Radiation enhancement and "temperature" in the collapse regime of gravitational scattering

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In collaboration with M. Ciafaloni, F. Coradeschi and G. Veneziano

Venice, 8 July 2017

Outline

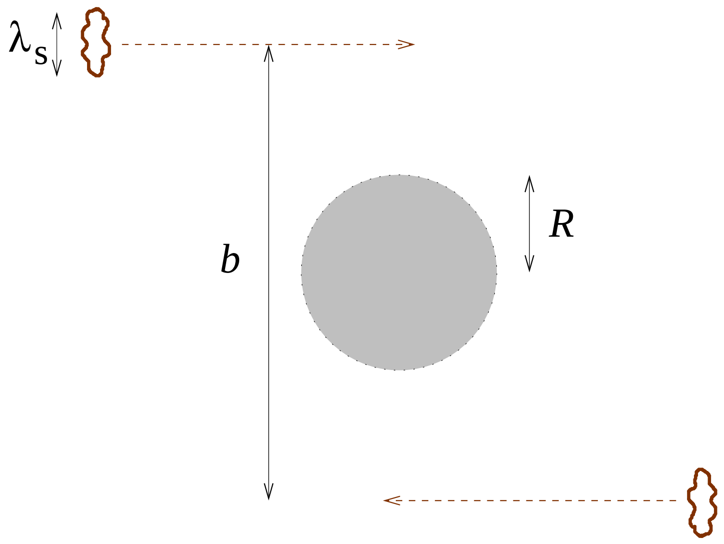
- Introduction
  - ACV method for string collisions at transplanckian energies
- Graviton radiation
  - based on a unified emission amplitude
    - central region (Regge limit)
    - soft region (Weinberg limit)
- Results
  - final state radiation is a unitary “pure” state
  - role of gravitational radius: $\langle\omega\rangle \sim R^{-1}$
  - radiation enhancement for $b \sim R$
  - “quasi-temperature” $T \sim T_H$

Introduction

String scattering

[Amati,Ciafaloni,Veneziano'88] considered scattering of 2 super-strings
at transplanckian CM energies $2E = \sqrt{s} \gg M_P \equiv \sqrt{\hbar/G}$ and impact parameter b

$l_P \equiv \sqrt{\hbar G}$ $\qquad$ $l_s \equiv \sqrt{\hbar\alpha'}$ $\qquad$ $R \equiv 2G\sqrt{s}$ $\qquad$ b



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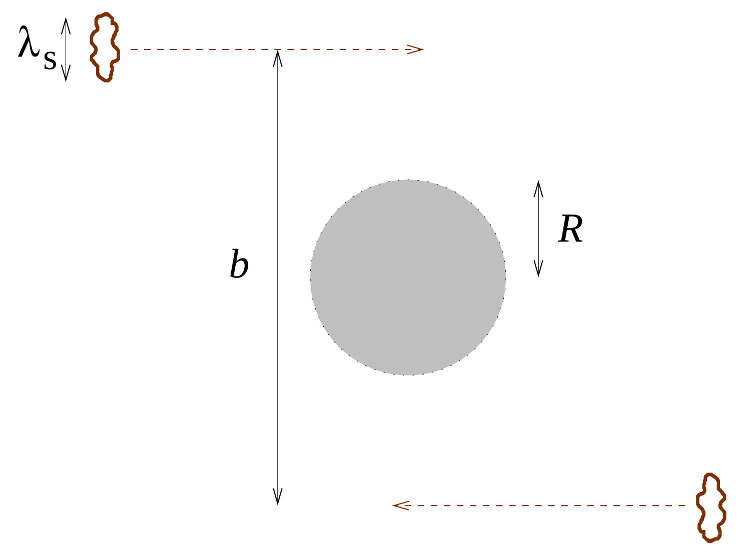
$$\sqrt{s} \gg M_P \qquad \Longleftrightarrow \qquad R \gg l_P$$



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$$\Longleftrightarrow \quad \alpha_G \equiv Gs/\hbar \gg 1$$
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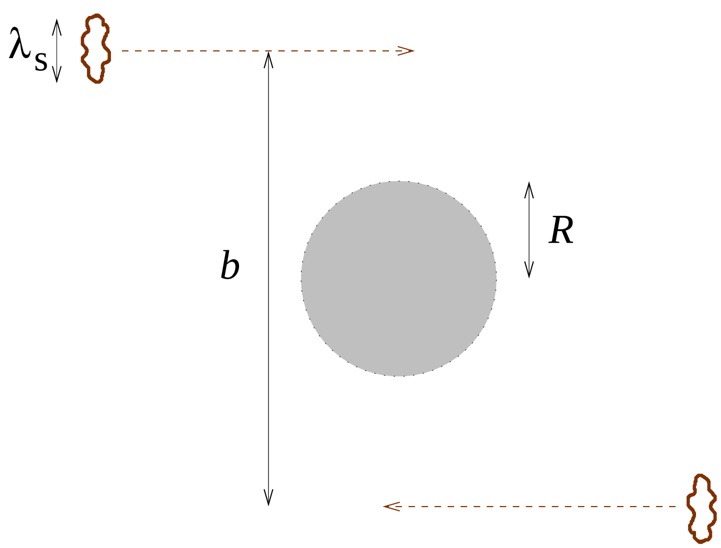
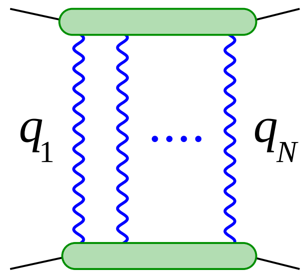


