EFFECTS OF RGE’S ON FERMION OBSERVABLES IN SO(10) MODELS

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Davide Meloni
Dipartimento di Matematica e Fisica
Roma Tre
in GUT theories the Lagrangian is written at the very large scale, so are the Yukawas

**but masses and mixing are measured at the EW scale**

Extrapolation by several order of magnitude must be done

**Question:** what if an energy threshold in non-SUSY GUT is present between EW and $10^{16}$ GeV?
What we have to explain

**Masses at ElectroWeak scale**

- **leptons**
  - from MeV to GeV

- **quarks**
  - from MeV to 100 GeV

**Mixing at ElectroWeak scale**

For neutrinos:

\[
\sum_{i=1}^{3} m_i \leq 1 \text{ (eV)}
\]

completely different patterns
Type-I see-saw (both left and right-handed helicity states)

\[ \mathcal{L}_m = -Y_{ij} \bar{L}_i (\tilde{H} \nu_{Rj}) + \frac{1}{2} \bar{\nu}^c_{Ri} M_{ij} \nu_{Rj} \]

\[ m_\nu = -m_D^T M^{-1} m_D \]

order of magnitude estimate:

\[ m_\nu \approx 1 \text{eV} \quad m_D \approx 100 \text{ GeV} \quad \Rightarrow \quad M \approx 10^{13} \text{ GeV} \]

neutrino masses can shed light on the physics at a much higher scale!
SO(10) as an example

16:

if masses are given by the Higgs mechanism, we need SU(2)_{L,Higgs} doublets

SO(10) colored states not relevant here

\[10 = (1, \boxed{2}, 2) \oplus (6, 1, 1),\]
\[126 = (6, 1, 1) \oplus (\overline{10}, 3, 1) \oplus (10, 1, 3) \oplus (15, \boxed{2}, 2)\]
SO(10) as an example

\[ 10 = (1, 2, 2) \oplus (6, 1, 1), \]
\[ 126 = (6, 1, 1) \oplus (\overline{10}, 3, 1) \oplus (10, 1, 3) \oplus (15, 2, 2) \]

Yukawa couplings: 3x3 symmetric matrices
Masses in $SO(10)$

as usual, masses in terms of Yukawas and vevs

\[
\begin{align*}
\langle (1,2,2) \rangle &= \mathbf{k}_{u,d} \\
\langle (15,2,2) \rangle &= \mathbf{v}_{u,d} \\
\langle (10,1,3) \rangle &= \mathbf{v}_R
\end{align*}
\]

\[
\begin{align*}
M_u &= h k_u + f v_u, \\
M_d &= h k_d + f v_d, \\
M^D_\nu &= h k_u - 3 f v_u, \\
M^M_\nu &= f v_R
\end{align*}
\]

these numbers at the GUT scale have been obtained using SM beta functions
Diagrams contributing to the RGE’s

Some typical diagrams

Contributions to the gauge coupling renormalization constants

Renormalization of the Yukawa couplings

Corrections to the Higgs couplings

Cheng, Eichten, Li (1974)
Intermediate energy scales

It is well known that SM itself does not unify

Pati-Salam intermediate group: $SU(4) \times SU(2)_L \times SU(2)_R$

Unification achieved with a large energy threshold
Numerical strategy

• First, the values of the Yukawas at $M_{\text{GUT}}$ are randomly generated according to some prior distribution $\log$ priors in $[10^{-15}, 10^{-1}]$

Multinest Feroz & Hobson 2008

• Then, they are evolved down to $M_Z$ after solving the RGEs

• Next, at $M_Z$, the observables can then be constructed and compared to experimental data

• Finally, the procedure is repeated with new randomly sampled parameter values from a reduced parameter space and the result is given when convergence on the point with largest likelihood occurs
Result I

$$\chi^2 \approx 13$$

19 free parameters, 17 observables

main message:
effects of $M_1$ not negligible in the 10-20% range

see this also in the opposite direction:
observables at the EW do not give the same results at GUT

dashed: SM evolution
solid: intermediate scale taken into account

intermediate scale

kinks at $M_1$

EW scale
Result II

\[ \chi^2 \approx 13 \]

19 free parameters, 17 observables

difference in r is huge
(see-saw and MI coincide)
One conclusion

• Extrapolation of neutrino (and fermion) observables from the EW to the GUT scale and vice versa are substantially modified by intermediate energy thresholds

• This effect can be as large as 20% for certain observables
RGE’s: dividing the problem in two energy regimes

at the GUT scale

\[ \mathcal{L} = 16 ( h \ 10_H + f \ 126_H ) \ 16 \]

