# Soft gluon resummation for the associated production of a top quark pair with a W or Z boson at the LHC 

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with a massive boson important processes: $p p \rightarrow t \bar{t} W / Z / H$
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- resummation: class of corrections beyond NLO


## Status of $t \bar{t} V$

- $t \bar{t} W / t \bar{t} Z$ : NLO QCD, matched to PS, EW NLO corrections
[Lazopoulos, Melnikov, Petriello, '08] [Lazopoulos, McElmurry, Melnikov, Petriello, '08] [Garzelli,
Kardos, Papadopoulos, Trocsanyi, '12] [Campbell, Ellis, '12] [Kardos, Trocsanyi, Papadopoulos
'12] [Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro '14]
[Frixione, Hirschi, Pagani, Shao, Zaro, '15]


## Status of $t \bar{t} V$

resummation:

- $t \bar{t} H$ :
- direct QCD approach (Mellin space approach) [Kulesza, Motyka, Stebel, Theeuwes, '15 '16 '17]
- SCET-based methods [Broggio, Ferroglia, Pecjak, Signer, Yang, '16] [Broggio, Ferroglia, Pecjak, Yang, '17]
- $t \bar{t} W / t \bar{t} Z$ :
- SCET-based methods [H. T. Li, c. s. Li, s. A. Li, '14] [Broggio, Ferroglia, Ossola, Pecjak, '16] [Broggio, Ferroglia, Ossola, Pecjak, Sameshima '17]


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- Mellin space for factorisation of phase space

$$
\sigma(N)=\int_{0}^{1} \tau^{N-1} \sigma(\tau)
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$$
\begin{aligned}
& \alpha_{S}^{m}\left(\frac{\log ^{n}(1-\hat{\tau})}{1-\hat{\tau}}\right)_{+} \quad m \leq 2 n-1 \\
& \int_{0}^{1} \mathrm{~d} x(f(x))_{+}=\int_{0}^{1} \mathrm{~d} x\left(f(x)-f\left(x_{0}\right)\right)
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- turn into $\log (N)=L$ in Mellin space


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calculations done in singlet octet colour basis

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\frac{\mathrm{d} \tilde{T}_{i j \in t+t}^{r e s} V}{\mathrm{~d} Q^{2}}=\operatorname{Tr}\left[\mathbf{H}_{i j \rightarrow t \bar{t} V} \mathbf{S}_{i j \rightarrow t \bar{t} V}\right] \Delta_{i} \Delta_{j}
$$

- $\Delta_{i}$ : soft and collinear radiation for incoming partons

$$
\Delta_{i}=\exp \left[\int_{0}^{1} \mathrm{~d} z \frac{z^{N-1}-1}{1-z} \int_{\mu^{2}}^{Q^{2}(1-z)^{2}} \frac{\mathrm{~d} q^{2}}{q^{2}} A_{i}\left(\alpha_{S}\left(q^{2}\right)\right)\right]
$$

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$$

- $\mathbf{S}_{i j \rightarrow t \bar{t} V}$ soft wide angle radiation, at NLL and in the basis in which the one-loop soft anomalous dimension matrix $\Gamma$ is diagonal:

$$
\mathbf{S}_{i j \rightarrow t \bar{t} V, R, I J}=\mathbf{S}_{i j \rightarrow t \bar{t} V, R, I J}^{(0)} \exp \left[\int_{\mu}^{Q / N} \frac{\mathrm{~d} q}{q}\left(\lambda_{R ; I}^{*}+\lambda_{R, J}\right)\right]
$$

- $\lambda_{R, J}$ : eigenvalues of $\Gamma$


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\frac{\mathrm{d} \tilde{\sigma}_{i j \rightarrow t \bar{t} V}^{r e s}}{\mathrm{~d} Q^{2}}=\operatorname{Tr}\left[\mathbf{H}_{i j \rightarrow t \bar{t}} V \mathbf{S}_{i j \rightarrow t \bar{t} V}\right] \Delta_{i} \Delta_{j}
$$

- $\mathbf{H}_{i j \rightarrow t \bar{t} V}=\mathbf{H}_{i j \rightarrow t \bar{t} V}^{(0)}+\frac{\alpha_{s}}{\pi} \mathbf{H}_{i j \rightarrow t \bar{t} V}^{(1)}+\ldots$ : hard contributions


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- at NLL accuracy $\mathbf{H}_{i j \rightarrow t \bar{t} V}=\mathbf{H}_{i j \rightarrow t \bar{t} V}^{(0)}$ (Born cross section)


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- at NLL accuracy $\mathbf{H}_{i j \rightarrow t \bar{t} V}=\mathbf{H}_{i j \rightarrow t \bar{t} V}^{(0)}$ (Born cross section)
- improvement beyond NLL: $\mathbf{H}_{i j \rightarrow t \bar{t} V}^{(1)}$ included (for full NNLL resummation $\mathrm{S}, \Delta_{i}, \Delta_{j}$ need to be upgraded to NNLL)


## Cross sections for $t \bar{t} W$

total inclusive cross sections, $\sqrt{S}=13 \mathrm{TeV}, \mu_{R}=\mu_{F}=m_{t}+\frac{m_{V}}{2}$, MMHT2014
NLO: [Garzelli, Kardos, Papadopoulos, Trocsanyi '11][Garzelli, Kardos,
Papadopoulos, Trócsányi '12]:

- $\sigma_{t \bar{t} W^{+}}=422.1_{-11.5 \%}^{+12.8 \%} \mathrm{fb}$
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NLL matched to NLO:

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NLL with $\mathbf{H}^{(1)}$ matched to NLO:

- $\sigma_{t \bar{t} W^{+}}=418.4_{-10.0 \%}^{+12.8 \%} \mathrm{fb}$
- $\sigma_{t \bar{t} W^{-}}=214.4_{-10.1 \%}^{+13.4 \%} \mathrm{fb}$

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- $\sigma_{t \bar{t} W^{+}}=329.9_{-11.1 \%}^{+12.5 \%} \mathrm{fb}$
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NLL with $\mathbf{H}^{(1)}$ matched to NLO:

- $\sigma_{t \bar{t} W^{+}}=341.1_{-8.6 \%}^{+10.7 \%} \mathrm{fb}$
- $\sigma_{t \bar{t} W^{-}}=175.3_{-8.4 \%}^{+9.9 \%} \mathrm{fb}$

Preliminary

Scale dependence $t \bar{t} W^{+} \mu=m_{t}+\frac{m_{W}}{2}$



Preliminary

## Scale dependence $t \bar{t} W^{+} \mu=Q$



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Preliminary

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Outlook:

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