Renormalization in large-N QCD is incompatible with open/closed string duality

M. B. arXiv: 1606.04546 [hep-th]

M. B. Phys. Rev. D 95 054010

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The first aim of this talk is to answer the following fundamental question that, surprisingly, has not been asked for more than 40 years.

We know that YM theory and QCD are not UV finite, but only renormalizable, in perturbation theory.

Yet, we may ask which are the renormalization properties of the YM theory and QCD (with massless quarks at first, for simplicity) nonperturbatively in the large-N 't Hooft expansion.

We recall that the large-N 't Hooft limit of SU(N) QCD (with massless quarks):

$$Z = \int \delta A \delta\bar{\psi}\delta\psi \, e^{-\frac{N}{2g^2}\int \sum_{\alpha\beta} Tr\left(F^2_{\alpha\beta}\right)+i\sum_f \bar{\psi}_f\gamma^\alpha D_\alpha\psi_f \, d^4x}$$

(G. 't Hooft 1974)

is a free theory of glueballs and mesons at leading 1/N order (planar theory), which become weakly coupled at the next order with couplings O(1/N) and O(1/sqrt N) respectively.

The answer to the previous question is that the large-N expansion of the YM S matrix is UV finite,
while the large-N 't Hooft expansion of the QCD S matrix is only renormalizable,
once the (planar) RG invariant, which is the only free parameter in the S matrix, has been made finite at the lowest $1/N$ order by gauge coupling renormalization.

Proof

In large-N YM the first-two coefficients of the beta function are only planar without 1/N corrections. Hence, the further 1/N corrections contribute at most only a finite change of renormalization scheme. Thus, glueball loops are UV finite in the S matrix since, if they were not, they would imply a divergent renormalization of the planar RG invariant, which is the only parameter in the S matrix

$$\Lambda_{QCD} = const\ \Lambda \exp(-\frac{1}{2\beta_0 g^2})(\beta_0 g^2)^{-\frac{\beta_1}{2\beta_0^2}}(1+...)$$

On the contrary, in large-N QCD the first-two coefficients of the beta function get corrections at order of N_f/N

$$\beta_0 = \beta_0^P + \beta_0^{NP} = \frac{1}{(4\pi)^2}\frac{11}{3} - \frac{1}{(4\pi)^2}\frac{2}{3}\frac{N_f}{N}$$

$$\beta_1 = \beta_1^P + \beta_1^{NP} = \frac{1}{(4\pi)^4}\frac{34}{3} - \frac{1}{(4\pi)^4}(\frac{13}{3} - \frac{1}{N^2})\frac{N_f}{N}$$

Thus, in large-N QCD the planar RG invariant gets a log-divergent renormalization starting from the order of N_f/N:

$$
\begin{aligned}
\sqrt{T} = \Lambda_{QCD} \sim \ & \Lambda \exp(-\frac{1}{2\beta_0 g^2}) \\
= \ & \Lambda \exp(-\frac{1}{2\beta_0^P(1+\frac{\beta_0^{NP}}{\beta_0^P})g^2}) \\
\sim \ & \Lambda \exp(-\frac{(1-\frac{\beta_0^{NP}}{\beta_0^P})}{2\beta_0^P g^2}) \\
\sim \ & \Lambda \exp(-\frac{1}{2\beta_0^P g^2}) \exp(\frac{\frac{\beta_0^{NP}}{\beta_0^P}}{2\beta_0^P g^2}) \\
\sim \ & \Lambda \exp(-\frac{1}{2\beta_0^P g^2})(1+\frac{\frac{\beta_0^{NP}}{\beta_0^P}}{2\beta_0^P g^2}+\cdots) \\
\sim \ & \Lambda \exp(-\frac{1}{2\beta_0^P g^2})(1+\frac{\beta_0^{NP}}{\beta_0^P}\log(\frac{\Lambda}{\Lambda_{QCD}^P})+\cdots) \\
= \ & \Lambda_{QCD}^P(1+\frac{\beta_0^{NP}}{\beta_0^P}\log(\frac{\Lambda}{\Lambda_{QCD}^P})+\cdots) \\
= \ & \sqrt{T^P}(1+\frac{\beta_0^{NP}}{\beta_0^P}\log(\frac{\Lambda}{\sqrt{T^P}})+\cdots)
\end{aligned}
$$

Hence, since the glueball and meson masses are proportional to the RG invariant, glueball and meson self-energies are log divergent in large-N QCD starting from the order of N_f/N.

