Charm Quark Mass with Calibrated Uncertainty

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Motivation

- m_c enters many QCD processes
- renormalization group running of α (0th moment!) ***JE 1999***
- running of $\sin^2\theta_W$ ***JE, Ramsey-Musolf 2005***
- SM prediction of $g_\mu - 2$ ***JE, Luo 2001***
- test of mass-Yukawa coupling relation in single Higgs SM
- can determine m_c with lattice, but second opinion wanted

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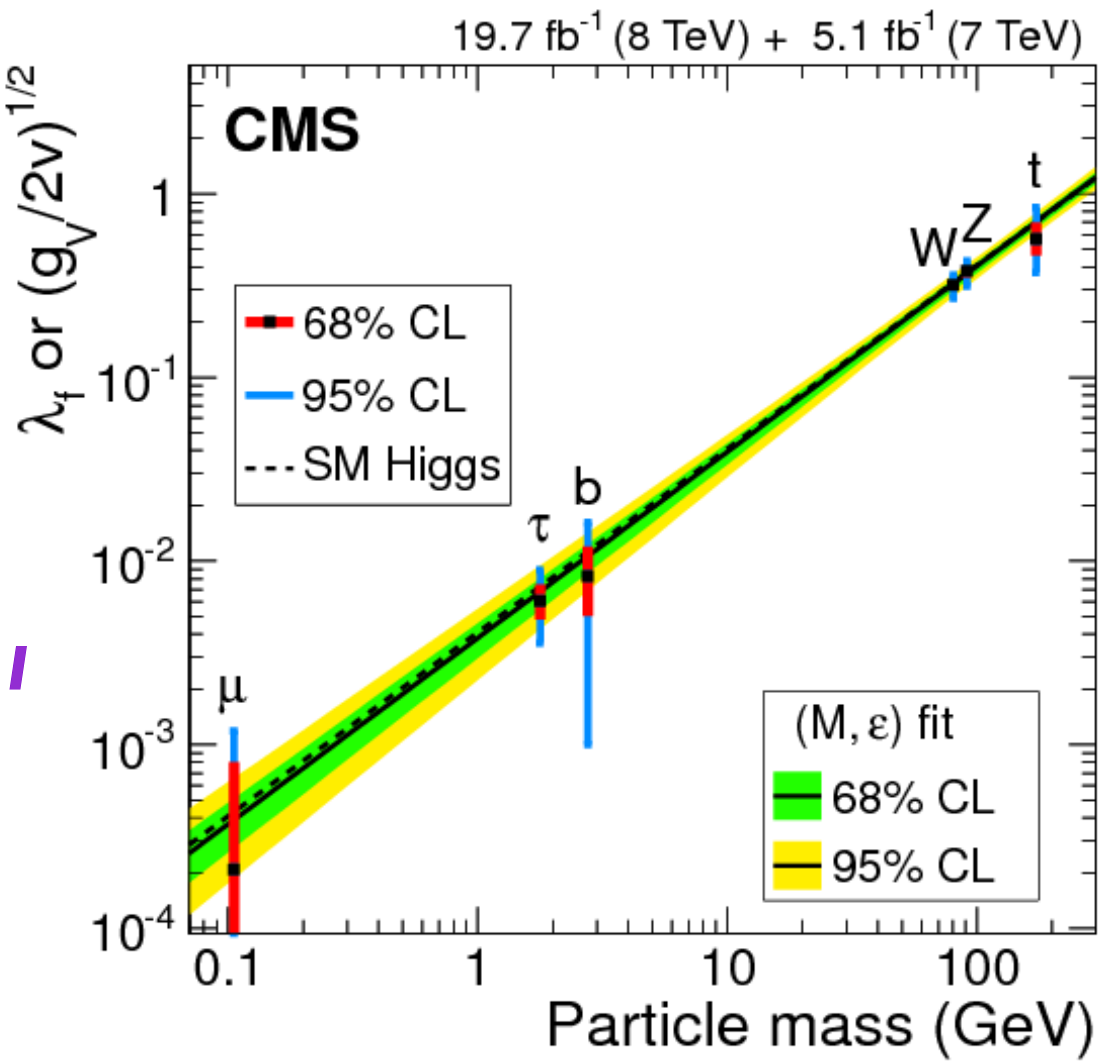

