Exotic hadrons at large Nc and in QCD sum rules

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1. QCD at large N_c and properties of the Green functions containing potential exotic poles

2. Implications for QCD sum rules for exotic states vs ordinary hadrons

Based on W.L., D.M, H.S., 1706.06003.

Supported by Austrian Science Fund (FWF): P29028

FWF Der Wissenschaftsfonds.

QCD at large Nc

$SU(N_c)$ gauge theory with $N_c \to \infty$ and $\alpha_s \sim 1/N_c$. At leading order, QCD Green functions have only non-interacting mesons as intermediate states; tetraquark bound states may emerge only in N_c-subleading diagrams. This fact was believed to provide the theoretical explanation of the non-existence of exotic tetraquarks.

However, even if the exotic tetraquark bound states appear only in subleading diagrams, the crucial question is their width: if narrow, they might be well observed in nature.

We discuss four-point Green functions of bilinear quark currents, depend on 6 variables p_1^2, p_2^2, $p_1'^2$, $p_2'^2$, $p = p_1 + p_2 = p_1' + p_2'$, and the two Mandelstam variables $s = p^2$ and $t = (p_1 - p_1')^2$.

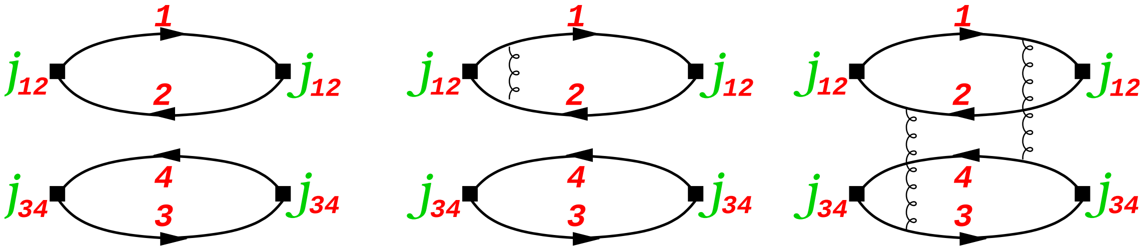
Criteria for selecting diagrams which potentially contribute to the tetraquark pole at $s = M_T^2$:

1. **The diagram should have a nontrivial (i.e., non-polynomial) dependence on the variable s.**
2. **The diagram should have a four-particle cut (i.e. threshold at $s = (m_1 + m_2 + m_3 + m_4)^2$), where m_i are the masses of the quarks forming the tetraquark bound state. The presence or absence of this cut is established by solving the Landau equations for the corresponding diagram.**

Flavour-exotic tetraquarks

Bilinear quark currents $j_{ij} = \bar{q}_i q_j$ **producing** M_{ij} **from the vacuum,** $\langle 0|j_{ij}|M_{ij}\rangle = f_{M_{ij}}$**,** $f_M \sim \sqrt{N_c}$**.**

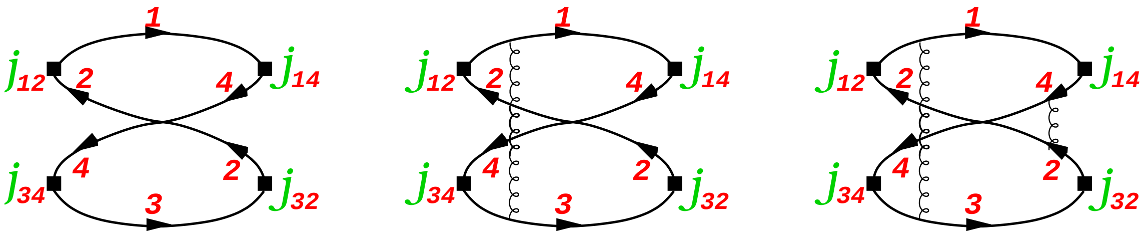
"Direct" 4-point functions $\Gamma_{\mathrm{I}}^{(\mathrm{dir})} = \langle j_{12}^\dagger j_{34}^\dagger j_{12} j_{34}\rangle$ **and** $\Gamma_{\mathrm{II}}^{(\mathrm{dir})} = \langle j_{14}^\dagger j_{32}^\dagger j_{14} j_{32}\rangle$**:**



$\sim N_c^2$ $\sim N_c^3 \alpha_s$ $\sim N_c^2 \alpha_s^2$

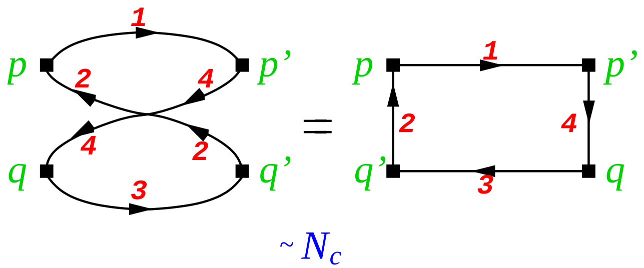
(a) (b) (c)

