

Thermalization of a strongly interacting non-Abelian plasma

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outline:

AdS/CFT approach to investigate

- relaxation towards equilibrium of a strongly interacting medium using
 - local probes (energy density, pressures)
 - non-local probes (geodesics, Wilson loops)
- quarkonium dissociation in a far-from-equilibrium plasma

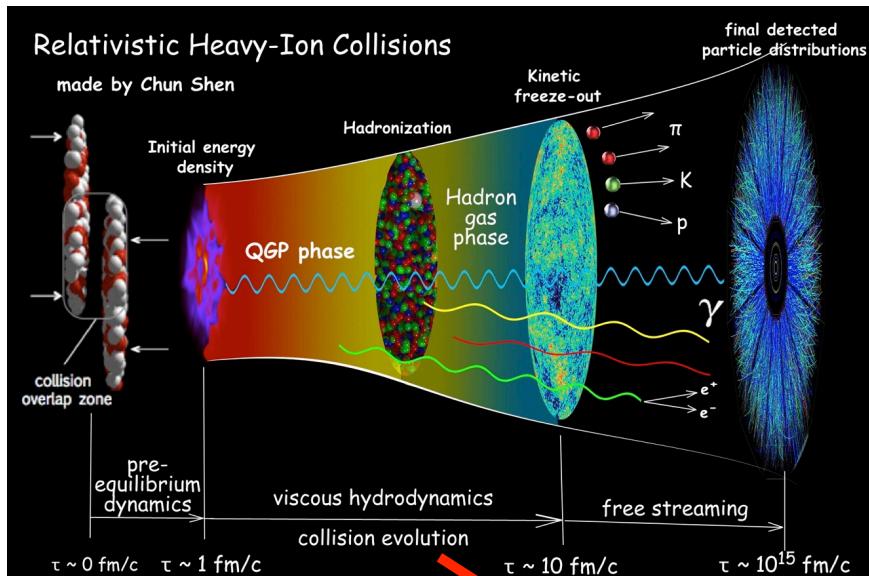
based on JHEP 07 (2015) 053
PRD 94 (2016) 025005
arXiv:1706.04809

in collaboration with

L. Bellantuono, P. Colangelo, F. Giannuzzi, S. Nicotri (Bari Univ. and INFN)

Quark-Gluon plasma (QGP)

- state of matter composed by deconfined quarks and gluons
- believed to be produced in heavy –ion collision experiments
- unknown evolution from pre-equilibrium condition to the final viscous hydrodynamic regime



indications from HI experiments (RHIC, LHC)

simulations reproducing the elliptic flow & pressure anisotropy: almost perfect fluid behaviour
tiny shear viscosity/entropy ratio



large values predicted by pQCD



plasma
strongly coupled, deconfined phase of QCD

fast thermalization:
 $\tau \approx O(1 \text{ fm}/c)$

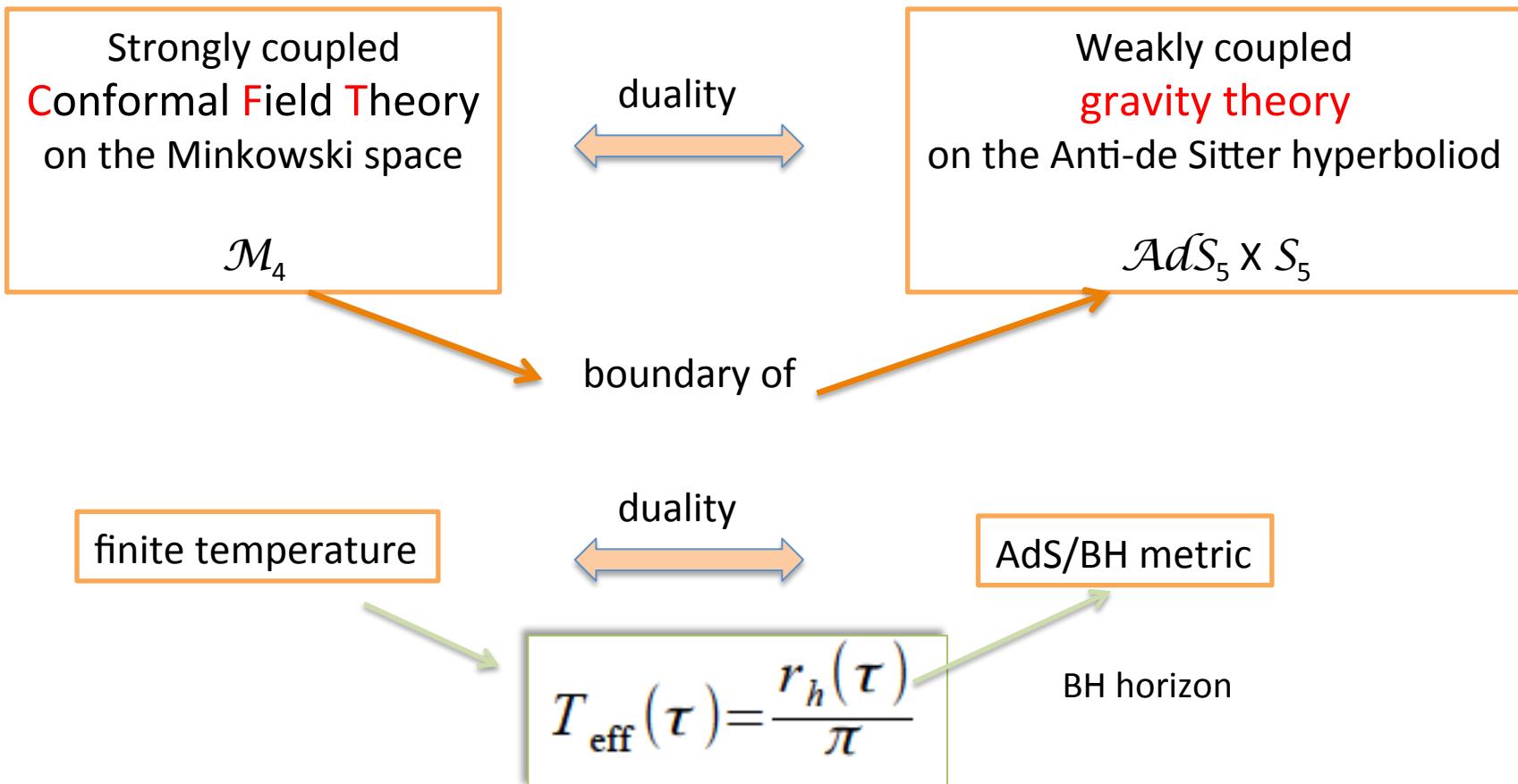
- HI collisions give raise to a dense interacting medium
- relevant degrees of freedom not individual partons
- description of the system as a fluid might be appropriate

use various probes

- to study equilibrium properties of the system
- to determine time scales to reach equilibrium

AdS/CFT correspondence

Maldacena
Gubser, Polyakov,
Klebanov, Witten



perfect fluid hydrodynamics

assumptions:

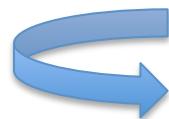
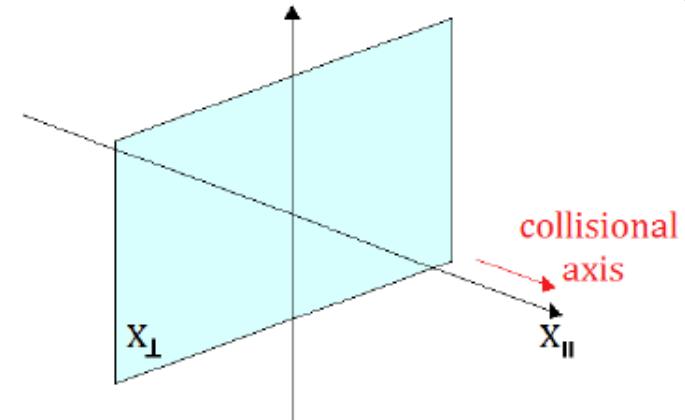
- boost invariance along the collision axis
- translation & rotation invariance in the transverse plane

coordinates and metric

proper time τ , rapidity y , transverse coordinates x_\perp

$$x^0 = \tau \cosh y \quad x^1 = \tau \sinh y.$$

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2.$$



diagonal stress-energy tensor $T_{\mu\nu}$
only depends on τ

stress-energy tensor in perfect fluid hydrodynamics

$$\epsilon(\tau) = \frac{const}{\tau^{4/3}}$$

$$p_{\parallel}(\tau) = -\epsilon(\tau) - \tau\epsilon'(\tau)$$

$$p_{\perp}(\tau) = \epsilon(\tau) + \frac{\tau}{2}\epsilon'(\tau) .$$

subleading corrections
-> viscous hydrodynamics

$$\epsilon(\tau) = \frac{3\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}} \left[1 - \frac{2c_1}{(\Lambda\tau)^{2/3}} + \frac{c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right) \right]$$

$$p_{\parallel}(\tau) = \frac{\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}} \left[1 - \frac{6c_1}{(\Lambda\tau)^{2/3}} + \frac{5c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right) \right]$$

$$p_{\perp}(\tau) = \frac{\pi^4\Lambda^4}{4(\Lambda\tau)^{4/3}} \left[1 - \frac{c_2}{(\Lambda\tau)^{4/3}} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^2}\right) \right] ,$$

an effective temperature can be defined

$$\epsilon(\tau) = \frac{3}{4}\pi^4 T_{eff}(\tau)^4$$

$$T_{eff}(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}} \left[1 - \frac{1}{6\pi(\Lambda\tau)^{2/3}} + \frac{-1 + \log 2}{36\pi^2(\Lambda\tau)^{4/3}} \right. \\ \left. + \frac{-21 + 2\pi^2 + 51\log 2 - 24(\log 2)^2}{1944\pi^3(\Lambda\tau)^2} + \mathcal{O}\left(\frac{1}{(\Lambda\tau)^{8/3}}\right) \right]$$

