POLARIZATION EFFECTS IN THE REACTIONS $p+\,^3He \to \pi^+ +\,^4He$, $\pi^+ +\,^4He \to p+\,^3He$ AND QUANTUM CHARACTER OF SPIN CORRELATIONS IN THE FINAL $(p,\,^3He)$ SYSTEM

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Abstract

The general consequences of T invariance for the direct and inverse binary reactions $a+b \to c+d$, $c+d \to a+b$ with spin-1/2 particles a, b and unpolarized particles c, d are considered. Using the formalism of helicity amplitudes, the polarization effects are studied in the reaction $p+\,^3He \to \pi^+ +\,^4He$ and in the inverse process $\pi^+ +\,^4He \to p+\,^3He$. It is shown that in the reaction $\pi^+ +\,^4He \to p+\,^3He$ the spins of the final proton and ^3He nucleus are strongly correlated. A structural expression through helicity amplitudes, corresponding to arbitrary emission angles, is obtained for the correlation tensor. It is established that in the reaction $\pi^+ +\,^4He \to p+\,^3He$ one of the "classical" incoherence inequalities of the Bell type for diagonal components of the correlation tensor is necessarily violated.

1 Consequences of T invariance for binary reactions

Let us consider the reaction $a+b \to c+d$, where a and b are the spin-1/2 particles and the particles c, d have arbitrary spins. The structure of the effective cross-section $\sigma_{a+b\to c+d}$ in the c.m. frame of the particles a and b, summed over spin projections of the final particles c, d, is as follows [1]:

$$\sigma_{a+b\to c+d}(\mathbf{k}_a, \mathbf{P}^{(a)}, \mathbf{P}^{(b)}; \mathbf{k}_c) = \sigma_0(E,\theta)\, L(\mathbf{k}_a, \mathbf{P}^{(a)}, \mathbf{P}^{(b)}; \mathbf{k}_c), \tag{1}$$

where $\sigma_0(E,\theta)$ is the respective cross-section for unpolarized particles a, b and L is the linear function of the polarization vectors $\mathbf{P}^{(a)}$ and $\mathbf{P}^{(b)}$:

$$\begin{aligned} L(\mathbf{k}_a, \mathbf{P}^{(a)}, \mathbf{P}^{(b)}; \mathbf{k}_c) = 1 + A(E,\theta)(\mathbf{P}^{(a)}\mathbf{n}) + B(E,\theta)(\mathbf{P}^{(b)}\mathbf{n}) + C(E,\theta)(\mathbf{P}^{(a)}\mathbf{P}^{(b)}) + \\ + D(E,\theta)(\mathbf{P}^{(a)}\mathbf{l})(\mathbf{P}^{(b)}\mathbf{l}) + F(E,\theta)(\mathbf{P}^{(a)}\mathbf{m})(\mathbf{P}^{(b)}\mathbf{m}) + \\ + G(E,\theta)(\mathbf{P}^{(a)}\mathbf{l})(\mathbf{P}^{(b)}\mathbf{m}) + H(E,\theta)(\mathbf{P}^{(a)}\mathbf{m})(\mathbf{P}^{(b)}\mathbf{l}). \end{aligned} \tag{2}$$

Here $\mathbf{l}$, $\mathbf{m}$, $\mathbf{n}$ are mutually orthogonal unit vectors, defined as:

$$\mathbf{l} = \mathbf{k}_a/k_a; \quad \mathbf{m} = \frac{\mathbf{l}' - \mathbf{l}(\mathbf{l}'\mathbf{l})}{\sin\theta}; \quad \mathbf{n} = \frac{\mathbf{l}\times\mathbf{l}'}{\sin\theta} \tag{3}$$

($\mathbf{l}' = \mathbf{k}_c/k_c$); $\mathbf{k}_a$, $\mathbf{k}_c$ are the respective momenta of the particles a, c, E is the total energy in the c.m. frame and $\theta = \arccos{(\mathbf{l}\,\mathbf{l}')}$ is the emission angle.

