4D REGULARIZATION OF H.O. COMPUTATIONS: FDU APPROACH

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Content

- Loop-tree duality
- LTD/FDU approach
  - Location of IR singularities and local UV counterterms
  - Real-virtual mapping
  - Application to a toy-model
- Physical example: $A^* \to q\bar{q}(g)$ @NLO
- Physical example: Higgs @NLO
- Conclusions and perspectives

Basic references for FDU/LTD:

1. *Catani et al, JHEP 09 (2008) 065*
2. *Rodrigo et al, Nucl.Phys.Proc.Suppl. 183:262-267 (2008)*
3. *Buchta et al, JHEP 11 (2014) 014*
4. **Rodrigo et al, JHEP 02 (2016) 044; JHEP 08 (2016) 160; JHEP 10 (2016) 162; arXiv:1702.07581 [hep-ph]**

Loop-tree duality

Dual representation of one-loop integrals

Loop Feynman integral

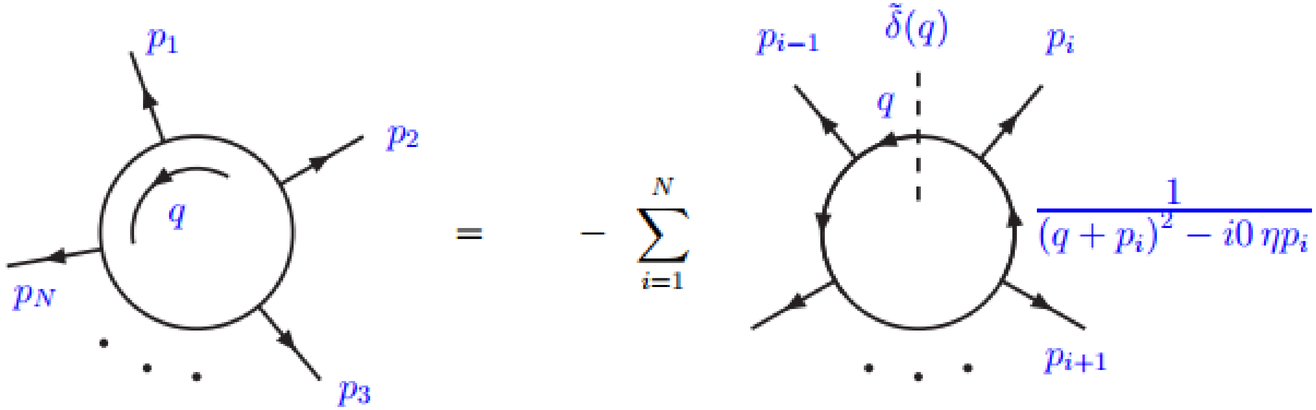
$$L^{(1)}(p_1,\dots,p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i) = \int_\ell \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + i0}$$

Dual integral

$$L^{(1)}(p_1,\dots,p_N) = -\sum_{i=1}^{N} \int_\ell \tilde{\delta}(q_i) \prod_{j=1, j\neq i}^{N} G_D(q_i;q_j)$$

Sum of phase-space integrals!

$$G_D(q_i,q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta(q_j - q_i)} \qquad \tilde{\delta}(q_i) = i2\pi\,\theta(q_{i,0})\,\delta(q_i^2 - m_i^2)$$



Even at higher-orders, the number of cuts is equal the number of loops

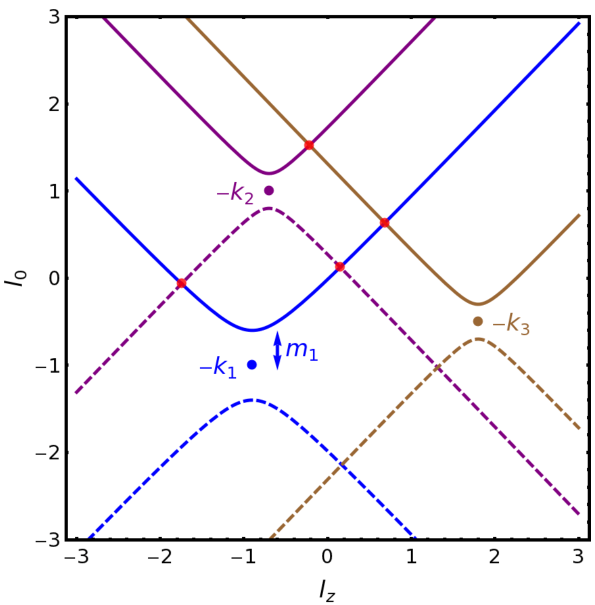
Catani et al, JHEP09(2008)065; Rodrigo et al, JHEP02(2016)044

LTD/FDU approach

Location of IR singularities in the dual-space

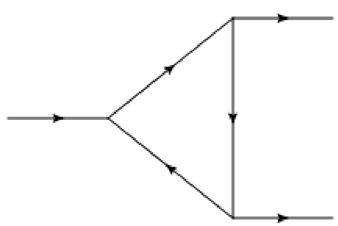
- Analize the dual integration region. It is obtained as the positive energy solution of the on-shell condition:

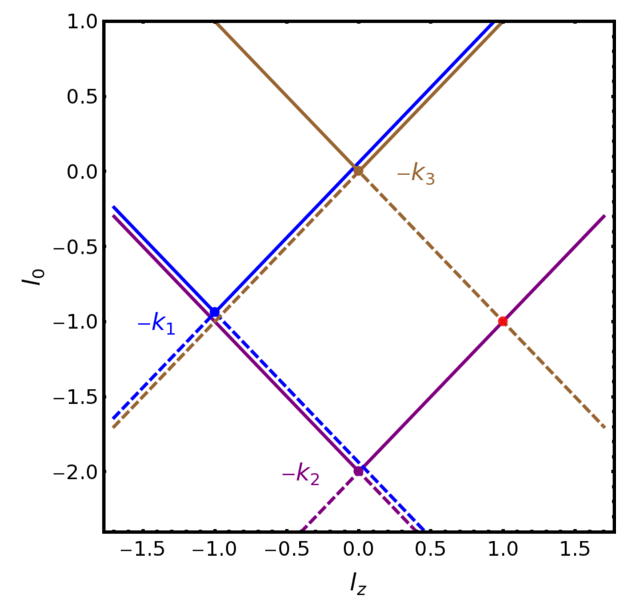
$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0 \quad \Rightarrow \quad q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$



Massive case: hyperboloids

- **Forward** (backward) on-shell hyperboloids associated with **positive** (negative) energy solutions.
- Degenerate to light-cones for massless propagators.
- *Dual integrands become singular at intersections* (two or more on-shell propagators)





