UV Properties of Higher Dimensional Operators in Higgs Effective Field Theories from Hidden Symmetries

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based on A.Q. Int.J.Mod.Phys. A32 (2017) no.16, 1750089

arXiv:1610.00150

EPS HEP2017 - Venice, 5-12 July 2017

Probing BSM Physics:

Higgs Effective Field Theories

Operators of higher dimension are added to the SM Lagrangian without violating the symmetries of the theory

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i^{(5)}}{\Lambda}\mathcal{O}_i^{(5)} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_i \frac{c_i^{(7)}}{\Lambda^3}\mathcal{O}_i^{(7)} + \sum_i \frac{c_i^{(8)}}{\Lambda^4}\mathcal{O}_i^{(8)} + \cdots$$

c are the Wilson coefficients, Λ is some large energy scale

UV Properties of HEFTs

HEFTs are renormalizable in the modern sense *à la* Gomis-Weinberg, i.e.:
- Power-counting renormalizability is lost
- Physical Unitarity (cancellation of ghost states) guaranteed by BRST symmetry & Slavnov-Taylor identities
- Froissart bound usually not respected

In general all possible terms allowed by symmetry must be included in an EFT approach

J.Gomis, S.Weinberg, Are nonrenormalizable gauge theories renormalizable?
Nucl.Phys. B469 (1996) 473-487

One-loop Anomalous Dimensions in the HEFTs

However a *tour de force* computation
of one-loop anomalous dimensions in general HEFTs
involving dim. six operators
has revealed surprising cancellations.

R.Alonso, E.Jenkins, A.Manohar, M.Trott
arXiv:1308.2627 , arXiv:1310.4838 , arXiv:1312.2014 , arXiv:1409.0868

Not all mixings in principle allowed by the symmetries
do indeed arise at one loop level.

Holomorphy

Basic idea: holomorphic operators do not mix
with anti-holomorphic and non-holomorphic operators.

True at the one-loop level
(up to some breaking proportional to Yukawa couplings)
on the S-matrix elements.

C.Cheung and C.Shen, arXiv: 1505.01844

Off-shell UV Patterns in HEFTs of $\Phi^\dagger\Phi$

The subclass of HEFT generated by higher-dimensional operators involving powers of $\Phi^\dagger\Phi$ and ordinary derivatives thereof only has some peculiar UV properties.

Use $\Phi^\dagger\Phi$ (after spontaneous symmetry breaking) as a new dynamical variable.

Some additional symmetries become apparent.

Extra Fields and the Scalar Constraint

$$\int d^4x \left[\frac{1}{v}(X_1 + X_2)(\Box + 2\lambda v^2)\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\right) - \frac{1}{2}(M^2 - 2\lambda v^2)X_2^2\right]$$

$\Phi = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} i\phi_1 + \phi_2 \\ \sigma + v - i\phi_3 \end{smallmatrix}\right)$ SU(2) doublet $\qquad$ X_2 SU(2) singlet

A suitable additional BRST symmetry ensures that the physical degrees of freedom are unchanged.

Equations of Motion for the Auxiliary Field

$$\frac{\delta S}{\delta X_1} = \frac{1}{v}(\Box + 2\lambda v^2)\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\right) = \frac{1}{v}(\Box + 2\lambda v^2)\frac{\delta S}{\delta \bar{c}^*}$$

The e.o.m. is satisfied by the constraint

$$X_2 = \frac{1}{2v}\sigma^2 + \sigma + \frac{1}{2v}\phi_a^2$$

$$= \Phi^\dagger \Phi - \frac{v^2}{2}$$

On-shell Equivalence with the Standard Formulation

By substituting the e.o.m. solution for the singlet field
in the classical action one gets back the ordinary quartic potential

$$S|_{\mathrm{on-shell}} = \int d^4x \left[\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + \frac{1}{2}\partial^\mu\phi_a\partial_\mu\phi_a - \frac{1}{2}\frac{M^2}{v^2}\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2\right)^2\right]$$

with coupling constant

$$\lambda = \frac{1}{2}\frac{M^2}{v^2}$$

right sign of the quartic
potential needed to ensure stability
from the sign of the mass term,
in turn fixed by the requirement
of the absence of tachyons

Dangereous Interactions for Renormalizability

The model contains derivative interactions of the schematic form

$$\chi \Box \chi^2$$

i.e. an operator of dimension 5.

Renormalizability?