"Recombination" functions $\Gamma^{(\mathrm{rec})} = \langle j_{12}^\dagger j_{34}^\dagger j_{14} j_{32}\rangle$ **and** $\Gamma^{(\mathrm{rec})\dagger}$**:**

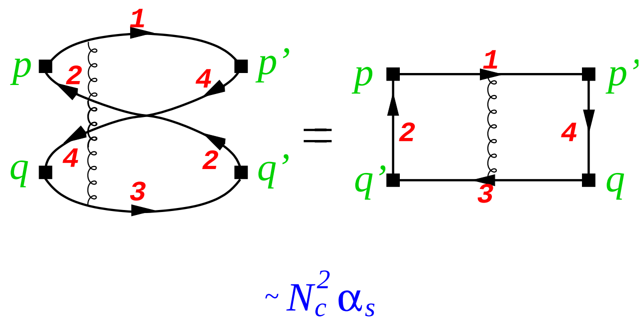


$\sim N_c$ $\sim N_c^2 \alpha_s$ $\sim N_c \alpha_s^2$

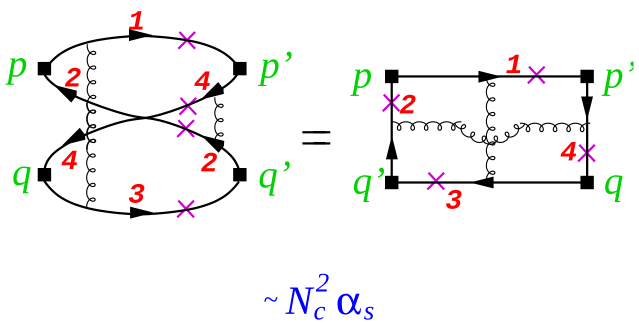
(a) (b) (c)



(a)



(b)



(c)

$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = \langle j^\dagger_{12} j^\dagger_{34} j_{12} j_{34}\rangle = O(N_c^0), \quad \Gamma^{(\mathrm{dir})}_{\mathrm{II},T} = \langle j^\dagger_{14} j^\dagger_{32} j_{14} j_{32}\rangle = O(N_c^0), \quad \Gamma^{(\mathrm{rec})}_{T} = \langle j^\dagger_{12} j^\dagger_{34} j_{14} j_{32}\rangle = O(N_c^{-1}).$$

The fact that “dir” and “rec” amplitudes have different behaviors in N_c requires at least two exotic poles:
T_A couples stronger to $M_{12}M_{34}$ channel, T_B couples stronger to $M_{14}M_{32}$ channel.

$$\begin{aligned}
\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} &= O(N_c^0) = f_M^4\left(\frac{|A(M_{12}M_{34}\to T_A)|^2}{p^2-M^2_{T_A}} + \frac{|A(M_{12}M_{34}\to T_B)|^2}{p^2-M^2_{T_B}}\right) + \cdots,\\
\Gamma^{(\mathrm{dir})}_{\mathrm{II},T} &= O(N_c^0) = f_M^4\left(\frac{|A(M_{14}M_{32}\to T_A)|^2}{p^2-M^2_{T_A}} + \frac{|A(M_{14}M_{32}\to T_B)|^2}{p^2-M^2_{T_B}}\right) + \cdots,\\
\Gamma^{(\mathrm{rec})}_{T} &= O(N_c^{-1}) = f_M^4\left(\frac{A(M_{12}M_{34}\to T_A)A(T_A\to M_{14}M_{32})}{p^2-M^2_{T_A}} + \frac{A(M_{12}M_{34}\to T_B)A(T_B\to M_{14}M_{32})}{p^2-M^2_{T_B}}\right) + \cdots.
\end{aligned}$$

We seek tetraquarks with finite mass at large N_c:

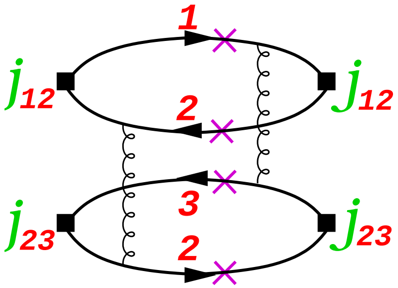
$$\begin{aligned}
A(T_A\to M_{12}M_{34}) &= O(N_c^{-1}), \qquad A(T_A\to M_{14}M_{32}) = O(N_c^{-2}),\\
A(T_B\to M_{12}M_{34}) &= O(N_c^{-2}), \qquad A(T_B\to M_{14}M_{32}) = O(N_c^{-1}).
\end{aligned}$$

The widths $\Gamma(T_{A,B}) = O(N_c^{-2})$.

