Predictions for Production and Decay of the Pseudoscalar Glueball from the Witten-Sakai-Sugimoto Model

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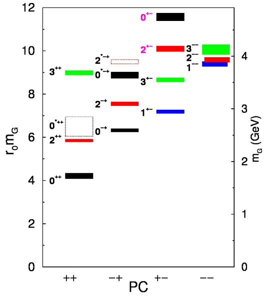
EPS-HEP Venice 2017

Ever elusive: Glueballs

Spectrum of *bare* glueballs
(prior to mixing with $q\bar{q}$ states)
more or less known from lattice:

$m_{0^{++}} \sim 1.7$ GeV
$m_{2^{++}} \sim 2.4$ GeV
$m_{0^{-+}} \sim 2.6$ GeV
...

Morningstar & Peardon hep-lat/9901004



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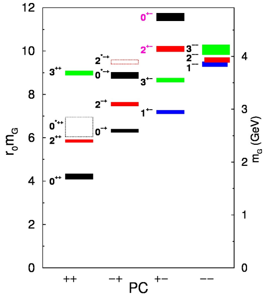

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Interactions of glueballs still unclear:

- Are glueballs broad or narrow?
- Do they mix with $q\bar{q}$ strongly or weakly?

$\rightarrow$ no conclusive identification of any glueball in meson spectrum

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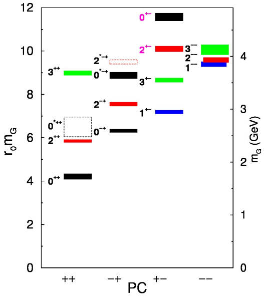

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most discussed lowest 0^{++} candidates:

narrow $f_0(1500)$ or $f_0(1710)$ vs. very broad background ("red dragon")

various phenomenological models describe $f_0(1500)$ or $f_0(1710)$
alternatingly as ~50-70% or ~75-90% glue

[G and two isoscalar $q\bar{q}$ states $u\bar{u} + d\bar{d}$ and $s\bar{s}$ can be shared by $f_0(1370)$, $f_0(1500)$, $f_0(1710)$]

Even more elusive: Pseudoscalar glueball

Pseudoscalar glueball ($\tilde{G}$):

- closely related to η' and the $U(1)_A$ problem
- in 1980: first glueball candidate the isoscalar pseudoscalar $\iota(1440)$, now listed as two states $\eta(1405)$ and $\eta(1475)$ in PDG
- together with $\eta(1295)$ $\Rightarrow$ a supernumerary state beyond the first radial excitations of the η and η' mesons, with $\eta(1405)$ singled out as glueball candidate
- BUT: lattice predicts $m(\tilde{G}) \sim 2.6$ GeV ! $\Rightarrow$ Still to be discovered

  indeed: evidence for three η states between 1.2 and 1.5 GeV under dispute

  ($\eta(1405)$ and $\eta(1475)$ could after all be one state $\eta(1440)$; also $\eta(1295)$ sometimes questioned)

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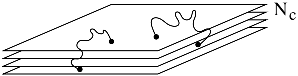
Seeking help from closest (top-down) holographic model of (large-N_c) QCD:
the **Witten-Sakai-Sugimoto model**

Qualitative + quantitative estimates of glueball decay pattern:

- K. Hashimoto, C.-I. Tan, S. Terashima, PRD77 (2008) 086001
- F. Brünner, D. Parganlija, AR, PRD91 (2015) 106002
- F. Brünner, AR, PRL115 (2015) 131601; PRD92 (2015) 121902
- F. Brünner, AR, PLB770 (2017) 124

Witten model: Holographic nonsupersymmetric QCD



E. Witten, Adv. Theor. Math. Phys. 2, 505 (1998):

Type-IIA string theory with $N_c \to \infty$ $D4$ branes
dual to $4+1$-dimensional super-Yang-Mills theory

supersymmetry completely broken by compactification
on "thermal-like" circle $x_4 \equiv x_4 + 2\pi/M_{\rm KK}$ (Kaluza−Klein)

- antisymmetric b.c. for adjoint fermions: masses $\sim M_{\rm KK}$
- adjoint scalars not protected by gauge symmetry: also masses $\sim M_{\rm KK}$

