Recent developments in the computation of scattering amplitudes beyond one loop

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Introduction and Outline

One loop automation has been successfully achieved over the past decade. As for the tree-level, most NLO calculations can be efficiently done by pushing a button (and eventually waiting to achieve statistics)

I will report on two projects, which are building on the work that was done at one loop both in terms of *understanding scattering amplitudes* via integrand-reduction and *developing automated code* for such calculations

- ★ Associated production of a top pair and a vector boson at **next-to-next-to-leading logarithmic accuracy**
- ★ Recent developments towards an integrand-reduction based approach to **two-loop** (and **higher order**) calculations

Part I

Associated production of a top-quark pair and a vector boson at NNLL accuracy

Work in collaboration with

Alessandro Broggio (TUM), Andrea Ferroglia (City Tech),
Nicolas Greiner (Zurich), Ben D. Pecjak (IPPP Durham),
Ray D. Sameshima (City Tech)

References and Goals

Associated production of a ***top pair*** *and a* ***Z boson*** *at the LHC to NNLL accuracy*,
A. Broggio, A. Ferroglia, GO, B.D. Pecjak, and R.D. Sameshima,
JHEP 1704, 105 (2017) [arXiv:1702.00800]

Associated production of a ***top pair*** *and a* ***W boson*** *at next-to-next-to-leading logarithmic accuracy* A. Broggio, A. Ferroglia, G. Ossola, and B.D. Pecjak,
JHEP 1609, 089 (2016) [arXiv:1607.05303]

Associated production of **top-quark pair** and **Higgs boson**:
A. Broggio, B.D. Pecjak, A. Signer, L.L. Yang
JHEP 1603 (2016) 124 [arXiv:1510.01914]
JHEP 1702 (2017) 127 [arXiv:1611.00049]

Goals of the Project: Analyze the factorization properties of these processes in the soft emission limit in order to:

1. Obtain *NNLL resummation formulas* for these processes
2. Evaluate *the total cross section and differential distributions* depending on the 4-momenta of the final state particles

Phenomenological motivations for tTW and tTZ

- Associated production of a ***top-quark pair*** and ***W** or **Z boson*** are *the two heaviest set of particles observed at the* LHC with center of mass energy of 7, 8, 13 TeV.
- Important to detect *anomalies in the top couplings* of the Z boson
- Both processes would be altered by a variety of *new physics* models
- Can be considered *background processes* in new physics searches
- Both processes have already been calculated at NLO accuracy in QCD

Garzelli, Kardos, Papadopoulos, Trocsanyi (2012)
Campbell, Ellis (2012)
Maltoni, Mangano, Tsinikos, Zaro (2011)

On the experimental side

- Process measured by CMS and ATLAS at 8 TeV

$$\sigma_{t\bar{t}W} = 382^{+117}_{-102} \text{ fb (CMS)} \qquad \sigma_{t\bar{t}W} = 369^{+100}_{-91} \text{ fb (ATLAS)}$$

$$\sigma_{t\bar{t}Z} = 242^{+65}_{-55} \text{ fb (CMS)} \qquad \sigma_{t\bar{t}Z} = 176^{+58}_{-52} \text{ fb (ATLAS)}$$

- ATLAS and CMS also already released preliminary measurements at 13 TeV

$$\sigma_{t\bar{t}W} = 980^{+320}_{-280} \text{ fb (CMS)} \qquad \sigma_{t\bar{t}W} = 1400^{+800}_{-800} \text{ fb (ATLAS)}$$

$$\sigma_{t\bar{t}Z} = 700^{+210}_{-190} \text{ fb (CMS)} \qquad \sigma_{t\bar{t}Z} = 900^{+300}_{-300} \text{ fb (ATLAS)}$$

W production measurements are
in agreement with each other
but about *1.5 σ larger than the NLO* prediction

Interesting to explore what happens beyond NLO…

Large logarithmic corrections

The partonic cross section for top pair production (+Higgs, W, or Z boson) receives potentially large corrections from soft gluon emission diagrams