Meantime, for the inverse reaction $c+d \to a+b$ with the unpolarized particles c, d and the fixed polarization vectors of the final particles $\boldsymbol{\zeta}^{(a)}$, $\boldsymbol{\zeta}^{(b)}$ the effective cross-section takes, due to T invariance and the principle of detailed balance, the following form [1]:

$$\sigma_{c+d\to a+b}(\mathbf{k}_c;\mathbf{k}^{(a)},\boldsymbol{\zeta}^{(a)},\boldsymbol{\zeta}^{(b)}) = \frac{1}{4}\widetilde{\sigma}_0(E,\theta)L(-\mathbf{k}_a,-\boldsymbol{\zeta}^{(a)},-\boldsymbol{\zeta}^{(b)};-\mathbf{k}_c), \qquad (4)$$

where

$$\widetilde{\sigma}_0(E,\theta) = \frac{4k_a^2}{k_c^2(2j_c+1)(2j_d+1)}\sigma_0(E,\theta) \qquad (5)$$

is the cross-section of the inverse reaction, summed over the spin projections of the final particles a, b. Further, the two-particle spin density matrix $\hat{\rho}^{(a,b)}$ for the final particles a, b can also be expressed through the same function L, replacing the polarization vectors by the vector Pauli operators:

$$\hat{\rho}^{(a,b)} = \frac{1}{4}[\hat{I}^{(a)}\otimes\hat{I}^{(b)} + (\mathbf{P}^{(a)}(E,\theta)\hat{\boldsymbol{\sigma}}^{(a)})\otimes\hat{I}^{(b)} + +\hat{I}^{(a)}\otimes(\mathbf{P}^{(b)}(E,\theta)\hat{\boldsymbol{\sigma}}^{(b)})+$$

$$+\sum_{i=1}^{3}\sum_{k=1}^{3}T_{ik}(E,\theta)\hat{\sigma}_i^{(a)}\otimes\hat{\sigma}_k^{(b)}] = \frac{1}{4}\hat{L}(-\mathbf{k}_a,-\hat{\boldsymbol{\sigma}}^{(a)},-\hat{\boldsymbol{\sigma}}^{(b)};-\mathbf{k}_c). \qquad (6)$$

In (6) $\hat{I}^a$, $\hat{I}^{(b)}$ are two-row unit matrices, $\mathbf{P}^{(a)}(E,\theta) = -A(E,\theta)\mathbf{n}$ and $\mathbf{P}^{(b)}(E,\theta) = -B(E,\theta)\mathbf{n}$ are the polarization vectors of the particles a, b,

$$T_{ik}(E,\theta) = C(E,\theta)\delta_{ik} + D(E,\theta)l_i l_k +$$

$$+F(E,\theta)m_i m_k + G(E,\theta)l_i m_k + H(E,\theta)m_i l_k \qquad (7)$$

are components of the correlation tensor describing spin correlations in the final two-particle system (a,b). In doing so, all the functions $A, B, C, D, F, G, H, \sigma_0$ and the unit vectors $\mathbf{l}$, $\mathbf{m}$, $\mathbf{n}$ in Eqs. (4)–(7) are *the same* as for the direct reaction $a+b\to c+d$.

Thus, due to T invariance, the dependence of the effective cross- section of the direct reaction $a+b \to c+d$ upon the polarizations of initial particles *completely determines* the polarization vectors and spin correlations for the same particles a, b produced in the inverse reaction $c+d \to a+b$ with unpolarized primary particles[1].

2 Polarization effects in the reaction $p + {}^3He \to \pi^+ + {}^4He$

This reaction belongs to the type $1/2 + 1/2 \to 0 + 0$ (the proton and ${}^3He$ nucleus have spin 1/2, π^+ and ${}^4He$ have zero spin). Thus, on account of the negative internal parity of the π^+-meson, this reaction can proceed *only from triplet* states of the system (p, ${}^3He$) [3,4,5] (as follows from the parity and angular momentum conservation).

