Decay Constants of the Heavy–Light Mesons $D^{(*)}$ and $B^{(*)}$: Isospin Breaking

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Borel QCD sum rules in local-duality limit

QCD sum rules are relations between observable properties of hadron states, such as masses or leptonic decay constants, and the fundamental parameters of quantum chromodynamics (QCD), the quantum field theory of the strong interactions, derived from correlators of adequate interpolating operators by insertion of a complete set of hadronic states, on the one hand, and use of the operator product expansion, on the other hand [1]. Aiming for the features of ground-state hadrons, application of Borel transformations serves to remove subtraction terms and to suppress the contributions of hadronic excitations.

Our ignorance about higher hadron states may be swept under the carpet by invoking quark–hadron duality by assuming QCD and hadron contributions to mutually cancel beyond effective thresholds. The accuracy [2] of emerging predictions is considerably raised if taking into account the dependence [3] of such thresholds on the parameters introduced by the Borel transformations.

This advanced QCD sum-rule framework prompted us to revisit the leptonic decay constants of heavy mesons [4], specifically, their relative magnitude [5]. Recently, we embarked on the study of isospin breaking induced in the decay constants of heavy–light mesons by the up-down quark-mass difference [6,7].

Things simplify considerably in the special case of a vanishing Borel variable (equivalent to its inverse, the Borel mass variable, becoming infinitely large), the "local-duality" limit: The QCD sum rule for the leptonic decay constant, f_{H_q}, of any meson H_q composed of a heavy quark $Q = b, c$ of mass m_Q and a light quark $q = u, d, s$ of mass m_q may be expressed by a dispersion integral,

$$f_{H_q}^2 = \int\limits_{(m_Q+m_q)^2}^{s_{\rm eff}(m_Q,m_q)} {\rm d}s\,\rho(s, m_Q, m_q, \alpha_{\rm s}) \ ,$$

over a spectral density $\rho(s, m_Q, m_q, \alpha_{\rm s})$ arising from QCD perturbatively in form of a series expansion in powers of $\alpha_{\rm s}$, the strong fine-structure constant; its effective threshold $s_{\rm eff}(m_Q, m_q)$ subsumes all the nonperturbative effects. As an observable, any decay constant must not depend on a renormalization scale, generically denoted by μ; inevitable truncations of perturbative series, however, entail a μ dependence of spectral densities and effective thresholds.

We derive the leptonic decay constants f_P and f_V of pseudoscalar mesons P and vector mesons V of masses M_P and M_V, resp., defined by these mesons' interpolating axialvector operators $A_\mu(x)$ and vector operators $V_\mu(x)$, resp.,

$$A_\mu(x) = \bar q(x)\,\gamma_\mu\,\gamma_5\,Q(x) \ , \qquad \langle 0|\,A_\mu(0)\,|P(p)\rangle = {\rm i}\,f_P\,p_\mu \ ,$$
$$V_\mu(x) = \bar q(x)\,\gamma_\mu\,Q(x) \ , \qquad \langle 0|\,V_\mu(0)\,|V(p)\rangle = f_V\,M_V\,\varepsilon_\mu(p) \ ,$$

from the pseudoscalar- and vector-meson ground-state pole contributions to the relevant Lorentz structures in the chosen currents' two-point correlators:

$$\int {\rm d}^4x\,\exp({\rm i}\,p\,x)\,\langle T(A_\mu(x)\,A_\nu^\dagger(0))\rangle \sim p_\mu\,p_\nu\,\frac{f_P^2}{M_P^2 - p^2} + \cdots \ ,$$
$$\int {\rm d}^4x\,\exp({\rm i}\,p\,x)\,\langle T(V_\mu(x)\,V_\nu^\dagger(0))\rangle \sim (g_{\mu\nu}\,M_V^2 - p_\mu\,p_\nu)\,\frac{f_V^2}{M_V^2 - p^2} + \cdots \ ;$$

we adopt the latters' spectral densities up to $O(\alpha_{\rm s}\,m_q^1)$ and $O(\alpha_{\rm s}^2\,m_q^0)$ [8–10].

Optimization of the perturbative convergence of the outcomes favours [9] the definition of all quark masses according to the modified minimal-subtraction renormalization scheme, signaled below by overlining each affected quantity.