$\to$ dual to pure-glue YM theory
3+1-dimensional at scales $\ll M_{\rm KK}$

but supergravity approximation needs weak curvature,
cannot take limit $M_{\rm KK} \to \infty$

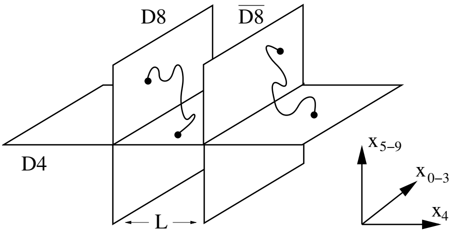
Glueballs: Constable & Myers 1999; Brower, Mathur & Tan 2000
- scalar and tensor glueballs corresponding to 5D dilaton Φ and graviton G_{ij}
  plus exotic scalar modes (discarded by us)
- pseudoscalar glueball from RR 1-form field C_1

Sakai-Sugimoto model: Adding chiral quarks

T. Sakai, S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005)

add N_f D8- and $\overline{\rm D8}$-branes, separated in x_4, $N_f \ll N_c$ (probe branes)

	0	1	2	3	4	5	6	7	8	9
D4	x	x	x	x	x					
D8/$\overline{\rm D8}$	x	x	x	x		x	x	x	x	x



4-8, 4-$\bar{8}$ strings
$\rightarrow$ fundamental, massless chiral fermions

flavor symmetry
$U(N_f)_L \times U(N_f)_R$

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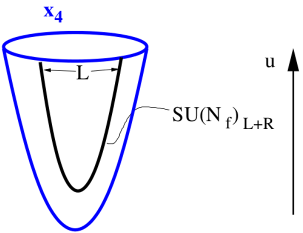

4-8, 4-$\bar{8}$ strings
→ fundamental, massless
chiral fermions

flavor symmetry
$U(N_f)_L \times U(N_f)_R$

spontaneously broken because D8-$\overline{\text{D8}}$ have to join in cigar-shaped topology

for now: maximal separation in x_4 (antipodal on x_4 circle): $L = \pi/M_{\text{KK}}$

Quantitative predictions

- Quite good parameter-free prediction of (axial-)vector meson mass pattern!

Other predictions depend on value of 't Hooft coupling λ at scale $M_{\rm KK}$:

Matching

1. $m_\rho \approx 776$ MeV fixes $\boxed{M_{\rm KK} = 949 \text{ MeV}}$ ($\Rightarrow T_{deconf} = 151$ MeV)
2. $f_\pi^2 = \frac{\lambda N_c}{54\pi^4} M_{\rm KK}^2$ gives $\boxed{\lambda = g_{\rm YM}^2 N_c \approx 16.63}$ [Sakai&Sugimoto 2005-7]
   (matching instead large-N_c lattice result [Bali et al. 2013] for $m_\rho/\sqrt{\sigma}$ gives $\lambda \approx 12.55$)

yields (for $N_c = 3$ and $\lambda = 16.63 \ldots 12.55$):

- LO decay rate of ρ meson $\sim \lambda^{-1} N_c^{-1}$
  $\Gamma_{\rho\to 2\pi}/m_\rho = 0.1535 \ldots 0.2034$ (exp.: 0.191(1))
- decay rate for $\omega \to 3\pi$ (from Chern-Simons part of D8 action) $\sim \lambda^{-4} N_c^{-2}$
  $\Gamma_{\omega\to 3\pi}/m_\omega = 0.0033 \ldots 0.0102$ (exp.: 0.0097(1))

WSS model also predictive regarding glueball decay pattern and rates?

