Precision Electroweak Measurements at FCC-ee (ILC, CLIC)

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on behalf of the FCC-ee study group

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High–energy $e^+e^-$ colliders

Compact Linear Collider (CLIC): CERN $e^+e^-$, $\sqrt{s} = 380$ GeV, 1 TeV, 3 TeV
Length = 11, 29, 50 km

Future Circular Collider (FCC): CERN $e^+e^-$, $\sqrt{s} = 90-350$ GeV; pp, $\sqrt{s} \approx 100$ TeV
Circumference = 97.5 km

International Linear Collider (ILC): Japan $e^+e^-$, $\sqrt{s} = 250, 350, 500$ GeV (1 TeV?)
Length = 31 km (50 km)

Circular Electron Positron Collider (CEPC): China $e^+e^-$, $\sqrt{s} = 90-250$ GeV; SPPC pp
Length = 100 km
Luminosity vs Energy

F. Zimmerman post-Berlin

Physics program: 88 to 370 GeV
Running scenarios

**ILC**

![Graph showing ILC integrated luminosities over time.]

Over all ~ 14 years
shorter time for same performances not excluded

<table>
<thead>
<tr>
<th>working point</th>
<th>luminosity/IP [10^{34} cm^{-2}s^{-1}]</th>
<th>geom. lumin.</th>
<th>luminosity/year</th>
<th>physics goal</th>
<th>run time [years]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z first 2 years</td>
<td>65</td>
<td>69</td>
<td>17 ab^{-1}/year</td>
<td>150 ab^{-1}</td>
<td>6</td>
</tr>
<tr>
<td>Z later</td>
<td>130</td>
<td>137</td>
<td>34 ab^{-1}/year</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W</td>
<td>20</td>
<td>30</td>
<td>5 ab^{-1}/year</td>
<td>8 - 10 ab^{-1}</td>
<td>2</td>
</tr>
<tr>
<td>H</td>
<td>7</td>
<td>8</td>
<td>1.8 ab^{-1}/year</td>
<td>5 ab^{-1}</td>
<td>3</td>
</tr>
<tr>
<td>top</td>
<td>2.0</td>
<td>2.1</td>
<td>0.5 ab^{-1}/year</td>
<td>1.5 ab^{-1}</td>
<td>3</td>
</tr>
</tbody>
</table>

*Each 5 to 7 years of running*

Dedicated to top mass threshold scan
Inputs to EW fits

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_H$ [GeV]</td>
<td>$125.14 \pm 0.24$</td>
</tr>
<tr>
<td>$M_W$ [GeV]</td>
<td>$80.385 \pm 0.015$</td>
</tr>
<tr>
<td>$\Gamma_W$ [GeV]</td>
<td>$2.085 \pm 0.042$</td>
</tr>
<tr>
<td>$M_Z$ [GeV]</td>
<td>$91.1875 \pm 0.0021$</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>$2.4952 \pm 0.0023$</td>
</tr>
<tr>
<td>$\sigma_{\text{had}}^0$ [nb]</td>
<td>$41.540 \pm 0.037$</td>
</tr>
<tr>
<td>$R^0_F$</td>
<td>$20.767 \pm 0.025$</td>
</tr>
<tr>
<td>$A^0_{\text{FB}}$</td>
<td>$0.0171 \pm 0.0010$</td>
</tr>
<tr>
<td>$A_{\ell}$</td>
<td>$0.1499 \pm 0.0018$</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}(Q_{\text{FB}})$</td>
<td>$0.2324 \pm 0.0012$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>$0.670 \pm 0.027$</td>
</tr>
<tr>
<td>$A_b$</td>
<td>$0.923 \pm 0.020$</td>
</tr>
<tr>
<td>$A^{0,c}_{\text{FB}}$</td>
<td>$0.0707 \pm 0.0035$</td>
</tr>
<tr>
<td>$A^{0,b}_{\text{FB}}$</td>
<td>$0.0992 \pm 0.0016$</td>
</tr>
<tr>
<td>$R^0_c$</td>
<td>$0.1721 \pm 0.0030$</td>
</tr>
<tr>
<td>$R^0_b$</td>
<td>$0.21629 \pm 0.00066$</td>
</tr>
</tbody>
</table>

