HIGHER-ORDER QED EFFECTS IN HADRONIC PROCESSES

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Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10 (2016) 056; and work in progress

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Outline

- Motivation and introduction
- QED corrections to splitting kernels
  - Extending DGLAP equations
  - QCD-QED corrections to AP kernels
- NLO QED effects to $\gamma\gamma$ production
- Conclusions and perspectives

Specific references:

1)- *H.O. QED splittings: de Florian, Rodrigo and GS, JHEP 01(2014)018, JHEP 10(2014)161; JHEP 03(2015)021*
2)- QCD-QED AP kernels: de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056
3)- *Diphoton production: Cieri, Ferrera and GS,* **work in progress**

Motivation and introduction

Why we need QED corrections?

- More precise experimental data is available!! We need to include (previously neglected) small theoretical effects!!
- NLO QCD is the standard; **NNLO QCD** calculations starting to appear!
- **QED effects might compete with NNLO QCD (since $\alpha_S^2 \sim \alpha$)**
- Inclusion of QED beyond LO could lead to novel effects:
  - Quark-gluon interacting with leptons and photons
  - Charge separation
  - **Dependence on the photon content of the proton!**

    Manohar, Nason, Salam, Zanderighi, '16
  - Enhanced contributions at high-energies (due to the running EM coupling)
- **Thus, QED corrections MUST be taken into account!**

Extending DGLAP equations

Introducing QED corrections

- *DGLAP equations dictate the evolution of PDFs*
- **QED** interactions connects **QCD** partons with **photons and leptons.**

New PDFs (leptons & photon) + *New splitting functions* → ***Modified DGLAP equations***

- **Extend original DGLAP equations to deal with new *objects*:**

$$\frac{dg}{dt} = \sum_f P_{gf} \otimes f + \sum_f P_{g\bar{f}} \otimes \bar{f} + P_{gg} \otimes g + P_{g\gamma} \otimes \gamma$$

$$\frac{d\gamma}{dt} = \sum_f P_{\gamma f} \otimes f + \sum_f P_{\gamma\bar{f}} \otimes \bar{f} + P_{\gamma g} \otimes g + P_{\gamma\gamma} \otimes \gamma$$

$$\frac{dq_i}{dt} = \sum_f P_{q_i f} \otimes f + \sum_f P_{q_i\bar{f}} \otimes \bar{f} + P_{q_i g} \otimes g + P_{q_i\gamma} \otimes \gamma$$

$$\frac{dl_i}{dt} = \sum_f P_{l_i f} \otimes f + \sum_f P_{l_i\bar{f}} \otimes \bar{f} + P_{l_i g} \otimes g + P_{l_i\gamma} \otimes \gamma$$

Kernels with fermions

Photon distributions

Lepton distributions

Kernels with photons

Kernels with leptons

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Extending DGLAP equations

Introducing QED corrections

- Change **PDFs basis** to simplify the system of coupled integro-differential equations *Roth, Weinzierl '04*

New lepton distributions!

$$\mathcal{B}_c = \{u, \bar{u}, \ldots, t, \bar{t}, e, \bar{e}, \ldots, \tau, \bar{\tau}, g, \gamma\}$$

Photon distribution!

CANONICAL BASIS

$$\Delta_{uc} = u + \bar{u} - c - \bar{c}\,,$$
$$\Delta_{ds} = d + \bar{d} - s - \bar{s}\,,$$
$$\Delta_{sb} = s + \bar{s} - b - \bar{b}\,,$$
$$\Delta_2^l = e + \bar{e} - \mu - \bar{\mu}\,,$$
$$\Delta_{UD} = u + \bar{u} + c + \bar{c} - d - \bar{d} - s - \bar{s} - b - \bar{b}$$
$$\Delta_3^l = e + \bar{e} + \mu + \bar{\mu} - 2(\tau + \bar{\tau})\,,$$

$$\Sigma = \sum_{i=1}^{n_F} (q_i + \bar{q}_i)\,,$$
$$\Sigma^l = \sum_{i=1}^{n_L} (l_i + \bar{l}_i)\,,$$

$$q_v = q_i - \bar{q}_i\,,\; l_v = l_i - \bar{l}_i\,,$$

Valence distributions

- Photon and gluon distributions are not altered
- Straightforward extension to deal with $n_F=6$

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Extending DGLAP equations

Introducing QED corrections

- New optimized DGLAP equations (I)

$$\frac{dq_{v_i}}{dt} = P^-_{q_i} \otimes q_{v_i} + \sum_{j=1}^{n_F} \Delta P^S_{q_i q_j} \otimes q_{v_j} + \Delta P^S_{q_i l} \otimes \left(\sum_{j=1}^{n_L} l_{v_j} \right),$$

$$\frac{dl_{v_i}}{dt} = P^-_{l} \otimes l_{v_i} + \sum_{j=1}^{n_F} \Delta P^S_{l q_j} \otimes q_{v_j} + \Delta P^S_{ll} \otimes \left(\sum_{j=1}^{n_L} l_{v_j} \right),$$

Valence PDFs

$$\frac{d\{\Delta_{uc}, \Delta_{ct}\}}{dt} = P^+_u \otimes \{\Delta_{uc}, \Delta_{ct}\},$$

$$\frac{d\{\Delta_{ds}, \Delta_{sb}\}}{dt} = P^+_d \otimes \{\Delta_{ds}, \Delta_{sb}\},$$

$$\frac{d\Delta^l_{\{2,3\}}}{dt} = P^+_l \otimes \Delta^l_{\{2,3\}},$$

Diagonal equations

$$\frac{d\Sigma}{dt} = \frac{P^+_u + P^+_d}{2} \otimes \Sigma + \frac{P^+_u - P^+_d}{2} \otimes \Delta_{UD} + \frac{n_u \bar{P}^S_{uu} + n_d \bar{P}^S_{dd} + (n_u + n_d)\bar{P}^S_{ud}}{2} \otimes \Sigma + 2(n_u P_{ug} + n_d P_{dg}) \otimes g$$
$$+ \frac{n_u \bar{P}^S_{uu} - n_d \bar{P}^S_{dd} - (n_u - n_d)\bar{P}^S_{ud}}{2} \otimes \Delta_{UD} + \left(n_u \bar{P}^S_{ul} + n_d \bar{P}^S_{dl}\right) \otimes \Sigma^l + 2(n_u P_{u\gamma} + n_d P_{d\gamma}) \otimes \gamma,$$

$$\frac{d\Sigma^l}{dt} = n_L \frac{\bar{P}^S_{lu} + \bar{P}^S_{ld}}{2} \otimes \Sigma + n_L \frac{\bar{P}^S_{lu} - \bar{P}^S_{ld}}{2} \otimes \Delta_{UD} + \left(P^+_l + n_L \bar{P}^S_{ll}\right) \otimes \Sigma^l + 2n_L (P_{lg} \otimes g + P_{l\gamma} \otimes \gamma),$$

