

# Soft gluon resummation for gaugino-gluino production

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Work done in collaboration with B. Fuks and M. Rothering



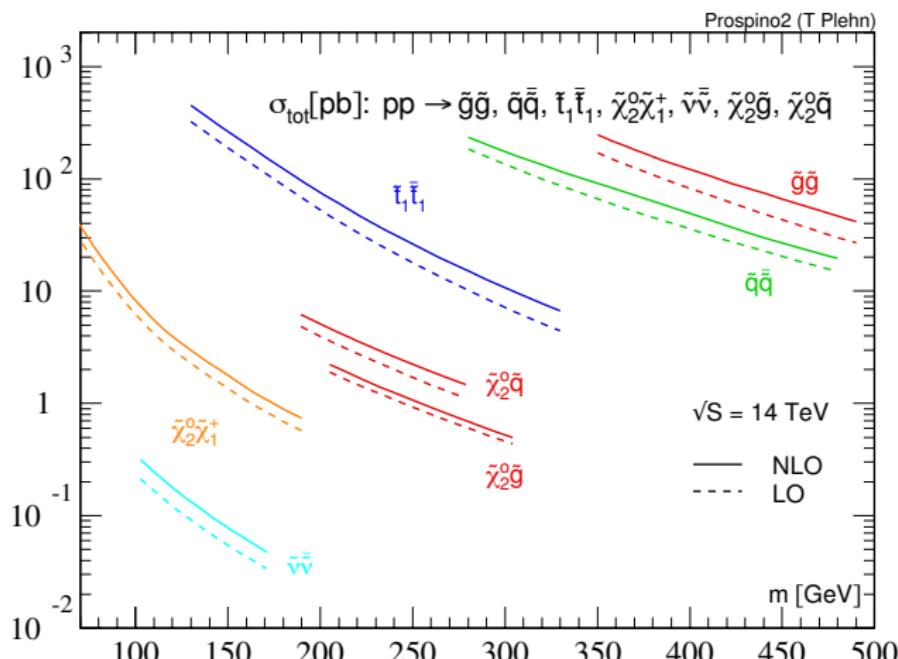
GEFÖRDERT VOM



Bundesministerium  
für Bildung  
und Forschung

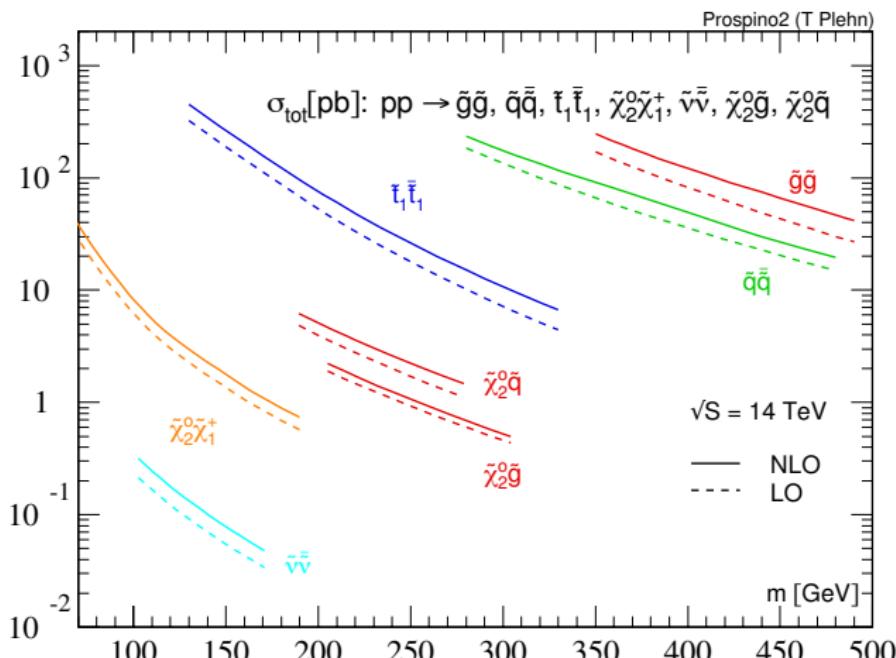
# Cross sections for SUSY particles at the LHC

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$\tilde{\chi}_i^0\tilde{g}$  channels subdominant, but maybe the only chance for heavy gluinos

## References

Current state of the art:

- W. Beenakker, C. Borschensky, M. Krämer, A. Kulesza, E. Laenen  
**NNLL-fast: Predictions for coloured supersymmetric particle production at the LHC with threshold and Coulomb resummation**  
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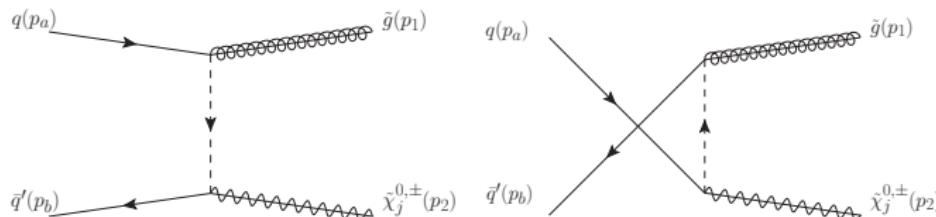
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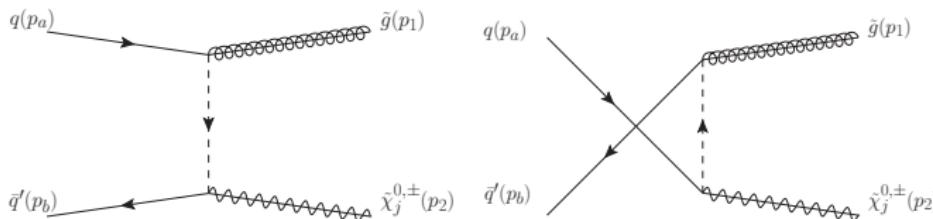
# Gaugino-gluino production at leading order in QCD

Feynman diagrams:



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Feynman diagrams:



Squared matrix elements:

$$\begin{aligned}
 \mathcal{M}_t \mathcal{M}_{tc}^* &= \frac{C_A C_F e g_s(\mu_r)}{(m_{\tilde{q}}^2 - t)(m_{\tilde{q}_c}^2 - t)} (\mathcal{L}' \mathcal{L}'_c + \mathcal{R}' \mathcal{R}'_c) (LL_c + RR_c) (m_{\tilde{g}}^2 - t) (m_{\tilde{\chi}}^2 - t), \\
 \mathcal{M}_u \mathcal{M}_{uc}^* &= \frac{C_A C_F e g_s(\mu_r)}{(m_{\tilde{q}}^2 - u)(m_{\tilde{q}_c}^2 - u)} (\mathcal{L} \mathcal{L}_c + \mathcal{R} \mathcal{R}_c) (L' L'_c + R' R'_c) (m_{\tilde{g}}^2 - u) (m_{\tilde{\chi}}^2 - u), \\
 \mathcal{M}_t \mathcal{M}_{uc}^* &= \frac{C_A C_F e g_s(\mu_r)}{(m_{\tilde{q}}^2 - t)(m_{\tilde{q}_c}^2 - u)} \left[ \left( -s^2 + t^2 + u^2 + (m_{\tilde{\chi}}^2 + m_{\tilde{g}}^2)(s - t - u) + 2m_{\tilde{g}}^2 m_{\tilde{\chi}}^2 \right) \right. \\
 &\quad \times \left. \left( LL_c \mathcal{L}' \mathcal{L}'_c + RR_c \mathcal{R}' \mathcal{R}'_c \right) + 2m_{\tilde{g}} m_{\tilde{\chi}} s (\mathcal{R} \mathcal{R}_c \mathcal{L}' L'_c + \mathcal{L} \mathcal{L}_c \mathcal{R}' R'_c) \right].
 \end{aligned}$$