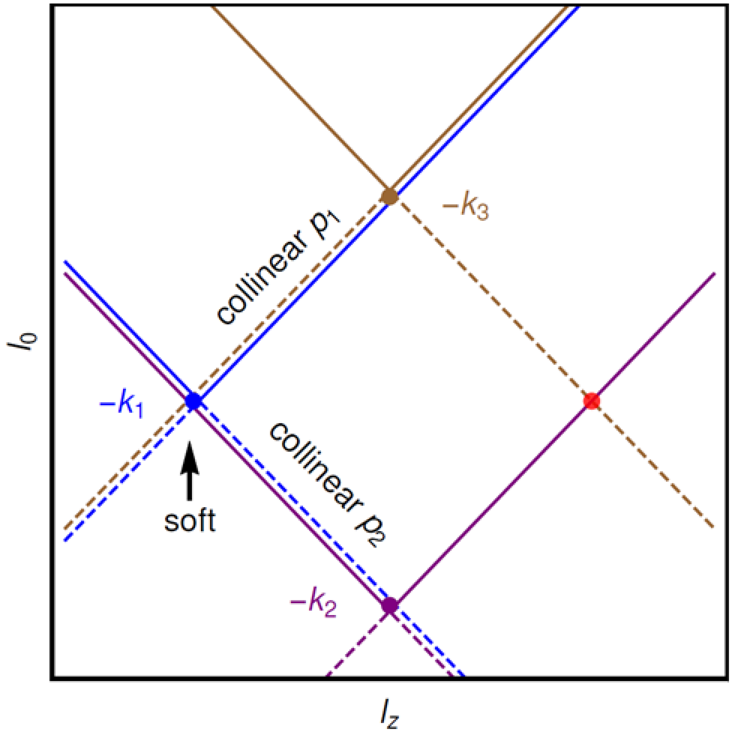
Massless case: light-cones

Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, JHEP08(2016)160

LTD/FDU approach

Location of IR singularities in the dual-space

- The application of LTD converts loop-integrals into PS ones: **integration over forward light-cones**.



- Only **forward-backward** interferences originate **threshold or IR poles** (*other propagators become singular in the integration domain*)
- **Forward-forward** singularities cancel among dual contributions
- Threshold and IR singularities associated with finite regions (i.e. contained in a **compact region**)
- **No threshold or IR singularity at large loop momentum**

- This structure suggests how to perform real-virtual combination! Also, how to overcome threshold singularities (integrable but numerically unstable)

Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, JHEP08(2016)160

LTD/FDU approach

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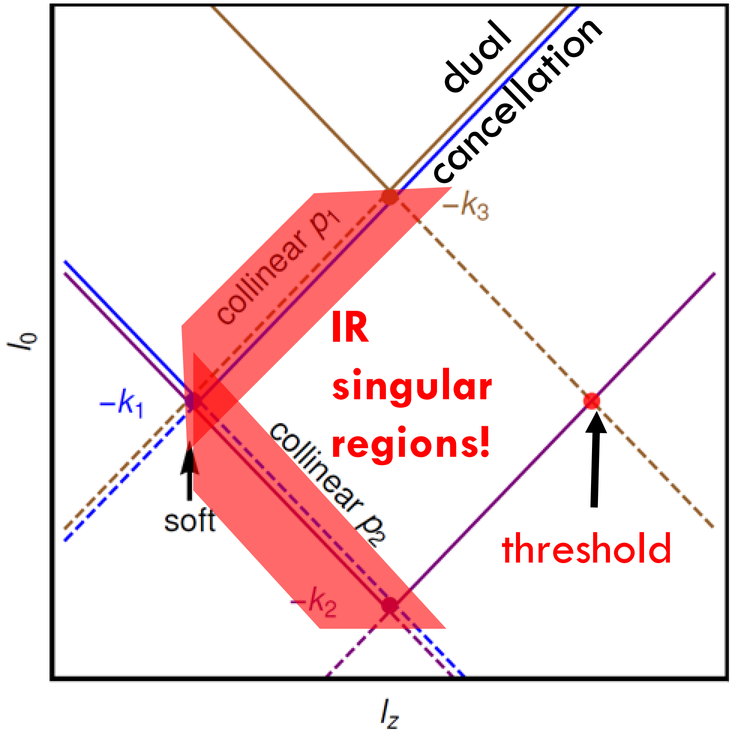

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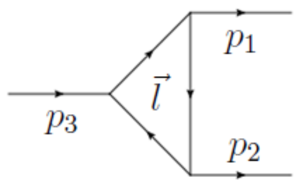
Buchta et al, JHEP11(2014)014; Rodrigo et al, JHEP02(2016)044, JHEP08(2016)160

LTD/FDU approach: toy model

Real-virtual momentum mapping

- Suppose **one-loop** scalar scattering amplitude given by the triangle (scalar toy-model!):

Virtual

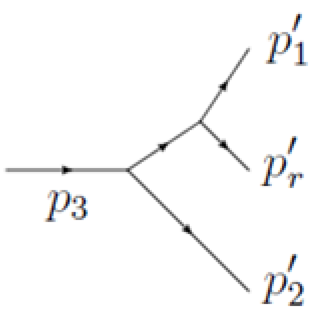


$$|\mathcal{M}^{(0)}(p_1,p_2;p_3)\rangle = ig$$
$$|\mathcal{M}^{(1)}(p_1,p_2;p_3)\rangle = -i\,g^3\,L^{(1)}(p_1,p_2,-p_3) \quad \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$$

- **1->2 one-loop process** → **1->3 with unresolved extra-parton**
- Add scalar tree-level contributions with one extra-particle; consider interference terms:

Opposite sign!

Real



$$|\mathcal{M}^{(0)}_{ir}(p_1',p_2',p_r';p_3)\rangle = -ig^2/s_{ir}' \Rightarrow \mathrm{Re}\,\langle\mathcal{M}^{(0)}_{ir}|\mathcal{M}^{(0)}_{jr}\rangle = \frac{g^4}{s_{ir}'\,s_{jr}'}$$

- Generate 1->3 kinematics starting from 1->2 configuration plus the loop three-momentum $\vec{l}$!!!

