Jet production at high precision using the CoLoRFulNNLO method

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Fixed order

Problem

$$\sigma_m^{\mathrm{NNLO}} = \sigma_{m+2}^{\mathrm{RR}} + \sigma_{m+1}^{\mathrm{RV}} + \sigma_m^{\mathrm{VV}}$$

$$\equiv \int_{m+2} \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} + \int_{m+1} \mathrm{d}\sigma_{m+1}^{\mathrm{RV}} J_{m+1} + \int_m \mathrm{d}\sigma_m^{\mathrm{VV}} J_m$$

- the three contributions are separately divergent in $d = 4$ dimensions:
  - in σ^{RR} kinematical singularities as one or two partons become unresolved yielding ϵ-poles at $O(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1})$ after integration over phase space, no explicit ϵ-poles
  - in σ^{RV} kinematical singularities as one parton becomes unresolved yielding ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$ after integration over phase space + explicit ϵ-poles at $O(\epsilon^{-2}, \epsilon^{-1})$
  - in σ^{VV} explicit ϵ-poles at $O(\epsilon^{-4}, \epsilon^{-3}, \epsilon^{-2}, \epsilon^{-1})$

How to combine to obtain finite cross section?

Structure of subtractions

...is governed by the jet functions

$$\sigma_m^{\mathrm{NNLO}} = \sigma_{m+2}^{\mathrm{RR}} + \sigma_{m+1}^{\mathrm{RV}} + \sigma_m^{\mathrm{VV}} = \sigma_{m+2} + \sigma_{m+1} + \sigma_m$$

$$\mathrm{d}\sigma_{m+2} = \left\{ \mathrm{d}\sigma_{m+2}^{\mathrm{RR}} J_{m+2} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} J_m - \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} J_{m+1} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} J_m \right] \right\}_{\epsilon=0}$$

$$\mathrm{d}\sigma_{m+1} = \left\{ \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV}} + \int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right] J_{m+1} - \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] J_m \right\}_{\epsilon=0}$$

$$\mathrm{d}\sigma_m = \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\}_{\epsilon=0} J_m$$

- RR,A_2 regularizes doubly-unresolved limits
- RR,A_1 regularizes singly-unresolved limits
- RR,A_{12} removes overlapping subtractions
- RV,A_1 regularizes singly-unresolved limits

CoLoRFulNNLO is a subtraction scheme with

- ✓ fully local counter-terms (efficiency and mathematical rigor)
- ✓ fully differential predictions (with jet functions defined in d = 4)
- ✓ explicit expressions including flavor and color (color space notation is used)
- ✓ completely general construction (valid in any order of perturbation theory)
- ✓ option to constrain subtraction near singular regions (important check)

Completely Local SubtRactions for Fully Differential Predictions@NNLO

$e^+e^- \rightarrow 3\text{jets}$

Jet rates at NNLO accuracy

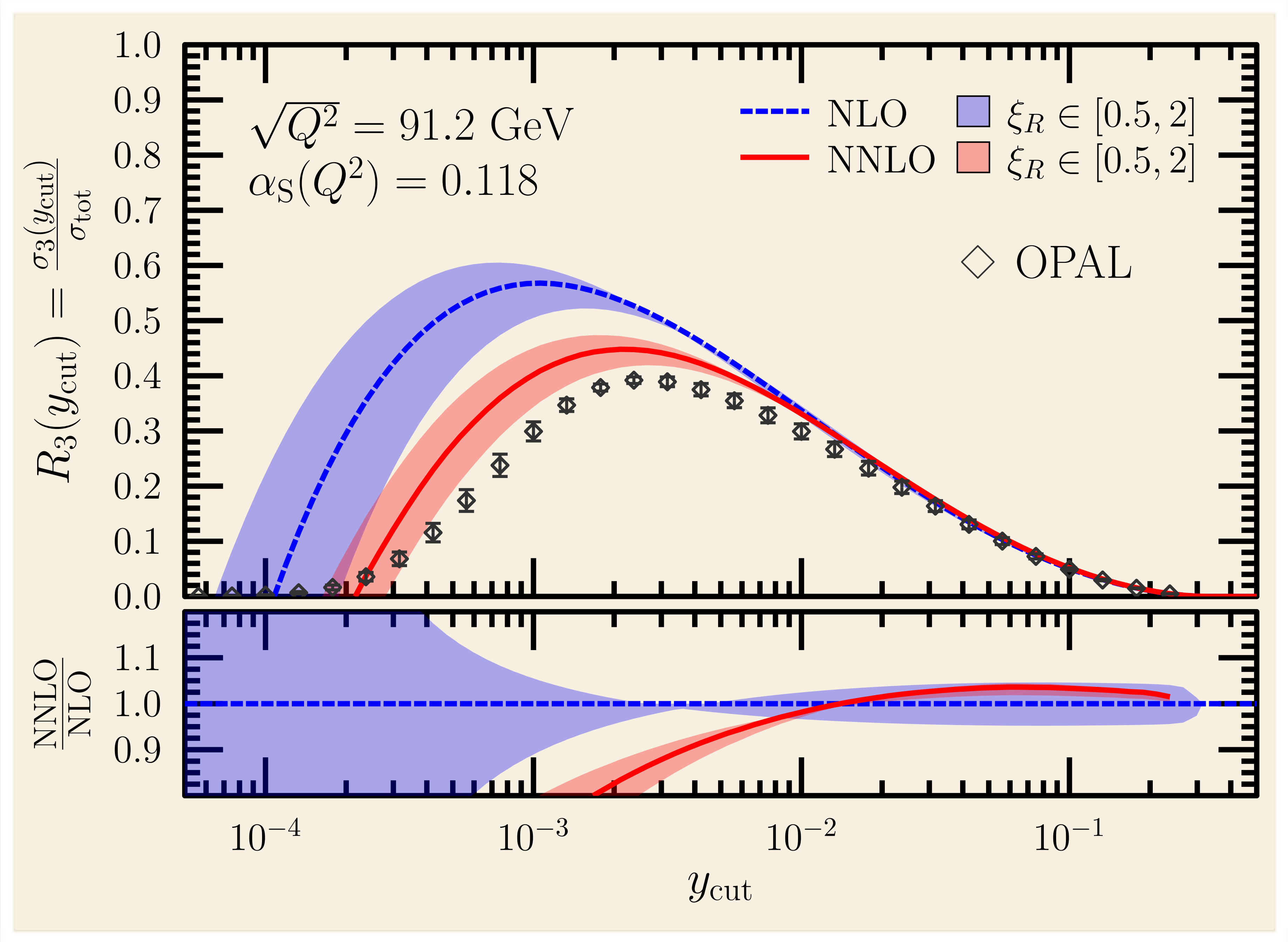
n-jet rate R_n is defined by the ratio

$$R_n(\vec{a}) = \frac{\sigma_{e^+e^- \to n\,\mathrm{jets}}(\vec{a})}{\sigma_{e^+e^- \to \mathrm{hadrons}}}$$