# Total cross section up to NLO QCD

Partonic cross section:

$$d\sigma_{ab}^{(0)} = \int_2 d\sigma^B = \int \frac{1}{2s} \frac{1}{4C_A^2} \sum_{\tilde{q}, \tilde{q}_c} \left( \mathcal{M}_t \mathcal{M}_{tc}^* + \mathcal{M}_u \mathcal{M}_{uc}^* - 2 \operatorname{Re}(\mathcal{M}_t \mathcal{M}_{uc}^*) \right) dPS^{(2)}$$

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Hadronic cross section:

$$\begin{aligned} \sigma_{AB} &= \int M^2 \frac{d\sigma_{AB}}{dM^2}(\tau) = \sum_{a,b} \int_0^1 dx_a dx_b dz [x_a f_{a/A}(x_a, \mu_f^2)] [x_b f_{b/B}(x_b, \mu_f^2)] \\ &\quad \times [z d\sigma_{ab}(z, M^2, \mu_r^2, \mu_f^2)] \delta(\tau - x_a x_b z) \end{aligned}$$

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NLO cross section:

$$d\sigma_{ab}^{(1)} = \sigma^{\{3\}} + \sigma^{\{2\}} + \sigma^C = \int_3 [d\sigma^R - d\sigma^A]_{\epsilon=0} + \int_2 [d\sigma^V + \int_1 d\sigma^A]_{\epsilon=0} + \sigma^C$$

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Agrees with E. Berger, MK, T. Tait, Phys. Rev. D 62 (2000) 095014.

# Threshold resummation formalism

Large logarithms:

$$z = \frac{M^2}{s} \rightarrow 1 \quad \Rightarrow \quad \left( \frac{\alpha_s}{2\pi} \right)^n \left[ \frac{\ln^m(1-z)}{1-z} \right]_+$$

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$$d\sigma_{ab \rightarrow ij}^{(\text{res.})}(N, M^2, \mu^2) = \sum_I \mathcal{H}_{ab \rightarrow ij, I}(M^2, \mu^2) \Delta_a(N, M^2, \mu^2) \Delta_b(N, M^2, \mu^2) \Delta_{ab \rightarrow ij, I}(N, M^2, \mu^2)$$

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Soft-collinear/soft functions:

$$\Delta_a \Delta_b \Delta_{ab \rightarrow ij, I} = \exp \left[ LG_{ab}^{(1)}(\lambda) + G_{ab \rightarrow ij, I}^{(2)}(\lambda, M^2/\mu^2) + \dots \right]$$

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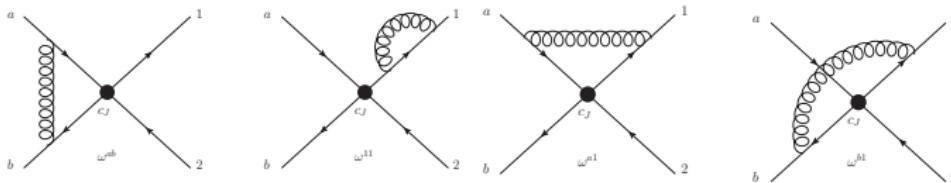
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Leading/next-to-leading logarithms:

$$\begin{aligned} G_{ab}^{(1)}(\lambda) &= g_a^{(1)}(\lambda) + g_b^{(1)}(\lambda), \\ G_{ab \rightarrow ij}^{(2)}(\lambda) &= g_a^{(2)}(\lambda, M^2, \mu_r^2, \mu_f^2) + g_b^{(2)}(\lambda, M^2, \mu_r^2, \mu_f^2) + \mathcal{H}_{ab \rightarrow ij, I}^{(2)}(\lambda). \end{aligned}$$

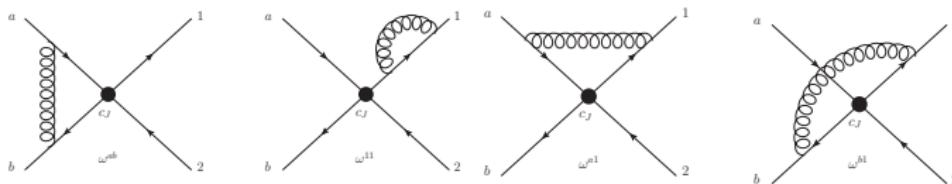
# Soft anomalous dimension

Feynman diagrams:



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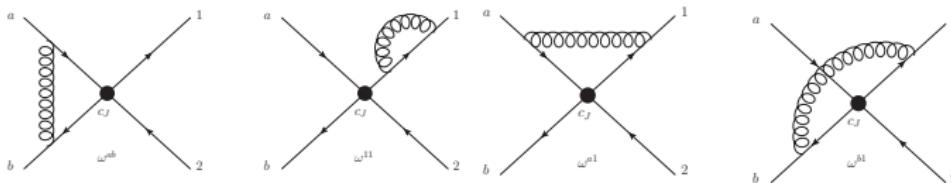


Calculation in axial gauge:

$$\begin{aligned}
 \omega^{ab} &= S_{ab} \frac{\alpha_s}{\pi \epsilon} \left[ -\ln \left( \frac{v_a \cdot v_b}{2} \right) + \frac{1}{2} \ln \left( \frac{(v_a \cdot n)^2}{|n|^2} \frac{(v_b \cdot n)^2}{|n|^2} \right) + i\pi - 1 \right], \\
 \omega^{a1} &= S_{a1} \frac{\alpha_s}{\pi \epsilon} \left[ -\frac{1}{2} \ln \left( \frac{(v_a \cdot v_1)^2 s}{2m_{\tilde{g}}^2} \right) + L_1 + \frac{1}{2} \ln \left( \frac{(v_a \cdot n)^2}{|n|^2} \right) - 1 \right], \\
 \omega^{b1} &= S_{b1} \frac{\alpha_s}{\pi \epsilon} \left[ -\frac{1}{2} \ln \left( \frac{(v_b \cdot v_1)^2 s}{2m_{\tilde{g}}^2} \right) + L_1 + \frac{1}{2} \ln \left( \frac{(v_b \cdot n)^2}{|n|^2} \right) - 1 \right], \\
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 \end{aligned}$$