- Semiclassical regime: $b, R \gg l_P, l_s$

Eikonal regime: elastic S-matrix

String amplitudes in Regge limit ($s \to \infty$, t fixed):

$$M_N(s,t) \xrightarrow{s \gg t}$$



$$\equiv \frac{\mathrm{i}^N s}{N!} \int \begin{array}{l} V_N(\boldsymbol{q}_1, \cdots, \boldsymbol{q}_N) \times \\ A_{\mathrm{el}}(s, \boldsymbol{q}_1) \cdots A_{\mathrm{el}}(s, \boldsymbol{q}_N) \;\; \delta(\boldsymbol{q} - \sum_n \boldsymbol{q}_n) \mathrm{d}\boldsymbol{q}_1 \cdots \mathrm{d}\boldsymbol{q}_N \\ \times V_N(\boldsymbol{q}_1, \cdots, \boldsymbol{q}_N) \end{array}$$

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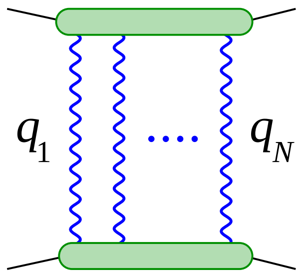
$M_N(s,t) \xrightarrow{s \gg t}$ 

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$\Rightarrow \boldsymbol{Q} \sim Gs/b$ can be large $\Rightarrow$ finite $\Theta_E \sim R/b$

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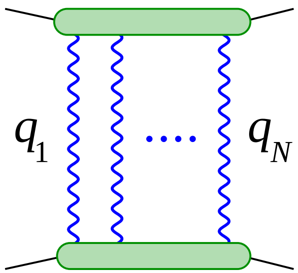
$M_N(s,t) \xrightarrow{s \gg t}$ q_1  q_N $\equiv \frac{\mathrm{i}^N s}{N!} \int$

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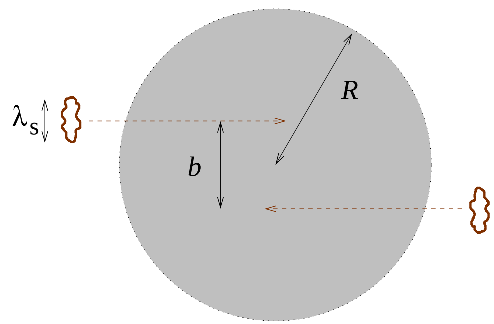
In semiclassical regime $b\,,\; R \gg \lambda_s$ strings are not excited $\Rightarrow V_N \to 1$ (point particles)
M_N is convolution in $\boldsymbol{q}$ space diagonalized in *impact-parameter* $\boldsymbol{b}$ space

$$S_{\mathrm{el}}(s, \boldsymbol{b}) = 1 + \mathrm{i}M(s, \boldsymbol{b}) = \sum_{N=0}^{\infty} \frac{(\mathrm{i}A_{\mathrm{el}}(s, \boldsymbol{b}))^N}{N!} = \mathrm{e}^{\mathrm{i}A_{\mathrm{el}}} = \mathrm{e}^{\mathrm{i}\,2\delta(s,\boldsymbol{b})}$$

Phase shift $\delta(s, \boldsymbol{b}) = \alpha_G \Delta(\boldsymbol{b})\,, \quad \Delta(\boldsymbol{b}) = \log \frac{L}{b}\;;$ $\qquad \Theta_E = \frac{\sqrt{s}}{4\hbar} \frac{\partial \delta}{\partial b} = \frac{2R}{b}$

Collapse region: $b \sim R \gg \lambda_s$



Gravity is strong

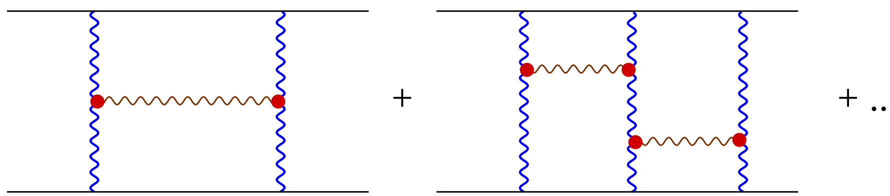
$\Rightarrow$ correction to eikonal are important