The log divergence arises because of

the asymptotic freedom (AF) of the planar theory,
i.e. of the YM theory,

and of the change of the beta function
at order of N_f/N due to the quark loops

The second aim of this talk is to work out the implications of these seemingly innocuous renormalization properties

for the existence of a would-be string solution

of large-N YM and QCD

Indeed,

the large-N limit of SU(N) QCD (with massless quarks):

$$Z = \int \delta A \delta\bar{\psi}\delta\psi \, e^{-\frac{N}{2g^2}\int \sum_{\alpha\beta} Tr\left(F^2_{\alpha\beta}\right) + i\sum_f \bar{\psi}_f \gamma^\alpha D_\alpha \psi_f \, d^4x}$$

(G. 't Hooft 1974)

is universally believed to be solved by a yet-to-be-found string theory, of closed strings in the glueball sector, and of open strings in the meson sector

The main evidence is the large-N counting of Feynman diagrams

In the glueball sector:

$$< \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n) >_{conn} \sim N^{2-n}$$

In the meson sector:

$$< \mathcal{M}_1(x_1)\mathcal{M}_2(x_2)\cdots\mathcal{M}_k(x_k) >_{conn} \sim N^{1-\frac{k}{2}}$$

In the meson/glueball sector:

$$< \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n)\mathcal{M}_1(x_1)\mathcal{M}_2(x_2)\cdots\mathcal{M}_k(x_k) >_{conn} \sim N^{1-n-\frac{k}{2}}$$

This is exactly the canonical counting that we would get from a string theory with string coupling: $g_s = 1/N$

of closed strings in the glueball sector:
a sphere with n punctures

of open strings in the meson sector:
a disk with k punctures on the boundary

and of open/closed strings in the meson/glueball sector:
a disk with k punctures on the boundary and n in the interior

This is the 't Hooft planar theory, that describes tree amplitudes

Then, unitarization introduces higher-genus contributions that correct the planar theory by string diagrams with a weight that is $1/N$ to a power equal to minus the Euler characteristic

Physically, this is the standard theory of confinement corresponding
to the picture that

mesons are bound states of quarks linked by a chromo-electric flux tube and

glueballs are rings of chromo-electric flux

since the string world-sheet is identified with the flux tube

Now, the UV finiteness of the large-N YM theory, due to AF and RG, which we have just found out on the gauge side, is

compatible

with the universally believed UV finiteness of (consistent) closed string theories, due to modular invariance on the string side

Thus a canonical string solution of the pure large-N YM theory may exist

But, contrary to the universal belief,
we prove in this talk a NO-GO THEOREM that the aforementioned renormalization properties in large-N QCD

+

the existence of the glueball mass gap at the lowest 1/N order,
i.e. in the planar theory,

are incompatible with the open/closed duality of a would-be canonical string solution (canonical means that matches topologically 't Hooft expansion)

As a consequence, the long sought-after canonical string solution of large-N QCD does not actually exist.

Open/closed string duality in a nutshell

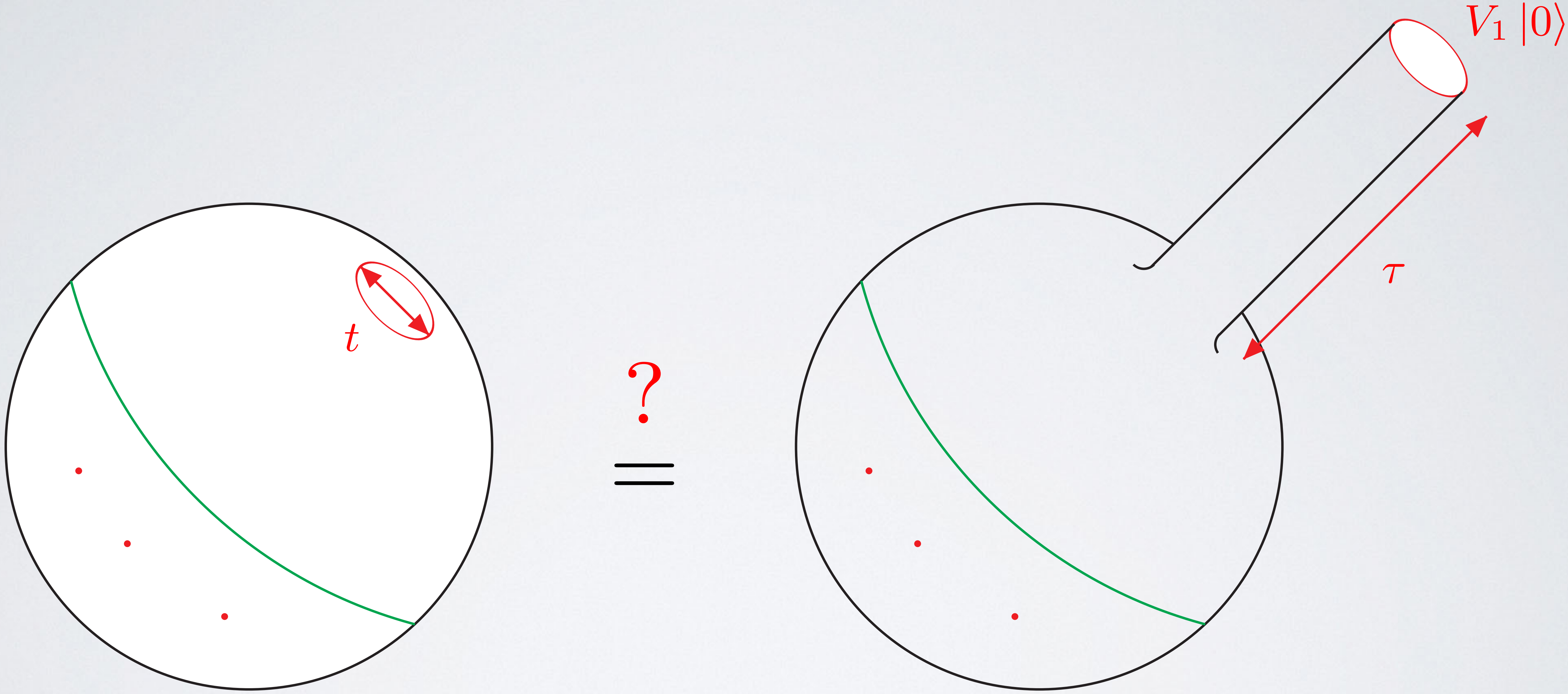
annulus= one-loop in the open-string sector

