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Light stringy states and the Yukawas

Pascal Anastasopoulos

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1110.5424
1601.07584
1609.09299

with M. Bianchi, D. Consoli, R. Richter

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- Particular interest have the intersecting D-brane scenarios.

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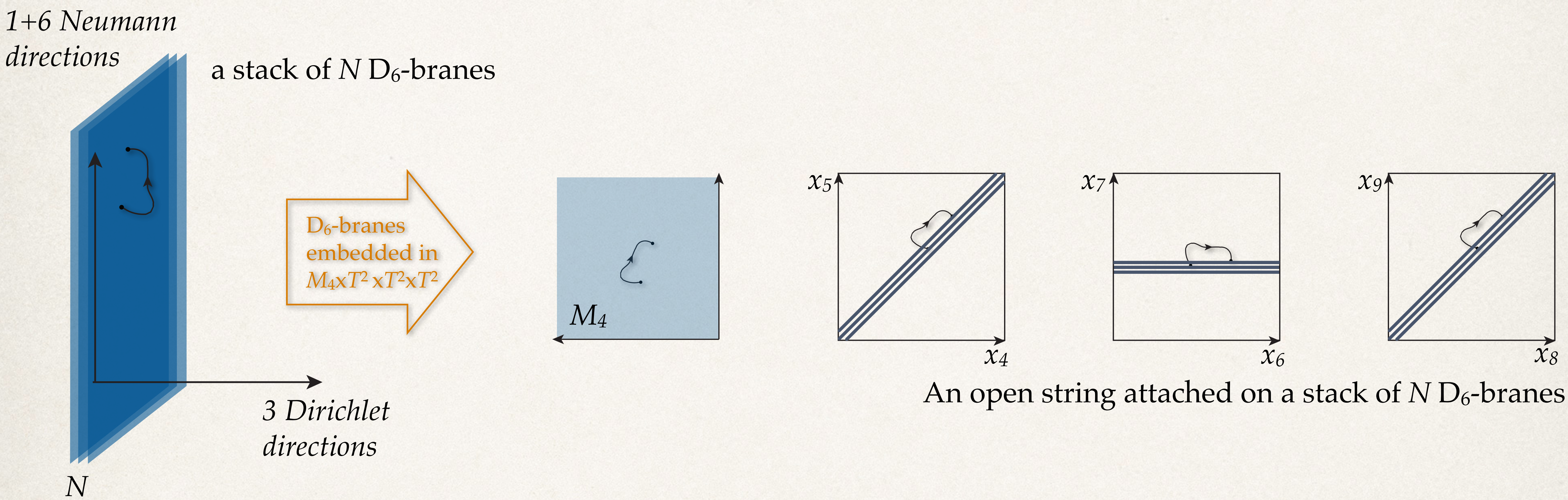
- Such models can be easily distinguished from KK models.

- It is very interesting to study their decay channels and their lifetimes.

Our tools

- We focus on type IIA constructions in a $T^2 \times T^2 \times T^2$ space with intersecting D6 branes:



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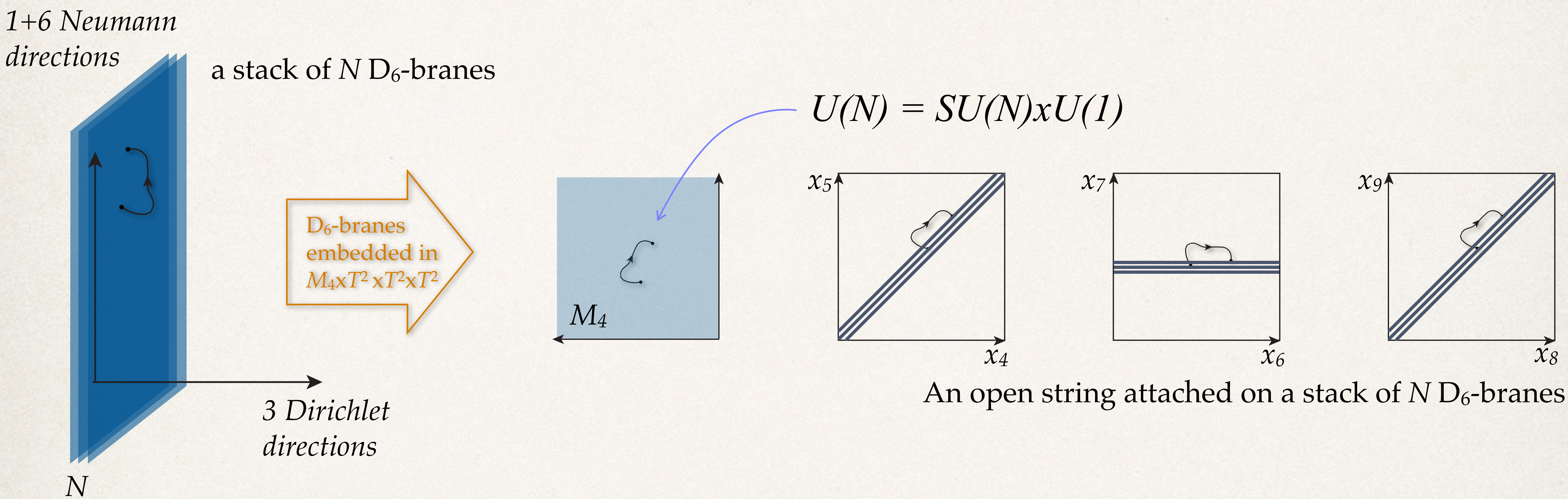

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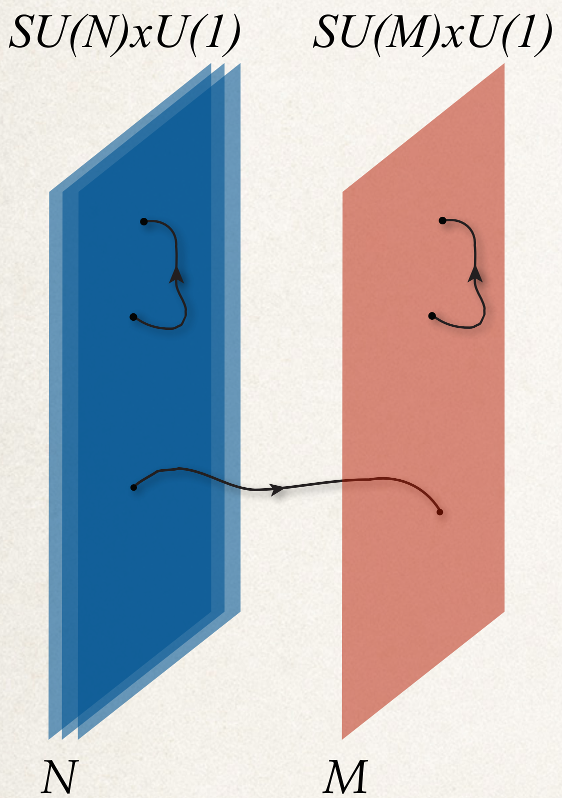

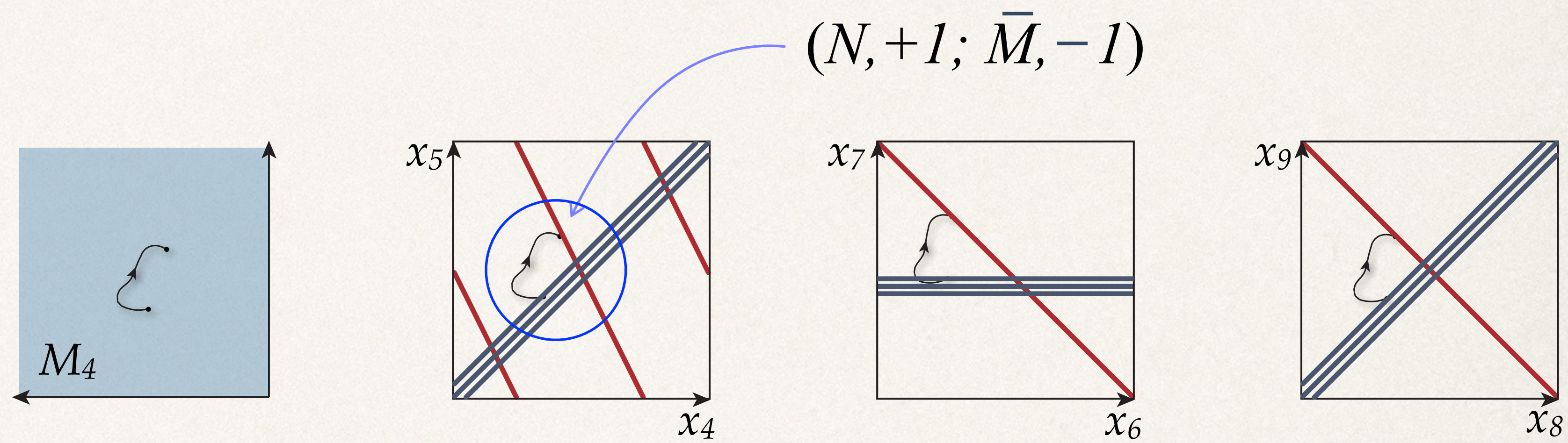

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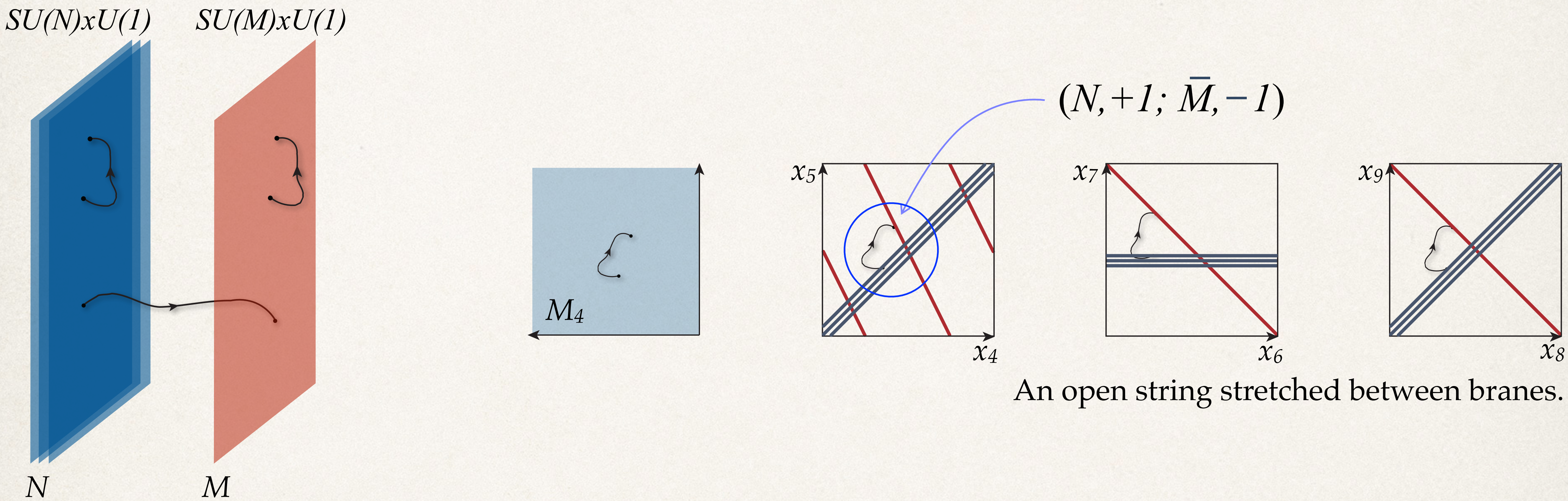