Cryptoexotic tetraquarks

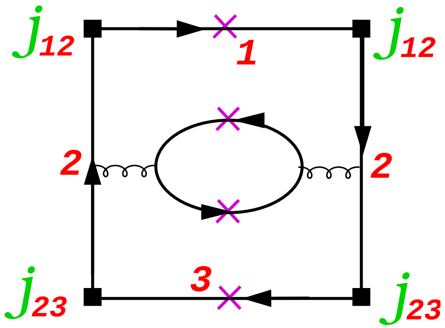
Diagrams of new topologies emerge.

For direct functions $\Gamma^{(\mathrm{dir})}_{(\mathrm{I,II}),T}$, new diagrams do not change leading large-N_c behavior:



$\sim N_c^2\,\alpha_s^2$

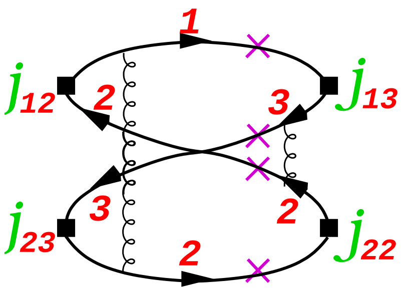
(a)



$\sim N_c^2\,\alpha_s^2$

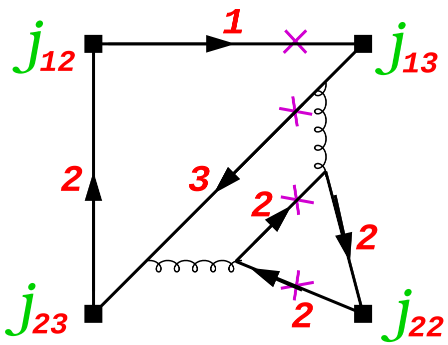
(b)

For recombination functions, the new diagram modifies leading large-N_c behavior



$\sim N_c\,\alpha_s^2$

(a)



$\sim N_c^2\,\alpha_s^2$

(b)

The new diagram modifies the leading large-N_c behavior:

$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = \langle j^\dagger_{12} j^\dagger_{23} j_{12} j_{23}\rangle = O(N_c^0), \quad \Gamma^{(\mathrm{dir})}_{\mathrm{II},T} = \langle j^\dagger_{13} j^\dagger_{22} j_{13} j_{22}\rangle = O(N_c^0), \quad \Gamma^{(\mathrm{rec})}_{T} = \langle j^\dagger_{12} j^\dagger_{23} j_{13} j_{22}\rangle = O(N_c^0).$$

"dir" and "rec" functions have the same behavior, and one exotic state T is enough:

$$A(T \to M_{12}M_{23}) \;=\; O(N_c^{-1}), \qquad A(T \to M_{13}M_{22}) = O(N_c^{-1}).$$

Its width is $\Gamma(T) = O(N_c^{-2})$.

T can mix with the ordinary meson M_{13}. The restriction on the mixing parameter $g_{TM_{13}}$:

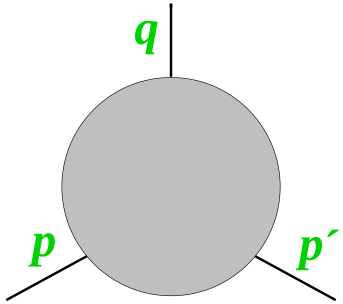
$$\Gamma^{(\mathrm{dir})}_{\mathrm{I},T} = O(N_c^0) = f_M^4 \left(\frac{A(M_{12}M_{23} \to T)}{p^2 - M_T^2} g_{TM_{13}} \frac{A(M_{13} \to M_{12}M_{23})}{p^2 - M_{M_{13}}^2} \right) + \cdots .$$

$A(M_{13} \to M_{12}M_{23}) \sim 1/\sqrt{N_c}$**, so** $g_{TM_{13}} \leq O(1/\sqrt{N_c})$**.**

$$\boxed{\text{The analysis of Green functions in large-}N_c\text{ QCD allows one to restrict some properties of the possible exotic states.}}$$

Strong decays from 3 – point vertex functions

• **The basic object:**



$$\Gamma(p,p',q)=\int\langle\Omega|T(J(x)j(0)j'(x')|\Omega\rangle\exp(ipx-ip'\,x')dxdx'$$