Schematically, the partonic cross section depends on logarithms of the ratio of two different scales:

$$L \equiv \ln\left(\frac{\text{“hard” scale}}{\text{“soft” scale}}\right)$$

where potentially $\alpha_s L \sim 1$

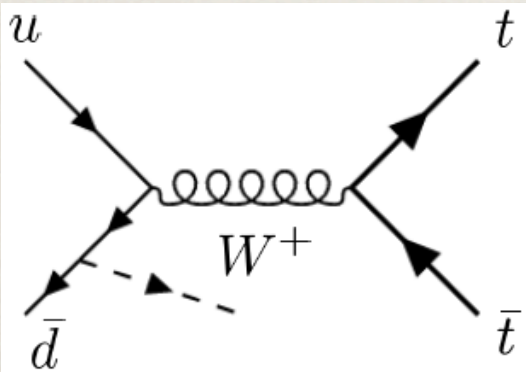
One needs to reorganize the perturbative series: **resummation** is needed.

The **resummation of soft emission corrections** can be carried out by means of *effective field theory methods*

Soft limit & factorization

$t\bar{t}W^+$ kinematics

$$q(p1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) + W^+(p_5)$$



We can define the invariants:

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

If real radiation in the final state is present, $\hat{s} \neq M^2$

$$z = \frac{M^2}{\hat{s}}$$

In the soft emission limit a clear scale hierarchy emerges:

$$\underbrace{\hat{s}, M^2, m_t^2}_{\text{Hard scales}} \gg \underbrace{\hat{s}(1-z)^2}_{\text{Soft scale}} \gg \Lambda^2_{\mathrm{QCD}}$$

In this limit, the partonic cross section factors into two parts:

$$C_{ij} = \mathrm{Tr}\Big[\underbrace{\mathbf{H}_{ij}\left(M, \{p_i\}, \mu\right)}_{\text{Hard function (virtual corrections)}} \underbrace{\mathbf{S}_{ij}\left(\sqrt{\hat{s}}(1-z), \{p_i\}, \mu\right)}_{\text{Soft function (real soft emission)}}\Big]$$

H and **S** are matrices in color space!

In order to evaluate the **NLO hard function** one needs to calculate **one-loop QCD amplitudes**. In doing this one need to separate the *various components of the amplitude in color space*.

The soft function can be calculated by evaluating diagrams involving the emission of soft gluons from the external legs

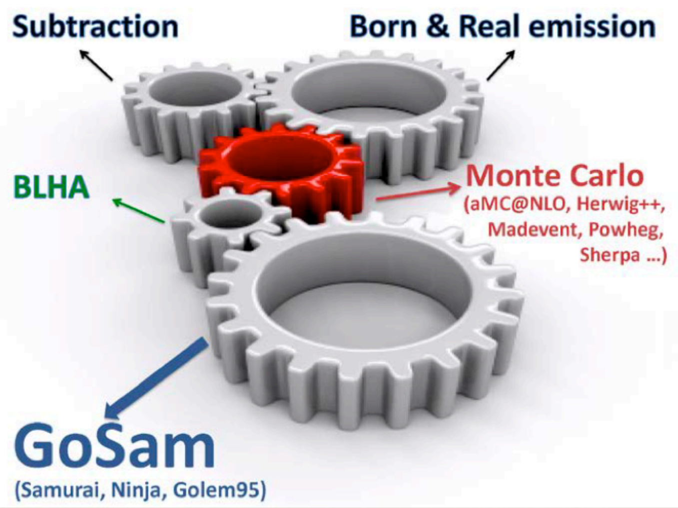
Solution: Modified version of **GoSam** and **Openloops**

The GoSam Project

http://gosam.hepforge.org/

GoSam Collaboration (Updated)
N. Greiner, G. Heinrich, S. Jahn, S. Jones, M. Kerner, G. Luisoni, P. Mastrolia,
GO , T. Peraro, J. Schlenk, L. Scyboz, F. Tramontano

Former members: G. Cullen, H. van Deurzen, E. Mirabella, J. Reichel, T. Reiter, J.F. von Soden-Fraunhofen



The GoSam framework is in continuous evolution
Modular Structure: Ability to incorporate new ideas and techniques