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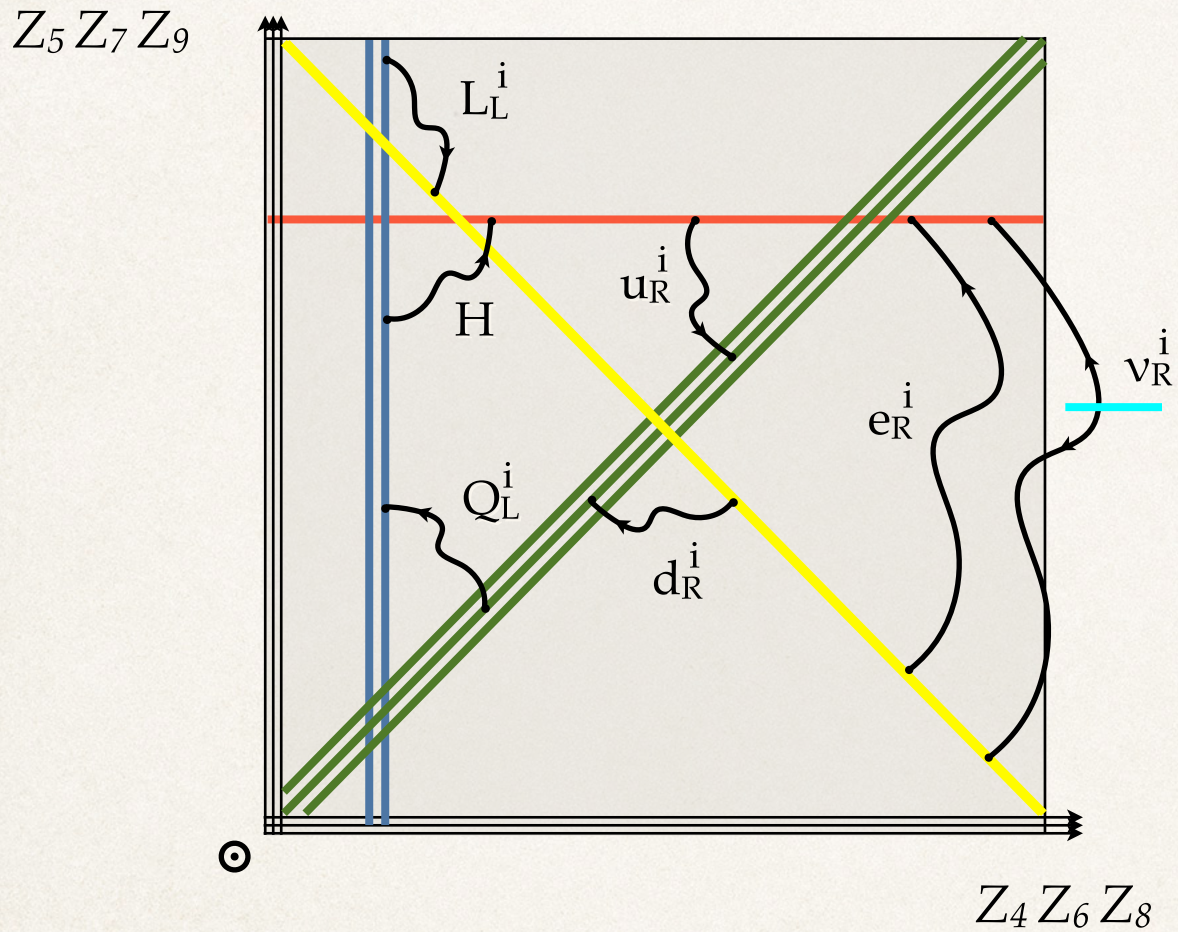
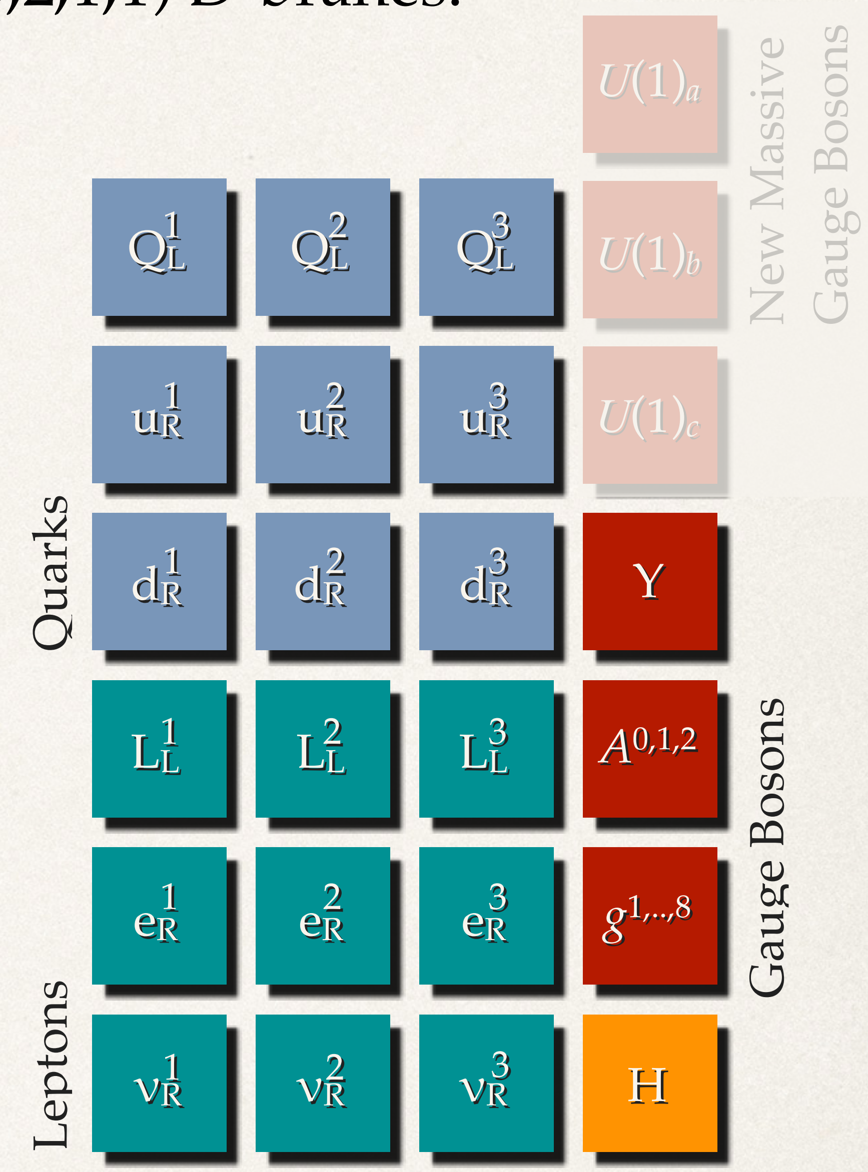
- Applying these rules we can built a local D-brane realisation of the SM.

Standard Model from open strings

- For the $SU(3) \times SU(2) \times U(1)_Y$ we need 4 stacks of (3,2,1,1) D-branes.
- Matter content at D-brane intersections.



M_4 (our 4 dimensions)



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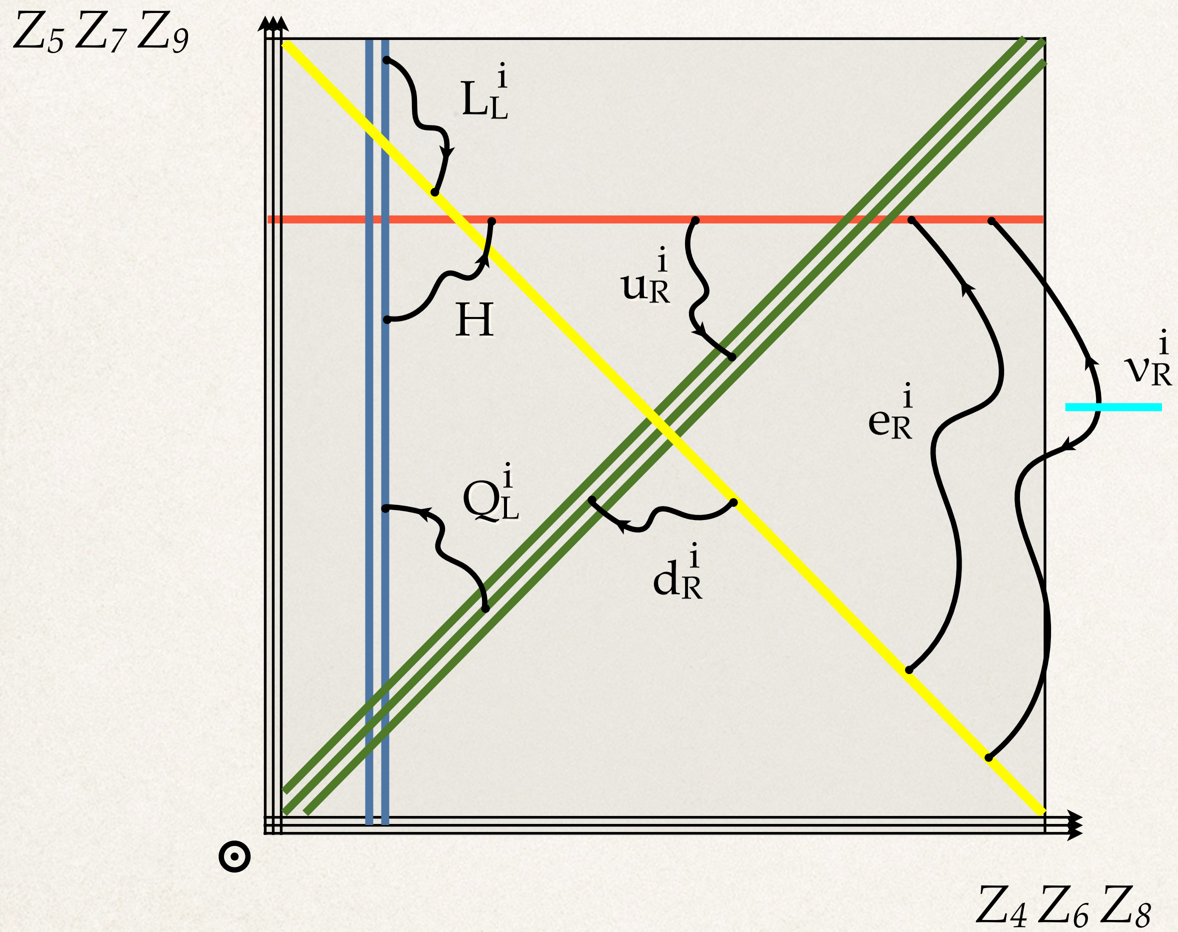