FCC-ee

expected to improve electroweak precision measurements by factors 20 to 50

Goals in JHEP 01 (2014) 164

- Clean environment
- Very Large integrated luminosity (Z pole & WW threshold)
- c.m. energy precisely known (natural transverse polarization up to WW threshold) 100 keV achievable through resonant depolarization
- Narrow luminosity spectrum around nominal $E_{CM}$

Global Ewfit to $m_Z$, $m_W$, $m_t$, $m_H$, $\alpha_s(m_Z^2)$, $G_F$, $\alpha_{\text{QED}}(m_Z^2)$

⇒ indirect search for NP up to 100 TeV achievable @ FCC-ee

well beyond the energy range accessible to direct searches
“Shopping” list

- Z Mass & Width from line-shape
- $\alpha_s$ from decay branching fractions
- Number of neutrino species
- $\sin^2 \theta_W$ & $\alpha_{\text{QED}}$ from asymmetries
- W Mass & Width from WW threshold scan
- $\alpha_s$ from decay branching fractions
- W Mass & Width from direct reconstruction
- W & WW cross-sections
- Constraints on gauge couplings
- Number of neutrino species from radiative Z events
- Top Mass from $t\bar{t}$ threshold scan
- Electroweak couplings of the top quark

More in FCC week 2017 in Berlin
Tera Z

precision measurements @ the Z pole: Mass & Width

\[ L \approx 2 \times 10^{36} \Rightarrow 4 \times 10^{12} \text{ Z decays} \]

Continuous \( E_{CM} \) calibration (resonant depolarization)

Z Mass & Width: 5 keV (stat) + 100 keV (syst)

Present relative theoretical uncertainty \( \approx 10^{-4} \)
(radiation function calculated up to \( O(\alpha^3) \))

\( \Rightarrow \) theoretical work to be done

Expected precision:

Z mass: \( \Delta_{rel}(m_Z) \approx 10^{-6} \)
Z width: \( \Delta_{rel}(\Gamma_Z) \approx 5 \times 10^{-5} \)

\( R_l \) (had/lep width): \( \Delta_{rel}(R_l) \approx 5 \times 10^{-5} \Rightarrow \Delta_{rel} \alpha_s(m_Z^2) \approx 2 \times 10^{-3} \)

\( \Delta N_\nu \approx 0.00008 \ (0.0003) \ \text{stat (syst)} \) from lineshape

\( \Delta R_b \approx 0.00001 \ (0.00005-0.0002) \ \text{stat (syst)} \) *

\( \Delta R_c \approx 0.00003 \ (0.0005) \ \text{stat (syst)} \) *

Gain factor w.r.t LEP
20
20
100 (stat), 20 (syst)

3-10
6

* work in progress
Tera Z

precision measurements @ the Z pole: Asymmetries

FCC-ee could give the final word
to long-standing differences between asymmetries:

LEP measurements:
$A_{FB}^{ll}$, $A_{l}(P_{\tau})$, $A_{FB}^{cc}$, $A_{FB}^{bb}$
were dominated by statistics

$\rightarrow$ large gain expected from $x \approx O(15)$

Expected precision:

$\Delta A_{b} \approx 0.00002 (0.0004) \text{ stat (syst) } ^{*}$
$\Delta A_{c} \approx 0.00003 (0.0004) \text{ stat (syst) } ^{*}$

from $\tau$ polarisation:

$\Delta A_{e} \approx 0.00005 (0.0001) \text{ stat (syst) }$
$\Delta A_{\tau} \approx 0.00004 (0.0003) \text{ stat (syst) }$

Gain factor w.r.t LEP

5
8
30
10

from $A_{FB}(\mu\mu)$: $\Delta \sin^{2}\theta_{eff} \approx 0.000006$

Gain factor w.r.t LEP $\approx 40$

also $A_{FB}(ee)$, $A_{FB}(\tau\tau)$

* work in progress
Tera Z

direct measurement of $\alpha_{\text{QED}}(m^2_Z)$ (1)

