Spontaneous mass generation and small dimensions of the Standard Model groups U(1), SU(2) and SU(3)

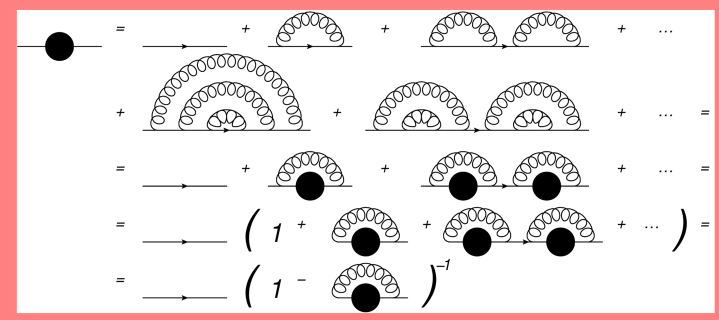
Felipe J. Llanes-Estrada*, Guillermo García, Jesús Guerrero Rojas
Dept. Física Teórica I, Univ. Complutense de Madrid,
28040 Madrid, Spain fllanes@fis.ucm.es

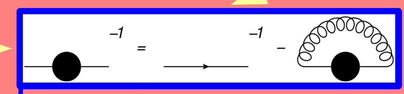
The Standard Model **gauge symmetry** is U(1) x $SU(2)_L$ x SU(3) for unknown reasons. One aspect that can be addressed is the **low dimensionality** of all its subgroups. Why not much larger groups like SU(7) or for that matter, SP(38) or E7?

We observe that fermions charged under **large groups** acquire **much bigger dynamical masses**, all things being equal at a high e.g. GUT scale, than ordinary quarks. Should such multicharged fermions exist, they are too heavy to be observed today and have either decayed early on (if they couple to the rest of the Standard Model) or become reliquial dark matter (if they don't). The result follows easily from strong antiscreening of the running coupling for the larger group together with scaling properties of the Dyson-Schwinger eq. for the fermion mass.

1) Dyson-Schwinger equations for the quark propagator: from interaction to mass

Rainbow approximation





Free fermion propagator ("bare")

$$S_0(p^2) = \frac{i}{\not p - m_c}$$

$$S(p^2) = \frac{i}{A(p^2)\not p - B(p^2)}$$

Full fermion propagator ("dressed")

Results in 1-D equation for running mass $M(p)=Mp=B/A$

$$M_p = m_c + \frac{C_F}{\pi^3}\int_0^\infty q^3 dq \frac{M_q}{|q|^2+M_q^2} g^2 D^0_{p-q}$$

With free gauge boson propagator, renormalization needed: Momentum subtraction scheme

$$M(p^2) = M(\mu^2) + \frac{g^2 C_F}{\pi^3}\int_0^\infty q^3 dq \int_{-1}^{1} dx\sqrt{1-x^2} \left(\frac{1}{|q-p|^2} - \frac{1}{|q-\mu|^2}\right)\frac{M(q^2)}{M^2(q^2)+|q|^2}.$$

Mass from interaction + energy scale

"DYNAMICAL MASS GENERATION"

(from current to constituent quarks)

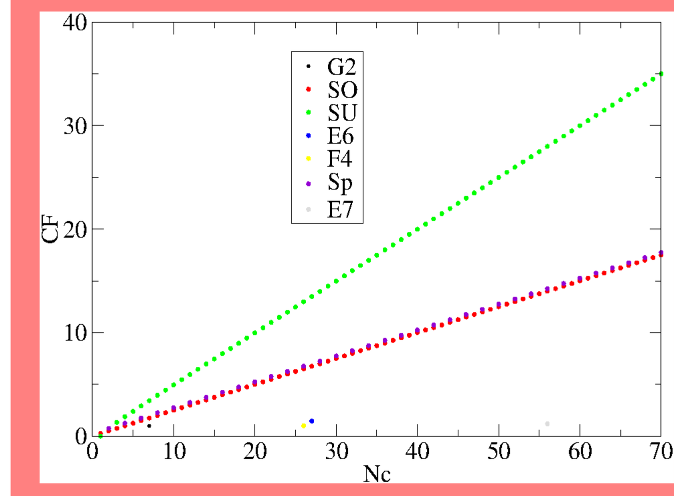
2) Color factors for various Lie groups

The color factor from the Casimir in the fundamental representation grows about linearly with Nc