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: toy model

Real-virtual momentum mapping

- **Mapping of momenta:** generate **1->3 real** emission kinematics (**3 external on-shell momenta**) starting from the variables available in the dual description of **1->2 virtual** contributions (**2 external on-shell momenta and 1 free three-momentum**)
  - ✓ Split the real phase space into two regions, i.e. $y'_{1r}<y'_{2r}$ and $y'_{2r}<y'_{1r}$, to separate the possible collinear singularities
  - ✓ Implement an optimized mapping in each region, to allow a fully local cancellation of IR singularities with those present in the dual terms

REGION 1:

$$p_r'^{\mu} = q_1^{\mu}\,, \qquad p_1'^{\mu} = p_1^{\mu} - q_1^{\mu} + \alpha_1\, p_2^{\mu}\,,$$
$$p_2'^{\mu} = (1-\alpha_1)\, p_2^{\mu}\,, \qquad \alpha_1 = \frac{q_3^2}{2 q_3 \cdot p_2}\,,$$

$$\Rightarrow \quad y'_{1r} = \frac{v_1\, \xi_{1,0}}{1-(1-v_1)\,\xi_{1,0}} \qquad y'_{12} = 1 - \xi_{1,0}$$
$$y'_{2r} = \frac{(1-v_1)(1-\xi_{1,0})\,\xi_{1,0}}{1-(1-v_1)\,\xi_{1,0}}$$

REGION 2:

$$p_2'^{\mu} = q_2^{\mu}\,, \qquad p_r'^{\mu} = p_2^{\mu} - q_2^{\mu} + \alpha_2\, p_1^{\mu}\,,$$
$$p_1'^{\mu} = (1-\alpha_2)\, p_1^{\mu}\,, \qquad \alpha_2 = \frac{q_1^2}{2 q_1 \cdot p_1}\,,$$

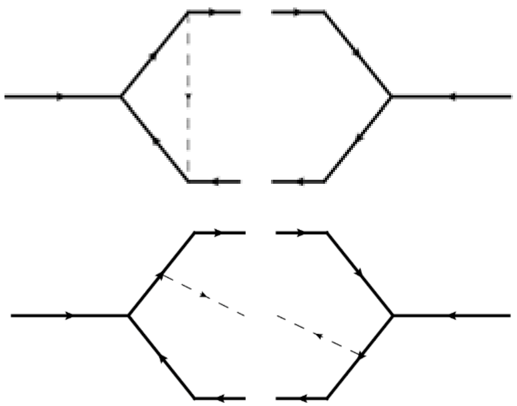
$$\Rightarrow \quad y'_{1r} = 1 - \xi_{2,0} \qquad y'_{2r} = \frac{(1-v_2)\,\xi_{2,0}}{1-v_2\,\xi_{2,0}}$$
$$y'_{12} = \frac{v_2\,(1-\xi_{2,0})\,\xi_{2,0}}{1-v_2\,\xi_{2,0}}$$

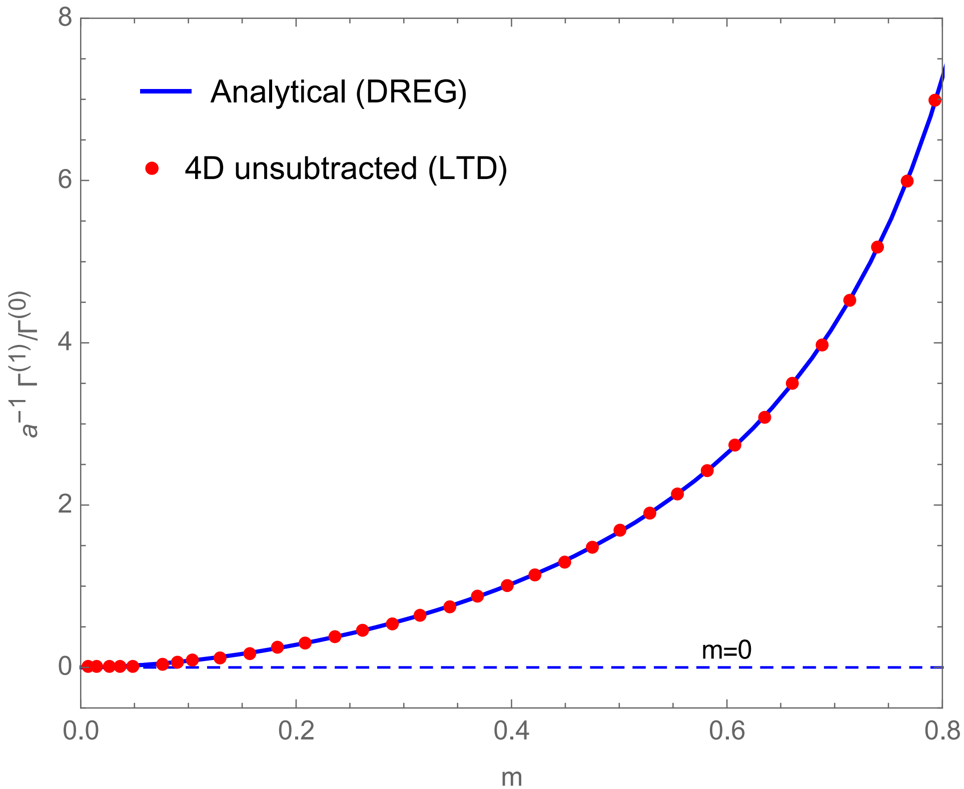
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD/FDU approach: toy model

Example: massive scalar three-point function (DREG vs LTD)

- We combine the dual contributions with the real terms (after applying the proper mapping) to get the total decay rate in the scalar toy-model.
  - The result agrees ***perfectly*** with standard DREG.
  - **Massless limit** is **smoothly** approached due to proper treatment of **quasi-collinear** configurations in the **RV mapping**





Rodrigo et al, JHEP10(2016)162

LTD/FDU approach: multileg

Real-virtual momentum mapping (GENERAL)

- **Real-virtual momentum mapping with massive particles:**
  - Consider **1** the **emitter**, **r** the **radiated particle** and **2** the **spectator**
  - Apply the PS partition and restrict to the only region where **1**//**r** is allowed (i.e. $\mathcal{R}_1 = \{y'_{1r} < \min y'_{kj}\}$)
  - Propose the following mapping:

$$
\boxed{
\begin{aligned}
p'^{\mu}_r &= q^{\mu}_1 \\
p'^{\mu}_1 &= (1-\alpha_1)\,\hat{p}^{\mu}_1 + (1-\gamma_1)\,\hat{p}^{\mu}_2 - q^{\mu}_1 \\
p'^{\mu}_2 &= \alpha_1\,\hat{p}^{\mu}_1 + \gamma_1\,\hat{p}^{\mu}_2
\end{aligned}
}
$$

Impose on-shell conditions to determine mapping parameters

with $\hat{p}_i$ massless four-vectors build using p_i (simplify the expressions)

  - *Express the loop three-momentum with the same parameterization used for describing the dual contributions!*

Repeat in each region of the partition…

Rodrigo et al, JHEP10(2016)162

LTD/FDU approach: renormalization

UV counterterms and local renormalization

- LTD must be applied to deal with **UV singularities** by building **local** versions of the usual UV counterterms.
- 1: **Expand** internal propagators around the *"UV propagator"*

Becker, Reuschle, Weinzierl, JHEP12(2010)013

$$
\frac{1}{q_i^2 - m_i^2 + \imath 0} = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} \times \left[1 - \frac{2 q_{\rm UV} \cdot k_{i,\rm UV} + k_{i,\rm UV}^2 - m_i^2 + \mu_{\rm UV}^2}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} + \frac{(2 q_{\rm UV} \cdot k_{i,\rm UV})^2}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0)^2} \right] + \mathcal{O}\left((q_{\rm UV}^2)^{-5/2} \right)
$$

- 2: Apply LTD to get the **dual representation** for the expanded UV expression, and **subtract** it from the **dual+real** combined integrand.