where $\vec{a}$ is a set of jet resolution parameters
in our example it is simply y_{cut} of
the exclusive k_T clustering algorithm

$$R_3^{\mathrm{FO}}(\vec{a},\mu) = \frac{\alpha_S(\mu)}{2\pi} A_3(\vec{a},\mu) + \left(\frac{\alpha_S(\mu)}{2\pi}\right)^2 B_3(\vec{a},\mu) + \left(\frac{\alpha_S(\mu)}{2\pi}\right)^3 C_3(\vec{a},\mu)$$

Jet rates at NNLO accuracy



Scale dependence of the three-jet rate R_3 with 3-loop running

unphysical for small y_{cut}

Resummation

Jet rates at NDL accuracy

$\alpha_s(M_Z)\log^2(1/y_{cut}) = 2.5$ for $\alpha_s(M_Z) = 0.118$ and $y_{cut} = 0.01$
$\alpha_s(M_Z)\log(1/y_{cut}) = 1$ for $\alpha_s(M_Z) = 0.118$ and $y_{cut} = 0.0001$

need to sum up at least leading and next-to-leading logarithms (L = log(1/ycut)) to all order in perturbation theory

NLL(...) resummation formula:

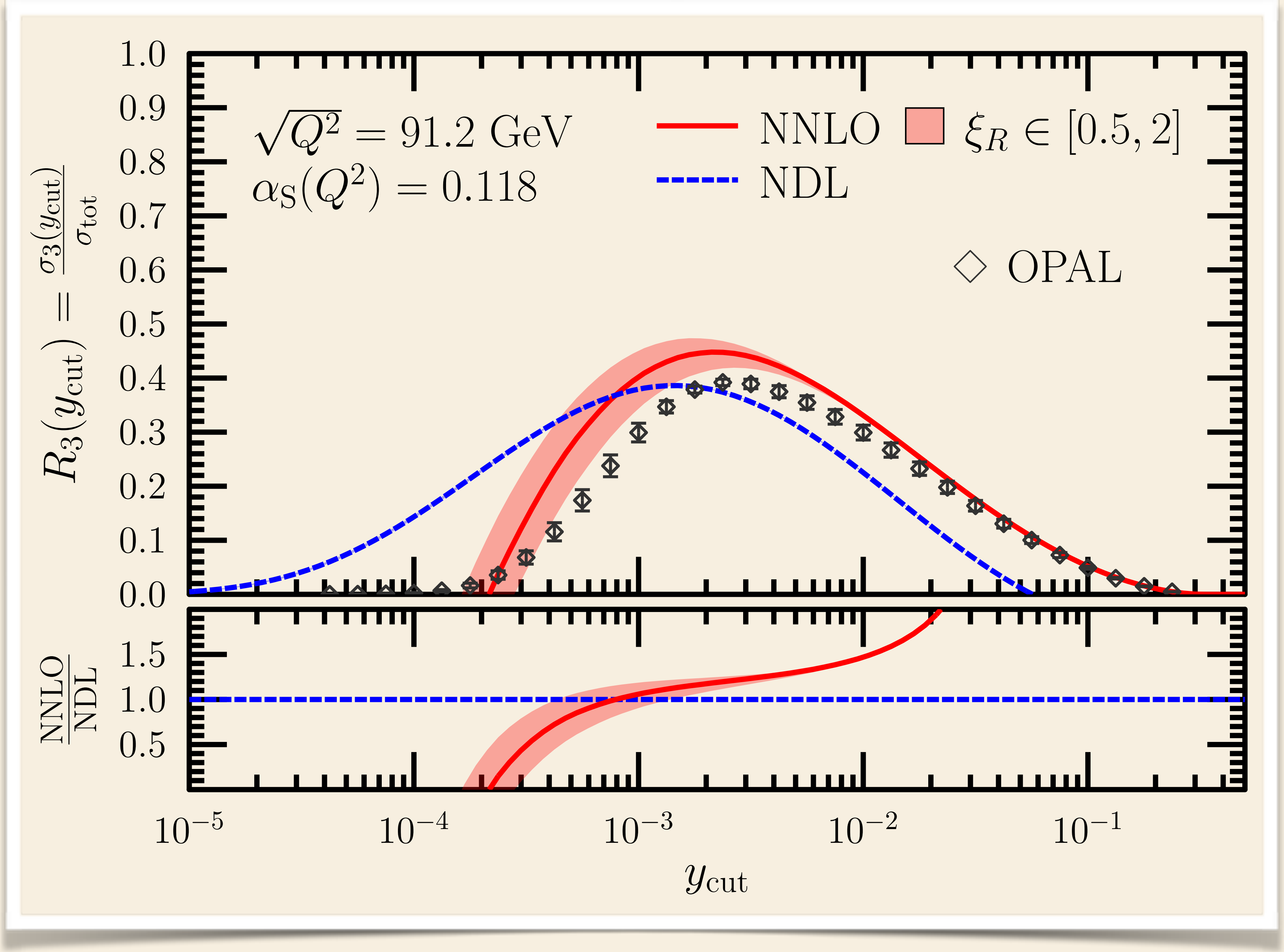
$$R^{\mathrm{NLL}}(L) = \exp(Lg_1(\alpha_{\mathrm{S}}L) + g_2(\alpha_{\mathrm{S}}L) + \dots)\mathcal{F}_{NLL}(L)$$

for jet-rates only NDL resummation formula is known:

$$R^{\mathrm{NDL}}(L) = \sum_{n=1}^{\infty} \alpha_{\mathrm{S}}^n \Big(G_{n,2n}L^{2n} + G_{n,2n-1}L^{2n-1} + \mathcal{O}(L^{2n-2}) \Big)$$

[Catani et al., Phys.Lett. B269 (1991) 432]

