EFFECTS OF RGE'S ON FERMION OBSERVABLES IN SO(10) MODELS

Based on 1409.3703 and 1612.07973
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EPS2017-Venezia

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Main point

in GUT theories the Lagrangian is written at the very large scale, so are the Yukawas

but masses and mixing are measured at the EW scale

$\Downarrow$

Extrapolation by several order of magnitude must be done

Question: what if an energy threshold in non-SUSY GUT is present between EW and 10^{16} GeV?

What we have to explain

Masses at ElectroWeak scale

leptons

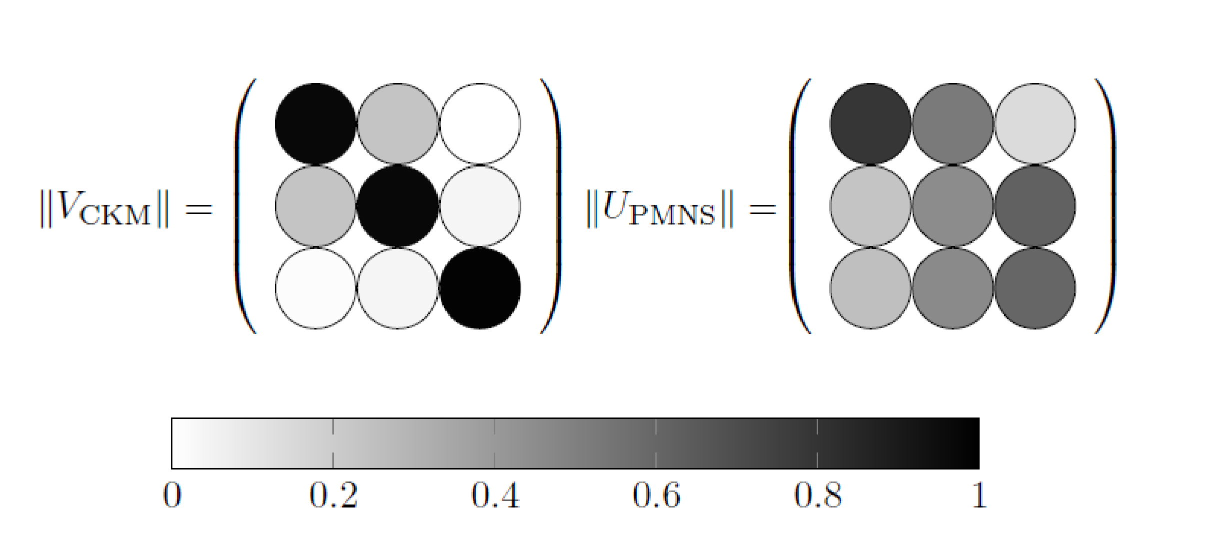
from MeV to GeV

quarks

from MeV to 100 GeV

for neutrinos: $\sum_{i=1}^{3} m_i \leq 1(\mathrm{e}V)$

Mixing at ElectroWeak scale



completely different patterns

for neutrinos

Type-I see-saw (both left and right-handed helicity states)

$$\mathcal{L}_m = -Y_{ij}\bar{L}_i(\tilde{H}\nu_{Rj}) + \frac{1}{2}\bar{\nu}^c_{Ri}M_{ij}\nu_{Rj}$$

m_D

$$\boxed{m_\nu = -m_D^T M^{-1} m_D}$$

order of magnitude estimate:

$m_\nu \approx 1 eV \qquad m_D \approx 100\ GeV \qquad \Longrightarrow \qquad M \approx 10^{13}\,GeV$

neutrino masses can shed light on the physics at a much higher scale!

SO(10) as an example

16 :

$u_r : \{-++\ +-\}$	$d_r : \{-++\ -+\}$	$u_r^c : \{+--\ ++\}$	$d_r^c : \{+--\ --\}$
$u_b : \{+-+\ +-\}$	$d_b : \{+-+\ -+\}$	$u_b^c : \{-+-\ ++\}$	$d_b^c : \{-+-\ --\}$
$u_g : \{++-\ +-\}$	$d_g : \{++-\ -+\}$	$u_g^c : \{--+\ ++\}$	$d_g^c : \{--+\ --\}$
$\nu : \{---\ +-\}$	$e : \{---\ -+\}$	$\nu^c : \{+++\ ++\}$	$e^c : \{+++\ --\}$

right-handed neutrino

if masses are given by the Higgs mechanism, we need $SU(2)_L$ Higgs doublets

SO(10)

colored states
not relevant here

$$\mathbf{10} = (1, \boxed{2}, 2) \oplus (6, 1, 1)\,,$$
$$\mathbf{126} = (6, 1, 1) \oplus (\overline{10}, 3, 1) \oplus (10, 1, 3) \oplus (15, \boxed{2}, 2)$$