Let us choose the axis of the total-spin quantization z along the vector $\mathbf{l} = \mathbf{k}_p/k_p$. There exist three possible triplet states of the (p, ${}^3He$)-system, characterized by the spin projections $+1$, -1 and 0 onto the axis z:

$$|+1,\mathbf{l}\rangle = |+1/2,\mathbf{l}\rangle^{(p)} \otimes |+1/2,\mathbf{l}\rangle^{(He)}, \quad |-1,\mathbf{l}\rangle = |-1/2,\mathbf{l}\rangle^{(p)} \otimes |-1/2,\mathbf{l}\rangle^{(He)},$$

$$|0,\mathbf{l}\rangle = \frac{1}{\sqrt{2}} \left(|+1/2,\mathbf{l}\rangle^{(p)} \otimes |-1/2,\mathbf{l}\rangle^{(He)} + |-1/2,\mathbf{l}\rangle^{(p)} \otimes |+1/2,\mathbf{l}\rangle^{(He)} \right). \tag{8}$$

The two-particle spin density matrix for the (p, ${}^3He$)-system is:

$$\hat{\rho}^{(p,He)} = \frac{1}{4}(\hat{I}^{(p)} + \mathbf{P}^{(p)}\boldsymbol{\sigma}^{(p)}) \otimes (\hat{I}^{(He)} + \mathbf{P}^{(He)}\boldsymbol{\sigma}^{(He)}), \tag{9}$$

($\mathbf{P}^{(p)}$ and $\mathbf{P}^{(He)}$ are the independent polarization vectors). Using the technique of helicity amplitudes (the helicity amplitude $R_\lambda(E,\theta)$, $\lambda = \pm 1, 0$, is the amplitude of the reaction $p + {}^3He \to \pi^+ + {}^4He$ proceeding from the state $|\lambda, \mathbf{l}\rangle$ (8)), we may write [1]:

[1] Previously a similar approach was used in [2] to study polarization effects in the scattering of spin-1/2 particles on an unpolarized target, basing on the T invariance of the differential cross-section of elastic scattering.

$$\sigma_{p+{}^3He\to\pi^+ +{}^4He} = \langle\psi|\hat{\rho}^{(p,He)}|\psi\rangle =$$

$$= \sum_{\lambda}\sum_{\lambda'} R_\lambda(E,\theta)\langle\lambda,\mathbf{l}\mid\hat{\rho}^{(p,He)}\mid\lambda',\mathbf{l}\rangle R^*_{\lambda'}(E,\theta); \tag{10}$$

$$|\psi\rangle = \sum_{\lambda=\pm1,0} R^*_\lambda(E,\theta)|\lambda,\mathbf{l}\rangle =$$

$$= R_1^*(E,\theta)\left(|+1/2,z\rangle^{(p)}\otimes|+1/2,z\rangle^{(He)} - |-1/2,z\rangle^{(p)}\otimes|-1/2,z\rangle^{(He)}\right) +$$

$$+\frac{1}{\sqrt{2}}R_0^*(E,\theta)\left(|+1/2,z\rangle^{(p)}\otimes|-1/2,z\rangle^{(He)} + |-1/2,z\rangle^{(p)}\otimes|+1/2,z\rangle^{(He)}\right) \tag{11}$$

is the non-normalized initial two-particle spin state selected by the reaction (due to parity conservation, $R_{+1}(E,\theta) = -R_{-1}(E,\theta) \equiv R_1(E,\theta)$).

Finally, using (11) and the formula (9) for the spin density matrix, we find that the cross-section $\sigma_{p+{}^3He\to\pi^+ +{}^4He}$ (10) is described by the general structural formula for $\sigma_{a+b\to c+d}$ (1,2), where the functions σ_0, A, B, C, D, F, G, H are bilinear combinations of the helicity amplitudes R_1, R_0 [1]:

$$\sigma_0(E,\theta) = \frac{1}{4}\langle\psi\mid\psi\rangle = \frac{1}{4}(|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2), \tag{12}$$

$$A(E,\theta) = B(E,\theta) = \frac{1}{\sqrt{2}\sigma_0(E,\theta)}\mathrm{Im}(R_1(E,\theta)R_0^*(E,\theta)), \tag{13}$$