1- region between \( M_{\text{GUT}} \) and \( M_1 \)

\[ \Phi \equiv (1, 2, 2), \quad \Sigma \equiv (15, 2, 2), \quad \Delta_R \equiv (10, 1, 3) \]

\[ -\mathcal{L}_Y = \sum_{i,j} \left( Y_{F_{ij}}^{(10)} F^T_{L} \Phi F^T_{R} \Phi^j + Y_{F_{ij}}^{(126)} F^T_{L} \Sigma F^j_{R} + Y_{R_{ij}}^{(126)} F^T_{R} \Delta_R F^j_{R} + \text{h.c.} \right) \]

matching conditions at \( M_{\text{GUT}} \)

\[ \frac{1}{\sqrt{2}} Y_{F_{ij}}^{(10)}(M_{\text{GUT}}) \equiv h, \]

\[ \frac{1}{4\sqrt{2}} Y_{F_{ij}}^{(126)}(M_{\text{GUT}}) = \frac{1}{4} Y_{R_{ij}}^{(126)}(M_{\text{GUT}}) \equiv f, \]

for neutrinos: see-saw formula

\[ m_\nu(\mu) = M^T_D(\mu) \ M^{-1}_R(\mu) \ M_D(\mu) \]
RGE’s: dividing the problem in two energy regimes

2- region between $M_I$ and $M_Z$

\[-\mathcal{L}_Y = \sum_{i,j} \left( Y_{u_{ij}}^{(10)} \bar{q}_L \phi_1 u_R^i + Y_{u_{ij}}^{(126)} \bar{q}_L \phi_2 u_R^i + Y_{d_{ij}}^{(10)} \bar{q}_L \phi_3 d_R^i + Y_{d_{ij}}^{(126)} \bar{q}_L \phi_4 d_R^i \\
+ Y_{\nu_{ij}}^{(10)} \bar{\ell}_L \phi_1 N_R^j + Y_{\nu_{ij}}^{(126)} \bar{\ell}_L \phi_2 N_R^j + Y_{e_{ij}}^{(10)} \bar{\ell}_L \phi_3 e_R^j + Y_{e_{ij}}^{(126)} \bar{\ell}_L \phi_4 e_R^j + h.c. \right)\]

matching conditions at $M_I$

\[
Y_u^{(10)}(M_I) = Y_d^{(10)}(M_I) = Y_\nu^{(10)}(M_I) = Y_e^{(10)}(M_I) = Y_F^{(10)},
\]

\[
Y_u^{(126)}(M_I) = Y_d^{(126)}(M_I) = -\frac{1}{3} Y_\nu^{(126)}(M_I) = -\frac{1}{3} Y_e^{(126)}(M_I) = Y_F^{(126)}
\]

for neutrinos: effective operator

\[
m_\nu = \frac{1}{2} \sum_{a,b=1,2} k_{a,b}^{(a,b)} u_\alpha^* u_\beta^*
\]

$v_{a,b}$ are vevs of the light Higgses, $k^{(a,b)}$ are matrices in flavor space subjected to matching conditions at $M_I$
\[ 16\pi^2 \frac{dY_{F}^{(10)}}{dt} = \left( Y_{F}^{(10)} Y_{F}^{(10)\dagger} + \frac{15}{4} Y_{F}^{(126)} Y_{F}^{(126)\dagger} \right) Y_{F}^{(10)} + Y_{F}^{(10)} \left\{ Y_{F}^{(10)\dagger} Y_{F}^{(10)} + \frac{15}{4} \left( Y_{F}^{(126)\dagger} Y_{F}^{(126)} + 2 Y_{R}^{(126)^\dagger} Y_{R}^{(126)} \right) \right\} \]
\[ + 4 \text{tr} \left( Y_{F}^{(10)\dagger} Y_{F}^{(10)} \right) Y_{F}^{(10)} - \frac{9}{4} \left( 2g_{2L}^2 + g_{2R}^2 + 5g_{4C}^2 \right) Y_{F}^{(10)}, \]

\[ 16\pi^2 \frac{dY_{F}^{(126)}}{dt} = \left( Y_{F}^{(10)} Y_{F}^{(10)\dagger} + \frac{15}{4} Y_{F}^{(126)} Y_{F}^{(126)\dagger} \right) Y_{F}^{(126)} + Y_{F}^{(126)} \left\{ Y_{F}^{(10)\dagger} Y_{F}^{(10)} + \frac{15}{4} \left( Y_{F}^{(126)\dagger} Y_{F}^{(126)} + 2 Y_{R}^{(126)^\dagger} Y_{R}^{(126)} \right) \right\} \]
\[ + \text{tr} \left( Y_{F}^{(126)\dagger} Y_{F}^{(126)} \right) Y_{F}^{(126)} - \frac{9}{4} \left( 2g_{2L}^2 + g_{2R}^2 + 5g_{4C}^2 \right) Y_{F}^{(126)}, \]