SU(4) x $SU(2)_L$ x $SU(2)_R$

SO(10) as an example

$u_r: \{-++\ +-\}$	$d_r: \{-++\ -+\}$	$u^c_r: \{+--\ ++\}$	$d^c_r: \{+--\ --\}$
$u_b: \{+-+\ +-\}$	$d_b: \{+-+\ -+\}$	$u^c_b: \{-+-\ ++\}$	$d^c_b: \{-+-\ --\}$
$u_g: \{++-\ +-\}$	$d_g: \{++-\ -+\}$	$u^c_g: \{--+\ ++\}$	$d^c_g: \{--+\ --\}$
$\nu: \{---\ +-\}$	$e: \{---\ -+\}$	$\nu^c: \{+++\ ++\}$	$e^c: \{+++\ --\}$

$$\mathbf{10} = (1,2,2) \oplus (6,1,1)\,,$$
$$\mathbf{126} = (6,1,1) \oplus (\overline{10},3,1) \oplus (10,1,3) \oplus (15,2,2)$$

$$\mathcal{L} = \mathbf{16}\left(h\,\mathbf{10}_H + f\,\overline{\mathbf{126}}_H\right)\mathbf{16}$$

Yukawa couplings: 3x3 symmetric matrices

Masses in SO(10)

as usual, masses in terms of Yukawas and vevs

$< (1,2,2) > = \mathbf{k}_{u,d}$

$< (15,2,2) > = \mathbf{v}_{u,d}$

$< (10,1,3) > = \mathbf{v}_R$

$$M_u = h\,k_u + f\,v_u, \qquad M_d = h\,k_d + f\,v_d$$

$$\boxed{M_\nu^D = h\,k_u - 3\,f\,v_u,} \qquad M_l = h\,k_d - 3\,f\,v_d, \qquad \boxed{M_\nu^M = f\,v_R}$$

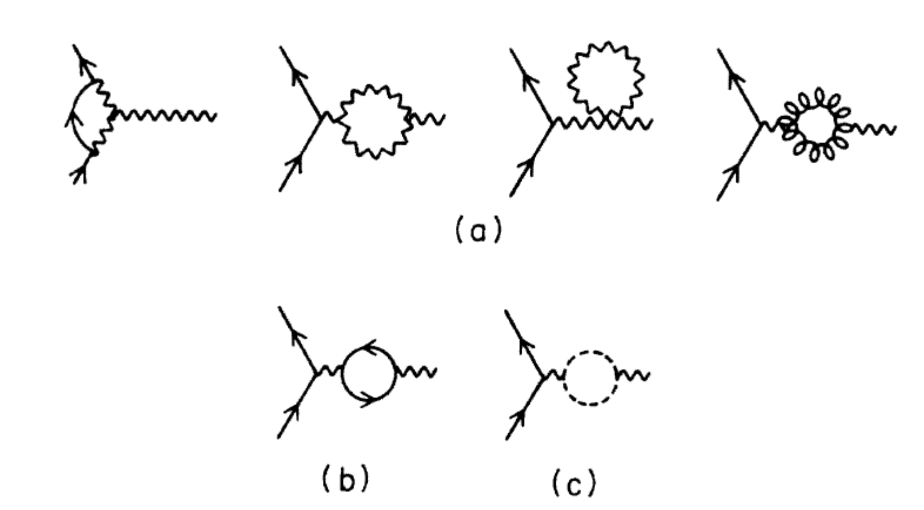
m_u (MeV)	0.495 ± 0.185	$\lvert V_{us}\rvert$	0.2254 ± 0.0006
m_d (MeV)	1.155 ± 0.495	$\lvert V_{cb}\rvert$	0.04194 ± 0.0006
m_s (MeV)	22.0 ± 7.0	$\lvert V_{ub}\rvert$	0.00369 ± 0.00013
m_c (GeV)	0.235 ± 0.035	J	$(3.16 \pm 0.1) \times 10^{-5}$
m_b (GeV)	1.00 ± 0.04	$\sin^2\theta^l_{12}$	0.308 ± 0.017
m_t (GeV)	74.15 ± 3.85	$\sin^2\theta^l_{23}$	0.3875 ± 0.0225
r	0.031 ± 0.001	$\sin^2\theta^l_{13}$	0.0241 ± 0.0025