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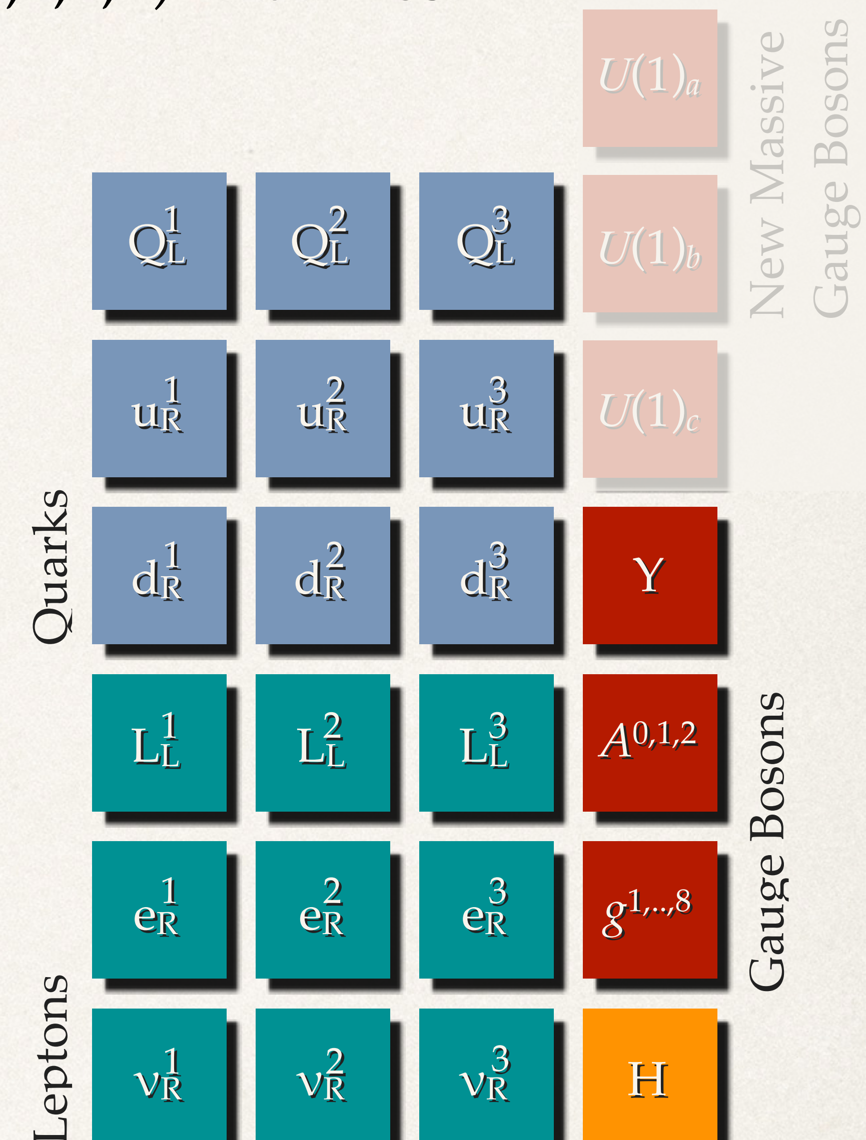

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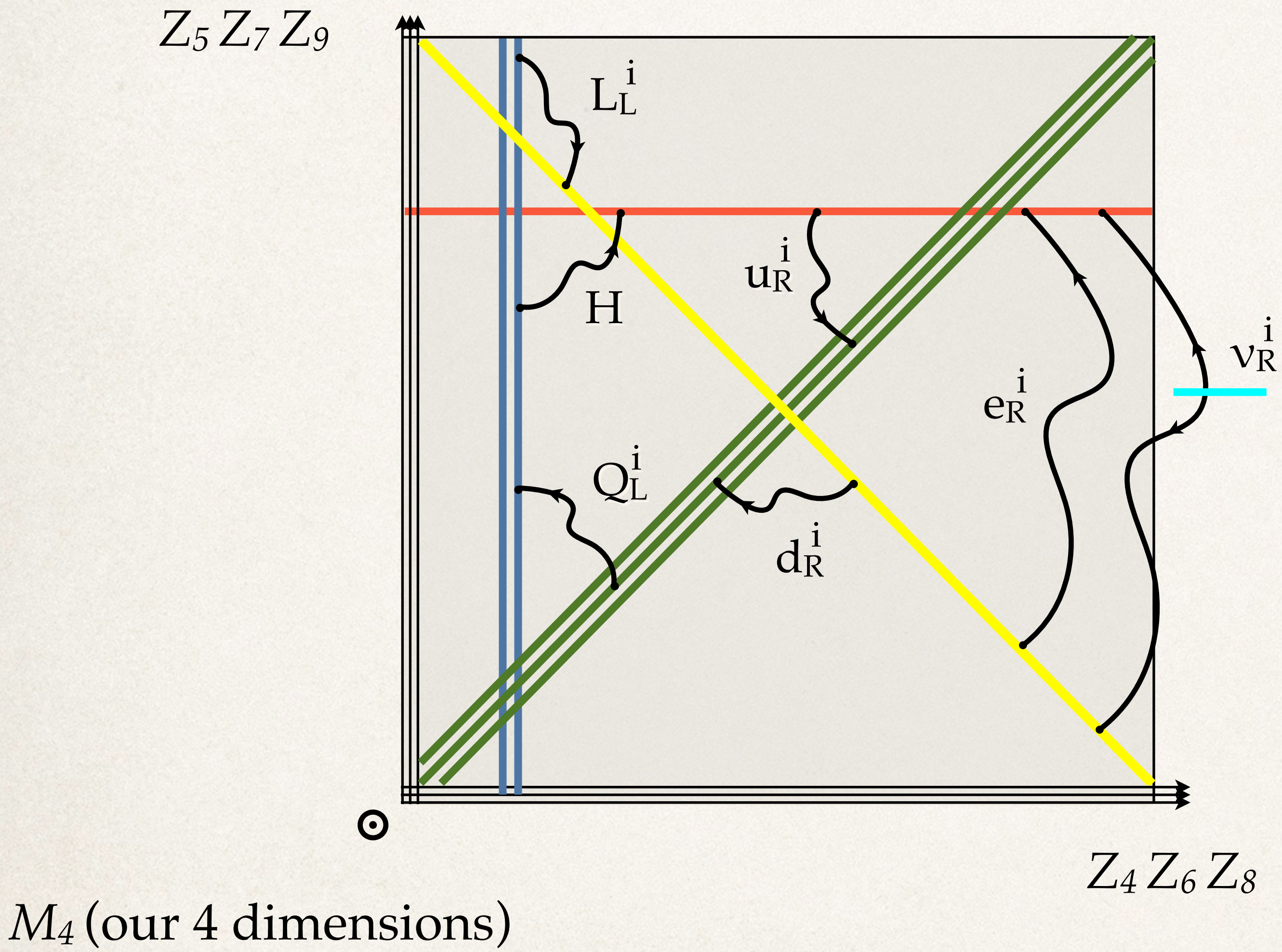

