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Neutrino spin precession and oscillations in transversal matter currents

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Introduction: neutrino spin oscillations in magnetic field

Massive neutrinos participate in electromagnetic interactions. The recent review on this topic is given in [1] . One of the most important phenomenon of nontrivial neutrino electromagnetic interactions is the neutrino magnetic moment precession and the corresponding spin oscillations in presence of external electromagnetic fields. The later effect has been studied in numerous papers published during several passed decades. Within this scope the neutrino spin oscillations $\nu_e^L \Leftrightarrow \nu_\mu^R$ induced by the neutrino magnetic moment interaction with the transversal magnetic field $\mathbf{B}_\perp$ was first considered in [2]. Then spin-favor oscillations $\nu_e^L \Leftrightarrow \nu_\mu^R$ in $\mathbf{B}_\perp$ in vacuum were discussed in [3], the importance of the matter effect was emphasized in [4]. The effect of the resonant amplification of neutrino spin oscillations in $\mathbf{B}_\perp$ in the presence of matter was proposed in [5, 6], the impact of the longitudinal magnetic field $\mathbf{B}_{||}$ was discussed in [7].
The neutrino spin oscillations in the presence of constant twisting magnetic field were considered in [8-13].

In [14] neutrino spin oscillations were considered in the presence of arbitrary constant electromagnetic fields $F_{\mu\nu}$. Neutrino spin oscillations in the presence of the field of circular and linearly polarized electromagnetic waves and superposition of an electromagnetic wave and constant magnetic field were considered in [15-17].

More general case of neutrino spin evolution in the case when neutrino is subjected to general types of non-derivative interactions with external scalar, pseoudoscalar, vector, axial-vector, tensor and pseudotensor fields was considered in [18]. From the general neutrino spin evolution equation, obtained in [18], it follows that neither scalar nor pseudoscalar nor vector fields can induce neutrino spin evolution. On the contrary, within the general consideration of neutrino spin evolution it was shown that electromagnetic (tensor) and weak (axial-vector) interactions can contribute to the neutrino spin evolution.

Recently we consider in details [19, 20] neutrino mixing and oscillations in arbitrary constant magnetic field that have $\mathbf{B}_\perp$ and $\mathbf{B}_{||}$ nonzero components in mass and flavour bases and derived an explicit expressions for the effective neutrino magnetic moments for the favour neutrinos in terms of the corresponding magnetic moments introduced in the neutrino mass basis.

New effect of spin oscillations – in transversal matter currents and matter polarization

For many years, until 2004, it was believed that a neutrino helicity precession and the spin oscillations can be induced by the neutrino magnetic interactions with an external electromagnetic field, by the transversal magnetic field in particular. A new and very interesting possibility for neutrino spin (and spin-favour) oscillations engendered by the neutrino interaction with matter background was proposed and investigated for first time in [21]. It was shown [21] that neutrino spin oscillations can be induced not only by the neutrino interaction with a magnetic field, as it was believed before, but also by neutrino interactions with matter in the case when there is a transversal matter current or matter polarization. This new effect has been explicitly highlighted in [21]:

"The possible emergence of neutrino spin oscillations owing to neutrino interaction with matter under the condition that there exists a nonzero transverse current component or matter polarization is the most important new effect that follows from the investigation of neutrino-spin oscillations in Section 4. So far, it has been assumed that neutrino-spin oscillations may arise only in the case where there exists a nonzero transverse magnetic field in the neutrino rest frame."

It should be noted that the predicted effect exist regardless of a source of the background matter transversal current or polarization (that can be a background magnetic field, for instance).

Note that the existence of the discussed effect of neutrino spin oscillations engendered by the transversal matter current and matter polarization and its possible impact in astrophysics have been considered in a series of recent papers [22-25].