$\Longrightarrow$ these numbers at the GUT scale have been obtained using SM beta functions

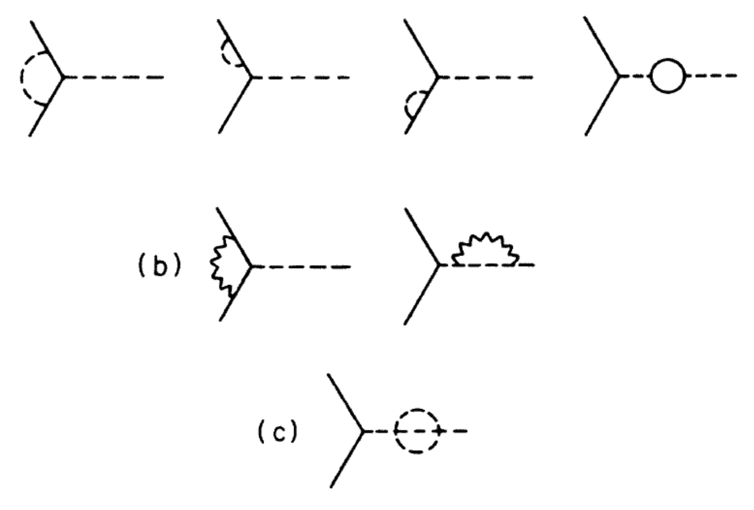
Diagrams contributing to the RGE's

Some typical diagrams

Contributions to the gauge coupling renormalization constants



Renormalization of the Yukawa couplings



Cheng,Eichten,Li (1974)

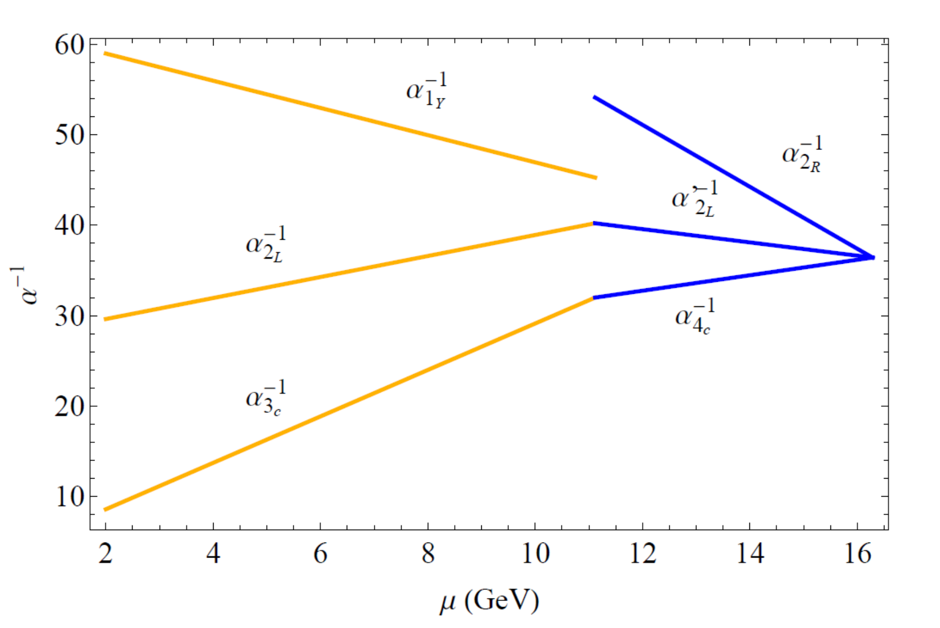
- corrections to the Higgs couplings

Intermediate energy scales

It is well known that SM itself does not unify

Pati-Salam intermediate group:
$SU(4) \times SU(2)_L \times SU(2)_R$

$$\mathrm{SO}(10) \xrightarrow{M_{\mathrm{GUT}} - 210_{\mathrm{H}}} 4_{\mathrm{C}}\, 2_{\mathrm{L}}\, 2_{\mathrm{R}} \xrightarrow{M_{\mathrm{I}} - 126_{\mathrm{H}}} 3_{\mathrm{C}}\, 2_{\mathrm{L}}\, 1_{\mathrm{Y}} \xrightarrow{M_{\mathrm{Z}} - 10_{\mathrm{H}}} 3_{\mathrm{C}}\, 1_{\mathrm{Y}}$$



$$M_{\mathrm{I}} = (1.5 \pm 0.2) \cdot 10^{12}\,\mathrm{GeV}\,, \qquad M_{\mathrm{GUT}} = (1.7 \pm 0.6) \cdot 10^{15}\,\mathrm{GeV}$$