Glueball decay rates in Sakai-Sugimoto model

F. Brünner, D. Parganlija, AR, PRD91 (2015) 106002

Full decay pattern of scalar (Dilatonic, as opposed to Exotic) glueball G_D

decay $G_D \to 4\pi$ suppressed (below 2ρ threshold): $\Gamma_{G\to 4\pi}/\Gamma_{G\to 2\pi} \sim \lambda^{-1} N_c^{-1}$,
while $f_0(1500) \to 4\pi$ dominant:

decay	Γ/M (PDG)	$\Gamma/M[G_D]$
$f_0(1500)$ (total)	0.072(5)	0.027...0.037
$f_0(1500) \to 4\pi$	0.036(3)	0.003...0.005
$f_0(1500) \to 2\pi$	0.025(2)	0.009...0.012
$f_0(1500) \to 2K$	0.006(1)	0.012...0.016
$f_0(1500) \to 2\eta$	0.004(1)	0.003...0.004

$\Rightarrow$ $f_0(1500)$ seemingly disfavored, at least when nearly pure glue

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$f_0(1710) \to \pi\pi$ OK: $\Gamma^{(\mathrm{ex})}(f_0(1710) \to \pi\pi)/(1722\mathrm{MeV}) \sim 0.01$
but $f_0(1710)$ decays predominantly into $K\bar{K}$!

— not reproduced by (chiral=flavor-symmetric) WSS model,
but may be due to mechanism of "chiral suppression of scalar glueball decay"
(Chanowitz 2005)

Nonchiral enhancement in mass-deformed WSS?

F. Brünner & AR, PRL 115 (2015) 131601 [1504.05815]

Current quark masses can be introduced in principle through deformations of the WSS model by either world-sheet instantons [Hashimoto, Hirayama, Liu & Yee 2008] or with bifundamental background scalar $\mathcal{T}$ [Bergman, Seki & Sonnenschein 2007]

both lead to

$$\int d^4x \int_{u_{\rm KK}}^{\infty} du\, h(u)\, \mathrm{Tr}\, \left(\mathcal{T}(u)\, \mathrm{P}\, e^{-i\int dz A_z(z,x)} + h.c. \right),$$

where $h(u)$ includes metric (glueball) fields

Choosing appropriate boundary conditions for $\mathcal{T}$, the quark mass matrix arises through

$$\int_{u_{\rm KK}}^{\infty} du\, h(u)\, \mathcal{T}(u) \propto \mathcal{M} = \mathrm{diag}(m_u, m_d, m_s),$$

thereby realizing a Gell-Mann-Oakes-Renner relation.

Witten-Veneziano mass term

Already in chiral model:

WSS contains (fully determined) Witten-Veneziano mass term for singlet η_0 pseudoscalar from U(1)$_A$ anomaly contributions $\sim 1/N_c$

$$m_0^2 = \frac{N_f}{27\pi^2 N_c} \lambda^2 M_{\rm KK}^2$$

from $S_{C_1} = -\frac{1}{4\pi(2\pi l_s)^6} \int d^{10}x \sqrt{-g} |\tilde{F}_2|^2$ with

$$\tilde{F}_2 = \frac{6\pi u_{\rm KK}^3 M_{\rm KK}^{-1}}{u^4} \left(\theta + \frac{\sqrt{2N_f}}{f_\pi} \eta_0 \right) du \wedge dx^4,$$

where θ is the QCD theta angle and $\eta_0(x) = \frac{f_\pi}{\sqrt{2N_f}} \int dz \, {\rm Tr} A_z(z,x)$.

With $N_f = N_c = 3$, $M_{\rm KK} = 949$ MeV, $\lambda = 16.63 \ldots 12.55$: $\boxed{m_0 = 967 \ldots 730 \text{ MeV}}$

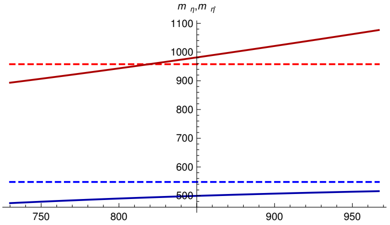
Witten-Veneziano mass term

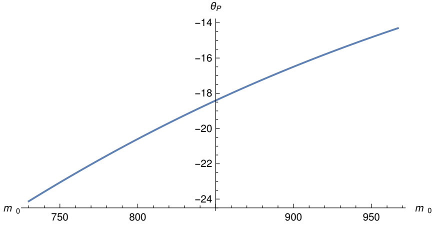
With finite quark masses η_0 and η_8 no longer mass eigenstates.