Singlet distributions

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Extending DGLAP equations

Introducing QED corrections

- New optimized DGLAP equations (II)

$$\frac{dg}{dt} = \sum_f P_{gf} \otimes f + \sum_f P_{g\bar{f}} \otimes \bar{f} + P_{gg} \otimes g + P_{g\gamma} \otimes \gamma \,,$$

Gluon PDF evolution

$$\frac{d\gamma}{dt} = \sum_f P_{\gamma f} \otimes f + \sum_f P_{\gamma\bar{f}} \otimes \bar{f} + P_{\gamma g} \otimes g + P_{\gamma\gamma} \otimes \gamma \,,$$

Photon PDF evolution

$$\frac{d\Delta_{UD}}{dt} = \frac{P_u^+ + P_d^+}{2} \otimes \Delta_{UD} + \frac{P_u^+ - P_d^+}{2} \otimes \Sigma + \frac{n_u \bar{P}^S_{uu} - n_d \bar{P}^S_{dd} + (n_u - n_d)\bar{P}^S_{ud}}{2} \otimes \Sigma + 2(n_u P_{ug} - n_d P_{dg}) \otimes g$$
$$+ \frac{n_u \bar{P}^S_{uu} + n_d \bar{P}^S_{dd} - (n_u + n_d)\bar{P}^S_{ud}}{2} \otimes \Delta_{UD} + \left(n_u \bar{P}^S_{ul} - n_d \bar{P}^S_{dl}\right) \otimes \Sigma^l + 2(n_u P_{u\gamma} - n_d P_{d\gamma}) \otimes \gamma \,,$$

- Splitting kernels definitions (*introduced to simplify the notation*)

$$P_{q_i q_k} = \delta_{ik} P^V_{qq} + P^S_{qq} \,, \qquad P_{l_i l_k} = \delta_{ik} P^V_{ll} + P^S_{ll} \,,$$

$$P_{q_i \bar{q}_k} = \delta_{ik} P^V_{q\bar{q}} + P^S_{q\bar{q}} \,, \qquad P_{l_i \bar{l}_k} = \delta_{ik} P^V_{l\bar{l}} + P^S_{l\bar{l}} \,,$$

$$P^\pm_q = P^V_{qq} \pm P^V_{q\bar{q}} \,, \qquad P^\pm_l = P^V_{ll} \pm P^V_{l\bar{l}} \,,$$

$$\Delta P^S_{fF} \equiv P^S_{fF} - P^S_{f\bar{F}} \,,$$

$$\bar{P}^S_{fF} \equiv P^S_{fF} + P^S_{f\bar{F}} \,,$$

Vanishes at $\mathcal{O}(\alpha\,\alpha_S)$ and $\mathcal{O}(\alpha^2)$

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Extending DGLAP equations

Introducing QED corrections

- **Extended sum rules** *(impose physical constraints in AP kernels)*
  - *Fermion number conservation* → $\int_0^1 dx\, P_f^- = 0$
  - *Momentum conservation* → $0 = \frac{dP}{dt} = \int_0^1 dx\, x \left(\frac{dg}{dt} + \frac{d\gamma}{dt} + \sum_f \left(\frac{df}{dt} + \frac{d\bar{f}}{dt} \right) \right)$

- Some general remarks:
  - **Charge separation** effects introduced by QED
  - Non-trivial **quark-lepton mixing** (although simplified in the optimized basis)
  - **Explicit formulae** involving AP kernels can be obtained by replacing the **evolution equations**
  - Sum rules allow to **fix the behaviour** of AP kernels in the end-point (**x=1**)
  - *Also, they are useful for checking the consistency of the results.*

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Definitions and previous results

- Perturbative expansion in QCD and QED couplings (non-trivial counting…)

$$P_{ij} = a_S\, P_{ij}^{(1,0)} + a\, P_{ij}^{(0,1)} + a_S^2\, P_{ij}^{(2,0)} + a_S\, a\, P_{ij}^{(1,1)} + a^2\, P_{ij}^{(0,2)} + \ldots$$

- Well-known LO results:

$$P_{qq}^{(1,0)}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\,\delta(1-x)\right] = C_F \left[p_{qq}(x) + \frac{3}{2}\,\delta(1-x)\right],$$

$$P_{qg}^{(1,0)}(x) = T_R \left[x^2 + (1-x)^2\right] = T_R\, p_{qg}(x)\,,$$

$$P_{gq}^{(1,0)}(x) = C_F \left[\frac{1+(1-x)^2}{x}\right] = C_F\, p_{gq}(x)\,,$$

$$P_{gg}^{(1,0)}(x) = 2\,C_A \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right] + \frac{\beta_0}{2}\,\delta(1-x)\,,$$

Standard QCD AP-kernels at LO

$$P_{ff}^{(0,1)}(x) = e_f^2 \left[p_{qq}(x) + \frac{3}{2}\delta(1-x)\right],$$

$$P_{f\gamma}^{(0,1)}(x) = e_f^2\, p_{qg}(x)\,,$$

$$P_{\gamma f}^{(0,1)}(x) = e_f^2\, p_{gq}(x)\,,$$

$$P_{\gamma\gamma}^{(0,1)}(x) = -\frac{2}{3}\sum_f e_f^2\,\delta(1-x)\,,$$

LO QED splitting functions

- Terms proportional to Dirac's **deltas** are originated by virtual (**loop**) corrections
- **Color** factors in **QCD** replaced with **EM charges** in **QED** → Motivates **Abelianization algorithm**!