Unification achieved with a large energy threshold ☺

Numerical strategy

- First, the values of the Yukawas at M_{GUT} are randomly generated according to some prior distribution $\longrightarrow$ log priors in $[10^{-15}, 10^{-1}]$

  Multinest Feroz&Hobson2008

- Then, they are evolved down to M_Z after solving the RGEs

- Next, at M_Z, the observables can then be constructed and compared to experimental data

- Finally, the procedure is repeated with new randomly sampled parameter values from a reduced parameter space and the result is given when convergence on the point with largest likelihood occurs

Result I

$\chi^2 \approx 13$

19 free parameters, 17 observables

solid: intermediate scale taken into account
dashed: SM evolution

main message:
effects of M_I not negligible
in the 10-20% range

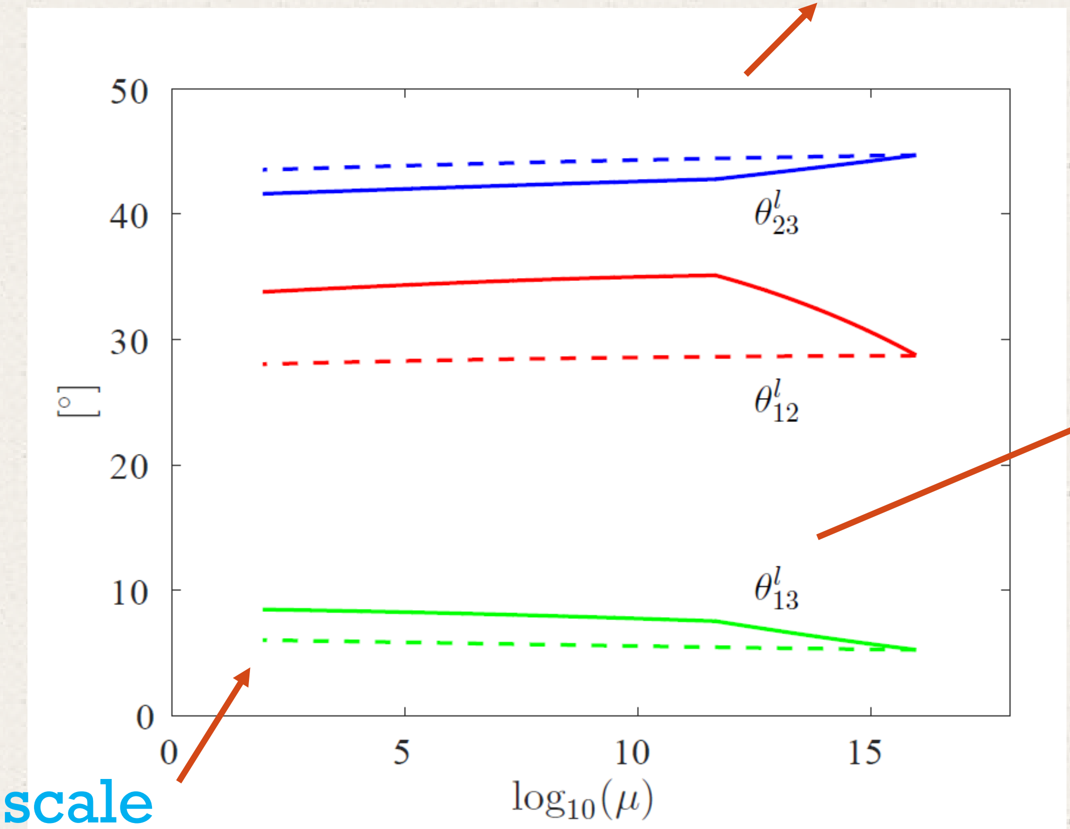
⇩

see this also in the opposite direction:
observables at the EW do not give the same results at GUT