Diagonalizing:

$N_f = N_c = 3$, $M_{\rm KK} = 949$ MeV, $\lambda = 16.63 \ldots 12.55$: $\boxed{m_0 = 967 \ldots 730 \text{ MeV}}$,
(with $\mathcal{M} = \mathrm{diag}(\hat{m}, \hat{m}, m_s)$, fixing $m_\pi = 140$ MeV and $m_K = 497$ MeV) $\to$

$$m_\eta = 518 \ldots 476 \text{ MeV}, \quad m_{\eta'} = 1077 \ldots 894 \text{ MeV},$$
$$\theta_P = -14.4^\circ \ldots -24.2^\circ,$$





nice ballpark:

light meson decays values favors [Ambrosini 2009, Pham 2015]: $\theta_P \approx -14^\circ$
radiative charmonium decay [Gerard 2004,2013]: $\theta_P \approx -21^\circ$
$\Gamma(\eta' \to 2\gamma)/\Gamma(\eta \to 2\gamma)$ leads to [PDG]: $\theta_P = (-18 \pm 2)^\circ$

Nonchiral enhancement in mass-deformed WSS!

Holographic realization of mass terms give additional vertices between glueballs and pseudoscalars
Rigorously calculable for $G_D \eta_0^2$,

$$\mathcal{L}^{\text{chiral}}_{G_D \eta_0 \eta_0} = \frac{3}{2} d_0 m_0^2 \, \eta_0^2 G_D , \qquad d_0 \approx \frac{17.915}{\lambda^{1/2} N_c M_{\text{KK}}}$$

but not (yet) fixed for octet.
Parametrize uncertainty by free parameter x:

$$\mathcal{L}^{\text{massive}}_{G_D \pi\pi} = \frac{3}{2} d_m G_D \mathcal{L}^{\mathcal{M}}_m , \qquad d_m \equiv x d_0$$

Most symmetric choice $x = 1$ ($\Leftrightarrow$ no $G_D \to \eta\eta'$)
$\to$ relatively strong enhancement factor for kaons and η mesons:

$$\Gamma^{\text{chiral}}_{G \to PP} \to \Gamma^{\text{chiral}}_{G \to PP} \times \left(1 - 4 \frac{m_P^2}{M_G^2}\right)^{1/2} \left(1 + 8.480 \frac{m_P^2}{M_G^2}\right)^2$$

Comparison with $f_0(1710)$

decay	Γ/M **(PDG)**	$\Gamma/M[G_D]$ (chiral)	$\Gamma/M[G_D]$ (massive)
$f_0(1710)$ (total)	**0.081(5)**	0.059...0.076	0.083...0.106
$f_0(1710) \to 2K$	$(^*)$ 0.029(10)	0.012...0.016	0.029...0.038
$f_0(1710) \to 2\eta$	0.014(6)	0.003...0.004	0.009...0.011
$f_0(1710) \to 2\pi$	$0.012(^{+5}_{-6})$	0.009...0.012	0.010...0.013
$f_0(1710) \to 2\rho, \rho\pi\pi \to 4\pi$	?	0.024...0.030	0.024...0.030
$f_0(1710) \to 2\omega$	$0.010(^{+6}_{-7})$	0.011...0.014	0.011...0.014
$f_0(1710) \to \eta\eta'$	?	0	if 0 : $\Uparrow$
$\Gamma(\pi\pi)/\Gamma(K\bar{K})$	**0.41** $^{+0.11}_{-0.17}$	**3/4**	**0.35**
$\Gamma(\eta\eta)/\Gamma(K\bar{K})$	**0.48**± 0.15	1/4	0.28

* PDG ratios for decay rates + $\mathrm{Br}(f_0(1710) \to KK) = 0.36(12)$ [Albaladejo&Oller 2008]

- decays into 2 pseudoscalars: massive WSS perfectly compatible with PDG data!
- significant decay into 4 pions (after extrapolation to beyond 2ρ threshold): falsifiable prediction of this model!
  ($f_0(1710) \to 2\rho^0$ forthcoming from CEP experiments at LHC!)