Corrections $(\lambda_s/b)^n$ are negligible

Corrections $(R/b)^n$ are essential:

[ACV '90]

H-diagrams



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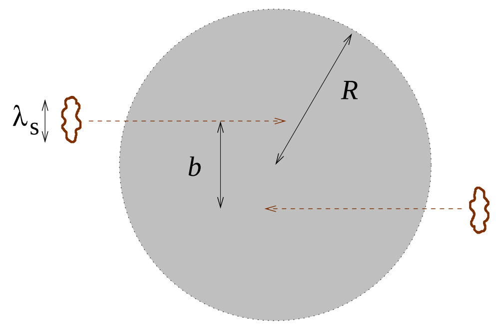

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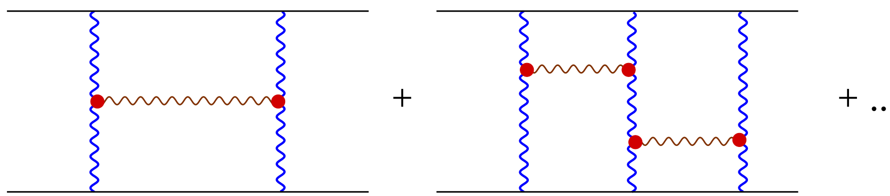

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- $|S_{\rm el}(s, \boldsymbol{b})| = \mathrm{e}^{2\mathrm{i}\delta} < 1$ $\qquad |S_{\rm el}(s, 0)| = \mathrm{e}^{-\pi\alpha_G} = \mathrm{e}^{-\pi G s/\hbar}$

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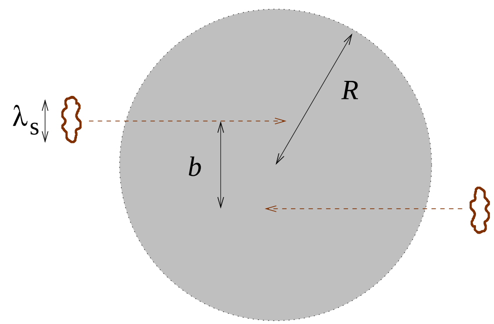

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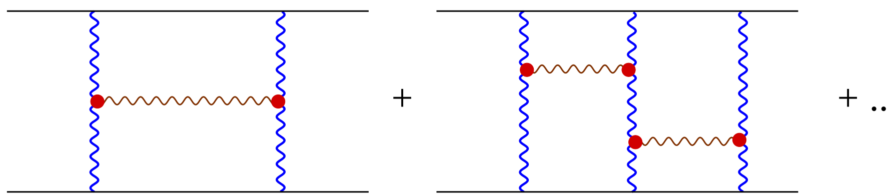

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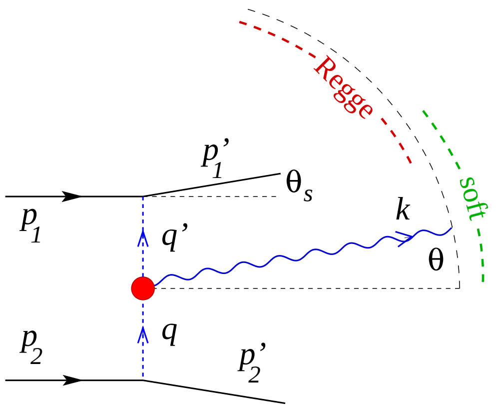
- Suppression of elastic ch. compensated by production of gravitons (unitarity)?
- Is b_c the signal of gravitational collapse? $\qquad 1 - |S_{\rm el}| \sim (b_c - b)^{3/2}$

Radiation

Graviton radiation: Unified amplitude

- Basic ingredient: Lipatov's vertex (accurate only if $\theta \gg \theta_s$)



$$\frac{M_{\text{Regge}}}{\kappa^3 s} = \frac{\boldsymbol{k}^* \boldsymbol{q} - \boldsymbol{k} \boldsymbol{q}^*}{\boldsymbol{k}\boldsymbol{k}^* \boldsymbol{q} \boldsymbol{q}'^*}$$

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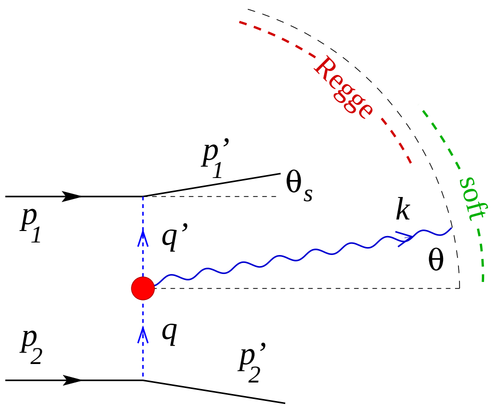