NEW

Electroweak corrections to diphoton plus jets
Chiesa, Greiner, Schoenherr, Tramontano
e-Print: arXiv:1706.09022

"Automated One-Loop Calculations with GoSam"
Cullen, Greiner, Heinrich, Luisoni, Mastrolia, GO, Reiter, Tramontano
Eur.Phys.J. C72 (2012) 1889 [arXiv:1111.2034]

"GOSAM-2.0: a tool for automated one-loop calculations within the Standard Model and beyond"
Cullen, van Deurzen, Greiner, Heinrich, Luisoni, Mastrolia, Mirabella, GO,
Peraro, Schlenk, von Soden-Fraunhofen, Tramontano
Eur.Phys.J. C74 (2014) 3001 [arXiv:1404.7096]

Hard function at NLO with GoSam

By default GoSam provides **squared amplitudes summed over colors**. To build the hard functions we need to combine color decomposed amplitudes

$$\mathbf{H}_{IJ}^{(0)} = \frac{1}{4}\frac{1}{\langle c_I|c_I\rangle\langle c_J|c_J\rangle}\left\langle c_I|\mathcal{M}_{\text{ren}}^{(0)}\right\rangle\left\langle\mathcal{M}_{\text{ren}}^{(0)}|c_J\right\rangle$$

$$\mathbf{H}_{IJ}^{(1)} = \frac{1}{4}\frac{1}{\langle c_I|c_I\rangle\langle c_J|c_J\rangle}\left[\left\langle c_I|\mathcal{M}_{\text{ren}}^{(0)}\right\rangle\left\langle\mathcal{M}_{\text{ren}}^{(1)}|c_J\right\rangle + \left\langle c_I|\mathcal{M}_{\text{ren}}^{(1)}\right\rangle\left\langle\mathcal{M}_{\text{ren}}^{(0)}|c_J\right\rangle\right]$$

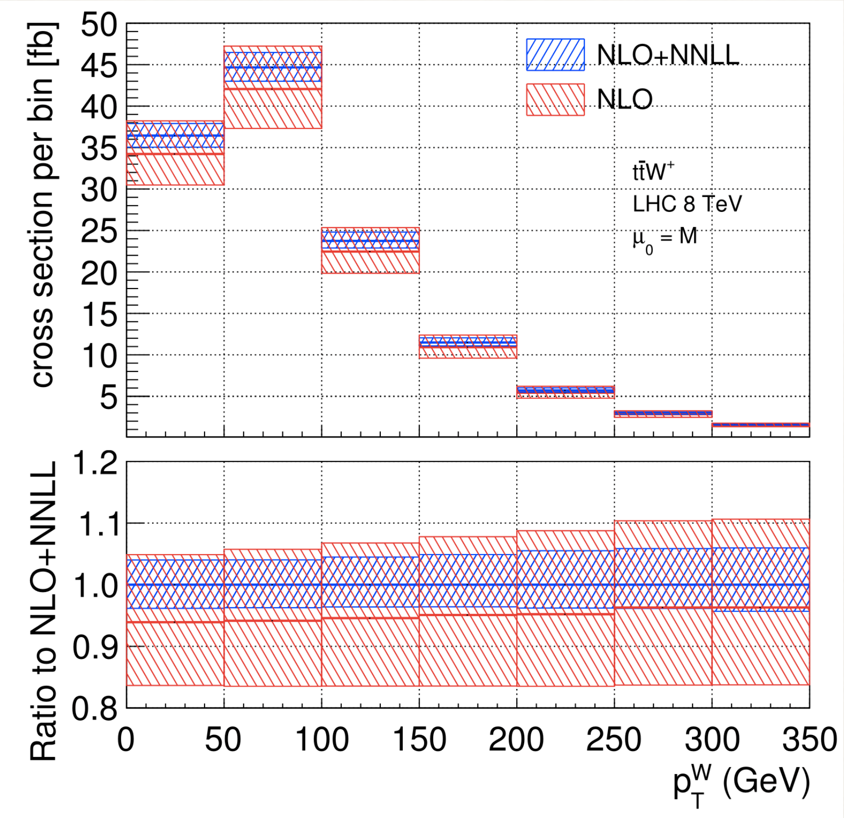
Solution: run **GoSam** with a new extension "*hardfunction*"