Jet rates at NDL accuracy



NDL resummation for the three-jet rates:

correct only for asymptotically small values of y_{cut}

R-matching

Jet rates at FO+NDL accuracy

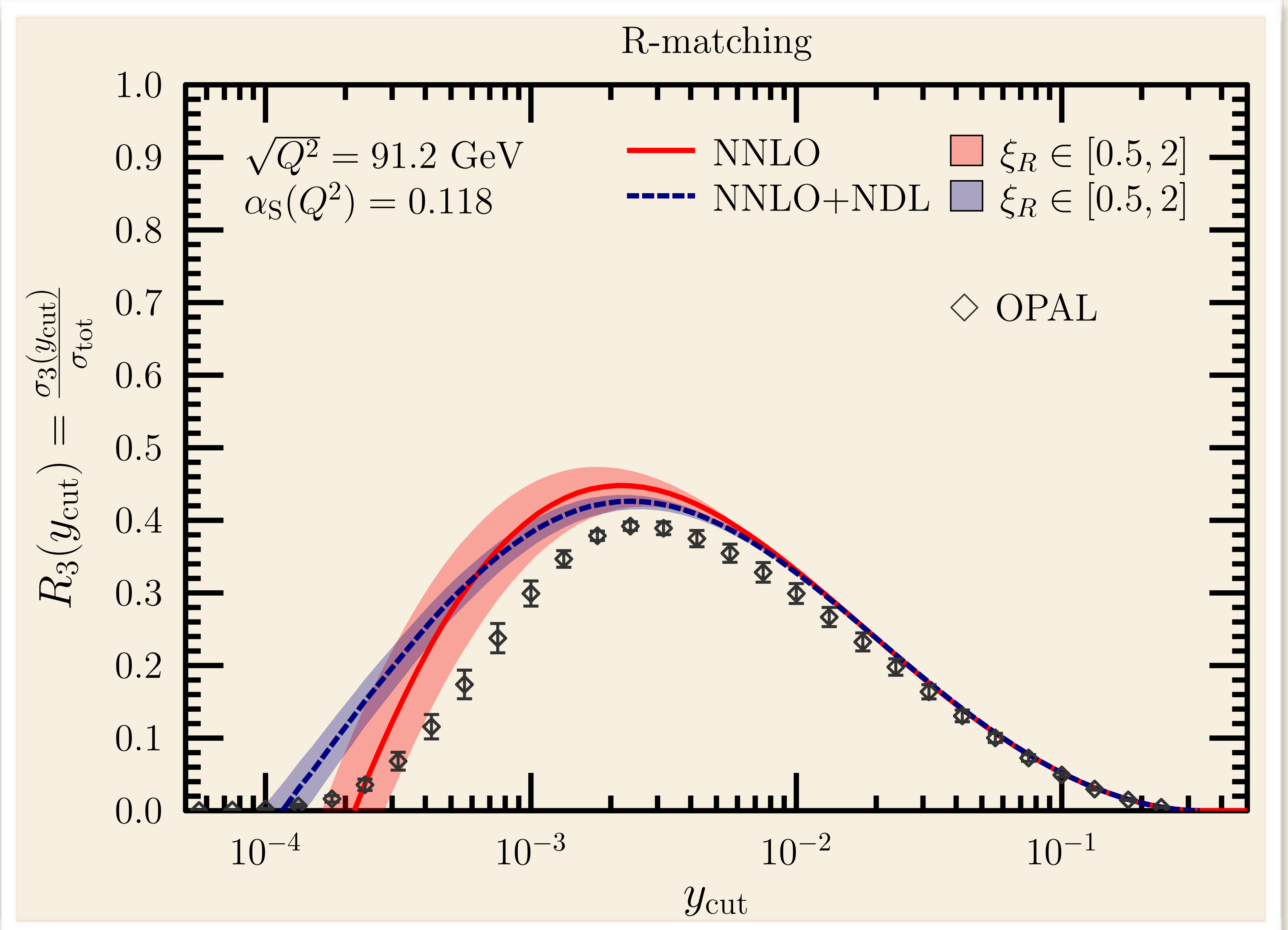
For NDL R-matching is the standard option, defined by

$$R^{\mathrm{FO+NDL}} = R^{\mathrm{NDL}} - R^{\mathrm{NDL}}_{exp} + R^{\mathrm{FO}}$$

with

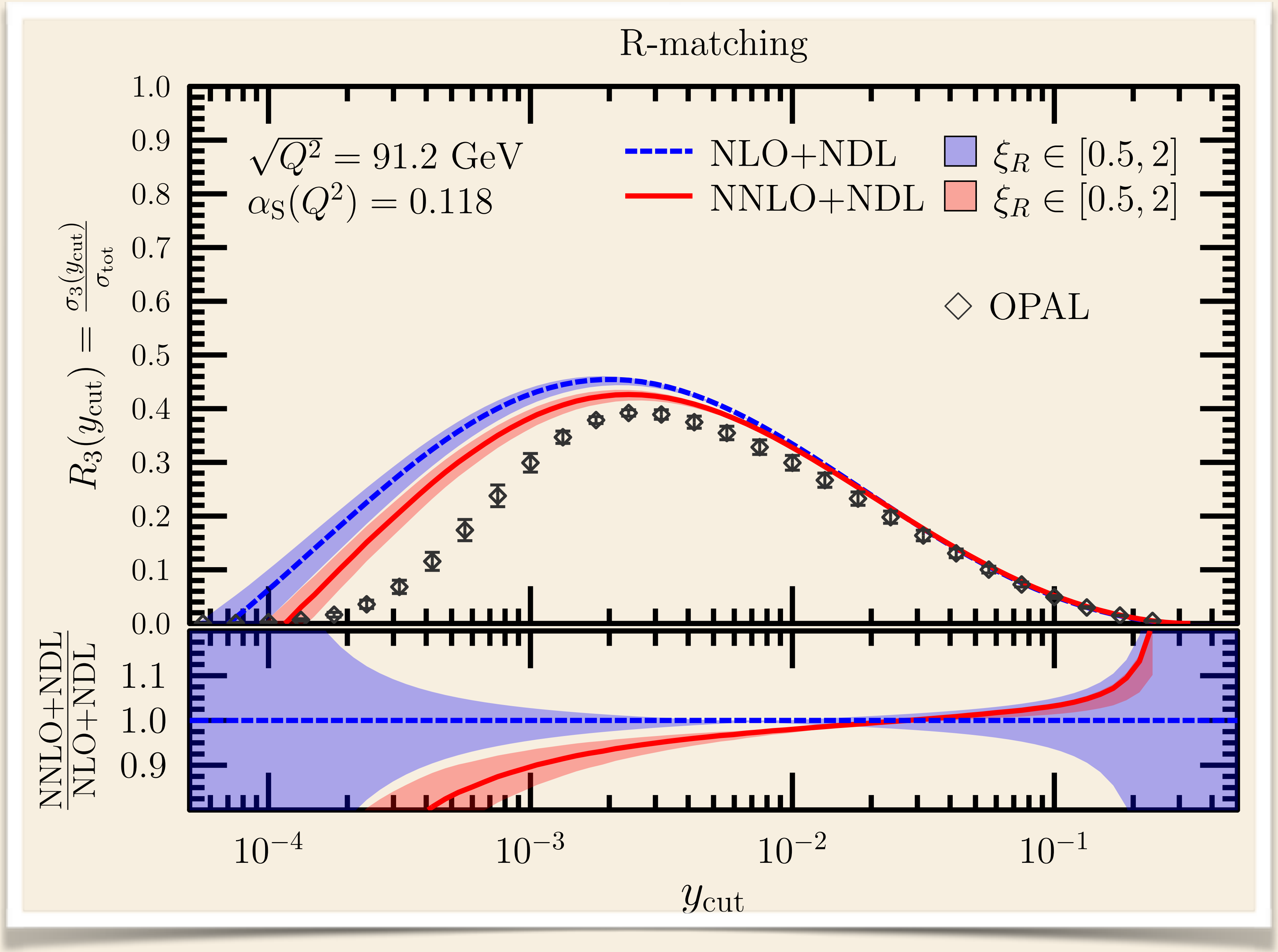
$$R^{\mathrm{NDL}}_{exp} = \frac{\alpha_{\mathrm{S}}}{2\pi} A_{exp} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^2 B_{exp} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^3 C_{exp}$$