Heller, Janik, Witaszczyk,
PRD 85 (12) 126002
Heller, Janik, PRD 76 (07) 025027
Baier et al, JHEP 0804 (08) 100

how to mimic HI collisions: introducing a perturbation (quench)

duality

operators in gauge theory

fields in $\text{AdS}_5 \times S_5$

$$T_{\mu\nu}$$

$$g_{MN}$$

modifications of the stress-energy tensor
are produced by modifications of the metric

the relation is provided by the procedure of
holographic renormalization

de Haro et al, Comm. Mat. Phys. 217 (01) 595
Kinoshita et al, Prog. Theor. Phys., 121 (09) 121

4D gauge theory driven out-of-equilibrium
 -> 4D metric deformed by a quench $\gamma(\tau)$

$$ds^2 = -d\tau^2 + e^{\gamma(\tau)} dx_\perp^2 + \tau^2 e^{-2\gamma(\tau)} dy^2$$



5D metric of the dual theory in Eddington-Finkelstein coordinates

$$ds^2 = 2drd\tau - Ad\tau^2 + \Sigma^2 e^B dx_\perp^2 + \Sigma^2 e^{-2B} dy^2$$

metric functions $A(r, \tau)$, $B(r, \tau)$, $\Sigma(r, \tau)$ obtained

solving the Einstein eqs

with as boundary condition

for $r \rightarrow \infty$

$$\Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 = 0$$

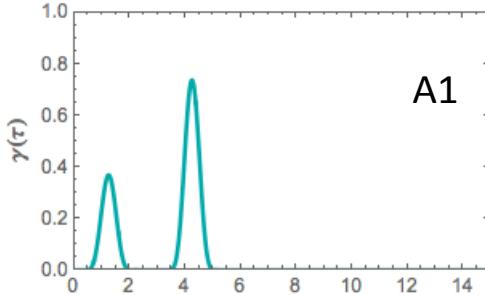
$$\Sigma(\dot{B})' + \frac{3}{2} (\Sigma'\dot{B} + B'\dot{\Sigma}) = 0$$

$$A'' + 3B'\dot{B} - 12\frac{\Sigma'\dot{\Sigma}}{\Sigma^2} + 4 = 0$$

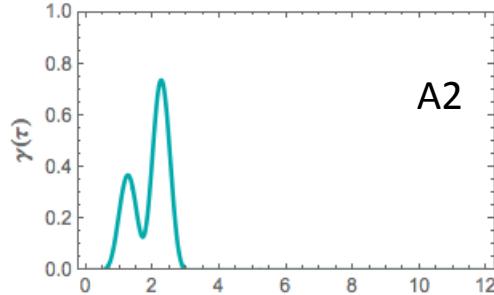
$$\ddot{\Sigma} + \frac{1}{2} (\dot{B}^2\Sigma - A'\dot{\Sigma}) = 0$$

$$\Sigma'' + \frac{1}{2} B'^2\Sigma = 0$$

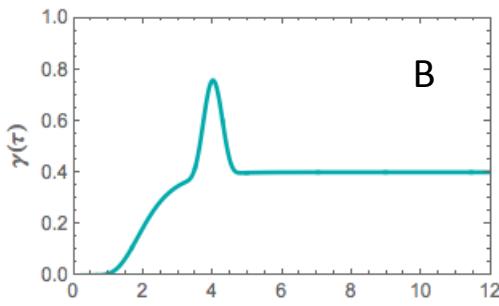
quench $\gamma(\tau)$ starts at $\tau = \tau_i \rightarrow$ AdS₅ metric at τ_i