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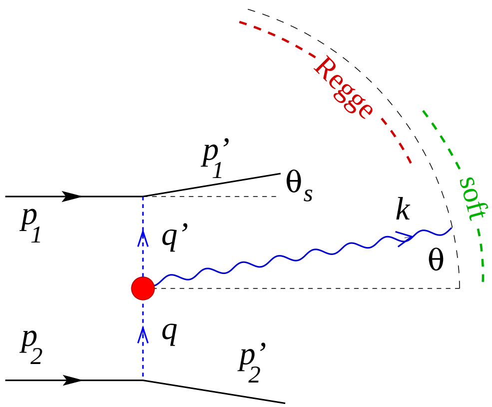

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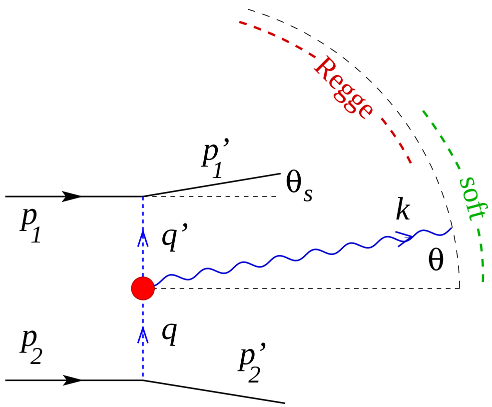

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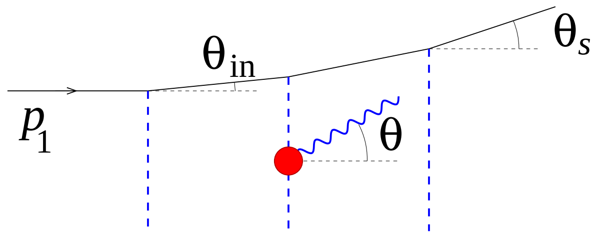
In coordinate space: "Soft-based" representation

$$M_{\text{unified}}(\boldsymbol{b}, \boldsymbol{x}) = \frac{1}{\boldsymbol{x}^2}\left[\frac{E}{\omega}\ln\left|\frac{\boldsymbol{b} - \frac{\omega}{E}\boldsymbol{x}}{\boldsymbol{b}}\right| - (E \to \omega)\right] = M_{\text{soft}}(E) - M_{\text{soft}}(\omega)$$

interpretation: graviton insertions on external + internal legs

Resummation

Total single-graviton emission amplitude resums emissions from the whole ladder



- "Local" incoming particle has direction $\boldsymbol{\theta}_{\rm in} \neq 0$
- Corresponding amplitude related by a rotation

$$M(\boldsymbol{\theta}_{\rm in}, \boldsymbol{\theta}) = \mathrm{e}^{\mathrm{i}2\phi_{\boldsymbol{\theta}}} M(\mathbf{0}, \boldsymbol{\theta} - \boldsymbol{\theta}_{\rm in}) \mathrm{e}^{-\mathrm{i}2\phi_{\boldsymbol{\theta}-\boldsymbol{\theta}_{\rm in}}}$$

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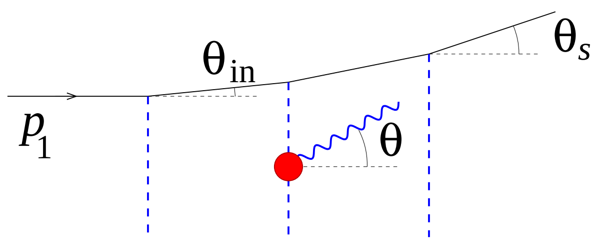

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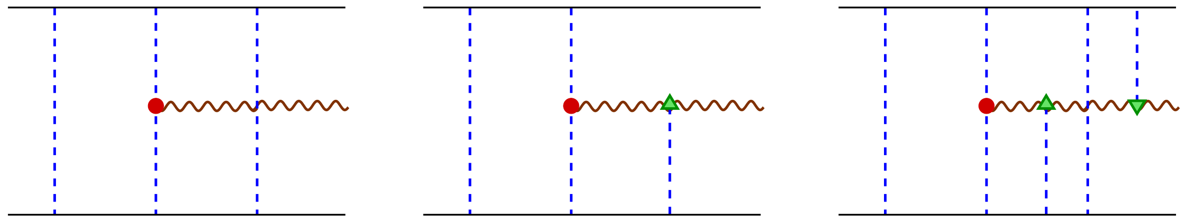

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interaction $= G\omega\sqrt{s}\Delta(\boldsymbol{b} - \boldsymbol{x})$