intermediate scale

kinks at M_I

EW scale



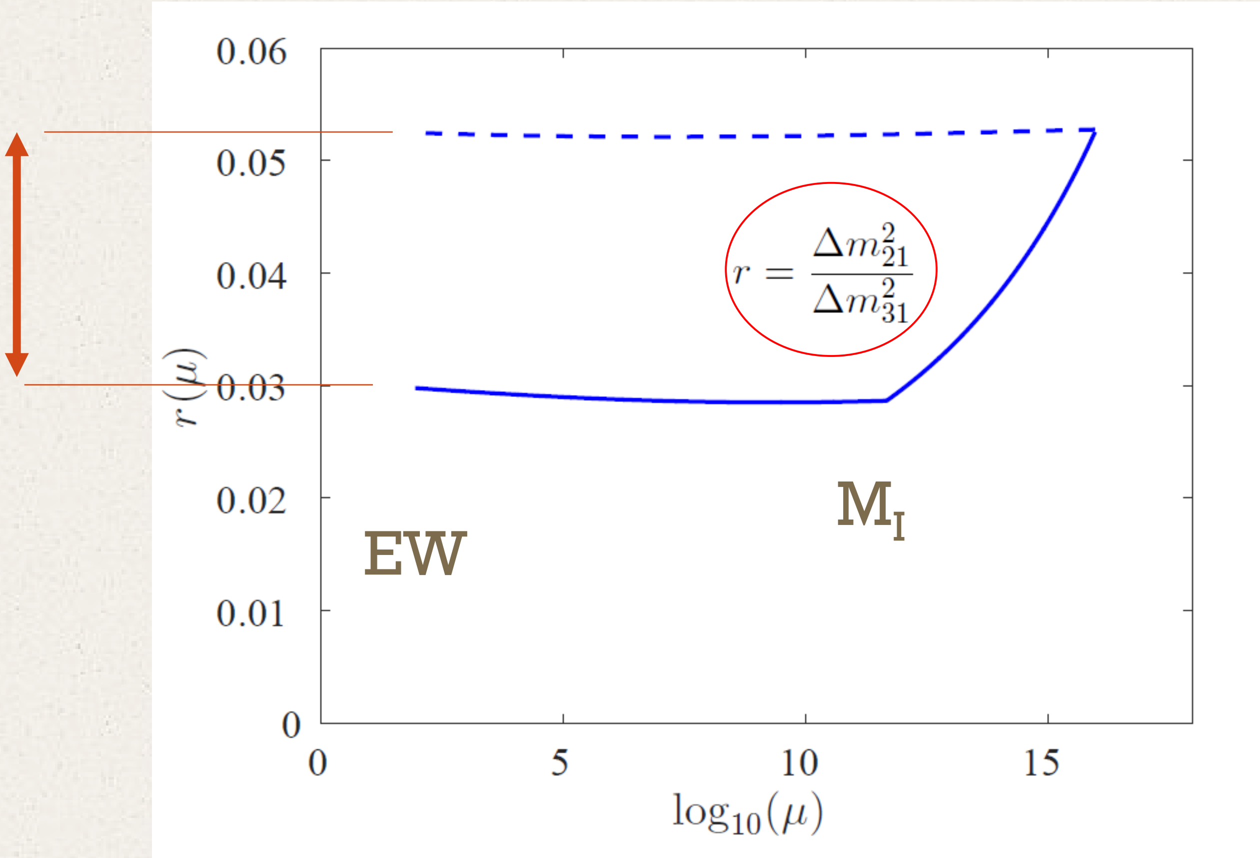
Result II

$\chi^2 \approx 13$

19 free parameters, 17 observables

difference in r is huge

(see-saw and MI coincide)



One conclusion

- Extrapolation of neutrino (and fermion) observables from the EW to the GUT scale and viceversa are substantially modified by intermediate energy thresholds
- This effect can be as large as 20% for certain observables

RGE's: dividing the problem in two energy regimes

at the GUT scale

$$\mathcal{L} = \mathbf{16}\left(h\,\mathbf{10}_H + f\,\overline{\mathbf{126}}_H\right)\mathbf{16}$$

1- region between M_{GUT} and M_I

$$\Phi \equiv (1,2,2)\,, \qquad \Sigma \equiv (15,2,2)\,, \qquad \overline{\Delta_R} \equiv (10,1,3)$$

$$-\mathcal{L}_{\mathrm{Y}} \;=\; \sum_{i,j}\left(Y^{(10)}_{F\,ij}\,F_{\mathrm{L}}^{iT}\Phi F_{\mathrm{R}}^{j} + Y^{(126)}_{F\,ij}\,F_{\mathrm{L}}^{iT}\Sigma F_{\mathrm{R}}^{j} + Y^{(126)}_{R\,ij}\,F_{\mathrm{R}}^{iT}\overline{\Delta_R}F_{\mathrm{R}}^{j} + \mathrm{h.c.}\right)$$

matching conditions at M_{GUT}

$$\frac{1}{\sqrt{2}}Y_F^{(10)}(M_{\mathrm{GUT}}) \equiv h\,,$$
$$\frac{1}{4\sqrt{2}}Y_F^{(126)}(M_{\mathrm{GUT}}) = \frac{1}{4}Y_R^{(126)}(M_{\mathrm{GUT}}) \equiv f\,,$$