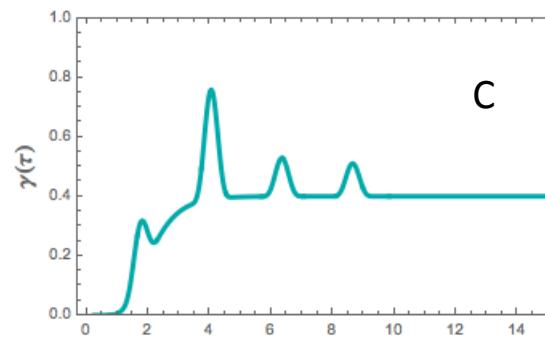
A1



A2



B



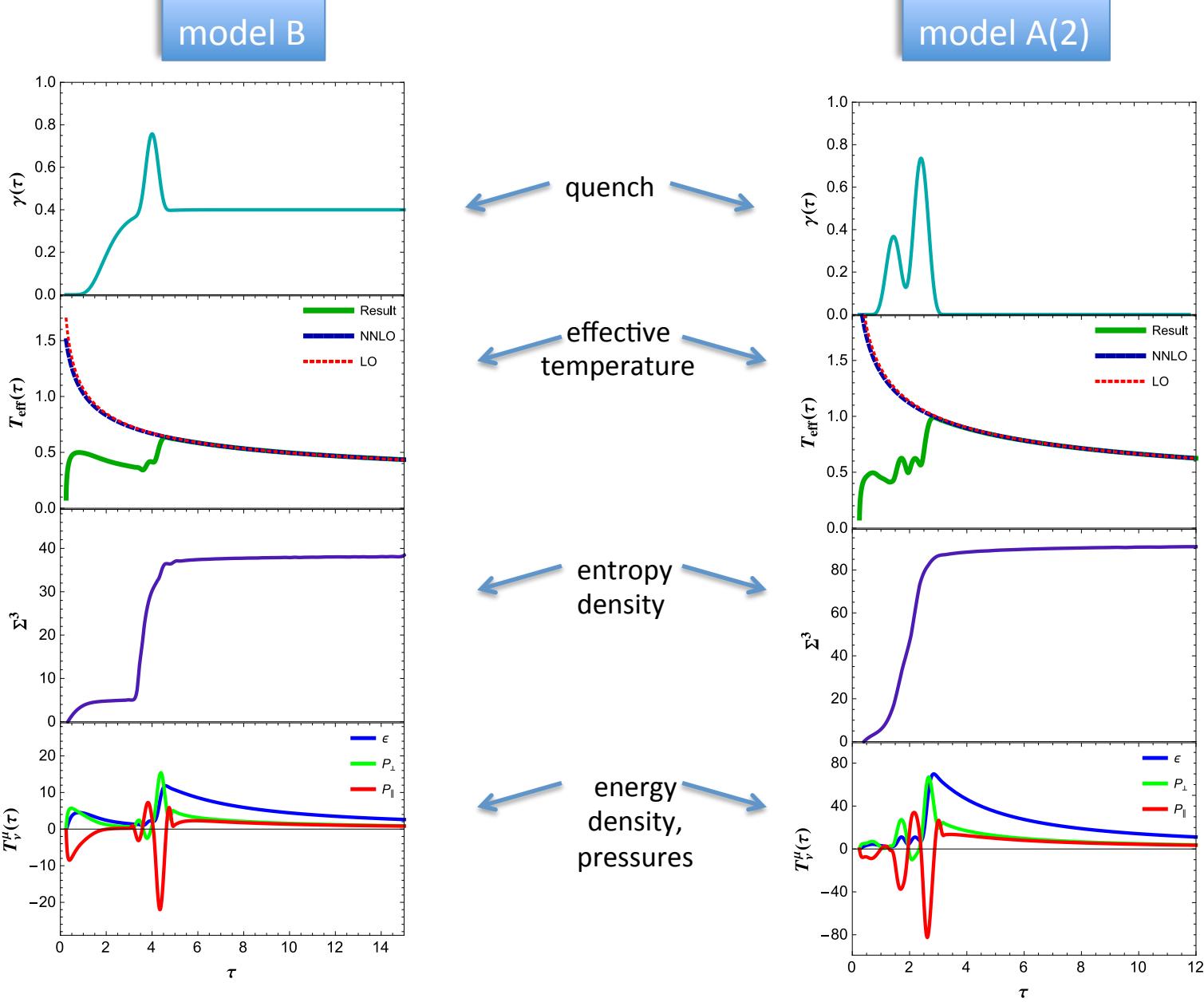
C

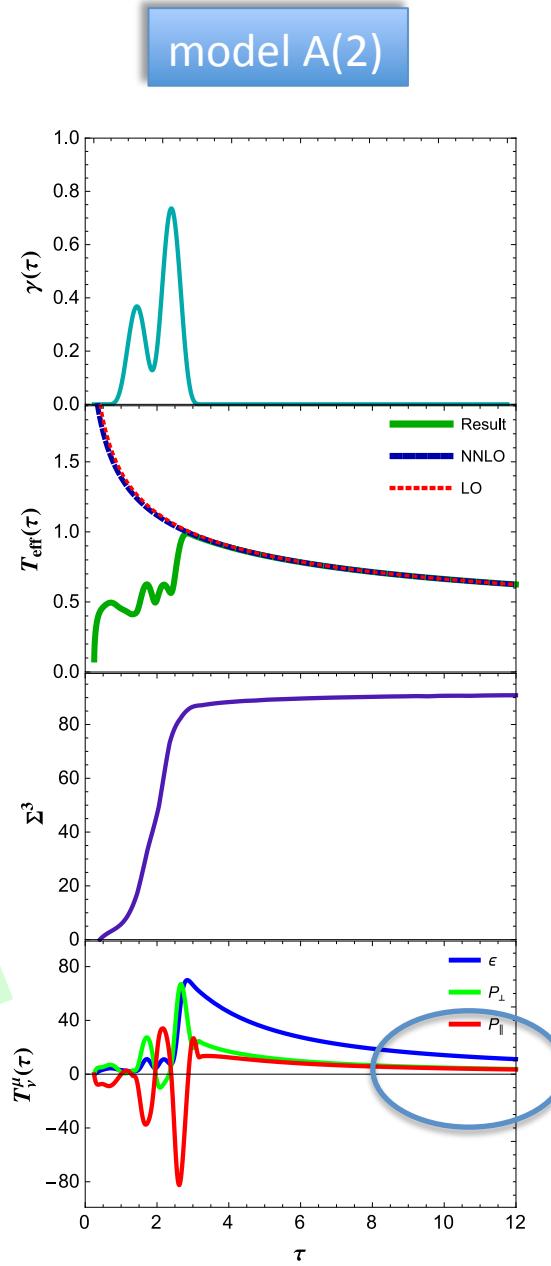
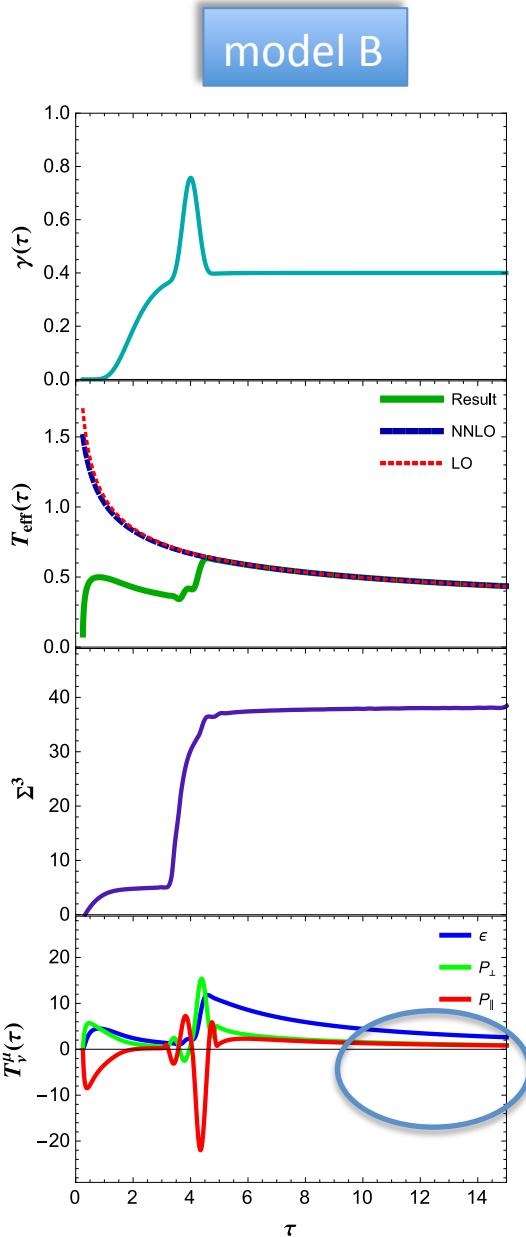
$$\begin{aligned} \Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 &= 0 \\ \Sigma(\dot{B})' + \frac{3}{2}(\Sigma'\dot{B} + B'\dot{\Sigma}) &= 0 \\ A'' + 3B'\dot{B} - 12\frac{\Sigma'\dot{\Sigma}}{\Sigma^2} + 4 &= 0 \\ \ddot{\Sigma} + \frac{1}{2}(\dot{B}^2\Sigma - A'\dot{\Sigma}) &= 0 \\ \Sigma'' + \frac{1}{2}B'^2\Sigma &= 0 \end{aligned}$$

Einstein equations:

3 dynamical and 2 constraint PDE

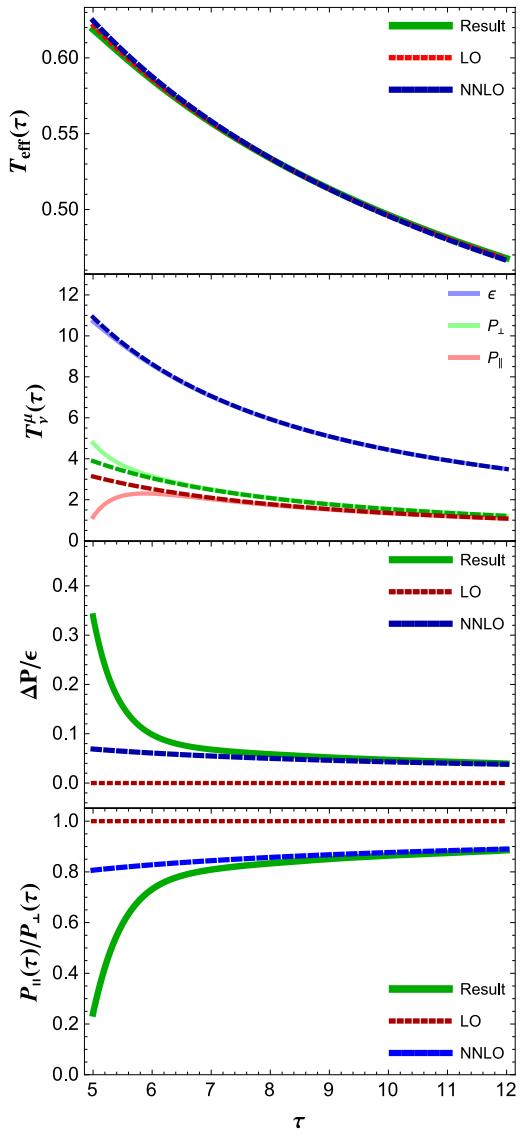
efficient algorithm for the solution



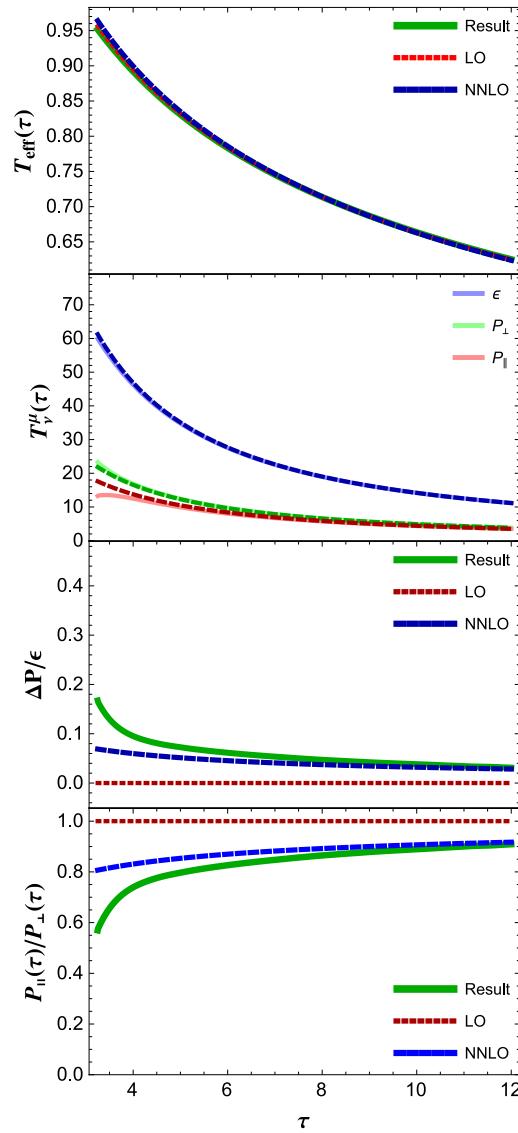


late time dynamics
thermalization?