Numerical values of the adopted modified-minimal-subtraction parameters:

Quantity	Numerical input value	Reference
$\overline{\alpha}_s(M_Z)$	0.1182 ± 0.0012	[11]
$(\overline{m}_u + \overline{m}_d)/2 \equiv \overline{m}_{ud}(2\text{ GeV})$	(3.70 ± 0.17) MeV	[11]
$(\overline{m}_d - \overline{m}_u)(2\text{ GeV})$	(2.67 ± 0.22) MeV	[11]
$\overline{m}_s(2\text{ GeV})$	(93.9 ± 1.1) MeV	[11]
$\overline{m}_c(\overline{m}_c)$	(1.275 ± 0.025) GeV	[12]
$\overline{m}_b(\overline{m}_b)$	(4.247 ± 0.034) GeV	[13]

Effective threshold: quark-mass behaviour

We fathom the discrepance of the leptonic decay constants f_{H_q} of isodoublet heavy–light mesons caused by the difference of the u- and d-quark masses by tracking the response of the local-duality QCD sum-rule outcome $f_H(m_q)$ to continuous variations of a generic light-quark mass m_q between zero and the strange-quark mass: $m_q \in [0, m_s]$. Given the perturbative spectral densities up to sufficiently high orders, the only remaining decisive ingredients are the effective thresholds' dependences $s_{\text{eff}} = s_{\text{eff}}(m_Q, m_q)$ on both quark masses.

Shuffling together available knowledge on heavy-quark expansion and chiral logarithmic corrections [14] suggests to use a parametrization of the form [15]

$$\sqrt{\overline{s}_{\text{eff}}(\mu)} = \overline{m}_Q(\mu) + \overline{m}_q(\mu) + \overline{z}_{\text{eff}}(\mu)$$

$$\overline{z}_{\text{eff}}(\mu) = \overline{z}_0(\mu) + \overline{z}_1(\mu)\,\overline{m}_q(\mu) + [\overline{z}_0(\mu) - \delta\overline{m}_Q(\mu)]\, z_{\text{CL}} + O(\overline{m}_q^2)\ ,$$

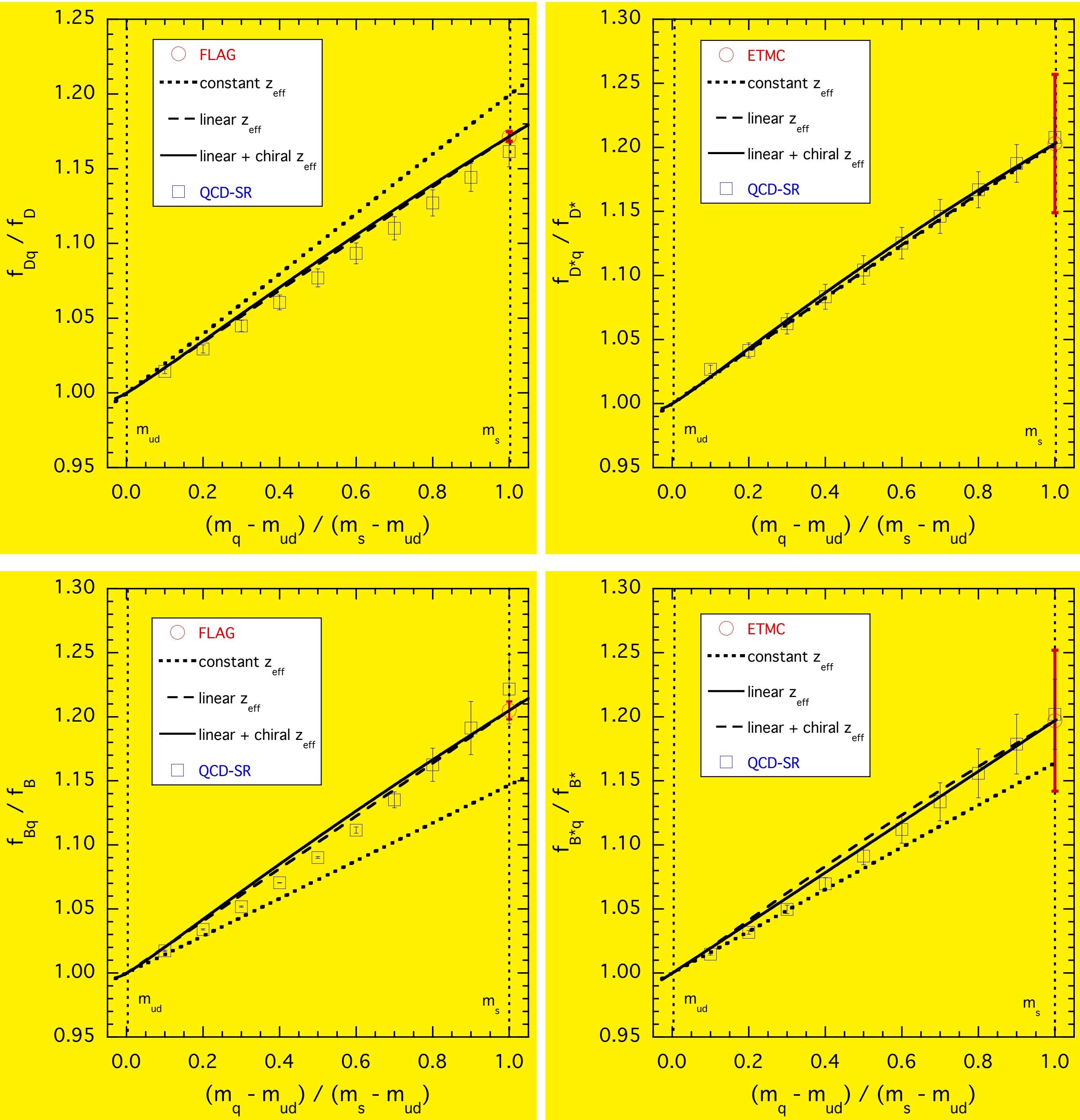
with $\delta\overline{m}_Q(\mu)$ the disparity of the heavy-quark mass in on-shell and modified minimal-subtraction renormalization scheme and z_{CL} the chiral logarithmic contributions in heavy-quark limit; $\overline{z}_0(\mu)$ and $\overline{z}_1(\mu)$ are two free parameters. For the decay constants of heavy–light-mesons, local duality thus entails [15]