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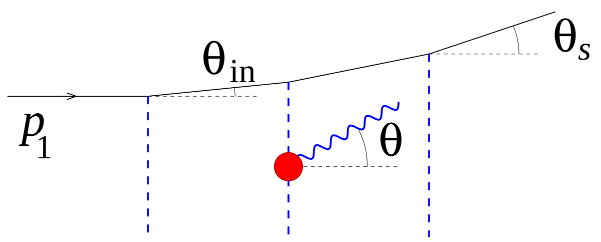

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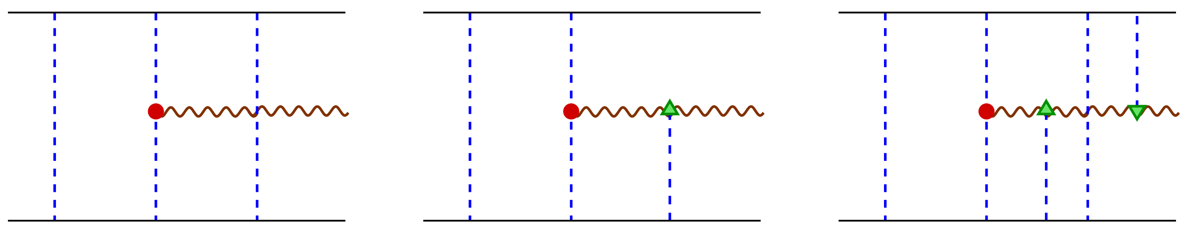

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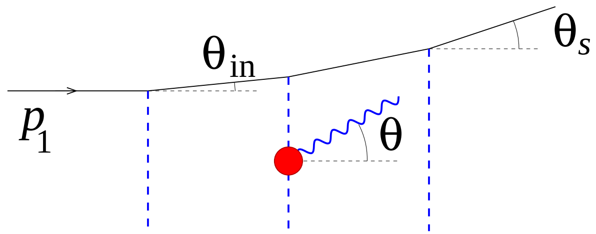

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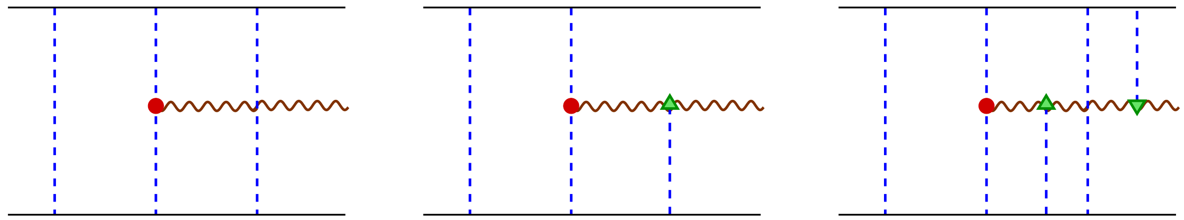

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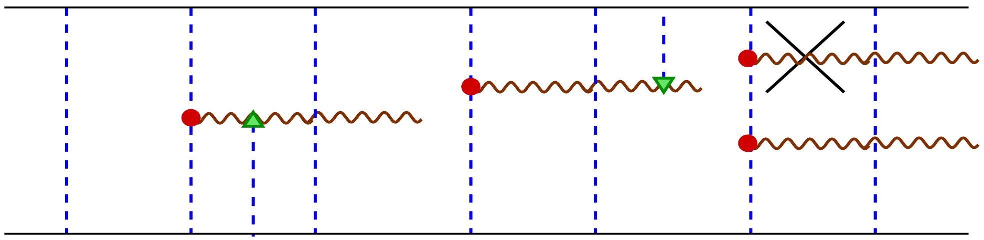
Multiple Emissions

In eikonal approximation $b \gg R$: multi-graviton emissions are independent



$$M_{\rm tot}(2 \to 2 + \boldsymbol{k}_1 \cdots \boldsymbol{k}_n) \equiv e^{i2\delta_0(b)} \frac{1}{n!} \mathfrak{M}(\boldsymbol{k}_1) \cdots \mathfrak{M}(\boldsymbol{k}_n)$$

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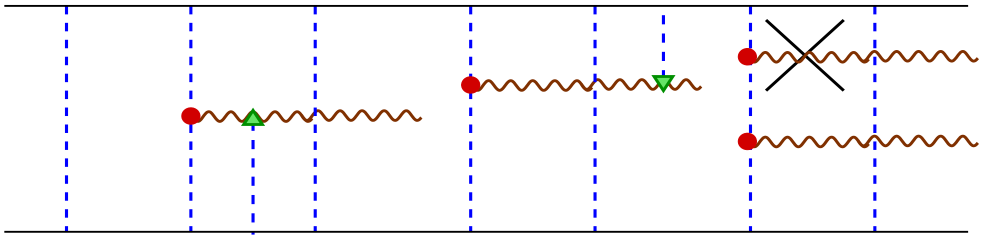