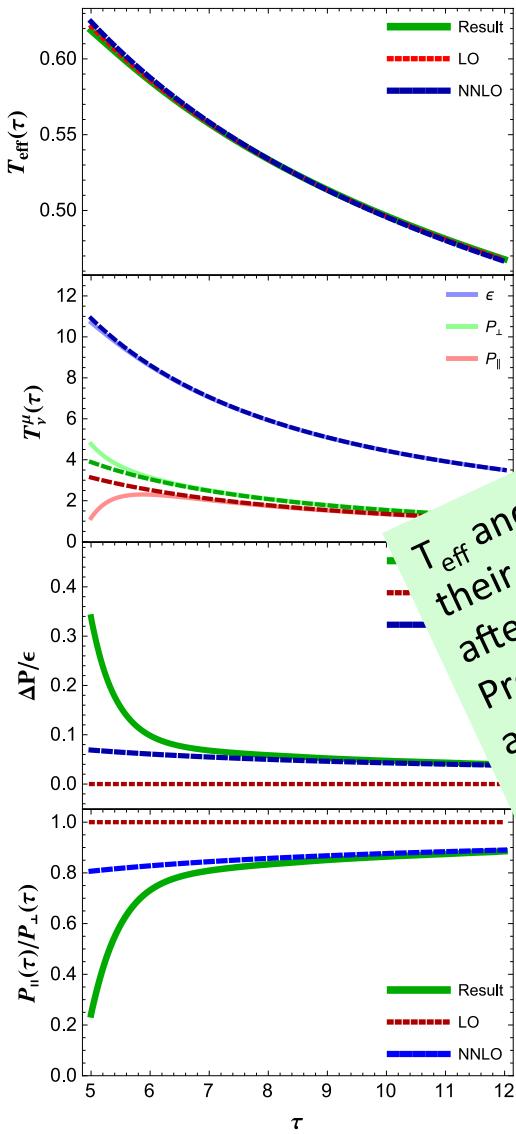
Model B



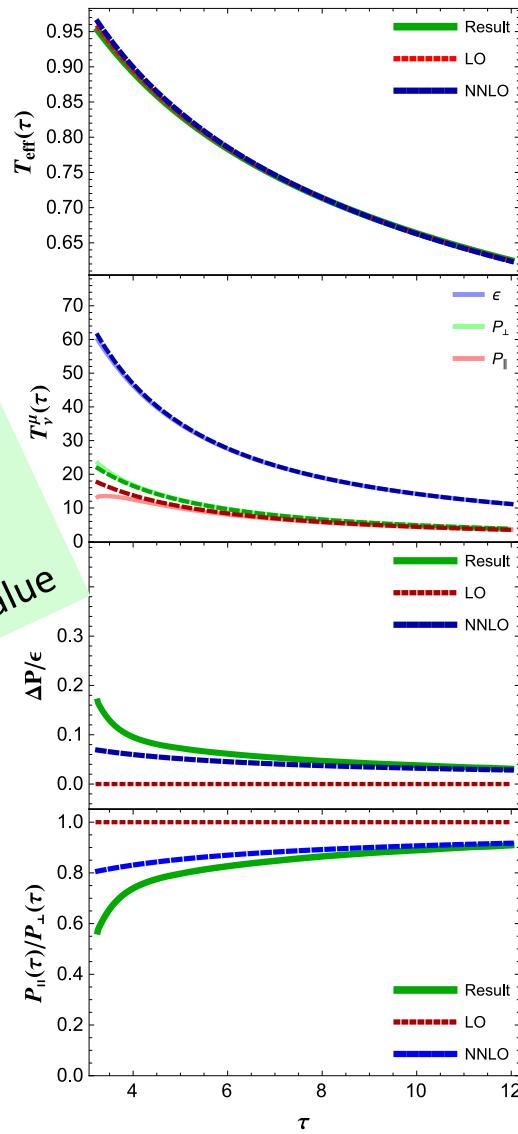
Model A(2)



model B



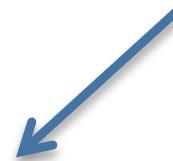
model A(2)



T_{eff} and ϵ reach their hydro value right after the quench (τ^*)
 Pressures take a bit longer:
 at τ_p they are within 5% of their hydro NNLO value

A scale can be introduced fixing T_{eff} at the end of the quench to 500 MeV

model	τ^*	τ_p	Λ	$\Delta\tau = \tau_p - \tau^*$ (fm/c)
\mathcal{A} (1)	5.25	6.8	2.25	0.60
\mathcal{A} (2)	3.25	6.0	1.73	1.03
\mathcal{B}	5	6.74	1.12	0.42
\mathcal{C}	9.45	10.24	1.59	0.20

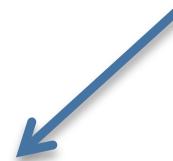


Thermalization times are a fraction of 1 fm

local observables sensitive to the metric close to the boundary

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Thermalization times are a fraction of 1 fm

local observables sensitive to the metric close to the boundary

computed in the dual space in terms of invariant geometric objects

related to minimal lengths, surfaces, volumes of various kinds in the bulk

probe deeper into the bulk spacetime, away from the boundary

nonlocal probes of thermalization: geodesics length

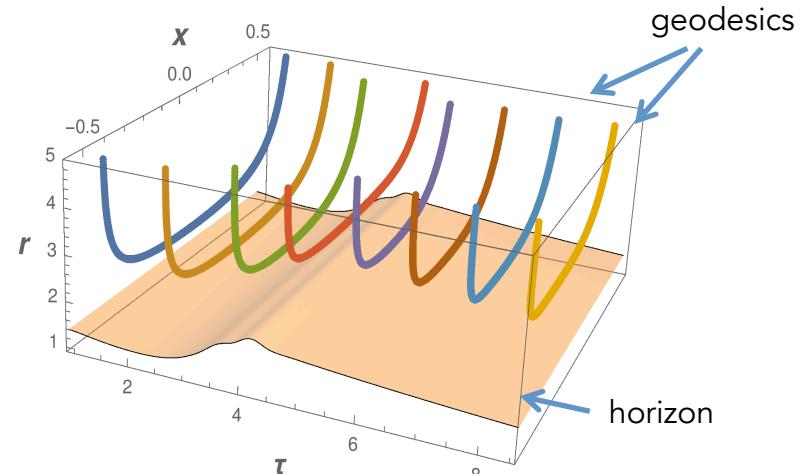
Spacelike geodesics connecting
two boundary points:

$$P = (t_0, -\ell/2, x_2, y) \text{ and } Q = (t_0, \ell/2, x_2, y)$$

(x_2, y) fixed

parametrized by $r(x), \tau(x)$

solutions of the geodesics equation extremizing



$$\mathcal{L} = \int_P^Q d\lambda \sqrt{\pm g_{MN} \dot{x}^M \dot{x}^N},$$

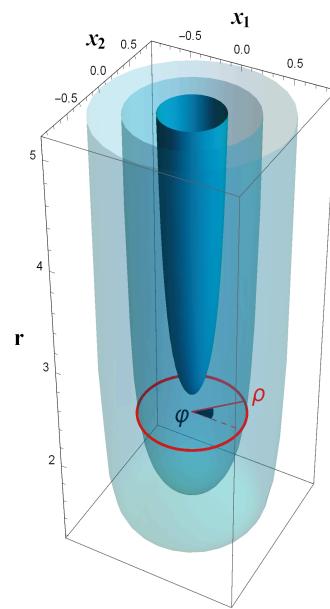
in terms of the computed metric functions: $\tilde{\Sigma}(r, \tau) \equiv \Sigma(r, \tau)^2 e^{B(r, \tau)}$

$$\mathcal{L} = \int_{-\ell/2}^{\ell/2} dx \frac{\tilde{\Sigma}(r, \tau)}{\sqrt{\tilde{\Sigma}(r_*, \tau_*)}}$$

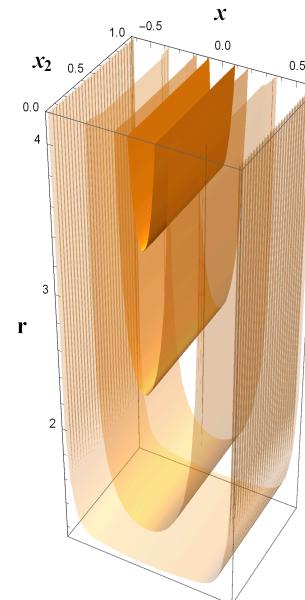
nonlocal probes of thermalization: extremal surfaces

extremal surfaces plugging in the bulk
having a Wilson loop as contour on the boundary

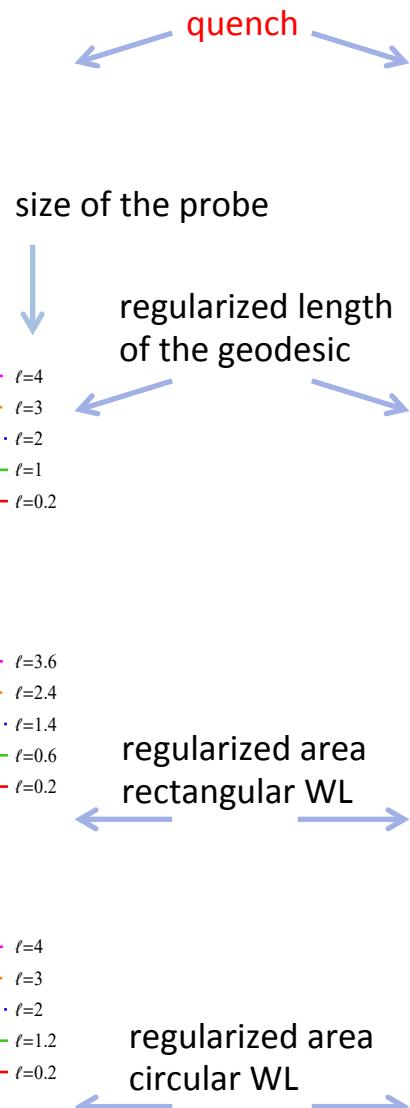
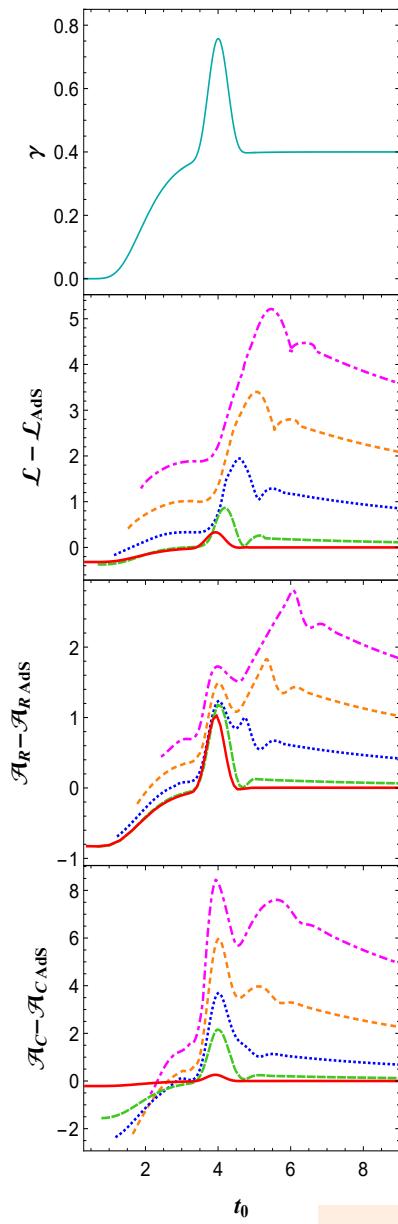
circular Wilson loop