LTD extended to deal with multiple poles

(use residue formula to obtain the dual representation)

- 3: Take into account **wave-function and vertex renormalization** constants (not trivial in the massive case!)

Rodrigo et al, JHEP10(2016)162

LTD/FDU approach: renormalization

UV counterterms and local renormalization

- Self-energy corrections with **on-shell renormalization** conditions

$$\Sigma_R(\not p_1 = M) = 0 \qquad \left.\frac{d\Sigma_R(\not p_1)}{d\not p_1}\right|_{\not p_1 = M} = 0$$

- ***Wave-function renormalization constant*** (**both IR and UV poles**):

$$\Delta Z_2(p_1) = -g_S^2 C_F \int_\ell G_F(q_1) G_F(q_3) \left((d-2)\frac{q_1 \cdot p_2}{p_1 \cdot p_2} + 4M^2 \left(1 - \frac{q_1 \cdot p_2}{p_1 \cdot p_2}\right) G_F(q_3) \right)$$

- ***Vertex renormalization (only UV):***

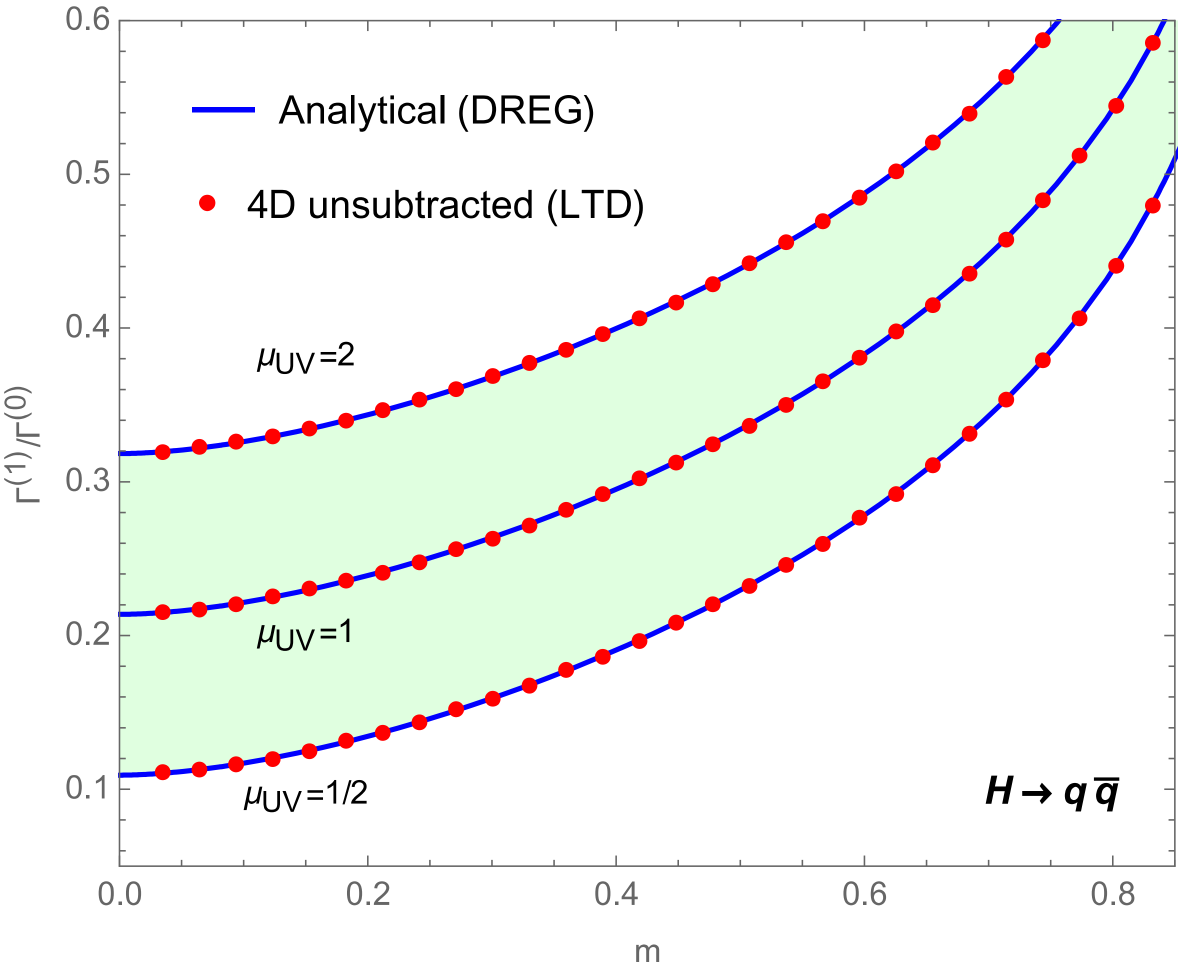
$$\mathbf{\Gamma}^{(1)}_{A,\mathrm{UV}} = g_S^2\, C_F \int_\ell (G_F(q_{\mathrm{UV}}))^3 \left[\gamma^\nu\, \not q_{\mathrm{UV}}\, \mathbf{\Gamma}^{(0)}_A\, \not q_{\mathrm{UV}}\, \gamma_\nu - d_{A,\mathrm{UV}}\, \mu^2_{\mathrm{UV}}\, \mathbf{\Gamma}^{(0)}_A \right]$$

- **Important features:**
  - Integrated results agrees with standard UV counter-terms!
  - **Smooth massless limit!**