Pseudoscalar glueball in Witten-Sakai-Sugimoto model

Pseudoscalar glueballs described by fluctuations of

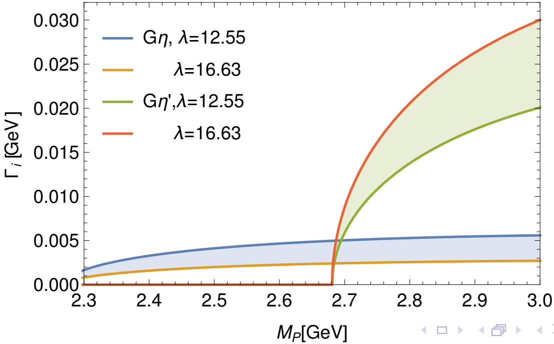
$$\text{RR field } \tilde{F}_2 = dC_1' + \frac{c}{U^4}\left(\theta + \frac{\sqrt{2N_f}}{f_\pi}\eta_0(x)\right) dU \wedge d\tau \qquad \text{(anomaly inflow)}$$

No direct coupling of C_1 to flavor D8 branes,

$$\boxed{\text{vertex } G\text{-}\tilde{G}\text{-}\eta_0 \propto \sqrt{\frac{N_f}{N_c}\frac{\sqrt{\lambda}}{N_c}}} \text{ from } -\frac{1}{4\pi(2\pi\ell_s)^6}\int d^{10}x\sqrt{-g}|\tilde{F}_2|^2$$

$\rightarrow$ **very narrow pseudoscalar glueball** with dominant decay pattern

$$\tilde{G} \rightarrow G(= f_0(1710)) + \eta(') \qquad \text{(resonant)}$$



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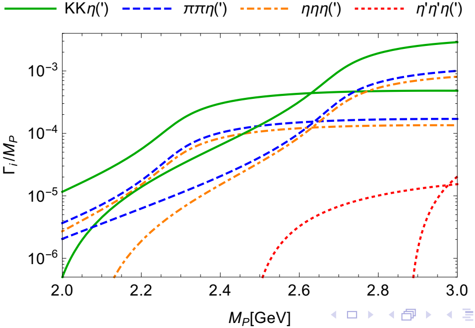
$$\tilde{G} \;\to\; G(= f_0(1710)) \,+\, \eta(') \;\to\; P\,P\,\eta(')$$



$\lambda = (12.55 + 16.63)/2$

Pseudoscalar glueball production

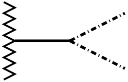
As with decay, production of $\tilde{G}$ involves $G{+}\eta(')$ or G+another $\tilde{G}$

(would explain why not yet observed in radiative J/ψ decays; needs excited ψ or Υ?)

- Another possibility: Central Exclusive Production in high-energy hadron collisions!

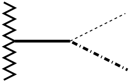
Parametric orders of the production amplitudes of pseudoscalar glueballs in double Pomeron or double Reggeon exchange

$\underline{\tilde{G}\tilde{G}}$:



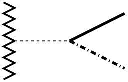
$\sim \lambda^{-1} N_c^{-2}$

$\underline{\eta(')\tilde{G}}$:



$\sim \lambda^0 N_f^{1/2} N_c^{-5/2}$

$G\tilde{G}$:



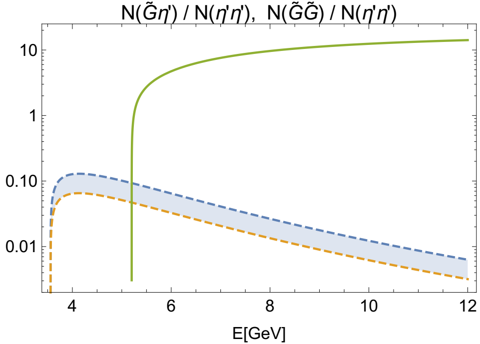
$\sim \lambda^{-1} N_f^1 N_c^{-3}$ . . . suppressed

(in the uppermost diagram the full line stands for G or G_T)

Production of $\tilde{G}\tilde{G}$ and $\tilde{G}\eta'$ pairs versus $\eta'\eta'$

Production from a virtual scalar glueball

for as functions of the c.o.m. energy of the produced pair (assuming $m(\tilde{G}) = 2.6$ GeV)



The full line gives $N(\tilde{G}\tilde{G})/N(\eta'\eta')$, which is independent of the 't Hooft coupling; upper and lower dashed lines correspond to $N(\tilde{G}\eta')/N(\eta'\eta')$ with 't Hooft coupling 12.55 and 16.63, respectively.