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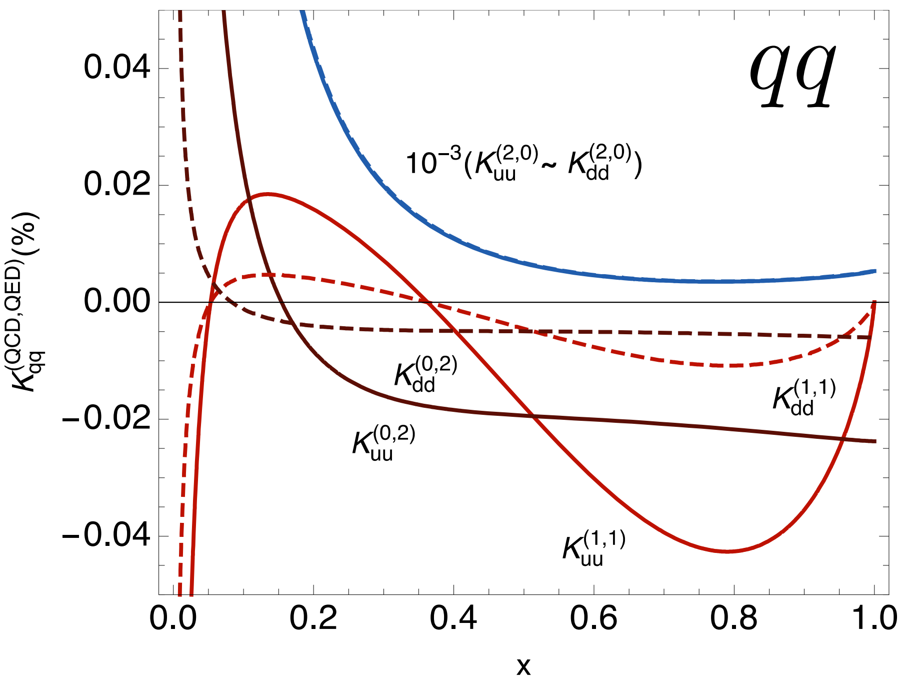
QCD-QED corrections to AP kernels

Phenomenological impact

- We define a ratio to quantify the effect of H.O. QED corrections

$$K_{ab}^{(i,j)} = a_S^i \, a^j \, \frac{P_{ab}^{(i,j)}(x)}{P_{ab}^{LO}(x)} \quad \text{with} \quad P_{ab}^{LO} = a_S \, P_{ab}^{(1,0)} + a \, P_{ab}^{(0,1)}$$

- Quark-quark splittings



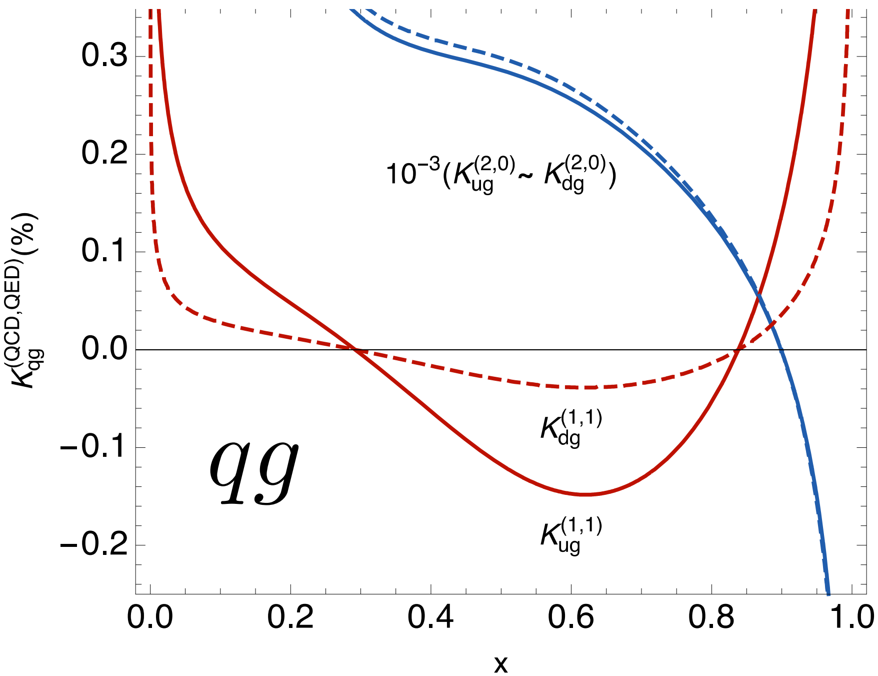
- Pure QCD contribution still dominant (x10^3)
- QED corrections introduce charge separation effects (specially at $\mathcal{O}(\alpha^2)$)
- Small corrections in intermediate x region

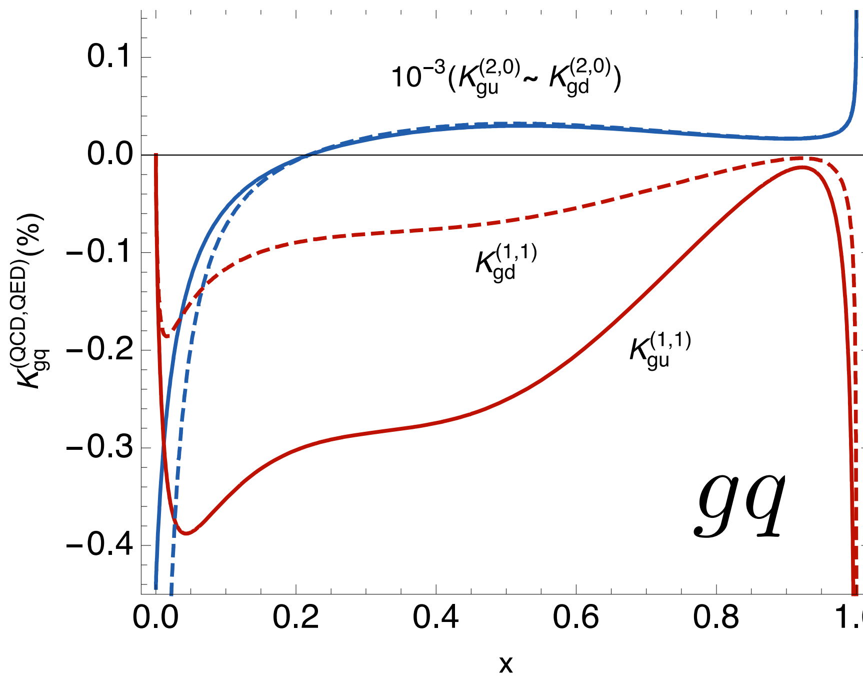
de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Phenomenological impact