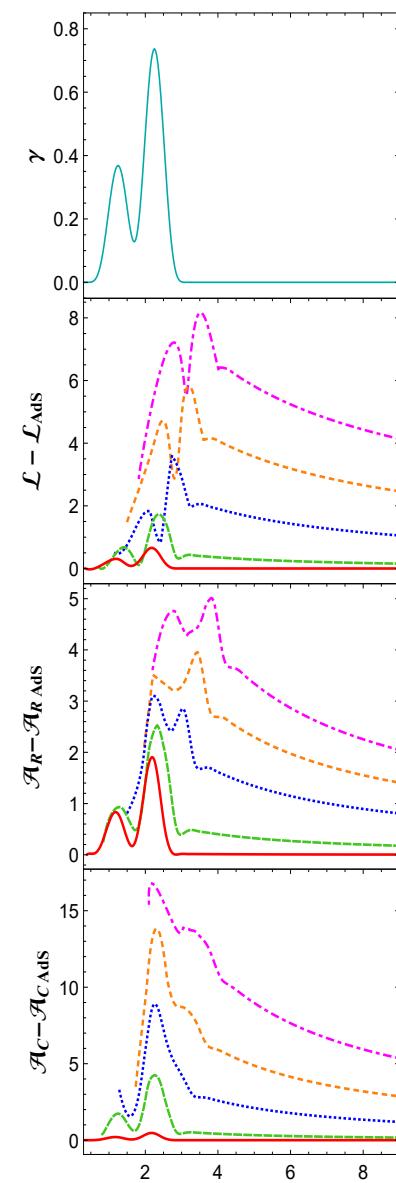
infinite rectangular Wilson loop



model B



model A (2)



PRD 94 (2016) 025005

quench profile followed in the three observables

thermalization of the nonlocal probes:
comparison with viscous hydrodynamics

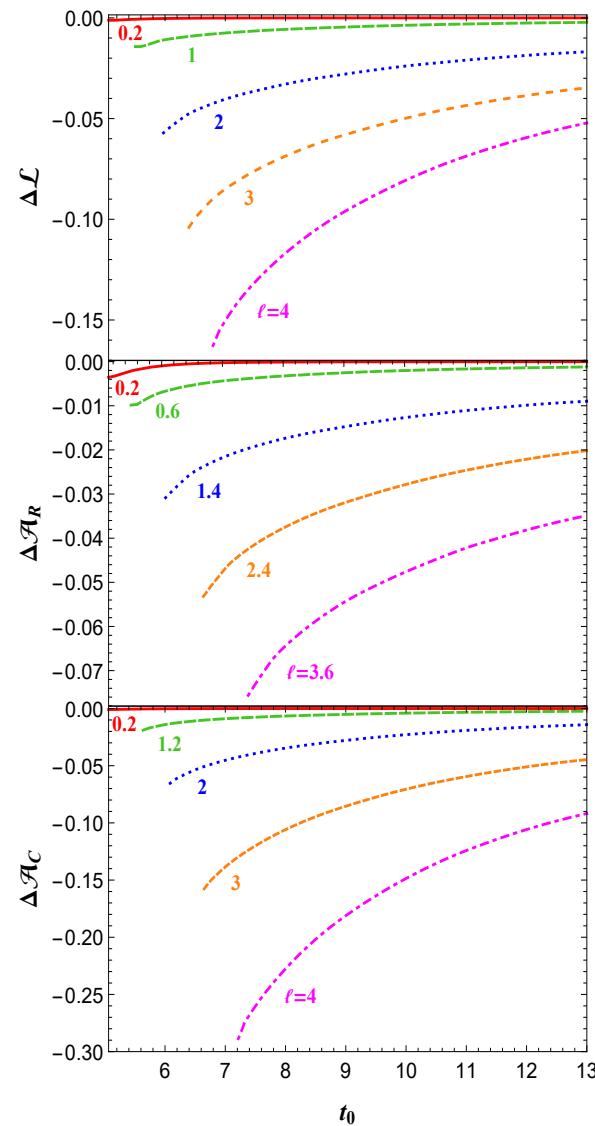
5D metric dual to the viscous hydrodynamics

$$ds_5^2 = 2drd\tau - A^H d\tau^2 + [\Sigma^H]^2 e^{B^H} dx_\perp^2 + [\Sigma^H]^2 e^{-2B^H} dy^2$$

$$A^H(r, \tau) = r^2 \left(1 - \frac{4}{3r^4} \varepsilon(\tau) \right)$$

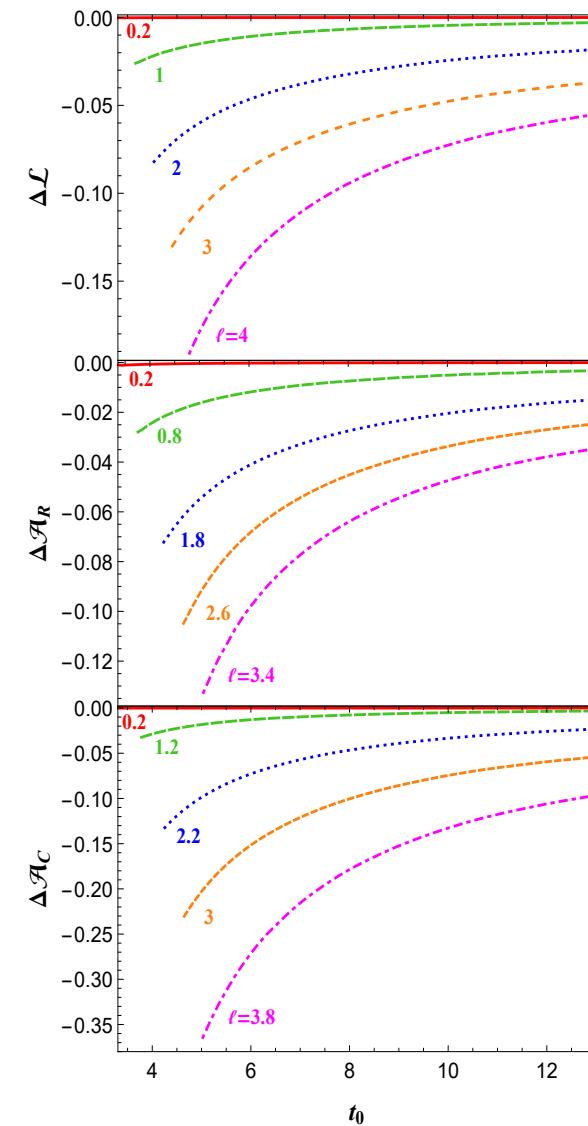
$$\Sigma^H(r, \tau) = r \left(\tau + \frac{1}{r} \right)^{1/3}$$

$$B^H(r, \tau) = \frac{1}{r^4} (p_\perp(\tau) - p_\parallel(\tau)) - \frac{2}{3} \log \left(\tau + \frac{1}{r} \right)$$



$$\Delta L = L - L_{hydro}$$

$$\Delta A = A - A_{hydro}$$

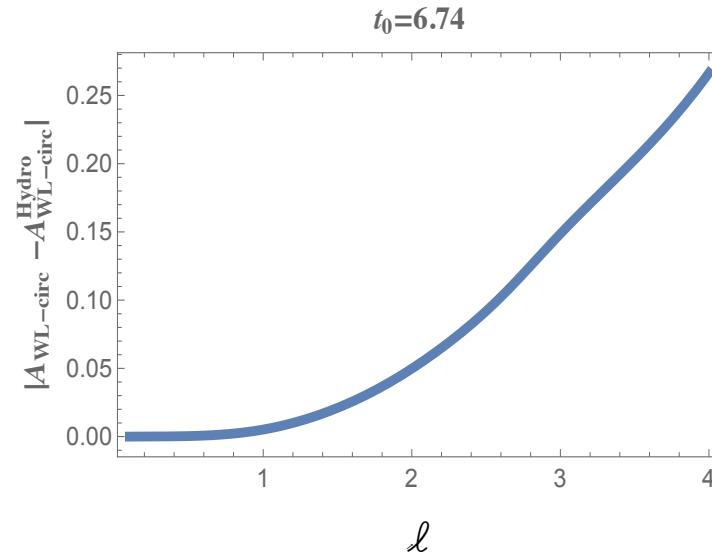
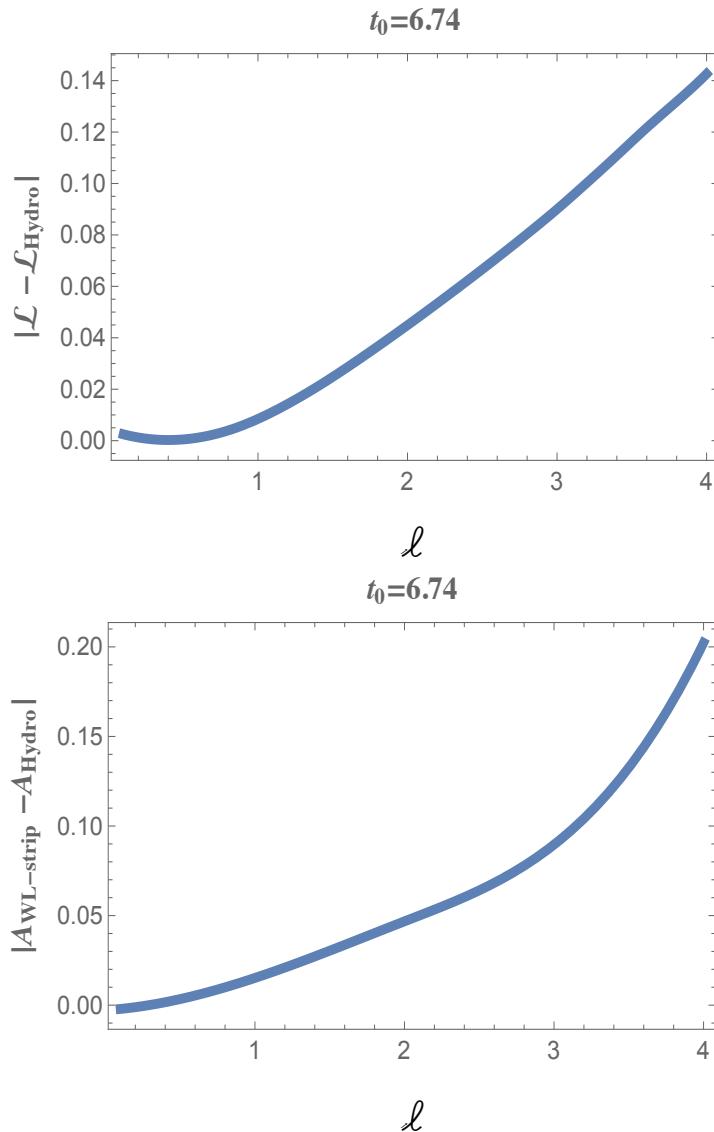


hydro regime reached at large time for large ℓ , almost immediately for small ℓ

local vs nonlocal probes of thermalization: typical time scales

model B

what happens at the time when local probes suggest thermalization has set in?