is topologically the same as the

cylinder = tree amplitude in the closed-string sector

Moreover, there is a conformal map, in fact, a modular transformation that exchanges the ultraviolet with the infrared, under which the annulus, i.e. a disk with a small hole, is mapped into a long cylinder and vice versa.
Thus, if conformal symmetry on the world-sheet is not anomalous, i.e. the string theory really exists, the annulus and the cylinder are identical.

An example of open/closed string duality



Now, we have just proved that in large-N QCD the one-loop graph on the left-hand side, which lives in the mixed glueball-meson sector, must be UV log divergent as the hole shrinks to a point

But, by open/closed duality, it may diverge only if the conformally equivalent tree glueball diagram on the right-hand side, that cannot be ultraviolet divergent because it is both a closed and a tree string diagram, has an infrared divergence corresponding to a scalar massless glueball propagating in the infinitely long cylinder on the right-hand side

.

But such a massless scalar glueball does not exists in planar large-N QCD. Hence, open/closed duality cannot hold, and the canonical string solution does not exist !

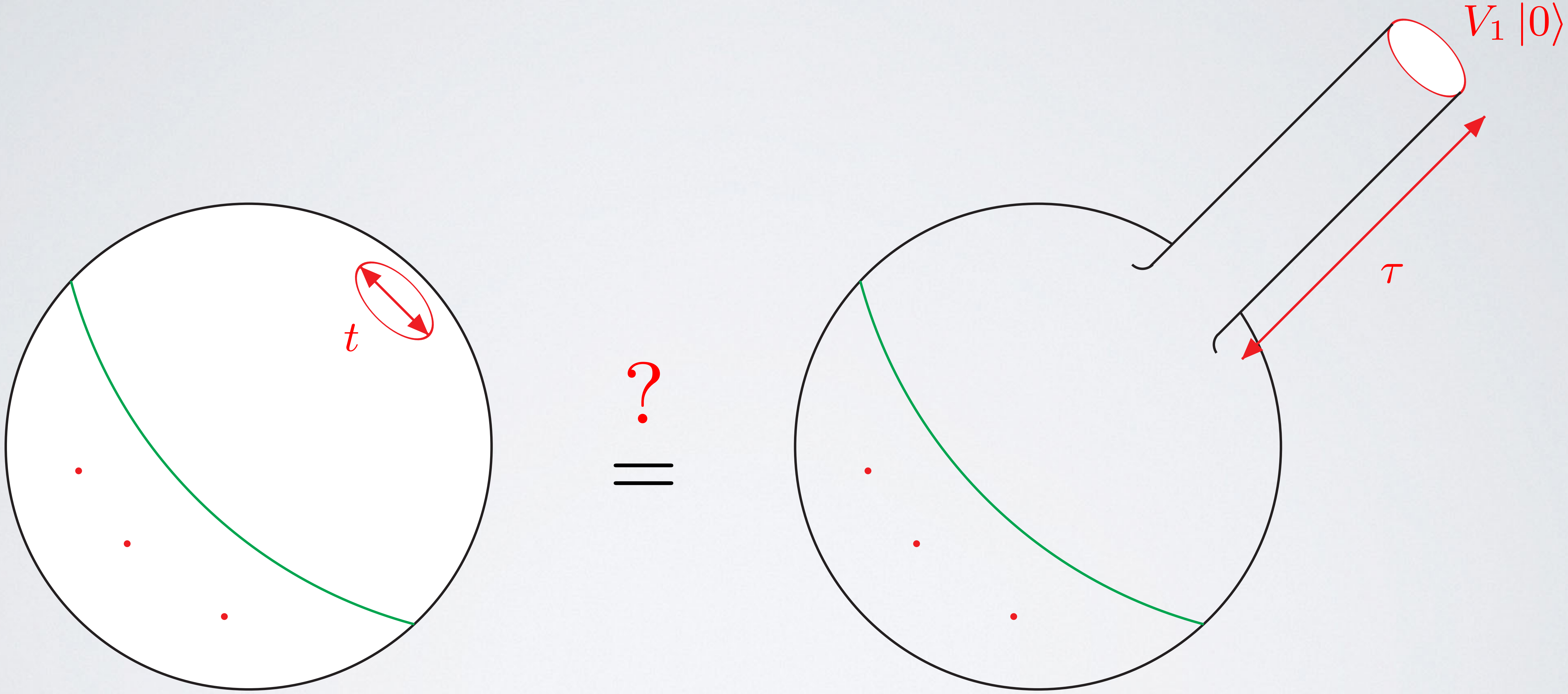
In fact, a stronger version of the NO-GO THEOREM holds
(arXiv: 1703.10176 [hep-th]),