Jet rates at NNLO+NDL accuracy



R-matching for the three-jet rates:
unphysical for $y_{cut} < 10^{-4}$
as a result of uncontrolled subleading logarithmic behavior

NLO+NDL vs NNLO+NDL

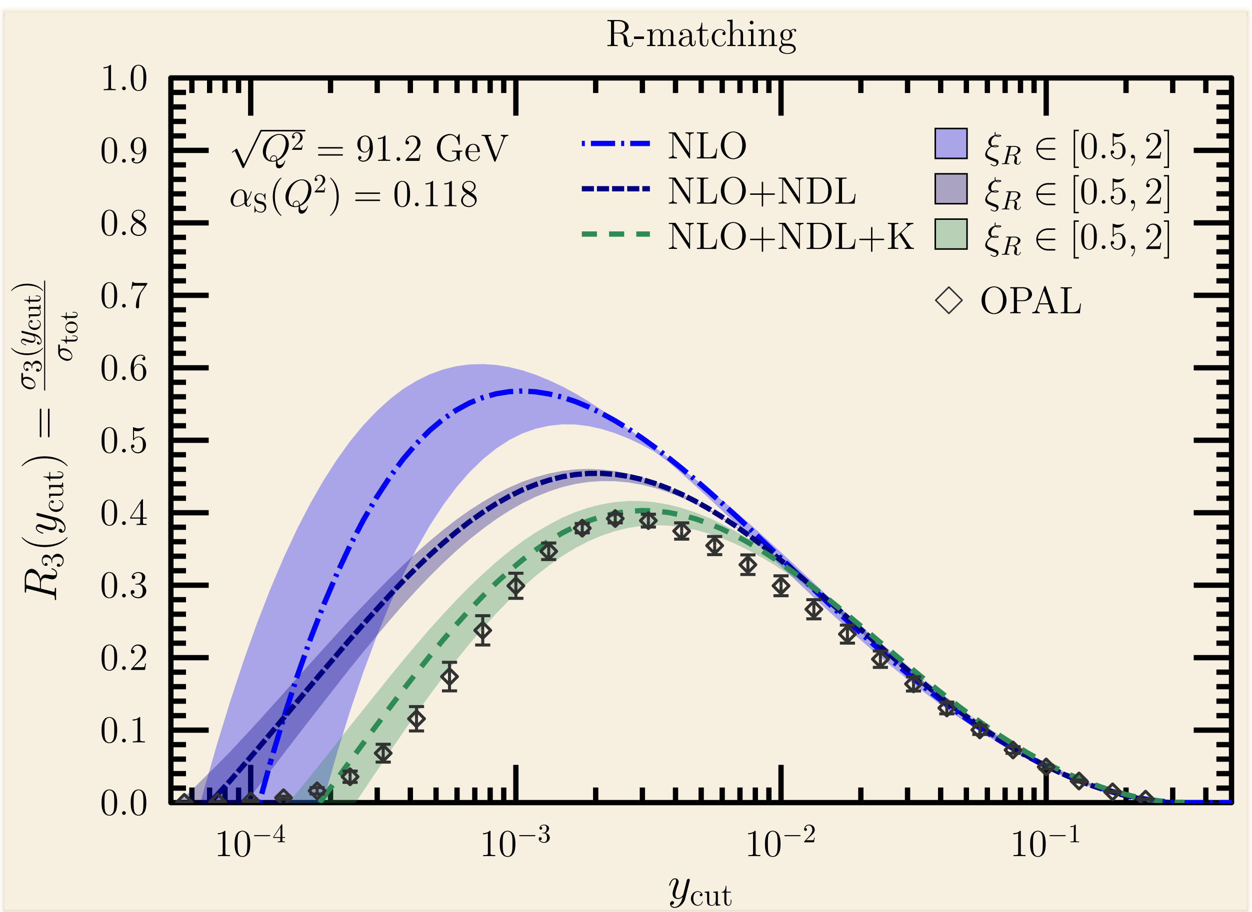


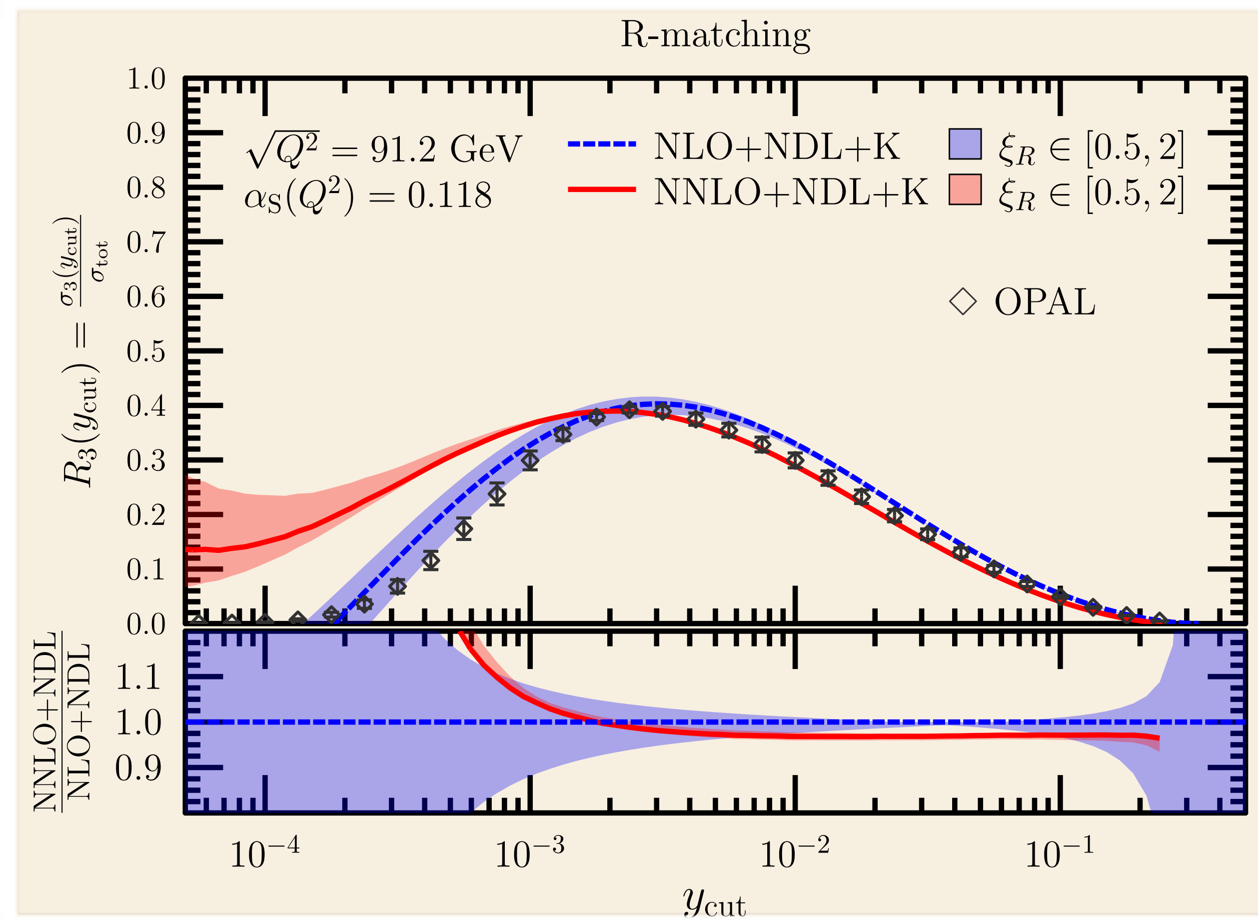
R-matching for the three-jet rates:
unphysical for $y_{cut} < 10^{-4}$
as a result of uncontrolled subleading logarithmic behavior

...with cusp anomalous dimension in splitting kernels

[Nagy and ZT hep-ph/9708344]

$$R_3^{\mathrm{NDL+K}}(y_{\mathrm{cut}}) = \sum_{n=1}^{\infty} \alpha_{\mathrm{S}}^n \Big(G_{n,2n} \log^{2n} y_{\mathrm{cut}} + G_{n,2n-1} \log^{n-1} y_{\mathrm{cut}} + \mathcal{O}(\log^{2n-2} y_{\mathrm{cut}}) \Big)$$