Total one-loop result:

$$h_{ab \rightarrow ij, I}^{(2)}(\lambda) = \frac{2\pi}{\alpha_s} \frac{\ln(1-2\lambda)}{2\beta_0} \operatorname{Re} \left\{ \frac{\alpha_s}{2\pi} C_A \left[ \ln 2 + i\pi - 1 + \ln \left( \frac{m_{\tilde{g}}^2 - t}{\sqrt{2}m_{\tilde{g}}\sqrt{s}} \right) + \ln \left( \frac{m_{\tilde{g}}^2 - u}{\sqrt{2}m_{\tilde{g}}\sqrt{s}} \right) \right] \right\}$$

# Hard matching coefficient

Perturbative expansion of the hard function:

$$\mathcal{H}_{ab \rightarrow ij, I}(M^2, \mu^2) = \sigma_{ab \rightarrow ij}^{(0)}(M^2) \left\{ 1 + \frac{\alpha_s(\mu^2)}{2\pi} C_{ab \rightarrow ij}^{(1)}(M^2, \mu^2) + \dots \right\}$$

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Hard matching coefficient  $C_{ab \rightarrow ij}^{(1)}(M^2, \mu^2)$ :

- $N$ -independent terms of the full NLO corrections in Mellin space
- $d\sigma^V$  and  $\int_1 d\sigma^A \propto \delta(1-z)$ , therefore constant in  $N$

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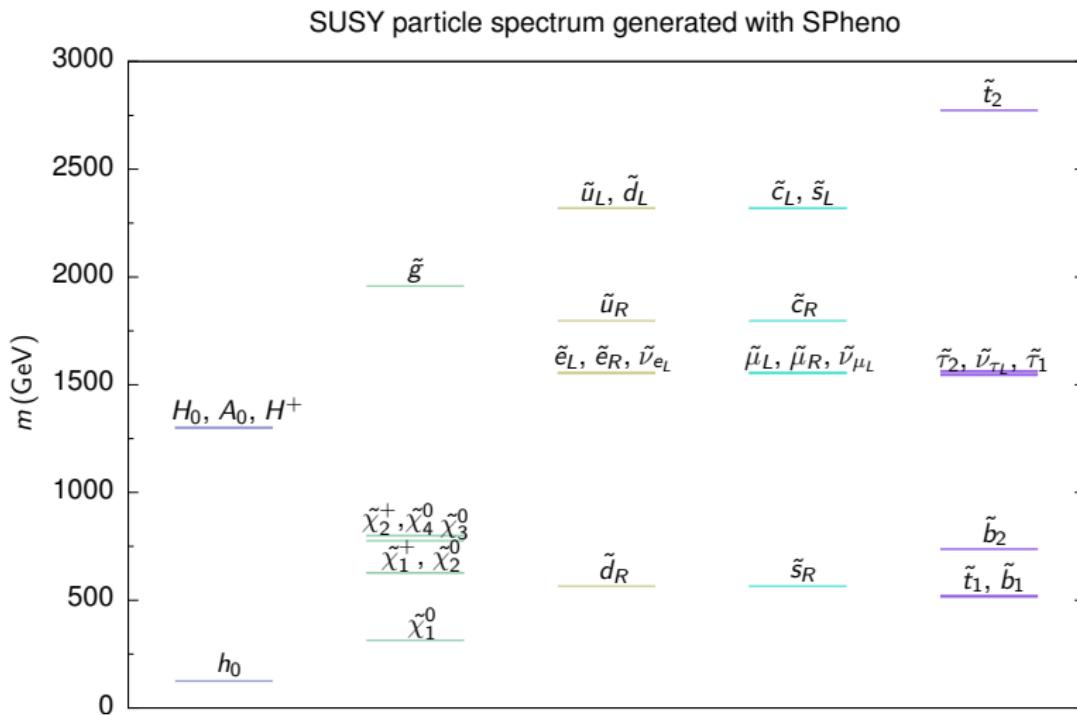
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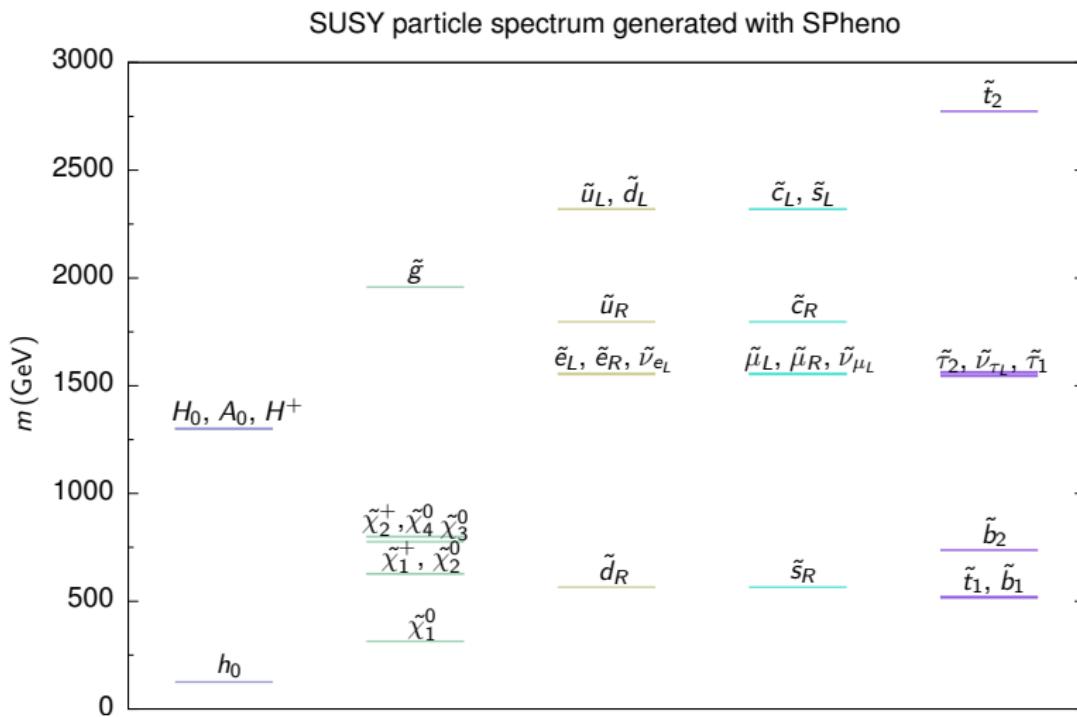
Collinear remainder:

$$\begin{aligned} \sigma^C &= \sum_{a'} \int_0^1 dx \int_2^1 [d\sigma_{a'b}^{(0)}(xp_a, p_b) \otimes \langle a' | \mathbf{P} + \mathbf{K} | a \rangle(x) + (a \leftrightarrow b)]_{\epsilon=0}, \\ \langle \mathbf{P}(N) \rangle &= \frac{\alpha_s}{2\pi} \left[ (2C_F - C_A) \ln \frac{\mu_f^2}{s} + C_A \ln \frac{\mu_f^2}{m_{\tilde{g}}^2 - t} \right] \left[ \ln \bar{N} - \frac{3}{4} \right] + \mathcal{O}\left(\frac{1}{N}\right), \\ \langle \mathbf{K}(N) \rangle &= \frac{\alpha_s}{2\pi} \left\{ 2C_F \ln^2 \bar{N} + C_A \left[ \ln \frac{m_{\tilde{g}}^2}{m_{\tilde{g}}^2 - t} + 1 \right] \ln \bar{N} + \frac{\pi^2}{2} C_F - \gamma_q - K_q \right. \\ &\quad + \frac{C_A}{4} \left[ 1 + 4 \text{Li}_2 \frac{2m_{\tilde{g}}^2 - t}{m_{\tilde{g}}^2} + \left( 1 + 4 \ln \frac{m_{\tilde{g}}^2}{m_{\tilde{g}}^2 - t} + 2 \frac{m_{\tilde{g}}^2}{m_{\tilde{g}}^2 - t} \right) \ln \frac{m_{\tilde{g}}^2}{2m_{\tilde{g}}^2 - t} \right. \\ &\quad \left. \left. + 3 \ln \left( 1 + \frac{2m_{\tilde{g}}}{m_{\tilde{g}}^2 - t} \left( m_{\tilde{g}} - \sqrt{2m_{\tilde{g}}^2 - t} \right) \right) + 6 \frac{m_{\tilde{g}}}{m_{\tilde{g}} + \sqrt{2m_{\tilde{g}}^2 - t}} - 3 \right] \right\} + \mathcal{O}\left(\frac{1}{N}\right). \end{aligned}$$

# pMSSM-13 benchmark scenario

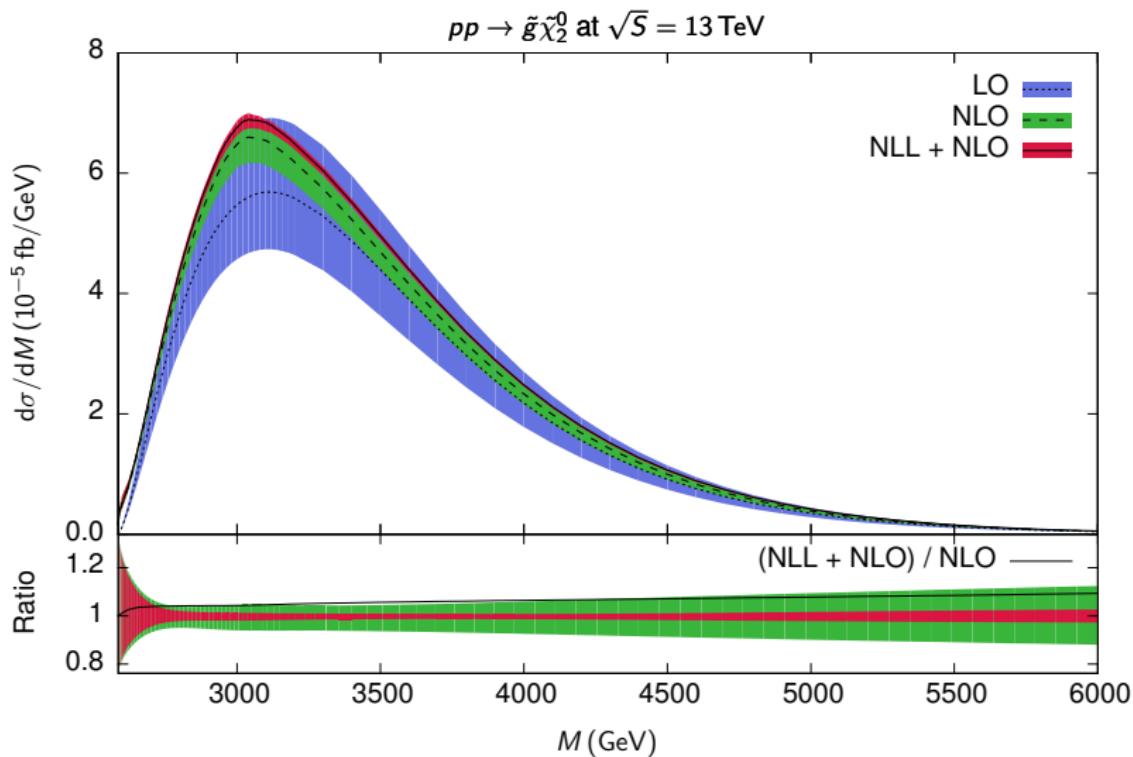


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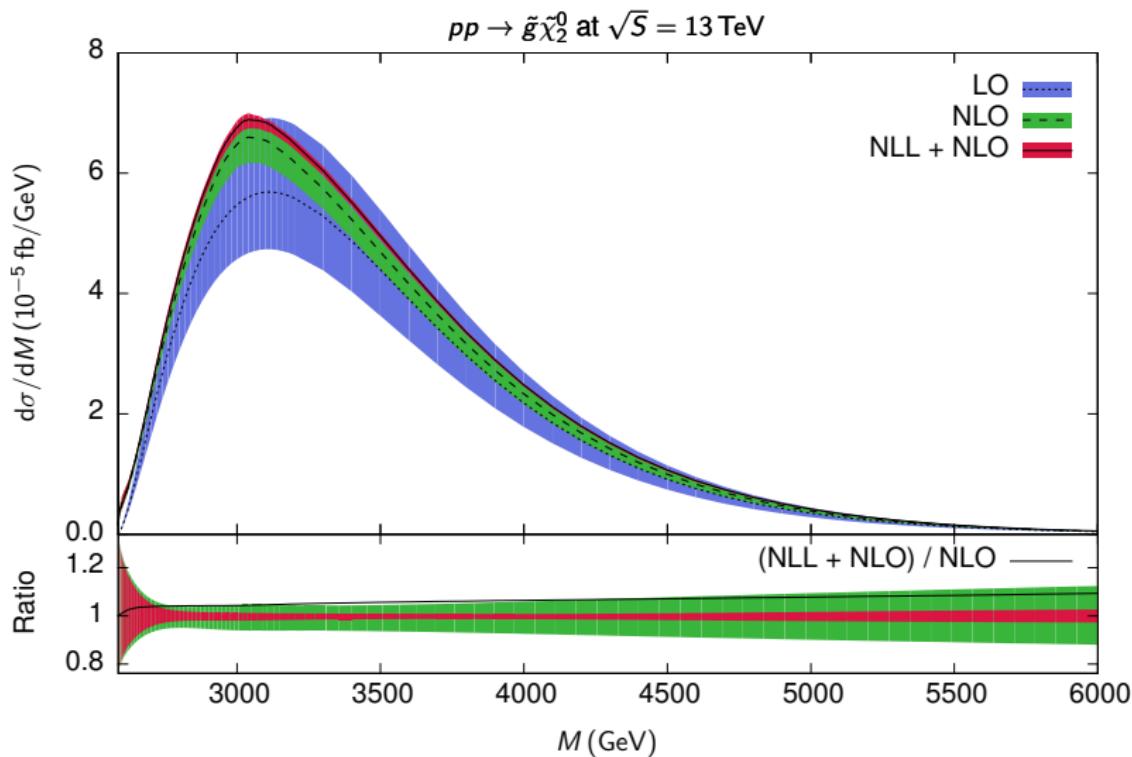


Vary  $M_1 \simeq M_2/2$  and  $M_3$ , impose  $m_{h^0}$ , FCNC, LHC, and DM constraints.