This correlator contains the triple-pole in the Minkowski region: namely

$$\Gamma(p,p',q)=\frac{ff'}{(p^2-M^2)(p'^2-M'^2)}F(q^2)+\cdots$$

where the form factor $F(q^2)$ contains pole at $q^2=M_q^2$:

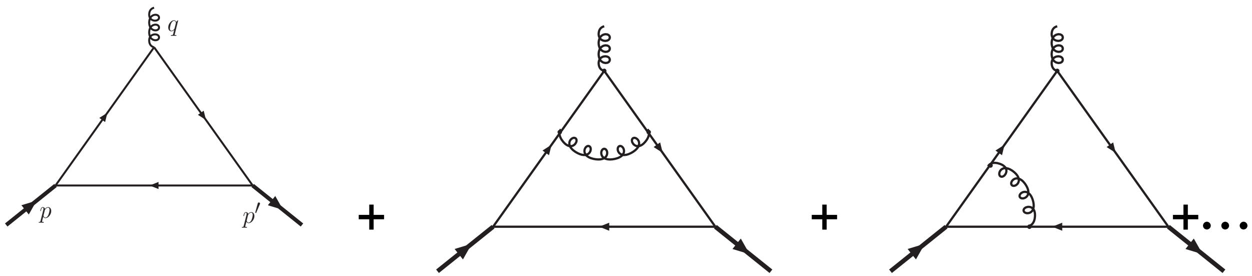
$$F(q^2)=\frac{f_q g_{MM'M_q}}{(q^2-M_q^2)}+\cdots$$

$g_{MM'M_q}$ **describes the $M\to M_1M_2$ strong transition;**

f**,** f'**, and** f_{M_q} **are the decay constants of the mesons** $\langle 0|j(0)|M\rangle=f_M$**.**

• **Normal hadrons:**

$$\Gamma_{\rm OPE}(p^2, p'^2, q^2) = \Gamma_0(p^2, p'^2, q^2) + \alpha_s\Gamma_1(p^2, p'^2, q^2) + \dots$$



Already one-loop lowest-order diagram has a nonzero double-spectral density, and therefore provides a nonzero contribution to the form factor.

• **Exotic hadrons:** $\langle T(\theta(x) j_1(0) j_2(y))\rangle$.

Many possibilities to write interpolating current for X, $\langle 0|\theta|X\rangle = f_X$, $\qquad f_X \neq 0$.

$$\theta = M(x)M(x), \qquad \theta = M^A(x)M^A(x), \qquad \theta = \bar{D}^a(x)D^a(x).$$

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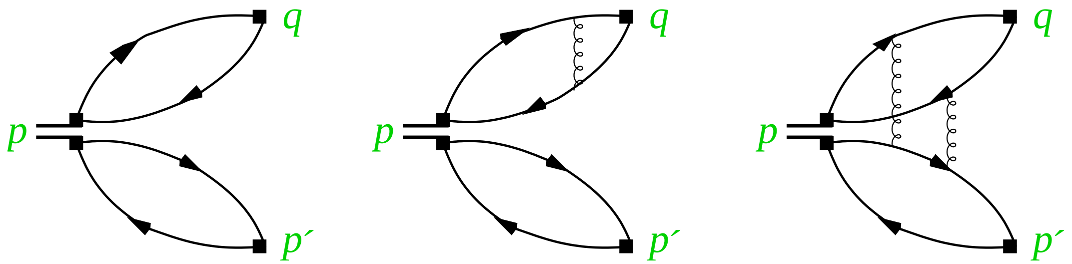

$$\Gamma_{\rm OPE}(p^2, p'^2, q^2) = \Pi(p'^2)\Pi(q^2) + (\alpha_s)^2\Gamma_{\rm connected}(p^2, p'^2, q^2)$$

Thus the LO contribution is not related to the exotic-state decay.

Conclusions

• *QCD at large N_c*:

Large-N_c QCD restricts properties of the exotic poles if such poles exist. E.g., two exotic $\bar{q}_1 q_2 \bar{q}_3 q_4$ narrow states $\Gamma \sim O(1/N_c^2)$, each decaying into one meson-meson channel.
One cryptoexotic state $\bar{q}_1 q_2 \bar{q}_2 q_3$ $\Gamma \sim O(1/N_c^2)$ decaying into various meson-meson channels with similar probabilities.

• *Dynamics of exotic-state decays and implications for QCD sum rules*:

Dynamics of fall-apart decays of exotic resonances has fundamental difference from dynamics of ordinary-meson decays: the appropriate contributions to Green functions describing decays of exotic states emerge only at subleading α_s orders; the leading order disconnected diagrams are not related to strong decays of exotic hadrons. This makes the calculation of α_s-corrections mandatory.

Many efforts for obtaining reliable predictions are necessary!