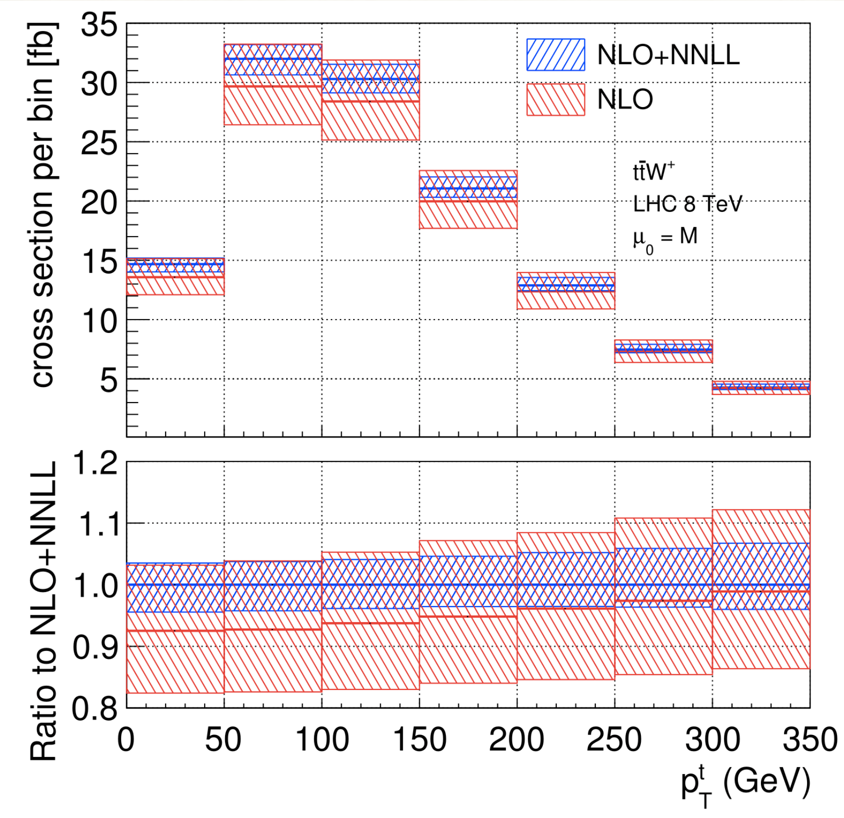
- The extension forces the code to compute all different elements of the Hard Function matrix by projecting on different elements of the color basis
- Change of basis can be performed according to the user's choice
- All results are provided as complex numbers

➢ GoSam generates a FORTRAN routine that allows, for each given phase space point, to compute the corresponding Hard Function at LO and NLO.

Broggio, Ferroglia, Greiner, GO
[more details will be provided in the proceedings]

tTW Distributions at NLO+NNLL





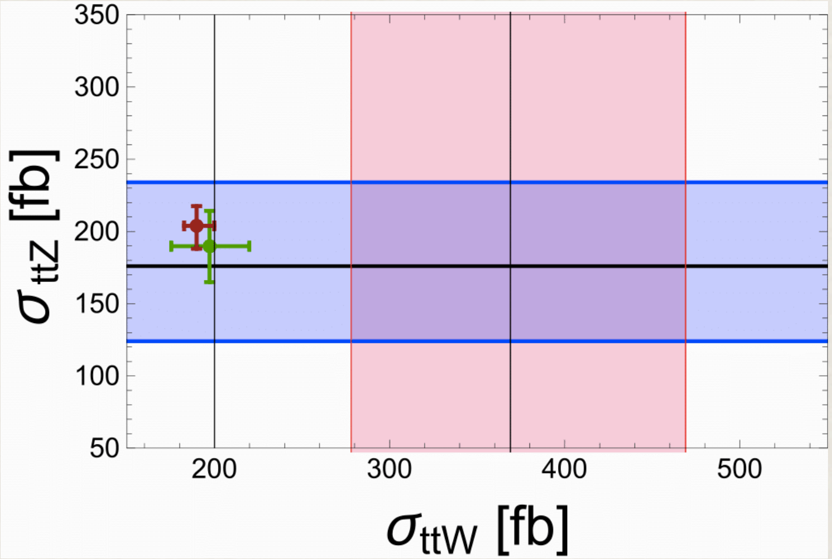
NLO+NNLL distributions overlap with the upper part of the NLO bands; NLO+NNLL bands are narrower than the NLO bands