# Invariant mass distribution

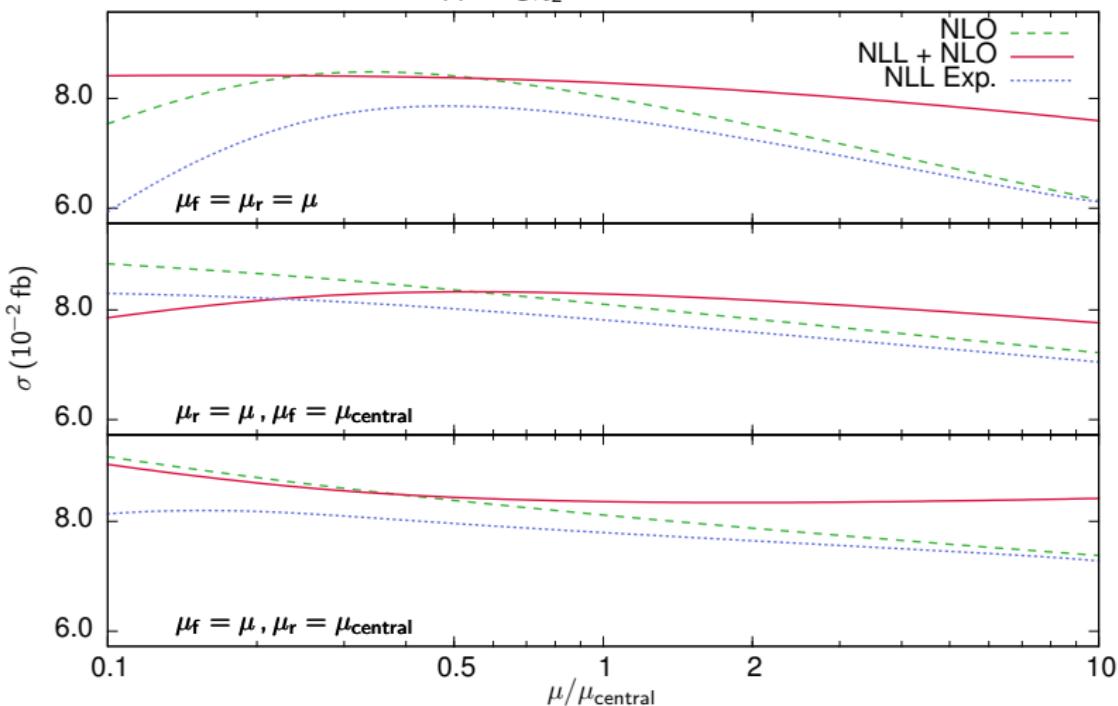


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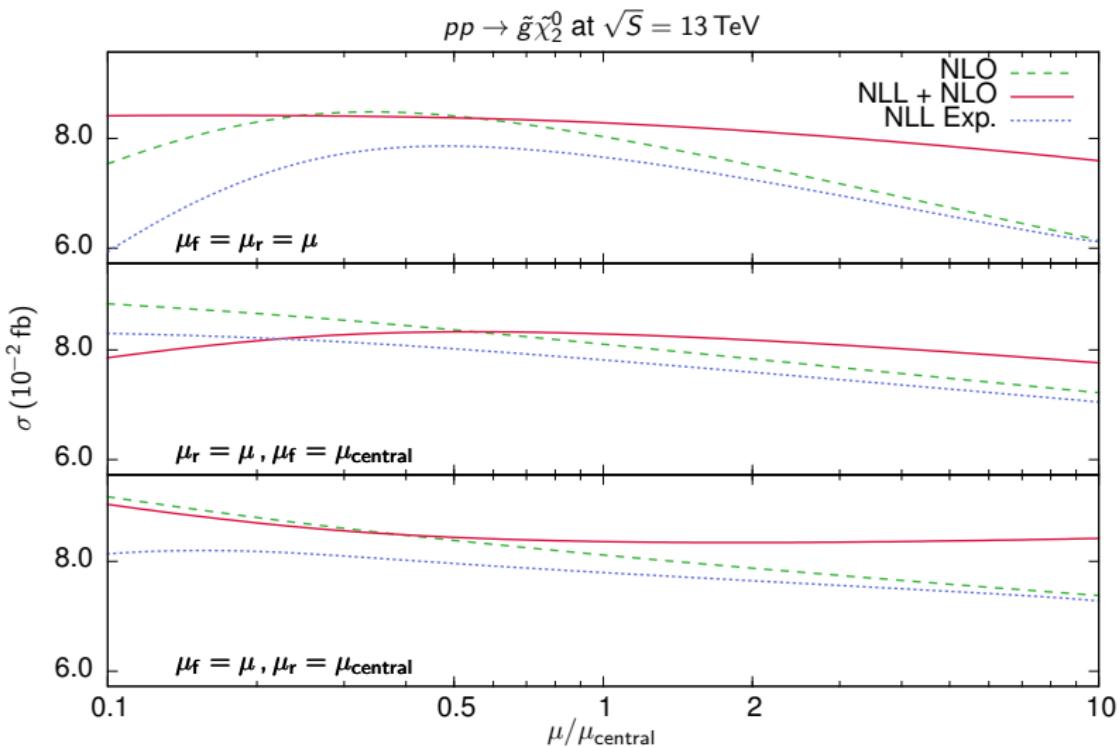


Additional radiation shifts maximum, reduces scale dependence.