that does not assume the existence of the glueball mass gap,

but follows only

from the renormalization properties of large-N QCD

and from a low-energy theorem of NSVZ type
proved in Phys. Rev. D 95 054010



t
$\overset{?}{=}$
$V_1\,|0\rangle$
τ

Proof

To say it in a nutshell, in the string interpretation the left-hand side is UV divergent because of the integration on the modular parameter, t, of the annulus.

By open/closed duality, the right-hand side must be divergent because of the integration on the dual modular parameter, tau, of the cylinder

On the contrary, the low-energy theorem in large-N QCD implies that the tau integration is, in fact, UV finite, while the boundary state V_110>

is UV divergent before integrating on tau because it is an UV-divergent counterterm due to the quark loops.

COMMENTS

1) The NO-GO theorem extends easily to large-N QCD with massive quarks and to a vast class of confining QCD-like AF theories such as n=1 SUSY QCD

2) The no-go theorem does not depend on the detailed realization of the would-be open/closed string (branes, extra dimensions and so on ...),

but only on the
2d conformal world-sheet structure
i.e.
there is no way to evade it
in the canonical string framework

Comments

3) The easy way-out to the NO-GO THEOREM is to give up AF, and to declare that the string is an effective description only in the infrared. For example, there is Luscher computation of the universality class of Wilson loops in the infrared.

4) An even cheaper way-out, again at the price of the QCD AF, is to modify QCD in the UV in such a way that the planar theory is not AF at a certain finite scale, in order for the string description to hold.

Comments

5) The NO-GO THEOREM tells us nothing about
the closed string sector alone, and therefore nothing about
gauge/gravity duality,
to the extent that we do not pretend to couple gravity to
open strings too,
but then we have to explain why glueballs are rings of flux
while mesons are not open strings

Comments

5) Yet, gauge/gravity is in fact presently totally empty for confining AF gauge theories, because there is no such a model based on g/g that is in fact AF, i.e. that reproduces the asymptotics of the 2-point correlator of the action density in an AF theory:

$$\int \langle \frac{\beta(g)}{gN} tr\big(\sum_{\alpha\beta} F^2_{\alpha\beta}(x)\big) \frac{\beta(g)}{gN} tr\big(\sum_{\alpha\beta} F^2_{\alpha\beta}(0)\big)\rangle_{conn} e^{ip\cdot x} d^4x$$

$$= C_S p^4 \left[\frac{1}{\beta_0 \log \frac{p^2}{\Lambda^2_{\overline{MS}}}} \left(1 - \frac{\beta_1}{\beta_0^2} \frac{\log\log \frac{p^2}{\Lambda^2_{\overline{MS}}}}{\log \frac{p^2}{\Lambda^2_{\overline{MS}}}} \right) + O\left(\frac{1}{\log^2 \frac{p^2}{\Lambda^2_{\overline{MS}}}} \right) \right]$$

$$= p^4 \sum_{k=1}^{\infty} \frac{g^4(m_k^2)\rho_0^{-1}(m_k^2)}{p^2 + m_k^2}$$

M.B. Nucl. Phys. B 875 (2013) 621 [hep-th/1305.0273]

A posteriori, it turns out that we have already proposed a noncanonical way-out to the NO-GO THEOREM: Hadron 2015, AIP Conf. Proc. 1735 (2016) 030004