R-matching for the three-jet rates:

still unphysical for $y_{\mathrm{cut}} < 10^{-4}$

as a result of uncontrolled subleading logarithmic behavior

logR-matching

log R matching for jet rates

For NDL log R-matching can be defined by

$$\log R^{\mathrm{FO+NDL}} = \log R^{\mathrm{NDL}} - (\log R^{\mathrm{NDL}})_{exp} + \tilde{R}^{\mathrm{FO}}$$

with

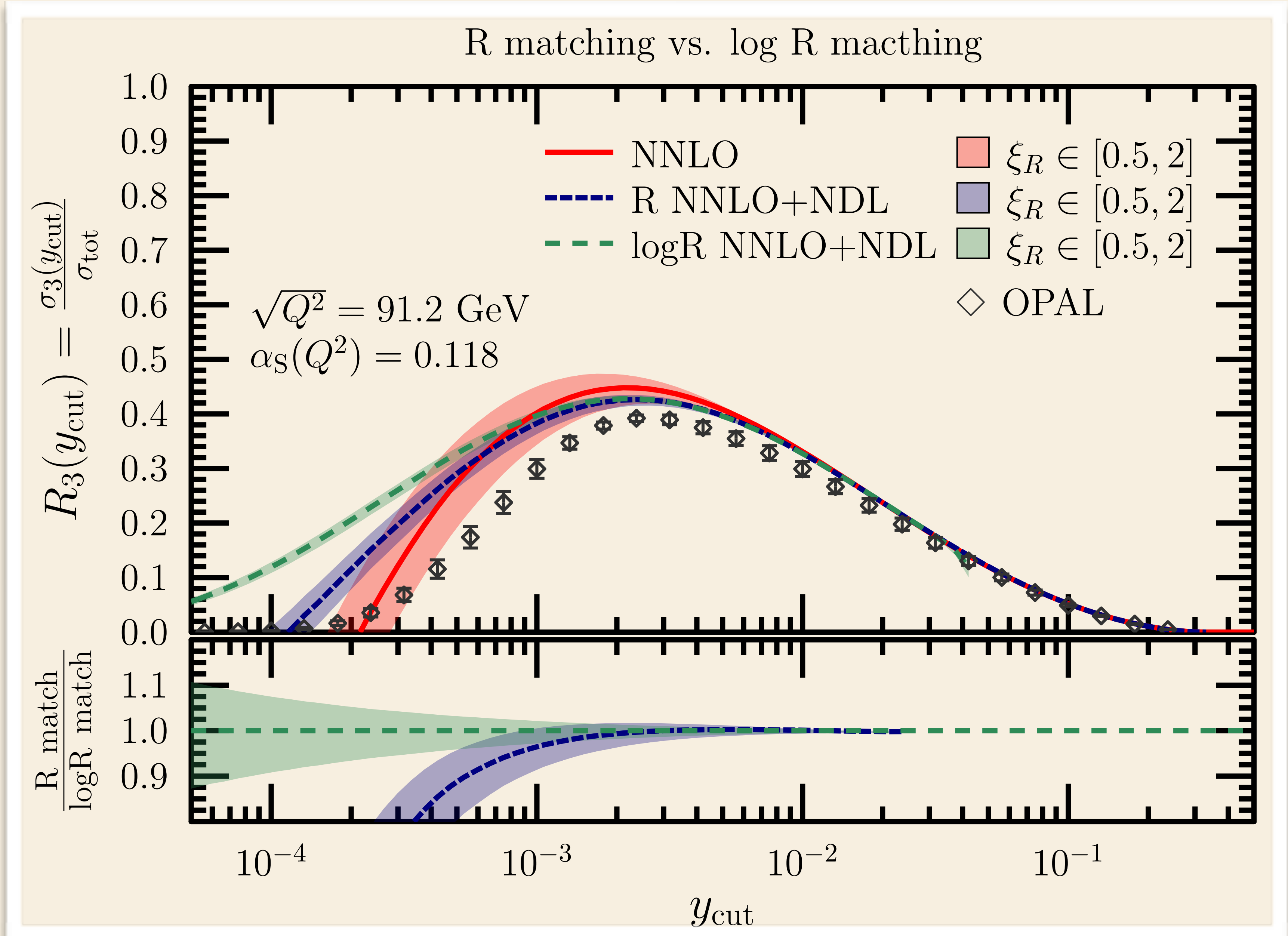
$$e^{\tilde{R}^{\mathrm{FO}}} \to \frac{\alpha_{\mathrm{S}}}{2\pi} A^{\mathrm{FO}} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^2 B^{\mathrm{FO}} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^3 C^{\mathrm{FO}}$$

