta| < 1.1 \)
- End (EC): 1.5 < |\eta| < 3.2
- CC-CC: \( p_T > 25 \text{ GeV/c} \) (248K events)
- CC-EC: \( p_T > 25 \text{ GeV/c} \) (241K events)
- EC-EC: \( p_T > 25 \text{ GeV/c} \) (71K events)
Asymmetry measurements corrected for direct fits to calculations

- angular-weighted event sums method [EPJ C 76, 321 (2010)]
- matrix unfolding of detector and QED FSR smearing; residual bias correction of few%

Simulation: PYTHIA 6.2(CTEQ5L) ⊕ PHOTOS 2.0 ⊕ CDF detector simulation

- Higher order QCD effect corrections applied to generated events

Templates: POWHEG-BOX(NLO) ⊕ NNPDF 3.0(NNLO) PDFs ⊕ PYTHIA 6.4
CDF Measurements

$sin^2 \theta_{\text{eff}}^l = 0.2315 \pm 0.0009 \text{ (stat)}$

$\pm 0.0002 \text{ (syst)}$

$\pm 0.0004 \text{ (PDF)}$

PRD 89, 072005 (2014)

$sin^2 \theta_{\text{eff}}^l = 0.23248 \pm 0.00049 \text{ (stat)}$

$\pm 0.00004 \text{ (syst)}$

$\pm 0.00019 \text{ (PDF)}$

PRD 93, 112016 (2016)
Asymmetries for electrons separately fit to CC-CC, CC-EC, EC-EC templates then best-fit values combined

Simulation: PYTHIA 6.323(CTEQ6L1), ALPGEN
- Higher order QCD effect corrections applied to generated events

Templates: PYTHIA 6.323 with NNPDF 2.3(ee) 3.0(μμ) NLO PDFs
- Higher order QCD effect corrections applied to generated events
- Detector simulation included
$\sin^2 \theta_{\text{eff}}^l = 0.23147 \pm 0.00043 \text{ (stat)}$
$\pm 0.00008 \text{ (syst)}$
$\pm 0.00017 \text{ (PDF)}$

$\sin^2 \theta_{\text{eff}}^l = 0.23016 \pm 0.00059 \text{ (stat)}$
$\pm 0.00006 \text{ (syst)}$
$\pm 0.00024 \text{ (PDF)}$

PRL 115, 041801 (2015)

D0NOTE 6500 (2017)
Weak Radiative Corrections

- **PYTHIA template:** single mixing angle and running $\alpha_{em}$

- **D0 ZGRAD+RESPBOS corrected results:** improved
  - ZGRAD+RESPBOS adjustment: improves accounting for differences of fermion-dependent ($u,d,l$) effective mixing angles @ $M_Z$

- **CDF ZFITTER based results:** improved even more
  - Complex-valued form-factors $\rho$ and $\kappa$ for Born Z-couplings
  - $g^f_V = \sqrt{\rho_f I_3 f}(1 - 4|Q_f|\kappa_f \sin^2 \theta_W)$  \quad $g^f_A = \sqrt{\rho_f I_3 f}$
  - $\rho_f/\kappa_f$: functions of fermion type, $M_{ll}^2$, $\sin^2 \theta_W$; 1-4% corrections
  - Photon-propagator form factor (real part aka running $\alpha_{em}$)
Standardizing Results for Combination

- Need common PDF and electroweak correction baselines for consistency
  - NNPDF 3.0: Includes LHC data, improved implementation for PDFs and ensembles
  - ZFITTER SM electroweak radiative corrections: Used by LEP-1 and SLD for standard-model analysis at Z pole

CDF: Results already at baseline

D0 standardization corrections

- $\Delta$(PDF), ee only: NNPDF v2.3 $\rightarrow$ v3.0 offset = $-0.00024 \pm 0.00004$
  - Found by comparing v3.0 pseudodata with v2.3 templates
- $\Delta$(RadCor): $+0.00008$ [ZGRAD+RESBOS ($u,d$ effect)] $+0.00014$
  - [ZGRAD+RESBOS $\rightarrow$ ZFITTER offset] = $+0.00022 \pm 0.00004$
- ee: ZGRAD+RESBOS already applied to published result, additional $-0.00010 \pm 0.00004$ correction required: $\sin^2 \theta^l_{eff} = 0.23137 \pm 0.00047$
- $\mu\mu$: full RadCor already applied to preliminary result
CDF Combination

**Systematic Uncertainties**

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sin^2 \theta_{\text{eff}}^{\text{lep}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy scale and resolution</td>
<td>$\pm 0.00002$</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>$\pm 0.00003$</td>
</tr>
<tr>
<td>QCD scale</td>
<td>$\pm 0.00006$</td>
</tr>
<tr>
<td>NNPDF-3.0 PDF</td>
<td>$\pm 0.00016$</td>
</tr>
</tbody>
</table>

$\sin^2 \theta_{\text{eff}}^{\text{l}} = 0.23221 \pm 0.00043 \text{ (stat)}$
$\pm 0.00007 \text{ (syst)}$
$\pm 0.00016 \text{ (PDF)}$