Neutrino spin oscillations in transversal matter current – semiclassical treatment

Consider, as an example, an electron neutrino spin precession in the case when neutrinos with the Standard Model interaction are propagating through moving and polarized matter composed of electrons (electron gas) in the presence of an electromagnetic field given by the electromagnetic field tensor $F_{\mu\nu} = (\mathbf{E}, \mathbf{B})$. Following the discussion in [21] to derive the neutrino spin oscillation probability in the transversal matter current we use the generalized Bargmann-Michel-Telegdi equation that describes the evolution of the three-dimensional neutrino spin vector $\mathbf{S}$,

$$\frac{d\mathbf{S}}{dt} = \frac{2}{\gamma}\Big[\boldsymbol{S} \times (\boldsymbol{B}_0 + \boldsymbol{M}_0)\Big], \qquad (1)$$

where the magnetic field $\mathbf{B_0}$ in the neutrino rest frame is determined by the transversal and longitudinal (with respect to the neutrino motion) magnetic and electric field components in the laboratory frame,

$$\mathbf{B}_0 = \gamma\Big(\mathbf{B}_\perp + \frac{1}{\gamma}\mathbf{B}_\parallel + \sqrt{1-\gamma^{-2}}\Big[\mathbf{E}_\perp \times \frac{\boldsymbol{\beta}}{\beta}\Big]\Big), \qquad (2)$$

$\gamma = (1-\beta^2)^{-\frac{1}{2}}$, $\boldsymbol{\beta}$ is the neutrino velocity. The matter term $\mathbf{M_0}$ in (2) is also composed of the transversal $\mathbf{M}_{0_\perp}$ and longitudinal $\mathbf{M}_{0_\parallel}$ parts, $\mathbf{M}_0 = \mathbf{M}_{0_\parallel} + \mathbf{M}_{0_\perp}$, (3)

$$\mathbf{M}_{0_\parallel} = \gamma\boldsymbol{\beta}\frac{n_0}{\sqrt{1-v_e^2}}\left\{\rho_e^{(1)}\left(1 - \frac{\mathbf{v}_e\boldsymbol{\beta}}{1-\gamma^{-2}}\right)\right\} - \frac{\rho_e^{(2)}}{1-\gamma^{-2}}\left\{\boldsymbol{\zeta}_e\boldsymbol{\beta}\sqrt{1-v_e^2} + \left(\boldsymbol{\zeta}_e\mathbf{v}_e\frac{(\boldsymbol{\beta}\mathbf{v}_e)}{1+\sqrt{1-v_e^2}}\right)\right\}, \qquad (4)$$

$$\mathbf{M}_{0_\perp} = -\frac{n_0}{\sqrt{1-v_e^2}}\left\{\mathbf{v}_{e_\perp}\left(\rho_e^{(1)} + \rho_e^{(2)}\frac{(\boldsymbol{\zeta}_e\mathbf{v}_e)}{1+\sqrt{1-v_e^2}}\right) + \boldsymbol{\zeta}_{e_\perp}\rho_e^{(2)}\sqrt{1-v_e^2}\right\}. \qquad (5)$$

Here $n_0 = n_e\sqrt{1-v_e^2}$ is the invariant number density of matter given in the reference frame for which the total speed of matter is zero. The vectors $\mathbf{v}_e$, and $\boldsymbol{\zeta}_e$ $(0 \le |\boldsymbol{\zeta}_e|^2 \le 1)$ denote, respectively, the speed of the reference frame in which the mean momentum of matter (electrons) is zero, and the mean value of the polarization vector of the background electrons in the above mentioned reference frame. The coefficients $\rho_e^{(1,2)}$ calculated within the extended Standard Model supplied with $SU(2)$ -singlet right-handed neutrino R are respectively, $\rho_e^{(1)} = \frac{\tilde{G}_F}{2\sqrt{2}\mu}$, $\rho_e^{(2)} = -\frac{G_F}{2\sqrt{2}\mu}$, where $\tilde{G}_F = G_F(1 + 4\sin^2\theta_W)$.

