# Implications of Strict Gauge Invariance for Particle Spectra and Precision Observables 

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- Physical spectrum: Observable particles
- Peaks in (experimental) cross-sections


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- Why does perturbation theory work?


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- Why does perturbation theory work?
- Test: Mass spectrum


## Mass relation - Higgs

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2) Expand Higgs field in fluctuations $h=v+\eta$

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$2 \times$ Higgs mass: Scattering state
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- Same poles to leading order
- Fröhlich-Morchio-Strocchi (FMS) mechanism
- Works also for the W/Z
- But: Physical state is a custodial triplet!
- Flavor has two components
- Global SU(3) generation
- Local SU(2) weak gauge (up/down distinction)
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- Yukawa terms break custodial symmetry
- Different masses for doublet members
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- Different masses for doublet members
- Test requires precision measurement


## How events looks like (LEP/ILC)



- Collision of bound states


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$e^{-}-\mathrm{H}$ bound state
$\mu-\mathrm{H}$ bound state


Z-H-H bound state
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- Collision of bound states - 'constituent' particles
- Higgs partners just spectators
- Similar to pp collisions


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- In non-Abelian gauge theories: Bound states
- FMS mechanism: Success of perturbation theory

Introduction: 1610.04188 These results: 1701.02881

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- Higgs admixture to many states
- Small effect...but may be testable!
- Must be accounted for in new physics searches
- Similar considerations for pp : Watch $\mathrm{pp} \rightarrow \overline{\mathrm{t}} \mathrm{t}$

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- Qualitative impact beyond the standard model

Introduction: 1610.04188 These results: 1701.02881