for neutrinos: see-saw formula

$$m_\nu(\mu) = M_D^T(\mu)\,M_R^{-1}(\mu)\,M_D(\mu)$$

RGE's: dividing the problem in two energy regimes

2- region between M_I and M_Z

$$
\begin{aligned}
-\mathcal{L}_Y = & \sum_{i,j}\Big(Y^{(10)}_{u\,ij}\,\overline{q^i_{\rm L}}\,\widetilde{\phi}_1 u^j_{\rm R} + Y^{(126)}_{u\,ij}\,\overline{q^i_{\rm L}}\,\widetilde{\phi}_2 u^j_{\rm R} + Y^{(10)}_{d\,ij}\,\overline{q^i_{\rm L}}\,\phi_3 d^j_{\rm R} + Y^{(126)}_{d\,ij}\,\overline{q^i_{\rm L}}\,\phi_4 d^j_{\rm R} \\
& + Y^{(10)}_{\nu\,ij}\,\overline{\ell^i_{\rm L}}\,\widetilde{\phi}_1 N^j_{\rm R} + Y^{(126)}_{\nu\,ij}\,\overline{\ell^i_L}\,\widetilde{\phi}_2 N^j_R + Y^{(10)}_{e\,ij}\,\overline{\ell^i_{\rm L}}\,\phi_3 e^j_{\rm R} + Y^{(126)}_{e\,ij}\,\overline{\ell^i_{\rm L}}\,\phi_4 e^j_{\rm R} + \text{h.c.}\Big)
\end{aligned}
$$

matching conditions at M_I

$$
\begin{aligned}
& Y_u^{(10)}(M_{\rm I}) = Y_d^{(10)}(M_{\rm I}) = Y_\nu^{(10)}(M_{\rm I}) = Y_e^{(10)}(M_{\rm I}) \equiv Y_F^{(10)}\,, \\
& Y_u^{(126)}(M_{\rm I}) = Y_d^{(126)}(M_{\rm I}) = -\frac{1}{3}Y_\nu^{(126)}(M_{\rm I}) = -\frac{1}{3}Y_e^{(126)}(M_{\rm I}) \equiv Y_F^{(126)}
\end{aligned}
$$

for neutrinos: effective operator

$$
m_\nu = \frac{1}{2}\sum_{a,b=1,2} \kappa^{(a,b)} v_a^* v_b^*
$$

$v_{a,b}$ are vevs of the light Higgses, $k^{(a,b)}$ are matrices in flavor space subjected to matching conditions at M_I

RGE's

Intermediate scale region

$$
\begin{aligned}
16\pi^2 \frac{\mathrm{d}Y_F^{(10)}}{\mathrm{d}t} &= \left(Y_F^{(10)} Y_F^{(10)\dagger} + \frac{15}{4} Y_F^{(126)} Y_F^{(126)\dagger}\right) Y_F^{(10)} \\
&+ Y_F^{(10)} \left\{ Y_F^{(10)\dagger} Y_F^{(10)} + \frac{15}{4} \left(Y_F^{(126)\dagger} Y_F^{(126)} + 2 Y_R^{(126)*} Y_R^{(126)} \right) \right\} \\
&+ 4\,\mathrm{tr}\left(Y_F^{(10)} Y_F^{(10)\dagger} \right) Y_F^{(10)} - \frac{9}{4} \left(g_{2L}^2 + g_{2R}^2 + 5 g_{4C}^2 \right) Y_F^{(10)} , \\
16\pi^2 \frac{\mathrm{d}Y_F^{(126)}}{\mathrm{d}t} &= \left(Y_F^{(10)} Y_F^{(10)\dagger} + \frac{15}{4} Y_F^{(126)} Y_F^{(126)\dagger}\right) Y_F^{(126)} \\
&+ Y_F^{(126)} \left\{ Y_F^{(10)\dagger} Y_F^{(10)} + \frac{15}{4} \left(Y_F^{(126)\dagger} Y_F^{(126)} + 2 Y_R^{(126)*} Y_R^{(126)} \right) \right\} \\
&+ \mathrm{tr}\left(Y_F^{(126)} Y_F^{(126)\dagger} \right) Y_F^{(126)} - \frac{9}{4} \left(g_{2L}^2 + g_{2R}^2 + 5 g_{4C}^2 \right) Y_F^{(126)} , \\
16\pi^2 \frac{\mathrm{d}Y_R^{(126)}}{\mathrm{d}t} &= \left\{ Y_F^{(10)T} Y_F^{(10)*} + \frac{15}{4} \left(Y_F^{(126)T} Y_F^{(126)*} + 2 Y_R^{(126)} Y_R^{(126)*} \right) \right\} Y_R^{(126)} \\
&+ Y_R^{(126)} \left\{ Y_F^{(10)\dagger} Y_F^{(10)} + \frac{15}{4} \left(Y_F^{(126)\dagger} Y_F^{(126)} + 2 Y_R^{(126)*} Y_R^{(126)} \right) \right\}
\end{aligned}
$$