D0 Combination

**Systematic Uncertainties**

<table>
<thead>
<tr>
<th>Source</th>
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</thead>
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<tr>
<td>Energy/Momentum calibration</td>
<td>$\pm 0.00001$</td>
</tr>
<tr>
<td>Energy/Momentum smearing</td>
<td>$\pm 0.00002$</td>
</tr>
<tr>
<td>Background</td>
<td>$\pm 0.00001$</td>
</tr>
<tr>
<td>Charge misidentification</td>
<td>$\pm 0.00002$</td>
</tr>
<tr>
<td>Lepton identification</td>
<td>$\pm 0.00005$</td>
</tr>
<tr>
<td>Fiducial asymmetry</td>
<td>$\pm 0.00001$</td>
</tr>
<tr>
<td>Correction (PDF and ZFITTER)</td>
<td>$\pm 0.00005$</td>
</tr>
<tr>
<td>NNPDF 2.3/NNPDF 3.0 PDF</td>
<td>$\pm 0.00019$</td>
</tr>
</tbody>
</table>

$\sin^2 \theta_{\text{eff}}^{\text{l}} = 0.23095 \pm 0.00035 \text{ (stat)}$
$\pm 0.00007 \text{ (syst)}$
$\pm 0.00019 \text{ (PDF)}$
Tevatron Combination

BLUE Method
- PDF uncertainty 100% correlated
- Other uncertainties uncorrelated

\[ \sin^2 \theta_{\text{eff}}^l = 0.23148 \pm 0.00027 \text{ (stat)} \]
\[ \pm 0.00005 \text{ (syst)} \]
\[ \pm 0.00018 \text{ (PDF)} \]

- Weight CDF/D0: 0.42/0.58
- Combination \( \chi^2 \) probability 2.6%
Inference of $sin^2 \theta_W$

On-shell renormalization scheme (ZFITTER)

1. $sin^2 \theta_W \equiv 1 - M_W^2 / M_Z^2$
2. $M_Z = 91.875 \pm 0.0021 \text{ GeV/c}^2$

ZFITTER SM conversion

1. $sin^2 \theta_W = sin^2 \theta_{eff} = Re[\kappa_l(M_Z^2, sin^2 \theta_W)] sin^2 \theta_W$
2. Input: $M_t = 173.2 \pm 0.9 \text{ GeV/c}^2$, $M_H = 125 \text{ GeV/c}^2$
3. Form factor $\sim 1.0371$
4. 0.00008 uncertainty on $sin^2 \theta_W$ from form factor

CDF

$sin^2 \theta_W = 0.22400 \pm 0.00041 \text{ (stat)}$
$\pm 0.00019 \text{ (syst)}$

D0

$sin^2 \theta_W = 0.22269 \pm 0.00034 \text{ (stat)}$
$\pm 0.00021 \text{ (syst)}$

Tevatron Combination

$sin^2 \theta_W = 0.22324 \pm 0.00026 \text{ (stat)}$
$\pm 0.00019 \text{ (syst)}$
### Inference of W Boson Mass

<table>
<thead>
<tr>
<th>Indirect measurements</th>
<th>CDF</th>
<th>D0</th>
<th>Tevatron Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEP-1 and SLD ($m_t$)</td>
<td>$M_W = 80.328 \pm 0.021$ (stat)</td>
<td>$M_W = 80.396 \pm 0.017$ (stat)</td>
<td>$M_W = 80.367 \pm 0.014$ (stat)</td>
</tr>
<tr>
<td>NuTeV</td>
<td>$80.135 \pm 0.085$</td>
<td>$80.396 \pm 0.021$</td>
<td></td>
</tr>
<tr>
<td>CDF $ee+\mu\mu$ 9 fb$^{-1}$</td>
<td>$80.328 \pm 0.024$</td>
<td>$80.396 \pm 0.021$</td>
<td></td>
</tr>
<tr>
<td>D0 $ee+\mu\mu$ 10 fb$^{-1}$</td>
<td>$80.367 \pm 0.017$</td>
<td>$80.396 \pm 0.017$</td>
<td></td>
</tr>
<tr>
<td>TeV combined: CDF+D0</td>
<td>$80.367 \pm 0.020$</td>
<td>$80.396 \pm 0.021$</td>
<td></td>
</tr>
</tbody>
</table>

| Direct measurements                    |                      |                    |                               |
| TeV and LEP-2                          | $80.385 \pm 0.015$    |                    |                               |
| ATLAS $e\nu+\mu\nu$ 5 fb$^{-1}$        | $80.370 \pm 0.019$    |                    |                               |

80 80.1 80.2 80.3 80.4 80.5 80.6  
W-boson mass (GeV/c$^2$)
CDF and D0 have measured $sin^2 \theta_{eff}^{l}$ from Drell-Yan lepton-pair asymmetries

- CDF electrons: PRD 93, 112016 (2016)
- CDF muons: PRD 89, 072005 (2014)
- D0 electrons: PRL 115, 041801 (2015)
- D0 muons: D0NOTE 6500 (2017)

Produced combination result

- $sin^2 \theta_{eff}^{l} = 0.23148 \pm 0.00033$

Using ZFITTER SM calculations, inferred W mass

- $M_W = 80.367 \pm 0.017$ GeV/c²

Tevatron legacy result

- TeV combination note: FERMILAB-CONF-17-201-E
- https://tevewwg.fnal.gov/wz/sw2eff17/
Great thanks to Accelerator Division for all the luminosity!!