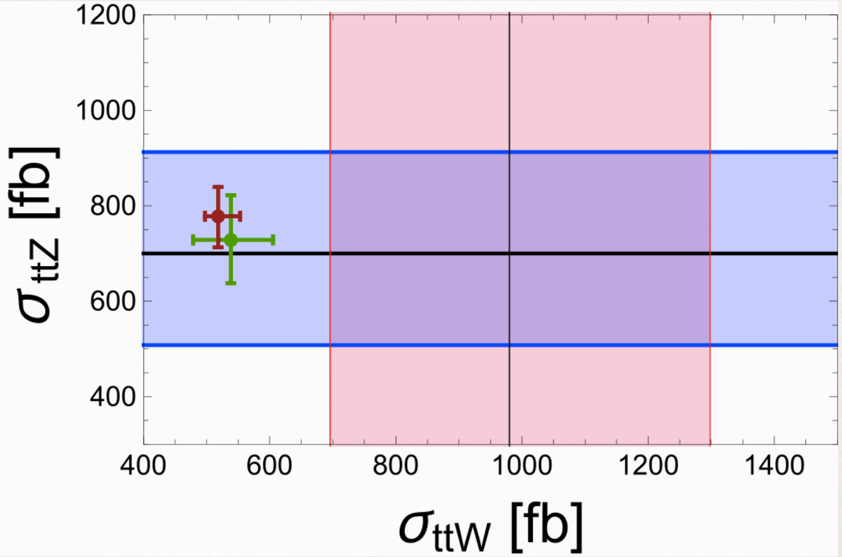
Broggio, Ferroglia, GO, Pecjak (2016)

tTZ/tTW Cross Sections vs Data

8 TeV - ATLAS data
arXiv:1509.05276

13 TeV - CMS data
CMS-PAS-TOP-16-017





Total cross section at NLO (**green cross**) and NLO+NNLL (**red cross**) compared to the ATLAS measurement (8 TeV) and CMS measurement (13 TeV).

Broggio, Ferroglia, GO, Pecjak, Sameshima (2017)

Part II

Scattering Amplitudes via Integrand Reduction at two loops (and higher orders)

Work in collaboration with

Pierpaolo Mastrolia, Amedeo Primo (Univ. Padova)
Tiziano Peraro (Edinburgh)
William J. Torres Bobadilla (Valencia)
Ray D. Sameshima (City Tech)

Integral vs Integrand Reduction

Let's consider a (two-loop) *Feynman integral* with n denominators:

$$\mathcal{I} = \int dq \int dk \; \mathcal{A}(q,k) = \int dq \int dk \; \frac{\mathcal{N}(q,k)}{D_1 D_2 \dots D_n}$$

Integral Level: Description in terms of Master Integrals

$$\mathcal{I} = \int dq \int dk \; \frac{\mathcal{N}(q,k)}{D_1 D_2 \dots D_n} = c_0 \, \mathcal{I}_0 + c_1 \, \mathcal{I}_1 + \dots + C_k \, \mathcal{I}_k$$

Integrand Level: Analyze the *integrand* A(q, k)

$$\mathcal{A}(q,k) = \frac{\mathcal{N}(q,k)}{D_1 D_2 \dots D_n}$$

Can we cast N(q, k) and A(q, k) in a *simpler form* than Feynman Integrals?
Will this lead to a simpler or *smaller set* of Master Integrals?