$$
+ 2\,\mathrm{tr}\left(Y_R^{(126)} Y_R^{(126)*} \right) Y_R^{(126)} - \frac{9}{4} \left(2 g_{2R}^2 + 5 g_{4C}^2 \right) Y_R^{(126)}
$$

RGE's

Intermediate scale region: masses of neutrinos

$$m_\nu = \frac{k_u^2}{2} Y_F^{(10)^T} M_R^{-1} Y_F^{(10)} - \frac{3}{8} k_u v_u \left\{ Y_F^{(126)^T} M_R^{-1} Y_F^{(10)} + Y_F^{(10)^T} M_R^{-1} Y_F^{(126)} \right\} + \frac{9}{32} v_u^2 Y_F^{(126)^T} M_R^{-1} Y_F^{(126)}$$

$$\begin{aligned} \kappa(M_{\mathrm{I}}) \equiv\ & Y_F^{(10)^T}(M_{\mathrm{I}})\, M_R^{-1}(M_{\mathrm{I}})\, Y_F^{(10)}(M_{\mathrm{I}}) \\ & - \frac{3}{4} \frac{v_u}{k_u} \left\{ Y_F^{(126)^T}(M_{\mathrm{I}})\, M_R^{-1}(M_{\mathrm{I}})\, Y_F^{(10)}(M_{\mathrm{I}}) + Y_F^{(10)^T}(M_{\mathrm{I}})\, M_R^{-1}(M_{\mathrm{I}})\, Y_F^{(126)}(M_{\mathrm{I}}) \right\} \\ & + \frac{9}{16} \frac{v_u^2}{k_u^2} Y_F^{(126)^T}(M_{\mathrm{I}})\, M_R^{-1}(M_{\mathrm{I}})\, Y_F^{(126)}(M_{\mathrm{I}}) \end{aligned}$$

ν masses

$$-\mathcal{L}_Y = \sum_{i,j}\Big(Y^{(10)}_{u\,ij}\,\overline{q^i_{\rm L}}\,\widetilde{\phi}_1 u^j_{\rm R} + Y^{(126)}_{u\,ij}\,\overline{q^i_{\rm L}}\,\widetilde{\phi}_2 u^j_{\rm R} + Y^{(10)}_{d\,ij}\,\overline{q^i_{\rm L}}\,\phi_3 d^j_{\rm R} + Y^{(126)}_{d\,ij}\,\overline{q^i_{\rm L}}\,\phi_4 d^j_{\rm R}$$
$$+\; Y^{(10)}_{\nu\,ij}\,\overline{\ell^i_{\rm L}}\,\widetilde{\phi}_1 N^j_R + Y^{(126)}_{\nu\,ij}\,\overline{\ell^i_{\rm L}}\,\widetilde{\phi}_2 N^j_R + Y^{(10)}_{e\,ij}\,\overline{\ell^i_{\rm L}}\,\phi_3 e^j_{\rm R} + Y^{(126)}_{e\,ij}\,\overline{\ell^i_{\rm L}}\,\phi_4 e^j_{\rm R} + \text{h.c.}\Big)$$