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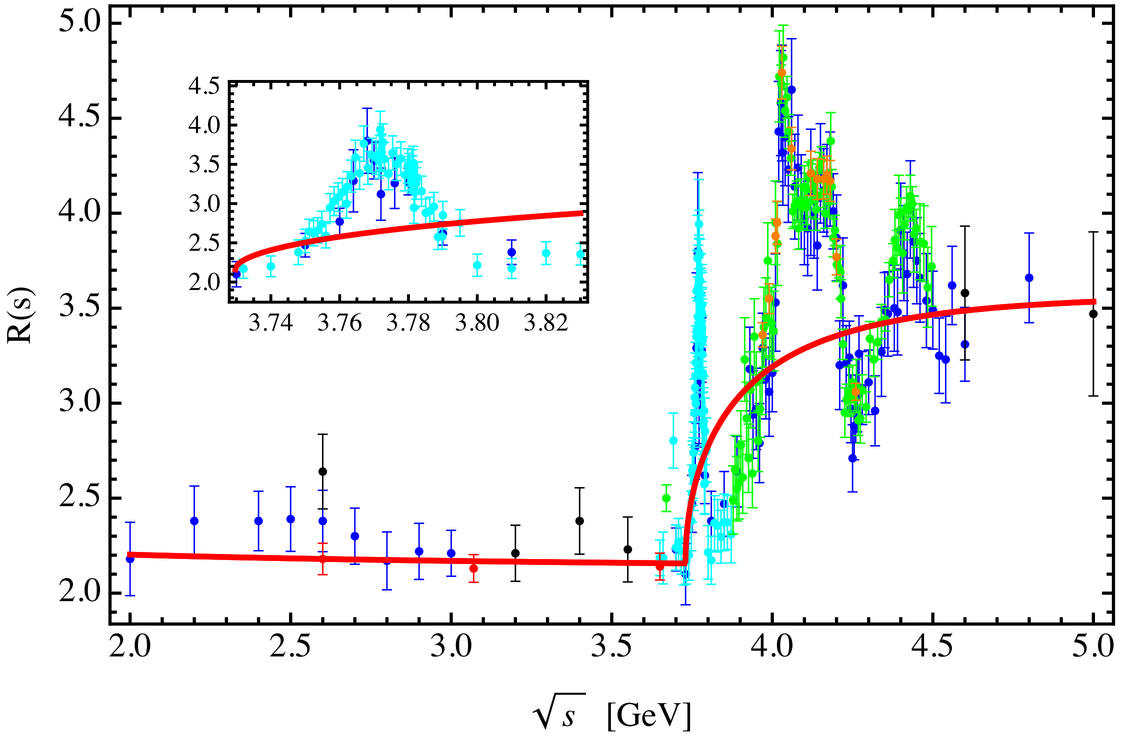
Relativistic sum rule formalism

$$12\pi^2 \frac{\hat{\Pi}_q(0) - \hat{\Pi}_q(-t)}{t} = \int_{4\hat{m}_q^2}^{\infty} \frac{\mathrm{d}s}{s} \frac{R_q(s)}{s+t}$$

- QCD sum rule of moments of the vector current correlator Π_q
- pQCD to $\mathcal{O}(\alpha_s^3)$ ***Chetyrkin, Kühn, Sturm 2006; Boughezal, Czakon, Schutzmeier 2006; Kniehl, Kotikov 2006; Maier, Maierhofer, Marquard 2008; Maier, Maierhofer, Marquard, Smirnov 2010***
- $t \to 0 \Rightarrow$ 1st moment sum rule $\mathcal{M}_1$
- differentiating $\Rightarrow$ higher moments $\mathcal{M}_n$ ***Novikov et al. 1978***
- $t \to \infty \Rightarrow$ 0th moment sum rule $\mathcal{M}_0$ ***JE, Luo 2003***
- regularization: subtract $R_c(s) = 4/3\ \lambda_1(s)$ at $m_c = 0$

Features of our approach

- only experimental input: electronic widths of J/ψ and ψ(2S)
- continuum contribution from self-consistency between sum rules
- include $\mathcal{M}_0$ → stronger (milder) sensitivity to continuum (m_c)