- ⌘ Integrands are ratios of polynomials in the integration variables
- ⌘ The pole structure is explicit in the integrand

While the *question is the same* as in the one-loop case, *the answer is more involved*

Integrand Reduction at Higher Orders

Mastrolia, GO (2011)

Let's consider a (two-loop) *Feynman integral* with n denominators:

$$\mathcal{I} = \int dq \int dk \; \mathcal{A}(q,k) = \int dq \int dk \; \frac{\mathcal{N}(q,k)}{D_1 D_2 \ldots D_n}$$

As for the one loop, we want to construct identities for the integrands:

$$\mathcal{N}(q,k) = \sum_{i_1 << i_8}^{n} \Delta_{i_1,\ldots,i_8}(q,k) \prod_{h \neq i_1,\ldots,i_8}^{n} D_h + \ldots + \sum_{i_1 << i_2}^{n} \Delta_{i_1,i_2}(q,k) \prod_{h \neq i_1,i_2}^{n} D_h$$

$$\mathcal{A}(q,k) = \sum_{i_1 << i_8}^{n} \frac{\Delta_{i_1,\ldots,i_8}(q,k)}{D_{i_1} D_{i_2} \ldots D_{i_8}} + \sum_{i_1 << i_7}^{n} \frac{\Delta_{i_1,\ldots,i_7}(q,k)}{D_{i_1} D_{i_2} \ldots D_{i_7}} + \ldots + \sum_{i_1 << i_2}^{n} \frac{\Delta_{i_1,i_2}(q,k)}{D_{i_1} D_{i_2}}$$

- Can we always achieve such decomposition?
- How many terms terms appear in the above expressions?
- What is the general form of the residues Δ?
- Can we detect the Master Integrals at the Integrand Level?

Unlike one-loop integrands, *at higher orders we have Irreducible Scalar Products* (ISPs) that do not integrate to zero, and lead to *additional Master Integrals*

Algebraic Geometry: Integrand Reduction via Multivariate Polynomial Division

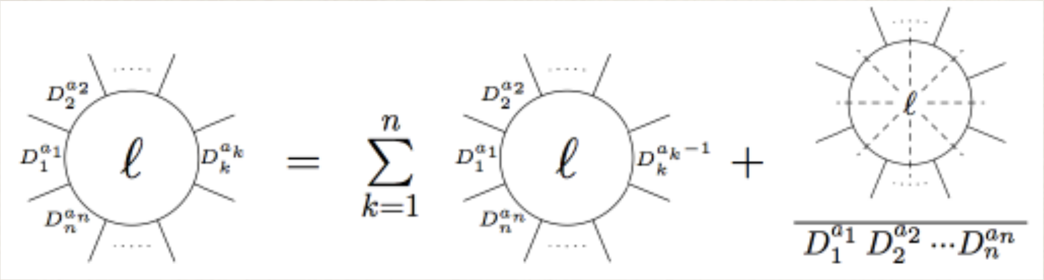
Zhang (2012); Badger, Frellesvig, Zhang (2012)
Mastrolia, Mirabella, GO, Peraro (2012, 2013)

Algebraic Geometry offered an answer to many (but not all) of the questions:

1. If the n-ple cut has a solution, then *there is a corresponding term* in the multi-pole expansion

2. *All residues can be computed by polynomial division* of the numerator modulo *the Gröbner basis of the ideal generated by the denominators* in each n-ple cut:
    a. The remainder of the division is the residue of the n-ple cut
    b. The quotients generate integrands with (n-1) denominators

3. Repeated use of polynomial division on all cuts provides a universal Recursive Formula for the decomposition of any Feynman integral



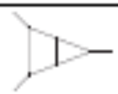
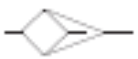
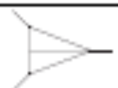
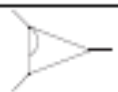
This general decomposition however *does not land on a minimal set* of Master Integrals

Adaptive Integrand Decomposition

Mastrolia, Peraro, Primo (2016)

The integration momenta, defined in d-dimensions, are *decomposed in parallel (physical) and orthogonal (non-physical) subspaces*, adapted according to the number of external leg in the diagram (less legs = more orthogonal space)

Non-physical degrees of freedom are integrated out, leading to **simpler expressions for the integrand-decomposition formulae**.