(Patrick Janot, arXiv:1512:05544, JHEP 2016(2) 1)

Now $\alpha_{\text{QED}}(M^2_Z)$ from the running of $\alpha$ ⇒ $\Delta \alpha/\alpha = 1.1 \times 10^{-4}$

rather get $\alpha_{\text{QED}}$ directly @ the Z pole from a self-normalized quantity, the forward-backward asymmetry uncertainties on efficiency, acceptance, luminosity cancel

$$A_{\text{FB}}^{\mu\mu} \approx A_{\text{FB},0}^{\mu\mu} + C_{\text{ste}}^* \alpha_{\text{QED}}^* (s-m^2_Z)/m^2_Z$$

$$\Delta \alpha_{\text{QED}} / \alpha_{\text{QED}} \approx \Delta A_{\text{FB}}^{\mu\mu} / A_{\text{FB}}^{\mu\mu}$$

$$\frac{\Delta \alpha_0}{\alpha_0} \approx 0.528 \frac{\Delta A_{\text{FB}}}{A_{\text{FB}}} (s_-) + 0.563 \frac{\Delta A_{\text{FB}}}{A_{\text{FB}}} (s_+)$$

$\alpha_0 = \alpha_{\text{QED}}(m^2_Z)$

$\Delta A_{\text{FB}}^{\mu\mu} / A_{\text{FB}}^{\mu\mu} (s_-) < 0$ & $\Delta A_{\text{FB}}^{\mu\mu} / A_{\text{FB}}^{\mu\mu} (s_+) > 0$

⇒ large cancellation of systematic uncertainties when combining measurements below and above Z peak

$\Rightarrow$ no sensitivity to $\alpha_{\text{QED}}$
Tera Z

direct measurement of $\alpha_{\text{QED}}(m^2_Z)$ (2)

(Patrick Janot, arXiv:1512.05544, JHEP 2016(2) 1)

$\sigma(\alpha)/\alpha$ for a year of running @ any $\sqrt{s}$:

<table>
<thead>
<tr>
<th>Type</th>
<th>Source</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>$E_{\text{beam}}$ calibration</td>
<td>$1 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>$E_{\text{beam}}$ spread</td>
<td>$&lt; 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>Acceptance and efficiency</td>
<td>negli.</td>
</tr>
<tr>
<td></td>
<td>Charge inversion</td>
<td>negli.</td>
</tr>
<tr>
<td></td>
<td>Backgrounds</td>
<td>negli.</td>
</tr>
<tr>
<td>Parametric</td>
<td>$m_Z$ and $\Gamma_Z$</td>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>$\sin^2 \theta_W$</td>
<td>$5 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>$G_F$</td>
<td>$5 \times 10^{-7}$</td>
</tr>
<tr>
<td>Theoretical</td>
<td>QED (ISR, FSR, IFT)</td>
<td>$&lt; 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>Missing EW higher orders</td>
<td>few $10^{-4}$</td>
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<tr>
<td></td>
<td>New physics in the running</td>
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</tr>
<tr>
<td>Total</td>
<td>Systematics</td>
<td>$1.2 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>Statistics</td>
<td>$3 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

for $3 \times 10^{-5}$ relative statistical uncertainty on $\alpha_{\text{QED}}$:

optimal: $\sqrt{s_-} = 87.9$ GeV & $\sqrt{s_+} = 94.3$ GeV

work on EWK theoretical corrections required to reach $3 \times 10^{-5}$
Oku W

WW threshold scan: $W$ mass & Width from WW cross-section (1)

At FCC-ee:

@ $\sqrt{s} = 161$ GeV, $L \approx 4 \times 10^{35}$, 8 ab$^{-1}$ $\Rightarrow$ $\approx 30 \times 10^{6}$ WW decays

@ $\sqrt{s} = 240$ GeV, $L \approx 0.9 \times 10^{35}$, 5 ab$^{-1}$ $\Rightarrow$ $\approx 80 \times 10^{6}$ WW decays