CEP of $\eta'\eta'$ in Durham [Harland-Lang et al. 2013]:
$\sigma(\eta'\eta')/\sigma(\pi^0\pi^0) \sim 10^3 \ldots 10^5$ at $\sqrt{s} = 1.96$ TeV

Conclusion

• With just one dimensionless parameter, top-down holographic QCD model of Witten, Sakai and Sugimoto very predictive and surprisingly successful quantitatively:

Meson spectrum and dynamics:
— vector and axial vector mesons masses, ρ and ω decay rates, anomalous m'_η, ...
with typically 10–30% errors

Glueball spectrum:
— if "exotic mode" discarded, scalar glueball mass close to lattice QCD prediction
tensor and pseudoscalar glueball ~ 30 % too light

• WSS model also perhaps good guide for glueball signatures!

Scalar glueball decay pattern consistent with $f_0(1710)$ as nearly pure glueball, if predictions for 4π and $\eta\eta'$ decays confirmed

Golden channel?: **very narrow pseudoscalar glueballs with characteristic decay and production pattern (would explain why not yet observed in radiative J/ψ decays)**

BACKUP SLIDES

Glueballs in the Witten model

$\exists$ scalar and tensor glueballs corresponding to 5D dilaton Φ and graviton G_{ij}
Csaki, Ooguri, Oz & Terning 1999

Type-IIA supergravity compactified on x_4-circle many more modes:
Constable & Myers 1999; Brower, Mathur & Tan 2000

Mode	S_4	T_4	V_4	N_4	M_4	L_4
Sugra fields	G_{44}	Φ, G_{ij}	C_1	B_{ij}	C_{ij4}	G^{α}_{α}
J^{PC}	0^{++}	$0^{++}/2^{++}$	0^{-+}	1^{+-}	1^{--}	0^{++}
n=0	7.30835	22.0966	31.9853	53.3758	83.0449	115.002
n=1	46.9855	55.5833	72.4793	109.446	143.581	189.632
n=2	94.4816	102.452	126.144	177.231	217.397	277.283
n=3	154.963	162.699	193.133	257.959	304.531	378.099
n=4	228.709	236.328	273.482	351.895	405.011	492.171

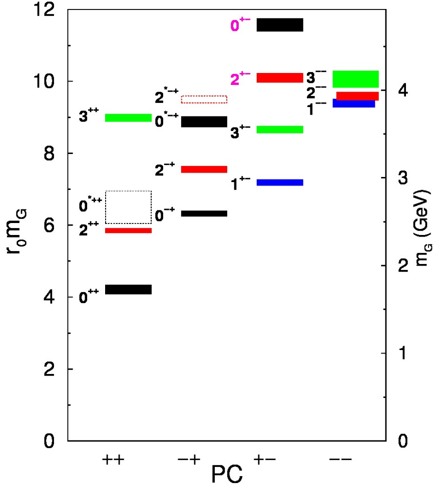
Lowest mode not from dilaton, but from "exotic polarization" – in 11D notation:

$$\underline{\delta g_{44}} = -\frac{r^2}{L^2} f\, H(r) G(x), \quad \delta g_{\mu\nu} = \frac{r^2}{L^2}\left[\frac{1}{4} H(r)\eta_{\mu\nu} - \left(\frac{1}{4} + \frac{3R^6}{5r^6 - 2R^6}\right) H(r)\frac{\partial_\mu \partial_\nu}{M^2}\right] G(x)$$

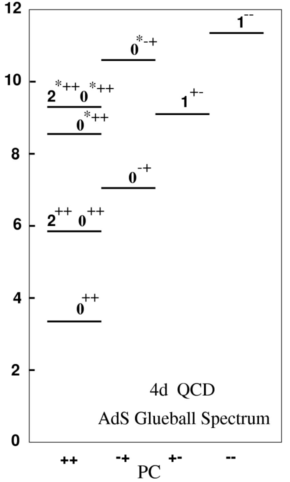
$$\delta g_{11,11} = \frac{r^2}{L^2}\frac{1}{4} H(r) G(x), \quad \delta g_{rr} = -\frac{L^2}{r^2} f^{-1}\frac{3R^6 H(r) G(r)}{5r^6 - 2R^6}, \quad \delta g_{r\mu} = \frac{90 r^7 R^6 H(r)\partial_\mu G(x)}{M^2 L^2 (5r^6 - 2R^6)^2}$$

Lattice glueballs vs. supergravity glueballs

Morningstar & Peardon hep-lat/9901004:



Brower, Mathur & Tan 2000:



(mass scales matched on 2^{++}) $\rightarrow$ seemingly good qualitative agreement!