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$$|\text{gravitons}\rangle = \mathrm{e}^{\mathrm{i}2\delta_0(b)} \exp\left\{ \mathrm{i} \sum_{\lambda=\pm} \int \frac{\mathrm{d}^3 k}{2\omega_k} \mathfrak{M}^{(\lambda)}(k) a^{(\lambda)\dagger}(k) + \text{h.c.} \right\} |0\rangle$$

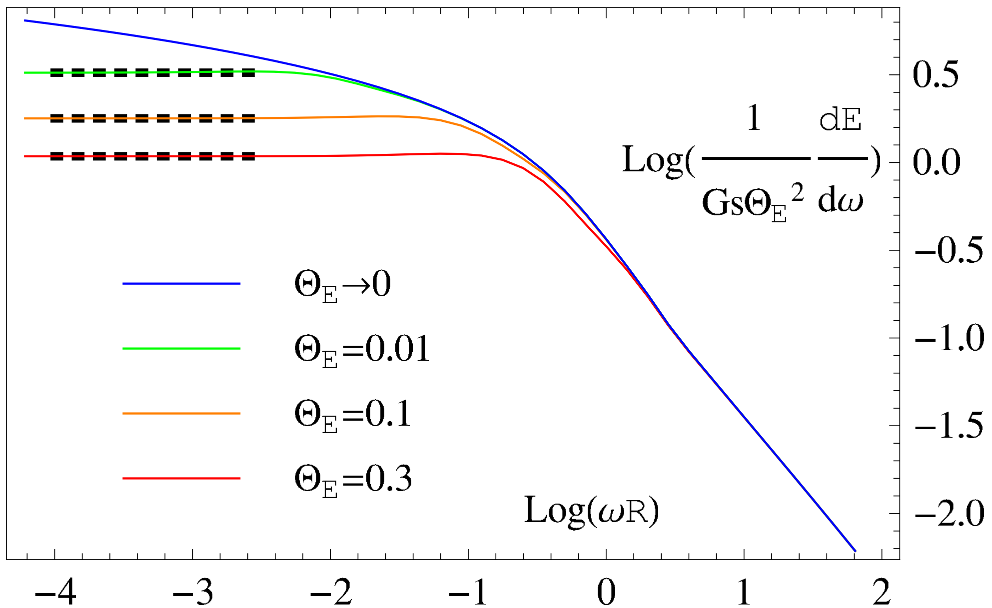
$$= S|0\rangle \qquad \text{virtual contribution à la Weinberg}$$

- Unitary S-matrix describing transplanckian collisions

Results

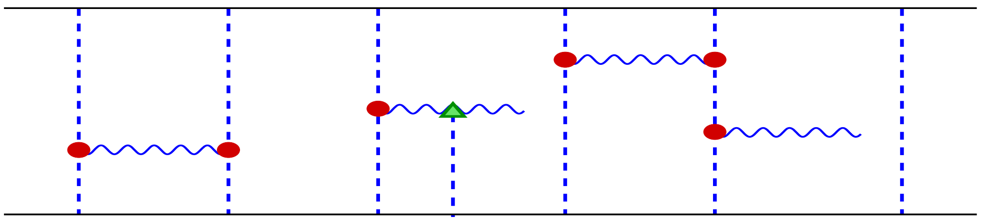
Spectrum: subplanckian from transplanckian

$$\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}\omega} = \omega \int \frac{\mathrm{d}^2\boldsymbol{\theta}}{(2\pi)^3} \sum_{\lambda=\pm} |\mathfrak{M}_\lambda(\boldsymbol{b}, \omega\boldsymbol{\theta})|^2 \xrightarrow[\omega R \gg 1]{} 0.2 \frac{Gs\,\boldsymbol{\Theta}_E^2}{\omega R}$$



- Agrees with Zero Frequency Limit ($\omega \to 0$)
- Almost universal (b-independent) shape, Characteristic frequency $\langle\omega\rangle \sim 1/R$
- $\langle\omega\rangle$ decreases as $\sqrt{s}$ increases, like Hawking radiation
- Agrees with *[Gruzinov,Veneziano 2015]* for $\hbar \to 0$

Towards $b \to R$



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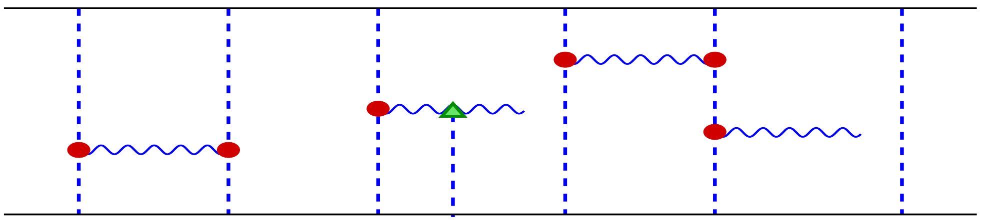