At LEP2 $\sqrt{s}=161$ GeV $\sigma=4\text{pb}$
$\epsilon=0.75$, $\sigma_B=300\text{ fb}$
$p=0.9$ : $\epsilon p \approx 0.68$ (@161)
$\Rightarrow m_W=80.40\pm0.21$ GeV
with $11/\text{pb} @ E_{\text{CM}}=161$ GeV
Oku W

**WW threshold scan: W mass & width from cross-section (2)**

Sensitivity to mass & width is different for different $\sqrt{s}$

- **max stat sensitivity** @ $\sqrt{s} \approx 2 M_W + 600$ MeV
- **statistical precision:**
  - $350$ MeV @ $L = 11$ pb$^{-1}$
  - $400$ keV @ $L = 8$ ab$^{-1}$
- **systematics controlled to:**
  - $\Delta E_{\text{beam}} < 400$ keV ($5 \times 10^{-6}$)
  - $\Delta \varepsilon / \varepsilon$, $\Delta L/L < 10^{-4}$
  - $\Delta \sigma_b < 0.7$ fb ($2 \times 10^{-3}$)

**max stat sensitivity** @ $\sqrt{s} \approx 2 M_W + 1.5$ GeV

- **$\Delta M_W$ from single fit**
- **$\Delta m_W$ with** $E_1 = 157.1$ GeV, $E_2 = 162.3$ GeV, $f = 0.4$
  - $\Delta m_W = 0.62$ MeV
  - $\Delta \Gamma_W = 1.5$ MeV
  - $\Delta m_W = 0.56$ (MeV)

**also $M_W$ from direct reconstruction:** $\Delta M_W = 0.5 \ (1?)$ MeV stat (syst)

* assume $\varepsilon$, $p$, $\sigma_b$ from LEP
Lepton universality test at the 2% level
$\text{Br}(\tau) > \text{Br}(e,\mu)$ at 2.7 $\sigma$

At FCC-ee: $\Delta_{\text{rel}} \text{Br}(e\nu,\mu\nu,\tau\nu) \approx 4 \times 10^{-4}$

Beware: channel cross-contamination $\Rightarrow$ better control of lepton id

$q/l$ universality test at the 0.6% level

$\Delta_{\text{rel}} \text{Br}(qq) \approx 10^{-4}$

$\Delta_{\text{rel}} \alpha_s(m^2_W) \approx (9\pi/2) \Delta_{\text{rel}} \text{Br}(qq) \approx 10^{-3}$

$\Rightarrow \Delta \alpha_s(m^2_W) \approx \pm 0.0001$

**Gain factor w.r.t LEP $\approx 40$ (10 w.r.t. today !!!)**

also coupling to c & b quarks ($V_{cs}, V_{cb}, ...$) with flavor tagging * 

* work in progress
Oku W

Neutrino counting

- @ LEP \( N_\nu = 2.984 \pm 0.008 \) from Z-line shape
  
  2 \( \sigma \) “low” \( \rightarrow \) non-unitarity of the PMNS matrix?

dominated by statistical uncertainty on normalization to small angle Bhabha cross-section \( \pm 0.0046 \) on \( N_\nu \)

Possible improvement @ FCC-ee (slide7)
by precisely measuring luminosity with \( e^+e^- \rightarrow \gamma\gamma \)

Another method:

above the Z pole radiative return to Z lead to a clean sample of \( \gamma \)-tagged Zs
Systematics on \( \gamma \) selection, luminosity, etc cancel in the ratio

After 5 years of running @ the WW threshold: \( \Delta N_\nu \approx 0.001 \)

Gain factor w.r.t LEP \( \approx 10 \)
**Mega Top**

**tt threshold scan: top mass**

Cross-section shape strongly depends on t-quark mass, width, $\alpha_s, Y_t$

**t-quark mass from threshold scan**

The threshold shape is affected by ISR & beam energy spread

FCC-ee has a very steep beam profile ➔ enhanced size of the top sample

Experimental systematic uncertainty:

$E_{\text{beam}}$ (few MeV) & $E_{\text{beam}}$ spread ➔ $\Delta m_t < 5$ MeV

$\Delta \alpha_s (@\text{FCC-ee}) \approx 0.0002$ ➔ $\Delta m_t < 20$ MeV

Theory systematic uncertainty:

1S/PS ➔ $\overline{\text{MS}}@4\text{loop}$ ➔ $\Delta m_t \approx 10$ MeV

other scale uncertainties under study

---

200 fb^{-1} ➔ $\Delta m_t \approx 10$ MeV (stat)
also above threshold: top mass

**Direct reconstruction**  

in continuum @ all energies above threshold

100 fb\(^{-1}\) @ 500 GeV  \(\Delta m_t \approx 80\) MeV (stat)

**Significant theory uncertainties** when converting to a particular mass scheme

**Radiative events**  
P. Gomis @ ECFA LC Workshop 2016

@ \(\sqrt{s} >> \) threshold, there is still sensitivity to \(t\bar{t}\) threshold in radiative events  
(rate of energetic ISR \(\gamma\) & FSR \(g\) strongly depends on \(m_t\))

500 fb\(^{-1}\) @ 380 GeV  \(\Delta m_t \approx 100\) MeV (stat)

3.5 ab\(^{-1}\) @ 1 TeV  \(\Delta m_t \approx 388\) MeV (stat)  
using ISR

well defined theoretical scheme

**Other considered methods**

• b-jet energy distribution  
  F. Franceschini @ TopLC’2016

• event shape analysis  
  A. Hoang @ TopLC’2016
Couplings of the top quark to $Z$ & $\gamma$ are very sensitive to effects from massive unknown particles

→ **NP discovery** beyond machine energy-scale or constraints to SM extensions

e.g. composite Higgs models

**Mega Top**

$t\bar{t}$ threshold scan: electroweak couplings of the top quark (1)
**Mega Top**

**t\bar{t} threshold scan: electroweak couplings of the top quark (2)**

Large stat & final state polarization (Br(t -> Wb) = 100%)

→ left & right couplings of the t-quark can be extracted with no need for initial state polarization

use **lv q-\bar{q} b-\bar{b}** final states

sensitivity to t electroweak couplings from **lepton angular & energy distributions**
Mega Top

t\bar{t} threshold scan: electroweak couplings of the top quark (2)

Parametrisation of the t\bar{t}X vertex (X = \gamma, Z):

$$\Gamma_{\mu}^{t\bar{t}X}(k^2, q, \bar{q}) = i e \left\{ \gamma_\mu \left( F_{1V}^X(k^2) + \gamma_5 F_{1A}^X(k^2) \right) - \frac{\sigma_{\mu\nu}}{2m_t}(q + \bar{q})^\nu \left( i F_{2V}^X(k^2) + \gamma_5 F_{2A}^X(k^2) \right) \right\}$$

in SM at tree level:

$$F_{1V}^{\gamma,SM} = \frac{2}{3}, \quad F_{1A}^{\gamma,SM} = 0, \quad F_{1V}^{Z,SM} = \frac{1}{4s_w c_w} \left( 1 - \frac{8}{3} s_w^2 \right), \quad F_{1A}^{Z,SM} = -\frac{1}{4s_w c_w}, \quad F_{2A,V}^{X} = 0$$

For the 6 CP conserving form factors:

Optimal \( v_s = 365 - 370 \) GeV  
No initial state polarization!

FCC-ee expected relative statistical precision \( \approx 10^{-2} - 10^{-3} \)
Conclusion:

Global ewk fit and sensitivity to new physics

\[ \mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i \]

Jorge de Blas

LHCP 2017

all inputs of SM experimentally measured

\( \Rightarrow \) any observable sensitive to ew radiative corrections unambiguously predicted

\( \Rightarrow \) any deviation from measurements reveals new weakly interacting particles

10 operators contributing to ew precision observables in the chosen basis

fit to ewpd, 1 operator at a time generated by NP

LEP limit: \( \Lambda_{\text{NP}} > 10 \text{ TeV} \)

FCC limit: \( \Lambda_{\text{NP}} > 100 \text{ TeV} ? \)
and also:

**Direct discoveries from:**

- rare decays
- flavor physics
- top decays
- very weakly coupled particles
- ....