$$v_1 = k_u\,, \qquad 4v_2 = v_u\,, \qquad v_3 = k_d\,, \qquad 4v_4 = v_d$$

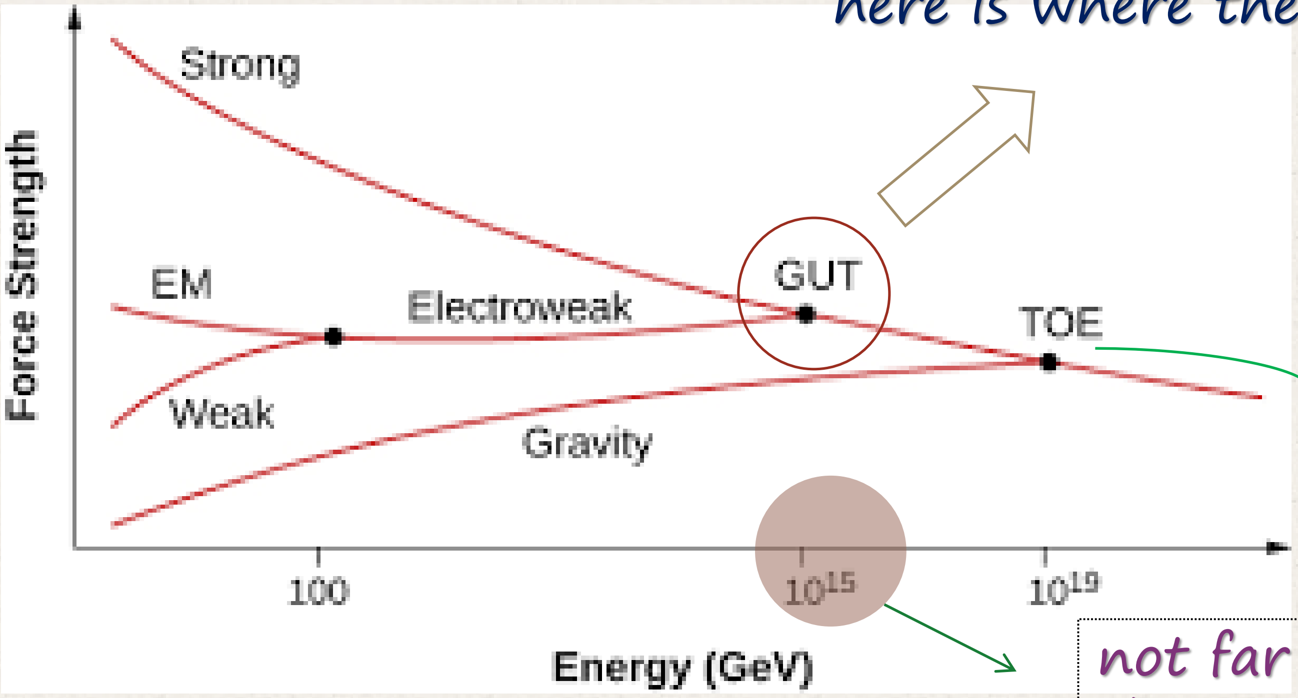
$$M_R = \frac{1}{4}\langle\overline{\Delta}_R\rangle\, Y_R^{(126)}$$

$$\begin{aligned}
\kappa^{(1,1)} &= Y_F^{T\,(10)} M_{\rm R}^{-1} Y_F^{(10)}\,, \\
\kappa^{(1,2)} &= -3Y_F^{T\,(126)} M_{\rm R}^{-1} Y_F^{(10)}\,, \\
\kappa^{(2,1)} &= -3Y_F^{T\,(10)} M_{\rm R}^{-1} Y_F^{(126)}\,, \\
\kappa^{(2,2)} &= 9Y_F^{T\,(126)} M_{\rm R}^{-1} Y_F^{(126)}\,.
\end{aligned}$$

ν masses pointing to Grand Unified Theories

basic idea: at some large energy scale, particles feel a single force

idealized situation



here is where the SM is unified

theory of everything (forget for the moment, we have problems at much smaller scales)

not far away from the see-saw estimate !

masses and Grand Unified Theories

difficult to fit everything but not impossible…

Altarelli, Meloni (2013)
Dueck, Rodejohann (2013)

obs.	fit	pull	obs.	fit	pull
m_u(MeV)	0.49	0.03	$\lvert V_{us}\rvert$	0.225	0.038
m_d(MeV)	0.78	0.75	$\lvert V_{cb}\rvert$	0.042	−0.208
m_s(MeV)	32.5	−1.50	$\lvert V_{ub}\rvert$	0.0038	−0.659
m_c(GeV)	0.287	−1.49	J	3.1×10^{-5}	0.589
m_b(GeV)	1.11	−2.77	$\sin^2\theta^l_{12}$	0.318	0.611
m_t(GeV)	71.4	0.70	$\sin^2\theta^l_{23}$	0.353	−1.548
r	0.031	0.10	$\sin^2\theta^l_{13}$	0.0222	−0.758
η_B	5.699×10^{-10}	−0.001			

and several interesting predictions

light ν *masses* (eV)	*heavy* ν *masses* $(10^{11}\ GeV)$	*phases* $(°)$	m_{ee} (eV)
.0046	1.00	$\delta = 88.6$	5×10^{-4}
.0098	1.09	$\phi_1 = -33.2$	
.0504	21.4	$\phi_2 = 15.7$	