Rodrigo et al, JHEP10(2016)162

Physical example: $A^* \to q\bar{q}(g)$@NLO

Results and comparison with DREG

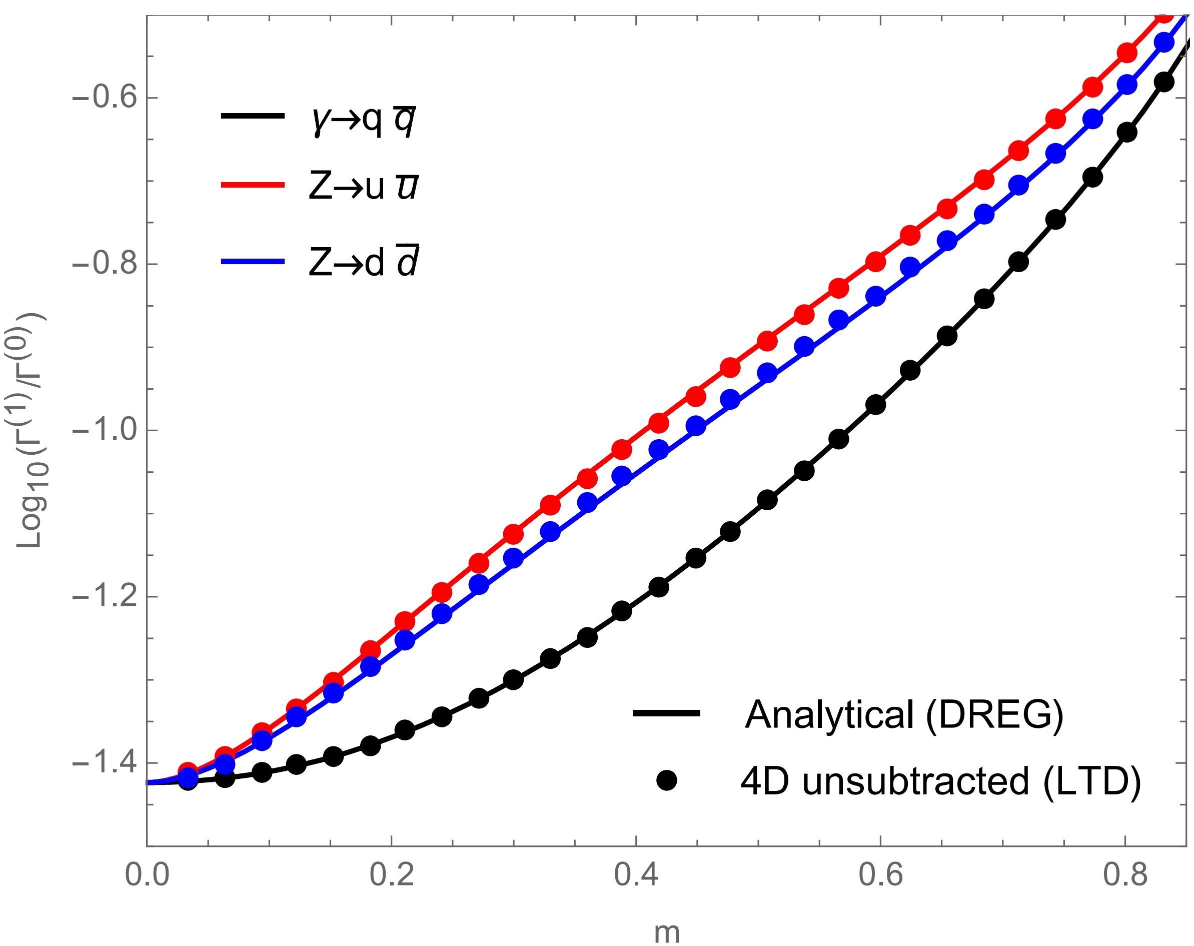


- *Total decay rate for Higgs into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Local version of UV counterterms succesfully reproduces the expected behaviour
  - Efficient numerical implementation

Rodrigo et al, JHEP10(2016)162

Physical example: $A^* \to q\bar{q}(g)$@NLO

Results and comparison with DREG



- *Total decay rate for a vector particle into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Efficient numerical implementation
  - Cancellation of UV log's (as in DREG…)

Rodrigo et al, JHEP10(2016)162

Physical example: $A^* \to q\bar{q}(g)$@NLO

Final remarks

- The total decay-rate can be expressed using purely **four-dimensional integrands**
- We recover the total NLO correction, while **avoiding dealing with DREG**

- **Main advantages:**
  - ✓ Direct **numerical** implementation (integrable functions for **ε=0**)

    Finite integral for ε=0 ~~⟹~~ Integrability with ε=0

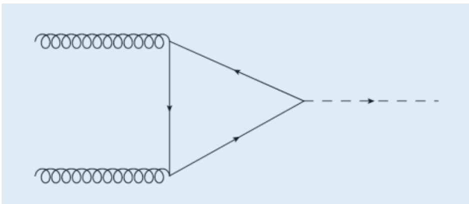
    With FDU is true!
  - ✓ No need of tensor reduction (**avoids the presence of Gram determinants,** which could introduce numerical instabilities)
  - ✓ **Smooth transition** to the massless limit (due to the efficient treatment of **quasi-collinear** configurations)
  - ✓ **Mapped real-contribution used as a fully local IR counter-term for the dual contribution!**

Rodrigo et al, JHEP10(2016)162

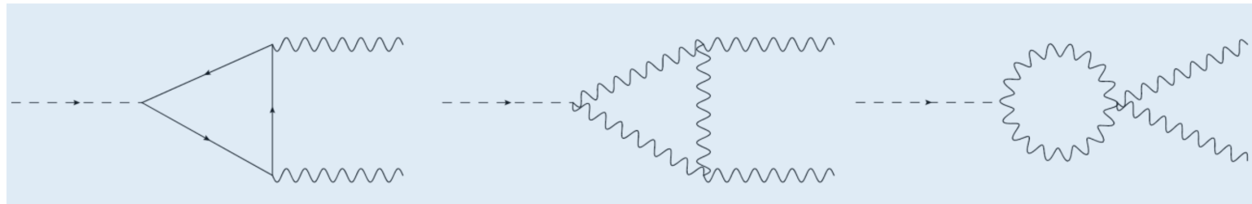
Physical example: Higgs@NLO

Using LTD to regularize finite amplitudes

- Application of LTD to compute one-loop Higgs amplitudes:



$gg \to H$



$H \to \gamma\gamma$

- They are IR/UV finite BUT still not well-defined in 4D!!! Hidden cancellation of singularities leads to potentially undefined results (scheme dependence!!!)
- We start by defining a tensor basis and projecting (amplitude level!):
- Combined expressions (use "0's in DREG" associated with Ward identities):

$$\mathcal{A}_1^{(1,f)} = g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2\,(2\ell \cdot p_{12})^2}{s_{12}^2 - (2\ell \cdot p_{12} - \imath 0)^2} \right) \frac{s_{12}\, M_f^2}{(2\,\ell \cdot p_1)(2\,\ell \cdot p_2)} \, c_1^{(f)} + \frac{2\, s_{12}^2}{s_{12}^2 - (2\ell \cdot p_{12} - \imath 0)^2} \, c_{23}^{(f)} \right]$$

Well defined in 4-d!!

UV divergent

$\to \mathcal{O}(\epsilon)$

Non-commutativity of limit and integration!!!!