$$C(E,\theta) = 1, \quad D(E,\theta) = -\frac{|R_0(E,\theta)|^2}{2\sigma_0(E,\theta)}, \quad F(E,\theta) = -\frac{|R_1(E,\theta)|^2}{2\sigma_0(E,\theta)}, \tag{14}$$

$$G(E,\theta) = H(E,\theta) = \frac{1}{\sqrt{2}\sigma_0(E,\theta)}\mathrm{Re}(R_1(E,\theta)R_0^*(E,\theta)). \tag{15}$$

For the particular cases $\theta = 0$ and $\theta = \pi$, when $R_1(E,\theta) = 0$ (owing to the conservation of the angular-momentum projection onto the reaction axis), the expression for $\sigma_{p+{}^3He\to\pi^+ + {}^4He}$ takes a considerably simpler form:

$$\sigma_{p+{}^3He\to\pi^+ + {}^4He} = \frac{1}{4}|R_0|^2 \left(1 + \mathbf{P}^{(p)}\mathbf{P}^{(He)} - 2(\mathbf{P}^{(p)}\mathbf{l})(\mathbf{P}^{(He)}\mathbf{l})\right), \qquad (16)$$

the coefficient at $|R_0|^2$ is the fraction of the state $|0,\mathbf{l}\rangle$ in the initial states.

3 Spin effects in the inverse reaction $\pi^+ + {}^4He \to p + {}^3He$

In the reaction $\pi^+ + {}^4He \to p + {}^3He$ the $(p,\, {}^3He)$-system is produced *in the triplet state only.* This state, normalized to unity, is as follows [1]:

$$| \widetilde{\psi}\rangle = \frac{1}{(|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2)^{1/2}}\times$$

$$\times[R_1(E,\theta)\left(| +1/2,\mathbf{l}\rangle^{(p)} \otimes | +1/2,\mathbf{l}\rangle^{(He)} - | -1/2,\mathbf{l}\rangle^{(p)} \otimes | -1/2,\mathbf{l}\rangle^{(He)}\right) +$$

$$+\frac{1}{\sqrt{2}}R_0(E,\theta)\left(| +1/2,\mathbf{l}\rangle^{(p)} \otimes | -1/2,\mathbf{l}\rangle^{(He)} + | -1/2,\mathbf{l}\rangle^{(p)} \otimes | +1/2,\mathbf{l}\rangle^{(He)}\right)], \qquad (17)$$

and it is symmetric under the interchange of spin quantum numbers of the proton and ${}^3He$. The state $|\widetilde{\psi}\rangle$ (17) is similar in structure to the initial triplet state $|\psi\rangle$ selected by the direct reaction $p + {}^3He \to \pi^+ + {}^4He$ (see Section 2), but differs by complex conjugation of helicity amplitudes.

Basing on the T invariance (see Section 1), we obtain [1]:

1) For the effective cross-section of the reaction $\pi^+ + {}^4He \to p + {}^3He$ in the c.m. frame, summed over the spin projections in the final state:

$$\widetilde{\sigma}_0(E,\theta) = (k_p/k_\pi)^2(|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2); \tag{18}$$

2) For the polarization vectors of the proton and ${}^3He$ in the final system:

$$\mathbf{P}^{(p)}(E,\theta) = \langle\widetilde{\psi}|\hat{\boldsymbol{\sigma}}^{(p)}|\widetilde{\psi}\rangle = \mathbf{P}^{(He)}(E,\theta) = \langle\widetilde{\psi}|\hat{\boldsymbol{\sigma}}^{(He)}|\widetilde{\psi}\rangle =$$

$$= -A(E,\theta)\mathbf{n} = -2\sqrt{2}\frac{\mathrm{Im}(R_1(E,\theta)R_0^*(E,\theta))}{|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2}\mathbf{n}; \tag{19}$$