- Quark-gluon splittings





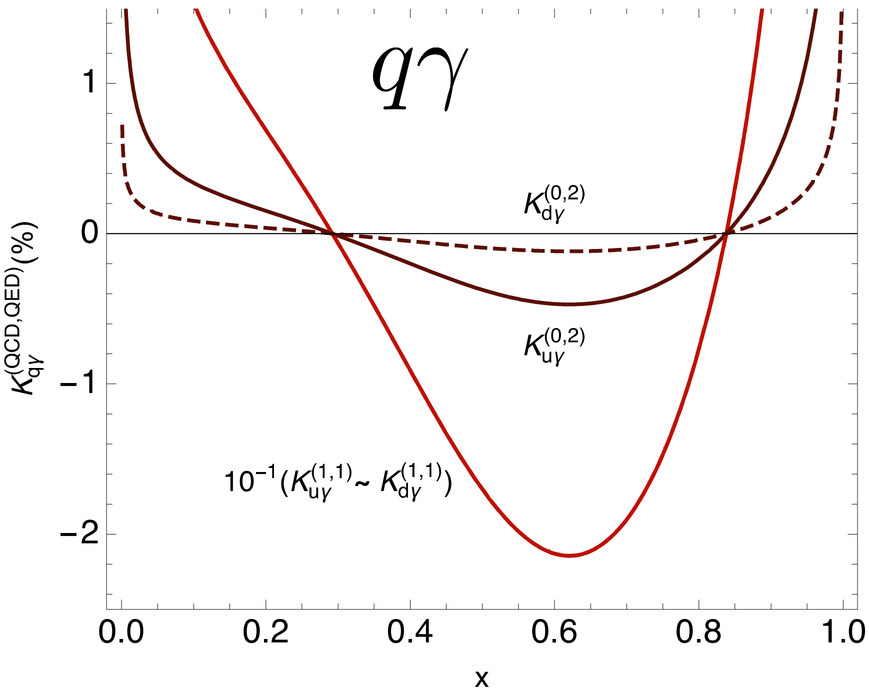
- Again, pure QCD contributions are dominant (x10^3)
- Small charge separation at $\mathcal{O}(\alpha\, \alpha_S)$ and no $\mathcal{O}(\alpha^2)$ corrections

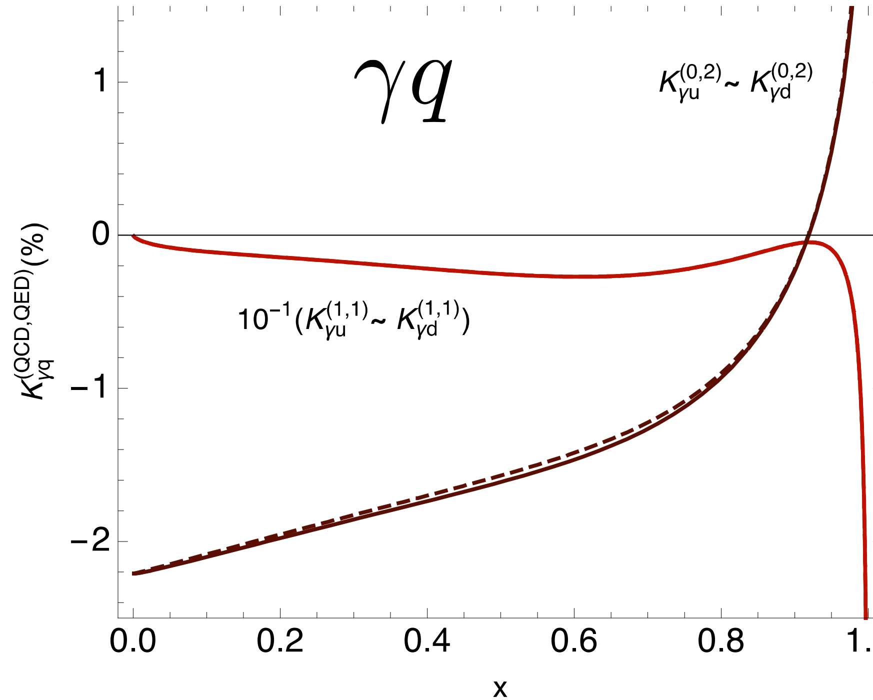
de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Phenomenological impact

- Quark-photon splittings (***new!***)