Quantitative predictions: vector meson spectrum

Parameter-free prediction of (axial-)vector meson mass pattern:

Isotriplet Meson	$\lambda_n = m^2/M_{\mathrm{KK}}^2$	m/m_ρ	$(m/m_\rho)^{\mathrm{exp.}}$	$(m/m_\rho)^{N\to\infty}$
1^{--} (ρ)	0.669314	1	1	1
1^{++} (a_1)	1.568766	1.531	1.59(5)	1.86(2)
1^{--} (ρ^*)	2.874323	2.072	1.89(3)	2.40(4)
1^{++} (a_1^*)	4.546104	2.606	2.12(3)	2.98(5)

(last column from lattice study by Bali et al. JHEP 06, 071 (2013))

agreement within $\lesssim 20\%$

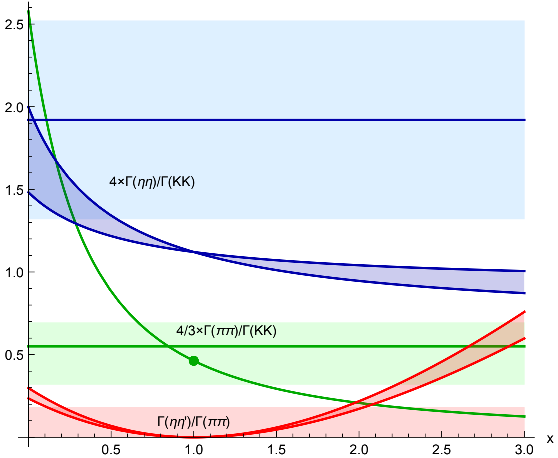
not bad, given that WSS is not yet large-N QCD (in particular at scales $\gtrsim M_{\mathrm{KK}}$)

(near-perfect agreement for m_{a_1}/m_ρ with real QCD certainly fortuitous)

Constraints on $\eta\eta'$ rates for $f_0(1710)$ as ≈pure glueball

Relaxing $x = 1$: [F. Brünner & AR, PRD92, 1510.07605]

WSS model gives *flavor asymmetries* consistent with experimental results for $f_0(1710)$ in as long as $\Gamma(G \to \eta\eta')/\Gamma(G \to \pi\pi) \lesssim 0.04$ (upper limit from WA102: < 0.18)



Tensor glueball decay rates in Sakai-Sugimoto model

Tensor glueball in WSS, and extrapolated to higher mass:

decay	M	$\Gamma/M[T(M)]$
$T \to 2\pi$	1487	0.013...0.018
$T \to 2K$	1487	0.004...0.006
$T \to 2\eta$	1487	0.0005...0.0007
T (total)	1487	$\approx 0.02 \ldots 0.03$
$T \to 2\rho \to 4\pi$	2000	0.135...0.178
$T \to 2K^* \to 2(K\pi)$	2000	0.119...0.177
$T \to 2\omega \to 6\pi$	2000	0.045...0.059
$T \to 2\pi$	2000	0.014...0.018
$T \to 2K$	2000	0.010...0.013
$T \to 2\eta$	2000	0.0018...0.0024
T (total)	2000	$\approx 0.3 \ldots 0.45$
T (total)	2400	$\approx 0.45 \ldots 0.6$

Very **broad tensor glueball**, if at 2.4 GeV (probably unobservable)

With a mass of 2 GeV, width larger but perhaps comparable with that of the rather broad tensor meson $f_2(1950)$, which has $\Gamma/M = 0.24(1)$.

Very narrow (unconfirmed) candidate $f_J(2220)$ not compatible with WSS