For neutrino evolution between two neutrino states $\nu_e^L \Leftrightarrow \nu_\mu^R$ in presence of the magnetic field and moving matter we get [21] the following equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L\\ \nu_e^R\end{pmatrix} = \mu\begin{pmatrix}\frac{1}{\gamma}|\mathbf{M}_{0\parallel} + \mathbf{B}_{0\parallel}| & |\mathbf{B}_\perp + \frac{1}{\gamma}\mathbf{M}_{0\perp}|\\ |\mathbf{B}_\perp + \frac{1}{\gamma}\mathbf{M}_{0\perp}| & -\frac{1}{\gamma}|\mathbf{M}_{0\parallel} + \mathbf{B}_{0\parallel}|\end{pmatrix}\begin{pmatrix}\nu_e^L\\ \nu_e^R\end{pmatrix}. \qquad (6)$$

Thus, the probability of the neutrino spin oscillations in the adiabatic approximation is given by [21]

$$P_{\nu_e^L \to \nu_e^R}(x) = \sin^2 2\theta_{\rm eff}\sin^2\frac{\pi x}{L_{\rm eff}}, \quad \sin^2 2\theta_{\rm eff} = \frac{E_{\rm eff}^2}{E_{\rm eff}^2 + \Delta_{\rm eff}^2}, \quad L_{\rm eff} = \frac{2\pi}{\sqrt{E_{\rm eff}^2 + \Delta_{\rm eff}^2}}, \qquad (7)$$

$$E_{\rm eff} = \mu\Big|\mathbf{B}_\perp + \frac{1}{\gamma}\mathbf{M}_{0\perp}\Big|, \quad \Delta_{\rm eff} = \frac{\mu}{\gamma}\big|\mathbf{M}_{0\parallel} + \mathbf{B}_{0\parallel}\big|. \qquad (8)$$

Thus, it follows that even without presence of an electromagnetic field, $\mathbf{B}_\perp = \mathbf{B}_{0\parallel} = 0$, neutrino spin oscillations $\nu_e^L \Leftrightarrow \nu_\mu^R$ can be induced in the presence of matter when the transverse matter term $\mathbf{M}_{0\perp}$ is not zero. If we neglect possible effects of matter polarization then the neutrino evolution equation (6) simplifies to

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L\\ \nu_e^R\end{pmatrix} = \frac{\mu}{\gamma}\begin{pmatrix}M_{0\parallel} & M_{0\perp}\\ M_{0\perp} & -M_{0\parallel}\end{pmatrix}\begin{pmatrix}\nu_e^L\\ \nu_e^R\end{pmatrix}, \qquad (9)$$

$$\mathbf{M}_{0_\parallel} = \gamma\boldsymbol{\beta}\rho_e^{(1)}\left(1 - \frac{\mathbf{v}_e\boldsymbol{\beta}}{1-\gamma^{-2}}\right)\frac{n_0}{\sqrt{1-v_e^2}}, \qquad \mathbf{M}_{0_\perp} = -\rho_e^{(1)}\mathbf{v}_{e_\perp}\frac{n_0}{\sqrt{1-v_e^2}}. \qquad (10)$$

The effective mixing angle and oscillation length in the neutrino spin oscillation probability (7) now are given by

$$\sin^2 2\theta_{\rm eff} = \frac{M_{0\perp}^2}{M_{0\parallel}^2 + M_{0\perp}^2}, \quad L_{\rm eff} = \frac{2\pi}{\mu M_0}\gamma. \qquad (11)$$

The above considerations can be applied to other types of neutrinos and various matter compositions. It is also obvious that for neutrinos with nonzero transition magnetic moments a similar effect for spin-flavour oscillations exists under the same background conditions.

Neutrino oscillations in mass basis

It will be clearly to start [19] from two neutrino physical states ν_i with masses m_i, i=1,2.

We use neutrino weak interaction Lagrangian

$$H_{weak} = f^\mu\left(\bar{\nu}\gamma_\mu\frac{1+\gamma^5}{2}\nu\right), \qquad (12)$$

where $f^\mu = G_F j^\mu/\sqrt{2}$, $j^\mu = n(1, \boldsymbol{v})/\sqrt{1-v^2}$ - medium current, G_F - Fermi constant of weak interaction.