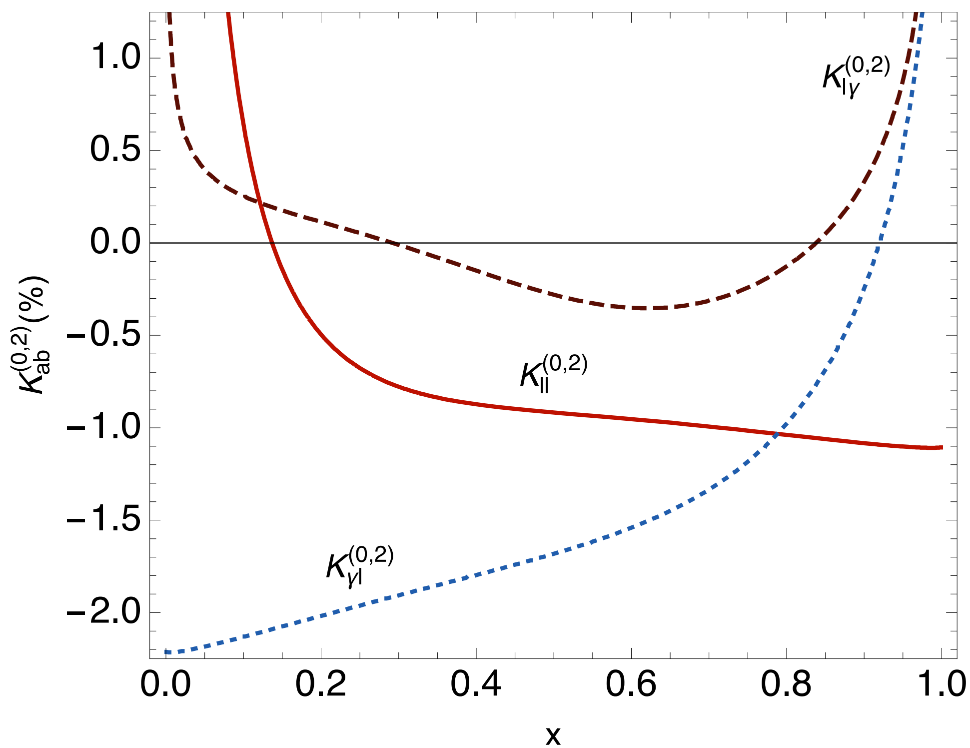
- No $\mathcal{O}(\alpha_S^2)$ contributions: QED corrections are crucial here!!!
- $\mathcal{O}(\alpha\,\alpha_S)$ is dominant but only $\mathcal{O}(\alpha^2)$ contributions are responsible of charge separation effects in $P_{q\gamma}$
- **Percent level corrections (could influence photon PDF)**

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

QCD-QED corrections to AP kernels

Phenomenological impact

- Splittings involving leptons (***new!***)



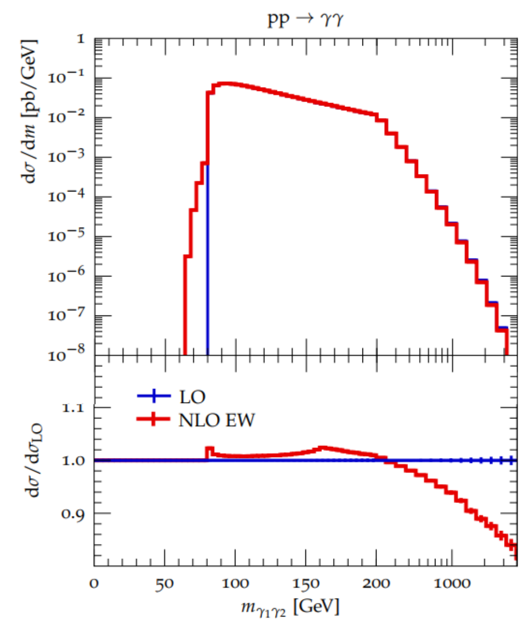
- Starting at $\mathcal{O}(\alpha)$
- Represent a few percent correction (no QCD contribution at LO)
- Lepton PDFs strongly suppressed (small phenomenological impact expected)

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

NLO QED corrections to $pp \to \gamma\gamma$

Applying QED Abelianization

- Application of Abelianization techniques to recover NLO QED.
- Full NLO EW corrections recently computed ➡ Non-negligible effects found in the high-invariant mass region!!!
- Some subtleties to take into account:
  - *QED running:* **only** on-shell final state photons are present at LO; no need to include full QED running.
  - *Photon-ordering:* presence of photon radiation, cuts imposed on the two hardest photons ➡ Dynamical constraint, minimum angular separation bigger than 120°!!!!)
  - *Photon-clustering:* collinear photons are merged; small phenomenological effects due to absence of collinear singularities (not the case in QCD…)