$\mathcal{I}_{i_1\cdots i_n}$	$\Delta_{i_1\cdots i_n}$	$\Delta^{\mathrm{int}}_{i_1\cdots i_k}$	$\Delta'_{i_1\cdots i_k}$
$\mathcal{I}^{\mathrm{P}}_{1356911}$ 	180 $\{1, x_{31}, x_{41}, x_{22}, x_{32}, x_{42}\}$	22 $\{1, x_{22}, \lambda_{11}, \lambda_{22}, \lambda_{12}\}$	4 $\{1, x_{22}\}$
$\mathcal{I}^{\mathrm{NP1}}_{15691011}$ 	240 $\{1, x_{31}, x_{41}, x_{22}, x_{32}, x_{42}\}$	30 $\{1, x_{22}, \lambda_{11}, \lambda_{22}, \lambda_{12}\}$	6 $\{1, x_{22}\}$
$\mathcal{I}^{\mathrm{P}}_{1571011}$ 	180 $\{1, x_{21}, x_{31}, x_{41}, x_{12}, x_{32}, x_{42}\}$	33 $\{1, x_{21}, x_{12}, \lambda_{11}, \lambda_{22}, \lambda_{12}\}$	13 $\{1, x_{21}, x_{12}\}$
$\mathcal{I}^{\mathrm{P}}_{1691011}$ 	115 $\{1, x_{31}, x_{41}, x_{12}, x_{22}, x_{32}, x_{42}\}$	20 $\{1, x_{11}, x_{22}\lambda_{11}, \lambda_{22}, \lambda_{12}\}$	6 $\{1, x_{12}, x_{22}\}$
$\mathcal{I}^{\mathrm{P}}_{361011}$ 	100 $\{1, x_{11}, x_{21}, x_{31}, x_{41}, x_{22}, x_{32}, x_{42}\}$	26 $\{1, x_{11}, x_{21}, x_{22}, \lambda_{11}, \lambda_{22}, \lambda_{12}\}$	16 $\{x_{11}, x_{21}, x_{22}\}$

Polynomial division reduced to a substitution rule: each reducible scalar product (or variable) is replaced by a *linear combination of denominators and ISPs*

Functional Reconstruction over Finite Fields

Many computer algebra systems use **finite fields** for solving problems such as polynomial factorization. The application of *finite fields in high-energy physics* is however very recent.

Schabinger, von Manteuffel (2014), Peraro (2016)
Maierhofer, Usovitsch, Uwer (2017)

Multivariate polynomials and rational functions which appear in many calculations in high-energy physics *can be reconstructed from their numerical evaluation at several values of its arguments*. These **techniques become particularly efficient**, when the numerical evaluation, and subsequent reconstruction, is performed **over finite fields**.

The well known *Newton's polynomial representation* and *Thiele's interpolation formulas* can be extended to the reconstruction of **multivariate polynomials**.

Using these methods, **the analytic calculation of any polynomial or rational function** can be turned into the problem of *providing an efficient numerical evaluation of the same function over finite fields*.

An alternative semi-automated path to NNLO Automation

Automated **Generation** of Feynman Integrals

i.e. extension of the GoSam framework

Reduction

by means of

Adaptive Integrand Decomposition

Work in progress!

Functional Reconstruction over Finite Fields

leading to

Analytic expressions

New handle on **IBPs**

Numerical or Analytic **Evaluation** of MIs

Conclusions and Future Outlook

Plenty of activity is going on beyond one-loop.

Traditional tools for IBP-based approaches have been upgraded with smarter techniques (finite fields reconstruction). Further improvements have been achieved in the analytic and numerical evaluation of Master Integrals.

New ideas emerged. New representations are explored. Algebraic geometry provided a new handle to understand the structure of mathematical objects which appear in these calculations. Unitarity at higher loops is under development.

It's a time of **theoretical improvements** and **mathematical explorations**, the feeling is that *'something is about to happen'* which will provide a better handle on more advanced calculations...

Thanks for your attention!