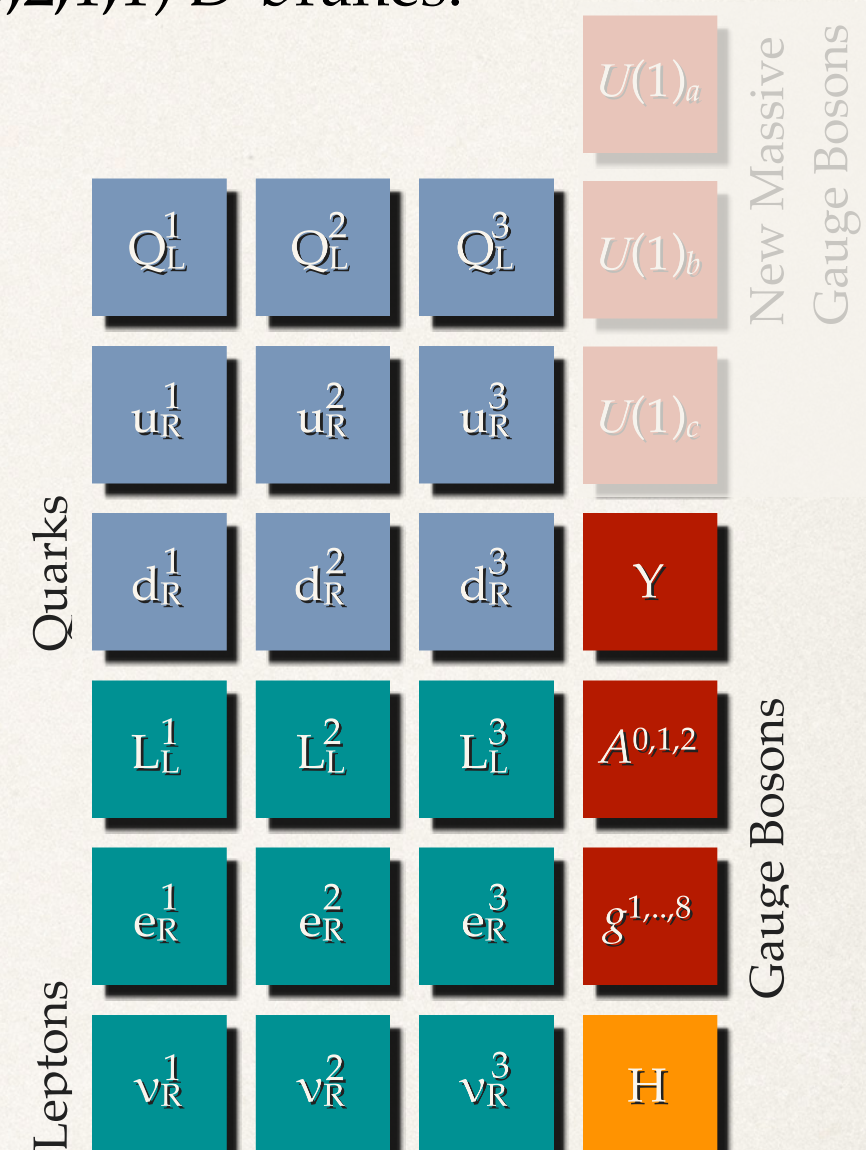

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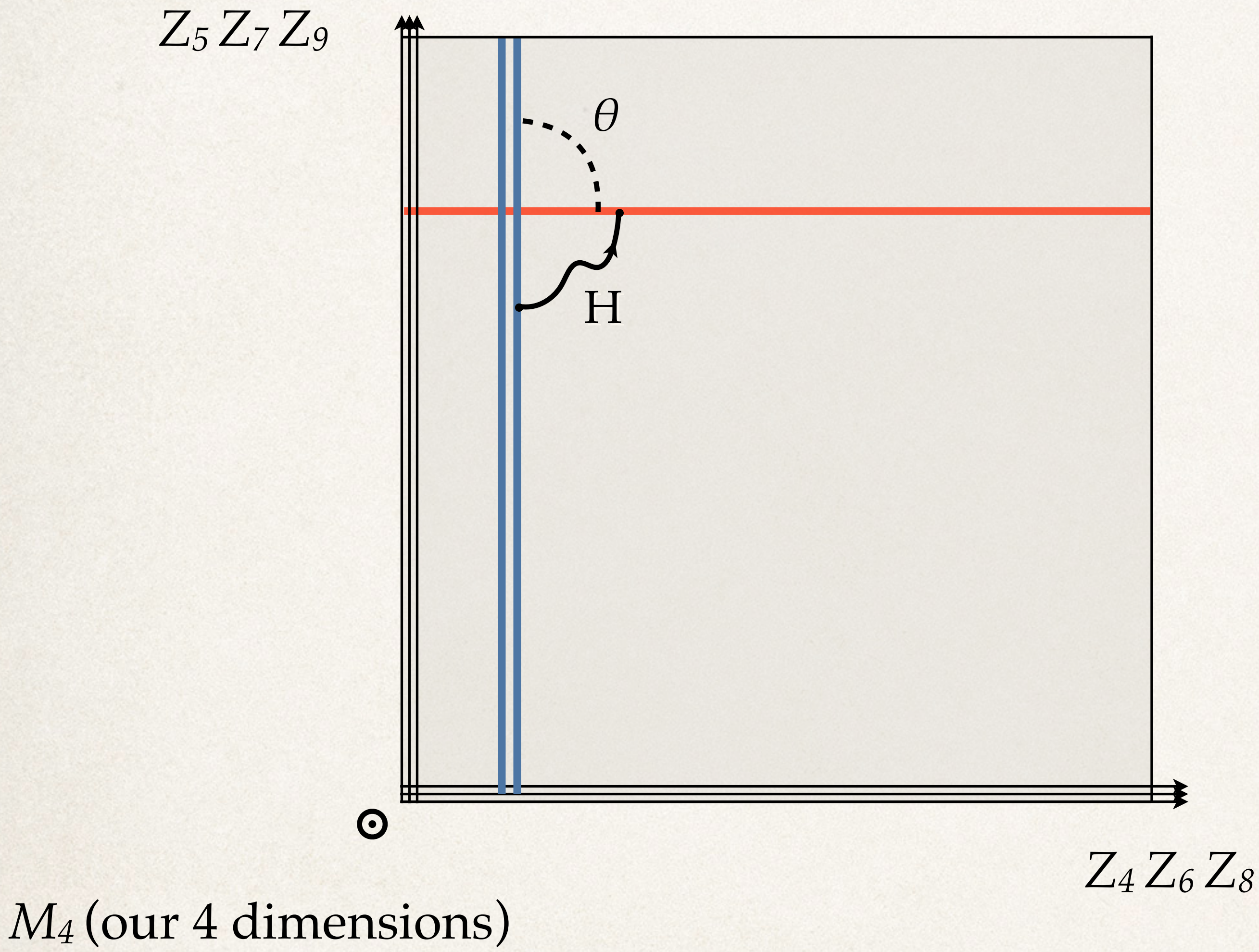
Towers of massive copies at intersections

- Strings vibrate. The frequencies are not random. They are proportional to the angle θ.



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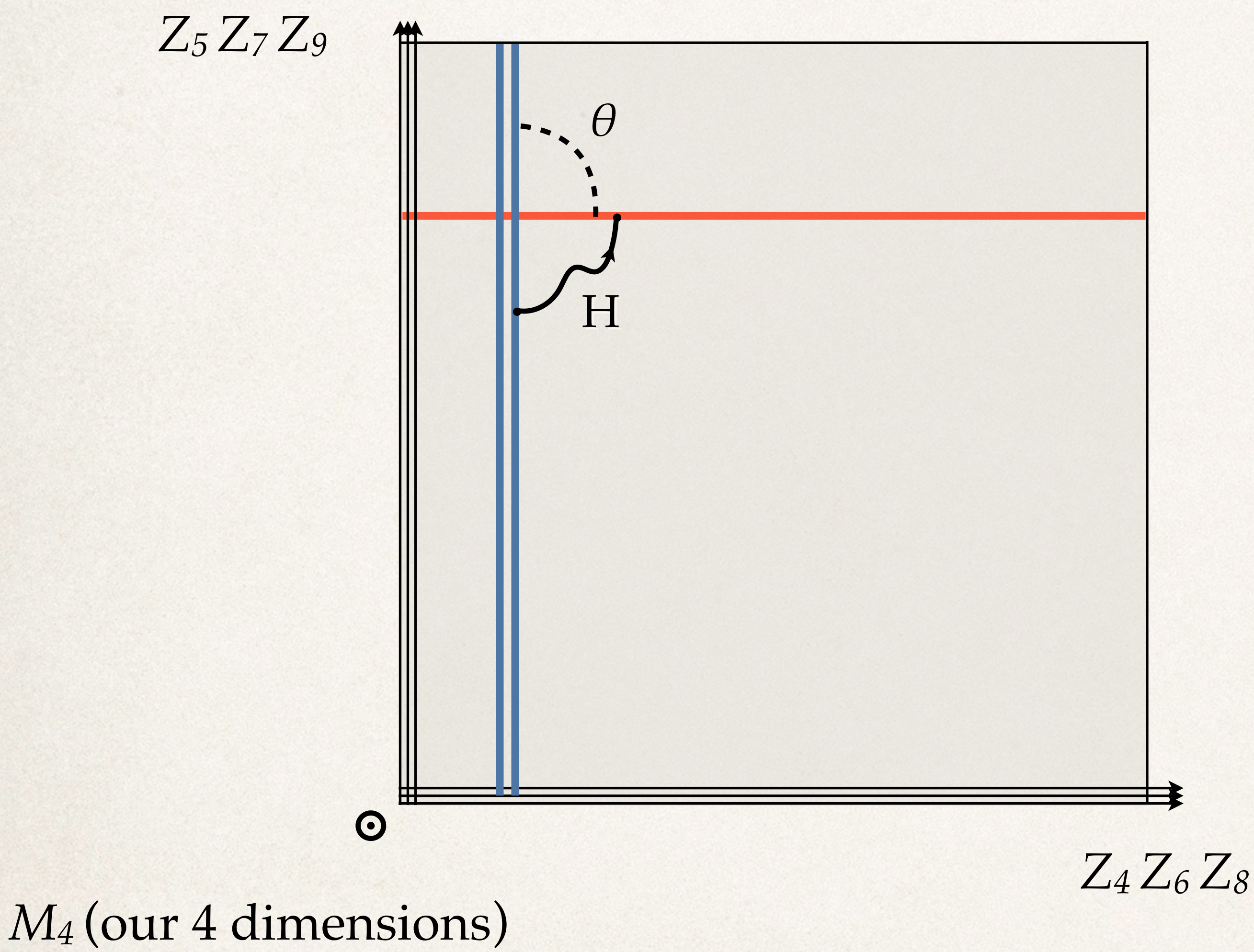

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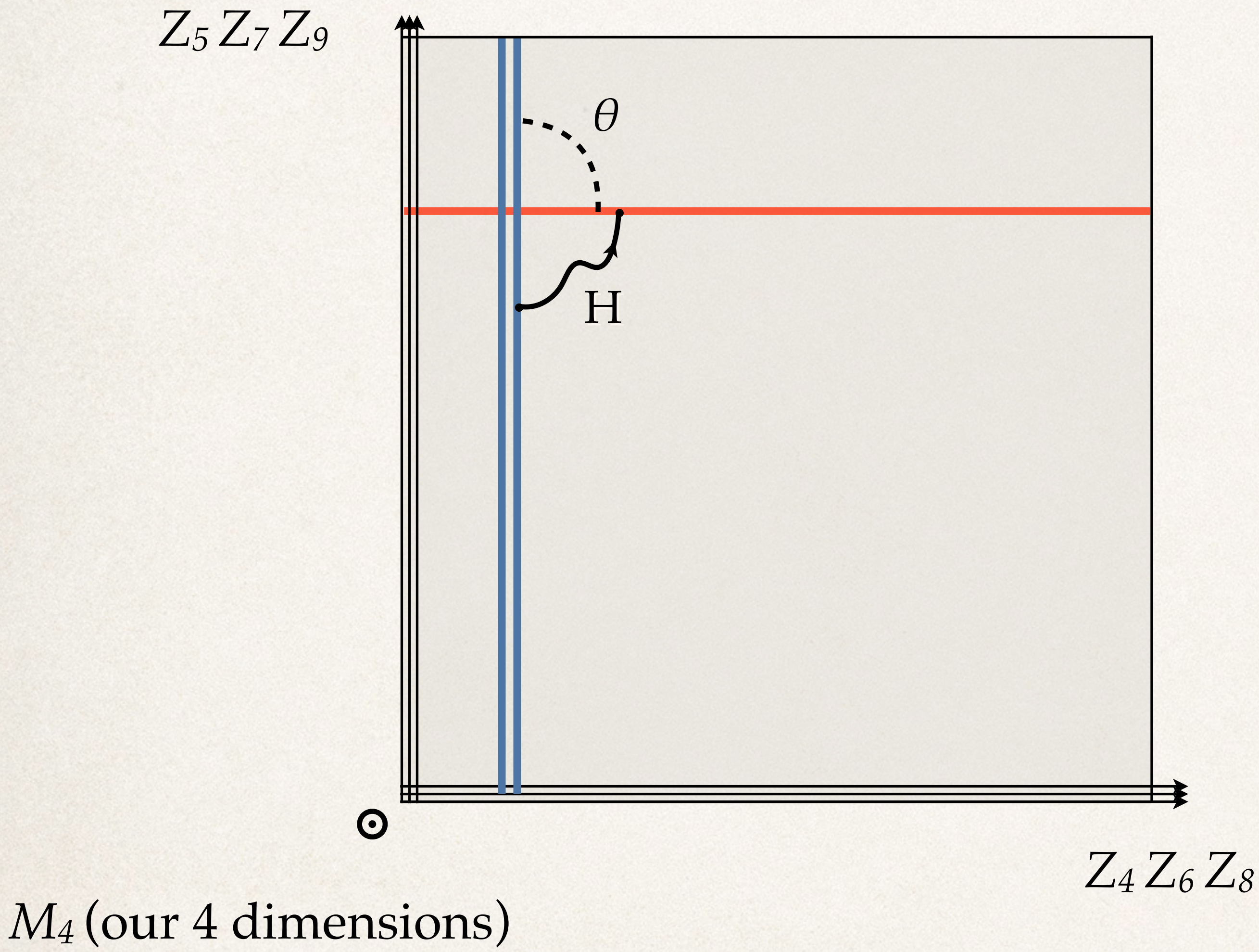