**giving access to New Physics**

**hardly accessible to hadron colliders**

perhaps

surprising

even striking discoveries!
backup
**E_{CM} calibration**

**Resonant depolarisation: a key ingredient!**

- @ WW threshold beam transverse polarization occurs naturally (10% is enough)
- @ Z pole use wigglers at the beginning of fills to shorten the polarization time
- not available @ E_{beam} > 90 GeV (due to increased energy spread $\alpha E^2$) but for H & top, ee -> Zg or ee-> ZZ, WW can be used ($\Delta E_{CM} \approx 5 \text{ MeV}$)

add fast oscillating B field to depolarize the bunch, the depolarization frequency corresponds to $<E_{beam}>$. Beam polarization measured by laser polarimeters.

- @ LEP
  
  Depolarization resonance very narrow: $\sim 100 \text{ keV precision}$ for each measurement
  
  But final systematic uncertainty was 1.5 MeV due to transport from dedicated polarization runs to the physics runs.

- @ FCC-ee
  
  continuous calibration with dedicated bunches $\Rightarrow$ no transport uncertainty

  $\Rightarrow \Delta E_{beam} << 100 \text{ keV} @ Z \text{ pole} \& \text{ WW threshold}$

  $\Delta M_Z, \Delta \Gamma_Z \approx 0.1 \text{ MeV} ; \Delta M_W \approx 0.5 \text{ MeV}$

  “EPOL” working group on polarization and beam energy:
  arxiv:1506.00933
Tera Z

precision measurements @ the Z pole: Asymmetries (b1)

Z -> ff : 3 observables from the direction and decay of outgoing fermion:

\[ A_f = \frac{2g_{Vf}g_{Af}}{(g_{Vf})^2 + (g_{Af})^2} \]

\[ \sin^2 \theta_{eff}^l = \frac{1}{4} \left( 1 - \frac{g_{Vl}}{g_{Al}} \right) \]

+ 2 observables with polarisation* of the initial state:

\[ A_{FB}^{pol} = \frac{\sigma_{F,l} - \sigma_{B,l} - \sigma_{F,r} + \sigma_{B,r}}{\sigma_{tot}} = \frac{3}{4} A_f \]

\[ A_{pol} = \frac{\sigma_{F,R} + \sigma_{B,R} - \sigma_{F,L} - \sigma_{B,L}}{\sigma_{tot}} = -A_f \]

\[ A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_{tot}} = \frac{3}{4} A_e A_f \quad \text{Can measure for } e, \mu, \tau, c, b \]

* not mandatory, as no significant gain and loss of luminosity
Tera Z

direct measurement of $\alpha_{\text{QED}}(m_Z^2)$ (b1)

(Patrick Janot, arXiv:1512:05544, JHEP 2016(2) 1)

Now $\alpha_{\text{QED}}(M_Z^2)$ from the running of $\alpha$:

$$\alpha_{\text{QED}}(m_Z^2) = \frac{\alpha_{\text{QED}}(0)}{1 - \Delta \alpha_f(m_Z^2) - \Delta \alpha_{\text{had}}(m_Z^2)}.$$

@ FCC-ee: direct measurement at $M_Z$:

$$\sigma_{\mu\mu} = G + Z + I$$

$$\alpha^2_{\text{QED}}(s) \quad G_F^2 \quad \alpha_{\text{QED}}(s) \times G_F$$

$$G = \frac{e^2}{s},$$

$$Z = \frac{c_Z^2 v^2 + a^2}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2},$$

$$I = \frac{2c_\gamma c_Z v^2 \times (s - m_Z^2)}{(s - m_Z^2)^2 + m_Z^2 \Gamma_Z^2},$$

$$c_\gamma = \sqrt{\frac{4\pi}{3}} \alpha_{\text{QED}}(s), \quad c_Z = \sqrt{\frac{4\pi}{3} \frac{m_Z^2 G_F}{2\pi},} \quad a = -\frac{1}{2}, \quad v = a \times (1 - 4\sin^2 \theta_W),$$