It is interesting to know evolution of chiral neutrino components within the common neutrino beam state. The neutrino evolution equation relevant weak interaction has the Shroedinger-like form

$$i\frac{d}{dt}\nu_m(t) = H_{eff}\nu_m(t). \qquad (13)$$

The effective Hamiltonian consists of the vacuum and interaction parts

$$H_{eff} = H + \Delta H, \qquad (14)$$

where the interaction part is composed with matrix elements of the field interaction Hamiltonian taken over the helicity neutrino state.

Let us calculate the effective interaction Hamiltonian under assumption that neutrino moves along z-axis. From the weak interaction Hamiltonian (without zero-component of medium current) we have:

$$\Delta H_{\alpha,\alpha'}^{s,s'} = \left\langle \nu_\alpha^s | H_{\rm weak} | \nu_{\alpha'}^{s'} \right\rangle = -\frac{nG_F/\sqrt{2}}{\sqrt{1-v^2}}\int d^4x \nu_\alpha^{s+}\gamma^0(\boldsymbol{v}, \boldsymbol{\gamma})\frac{1+\gamma^5}{2}\nu_{\alpha'}^{s'}. \qquad (15)$$

In the spinor representation the free neutrino states are given by

$$\nu_\alpha^s = C_\alpha\begin{pmatrix}u_\alpha^s\\ \frac{\boldsymbol{\sigma}\boldsymbol{p}_\alpha}{E_\alpha + m_\alpha}u_\alpha^s\end{pmatrix}\sqrt{\frac{(E_\alpha + m_\alpha)}{2E_\alpha}}\exp(i\boldsymbol{p}_\alpha\boldsymbol{x}). \qquad (16)$$

The two-component spinors define neutrino helicity states, and are given by

$$u_\alpha^{s=1} = \begin{pmatrix}1\\0\end{pmatrix}, u_\alpha^{s=-1} = \begin{pmatrix}0\\1\end{pmatrix}. \qquad (17)$$

Recall that in the ultrarelativistic limit these are correspondent to the right-handed ν_R and left-handed ν_L chiral neutrinos, respectively.

Assuming that $\boldsymbol{v}_\perp$ is aligned along the x-axis and reminding zero-component of medium current we finely obtain:

$$\Delta H_{\alpha,\alpha'}^{s,s'} = \frac{nG_F/\sqrt{2}}{2\sqrt{1-v^2}}u_\alpha^{s+}\left\{(1-v_\parallel)\begin{pmatrix}0 & 0\\ 0 & 2\end{pmatrix} + v_\perp\begin{pmatrix}0 & \frac{m_\alpha}{E_\alpha}\\ \frac{m_{\alpha'}}{E_{\alpha'}} & 0\end{pmatrix}\right\}u_{\alpha'}^{s'}. \qquad (18)$$

One can put spinors in this formula and easily find for example

$$\Delta H_{\alpha,\alpha'}^{1,-1} = \frac{nG_F/\sqrt{2}}{2\sqrt{1-v^2}}v_\perp\frac{m_\alpha}{E_\alpha}, \quad \Delta H_{\alpha,\alpha'}^{-1,-1} = \frac{nG_F/\sqrt{2}}{2\sqrt{1-v^2}}2(1-v_\parallel). \qquad (19)$$

As it was expected, in neutrino transitions without change of helicity only $v_\parallel$ component of the media velocity contribute to the effective potential, whereas in transitions with change of the neutrino helicity the transversal component $v_\perp$ matters.

One can write out the ΔH matrix. For the effective Hamiltonian with the diagonal vacuum part H we get

$$i\frac{d}{dt}\begin{pmatrix}\nu_1^{s=1}\\ \nu_1^{s=-1}\\ \nu_2^{s=1}\\ \nu_2^{s=-1}\end{pmatrix} = \left[H + \frac{nG_F/\sqrt{2}}{2\sqrt{1-v^2}}\begin{pmatrix}0 & v_\perp\frac{m_1}{E_1} & 0 & v_\perp\frac{m_1}{E_1}\\ v_\perp\frac{m_1}{E_1} & 2(1-v_\parallel) & v_\perp\frac{m_2}{E_2} & 2(1-v_\parallel)\\ 0 & v_\perp\frac{m_2}{E_2} & 0 & v_\perp\frac{m_2}{E_2}\\ v_\perp\frac{m_1}{E_1} & 2(1-v_\parallel) & v_\perp\frac{m_2}{E_2} & 2(1-v_\parallel)\end{pmatrix}\right]\begin{pmatrix}\nu_1^{s=1}\\ \nu_1^{s=-1}\\ \nu_2^{s=1}\\ \nu_2^{s=-1}\end{pmatrix} \qquad (20)$$