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]

Physical example: Higgs@NLO

Using LTD to regularize finite amplitudes

- Use local renormalization (compatible with Dyson's prescription)

$$\mathcal{A}^{(1,f)}_{1,\mathrm{R}}\Big|_{d=4} = \left(\mathcal{A}^{(1,f)}_{1} - \mathcal{A}^{(1,f)}_{1,\mathrm{UV}}\right)_{d=4} \qquad \mathcal{A}^{(1,f)}_{1,\mathrm{UV}} = -g_f \int_\ell \frac{\tilde{\delta}(\ell)\, \ell_0^{(+)}\, s_{12}}{2(q^{(+)}_{\mathrm{UV},0})^3} \left(1 + \frac{1}{(q^{(+)}_{\mathrm{UV},0})^2}\frac{3\,\mu^2_{\mathrm{UV}}}{d-4}\right) c^{(f)}_{23}$$

  - Counter-term mimics UV behaviour at integrand level.
  - Term proportional to μ^2_{UV} used to fix DREG scheme (vanishing counter-term in d-dim!!)
  - Valid also for W amplitudes in unitary-gauge (naive Dyson's prescription fails to subtract subleading terms due to enhanced UV divergences)

$$-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)\frac{1}{q_i^2 - M_W^2 + \imath 0}$$

  - *Universal expressions for scalar, fermions and bosons (inside the loop; codified in the **c's**)*

- **Important:** *Dual amplitudes defined in Euclid space. Asymptotic expansions straighforwardly implemented within at integrand level!!*

$$\boxed{\tilde{\delta}(q_3)\; G_D(q_3;q_2) = \frac{\tilde{\delta}(q_3)}{2q_3\cdot p_{12}} \sum_{n=0}^{\infty}\left(\frac{-s_{12}}{2q_3\cdot p_{12}}\right)^n}$$

More details: POSTER SESSION!!

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]

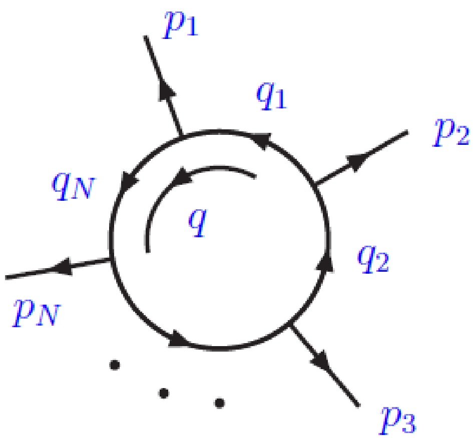
Conclusions and perspectives

- ✓ Loop-tree duality allows to treat **virtual and real** contributions **simultaneously** (implementation simplified)
- ✓ Physical interpretation of **IR/UV singularities** in loop integrals
- ✓ **Combined virtual-real terms are integrable in four space-time dimensions!!**
- ✓ **First (realistic) physical implementation!!!**
- ✓ **Universal & compact expressions for Higgs amplitudes**

- **Perspectives:**
  - Automation of multileg processes and extension to NNLO
  - Generalization of universality relations
  - Exploit simplifications due to easier asymptotic expansions
  - *Carefull comparison with other schemes* → "Workstop-Thinkstart meeting" ***UZH, Zurich, Sep. 2016*** ***arXiv:1705.01827 [hep-ph]***

Thanks!!!

Cauchy's theorem and prescriptions



$$L_R^{(N)}(p_1, p_2, \cdots, p_N) = -\imath \int \frac{d^d\ell}{(2\pi)^d} \prod_{i=1}^{N} G_R(q_i)$$

Generic one-loop Feynman integral

$$q_i = \ell + \sum_{k=1}^{i} p_k$$

Momenta definition

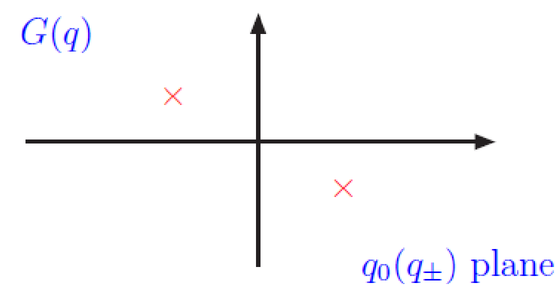
Prescriptions are useful to avoid poles. Different prescriptions are possible; connection between FTT and LTD theorems!

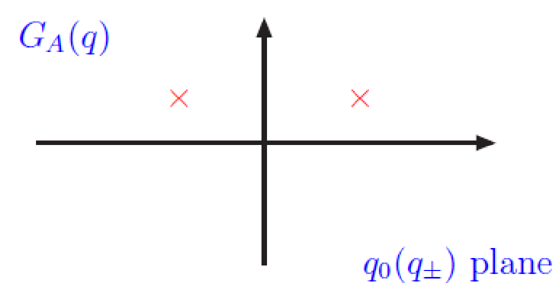
Feynman propagator

$$G(q) \equiv \frac{1}{q^2 + \imath 0}$$

Advanced propagator

$$G_A(q) \equiv \frac{1}{q^2 - \imath 0\, q_0}$$

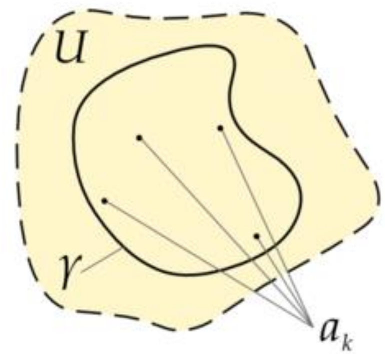




Cauchy's theorem and prescriptions

Residue theorem (*from Wikipedia*)

$$\oint_\gamma f(z)\,dz = 2\pi i \sum \mathrm{Res}(f, a_k)$$

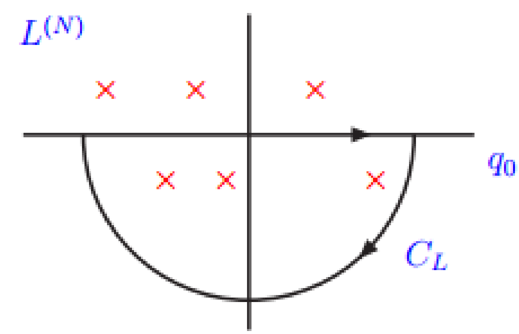


«If f is a holomorphic function in U/{a_i}, and γ a simple positively oriented curve, then the integral is given by the sum of the residues at each singular point a_i»

Residue theorem can be used to compute integrals involving propagators: the prescription and the contour that we choose determine the result!