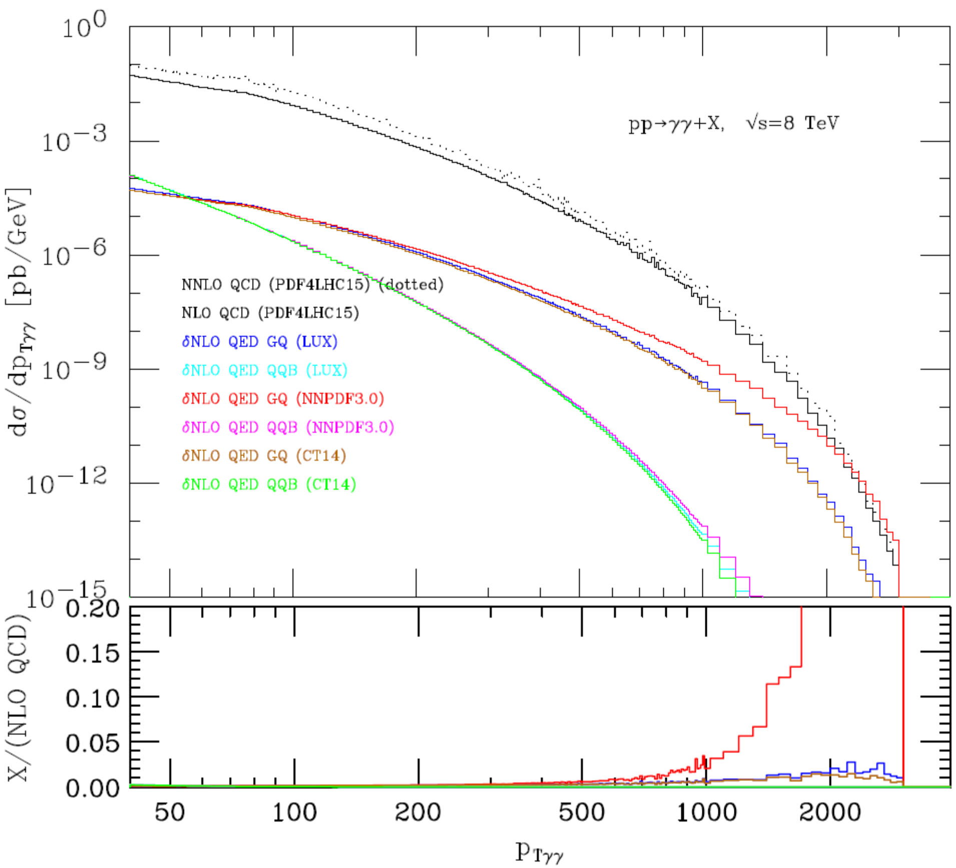
Chiesa et al, arXiv:1706.09022

Cieri, Ferrera and GS, work in progress

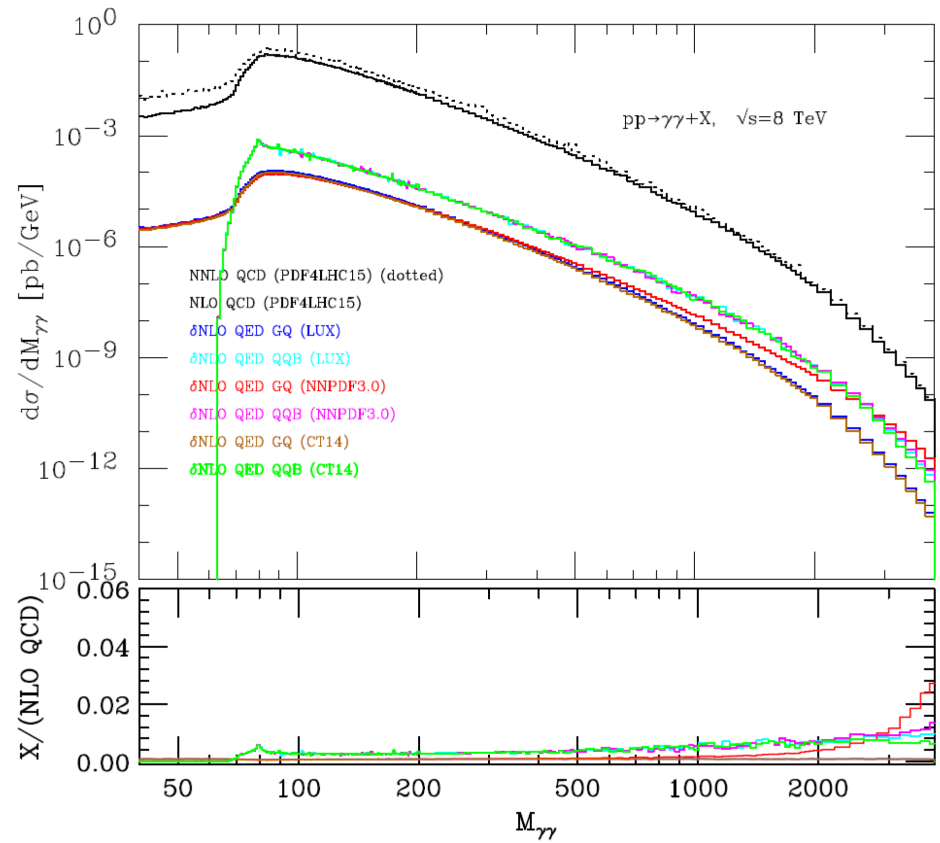
NLO QED corrections to $pp \to \gamma\gamma$

Applying QED Abelianization

- Some plots (comparison with NNLO and NLO QCD corrections)



Transverse-momentum distribution



Invariant mass distribution (hardest photons)

Cieri, Ferrera and GS, work in progress

Conclusions and perspectives

- ✓ Splitting kernels are crucial to describe collinear limits (IR subtraction) and control PDF/FF evolution
- ✓ Mixed QCD-QED corrections computed!
- ✓ Fully consistent treatment of IR factorization
- ✓ **Percent level contributions to photon PDF evolution (potential consequence)**
- ✓ Physical example: NLO QED corrections to diphoton production!
- ✓ Additional subtleties due to photon radiation (ordering, merging, identification)
- ✓ **QED corrections for the high-invariant mass region (a few percent level) ⇒ Full EW is crucial!**

Thanks !!!

Backup: Extended DGLAP equations

 Introducing QED corrections: explicit expressions

- **Simplified formulae at** $\mathcal{O}(\alpha\,\alpha_S)$
  - Lepton distributions decouple
  - Charge separation still present
  - $\Delta P^S = 0$ after explicit computation

$$\frac{dq_{v_i}}{dt} = P^-_{q_i} \otimes q_{v_i} + \sum_{j=1}^{n_F} \Delta P^S \otimes q_{v_j}\,,$$

$$\frac{d\{\Delta_{uc},\Delta_{ct}\}}{dt} = P^+_u \otimes \{\Delta_{uc},\Delta_{ct}\}\,,$$

$$\frac{d\{\Delta_{ds},\Delta_{sb}\}}{dt} = P^+_d \otimes \{\Delta_{ds},\Delta_{sb}\}\,,$$

$$\frac{d\Delta_{UD}}{dt} = \frac{P^+_u + P^+_d}{2} \otimes \Delta_{UD} + \frac{P^+_u - P^+_d}{2} \otimes \Sigma + (n_u - n_d) P^S \otimes \Sigma$$
$$+ 2(n_u P_{ug} - n_d P_{dg}) \otimes g + 2(n_u P_{u\gamma} - n_d P_{d\gamma}) \otimes \gamma\,,$$

$$\frac{d\Sigma}{dt} = \frac{P^+_u + P^+_d}{2} \otimes \Sigma + \frac{P^+_u - P^+_d}{2} \otimes \Delta_{UD} + n_F\, P^S \otimes \Sigma$$
$$+ 2(n_u P_{ug} + n_d P_{dg}) \otimes g + 2(n_u P_{u\gamma} + n_d P_{d\gamma}) \otimes \gamma\,.$$