A noncanonical nontrivial S-matrix, evading the no-go theorem, arises by summing world-sheet instantons in a certain twistorial topological string theory on (non-commutative) twistor space, in such a way that the open/closed duality is spoiled for the space-time one-loop effective action together with the mechanism that leads to the inconsistency:

$$S(B) = \frac{k}{2\pi}\int tr(B \wedge dB + \frac{2}{3} B \wedge B \wedge B) - \sum_p tr_f \log(1 - \exp \int_{C_p} B_\lambda)$$

$$\Gamma = -\sum_p tr_f \log(1 - \exp \int_{C_p} B_\lambda) = -\sum_p Tr \log(\frac{d}{d\lambda} + B_\lambda)|_{C_p} = -\log Det(\frac{d}{d\lambda} + B_\lambda)$$

Proof

Each next term in the expansion of the functional determinant can be interpreted as the insertion of an extra Wilson loop, that is equivalent to the insertion of an extra hole, i.e. of an open-string loop, or, by open/closed duality, of a closed-string cylinder.

Thus, though each term satisfies open/closed duality, the space-time one-loop effective action arises from an infinite sum of string loops or trees, and not from just one open-string loop or one closed-string tree, as it would occur canonically.

Hence, the space-time one-loop effective action is noncanonically defined, and no inconsistency with large-N renormalization arises.

The space-time one-loop effective action arises from the functional determinant on the fiber of twistor space in the TTST

$$\Gamma = \log Det_f(\frac{d}{d\lambda} + B_\lambda) = \log Det_f(\frac{d}{d\lambda} + \tilde{B}_\lambda + \delta B_\lambda)$$
$$= \log Det_f(\frac{d}{d(\frac{\hat{P}^+}{\frac{1}{2}\Lambda^2_{QCD}})} + \tilde{B}_{p^+} + \delta B_{p^+})$$

with the background (flat) connection B determined by the equation of motion of the TTST

$$\tilde{B}_{p^+} = \hat{P}_- + \frac{(\frac{1}{2}\Lambda^2_{QCD})^{-1}\hat{P}_u\hat{P}_{\bar{u}} - \hat{P}_-\frac{d}{d\hat{P}_-} + Q}{\frac{\hat{P}^+}{\frac{1}{2}\Lambda^2_{QCD}}}$$

After some trivial manipulation we find:

$$\Gamma = \log Det_f\left(\frac{d}{d\left(\frac{\hat{P}^+}{(\frac{1}{2}\Lambda^2_{QCD})^{-1}}\right)} + \hat{P}_- + \frac{(\frac{1}{2}\Lambda^2_{QCD})^{-1}\hat{P}_u\hat{P}_{\bar{u}} - \hat{P}_-\frac{d}{d\hat{P}_-} + Q}{\frac{\hat{P}^+}{\frac{1}{2}\Lambda^2_{QCD}}} + \delta B_{p+}\right)$$

$$= \log Det_f\left(\hat{P}_+\frac{d}{d\hat{P}_+} - \hat{P}_-\frac{d}{d\hat{P}_-} + (\frac{1}{2}\Lambda^2_{QCD})^{-1}(\hat{P}_+\hat{P}_- + \hat{P}_u\hat{P}_{\bar{u}}) + Q + \delta B'_{p+}\right)$$

$$- \log Det_f\left(\frac{\hat{P}^+}{\frac{1}{2}\Lambda^2_{QCD}}\right)$$

$$= \log Det_f\left(\hat{P}_+\hat{P}_- + \hat{P}_u\hat{P}_{\bar{u}} + \frac{1}{2}\Lambda^2_{QCD}(\hat{P}_+\frac{d}{d\hat{P}_+} - \hat{P}_-\frac{d}{d\hat{P}_-} + Q) + \delta B''_{p+}\right)$$

$$- \log Det_f\left(\frac{\hat{P}^+}{\frac{1}{2}\Lambda^2_{QCD}}\right) - \log Det_f\left(\frac{1}{2}\Lambda^2_{QCD}\right)$$

with the mass spectrum determined by the eigenvalues of the spin operator and of Q, that is (semi-)integer valued, since it has a cohomological origin, arising by the infinite non-commutative Hodge structure underlying the TTST