multiplicative matching instead of additive:

provides correct asymptotic behavior at small y_{cut}

unphysical above the LO kinematic limit

R matching vs log R matching



the R matching and log R matching prescriptions give consistent predictions for $y_{cut} > 10^{-3}$

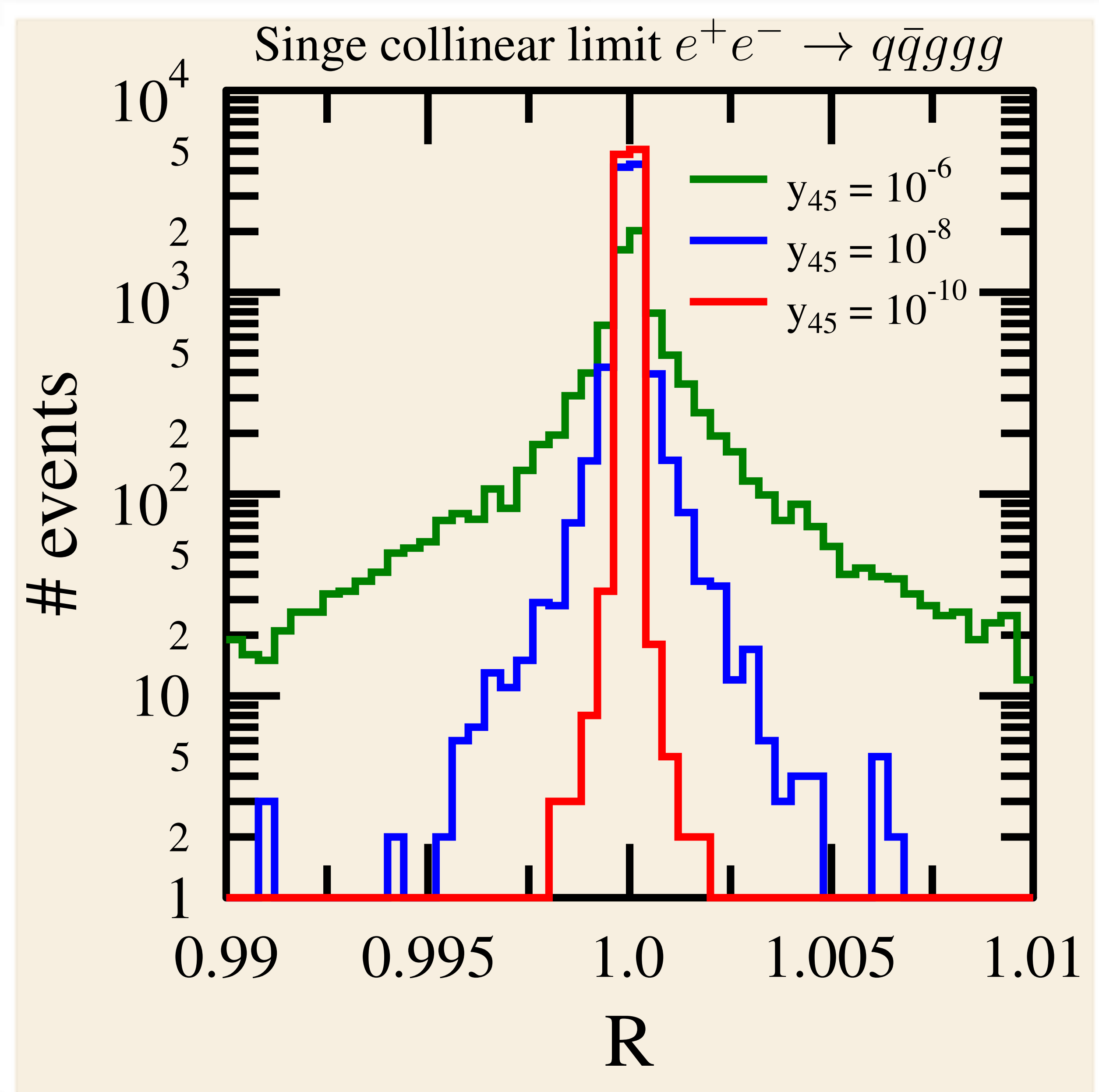
Conclusions

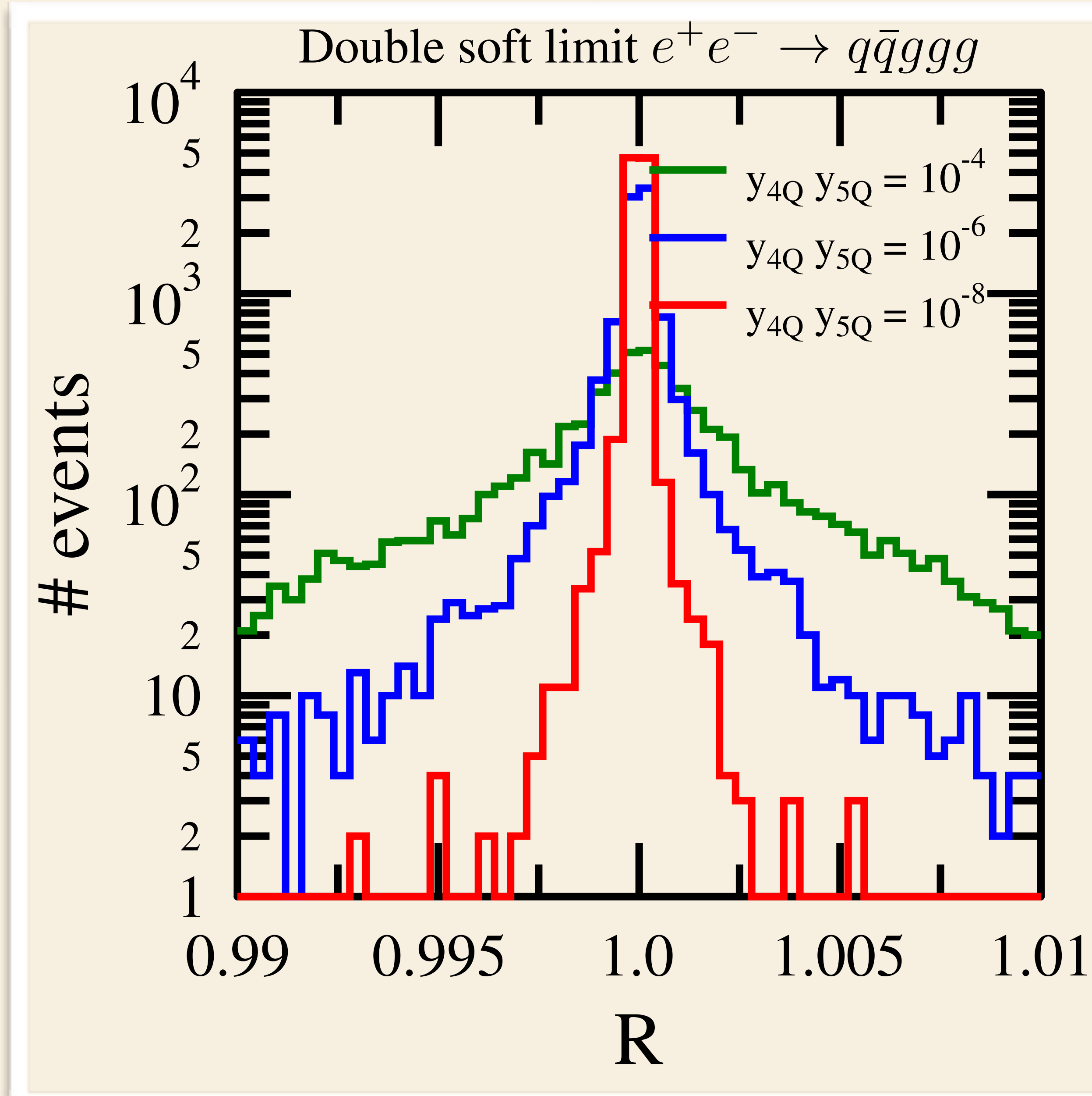
Conclusions

- ✓ Defined a general subtraction scheme for computing NNLO fully differential jet cross sections (presently only for processes with no colored particles in the initial state, extension to hadronic collisions is in progress)
- ✓ Subtractions are
  - ✓ fully local
  - ✓ exact and explicit in color (using color state formalism)
- ✓ Application: three rates in electron-positron annihilation
  - ✓ numerical precision matches formal precision well
  - ✓ NNLO matched to NDL with R and log R matching prescriptions
  - ➡ higher logarithmic accuracy needed

