Radiation enhancement and “temperature” in the collapse regime of gravitational scattering

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Outline

- Introduction
  - ACV method for string collisions at transplanckian energies

- Graviton radiation
  - based on a unified emission amplitude
    - central region (Regge limit)
    - soft region (Weinberg limit)

- Results
  - final state radiation is a unitary “pure” state
  - role of gravitational radius: $\langle \omega \rangle \sim R^{-1}$
  - radiation enhancement for $b \sim R$
  - “quasi-temperature” $T \sim T_H$
Introduction
[Amati, Ciafaloni, Veneziano’88] considered scattering of 2 super-strings at transplanckian CM energies $2E = \sqrt{s} \gg M_P \equiv \sqrt{\hbar/G}$ and impact parameter $b$

\[ l_P \equiv \sqrt{\hbar G} \quad l_s \equiv \sqrt{\hbar \alpha'} \quad R \equiv 2G\sqrt{s} \quad b \]
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- According to Gen.Rel.
  formation of a macroscopic BH

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$$\sqrt{s} \gg M_P \iff R \gg l_P \iff \alpha_G \equiv Gs/\hbar \gg 1$$

  action/\hbar

- Semiclassical regime: $b, R \gg l_P, l_s$
Eikonal regime: elastic $S$-matrix

String amplitudes in Regge limit ($s \to \infty$, $t$ fixed):

\[ M_N(s, t) \xrightarrow{s \gg t} q_1 \cdots q_N \equiv \frac{i^N s}{N!} \int \mathcal{V}_N(q_1, \cdots, q_N) \times \]
\[ A_{el}(s, q_1) \cdots A_{el}(s, q_N) \delta(q - \sum_n q_n) dq_1 \cdots dq_N \times \mathcal{V}_N(q_1, \cdots, q_N) \]

(reggeized) graviton exchanges between (excited) string states
Eikonal regime: elastic $S$-matrix

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(Strings are extended objs: $q \sim \hbar/b$ are soft, $\langle N \rangle \sim Gs/\hbar$ is large)

$Q \sim Gs/b$ can be large $\Rightarrow$ finite $\Theta_E \sim R/b$
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(reggeized) graviton exchanges between (excited) string states

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$\Rightarrow Q \sim Gs/b$ can be large $\Rightarrow$ finite $\Theta_E \sim R/b$

In semiclassical regime $b, R \gg \lambda_s$ strings are not excited $\Rightarrow V_N \to 1$ (point particles)

$M_N$ is convolution in $q$ space diagonalized in impact-parameter $b$ space

$$S_{el}(s, b) = 1 + iM(s, b) = \sum_{N=0}^{\infty} \frac{(iA_{el}(s, b))^N}{N!} = e^{iA_{el}} = e^{i2\delta(s, b)}$$

Phase shift $\delta(s, b) = \alpha_G \Delta(b)$, $\Delta(b) = \log \frac{L}{b}$; $\Theta_E = \frac{\sqrt{s} \partial \delta}{\partial b} = \frac{2R}{b}$
Gravity is strong
⇒ correction to eikonal are important
Corrections \((\lambda_s / b)^n\) are negligible
Corrections \((R/b)^n\) are essential:

[ACV ’90]

H-diagrams

\(\bullet = \text{Lipatov’s vertex}\)
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- Phase shift \(\delta(s, b)\) acquires an imaginary part for \(b < b_c = 1.6R\)
- \(|S_{\text{el}}(s, b)| = e^{2i\delta} < 1\)
- \(|S_{\text{el}}(s, 0)| = e^{-\pi\alpha_G} = e^{-\pi G s/\hbar}\)
Collapse region: \( b \sim R \gg \lambda_s \)

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- Suppression of elastic ch. compensated by production of gravitons (unitarity)?
- Is \( b_c \) the signal of gravitational collapse?
\[ 1 - |S_{el}| \sim (b_c - b)^{3/2} \]
Radiation
Graviton radiation: Unified amplitude

- Basic ingredient: Lipatov’s vertex (accurate only if $\theta \gg \theta_s$)

\[
\frac{M_{\text{Regge}}}{\kappa^3 s} = \frac{k^* q - k q^*}{kk^* qq'^*}
\]
Graviton radiation: Unified amplitude