- quark-hadron duality needed only in finite region (not locally)
- can estimate effect from correlated errors across various $\mathcal{M}_n$

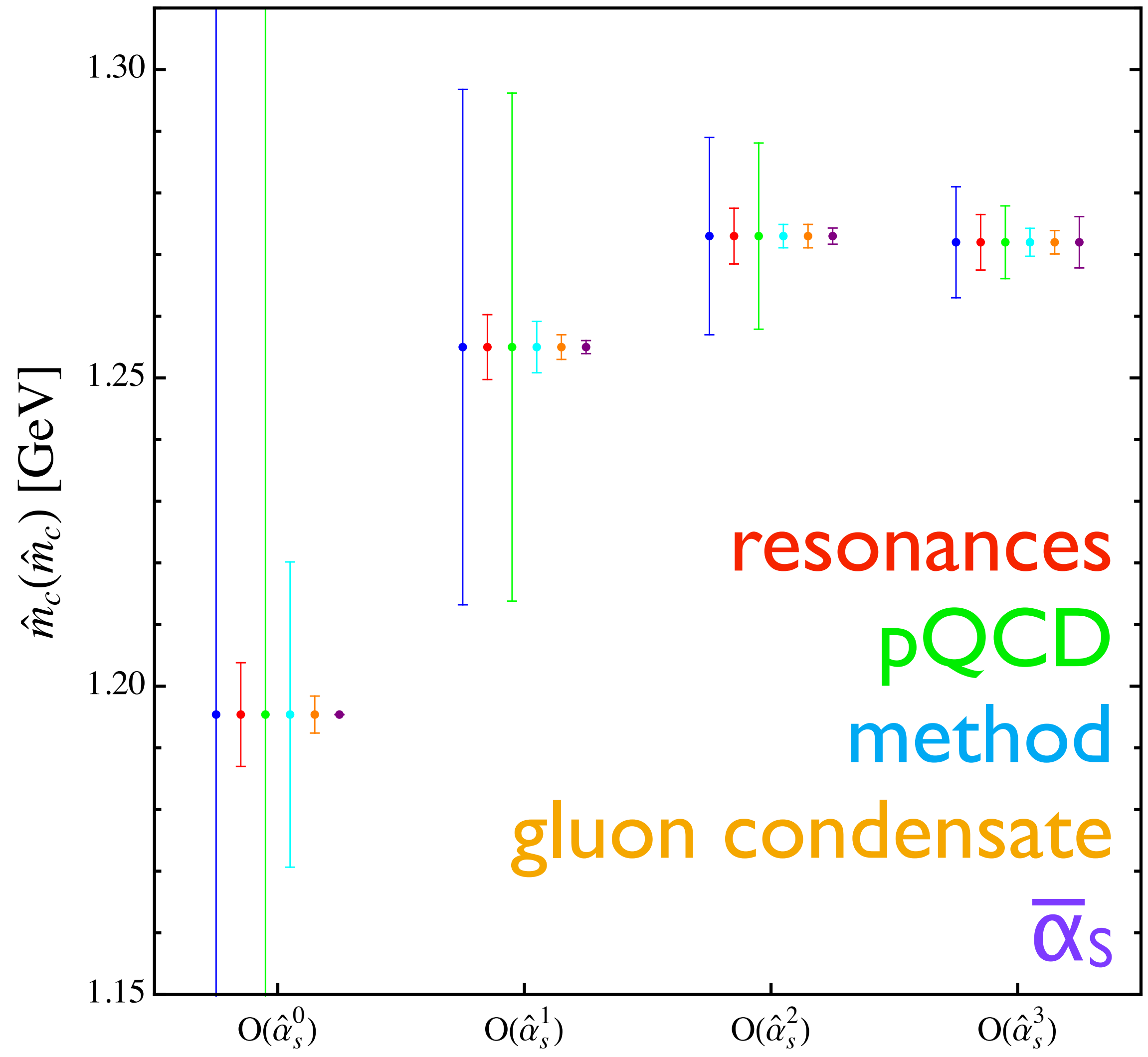
Result

$$\overline{m}_c(\overline{m}_c) = 1272 \pm 8 + 2616\,[\overline{\alpha}_S(M_Z) - 0.1182]\ \mathrm{MeV}$$