$$\Delta \sigma_{\mu\mu} = \frac{\Delta \alpha}{\alpha} (I + 2G) \Rightarrow \frac{\Delta \alpha}{\alpha} \approx \frac{\Delta \sigma_{\mu\mu}}{2G} \approx \frac{1}{2} \frac{\Delta \sigma_{\mu\mu}}{\sigma_{\mu\mu}} \left(1 + \frac{Z}{G}\right) \Rightarrow \Delta \alpha \approx 2 \times 10^{-5}$$

with $N_{\mu\mu} > 10^9$
Tera Z

direct measurement of $\alpha_{\text{QED}}(m_Z^2)$ (b2)

(Patrick Janot, arXiv:1512.05544, JHEP 2016(2) 1)

\[
A_{\text{FB}}^{\mu\mu} = \frac{\sigma_{\mu\mu}^F - \sigma_{\mu\mu}^B}{\sigma_{\mu\mu}^F + \sigma_{\mu\mu}^B}
\]

uncertainties on efficiency, acceptance, luminosity cancel

\[
\frac{d\sigma_{\mu\mu}}{d\cos\theta}(s) \propto G_1(s) \times (1 + \cos^2\theta) + G_3(s) \times 2\cos\theta,
\]

\[
G_1(s) = G + I + Z
\]

\[
G_3(s) = \frac{a^2}{\nu^2} \left\{ I + \frac{4\nu^4/a^4}{(1 + \nu^2/a^2)^2} Z \right\}
\]

\[
A_{\text{FB}}^{\mu\mu}(s) = \frac{3}{4} \frac{G_3(s)}{G_1(s)}
\]

\[
A_{\text{FB}}^{\mu\mu} = A_{\text{FB},0}^{\mu\mu} + \frac{3}{4} \frac{a^2}{\nu^2} \frac{I}{G + Z}
\]

\[
A_{\text{FB},0}^{\mu\mu} = \frac{3}{4} \times 4\nu^2 a^2/(a^2 + \nu^2)^2 \simeq 0.016
\]

\[
\Delta A_{\text{FB}}^{\mu\mu} = \frac{\Delta \alpha}{\alpha} \times \frac{3}{4} \frac{a^2 I(Z - G)}{\nu^2 (G + Z)^2} = \left( A_{\text{FB}}^{\mu\mu} - A_{\text{FB},0}^{\mu\mu} \right) \times \frac{Z - G}{Z + G} \times \frac{\Delta \alpha}{\alpha}
\]

$A_{\text{FB}}$ insensitive to $\alpha$ in these 3 points

\[78\]
\[112\]
Tera Z

direct measurement of $\alpha_{\text{QED}}(m^2_Z)$ (b3)

(Patrick Janot, arXiv:1512:05544, JHEP 2016(2) 1)

sets the minimum accuracy of $A_{FB}$ to start improving $\alpha$ accuracy off the Z peak

removes model dependence & theory uncertainty

can also use $ee, \tau\tau$
Mega Top

**tt threshold scan: top mass (b1)**

![Graph showing the relationship between $\alpha_s$ prior uncertainty and $\Delta m_t$](image)

**Experimental systematic uncertainty:**
- $E_{\text{beam}}$ (few MeV) & $E_{\text{beam}}$ spread $\Rightarrow \Delta m_t < 5$ MeV
- $\Delta \alpha_s (@\text{FCC-ee}) \approx 2 \times 10^{-4} \Rightarrow \Delta m_t < 20$ MeV

**Theory systematic uncertainty:**
- $1S/PS \Rightarrow \overline{\text{MS}}@4\text{loop} \Rightarrow \Delta m_t \approx 10$ MeV
- other scale uncertainties under study

Conservative upper limit from expected precision on $\alpha_s$ @FCC-ee

Uncertainty on line shape & parametric uncertainty in mass conversion added in quadrature