This equation governs all possible oscillations of the four neutrino mass states determined by the masses m_1 and m_2 and helicities s = 1 and s = −1 in the presence of a moving media. Thus, it follows that: 1) the change of helicity is due to the weak interaction with transversally moving media with velocity $\boldsymbol{v}_\perp$, 2) the media moving with longitudinal velocity $\boldsymbol{v}_\parallel$ shifts the neutrino energy, 3) an additional mixing between neutrino states with different masses is induced by weak interaction with media with velocity $\boldsymbol{v}_\parallel$

Neutrino oscillations in flavour basis

Once having physics in the mass basis in hands, our next step is to bring it to observational terms [19]. This means that we must elaborate a generalization of the mixing matrix for transitions between neutrino vector written in two-component base ν_m and $\nu_f = (\nu_e^L\nu_e^R\nu_\mu^L\nu_\mu^R)^T$ so that

$$\nu_f = U\nu_m \qquad (21)$$

This procedure appears to be not quite direct since we should hold the condition that the polarization of the fields must preserve under transformation of the bases elements. That is why we put (still keeping in mind that chiral components are almost helicity ones):

$$\nu_e^{R,L} = \nu_1^{s=\pm1}\cos\theta + \nu_2^{s=\pm1}\sin\theta, \nu_\mu^{R,L} = -\nu_1^{s=\pm1}\sin\theta + \nu_2^{s=\pm1}\cos\theta, \qquad (22)$$

It is easy to obtain that

$$U = \begin{pmatrix}\cos\theta & 0 & \sin\theta & 0\\ 0 & \cos\theta & 0 & \sin\theta\\ -\sin\theta & 0 & \cos\theta & 0\\ 0 & -\sin\theta & 0 & \cos\theta\end{pmatrix}. \qquad (23)$$

Given the transition matrix, derivation of the evolution equation in the flavour basis is straightforward:

$$i\frac{d}{dt}\nu_f = UH_{eff}U^+\nu_f, \qquad (24)$$

so that the effective interaction Hamiltonian $\Delta\tilde{H}^f \equiv U\Delta HU^+$ has the following structure,

$$\Delta\tilde{H}^f = \frac{nG_F/\sqrt{2}}{2\sqrt{1-v^2}}\begin{pmatrix}0 & v_\perp(\frac{\eta}{\gamma})_{ee} & 0 & v_\perp(\frac{\eta}{\gamma})_{e\mu}\\ v_\perp(\frac{\eta}{\gamma})_{ee} & 2(1-v_\parallel)\eta_{ee} & v_\perp(\frac{\eta}{\gamma})_{\mu e} & 2(1-v_\parallel)\eta_{e\mu}\\ 0 & v_\perp(\frac{\eta}{\gamma})_{\mu e} & 0 & v_\perp(\frac{\eta}{\gamma})_{\mu\mu}\\ v_\perp(\frac{\eta}{\gamma})_{e\mu} & 2(1-v_\parallel)\eta_{e\mu} & v_\perp(\frac{\eta}{\gamma})_{\mu\mu} & 2(1-v_\parallel)\eta_{\mu\mu}\end{pmatrix}. \qquad (25)$$