The one-loop non-planar (scalar) vertex in the TTST:

$$Tr\big((\hat{P}^2+\frac{1}{2}\Lambda^2_{QCD}Q)^{-1}\delta B_{p+}(\hat{P}^2+\frac{1}{2}\Lambda^2_{QCD}Q)^{-1}\delta B_{p+}(\hat{P}^2+\frac{1}{2}\Lambda^2_{QCD}Q)^{-1}\delta B_{p+}\big)$$
$$=\sum_{ijk}\hat{tr}((\hat{P}^2+\frac{1}{2}\Lambda^2_{QCD}Q_i)^{-1}\delta B^{ij}_{p+}(\hat{P}^2+\frac{1}{2}\Lambda^2_{QCD}Q_j)^{-1}\delta B^{jk}_{p+}(\hat{P}^2+\frac{1}{2}\Lambda^2_{QCD}Q_k)^{-1}\delta B^{ki}_{p+})$$

arises by an AF theory,

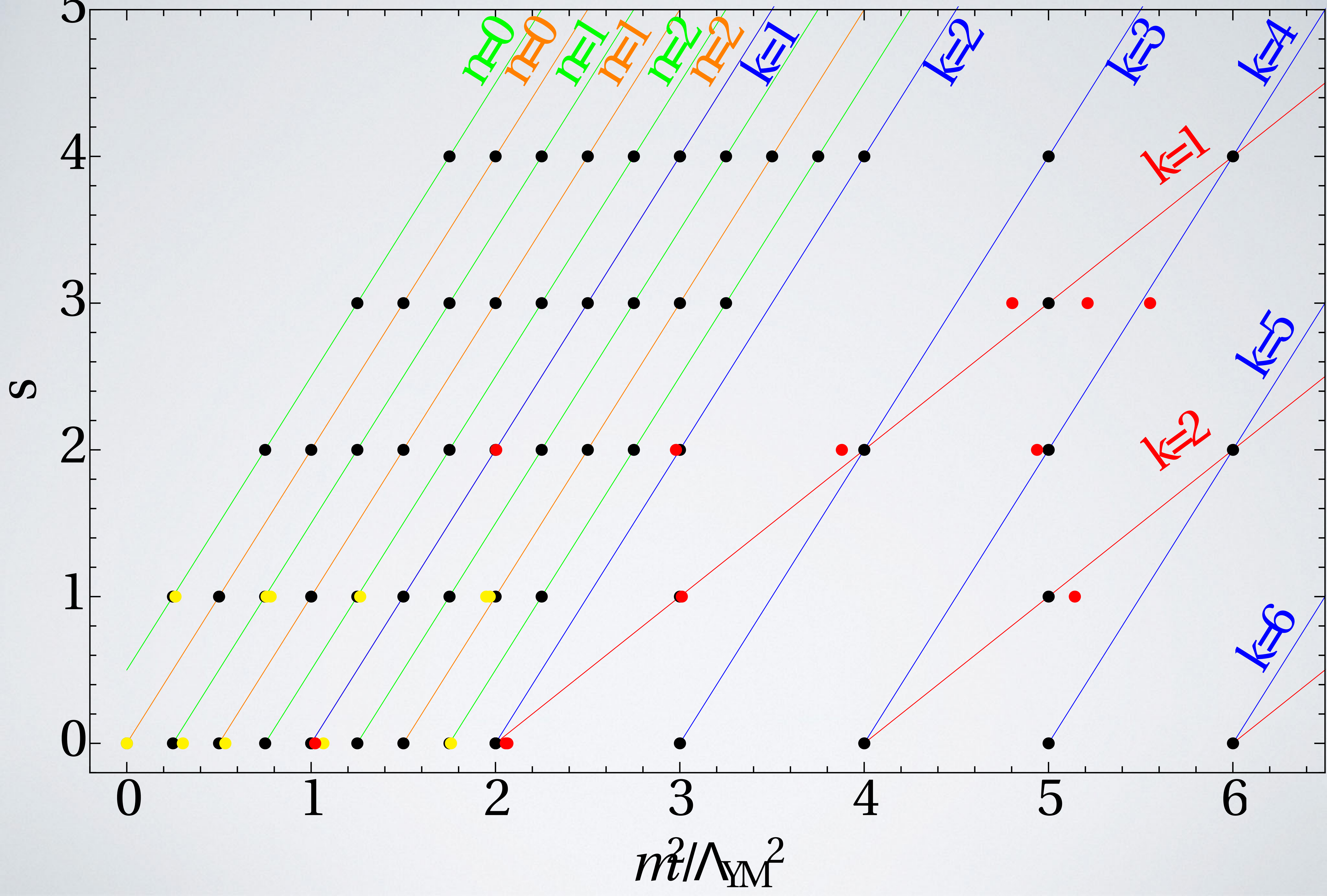
as follows by comparison with the one-loop nonplanar vertex from the AF bootstrap (M. B. EPJ Web of Conferences 80, (2014) 00010, arXiv:1409.5144 [hep-th]):

$$\int dq_1 dq_2 dq_3 \delta(q_1+q_2+q_3)\int \sum_{n_1=1}^{\infty}\frac{\rho^{-\frac{1}{2}}(m^2_{n_1})\Phi_{n_1}(q_2)}{p^2+m^2_{n_1}}\sum_{n_2=1}^{\infty}\frac{\rho^{-\frac{1}{2}}(m^2_{n_2})\Phi_{n_2}(q_3)}{(p+q_2)^2+m^2_{n_2}}\sum_{n_3=1}^{\infty}\frac{\rho^{-\frac{1}{2}}(m^2_{n_3})\Phi_{n_3}(q_1)}{(p+q_2+q_3)^2+m^2_{n_3}}dp$$