- uses $\mathcal{M}_0$ and $\mathcal{M}_2$ (assumed uncorrelated)
- central value in good agreement with other recent sum rule determinations
- less agreement regarding theory dominated uncertainty

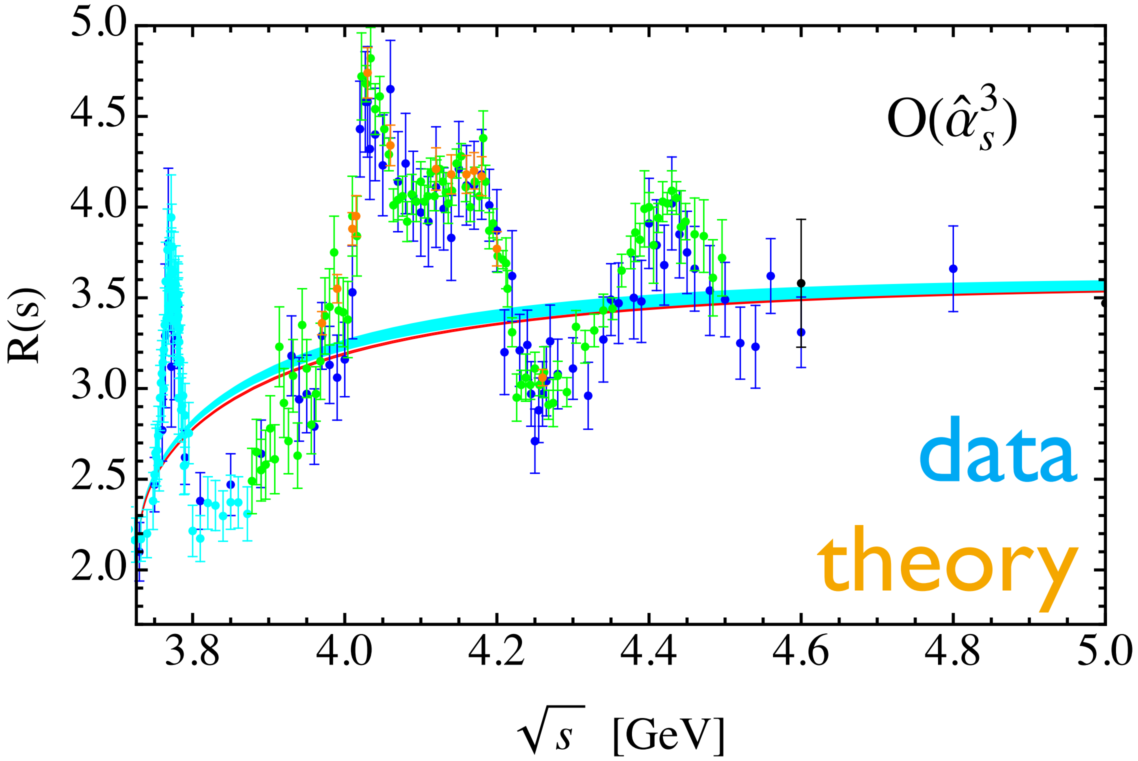
Error calibration

- experimental input error
- truncation error (we use more conservative estimate than taking last computed term)
- we use $e^+ e^- \to$ hadron data to control method (higher order in OPE & quark-hadron duality violations)
- parametric uncertainty (100%)
- $\overline{\alpha}_S(M_Z) = 0.1182 \pm 0.0016$