Here we introduce the following formal notations intended to manifest an analogy with the standard spin and spin-flavour formalism,

$$\Big(\frac{\eta}{\gamma}\Big)_{ee} = \frac{\cos^2\theta}{\gamma_{11}} + \frac{\sin^2\theta}{\gamma_{22}} + \frac{\sin 2\theta}{\gamma_{12}}, \quad \Big(\frac{\eta}{\gamma}\Big)_{e\mu} = \frac{\cos^2\theta}{\gamma_{11}} - \frac{\sin^2\theta}{\gamma_{22}} + \frac{\sin 2\theta}{\gamma_{21}}, \quad \eta_{ee} = 1 + \sin 2\theta$$

$$\Big(\frac{\eta}{\gamma}\Big)_{\mu\mu} = \frac{\sin^2\theta}{\gamma_{11}} + \frac{\cos^2\theta}{\gamma_{22}} - \frac{\sin 2\theta}{\gamma_{12}}, \quad \Big(\frac{\eta}{\gamma}\Big)_{\mu e} = \frac{\cos^2\theta}{\gamma_{22}} - \frac{\sin^2\theta}{\gamma_{11}} + \frac{\sin 2\theta}{\gamma_{21}}, \quad \eta_{e\mu} = \cos 2\theta, \quad \eta_{\mu\mu} = 1 - \sin 2\theta \qquad (26)$$

It is interesting to consider a particular case. For example, one would have two states (ν_e^R, ν_μ^L) mixed in accordance with the equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^R\\ \nu_\mu^L\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{nG_F/\sqrt{2}}{2\sqrt{1-v^2}} \times v_\perp(\frac{\eta}{\gamma})_{e\mu}\\ \frac{nG_F/\sqrt{2}}{2\sqrt{1-v^2}} \times v_\perp(\frac{\eta}{\gamma})_{e\mu} & \frac{\Delta m^2}{4E}\cos 2\theta + \frac{nG_F/\sqrt{2}}{2\sqrt{1-v^2}} \times 2(1-v_\parallel)\eta_{\mu\mu}\end{pmatrix}\begin{pmatrix}\nu_e^R\\ \nu_\mu^L\end{pmatrix}. \qquad (27)$$

From this it follows that the neutrino interacts with moving media and can generate the neutrino flavour mixing (an additional mixing to the usual effect due to neutrino mixing angle θ) with changing neutrino chirality. For the spin-flavour neutrino oscillation probability in the adiabatic case we get

$$P_{\nu_e^R \leftrightarrow \nu_\mu^L} = \frac{\left(\frac{nG_F/\sqrt{2}}{\sqrt{1-v^2}}v_\perp(\frac{\eta}{\gamma})_{e\mu}\right)^2}{\left(\frac{nG_F/\sqrt{2}}{\sqrt{1-v^2}}v_\perp(\frac{\eta}{\gamma})_{e\mu}\right)^2 + \left(\frac{\Delta m^2}{2E}\cos 2\theta + \frac{nG_F/\sqrt{2}}{2\sqrt{1-v^2}}(1-v_\parallel)\eta_{\mu\mu}\right)^2}\sin^2\Big(\frac{1}{2}\sqrt{D}x\Big), \qquad (28)$$

where $D = \left(\frac{nG_F/\sqrt{2}}{\sqrt{1-v^2}}v_\perp(\frac{\eta}{\gamma})_{e\mu}\right)^2 + \left(\frac{\Delta m^2}{2E}\cos 2\theta + \frac{nG_F/\sqrt{2}}{2\sqrt{1-v^2}}(1-v_\parallel)\eta_{\mu\mu}\right)^2$. It follows that $\boldsymbol{v}$ components not only generates spin-flavour neutrino mixing but also can produce the resonance amplification of the corresponding oscillations.

Relativistic neutrino quantum states in arbitrary moving matter

In this section our interest is exact solution of modified Dirac equation for neutrino propagating in moving matter. Neutrino quantum states in nonmoving matter were found in [25]. Generalization on case when matter velocity is parallel to neutrino momentum is trivial. Neutrino interaction with arbitrary moving external media is described by the following modified Dirac equation:

$$\Big(i\gamma^\mu\partial_\mu - \frac{1}{2}\gamma^\mu(1+\gamma^5)f_\mu - m\Big)\psi(x) = 0. \qquad (29)$$

We can rewrite it in Hamiltonian form:

$i\frac{\partial\psi(x)}{\partial t} = \hat{H}_{matt}\psi(x)$, where Hamiltonian is given by

$$\hat{H}_{matt} = \left(\hat{\boldsymbol{\alpha}}\boldsymbol{p} + \hat{\beta}m + \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}(1+\gamma^5) - \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}\hat{\boldsymbol{\alpha}}\boldsymbol{v}(1+\gamma^5)\right) \qquad (30)$$

and $\boldsymbol{v}$ is matter velocity.