More symmetries needed

Propagators

The quadratic part is diagonalized by $\sigma = \sigma' + X_1 + X_2$

$$\Delta_{\sigma'\sigma'} = \frac{i}{p^2}\,, \quad \Delta_{\phi_a\phi_b} = \frac{i\delta_{ab}}{p^2}\,, \quad \Delta_{\bar{c}c} = \frac{i}{p^2}$$

$$\Delta_{X_1X_1} = -\frac{i}{p^2}\,, \quad \Delta_{X_2X_2} = \frac{i}{p^2 - M^2}\,.$$

The derivative interaction only depends on $X = X_1 + X_2$
whose propagator has an improved UV behaviour

$$\Delta_{XX} = \frac{iM^2}{p^2(p^2 - M^2)}$$

Thus the derivative interaction
is harmless and p.c. renormalizability still holds

Functional identities & Renormalization

X_1 equation

$$\frac{\delta\Gamma}{\delta X_1} = \frac{1}{v}(\Box + 2\lambda v^2)\frac{\delta\Gamma}{\delta\bar{c}^*}$$

X_2 equation & shift symmetry $\delta X_1(x) = \alpha(x)\,, \quad \delta X_2(x) = -\alpha(x)$

$$\frac{\delta\Gamma}{\delta X_2} = \frac{1}{v}(\Box + 2\lambda v^2)\frac{\delta\Gamma}{\delta\bar{c}^*} + (\Box + 2\lambda v^2)(X_1 + X_2) - (M^2 - 2\lambda v^2)X_2 - v\bar{c}^*$$

BSM Extensions: dim.6 operators

The X_2 equation is not the most general functional symmetry holding true for the vertex functional.
The breaking term on the R.H.S. of the shift symmetry stays linear in the quantum fields even if one adds a kinetic term for the scalar singlet

$$\int d^4x \, \frac{z}{2} \partial^\mu X_2 \partial_\mu X_2$$

Upon integration over the auxiliary field this is equivalent to the addition of the dimension-six operator

$$\int d^4x \; \frac{z}{v^2} \partial_\mu \Phi^\dagger \Phi \partial^\mu \Phi^\dagger \Phi$$

The X2-equation in the presence of a cubic interaction

$$g_6 X_2^3$$

One needs one more external source L
to define the X2-equation in the presence of a cubic interaction

$$\int d^4x \, L X_2^2$$

The X2-equation becomes

$$\begin{aligned}\frac{\delta\Gamma}{\delta X_2} =& \frac{1}{v}(\Box + 2\lambda v^2)\frac{\delta\Gamma}{\delta \bar{c}^*} + 3g_6\frac{\delta\Gamma}{\delta L} \\ &+ (\Box + 2\lambda v^2)(X_1 + (1-z)X_2) - (M^2 - 2\lambda v^2)X_2 - v\bar{c}^*\end{aligned}$$

valid to all orders in the loop expansion

Mapping on the HEFT

X2-theory

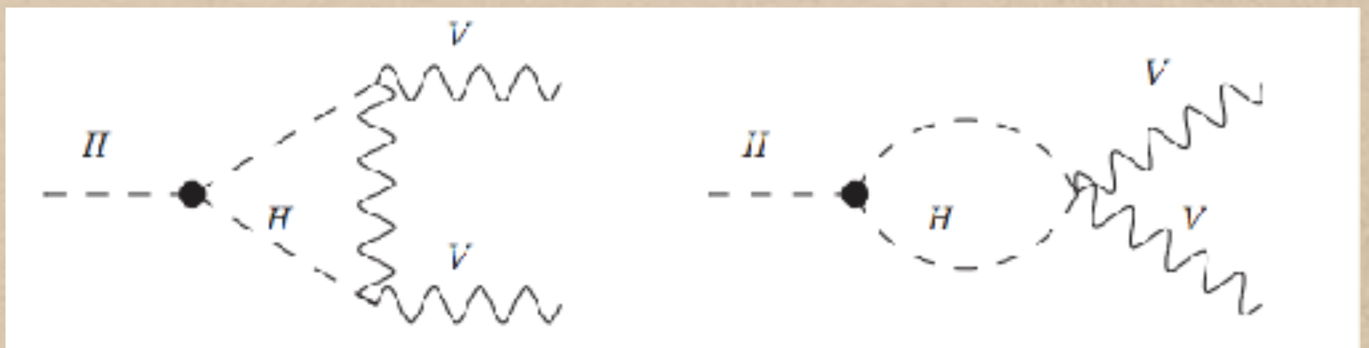
F.eqs. governing amplitudes involving the new dynamical variables in terms of ext. sources

Diagrammatic isolation of BSM operators

Transition function

HEFT

Do we generate derivative dim.6 ops if we add the third power of $\Phi^\dagger \Phi$?



What is the off-shell pattern of ops. mixing?

The mapping works as follows:

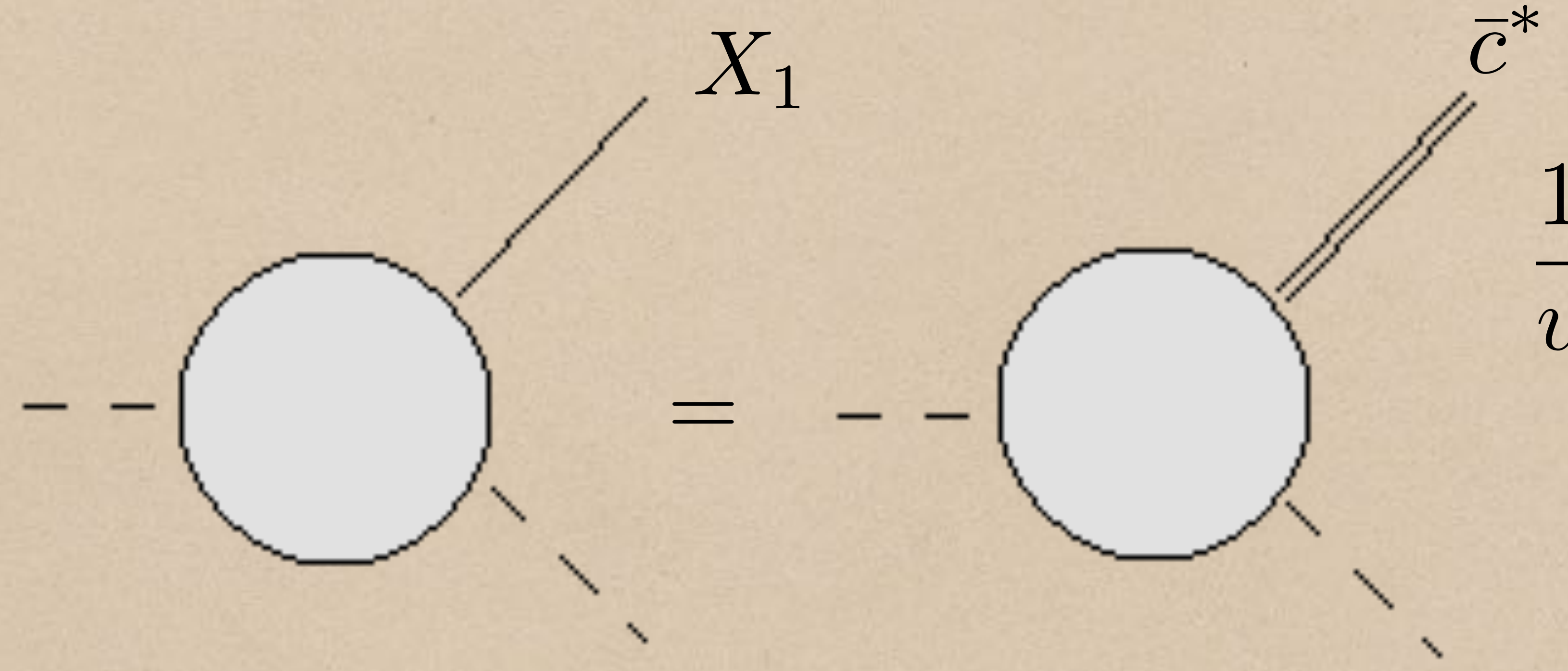
- replace all 1-PI insertions of X_1 with insertions of the constraint ext. source through the X_1 functional equation
- eliminate X_1 and X_2 in terms of the components of the scalar SU(2) Higgs doublet through their e.o.m.
- the resulting functional is conjectured to be the off-shell 1-PI vertex functional of the HEFT

Paper in preparation

Checks on the two and three-point off-shell amplitudes

Example of the three-point sigma amplitude

X1 insertions give contributions vanishing on-shell



$$m^2 = 2\lambda v^2$$

$$\frac{1}{v}(\Box + m^2)X_1 =|_{e.o.m.} -(\Box + M^2)X_2$$

Eliminate X2 via
the X1-e.o.m.

$$\Gamma_{X_1 \ldots} = \frac{1}{v}(\Box + m^2)\Gamma_{\bar{c}^* \ldots}$$

$$X_2 =|_{e.o.m.} \sigma + \frac{1}{2v}\sigma^2 + \ldots$$

X2 amplitudes yield the full g6-dependence

On-shell the g6-dependent part is confined into the three-point X2 amplitude in the mass (diagonal) basis

Checked
the correspondence
of the g6-dep. terms
off-shell

UV divergences in vertex functions

Again controlled by the X2- and X1-equations

More complicated relations
involving external sources insertions

The latter have better UV properties
than the field X2

Valid off-shell. Disentangle the SM part
from the BSM contributions in an algebraic way
(counterpart of diagrammatic separation)

Outlook

- HEFTs based on powers of $\Phi^\dagger\Phi$ and ordinary derivatives thereof have some nice UV properties rooted in some functional identities which become transparent if one uses the field X_2
- Some applications: off-shell operator mixing, consistent set of higher dimensional operators, resummation

Back-up slides

BRST implementation of the on-shell constraint

Off-shell there is one more scalar field X_1.
What about this field? Physical or unphysical?

BRST symmetry (it does not originate from gauge invariance)

$$sX_1 = vc\,, \qquad sc = 0\,, \qquad s\sigma = s\phi_a = sX_2 = 0\,,$$
$$s\bar{c} = \frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2\,.$$

Ghost action

$$S_{ghost} = -\int d^4x\,\bar{c}\Box c\,.$$

Invariance under the nilpotent
BRST symmetry
formally associated with a $U(1)_{constr}$ group

A.Q., Phys.Rev. D73 (2006) 065024