$$f_P^2\,\overline{m}_Q = \frac{\overline{z}_{\text{eff}}^3(\mu)}{\pi^2}\left\{1 + \frac{\overline{\alpha}_s(\mu)}{36\,\pi}\left[33 + 4\,\pi^2 - 18\log 2 + O(\overline{m}_q)\right] + O(\alpha_s^2)\right\},$$

$$f_V^2\,\overline{m}_Q = \frac{\overline{z}_{\text{eff}}^3(\mu)}{\pi^2}\left\{1 + \frac{\overline{\alpha}_s(\mu)}{36\,\pi}\left[21 + 4\,\pi^2 - 18\log 2 + O(\overline{m}_q)\right] + O(\alpha_s^2)\right\}.$$

By imposing constraints, we focus to three variants of $\overline{z}_{\rm eff}(\mu)$ called constant — $\overline{z}_1(\mu) = z_{\rm CL} = 0$, linear — $z_{\rm CL} = 0$, and linear + chiral — no constraints.

Dependence on a shifted and rescaled continuously varying light-quark mass m_q of the decay constants f_{H_q} (normalized to their isospin-symmetric values f_H) of the ground-state pseudoscalar (left) and vector (right) charmed (top) and beauty (bottom) mesons $H = D^{(*)}, B^{(*)}$ [15], compared with findings [7] of QCD sum rules relying on Borel-variable-dependent thresholds (squares):



The parameters $\overline{z}_{0,1}(\mu)$ can be determined by comparing our QCD sum-rule outcome for the decay-constant function $f_H(m_q)$, at the quark-mass average

$$m_{ud} \equiv \frac{m_u + m_d}{2}$$

— which yields isospin-symmetric decay-constant predictions labelled f_H — and, where necessary, the strange-quark mass m_s, with lattice-QCD results.

Lattice-QCD estimates of the leptonic decay constants of isospin-symmetric (f_H) and strange (f_{H_s}) heavy–light mesons H_q and of their respective ratios:

Meson H_q	f_H [MeV]	f_{H_s} [MeV]	$\frac{f_{H_s}}{f_H}$	Reference
$D_{(s)}$	212.15 ± 1.45	248.83 ± 1.27	1.1716 ± 0.0032	[11]
$D^*_{(s)}$	223.5 ± 8.7	268.8 ± 6.5	1.203 ± 0.054	[16]
$B_{(s)}$	186.0 ± 4.0	224.0 ± 5.0	1.205 ± 0.007	[11]
$B^*_{(s)}$	186.4 ± 7.1	223.1 ± 5.6	1.197 ± 0.055	[16]

From the m_q dependences of f_{H_q}, we extract the isospin-breaking differences of the heavy-meson decay constants by allowing the scales μ to vary over the ranges $(1\ \text{GeV}, 3\ \text{GeV})$ for charmed and $(3\ \text{GeV}, 6\ \text{GeV})$ for bottom mesons and averaging over the figures emerging from linear or linear + chiral $\overline{z}_{\text{eff}}(\mu)$.

QCD sum-rule results for isospin violations in heavy-meson decay constants:

	$f_{H_d} - f_{H_u}$ [MeV]		
Mesons H_q	Borelized threshold [7]	Local-duality limit [15]	Lattice QCD
$D^{\pm,0}$	0.97 ± 0.13	0.96 ± 0.09	$0.94^{+0.50}_{-0.12}$ [17]
$D^{*\pm,0}$	1.73 ± 0.27	1.18 ± 0.35	—
$B^{0,\pm}$	0.90 ± 0.13	1.01 ± 0.10	3.8 ± 1.0 [18]
$B^{*0,\pm}$	0.81 ± 0.11	0.89 ± 0.30	—

Apart from the D^* case (which is anyway affected by sizeable uncertainties), we find an excellent agreement with earlier predictions [7] of our advanced [3] QCD sum rules. Comparing with available lattice-QCD findings, we observe a perfect agreement for the D mesons but a strong tension for the B mesons.

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