## Scale dependence

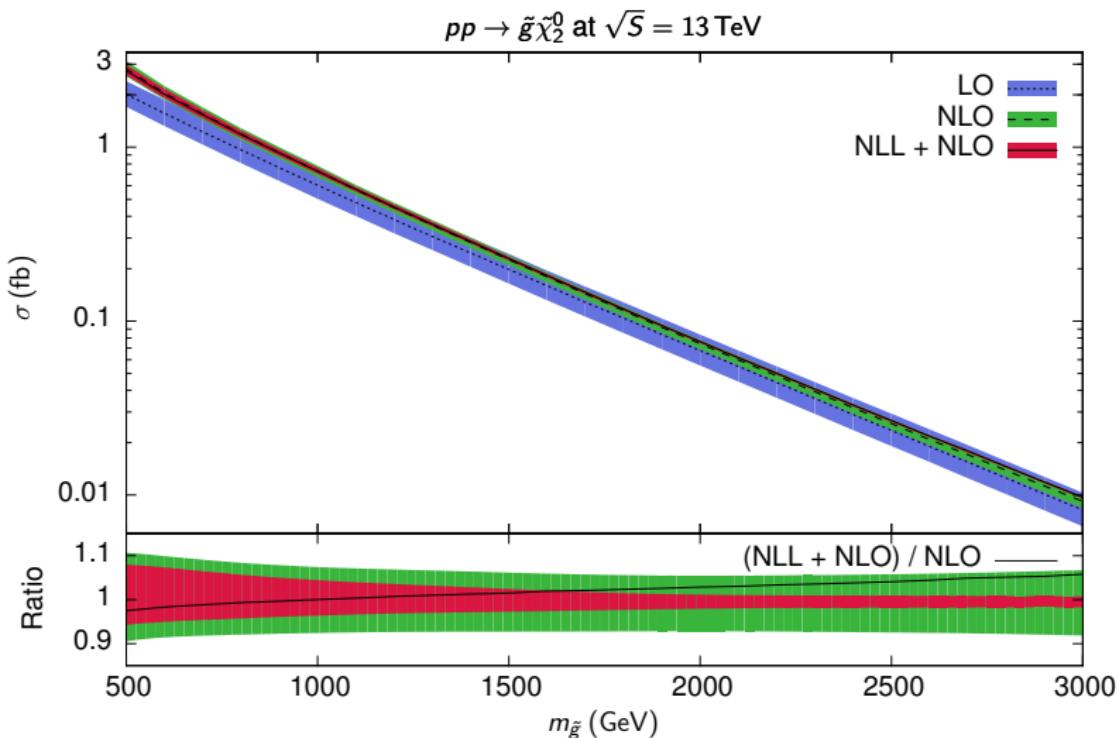
 $pp \rightarrow \tilde{g}\chi_2^0$  at  $\sqrt{S} = 13$  TeV

## Scale dependence

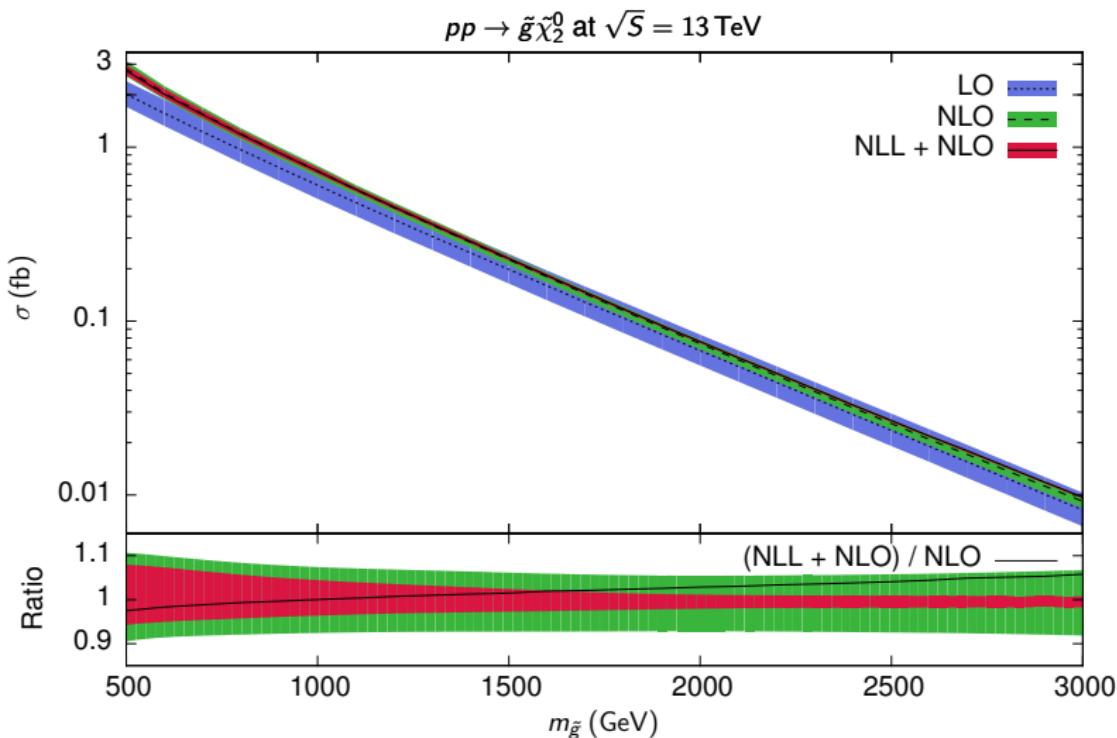


Stable predictions at NLL+NLO, expansion of NLL approximates NLO.