3) For the correlation tensor of the $(p,\, {}^3He)$-system:

$$T_{ik}(E,\theta) = \langle\widetilde{\psi}|\hat{\sigma}_i^{(p)}\hat{\sigma}_k^{(He)}|\widetilde{\psi}\rangle = \delta_{ik} - \frac{2}{|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2}\times$$

$$\times\Big[|R_0(E,\theta)|^2 l_i l_k + 2|R_1(E,\theta)|^2 m_i m_k -$$

$$-\sqrt{2}\mathrm{Re}(R_1(E,\theta)R_0^*(E,\theta))(l_i m_k + m_i l_k)\Big]. \tag{20}$$

In all the expressions (17)-(20) the helicity amplitudes $R_0(E,\theta)$, $R_1(E,\theta)$ and the unit vectors $\mathbf{l}$, $\mathbf{m}$, $\mathbf{n}$ are *the same* as for the previously considered direct process $p + {}^3He \to \pi^+ + {}^4He$.

In accordance with (19,20): 1) the polarization of the ${}^3He$ nucleus along the normal to the reaction plane is identical to that of the proton; 2) the correlation tensor $T_{ik}(E,\theta)$, describing the spin correlations in the $(p,\, {}^3He)$-system, is symmetric. Thus, the spins of the proton and the ${}^3He$ nucleus in the reaction $\pi^+ + {}^4He \to p + {}^3He$ must be *tightly correlated*, which enables one, in principle, to prepare a beam of ${}^3He$ nuclei with controllable polarization without acting directly on these nuclei (see [1] for more details).

4 Violation of the incoherence inequalities for the correlation tensor

As it was established in the paper [6], in the case of incoherent mixtures of factorizable two-particle states of spin-1/2 fermions the following inequalities for the diagonal components of the correlation tensor should be satisfied:

$$|T_{11}+T_{22}+T_{33}| \le 1; \quad |T_{11}+T_{22}| \le 1; \quad |T_{11}+T_{33}| \le 1; \quad |T_{22}+T_{33}| \le 1; \tag{21}$$

However, for *non-factorizable* quantum-mechanical superpositions these inequalities may be violated. The triplet state $|\widetilde{\psi}\rangle$ (17) of the final system in the reaction $\pi^+ + {}^4He \to p + {}^3He$ represents a characteristic example of such non-factorizable spin states (it is well seen that the state $|\widetilde{\psi}\rangle$ cannot be reduced to the product of one-particle spin states).

Let us calculate the components of the correlation tensor T_{ik} (20) for the system $(p, {}^3He)$ in the coordinate frame with $z \parallel \mathbf{l}$, $x \parallel \mathbf{m}$, $y \parallel \mathbf{n}$. Finally, we obtain the following expressions (indexes: $1 \to x$, $2 \to y$, $3 \to z$) :

$$T_{11} = \frac{|R_0(E,\theta)|^2 - 2|R_1(E,\theta)|^2}{|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2}; \quad T_{22} = 1;$$

$$T_{33} = \frac{2|R_1(E,\theta)|^2 - |R_0(E,\theta)|^2}{|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2} = -T_{11}; \tag{22}$$

$$T_{13} = T_{31} = \frac{2\sqrt{2}}{|R_0(E,\theta)|^2 + 2|R_1(E,\theta)|^2}\mathrm{Re}(R_1 R_0^*);$$

$$T_{12} = T_{21} = T_{23} = T_{32} = 0; \tag{23}$$

in doing so, $tr(T) = 1$.

Thus, as follows from Eq. (22), in the reaction $\pi^+ + {}^4He \to p + {}^3He$ *one* of the incoherence inequalities (21) for the diagonal components of the correlation tensor is *necessarily* violated, irrespective of the concrete mechanism of generation of the system $(p, {}^3He)$. Indeed, if $|R_0|^2 > 2|R_1|^2$, we obtain that $|T_{11} + T_{22}| > 1$; if, on the contrary, $|R_0|^2 < 2|R_1|^2$, then we have: $|T_{22} + T_{33}| > 1$. Meantime, in both the cases the other three incoherence inequalities (21) are satisfied.

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