\[ 16\pi^2 \frac{dY_{R}^{(126)}}{dt} = \left\{ Y_{F}^{(10)T} Y_{F}^{(10)^*} + \frac{15}{4} \left( Y_{F}^{(126)T} Y_{F}^{(126)^*} + 2 Y_{R}^{(126)T} Y_{R}^{(126)^*} \right) \right\} Y_{R}^{(126)} \]
\[ + Y_{R}^{(126)} \left\{ Y_{F}^{(10)\dagger} Y_{F}^{(10)} + \frac{15}{4} \left( Y_{F}^{(126)\dagger} Y_{F}^{(126)} + 2 Y_{R}^{(126)^\dagger} Y_{R}^{(126)} \right) \right\} \]
\[ + 2 \text{tr} \left( Y_{R}^{(126)^\dagger} Y_{R}^{(126)^*} \right) Y_{R}^{(126)} - \frac{9}{4} \left( 2g_{2R}^2 + 5g_{4C}^2 \right) Y_{R}^{(126)}. \]
Intermediate scale region: masses of neutrinos

\[ m_\nu = \frac{k^2_{uu}}{2} Y_F^{(10)} M_R^{-1} Y_F^{(10)} - \frac{3}{8} k_{uu} v_u \left\{ Y_F^{(126)} M_R^{-1} Y_F^{(10)} + Y_F^{(10)} M_R^{-1} Y_F^{(126)} \right\} + \frac{9}{32} v_{uu}^2 Y_F^{(126)} M_R^{-1} Y_F^{(126)} \]

\[ \kappa(M_1) \equiv Y_F^{(10)} (M_1) M_R^{-1} (M_1) Y_F^{(10)} (M_1) \\
- \frac{3}{4} \frac{v_u}{k_{uu}} \left\{ Y_F^{(126)} (M_1) M_R^{-1} (M_1) Y_F^{(10)} (M_1) + Y_F^{(10)} (M_1) M_R^{-1} (M_1) Y_F^{(126)} (M_1) \right\} \\
+ \frac{9}{16} \frac{v_{uu}^2}{k^2_u} Y_F^{(126)} (M_1) M_R^{-1} (M_1) Y_F^{(126)} (M_1) \]
\[ -\mathcal{L}_Y = \sum_{i,j} \left( Y_{uij}^{(10)} \bar{q}_L \phi_1 u_R^i + Y_{uij}^{(126)} \bar{q}_L \phi_2 u_R^i + Y_{dij}^{(10)} \bar{q}_L \phi_3 d_R^j + Y_{dij}^{(126)} \bar{q}_L \phi_4 d_R^j \right) \\
+ Y_{\nu ij}^{(10)} \bar{\ell}_L \phi_1 N_R^j + Y_{\nu ij}^{(126)} \bar{\ell}_L \phi_2 N_R^j + Y_{eij}^{(10)} \bar{\ell}_L \phi_3 e_R^j + Y_{eij}^{(126)} \bar{\ell}_L \phi_4 e_R^j + \text{h.c.} \right) \]

\[ v_1 = k_u, \quad 4v_2 = v_u, \quad v_3 = k_d, \quad 4v_4 = v_d \]

\[ M_R = \frac{1}{4} \left\langle \Delta R \right\rangle Y_R^{(126)} \]

\[ \kappa_{(1,1)}^{(1,1)} = Y_F^{T(10)} M_R^{-1} Y_F^{(10)}, \]
\[ \kappa_{(1,2)}^{(1,2)} = -3 Y_F^{T(126)} M_R^{-1} Y_F^{(10)}, \]
\[ \kappa_{(2,1)}^{(2,1)} = -3 Y_F^{T(10)} M_R^{-1} Y_F^{(126)}, \]
\[ \kappa_{(2,2)}^{(2,2)} = 9 Y_F^{T(126)} M_R^{-1} Y_F^{(126)}. \]
basic idea: at some large energy scale, particles feel a single force

idealized situation

here is where the SM is unified

theory of everything (forget for the moment, we have problems at much smaller scales)

ν masses pointing to Grand Unified Theories

not far away from the see-saw estimate!
**masses and Grand Unified Theories**

difficult to fit everything but not impossible…

Altarelli, Meloni (2013)
Dueck, Rodejohann (2013)

![](image)

and several interesting predictions