# Gluino mass dependence

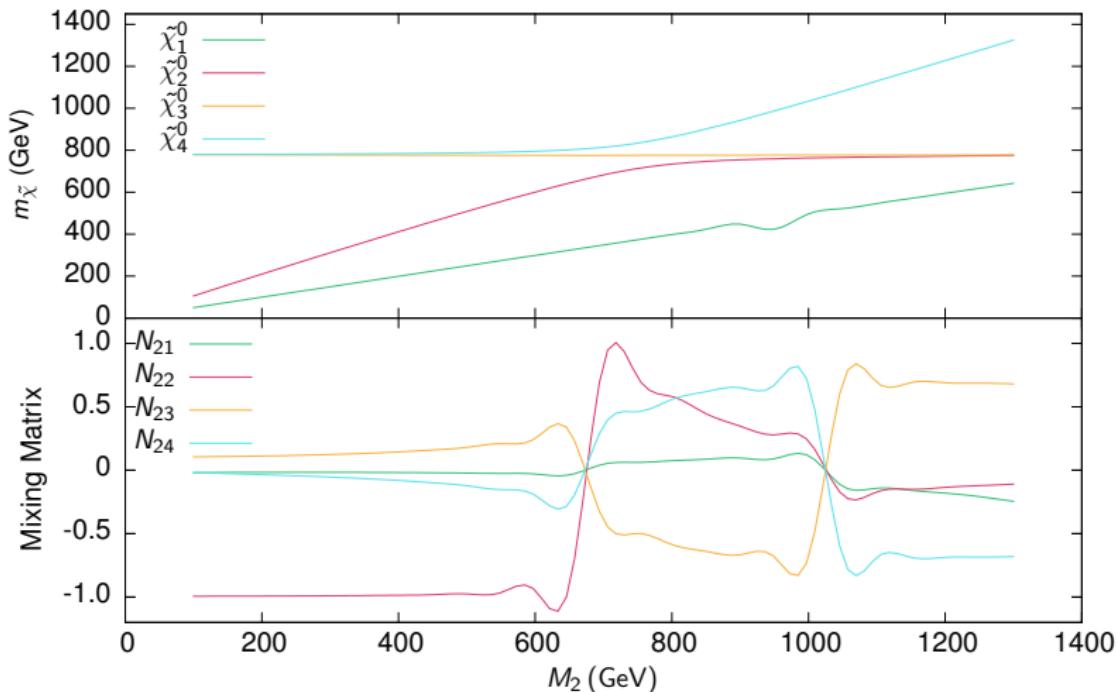


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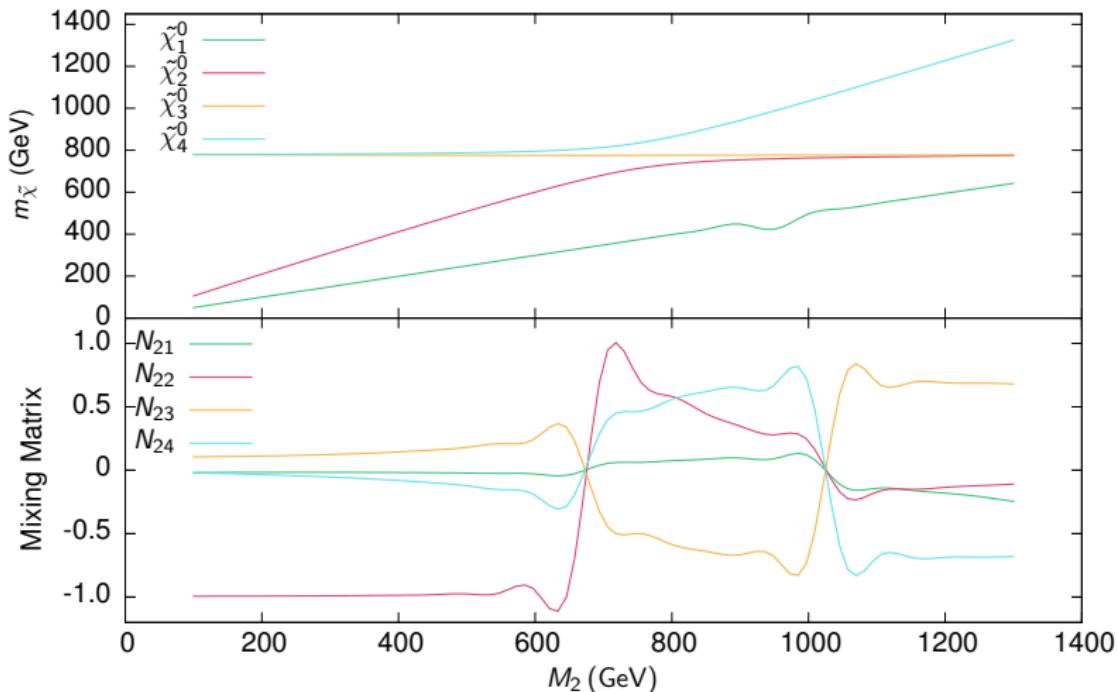


Cross sections will soon be observable at the LHC with  $\mathcal{L} = 100$   $\text{fb}^{-1}$ .

# Gaugino decomposition



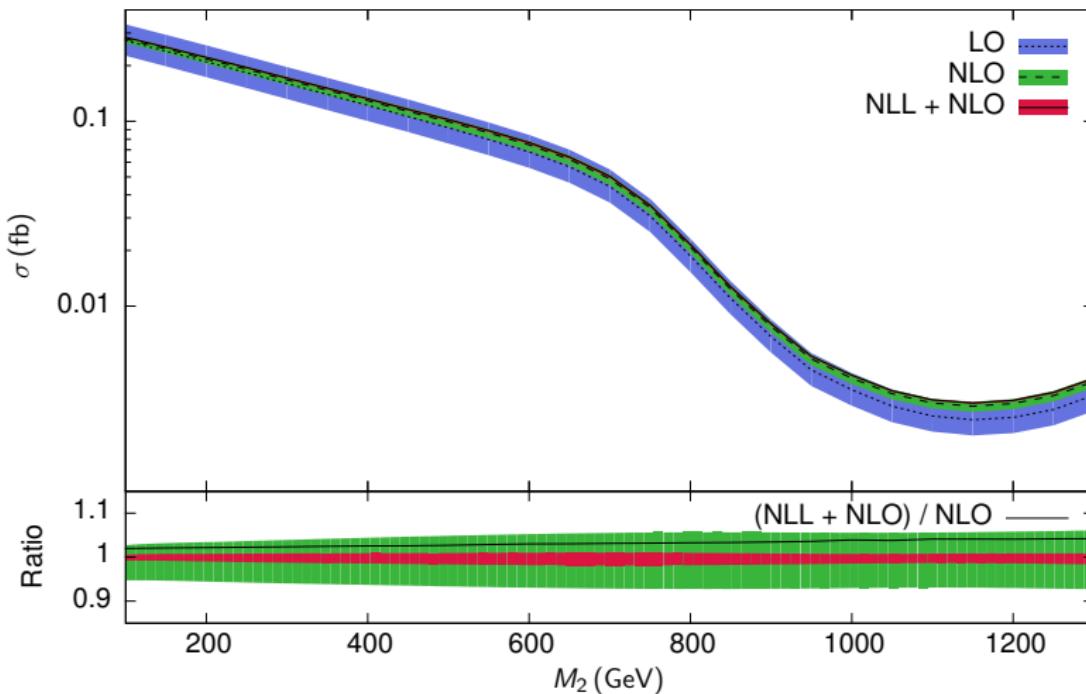
# Gaugino decomposition



Avoided crossings at  $M_2 = \mu = 773$  GeV (also for charginos).