only for small ℓ thermalization has occurred
 geodesics lenght for approx $\ell < 0.5$
 circular WI area for $\ell < 1$
 rectangular WL area only for very small ℓ

in gauge/gravity framework

- quarks treated as dual to open strings
- evolution of the system: solve eqs of motion stemming from Nambu-Goto action

quarkonium dissociation

- follow the evolution of a string with endpoints kept close to the boundary
- string falls down under gravity
- at finite temperature, horizon is reached: dissociation

Nambu-Goto action

$$S_{NG} = -T_f \int dt dw \sqrt{\Sigma_w(t, r) (A(t, r) - 2 \partial_t r) + (\partial_w r)^2}$$



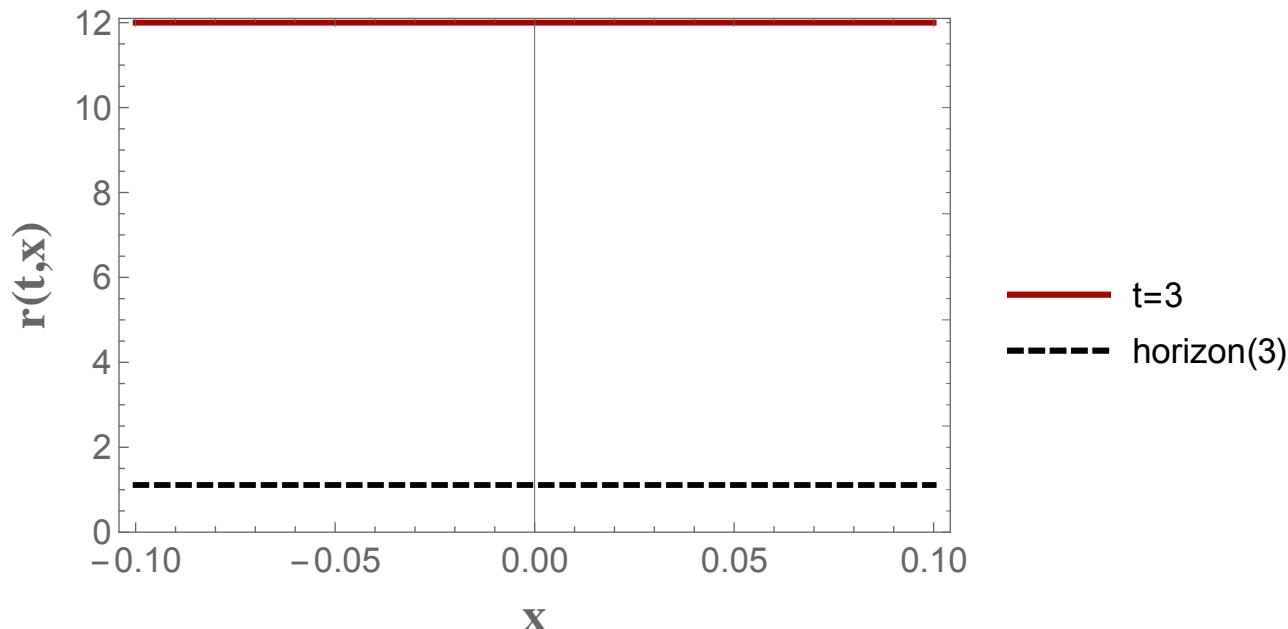
w=x (transverse string)
w=y (longitudinal direction)

Nambu-Goto action

$$S_{NG} = -T_f \int dt dw \sqrt{\Sigma_w(t, r) (A(t, r) - 2 \partial_t r) + (\partial_w r)^2}$$



w=x (transverse string)
w=y (longitudinal direction)

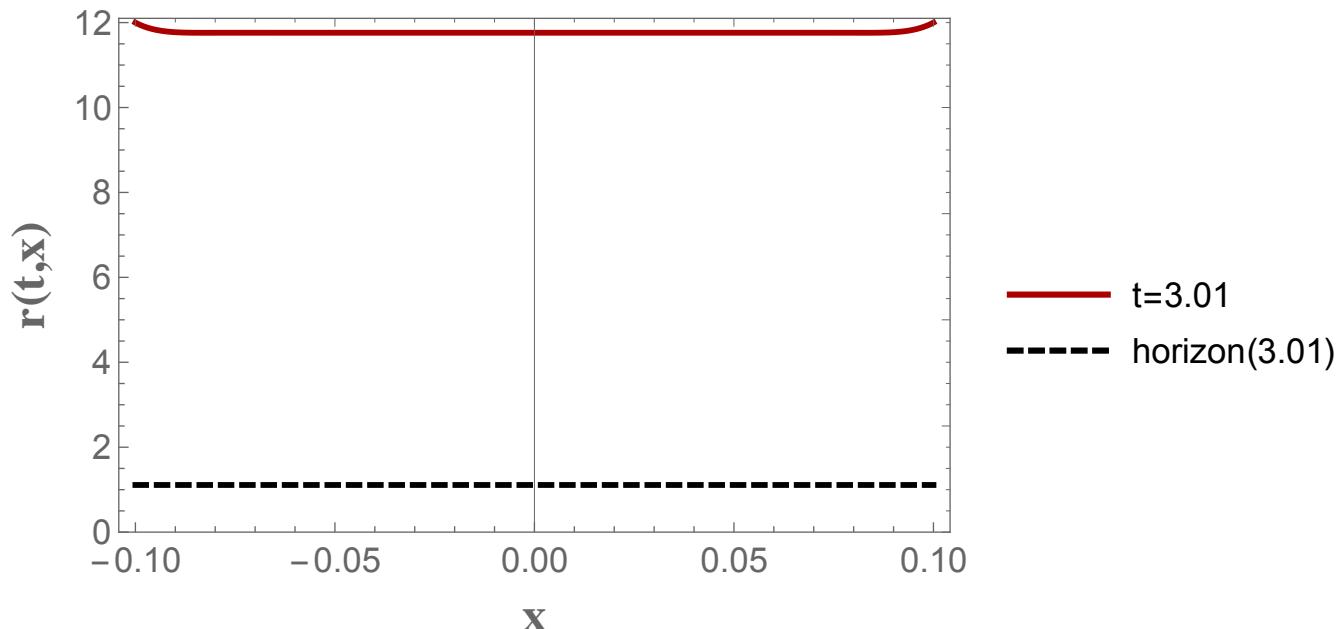


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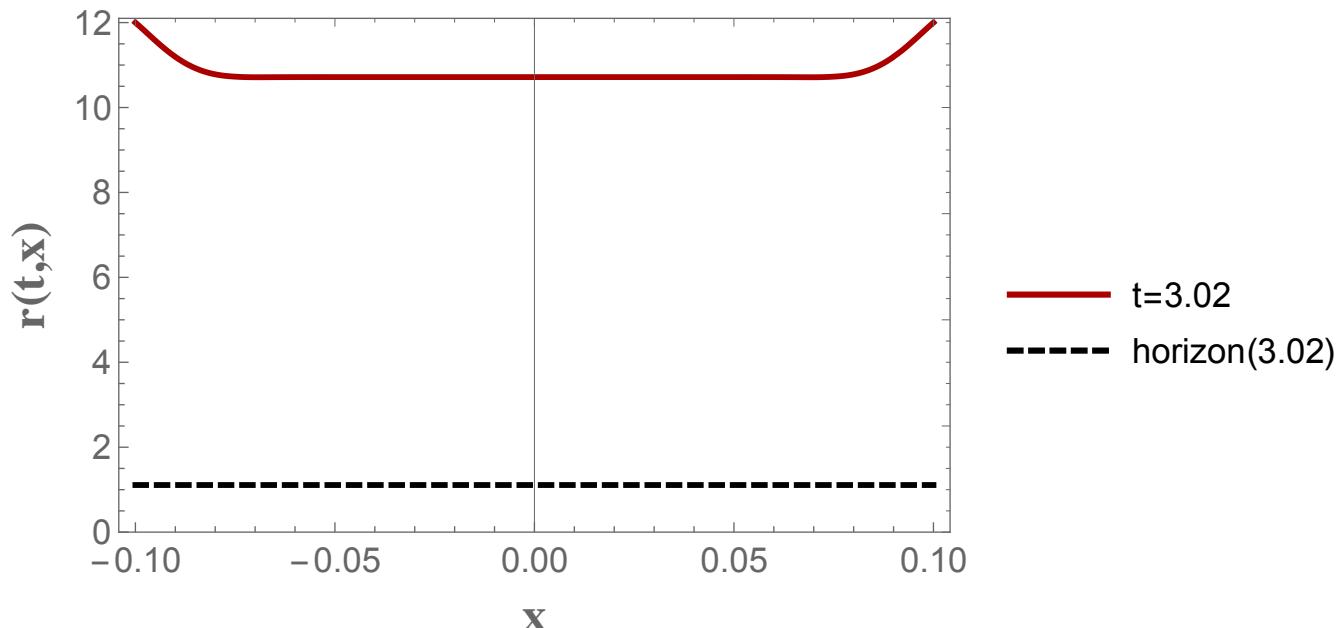
w=x (transverse string)
w=y (longitudinal direction)