We consider case of uniform matter moving with constant velocity. Therefore, energy and momentum are integrals of motion. But in general case, helicity operator doesn't commute with this Hamiltonian. So let's introduce new operator:

$$\hat{s}_v = \frac{1}{p}\left[\hat{\boldsymbol{\Sigma}}\boldsymbol{p} + \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}(1+\gamma^5)\hat{\boldsymbol{\Sigma}}\boldsymbol{v}\right], \ p = |\boldsymbol{p}|. \qquad (31)$$

It satisfies commutational relation $[\hat{s}_v, \hat{H}_D] = \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}\frac{m}{p}\hat{\boldsymbol{\Sigma}}\boldsymbol{v}[\gamma^5, \gamma^0]$, and therefore in massless case, which corresponds to ultra-relativistic limit, is conserved. We will call it spin operator of relativistic neutrino in moving matter. Eigenstates of this operator has following form:

$$s_v = \pm 1; \pm\frac{1}{p}\left|\boldsymbol{p} + \frac{G_F n/\sqrt{2}}{\sqrt{1-v^2}}\boldsymbol{v}\right|. \qquad (32)$$

Unlike ordinary helicity operator, our spin operator has four eigenvalues. In moving matter degeneration is removed. We showed that different spin numbers realizes for different signs of energy:

$$s_v = \begin{cases}\pm 1, \text{for } \varepsilon = 1\\ \pm\frac{1}{p}\left|\boldsymbol{p} + \frac{G_F n/\sqrt{2}}{\sqrt{1-v^2}}\boldsymbol{v}\right|, \text{for } \varepsilon = -1.\end{cases} \qquad (33)$$

In case of uniform matter and constant velocity energy and momentum are conserved. So, for stationary states we get:

$$\psi(x) = \frac{e^{-i(Et - \boldsymbol{p}\boldsymbol{r})}}{L^{\frac{3}{2}}}u(p). \qquad (34)$$

$u(p)$ is spin operator eigenstate, which is independent on spatial coordinates and time.

Using our spin operator, we can find energy spectrum and exact solution of modified Dirac equation for ultra-relativistic neutrino in arbitrary moving matter:

$$\psi(x) = \frac{e^{-i(E_{\varepsilon,s}t - \boldsymbol{p}\boldsymbol{r})}}{2L^{\frac{3}{2}}}\begin{pmatrix}\sqrt{1 + s\frac{p_3 + \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}(1-s\varepsilon)v_3}{\left|\boldsymbol{p} + \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}(1-s\varepsilon)\boldsymbol{v}\right|}}\\ s\sqrt{1 - s\frac{p_3 + \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}(1-s\varepsilon)v_3}{\left|\boldsymbol{p} + \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}(1-s\varepsilon)\boldsymbol{v}\right|}}e^{i\delta}\\ s\varepsilon\sqrt{1 + s\frac{p_3 + \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}(1-s\varepsilon)v_3}{\left|\boldsymbol{p} + \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}(1-s\varepsilon)\boldsymbol{v}\right|}}\\ \varepsilon\sqrt{1 - s\frac{p_3 + \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}(1-s\varepsilon)v_3}{\left|\boldsymbol{p} + \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}(1-s\varepsilon)\boldsymbol{v}\right|}}e^{i\delta}\end{pmatrix}, \qquad (35)$$

$$E_{\varepsilon,s} = \frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}} + \varepsilon\left(\sqrt{\boldsymbol{p}^2 + (1-\varepsilon s)\left[\frac{G_F n/\sqrt{2}}{\sqrt{1-v^2}}\boldsymbol{p}\boldsymbol{v} + \frac{G_F^2 n^2\boldsymbol{v}^2}{4(1-v^2)}\right]} - s\frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}\right). \qquad (36)$$

where $s, \varepsilon = \pm 1$ are spin numbers and energy sigh , $\tan\delta = \frac{p_1 + (1-s\varepsilon)\frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}v_1}{p_2 + (1-s\varepsilon)\frac{G_F n/\sqrt{2}}{2\sqrt{1-v^2}}v_2}$.