Continuum

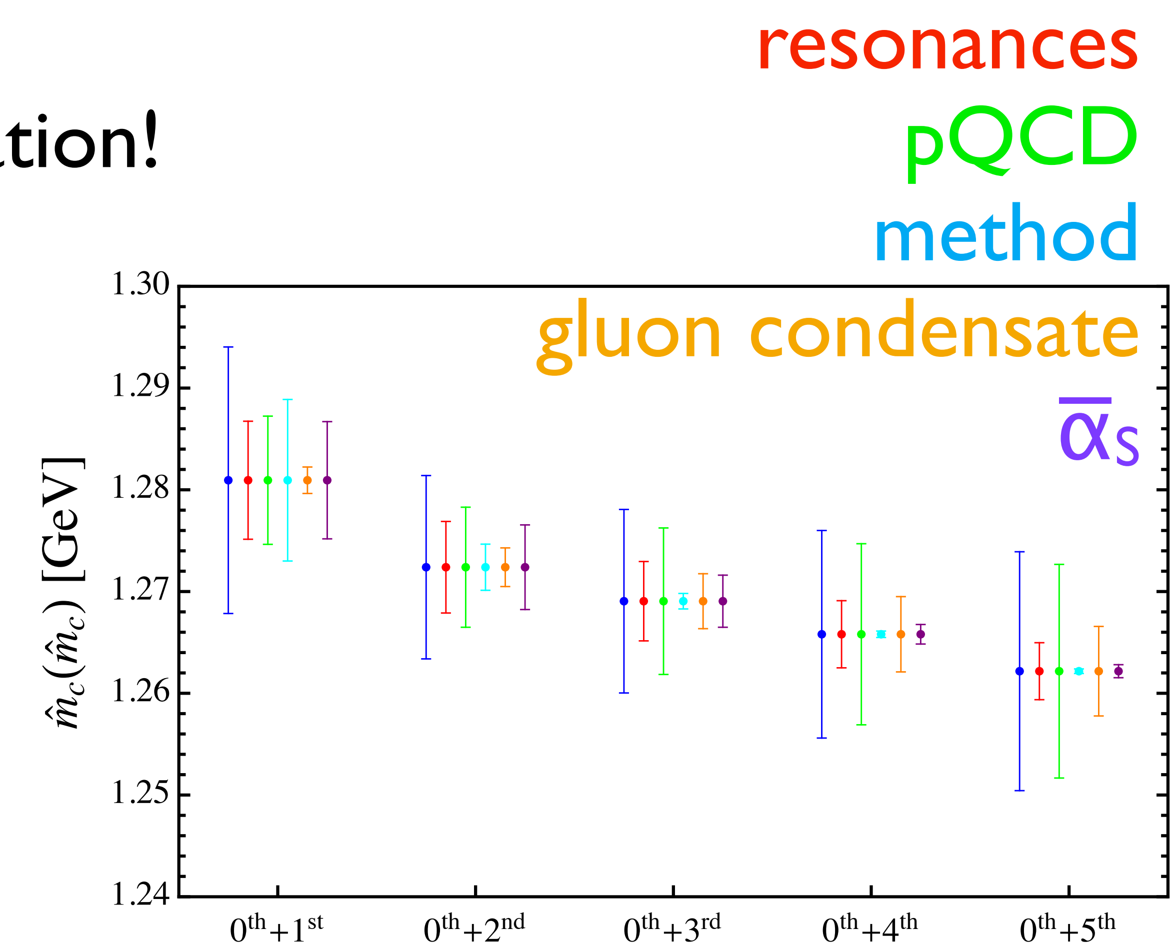
- $R_c^{cont} = 4/3\ \lambda_1(s)\ [\ 1 - 4\ \overline{m}^2(2M_D)/s'\ ]^{1/2}\ [\ 1 + 2\ \lambda_3\ \overline{m}^2(2M_D)/s'\ ]$
- $s' \equiv s + 4\ [\ \overline{m}^2(2M) - M^2\ ]$
- λ_1 known asymptotic behaviour
- λ_3 free parameter (expect ≈ 1)
- $\mathcal{M}_0$ & $\mathcal{M}_2 \Rightarrow \lambda_3 = 1.23(6)$



- removing background from light quarks and (small) singlet contributions from ***Crystal Ball***, ***BES*** & ***CLEO*** data $\Rightarrow \lambda_3 = 1.34(17)$
- or fit normalization of sub-continuum data to pQCD $\Rightarrow \lambda_3 = 1.15(16)$