Spectrum of the TTST

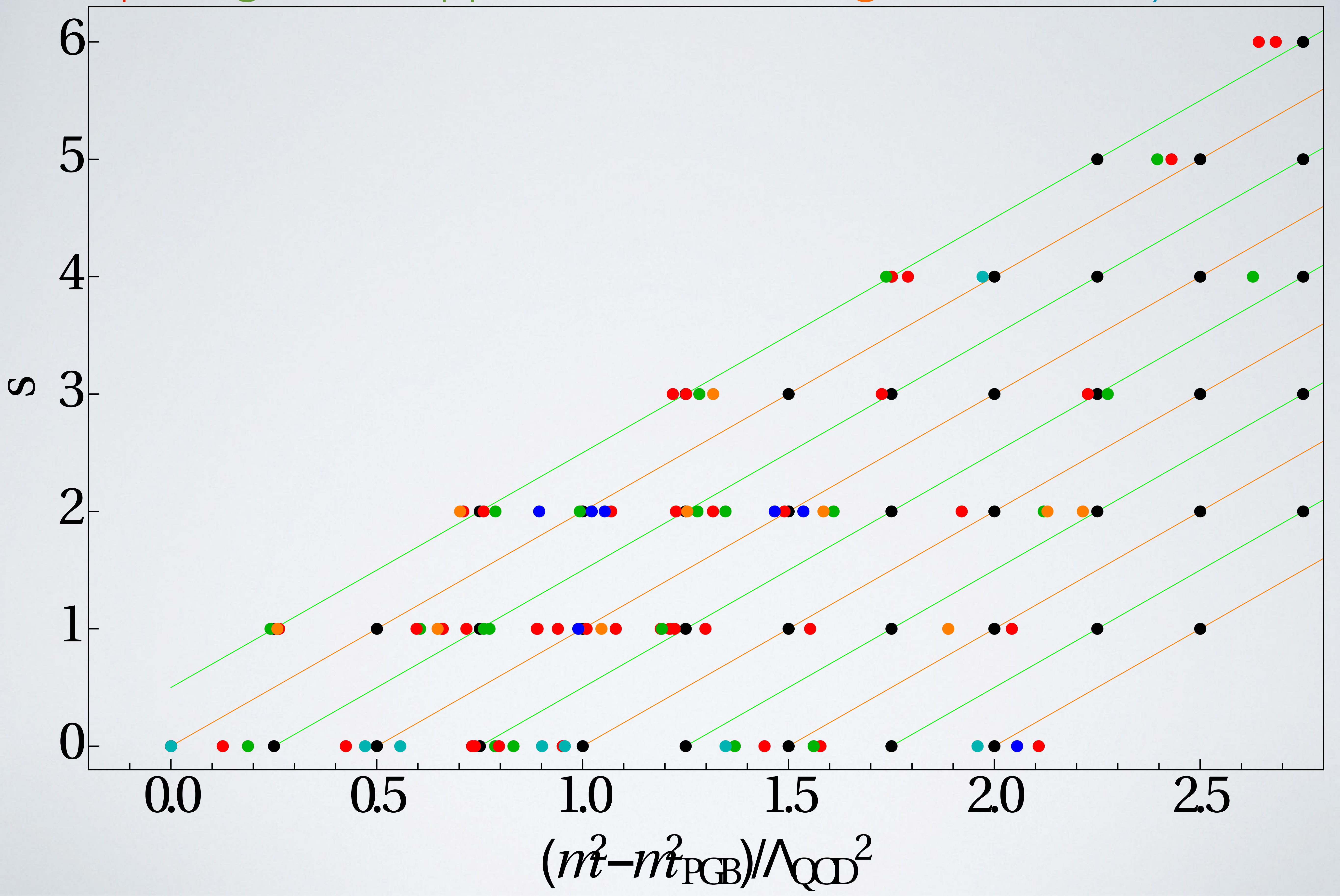
$$
\begin{aligned}
m_k^{(s)2} &= (k+\frac{s}{2})\Lambda_{QCD}^2 \;;s\;even;\;\; k=1,2,\cdots for\,glueballs \\
m_k^{(s)2} &= 2(k+\frac{s}{2})\Lambda_{QCD}^2 \;;s\;odd;\;\; k=1,2,\cdots for\,glueballs \\
m_k^{(s)2}-m_{PGB}^2 &= \frac{1}{2}(k+s)\Lambda_{QCD}^2 \;;s=0,1,\cdots\;;k=0,1,\cdots for\,mesons \\
m_k^{(s)2}-m_{PGB}^2 &= \frac{1}{2}(k+s-\frac{1}{2})\Lambda_{QCD}^2 \;;s=1,\cdots\;;k=0,1,\cdots for\,mesons
\end{aligned}
$$