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$$e^{i2\delta_0} \to e^{i2\delta}$$

- emissions from H diagrams

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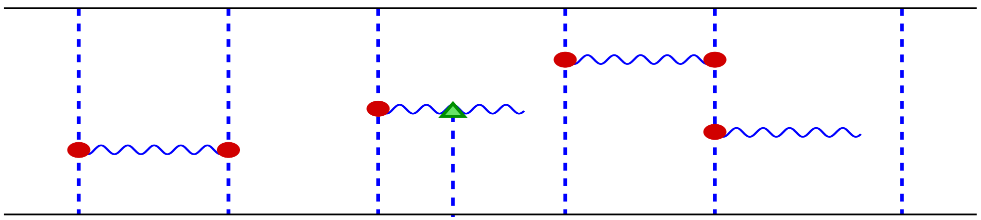

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$e^{i2\delta_0} \to e^{i2\delta}$

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evaluated using "soft-based" representation

$$M_H = M_{H,\text{soft}} - M_{H,\text{soft}}\big|_{E\to\omega}$$

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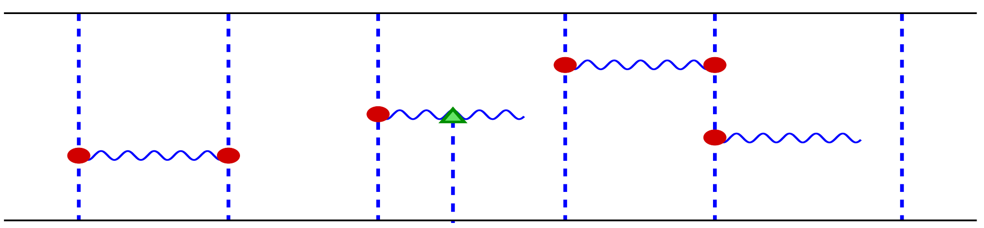

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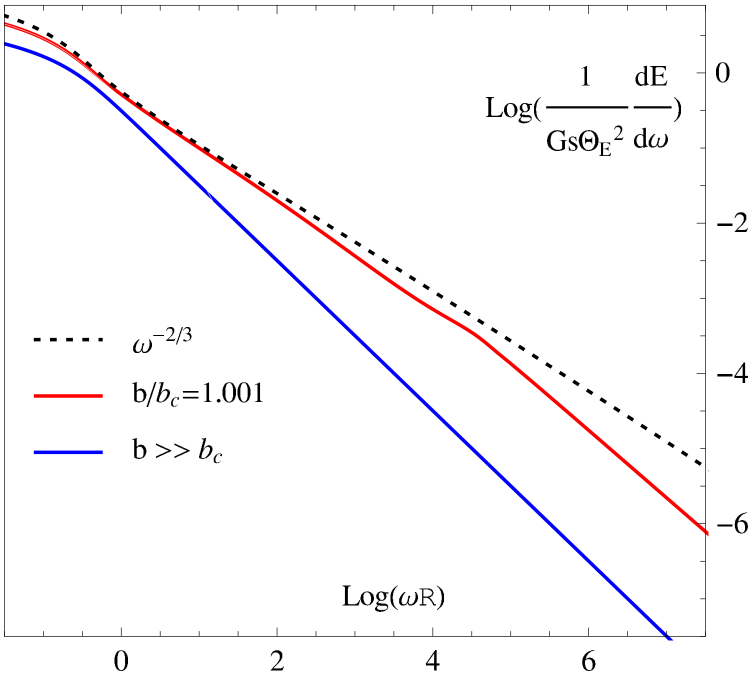
Neglecting correlations, we end up with the same formulas, if $\Delta_0(b) \to \Delta_0(b) + \Delta_H(b) \equiv \Delta(b)$
Δ has the branch-cut at $b_c = 1.6R$

- For $b \to b_c$ significant enhancement of radiation $\sim \omega^{-2/3}$ instead of ω^{-1} due to large tidal forces
- increasing fraction of energy is radiated off



Energy conservation and "temperature"

Due to radiation enhancement, it is important to take into account correlations due to energy conservation

Following *[Veneziano'05]*, we impose en.cons. event by event by requiring $\sum_j \omega_j \leq E$ and extending this bound to virtual corrections on the basis of *[AGK'72]* cutting rules