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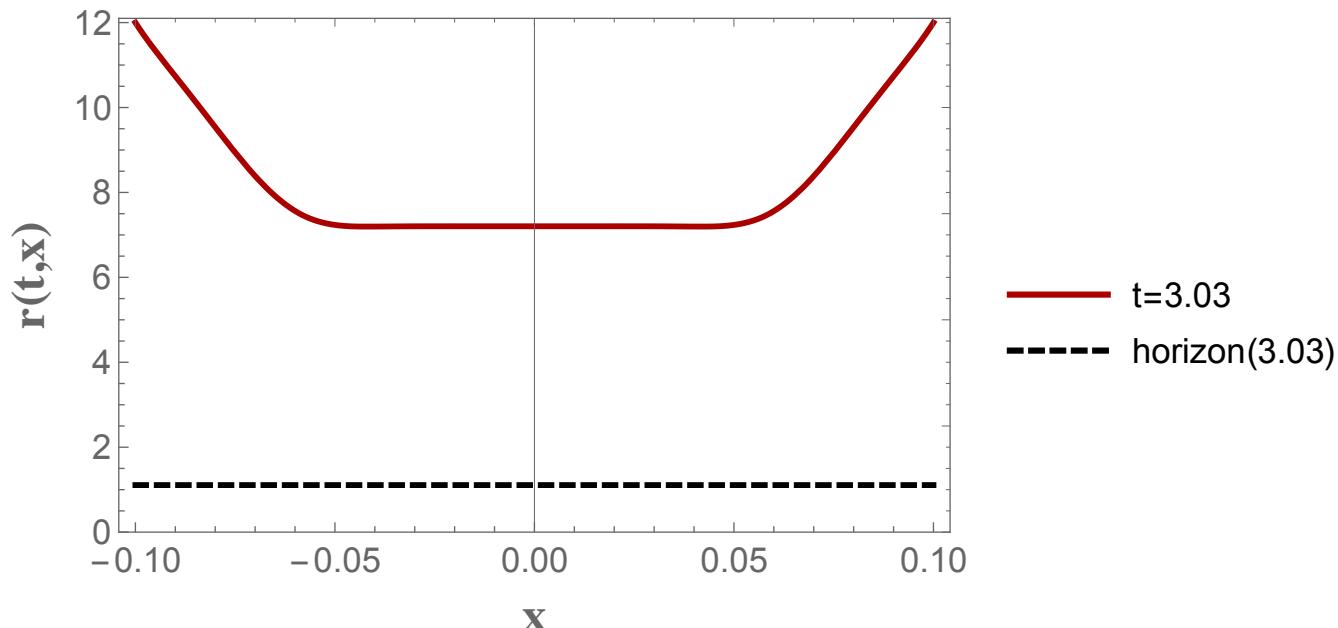


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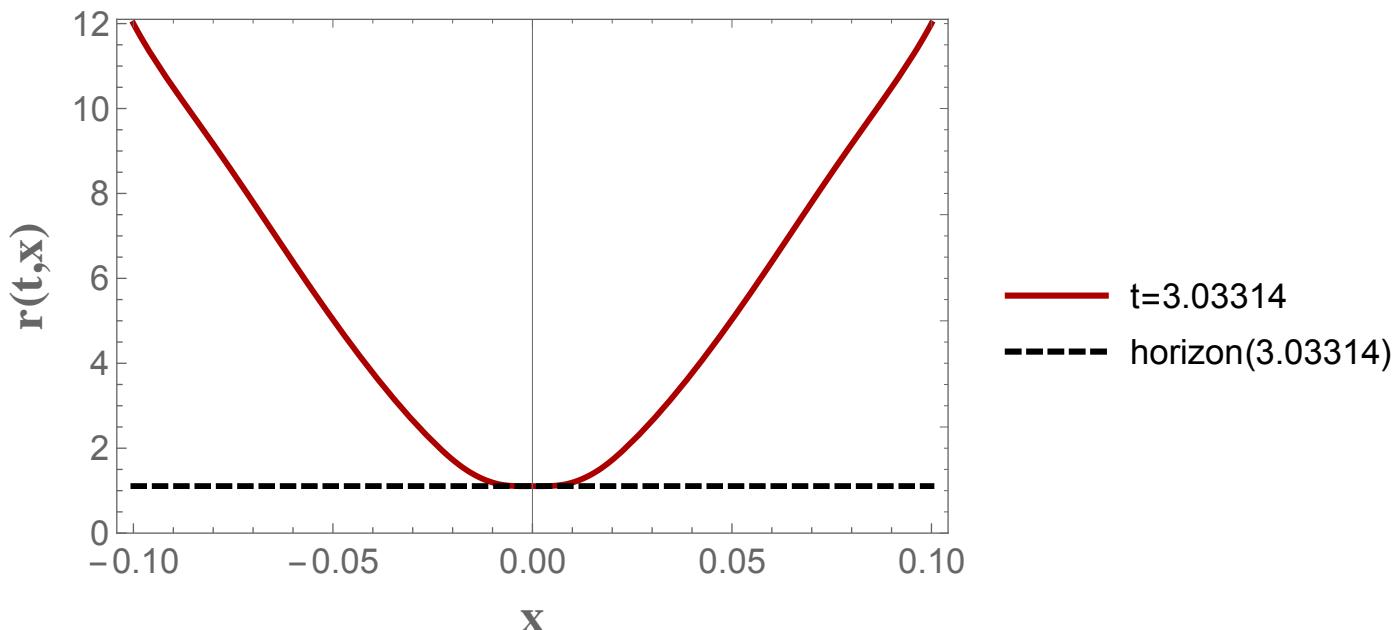


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w=x (transverse string)
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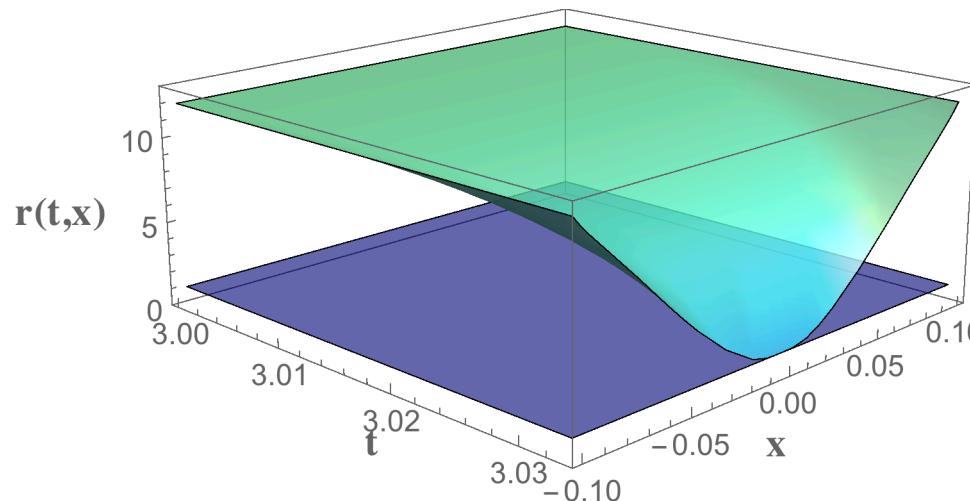


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↓

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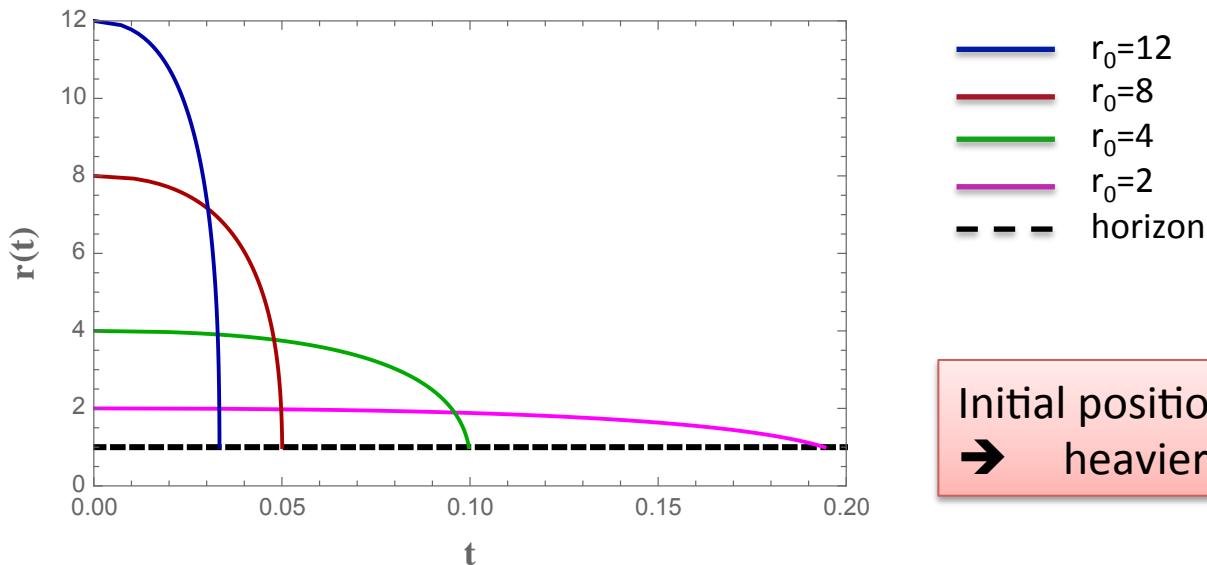