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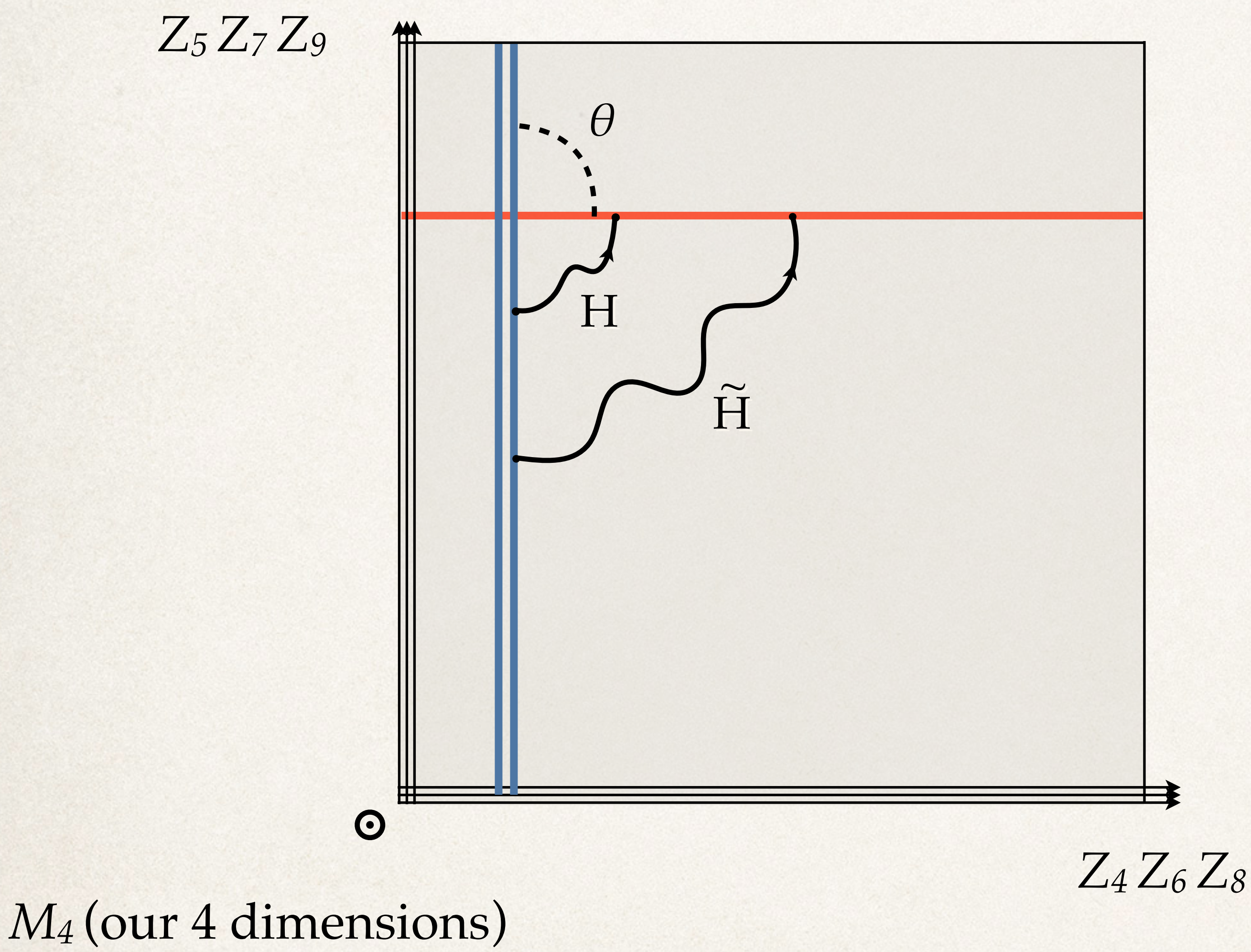

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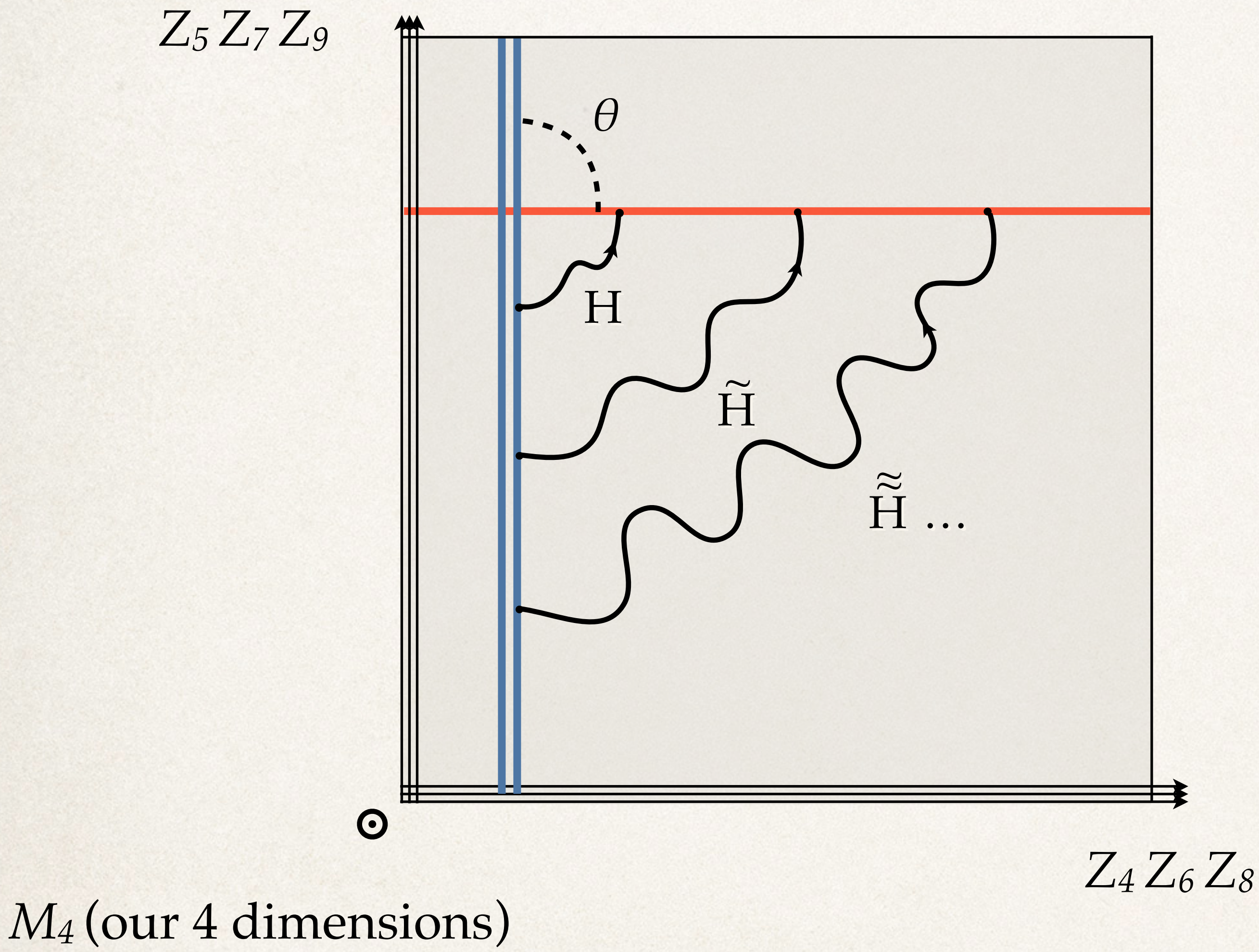

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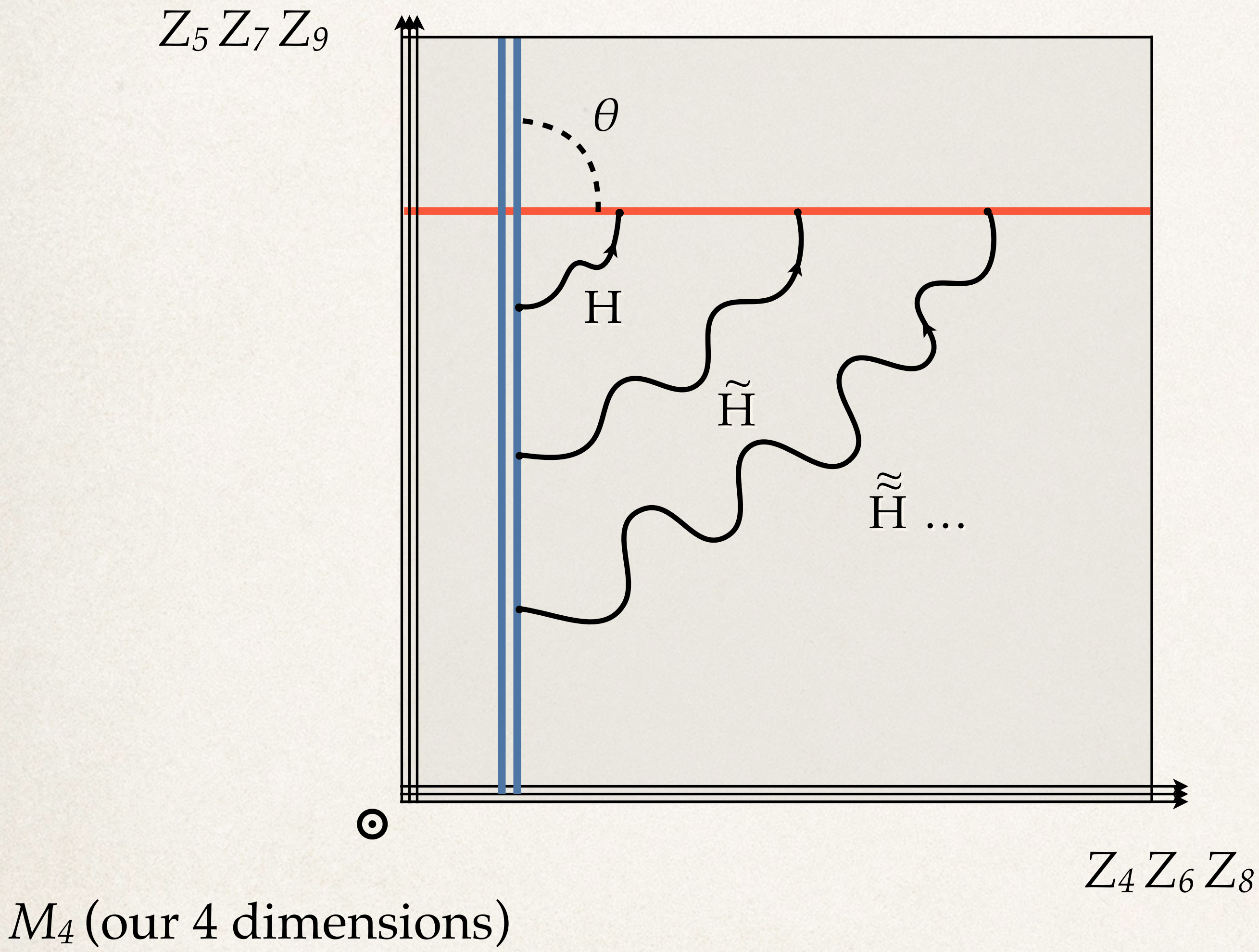