In particular case of zero neutron density it coincides with solutions of Dirac equation in vacuum. Let's also note that when $s\varepsilon = 1$ there is no interaction with matter. Corresponding solutions describes right neutrino and left antineutrino in vacuum. Emergence of sterile particle is consequence of ultra-relativistic limit. In this limit helicity states is exactly chiral states. Due to *V-A* structure of weak interaction Lagrangian, right particles, i.e. right neutrino and left antineutrino, doesn't interact. In reality interaction will be suppressed by neutrino Lorentz-factor.

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[1] C. Giunti and A. Studenikin, Rev. Mod. Phys. 87 (2015) 531.
[2] A. Cisneros, Astrophys. Space Sci. 10 (1971) 87.
[3] J. Schechter and J. W. F. Valle, Phys. Rev. D 24 (1981) 1883.
[4] L. Okun, M. Voloshin, M. Vysotsky, Sov. J. Nucl. Phys. 44 (1986) 440.
[5] E. Akhmedov, Phys. Lett. B 213 (1988) 64.
[6] C. S. Lim and W. J. Marciano, Phys. Rev. D 37 (1988) 1368.
[7] E. Akhmedov, M. Khlopov, Mod. Phys. Lett. A 3 (1988) 451.
[8] J. Vidal and J. Wudka, Phys. Lett. B 249 (1990) 473
[9] A. Y. Smirnov, Phys. Lett. B 260 (1991) 161.
[10] E. K. Akhmedov, S. T. Petcov and A. Y. Smirnov, Phys. Rev. D 48 (1993) 2167 [hep-ph/9301211].
[11] G. G. Likhachev and A. I. Studenikin, J. Exp. Theor. Phys. 81 (1995) 419 [Zh. Eksp. Teor. Fiz. 108 (1995) 769].
[12] M. Dvornikov, J. Phys. G 35 (2008) 025003 [arXiv:0708.2328 [hep-ph]].
[13] A. Dmitriev, R. Fabbricatore and A. Studenikin, PoS CORFU 2014 (2015) 050 [arXiv:1506.05311 [hep-ph]].
[14] A. Egorov, A. Lobanov and A. Studenikin, Phys. Lett. B 491 (2000) 137.
[15] A. Lobanov and A. Studenikin, Phys. Lett. B 515 (2001) 94.
[16] M. S. Dvornikov and A. I. Studenikin, Phys. Atom. Nucl. 64 (2001) 1624 [Yad. Fiz. 64 (2001) 1705].
[17] M. S. Dvornikov and A. I. Studenikin, Phys. Atom. Nucl. 67 (2004) 719 [Yad. Fiz. 67 (2004) 741].
[18] M. Dvornikov, A. Studenikin, JHEP 09 (2002) 016.
[19] R. Fabbricatore, A. Grigoriev and A. Studenikin, J. Phys. Conf. Ser. 718 (2016) no.6, 062058 [arXiv:1604.01245 [hep-ph]].
[20] A. Studenikin, Phys. Atom. Nucl. 67 (2004) 993.
[21] A. Studenikin, hep-ph/0407010.
[22] V. Cirigliano, G. Fuller, A. Vlasenko, Phys. Lett. B 747 (2015) 27.
[23] C. Volpe, Int. J. Mod. Phys. E 24 (2015) 1541009.
[24] A. Kartavtsev, G. Raffelt, H. Vogel, Phys. Rev. D 91 (2015) 125020.
[25] A. I. Studenikin and A. I. Ternov, Phys. Lett. B 608 (2005) 107; [hep-ph/0410297].

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