Analytic result for the string profile in a simplified case: AdS-BH

$$t^{BH} = \frac{2\sqrt{2}}{3r_0(1+Y)} \left[F_1\left(\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -1, \frac{1-Y}{1+Y}\right) - \frac{(1-P^2)^{3/4}}{(1+P)^{3/2}} F_1\left(\frac{3}{4}, \frac{1}{2}, 1, \frac{7}{4}, -\frac{1-P}{1+P}, \frac{1-P}{1+P} \frac{1-Y}{1+Y}\right) \right]$$

$$Y = \sqrt{1 - \frac{r_H^4}{r_0^4}}$$

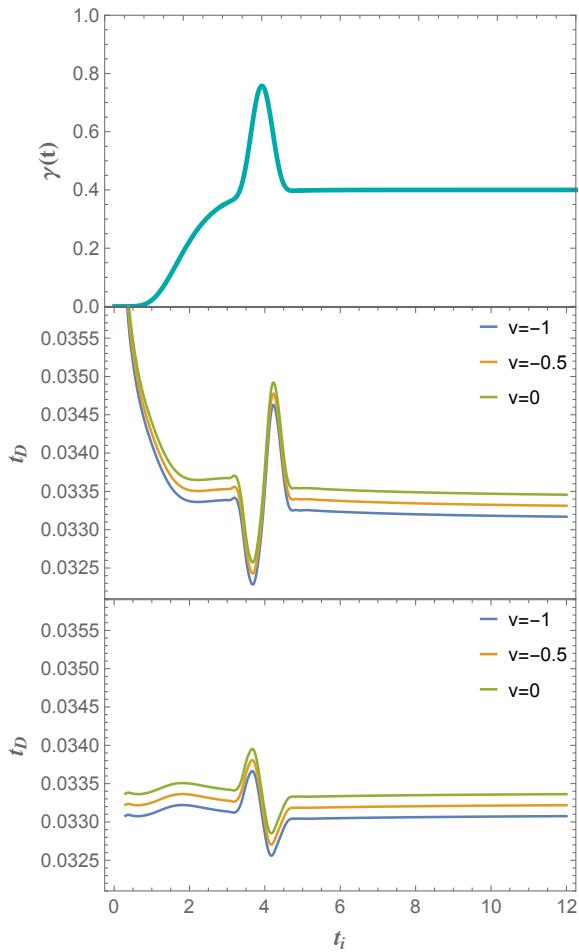
$$P = \sqrt{1 - \frac{r^4}{r_0^4}}$$



Initial position identified with the q mass
 → heavier quarkonia dissociate faster

dissociation time

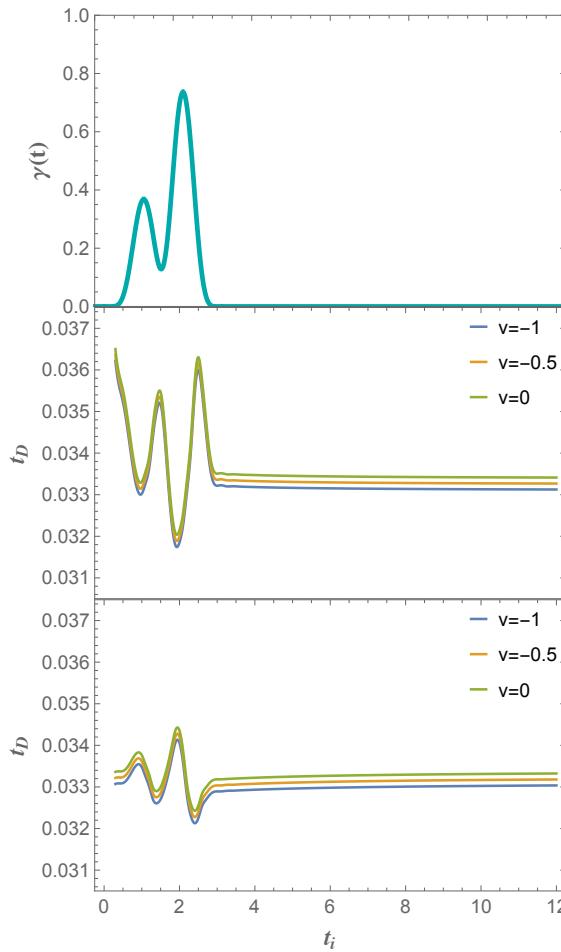
Bellantuono, Colangelo, Giannuzzi,
Nicotri, FDF
arXiv:1706.04809



quench

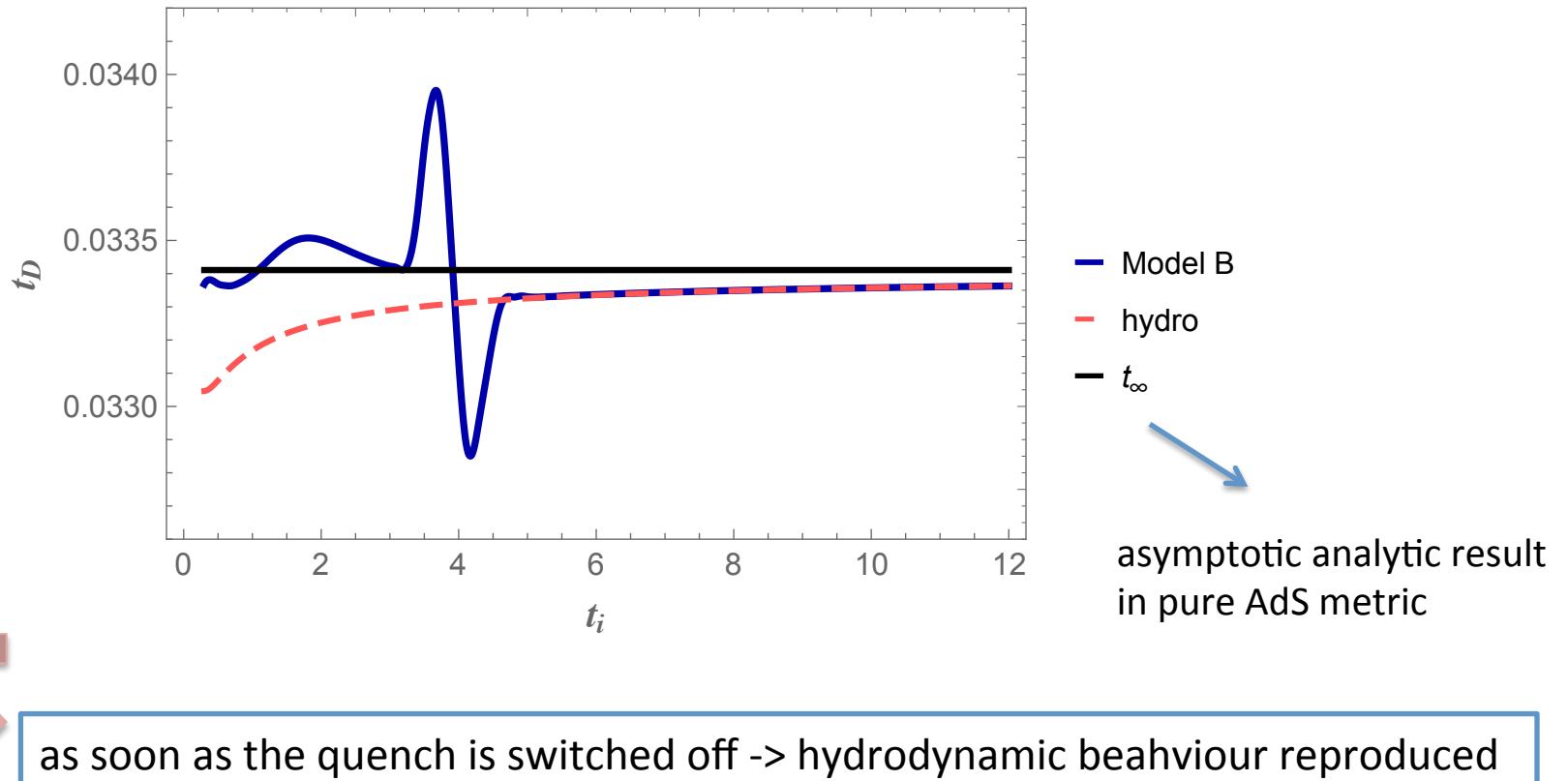
$w=y$

$w=x$



- metric functions are t -dependent:
 t_D depends on the initial time t_i when the string is kept on the boundary
- dissociation faster for a string placed in the transverse plane
- after the quench t_D tends to a constant value in both models

dissociation time: comparison with hydrodynamics



Conclusions

- present data on HI collisions indicate the occurrence of a collective flow
- regimes to be studied: fluid carried out of equilibrium / late time dynamics

highly non perturbative phenomenon

hydrodynamics seems
an appropriate description here

Gauge/Gravity duality may be used to mimic the situation

Main findings:

- the system evolves towards an equilibrium phase
- equilibration of local observables: $\tau \approx O(1 \text{ fm})$
- non-local observables: larger sizes of the probes require larger times
- quarkonium dissociation: fast phenomenon,
hydrodynamic regime reached immediately after the perturbation ends