$$C_F\delta^i_k = \sum_{a,b,j}(T_a)^i_j\delta_{ab}(T_b)^j_k = \sum_{a,j}(T_a)^i_j(T_a)^j_k \qquad Tr(T_aT_b) = \kappa\delta_{ab}$$

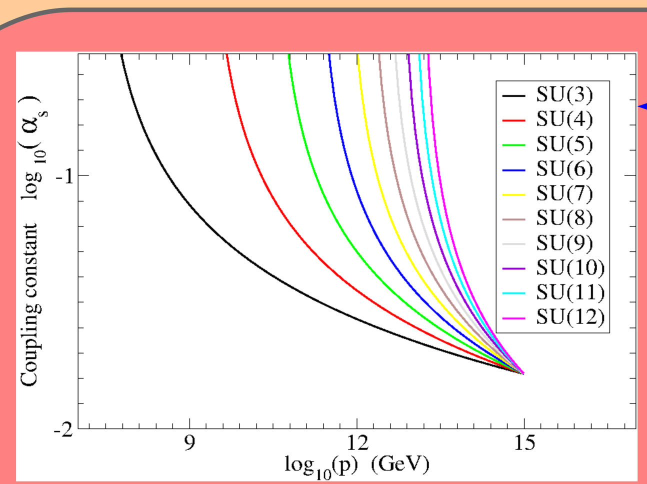
1/2

Group	Color Factor (C_F)
$SU(N_c)$	$\frac{1}{2}\left(N_c - \frac{1}{N_c}\right)$ $\forall N_c \in \mathbb{N}$
$SO(N_c)$	$\frac{1}{4}\left(N_c - 1\right)$ $\forall N_c \in \mathbb{N}$
$Sp(N_c)$	$\frac{1}{4}\left(N_c + 1\right)$ $N_c = 2n$ $n \in \mathbb{N}$
$E6$	$\frac{1}{12}\left(N_c - \frac{29}{3}\right)$ $N_c = 27$
$F4$	$\frac{1}{18}\left(N_c - 8\right)$ $N_c = 26$
$G2$	$\frac{1}{4}\left(N_c - 3\right)$ $N_c = 7$
$E7$	$\frac{1}{48}\left(N_c + 1\right)$ $N_c = 56$