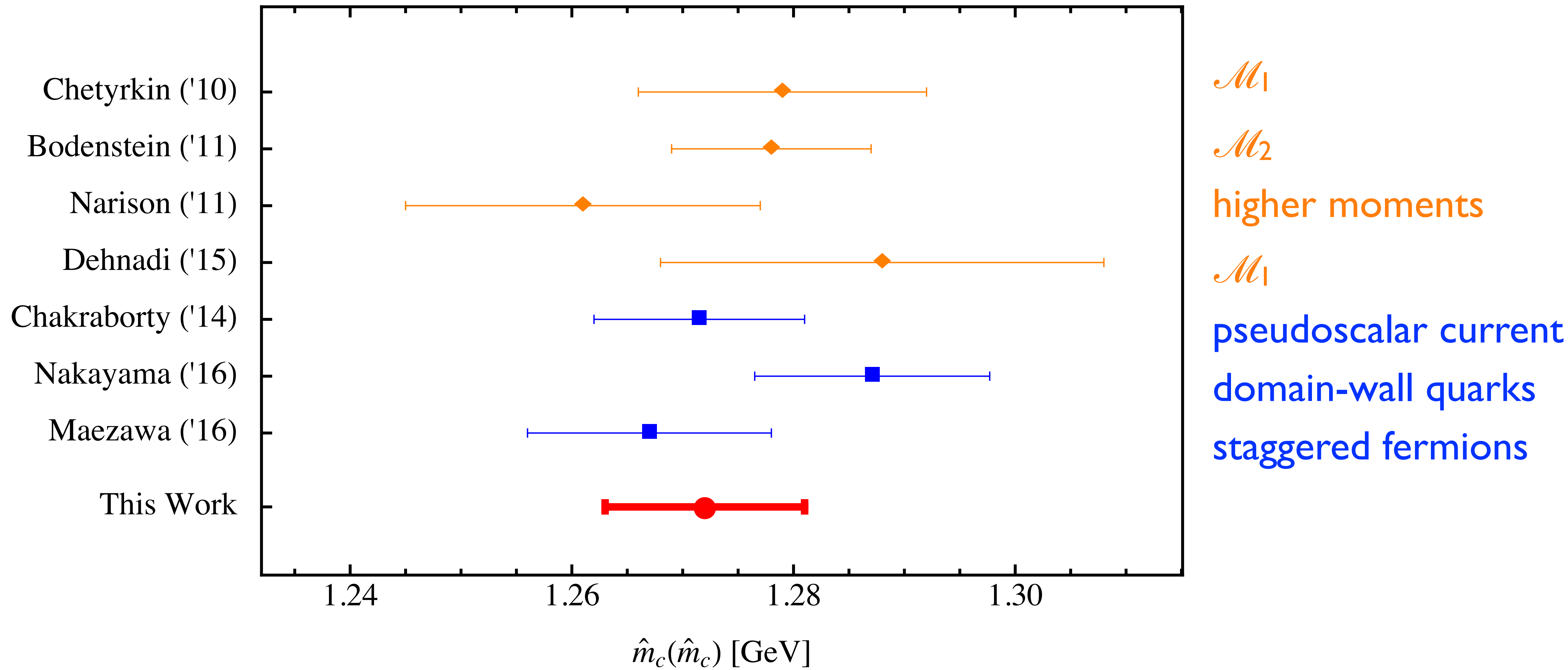
Alternative fits

- $\mathcal{M}_0$, $\mathcal{M}_1$: continuum region!
- $\mathcal{M}_0$, $\mathcal{M}_3$ or $\mathcal{M}_1$, $\mathcal{M}_2$: OPE truncation!
- $\mathcal{M}_0$, $\mathcal{M}_2$: comparable errors
- $(\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2)_\rho$
- $\mathcal{M}_0$, $(\mathcal{M}_1, \mathcal{M}_2)_\rho$
- $\mathcal{M}_0$, $(\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3)_\rho$



- these and other options differ by ≲ 4 MeV in $\overline{m}_c(\overline{m}_c)$

Recent m_c determinations



Conclusions & outlook

$$\overline{m}_c(\overline{m}_c) = 1272 \pm 8 + 2616\,[\overline{\alpha}_S(M_Z) - 0.1182]\ \text{MeV}$$

- physically motivated continuum *ansatz* reproduces experimental data (normalization and moment dependence) very well

- < 0.7% theory uncertainty from pQCD near $\mu \approx 1$ GeV may seem optimistic

- but it is really ≈ 3% in $\frac{1}{2} M_{J/\psi} - \overline{m}_c(\overline{m}_c)$

➡ expect ≈ 15 MeV in $\frac{1}{2} M_{\Upsilon(1S)} - \overline{m}_b(\overline{m}_b)$ (***in preparation***)