Evolution equations

$$\int_0^1 dx \; \left(P^{V\,(1,1)}_{qq} - P^{V\,(1,1)}_{q\bar{q}}\right) = 0\,,$$

$$\int_0^1 dx\,x \left(2\,n_u P^{(1,1)}_{ug} + 2\,n_d P^{(1,1)}_{dg} + P^{(1,1)}_{\gamma g} + P^{(1,1)}_{gg}\right) = 0\,,$$

$$\int_0^1 dx\,x \left(2\,n_u P^{(1,1)}_{u\gamma} + 2\,n_d P^{(1,1)}_{d\gamma} + P^{(1,1)}_{g\gamma} + P^{(1,1)}_{\gamma\gamma}\right) = 0\,,$$

$$\int_0^1 dx\,x \left(P^{V\,(1,1)}_{qq} + P^{V\,(1,1)}_{q\bar{q}} + P^{(1,1)}_{gq} + P^{(1,1)}_{\gamma q}\right) = 0\,.$$

Sum rules

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Backup: Extended DGLAP equations

Introducing QED corrections: complete sum rules

- **Explicit formulae at** $\mathcal{O}(\alpha^2)$

$$\int_0^1 dx\, x \left(\frac{P_u^+ - P_d^+}{2} + n_L \frac{\bar{P}_{lu}^S - \bar{P}_{ld}^S}{2} + \frac{n_u \bar{P}_{uu}^S - n_d \bar{P}_{dd}^S}{2} - \frac{(n_u - n_d)\bar{P}_{ud}^S}{2} + \frac{P_{gu} - P_{gd}}{2} + \frac{P_{\gamma u} - P_{\gamma d}}{2} \right) = 0\,,$$

From Δ_{UD} evolution

$$\int_0^1 dx\, x\, (2n_d P_{dg} + 2n_u P_{ug} + 2n_L P_{lg} + P_{\gamma g} + P_{gg}) = 0\,,$$

$$\int_0^1 dx\, x\, (2n_d P_{d\gamma} + 2n_u P_{u\gamma} + 2n_L P_{l\gamma} + P_{g\gamma} + P_{\gamma\gamma}) = 0\,;$$

From gluon and photon evolution

$$\int_0^1 dx\, x \left(\frac{P_u^+ + P_d^+}{2} + n_L \frac{\bar{P}_{lu}^S + \bar{P}_{ld}^S}{2} + \frac{n_u \bar{P}_{uu}^S + n_d \bar{P}_{dd}^S}{2} + \frac{n_F \bar{P}_{ud}^S}{2} + \frac{P_{gu} + P_{gd}}{2} + \frac{P_{\gamma u} + P_{\gamma d}}{2} \right) = 0\,,$$

From singlet evolution

$$\int_0^1 dx\, x\, (n_u \bar{P}_{ul}^S + n_d \bar{P}_{dl}^S + n_L \bar{P}_{ll}^S + P_l^+ + P_{gl} + P_{\gamma l}) = 0\,.$$

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282; JHEP 10(2016)056

Backup slides: $\mathcal{O}(\alpha\,\alpha_S)$ splittings

Explicit formulae (I)

$$P_{q\gamma}^{(1,1)} = \frac{C_F\,C_A\,e_q^2}{2}\left\{4 - 9x - (1-4x)\ln(x) - (1-2x)\ln^2(x) + 4\ln(1-x)\right.$$
$$\left. + \, p_{qg}(x)\left[2\ln^2\left(\frac{1-x}{x}\right) - 4\ln\left(\frac{1-x}{x}\right) - \frac{2\pi^2}{3} + 10\right]\right\},$$

$$P_{g\gamma}^{(1,1)} = C_F\,C_A\left(\sum_{j=1}^{n_F} e_{q_j}^2\right)\left\{-16 + 8x + \frac{20}{3}x^2 + \frac{4}{3x} - (6+10x)\ln(x) - 2(1+x)\ln^2(x)\right\},$$

$$P_{\gamma\gamma}^{(1,1)} = -C_F\,C_A\left(\sum_{j=1}^{n_F} e_{q_j}^2\right)\delta(1-x)\,,$$

$$P_{qg}^{(1,1)} = \frac{T_R\,e_q^2}{2}\left\{4 - 9x - (1-4x)\ln(x) - (1-2x)\ln^2(x) + 4\ln(1-x)\right.$$
$$\left. + \, p_{qg}(x)\left[2\ln^2\left(\frac{1-x}{x}\right) - 4\ln\left(\frac{1-x}{x}\right) - \frac{2\pi^2}{3} + 10\right]\right\},$$

$$P_{\gamma g}^{(1,1)} = T_R\left(\sum_{j=1}^{n_F} e_{q_j}^2\right)\left\{-16 + 8x + \frac{20}{3}x^2 + \frac{4}{3x} - (6+10x)\ln(x) - 2(1+x)\ln^2(x)\right\},$$

$$P_{gg}^{(1,1)} = -T_R\left(\sum_{j=1}^{n_F} e_{q_j}^2\right)\delta(1-x)\,,$$

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282

Backup slides: $\mathcal{O}(\alpha\,\alpha_S)$ splittings

Explicit formulae (II)

$$P_{qq}^{S(1,1)} = P_{q\bar{q}}^{S(1,1)} = 0\,,$$

$$P_{qq}^{V(1,1)} = -2\,C_F\,e_q^2\left[\left(2\ln(1-x)+\frac{3}{2}\right)\ln(x)\,p_{qq}(x)+\frac{3+7x}{2}\ln(x)+\frac{1+x}{2}\ln^2(x)\right.$$
$$\left.+\,5(1-x)+\left(\frac{\pi^2}{2}-\frac{3}{8}-6\zeta_3\right)\delta(1-x)\right]\,,$$

$$P_{q\bar{q}}^{V(1,1)} = 2\,C_F\,e_q^2\left[4(1-x)+2(1+x)\ln(x)+2p_{qq}(-x)S_2(x)\right]\,,$$

$$P_{gq}^{(1,1)} = C_F\,e_q^2\left[-(3\ln(1-x)+\ln^2(1-x))p_{gq}(x)+\left(2+\frac{7}{2}x\right)\ln(x)\right.$$
$$\left.-\left(1-\frac{x}{2}\right)\ln^2(x)-2x\ln(1-x)-\frac{7}{2}x-\frac{5}{2}\right]\,,$$