Color factor for classical Lie groups and sample special groups

3) Running coupling and mass in perturbation theory near the GUT scale

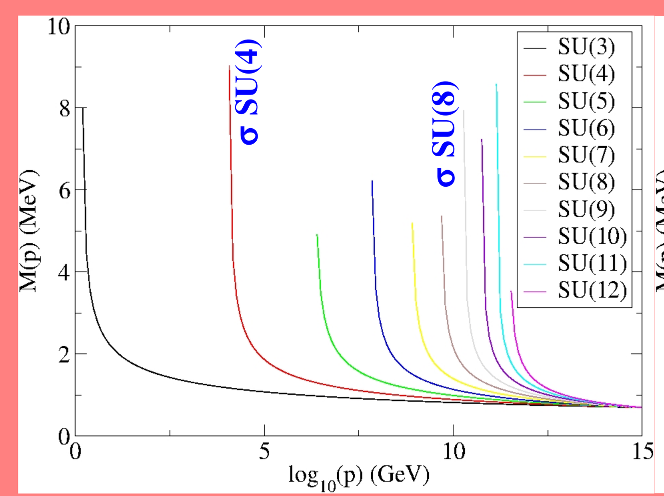


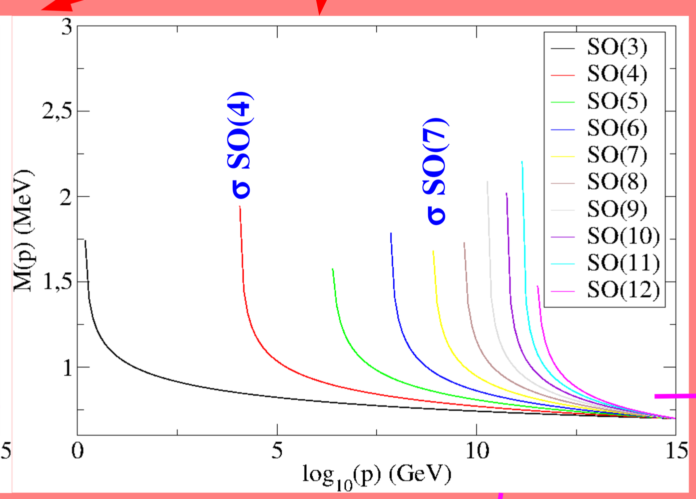
$$\alpha_s(\mu_2) = \alpha_s(\mu_1)\frac{1}{1+\frac{\alpha_s(\mu_1)}{\pi}\beta_1 \ln\frac{\mu_2}{\mu_1}}$$

Running coupling to 1-loop

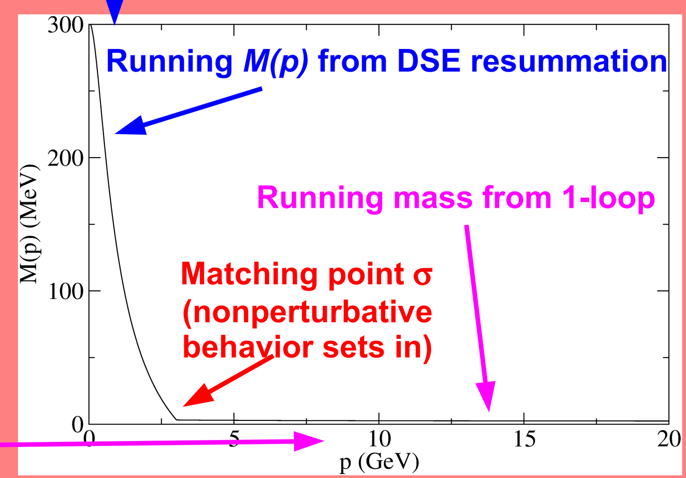
$$m_s(\mu_2) = m_s(\mu_1)\left(\frac{1}{1+\frac{\alpha_s(\mu_1)}{\pi}\beta_1 \ln\frac{\mu_2}{\mu_1}}\right)^{\frac{\gamma_1}{\beta_1}}$$

Running Mass to 1-loop





4) Matched mass function: perturbation theory at large scale + DSE at small scale



Starting condition for all groups:

$\alpha_s(\mu_{GUT})=0.017$

$m(\mu_{GUT})=1$ MeV

Solution rescaling:
Once DSE numerically solved for a given group -like QCD's SU(3) one can rescale solution to find any other

$$p^2 \to \lambda^2 p^2 \qquad \sigma^2 \to \lambda^2\sigma^2, \qquad \frac{\sigma_{group}}{\sigma_{SU(3)}} = \lambda$$

$$M(0) = \frac{\tilde M(0)}{\lambda} \qquad \frac{M_{group}(0)}{M_{SU(3)}(0)} = \lambda$$

5) Dependence on N_f (number of "flavors") and exponential in Nc

$$\beta_1 = \frac{1}{6}(11N_c - 2N_f) \qquad \gamma_1 = \frac{3}{2}C_F,$$

If N_f large enough, the vacuum becomes screening instead of antiscreening:
No dynamical mass generation (for SU(4), N_f=22)

Even for a smaller N_f called "critical N_f",
Dynamical Chiral Symmetry Breaking ceases. M(0) is only radiatively (logarithmically) evolved from M(GUT) -> Light fermions!
(For SU(3) QCD, N_f^{crit} = 8-11 instead of the screening 17)