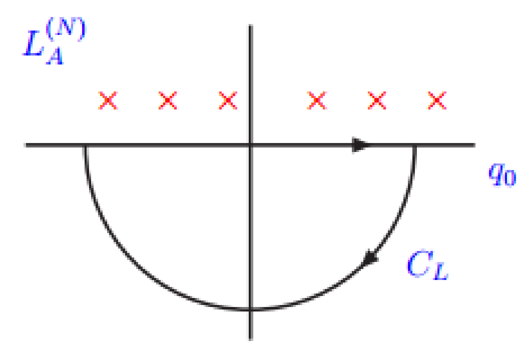
Feynman propagator

$$[G(q)]^{-1} = 0 \quad \Longrightarrow \quad q_0 = \pm\sqrt{\mathbf{q}^2 - i0}$$



Advanced propagator

$$[G_A(q)]^{-1} = 0 \quad \Longrightarrow \quad q_0 \simeq \pm\sqrt{\mathbf{q}^2} + i0$$



NO POLES CLOSED BY C_L!

Loop-tree duality

Derivation (one-loop)

- **Idea:** *«Sum over all possible 1-cuts»* (but with a **modified prescription…**)
  - Apply Cauchy's residue theorem to the Feynman integral:

$$L^{(N)}(p_1, p_2, \ldots, p_N) = \int_{\mathbf{q}} \int dq_0 \prod_{i=1}^{N} G(q_i) = \int_{\mathbf{q}} \int_{C_L} dq_0 \prod_{i=1}^{N} G(q_i) = -2\pi i \int_{\mathbf{q}} \sum \mathrm{Res}_{\{\mathrm{Im}\, q_0 < 0\}} \left[\prod_{i=1}^{N} G(q_i) \right]$$

  - Compute the residue in the poles with negative imaginary part:

$$\mathrm{Res}_{\{i\mathrm{-th\ pole}\}} \left[\prod_{j=1}^{N} G(q_j) \right] = \left[\mathrm{Res}_{\{i\mathrm{-th\ pole}\}}\, G(q_i) \right] \left[\prod_{\substack{j=1 \\ j \neq i}}^{N} G(q_j) \right]_{\{i\mathrm{-th\ pole}\}}$$

$$\left[\mathrm{Res}_{\{i\mathrm{-th\ pole}\}} \frac{1}{q_i^2 + i0} \right] = \int dq_0\, \delta_+(q_i^2)$$

Put on-shell the particle crossed by the cut

$$\left[\prod_{j \neq i} G(q_j) \right]_{\{i\mathrm{-th\ pole}\}} = \prod_{j \neq i} \frac{1}{q_j^2 - i0\, \eta(q_j - q_i)}$$

Introduction of «dual propagators» (η prescription, a future- or light-like vector)

Catani et al, JHEP09(2008)065; Rodrigo et al, JHEP02(2016)044

Loop-tree duality

Derivation (general facts)

- *It is crucial to keep track of the prescription!* Duality relation involves the presence of dual propagators:

$$L^{(N)}(p_1, p_2, \dots, p_N) = -\int_q \sum_{i=1}^{N} \tilde{\delta}(q_i) \prod_{\substack{j=1 \\ j\neq i}}^{N} \frac{1}{q_j^2 - i0\,\eta(q_j - q_i)}$$

  - The prescription involves a future- or light-like vector (arbitrary) and could depend on the loop momenta (at 1-loop is always independent of *q*). It is related with the finite value of **i0** in intermediate steps
  - *Connection with **Feynman Tree Theorem***: **dual prescription** encodes the information contained in **multiple cuts**
  - Implement a shift in each term of the sum to have the same measure: the loop integral becomes a phase-space integral!
  - The unification of coordinates allows a cancellation of singularities among dual components (*UV and soft/collinear divergences remaining*)

Catani et al, JHEP09(2008)065; Rodrigo et al, JHEP02(2016)044

Feynman tree theorem

Derivation

- **Idea:** *«Sum over all possible m-cuts»*

$$L_A^{(N)}(p_1, p_2, \dots, p_N) = \int_q \prod_{i=1}^{N} G_A(q_i) \; = 0$$

Residue theorem (using a proper integration path)

$$G_A(q) = G(q) + \tilde{\delta}(q)$$

Using PV prescription

$$L_A^{(N)}(p_1, p_2, \dots, p_N) = \int_q \prod_{i=1}^{N} \left[G(q_i) + \tilde{\delta}(q_i) \right]$$

$$= L^{(N)}(p_1, p_2, \dots, p_N) + L^{(N)}_{\text{1-cut}}(p_1, p_2, \dots, p_N) + \dots + L^{(N)}_{\text{N-cut}}(p_1, p_2, \dots, p_N)$$

m-cut definition:

$$L^{(N)}_{\text{m-cut}}(p_1, p_2, \dots, p_N) = \int_q \left\{ \tilde{\delta}(q_1) \dots \tilde{\delta}(q_m)\, G(q_{m+1}) \dots G(q_N) + \text{uneq. perms.} \right\}$$

$$\boxed{L^{(N)}(p_1, p_2, \dots, p_N) = - \left[\, L^{(N)}_{\text{1-cut}}(p_1, p_2, \dots, p_N) + \dots + L^{(N)}_{\text{N-cut}}(p_1, p_2, \dots, p_N) \, \right]}$$

Feynman tree theorem

Derivation

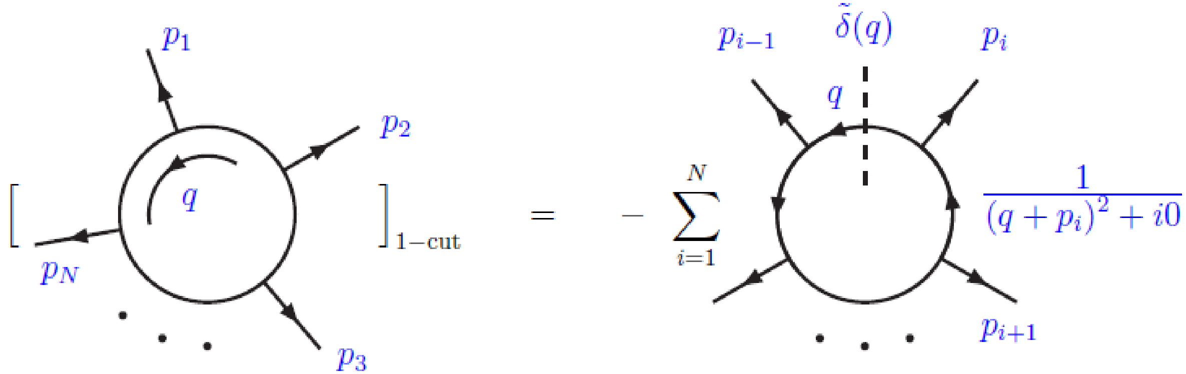
- Some remarks:
  - Making m-cuts decomposes the original **one-loop** diagram into **m-tree level terms,** all of them using the **same prescription**
  - *1-cut = sum over «tree level» terms*

$$L^{(N)}_{1-\mathrm{cut}}(p_1, p_2, \ldots, p_N) = \int_q \sum_{i=1}^{N} \widetilde{\delta}(q_i) \prod_{\substack{j=1 \\ j\neq i}}^{N} G(q_j)$$

$$I^{(n)}_{1-\mathrm{cut}}(k_1, k_2, \ldots, k_n) = \int_q \widetilde{\delta}(q) \prod_{j=1}^{n} G(q+k_j) = \int_q \widetilde{\delta}(q) \prod_{j=1}^{n} \frac{1}{2qk_j + k_j^2 + i0}$$