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- In complementary region $\theta \sim \theta_s$ (and $\omega \ll \sqrt{s}$) [Weinberg '65] current is OK

$$\frac{M_{\text{Regge}}}{\kappa^3 s} = \frac{k^* q - kq^*}{kk^* qq'^*}$$

$$\frac{M_{\text{soft}}}{\kappa^3 s} = \frac{k^* q - kq^*}{(k - \frac{\omega}{E} q)k^* qq'^*}$$
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\]

In coordinate space: “Soft-based” representation

\[
M_{\text{unified}}(b, x) = \frac{1}{x^2} \left[ \frac{E}{\omega} \ln \left| \frac{b - \frac{\omega}{E} x}{b} \right| - (E \to \omega) \right] = M_{\text{soft}}(E) - M_{\text{soft}}(\omega)
\]

interpretation: graviton insertions on external + internal legs
Resummation

Total single-graviton emission amplitude resums emissions from the whole ladder

- “Local” incoming particle has direction $\theta_{\text{in}} \neq 0$
- Corresponding amplitude related by a rotation

$$M(\theta_{\text{in}}, \theta) = e^{i2\phi_\theta} M(0, \theta - \theta_{\text{in}}) e^{-i2\phi_\theta - \theta_{\text{in}}}$$

- Amplitudes interfere constructively only if $\theta \gg \theta_{\text{in}} \sim \theta_s$
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- Emitted graviton can rescatter with incident particles

$x = \text{graviton transv. position}$
$b = \text{opposite particle}$
interaction $= G\omega\sqrt{s}\Delta(b - x)$
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interaction = $G\omega \sqrt{s} \Delta(b - x)$

\[
M_{\text{tot}}(b, k) = e^{i2\alpha G \Delta(b)} M_\lambda(b, k) \quad (k = \omega \theta)
\]

\[
M_\lambda(b, k) = \frac{e^{i\lambda \phi_\theta}}{(2\pi)^2} \frac{\sqrt{\alpha G}}{i\omega} \int \frac{d^2x}{x^2} \frac{e^{iq \cdot x}}{e^{i\lambda \phi_x}} \left\{ e^{i2\alpha G \left[ \Delta(b - \frac{\omega}{E} x) - \Delta(b) \right]} - e^{-i2\omega R \left[ \Delta(b) - \Delta(b - x) \right]} \right\}
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\[ M_{tot}(b, k) = e^{i2\alpha_G \Delta(b)} M_\lambda(b, k) \quad (k = \omega \theta) \quad \text{note!} \]

\[ M_\lambda(b, k) = \frac{e^{i\lambda \phi_{\theta}}}{(2\pi)^2} \frac{\sqrt{\alpha_G}}{i\omega} \int \frac{d^2 x}{x^2} \frac{e^{i q \cdot x}}{e^{i\lambda \phi_{\theta}}} \left\{ e^{i2\alpha_G [\Delta(b - \frac{\omega}{E} x) - \Delta(b)]} - e^{-i2\omega R [\Delta(b) - \Delta(b - x)]} \right\} \]

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$b =$ opposite particle

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Multiple Emissions

In eikonal approximation $b \gg R$: multi-graviton emissions are independent

$$M_{\text{tot}}(2 \rightarrow 2 + k_1 \cdots k_n) \equiv e^{i2\delta_0(b)} \frac{1}{n!} M(k_1) \cdots M(k_n)$$
Multiple Emissions

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\[ |\text{gravitons}\rangle = e^{i2\delta_0(b)} \exp \left\{ i \sum_{\lambda=\pm} \int \frac{d^3k}{2\omega_k} \mathcal{M}^{(\lambda)}(k)a^{(\lambda)\dagger}(k) + \text{h.c.} \right\} |0\rangle 
= S|0\rangle \quad \text{virtual contribution à la Weinberg} \]

- Unitary $S$-matrix describing transplanckian collisions
Results
Spectrum: subplanckian from transplanckian

\[
\frac{dE_{GW}}{d\omega} = \omega \int \frac{d^2\theta}{(2\pi)^3} \sum_{\lambda=\pm} |M_{\lambda}(b, \omega\theta)|^2 \xrightarrow{\omega R \gg 1} 0.2 \frac{G_s \Theta_E^2}{\omega R}
\]