To be specific, we choose σ such that $\alpha_s(\sigma)=0.4/CF$

Because of logarithmic running, the matching point where non perturbative physics sets in depends exponentially on the number of colors

$$\sigma = \mu_{GUT} \times e^{\frac{\pi}{\beta_1}\left(\frac{1}{\alpha_s(\sigma)} - \frac{1}{\alpha_s(\mu_{GUT})}\right)}$$

Since M(0) ∝ σ the constituent fermion mass exponentiates with *Nc* (actually, with CF)
(just like the Landau pole position Λ(Nc))

6) MAIN RESULT: BEHAVIOR OF DYNAMICAL FERMION MASS FOR DIFFERENT GROUPS

$$\frac{M_{group}(0)}{M_{SU(3)}(0)} = \frac{\sigma_{group}}{\sigma_{SU(3)}}$$

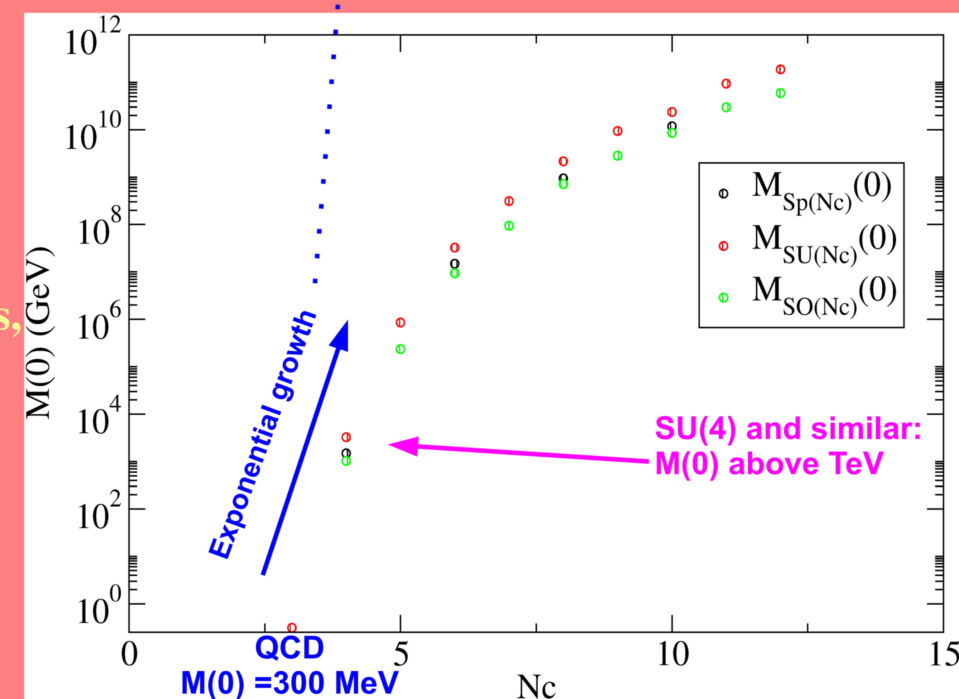
$$M(0)_{N_c} \propto e^{N_c} \times \theta(N_f^{\rm critical} - N_f)$$

CONCLUSION:

Fermions charged under large groups, if all start equal at GUT scale

ARE VERY MASSIVE

which would naturally lead to small dimensional Standard Model groups at low scales (provided not too many flavors)



It would be fun to find at an accelerator fermions charged under SU(4), Sp(4) or SO(4) at the TeV scale (groups of higher dimension seem out of reach)

Just a bit less fun would be to repeat the entire exercise matching Two-loop perturbation theory and an improved DSE treatment

Based on arXiv:1507.08143v2 [hep-ph] (Nuclear Physics B, in press)

Nota bene: this work has nothing to do with t'Hooft's large-Nc expansion as the bare coupling, g, is for us independent of Nc as opposed to $Nc^{-1/2}$