M. Perello & M. Vos
### Summary table

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$m_Z$ (MeV)</td>
<td>Lineshape</td>
<td>91187.5 ± 2.1</td>
<td>0.005</td>
<td>&lt; 0.1</td>
<td>QED corr.</td>
</tr>
<tr>
<td>$\Gamma_Z$ (MeV)</td>
<td>Lineshape</td>
<td>2495.2 ± 2.3</td>
<td>0.008</td>
<td>&lt; 0.1</td>
<td>QED corr.</td>
</tr>
<tr>
<td>$R_l$</td>
<td>Peak</td>
<td>20.767 ± 0.025</td>
<td>0.0001</td>
<td>&lt; 0.001</td>
<td>Statistics</td>
</tr>
<tr>
<td>$R_b$</td>
<td>Peak</td>
<td>0.21629 ± 0.00066</td>
<td>0.000003</td>
<td>&lt; 0.00006</td>
<td>$g \to bb$</td>
</tr>
<tr>
<td>$N_{\nu}$</td>
<td>Peak</td>
<td>2.984 ± 0.008</td>
<td>0.00004</td>
<td>0.004</td>
<td>Lumi meast.</td>
</tr>
<tr>
<td>$A_{FB}^{\mu\mu}$</td>
<td>Peak</td>
<td>0.0171 ± 0.0010</td>
<td>0.000004</td>
<td>&lt; 0.0001</td>
<td>$E_{beam}$ meast.</td>
</tr>
<tr>
<td>$\alpha_s(m_Z)$</td>
<td>$R_l$</td>
<td>0.1190 ± 0.0025</td>
<td>0.000001</td>
<td>0.00015</td>
<td>New Physics</td>
</tr>
<tr>
<td>$m_W$ (MeV)</td>
<td>Threshold scan</td>
<td>80385 ± 15</td>
<td>0.3</td>
<td>&lt; 1</td>
<td>QED corr.</td>
</tr>
<tr>
<td>$N_{\nu}$</td>
<td>Radiative return</td>
<td>2.92 ± 0.05</td>
<td>0.0008</td>
<td>&lt; 0.001</td>
<td>?</td>
</tr>
<tr>
<td>$\alpha_s(m_W)$</td>
<td>$B_{had} = (\Gamma_{had}/\Gamma_{tot})W$</td>
<td>$B_{had} = 67.41 ± 0.27$</td>
<td>0.00018</td>
<td>0.00015</td>
<td>CKM Matrix</td>
</tr>
<tr>
<td>$m_{top}$ (MeV)</td>
<td>Threshold scan</td>
<td>173200 ± 900</td>
<td>10</td>
<td>10</td>
<td>QCD (~40 MeV)</td>
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</tbody>
</table>
Global ewk fit and sensitivity to new physics

\[ \mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^2} \mathcal{O}_i \]

Jorge de Blas
LHCP 2017

- 2 bosonic interactions:
  \[ \mathcal{O}_{\phi D} = \left| \phi^\dagger D^\mu \phi \right|^2, \quad \mathcal{O}_{\phi WB} = \left( \phi^\dagger \sigma_\alpha \phi \right) W^a_{\mu\nu} B^{\mu\nu} \]
  giving rise to tree level contributions to S & T parameters
  \[ \alpha_{\text{em}} S = 4 \sin \theta_W \cos \theta_W c_{\phi WB} v^2 / \Lambda^2, \quad \alpha_{\text{em}} T = -c_{\phi D} v^2 / (2\Lambda^2) \]

- 7 fermionic currents:
  \[ \mathcal{O}_{\phi \psi}^{(1)} = \left( \phi^\dagger D^\mu \phi \right) (\bar{\psi} \gamma_\mu \psi) (\psi = l, e, q, u, d) \]
  and \[ \mathcal{O}_{\phi F}^{(3)} = \left( \phi^\dagger \sigma_\alpha D^\mu \phi \right) (\bar{F} \gamma_\mu \sigma_\alpha F) (F = l, q) \]
  inducing corrections to the neutral and charged current vertices

- 1 four-lepton operator:
  \[ \mathcal{O}_{ll} = \left( \bar{l} \gamma_\mu l \right) \left( \bar{l} \gamma^\mu l \right) \]
  modifying the amplitude the amplitude of muon decay (used to extract \( G_F \))