$$P_{\gamma q}^{(1,1)} = P_{gq}^{(1,1)}\,,$$

de Florian, Rodrigo and GS, Eur. Phys. J. C76 (2016) no.5, 282

Backup slides: $\mathcal{O}(\alpha^2)$ splittings

Explicit formulae (I)

$$P^{(0,2)}_{q\gamma} = \frac{C_A\, e_q^4}{2} \Big\{ 4 - 9x - (1-4x)\ln(x) - (1-2x)\ln^2(x) + 4\ln(1-x) + p_{qg}(x) \left[2\ln^2\left(\frac{1-x}{x}\right) - 4\ln\left(\frac{1-x}{x}\right) - \frac{2\pi^2}{3} + 10 \right] \Big\} ,$$

$$P^{(0,2)}_{\gamma q} = e_q^4 \Big[-\left(3\ln(1-x) + \ln^2(1-x)\right) p_{gq}(x) + \left(2 + \frac{7}{2}x\right)\ln(x) - \left(1 - \frac{x}{2}\right)\ln^2(x) - 2x\ln(1-x) - \frac{7}{2}x - \frac{5}{2} \Big] - e_q^2 \left(\sum_f e_f^2 \right) \left[\frac{4}{3}x + p_{gq}(x) \left(\frac{20}{9} + \frac{4}{3}\ln(1-x) \right) \right] ,$$

$$P^{V(0,2)}_{qq} = -e_q^4 \Big[\left(2\ln(x)\ln(1-x) + \frac{3}{2}\ln(x) \right) p_{qq}(x) + \frac{3+7x}{2}\ln(x) + \frac{1+x}{2}\ln^2(x) + 5(1-x) + \left(\frac{\pi^2}{2} - \frac{3}{8} - 6\zeta_3 \right)\delta(1-x) \Big] - e_q^2 \left(\sum_f e_f^2 \right) \left[\frac{4}{3}(1-x) + p_{qq}(x)\left(\frac{2}{3}\ln(x) + \frac{10}{9} \right) + \left(\frac{2\pi^2}{9} + \frac{1}{6} \right)\delta(1-x) \right] ,$$

$$P^{V(0,2)}_{q\bar{q}} = e_q^4 \left[4(1-x) + 2(1+x)\ln(x) + 2p_{qq}(-x)S_2(x) \right] ,$$

$$P^{S(0,2)}_{qQ} = P^{S(0,2)}_{q\bar{Q}} = C_A\, e_q^2\, e_Q^2\, p_s(x) ,$$

de Florian, Rodrigo and GS, JHEP 10(2016)056

Backup slides: $\mathcal{O}(\alpha^2)$ splittings

Explicit formulae (II)

$$P^{(0,2)}_{l\gamma} = \frac{e_l^4}{C_A\, e_q^4}\, P^{(0,2)}_{q\gamma}\,,$$

$$P^{(0,2)}_{\gamma l} = e_l^4\,\Big[-(3\ln(1-x)+\ln^2(1-x))p_{gq}(x) + \left(2+\frac{7}{2}x\right)\ln(x) - \left(1-\frac{x}{2}\right)\ln^2(x) - 2x\ln(1-x) - \frac{7}{2}x - \frac{5}{2}\Big] - e_l^2\left(\sum_f e_f^2\right)\left[\frac{4}{3}x + p_{gq}(x)\left(\frac{20}{9}+\frac{4}{3}\ln(1-x)\right)\right]\,,$$

$$P^{V(0,2)}_{ll} = -e_l^4\,\Big[\left(2\ln(x)\ln(1-x)+\frac{3}{2}\ln(x)\right)p_{qq}(x) + \frac{3+7x}{2}\ln(x) + \frac{1+x}{2}\ln^2(x) + 5(1-x) + \left(\frac{\pi^2}{2}-\frac{3}{8}-6\zeta_3\right)\delta(1-x)\Big] - e_l^2\left(\sum_f e_f^2\right)\left[\frac{4}{3}(1-x) + p_{qq}(x)\left(\frac{2}{3}\ln(x)+\frac{10}{9}\right) + \left(\frac{2\pi^2}{9}+\frac{1}{6}\right)\delta(1-x)\right]\,,$$

$$P^{V(0,2)}_{l\bar{l}} = \frac{e_l^4}{e_q^4}\, P^{V(0,2)}_{q\bar{q}}\,,$$

$$P^{S(0,2)}_{lL} = P^{S(0,2)}_{l\bar{L}} = e_l^2\, e_L^2\, p_s(x)\,.$$

de Florian, Rodrigo and GS, JHEP 10(2016)056

Backup slides: $\mathcal{O}(\alpha^2)$ splittings

Explicit formulae (III)

$$P^{(0,2)}_{\gamma\gamma} = \left(\sum_f e_f^4\right)\left[-16 + 8x + \frac{20}{3}x^2 + \frac{4}{3x} - (6+10x)\ln(x) - 2(1+x)\ln^2(x) - \delta(1-x)\right],$$

$$P^{(0,2)}_{fg} = 0\,, \quad P^{(0,2)}_{gf} = 0\,, \quad P^{(0,2)}_{\gamma g} = 0\,, \quad P^{S(0,2)}_{lq} = P^{S(0,2)}_{l\bar{q}} = e_l^2\, e_q^2\, p_s(x)\,,$$

$$P^{(0,2)}_{g\gamma} = 0\,, \quad P^{(0,2)}_{gg} = 0\,. \quad P^{S(0,2)}_{ql} = P^{S(0,2)}_{q\bar{l}} = C_A\, e_l^2\, e_q^2\, p_s(x)\,,$$

$$p_s(x) = \frac{20}{9x} - 2 + 6x - \frac{56}{9}x^2 + \left(1 + 5x + \frac{8}{3}x^2\right)\ln(x) - (1+x)\ln^2(x)$$

de Florian, Rodrigo and GS, JHEP 10(2016)056