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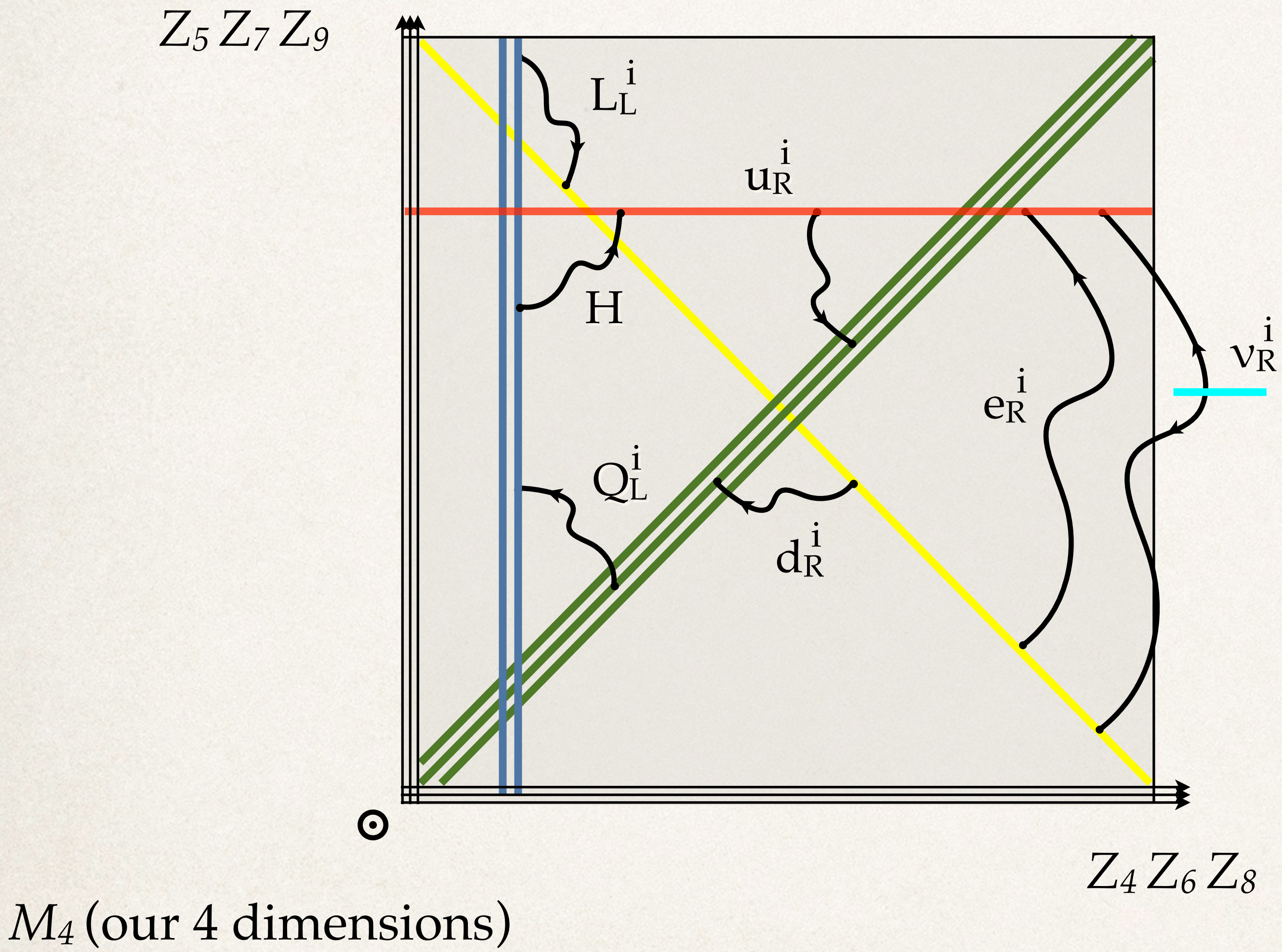
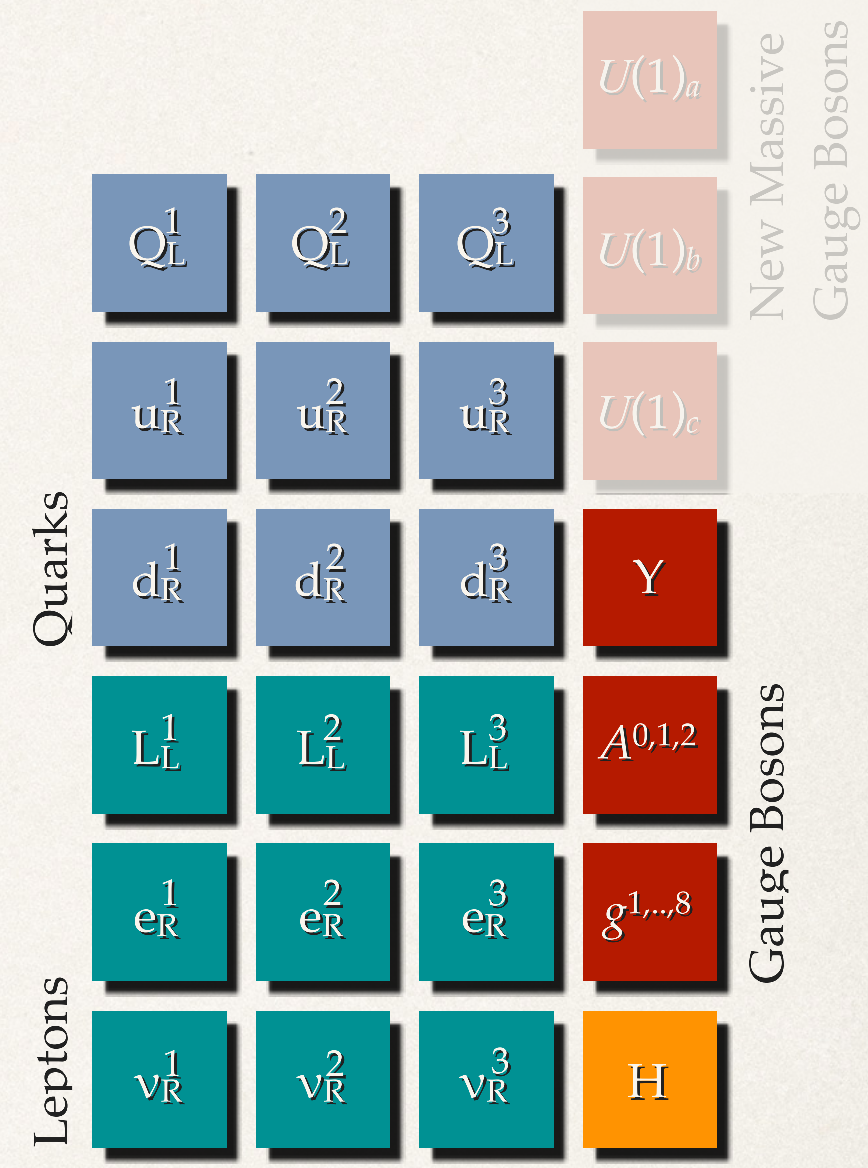
- Such towers of states appear at each intersection.

Consequences and predictions

- The Standard Model revised.





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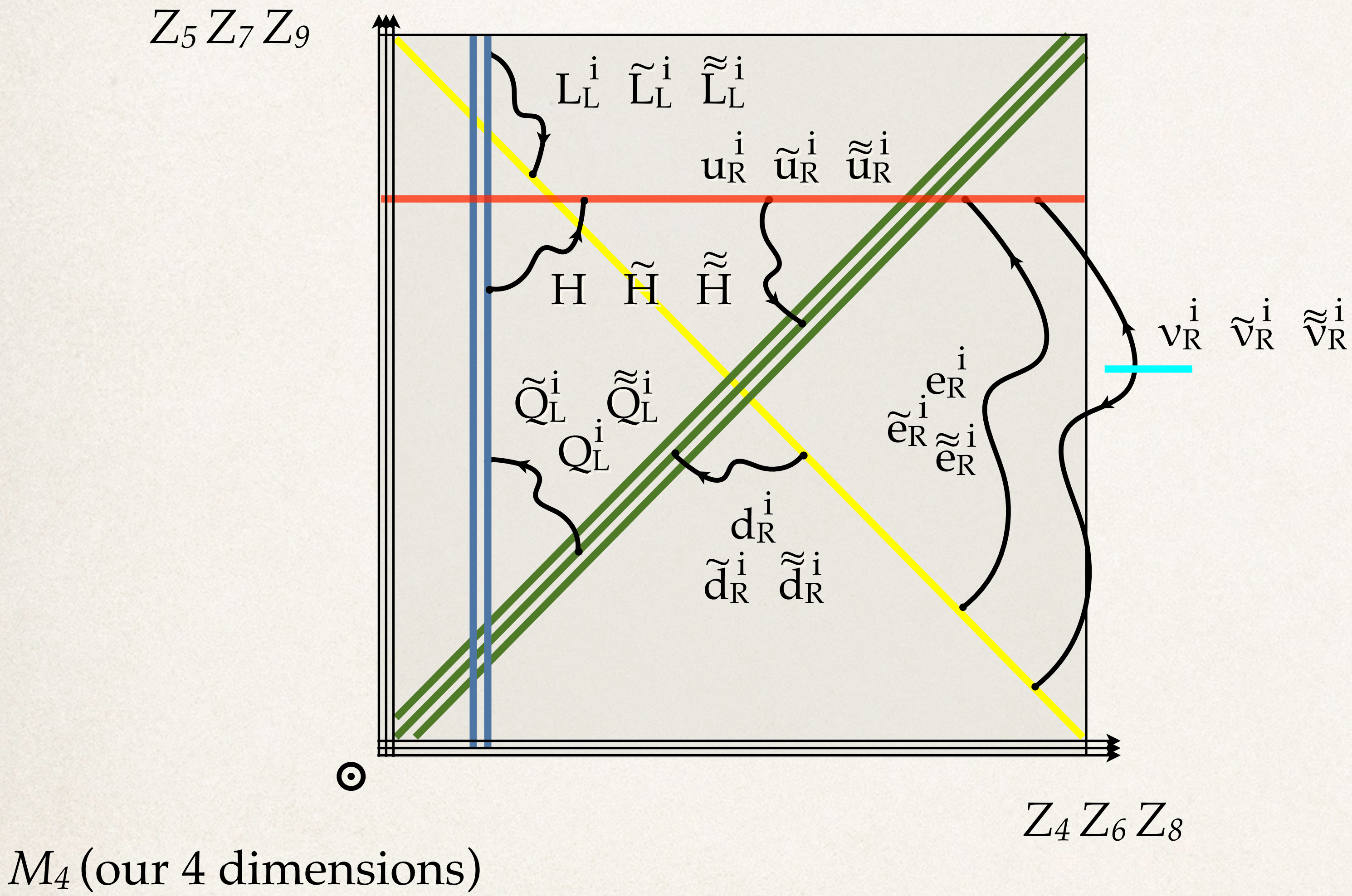