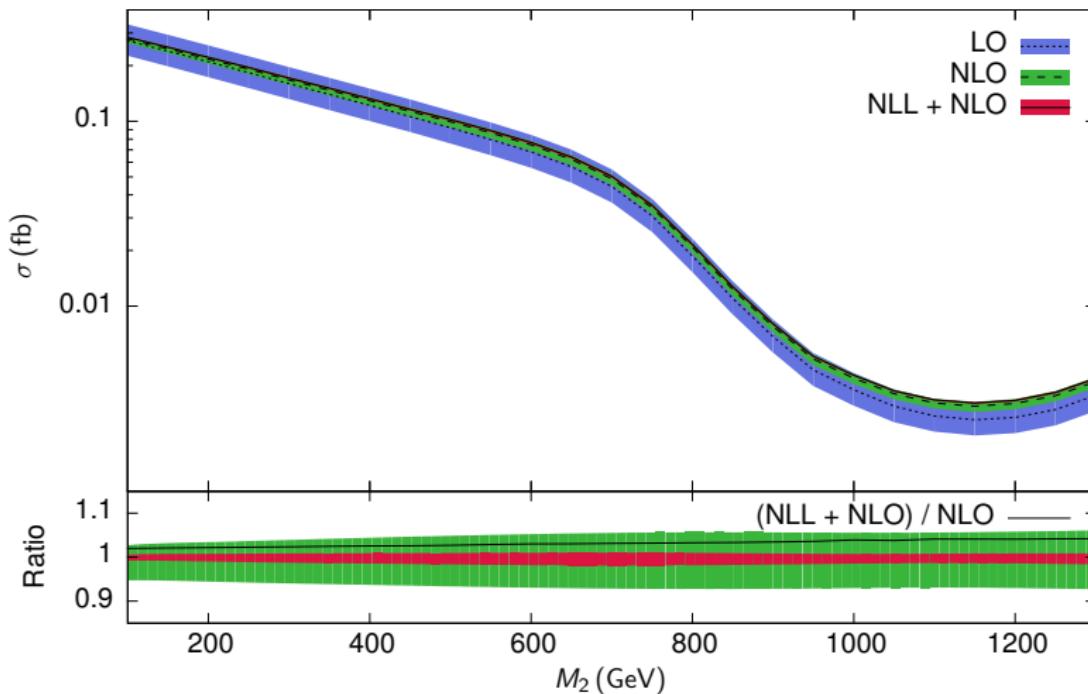
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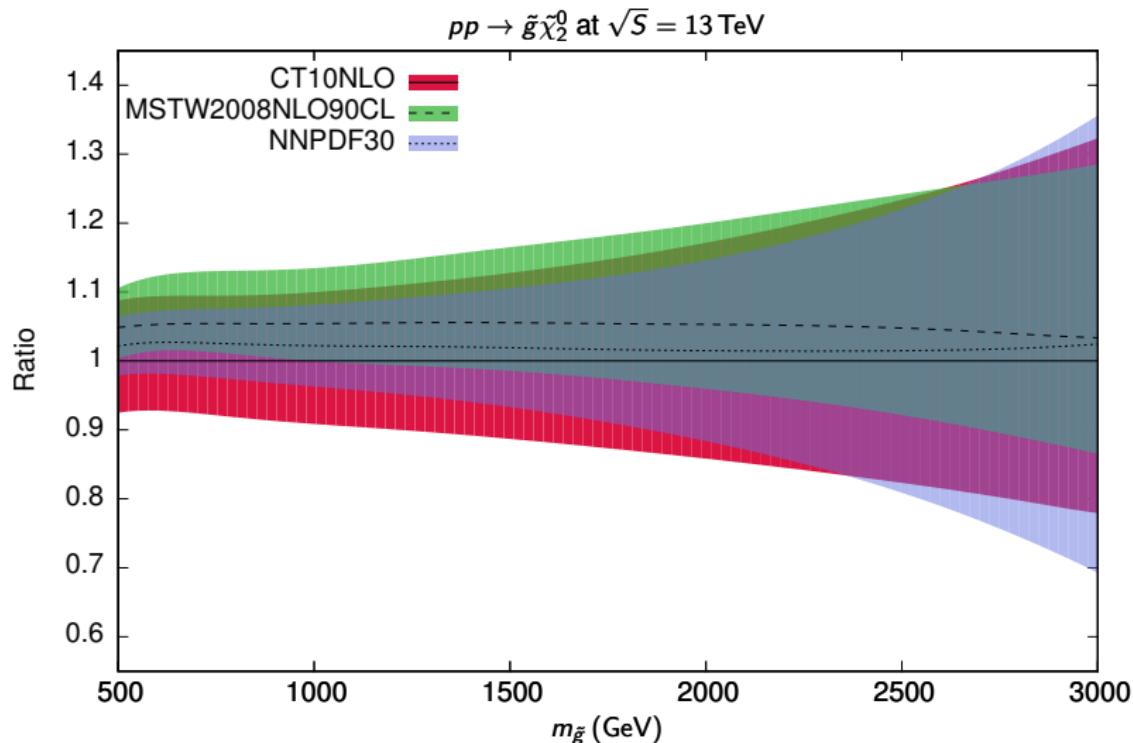
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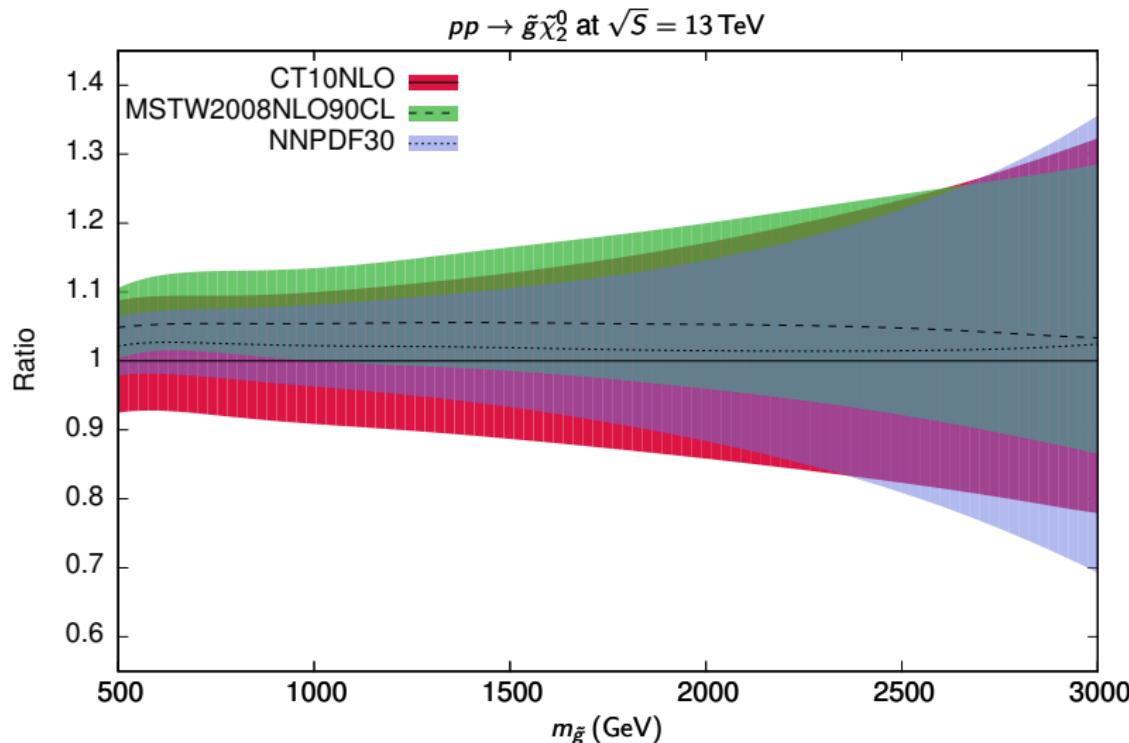


Beyond  $M_2 > \mu$ , decomposition changes more than the mass.

## Relative PDF uncertainties



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Large uncertainties at large  $x \rightarrow$  employ threshold improved PDFs.

# Conclusion

Motivation:

- Semi-weak process, will become relevant if gluinos are heavy
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Results:

- NLL increases NLO invariant mass distribution by up to 10%
- Total scale dependence reduced from 30% to 5%
- PDF uncertainty not reduced, needs threshold-improved PDFs