Large-N lattice glueball and meson spectrum versus TTST

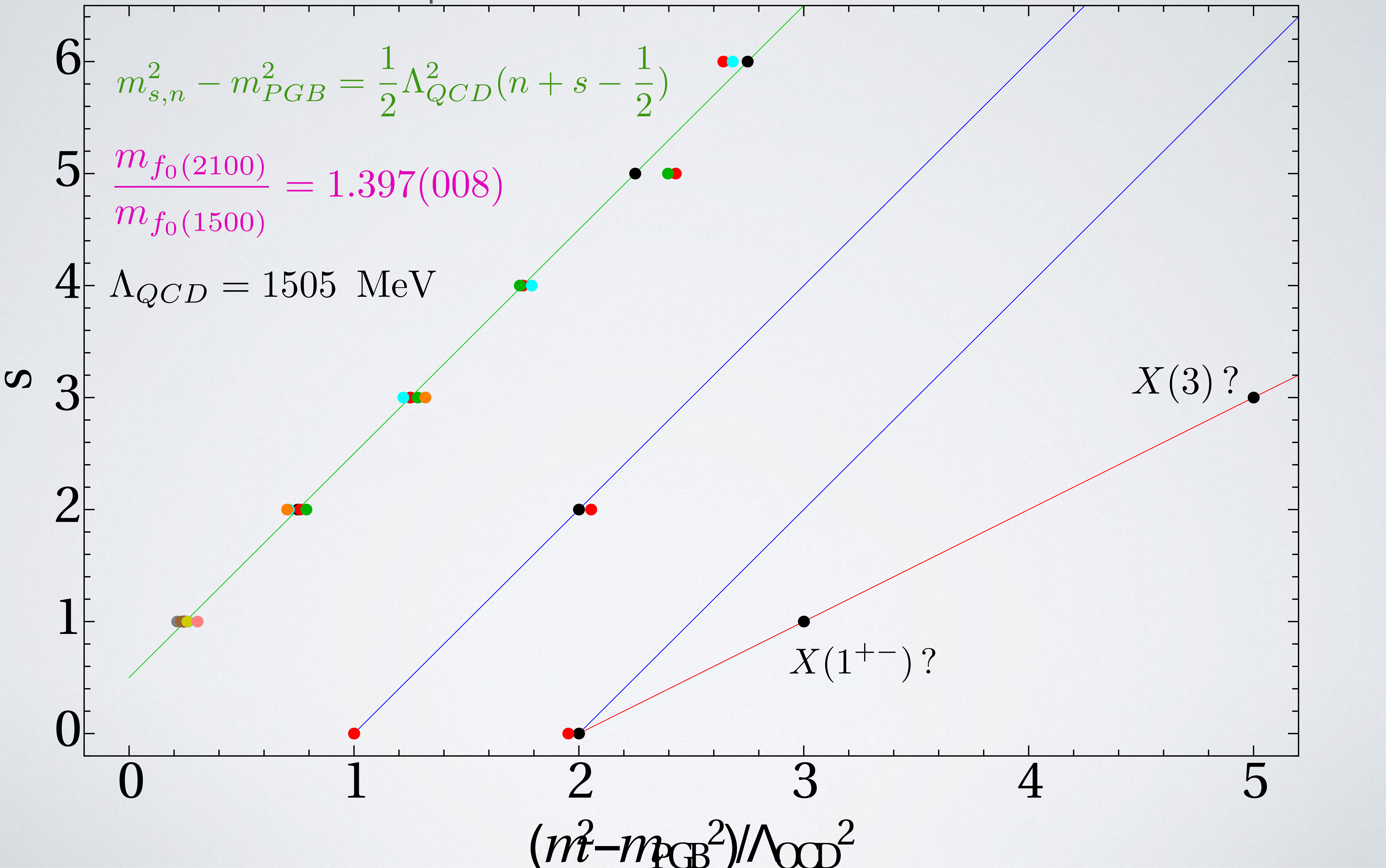


PDG (2015) versus the TTST

red=pion, green=kappa, blue=eta, orange=eta+eta',cyan=eta'



The actual glueball (purple and blue) and meson leading Regge trajectories for any flavor (other colors) implied by Particle Data Group and BES collaboration versus the TTST



In fact, the noncanonical twistor string
is the only presently known string theory that may be
consistent
with the fundamental principles of QCD !