Appendix

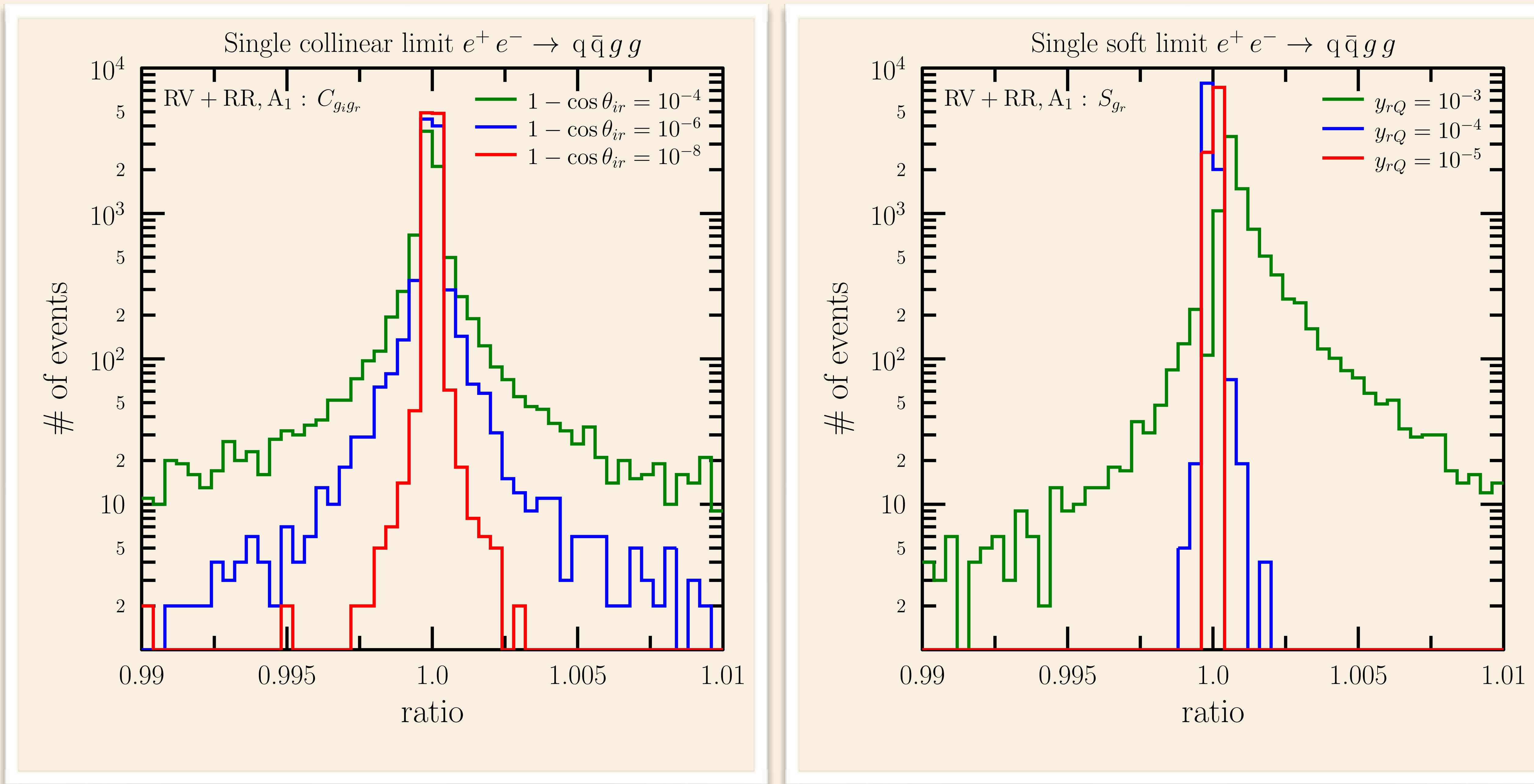
Kinematic singularities cancel in RR





R = subtraction/RR

Kinematic singularities cancel in RV



R = subtraction/(RV+RR,A_1)

Regularized RR & RV contributions

can now be computed by numerical Monte Carlo integrations

(generalization to colored initial states is in progress)

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} + \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR,A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR,A}_{12}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV,A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR,A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] \right\} J_m$$

G. Somogyi, ZT hep-ph/0609041, hep-ph/0609043
G. Somogyi, ZT, V. Del Duca hep-ph/0502226, hep-ph/0609042
Z. Nagy, G. Somogyi, ZT hep-ph/0702273

Integrated approximate xsections

$$\sigma^{\mathrm{NNLO}} = \sigma^{\mathrm{RR}}_{m+2} + \sigma^{\mathrm{RV}}_{m+1} + \sigma^{\mathrm{VV}}_{m} = \sigma^{\mathrm{NNLO}}_{m+2} + \sigma^{\mathrm{NNLO}}_{m+1} + \sigma^{\mathrm{NNLO}}_{m}$$

$$\sigma^{\mathrm{NNLO}}_{m+2} = \int_{m+2} \left\{ \mathrm{d}\sigma^{\mathrm{RR}}_{m+2} J_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} J_m - \left(\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} J_{m+1} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} J_m \right) \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m+1} = \int_{m+1} \left\{ \left(\mathrm{d}\sigma^{\mathrm{RV}}_{m+1} + \boxed{\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2}} \right) J_{m+1} - \left[\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right)^{\mathrm{A}_1} \right] J_m \right\}$$

$$\sigma^{\mathrm{NNLO}}_{m} = \int_{m} \left\{ \mathrm{d}\sigma^{\mathrm{VV}}_{m} \boxed{+ \int_2 \left(\mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_2}_{m+2} - \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_{12}}_{m+2} \right) + \int_1 \left[\mathrm{d}\sigma^{\mathrm{RV},\mathrm{A}_1}_{m+1} + \left(\int_1 \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_1}_{m+2} \right)^{\mathrm{A}_1} \right]} \right\} J_m$$

After integrating over unresolved momenta & summing over unresolved colors and flavors, the subtraction terms can be written as products of insertion operators (in color space) and lower point cross sections:

$$\int_p \mathrm{d}\sigma^{\mathrm{RR},\mathrm{A}_p} = \boldsymbol{I}^{(0)}_p(\{p\}_n;\epsilon) \otimes \mathrm{d}\sigma^{\mathrm{B}}_n$$

Example: $e^+e^- \to$ m(=3) jets at $\mu^2 = s$

$$\sigma_m^{\mathrm{NNLO}} = \int_m \left\{ \mathrm{d}\sigma_m^{\mathrm{VV}} + \int_2 \left[\mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_2}} - \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_{12}}} \right] + \int_1 \left[\mathrm{d}\sigma_{m+1}^{\mathrm{RV,A_1}} + \left(\int_1 \mathrm{d}\sigma_{m+2}^{\mathrm{RR,A_1}} \right)^{\mathrm{A_1}} \right] \right\} J_m$$

$$\mathrm{d}\sigma_3^{\mathrm{VV}} = \mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{F}inite\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right)$$

$$\mathcal{P}oles\left(A_3^{(2\times0)} + A_3^{(1\times1)}\right) + \mathcal{P}oles \sum \int \mathrm{d}\sigma^{\mathrm{A}} =$$ 200k Mathematica lines

= zero numerically in any phase space point:

```
                    0.              2    0. nf
                0. + --- + 0. Nc   + ----- + 0. Nc nf
                       2                Nc
                     Nc
Out[1]= ---------------------------------- +
                                  2
                                 e

                0.              2   0. nf
          0. + --- + 0. Nc   + ----- + 0. Nc nf
                 2               Nc
               Nc                                          0
---------------------------------------------- + O[e]
                           e
```

log R matching formulae

$$(\log R^{\mathrm{NDL}})_{exp} = \log \frac{\alpha_{\mathrm{S}}}{2\pi} + \log A_{exp} + \frac{\alpha_{\mathrm{S}}}{2\pi} \frac{B_{exp}}{A_{exp}} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^2 \frac{2A_{exp}C_{exp} - B_{exp}^2}{2A_{exp}^2} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^3 \frac{B_{exp}^3 - 3A_{exp}B_{exp}C_{exp} + 2A_{exp}^2 D_{exp}}{3A_{exp}^3}$$

$$\tilde{R}^{\mathrm{FO}} = \log \frac{\alpha_{\mathrm{S}}}{2\pi} + \log A_{FO} + \frac{\alpha_{\mathrm{S}}}{2\pi} \frac{B_{FO}}{A_{FO}} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^2 \frac{2A_{FO}C_{FO} - B_{FO}^2}{2A_{FO}^2} + \left(\frac{\alpha_{\mathrm{S}}}{2\pi}\right)^3 \frac{B_{FO}^3 - 3A_{FO}B_{FO}C_{FO}}{3A_{FO}^3}$$