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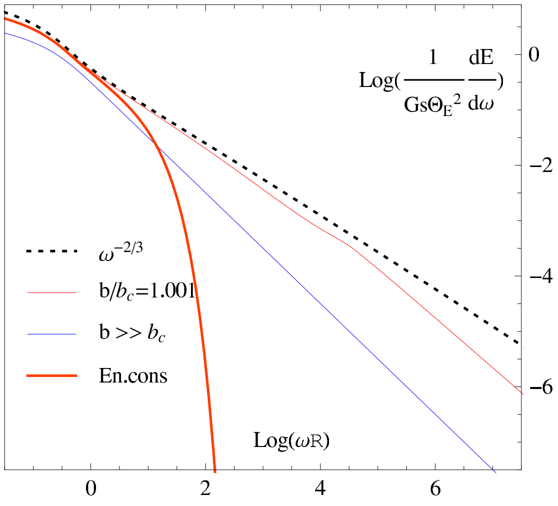
$$P_n = P_0 \frac{1}{n!} \left(\prod_{j=1}^{n} \int \mathrm{d}^3 k_j \; |\mathfrak{M}(k_j)|^2 \right) \; \Theta\big(E - (\omega_1 + \cdots + \omega_n)\big)$$

$$N(E) \equiv \sum_{n=0}^{\infty} P_n = \int_{-\mathrm{i}\infty}^{+\mathrm{i}\infty} \frac{\mathrm{d}\lambda}{2\pi\mathrm{i}} \, \frac{\mathrm{e}^{\lambda E}}{\lambda + \varepsilon} \exp\left\{ \int \mathrm{d}^3 k \; |\mathfrak{M}(k)|^2 [\mathrm{e}^{\omega\lambda} - 1] \right\}$$

$$\left.\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}\omega}\right|_{\substack{\text{ener}\\ \text{cons}}} \simeq \left.\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}\omega}\right|_0 \times \frac{N(E-\omega_k)}{N(E)}$$

$$\simeq \left.\frac{\mathrm{d}E^{\mathrm{GW}}}{\mathrm{d}\omega}\right|_0 \times \mathrm{e}^{-\omega/\tau}$$

$$\tau \overset{b\to b_c}{\sim} \frac{\hbar}{R} \simeq T_{\text{Hawking}}(0.1\sqrt{s})$$



Collapse region: $b < b_c$

Finally we cross the critical impact parameter and reach $b = 0$

- Elastic amplitude provides exponential suppression: $\Delta(\boldsymbol{b} = 0) = \mathrm{i}\pi/2$
  $\sim \exp(-\pi G s) = \exp(-\pi E R)$
- Rescattering term of emission factor is enhanced $\sim \exp(+\pi\,\omega\,R)$

$$M_n(\boldsymbol{b}, \boldsymbol{k}_1, \cdots, \boldsymbol{k}_n) = \mathrm{e}^{\mathrm{i}2\alpha_G \Delta(\boldsymbol{b})} \prod_{j=1}^{n} \mathfrak{M}_{\lambda_j}(\boldsymbol{b}, \boldsymbol{k}_j)$$

$$\mathfrak{M}_\lambda(\boldsymbol{b}, \boldsymbol{k}) = \frac{\mathrm{e}^{\mathrm{i}\lambda\phi_{\boldsymbol{\theta}}}}{(2\pi)^2} \frac{\sqrt{\alpha_G}}{\mathrm{i}\omega} \int \frac{\mathrm{d}^2\boldsymbol{x}}{\boldsymbol{x}^2} \frac{\mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}}}{\mathrm{e}^{\mathrm{i}\lambda\phi_{\boldsymbol{x}}}} \left\{ \mathrm{e}^{\mathrm{i}2\alpha_G\left[\Delta(\boldsymbol{b}-\frac{\omega}{E}\boldsymbol{x})-\Delta(\boldsymbol{b})\right]} - \mathrm{e}^{-\mathrm{i}2\omega R[\Delta(\boldsymbol{b})-\Delta(\boldsymbol{b}-\boldsymbol{x})]} \right\}$$

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- If $\sum_j \omega_j = E$ enhacement compensates suppression: unitarity is possible
- First preliminary estimate of quasi-temperature yields ($k_B = 1$)
  $T \simeq 0.7\frac{\hbar}{R} \simeq T_H(\sqrt{s}/9)$ similar to T at $b \simeq b_c$

Conclusions

- We can compute graviton radiation in transplanckian collisions
  - Determined unified limiting form of graviton emission amplitudes
  - Resolution of energy crisis: $\sim 1/\omega$ spectrum for $\omega > R^{-1}$
- We see the role of $R^{-1} = \langle\omega\rangle$ in spectrum like in Hawking radiation $\forall s\,,\ b$
- For $b \lesssim R$ non-linear effects (tidal forces) provide enhanced emission; all energy is radiated off
- Requiring energy conservation:
  - coherent radiation sample $\Rightarrow$ no information loss
  - Spectrum is exponentially suppressed, like thermal radiation
  - "quasi-temperature" $\simeq$ Hawking's T_H for a BH somewhat lighter than $\sqrt{s}$
- Suggests a possible mechanism of solving the information paradox