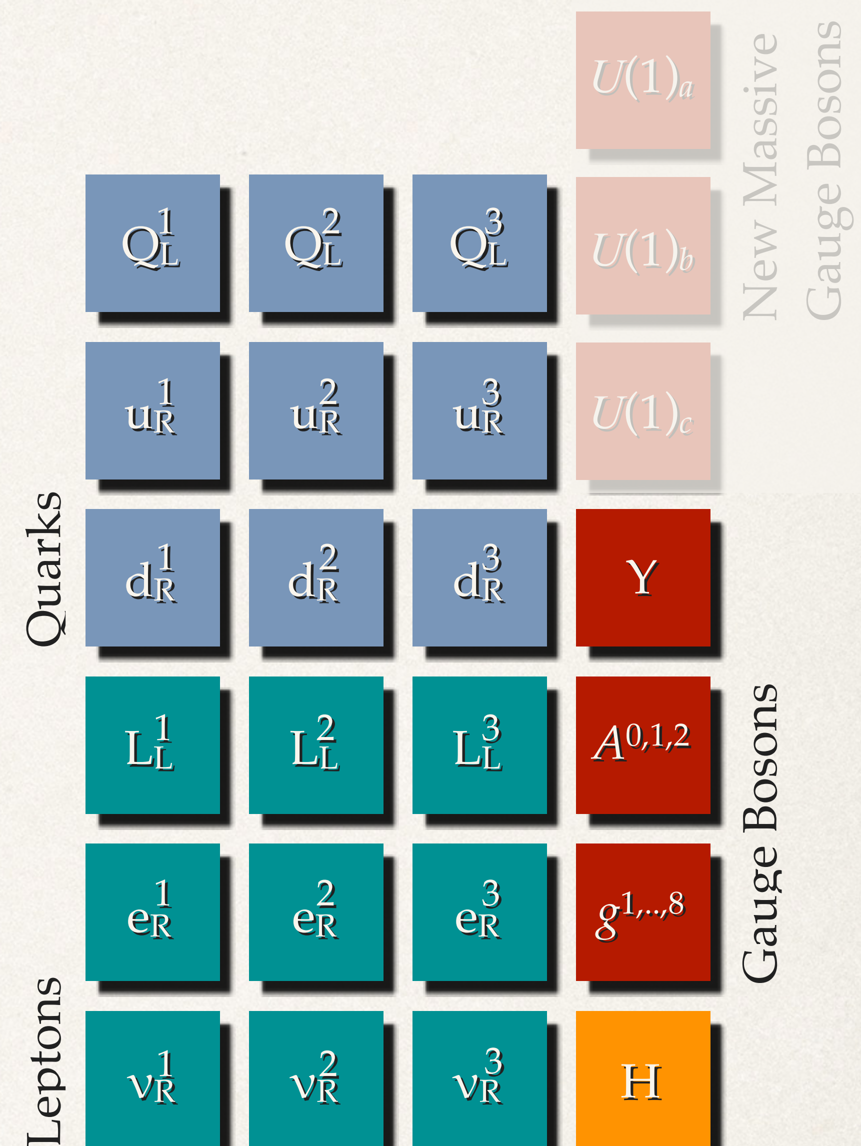

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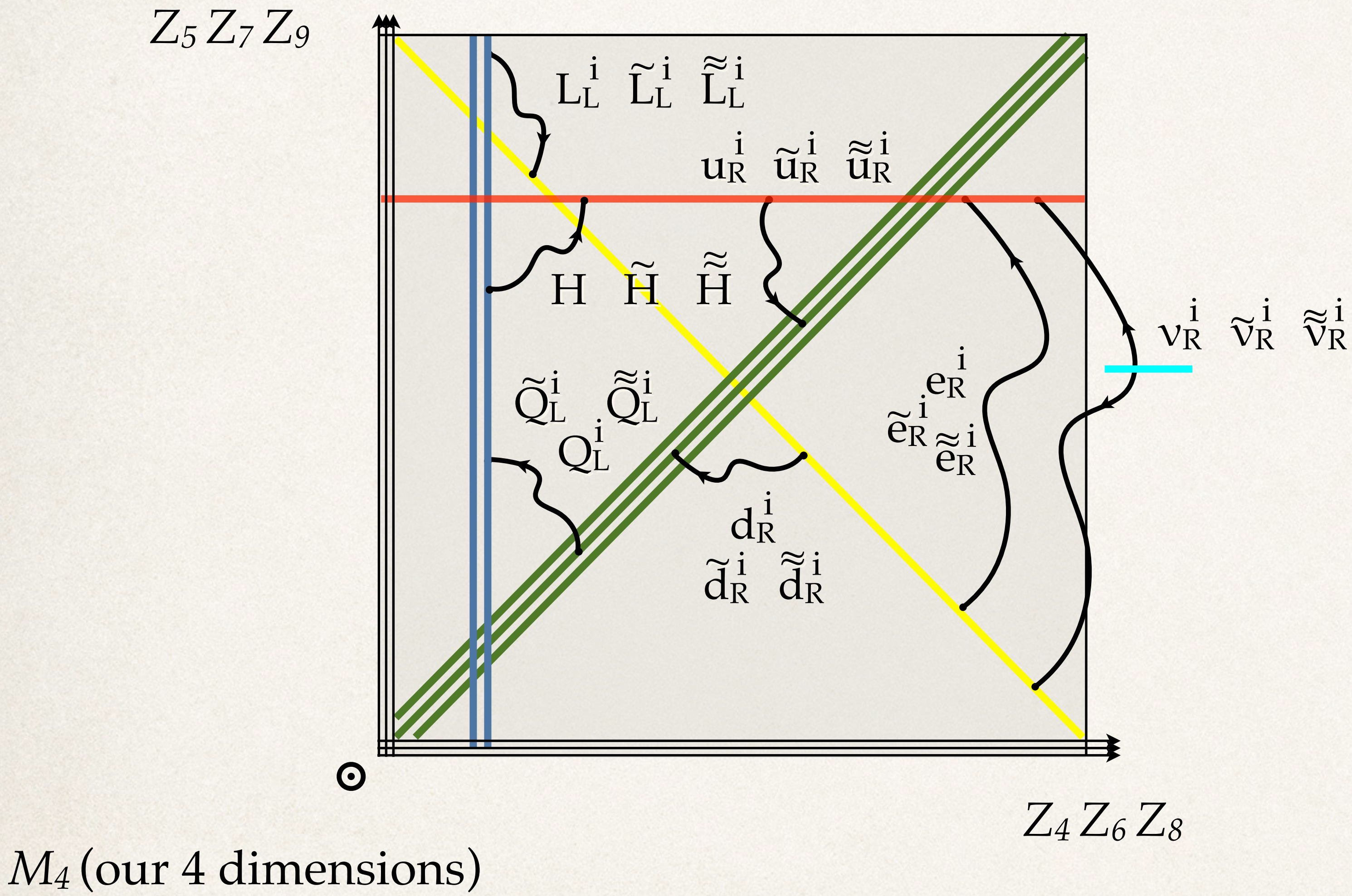
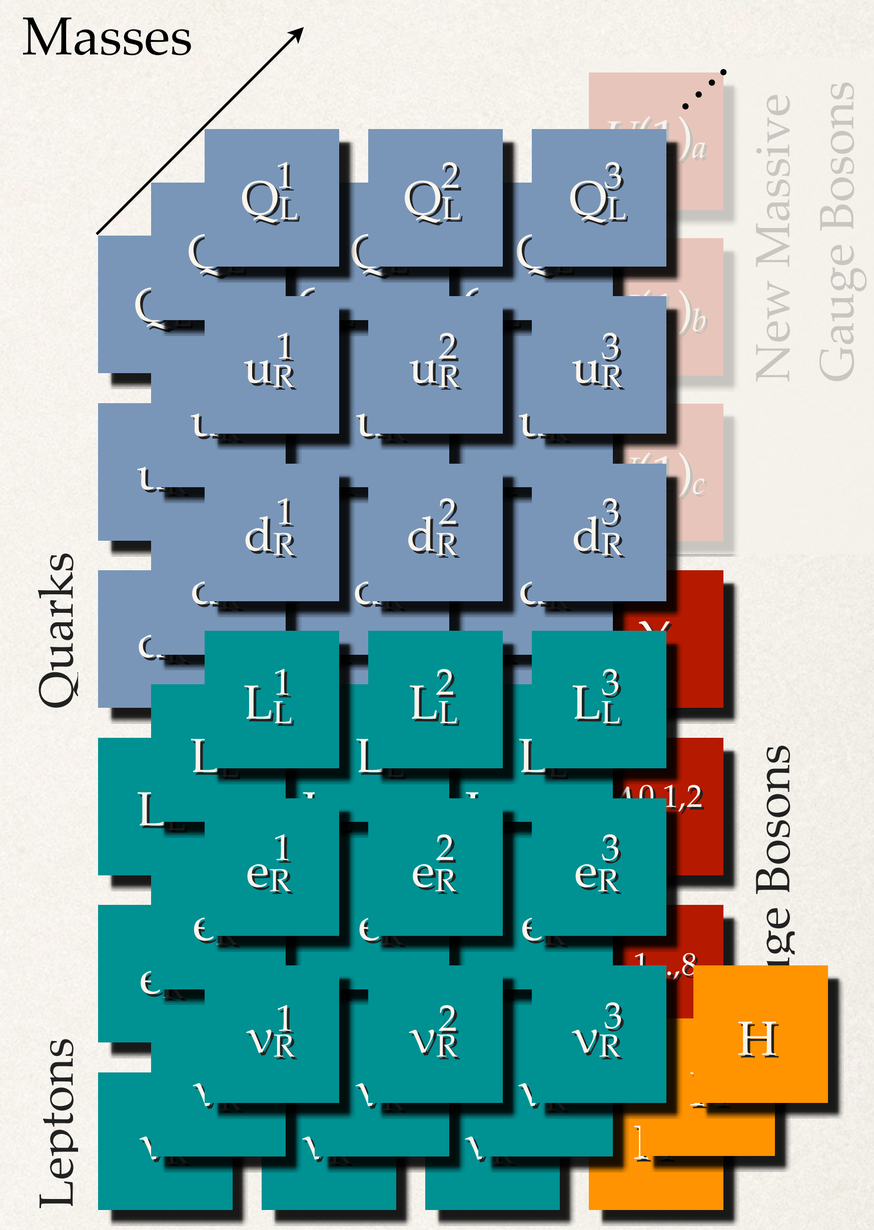
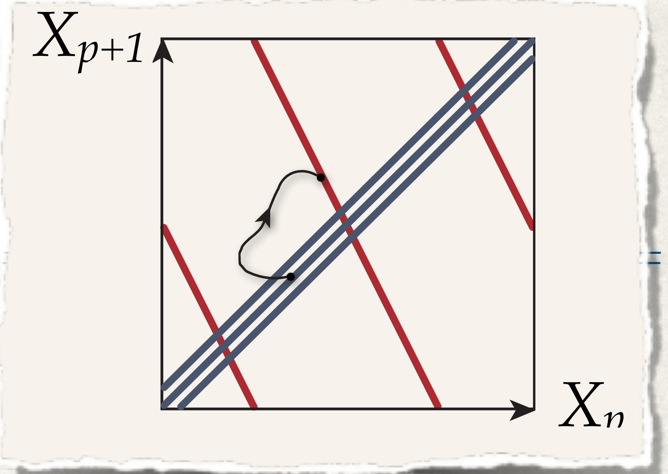




- Our aim is to study the phenomenological consequences of these massive copies of the Standard Model matter particles.