- Agrees with Zero Frequency Limit \( (\omega \to 0) \)
- Almost universal \((b\)-independent\) shape, Characteristic frequency \( \langle \omega \rangle \sim 1/R \)
- \( \langle \omega \rangle \) decreases as \( \sqrt{s} \) increases, like Hawking radiation
- Agrees with \([Gruzinov, Veneziano 2015]\) for \( \hbar \to 0 \)
Towards $b \rightarrow R$

We have to take into account

- H diagrams along the ladder
- emissions from H diagrams
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\[ e^{i2\delta_0} \rightarrow e^{i2\delta} \]
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evaluated using "soft-based" representation

\[ M_H = M_{H,\text{soft}} - M_{H,\text{soft}}|_{E \rightarrow \omega} \]
Towards $b \to R$

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$e^{i2\delta_0} \to e^{i2\delta}$

evaluated using “soft-based” representation

$M_H = M_{H,\text{soft}} - M_{H,\text{soft}} \big| E \to \omega$

Neglecting correlations, we end up with the same formulas, if $\Delta_0(b) \to \Delta_0(b) + \Delta_H(b) \equiv \Delta(b)$

$\Delta$ has the branch-cut at $b_c = 1.6R$

- For $b \to b_c$ significant enhancement of radiation $\sim \omega^{-2/3}$ instead of $\omega^{-1}$
  due to large tidal forces
- increasing fraction of energy is radiated off
Energy conservation and “temperature”

Due to radiation enhancement, it is important to take into account correlations due to energy conservation.

Following [Veneziano’05], we impose en.cons. event by event by requiring $\sum_j \omega_j \leq E$ and extending this bound to virtual corrections on the basis of [AGK’72] cutting rules.
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\[
P_n = P_0 \frac{1}{n!} \left( \prod_{j=1}^{n} \int d^3 k_j \ |M(k_j)|^2 \right) \Theta(E - (\omega_1 + \cdots + \omega_n))
\]

\[
N(E) \equiv \sum_{n=0}^{\infty} P_n = \int_{-i\infty}^{+i\infty} \frac{d\lambda}{2\pi i} \frac{e^{\lambda E}}{\lambda + \varepsilon} \exp \left\{ \int d^3 k \ |M(k)|^2 [e^{\omega \lambda} - 1] \right\}
\]

\[
\frac{dE^{GW}}{d\omega} \bigg|_{\text{ener cons}} \simeq \frac{dE^{GW}}{d\omega} \bigg|_0 \times \frac{N(E - \omega_k)}{N(E)}
\]

\[
\simeq \frac{dE^{GW}}{d\omega} \bigg|_0 \times e^{-\omega/\tau}
\]

\[
\tau \sim b \to b_c \frac{\hbar}{R} \simeq T_{\text{Hawking}}(0.1 \sqrt{s})
\]
Finally we cross the critical impact parameter and reach \( b = 0 \)

- Elastic amplitude provides exponential suppression: \( \Delta(b = 0) = i\pi/2 \)
  \[
  \sim \exp(-\pi Gs) = \exp(-\pi ER)
  \]

- Rescattering term of emission factor is enhanced \( \sim \exp(+\pi \omega R) \)

\[
M_n(b, k_1, \ldots, k_n) = e^{i2\alpha G \Delta(b)} \prod_{j=1}^{n} M_{\lambda_j}(b, k_j)
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Collapse region: \( b < b_c \)

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\]

- If \( \sum_j \omega_j = E \) enhancement compensates suppression: unitarity is possible

- First preliminary estimate of quasi-temperature yields \( (k_B = 1) \)
  \[ T \simeq 0.7 \frac{\hbar}{R} \simeq T_H (\sqrt{s}/9) \] similar to \( T \) at \( b \simeq b_c \)
Conclusions

- We can compute graviton radiation in transplanckian collisions
  - Determined unified limiting form of graviton emission amplitudes
  - Resolution of energy crisis: $\sim 1/\omega$ spectrum for $\omega > R^{-1}$

- We see the role of $R^{-1} = \langle \omega \rangle$ in spectrum like in Hawking radiation $\forall s, b$

- For $b \lesssim R$ non-linear effects (tidal forces) provide enhanced emission; all energy is radiated off

- Requiring energy conservation:
  - coherent radiation sample $\Rightarrow$ no information loss
  - Spectrum is exponentially suppressed, like thermal radiation
  - “quasi-temperature” $\sim$ Hawking’s $T_H$ for a BH somewhat lighter than $\sqrt{s}$

- Suggests a possible mechanism of solving the information paradox