Basic 1-cut integral (shift in loop momentum)

Loop



Sum of multiple cuts

$\gamma \to q\bar{q}$@NLO: 4D formulae

- Integration regions:

$$\mathcal{R}_1(\xi_0, v) = \theta(1-2v_1)\,\theta\left(\frac{1-2v_1}{1-v_1} - \xi_{1,0}\right)\Big|_{\{\xi_{1,0},v_1\}\to\{\xi_{3,0},v_3\}=\{\xi_0,v\}}$$

$$\mathcal{R}_2(\xi_0, v) = \theta\left(\frac{1}{1+\sqrt{1-v}} - \xi_0\right)$$

- Four-dimensional cross-sections:

$$\widetilde{\sigma}_1^{(1)} = \sigma^{(0)}\,\frac{\alpha_S}{4\pi}\,C_F \int_0^1 d\xi_{1,0} \int_0^{1/2} dv_1\, 4\,\mathcal{R}_1(\xi_{1,0}, v_1) \left[2\left(\xi_{1,0} - (1-v_1)^{-1}\right) - \frac{\xi_{1,0}(1-\xi_{1,0})}{(1-(1-v_1)\,\xi_{1,0})^2}\right]$$

$$\widetilde{\sigma}_2^{(1)} = \sigma^{(0)}\,\frac{\alpha_S}{4\pi}\,C_F \int_0^1 d\xi_{2,0} \int_0^1 dv_2\, 2\,\mathcal{R}_2(\xi_{2,0}, v_2)\,(1-v_2)^{-1} \left[\frac{2\,v_2\,\xi_{2,0}\,(\xi_{2,0}(1-v_2)-1)}{1-\xi_{2,0}}\right]$$

$$\begin{aligned}
\overline{\sigma}_V^{(1)} = {} & \sigma^{(0)}\,\frac{\alpha_S}{4\pi}\,C_F \int_0^\infty d\xi \int_0^1 dv \left\{ -2\,(1-\mathcal{R}_1(\xi,v))\,v^{-1}(1-v)^{-1}\,\frac{\xi^2(1-2v)^2+1}{\sqrt{(1+\xi)^2-4v\,\xi}} \right. \\
& + 2\,(1-\mathcal{R}_2(\xi,v))\,(1-v)^{-1}\left[2\,v\,\xi\,(\xi(1-v)-1)\left(\frac{1}{1-\xi+\imath 0} + \imath\pi\delta(1-\xi)\right) - 1 + v\,\xi\right] \\
& + 2\,v^{-1}\left(\frac{\xi(1-v)(\xi(1-2v)-1)}{1+\xi} + 1\right) - \frac{(1-2v)\,\xi^3\,(12-7m_{\rm UV}^2-4\xi^2)}{(\xi^2+m_{\rm UV}^2)^{5/2}} \\
& \left. - \frac{2\,\xi^2(m_{\rm UV}^2+4\xi^2(1-6v(1-v)))}{(\xi^2+m_{\rm UV}^2)^{5/2}} \right\}
\end{aligned}$$

Higgs@NLO: Formulae

 Using LTD to regularize finite amplitudes

- Application of LTD to compute one-loop Higgs amplitudes:

$$
\begin{aligned}
\mathcal{A}_1^{(1,f)} &= g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2\,(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \right) \left(\frac{s_{12}\, M_f^2}{(2\,\ell\cdot p_1)(2\,\ell\cdot p_2)}\, c_1^{(f)} \right.\right. \\
&\qquad \left.\left. + c_2^{(f)} \right) + \frac{2\, s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2}\, c_3^{(f)} \right] \\
\mathcal{A}_2^{(1,f)} &= g_f \frac{c_3^{(f)}}{2} \int_\ell \tilde{\delta}(\ell) \left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} - 2 \right)
\end{aligned}
$$

with $q_1 = \ell + p_1,\ q_2 = \ell + p_{12}$ and $q_{1,0}^{(+)} = \sqrt{(\boldsymbol{\ell} + \mathbf{p}_1)^2 + M_f^2}\,,\quad q_{4,0}^{(+)} = \sqrt{(\boldsymbol{\ell} + \mathbf{p}_2)^2 + M_f^2}\,,$

$q_3 = \ell,\ q_4 = \ell + p_2$ $\qquad \ell_0^{(+)} = q_{2,0}^{(+)} = q_{3,0}^{(+)} = \sqrt{\boldsymbol{\ell}^2 + M_f^2}\,.$

- **Comments:**
  - Generic result valid for $gg \to H$ and $H \to \gamma\gamma$!!
  - Process dependence codified in the coefficients. **Valid for scalar, fermion and vector massive particles inside the loop!!!**

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]

Higgs@NLO: Coefficients

The coefficients $c_i^{(f)}$ are written as $c_i^{(f)} = c_{i,0}^{(f)} + r_f\, c_{i,1}^{(f)}$ with $r_f = s_{12}/M_f^2$, and

$$g_f = \frac{2\, M_f^2}{\langle v \rangle\, s_{12}} \,, \quad c_{23,0}^{(f)} = (d-4)\frac{c_{1,0}^{(f)}}{2} \,, \quad c_{1,0}^{(\phi)} = \frac{2}{d-2} \,, \quad c_{1,0}^{(W)} = \frac{4\,(d-1)}{d-2} \,, \quad c_{1,1}^{(W)} = -\frac{2(2\,d-5)}{d-2} \,,$$

$$c_{3,0}^{(\phi)} = 2 \,, \quad c_{1,0}^{(t)} = \frac{8}{d-2} \,, \quad c_{1,1}^{(t)} = -1 \,, \quad c_{3,0}^{(t)} = 8 \,, \quad c_{23,1}^{(W)} = \frac{d-4}{d-2} \,, \quad c_{3,0}^{(W)} = 4(d-1) \,,$$

with $c_2^{(f)} = c_{23}^{(f)} - c_3^{(f)}$ and $c_{1,1}^{(\phi)} = c_{23,1}^{(\phi)} = c_{3,1}^{(\phi)} = c_{23,1}^{(t)} = c_{3,1}^{(t)} = c_{3,1}^{(W)} = 0$.

Driencourt-Mangin, Rodrigo and G.S., arXiv:1702.07581 [hep-ph]