Quantization at angles



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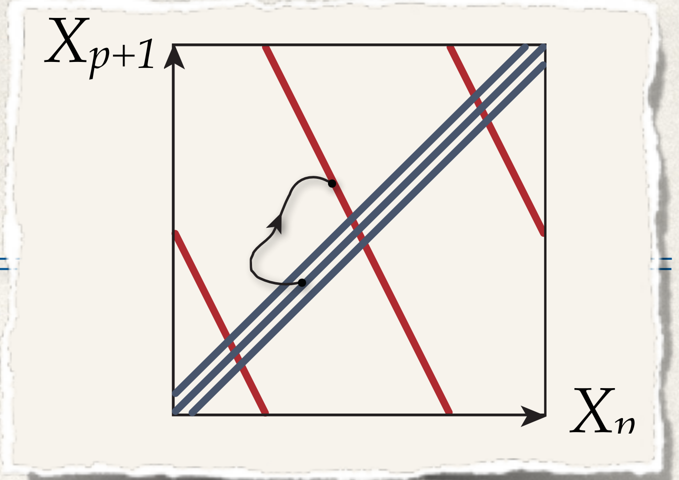

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- Mode expansion ($Z^p = X^p + iX^{p+1}$):

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$$\Psi^I(z) = \sum_{r\in Z+\nu} \psi^I_{r-a_I} z^{-r-\frac{1}{2}+a_I} \qquad \bar{\Psi}^I(z) = \sum_{r\in Z+\nu} \psi^I_{r+a_I} z^{-n-\frac{1}{2}-a_I}$$

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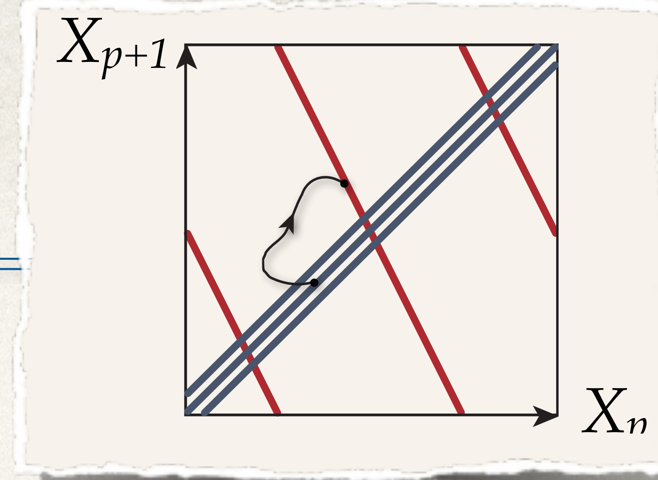

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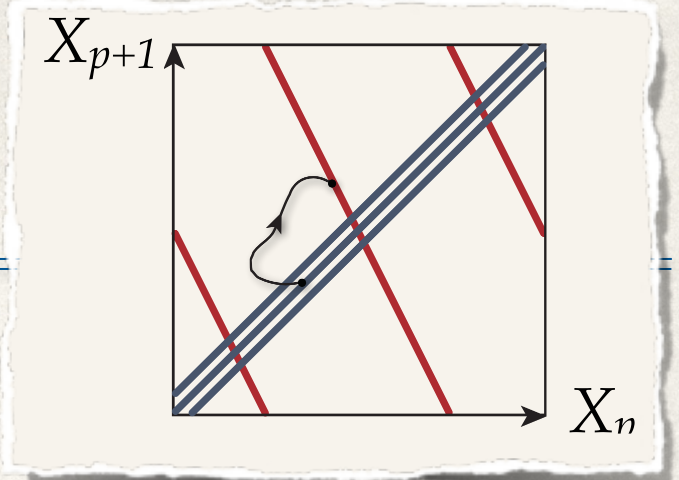

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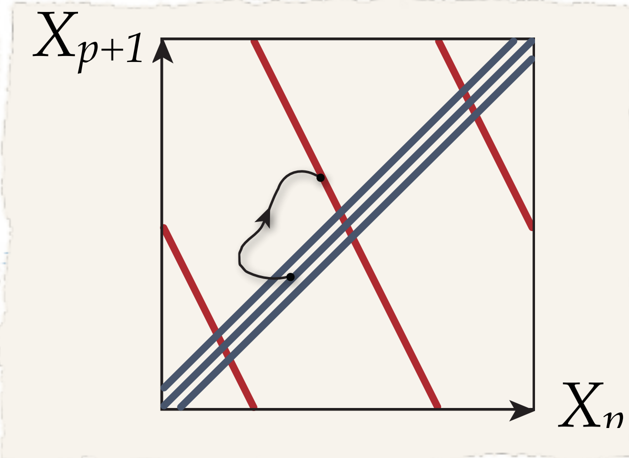
- And the three vacua that these states act on:

WS bosonic: $|a^1; a^2; a^3\rangle_B$

WS fermionic (NS): $|a^1; a^2; a^3\rangle_{NS}$

WS fermionic (R): $|a^1; a^2; a^3\rangle_R$

Sectors at the intersections



- Fock space at intersections (at the *I-torus*):

  - NS sector (spacetime bosons)

odd number

$$(a^I_{-n+a_I})^m (\psi^I_{-r-\frac{1}{2}+a_I})^s ... |a_I\rangle_{B\otimes NS} \qquad 1 \leq n, \; 0 \leq r$$

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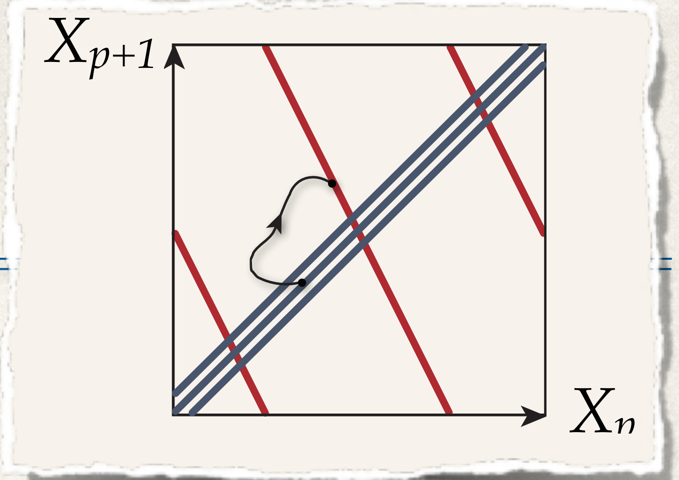

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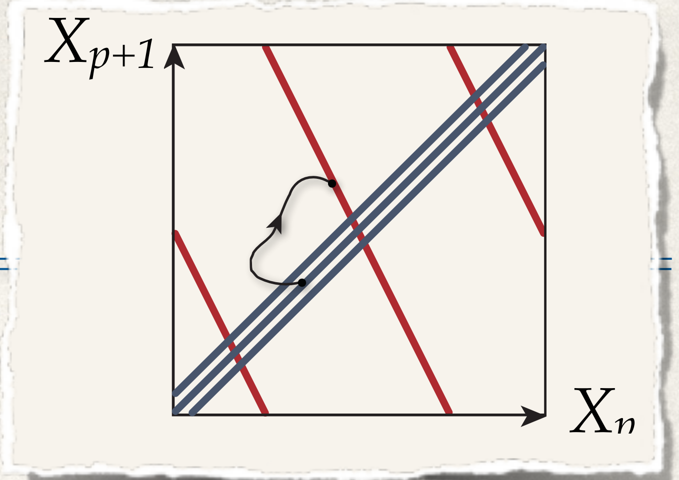

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- The mass formula is given by:

$$M^2 = \sum_{\mu=1}^{2}\Big(\sum_{n\in Z} a^\mu_{-n}a_{n\mu} + \sum_{r\in Z+\nu} r\psi^\mu_{-r}\psi_{r\mu}\Big)$$
$$+\sum_{I=1}^{3}\Big(\sum_{m\in Z} a^I_{-m+a_I}a^I_{m-a_I} + \sum_{r\in Z+\nu}(r-a_I)\psi^I_{-r+a_I}\psi^I_{r-a_I}\Big)$$
$$+2\nu\Big(-\frac{1}{2}+\frac{1}{2}(\pm a_1 \pm a_2 \pm a_3)\Big)$$

VO: A dictionary for twisted states

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- To check that we express states to their VO's using the dictionary:

- For the NS-sector:

Positive angle θ		*Negative angle* θ	
$\lvert\theta\rangle_{B\otimes NS}$	$e^{i\theta H}\ \sigma^{+}_{\theta}$	$\lvert\theta\rangle_{B\otimes NS}$	$e^{i\theta H}\ \sigma^{-}_{-\theta}$
$\alpha_{-\theta}\lvert\theta\rangle_{B\otimes NS}$	$e^{i\theta H}\ \tau^{+}_{\theta}$	$\alpha_{\theta}\lvert\theta\rangle_{B\otimes NS}$	$e^{i\theta H}\ \tau^{-}_{-\theta}$
$(\alpha_{-\theta})^2\lvert\theta\rangle_{B\otimes NS}$	$e^{i\theta H}\ \omega^{+}_{\theta}$	$(\alpha_{\theta})^2\lvert\theta\rangle_{B\otimes NS}$	$e^{i\theta H}\ \omega^{-}_{-\theta}$
$\psi_{-\frac{1}{2}+\theta}\lvert\theta\rangle_{B\otimes NS}$	$e^{i(\theta-1)H}\ \sigma^{+}_{\theta}$	$\psi_{-\frac{1}{2}-\theta}\lvert\theta\rangle_{B\otimes NS}$	$e^{i(\theta+1)H}\ \sigma^{-}_{-\theta}$
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- For the R-sector:

Positive angle θ		*Negative angle* θ	
$\lvert\theta\rangle_{B\otimes R}$	$e^{i(\theta-1/2)H}\ \sigma^{+}_{\theta}$	$\lvert\theta\rangle_{B\otimes R}$	$e^{i(\theta+1/2)H}\ \sigma^{-}_{-\theta}$

VO: A dictionary for twisted states

- Therefore, not all previous states are physical.
- To check that we express states to their VO's using the dictionary:

  - For the NS-sector:

Positive angle θ		*Negative angle* θ	
$\vert\theta\rangle_{B\otimes NS}$	$: \; e^{i\theta H}\,\sigma^+_\theta$	$\vert\theta\rangle_{B\otimes NS}$	$: \; e^{i\theta H}\,\sigma^-_{-\theta}$
$\alpha_{-\theta}\vert\theta\rangle_{B\otimes NS}$	$: \; e^{i\theta H}\,\tau^+_\theta$	$\alpha_\theta\vert\theta\rangle_{B\otimes NS}$	$: \; e^{i\theta H}\,\tau^-_{-\theta}$
$(\alpha_{-\theta})^2\vert\theta\rangle_{B\otimes NS}$	$: \; e^{i\theta H}\,\omega^+_\theta$	$(\alpha_\theta)^2\vert\theta\rangle_{B\otimes NS}$	$: \; e^{i\theta H}\,\omega^-_{-\theta}$
$\psi_{-\frac{1}{2}+\theta}\vert\theta\rangle_{B\otimes NS}$	$: \; e^{i(\theta-1)H}\,\sigma^+_\theta$	$\psi_{-\frac{1}{2}-\theta}\vert\theta\rangle_{B\otimes NS}$	$: \; e^{i(\theta+1)H}\,\sigma^-_{-\theta}$
$\alpha_{-\theta}\,\psi_{-\frac{1}{2}+\theta}\vert\theta\rangle_{B\otimes NS}$	$: \; e^{i(\theta-1)H}\,\tau^+_\theta$	$\alpha_\theta\,\psi_{-\frac{1}{2}-\theta}\vert\theta\rangle_{B\otimes NS}$	$: \; e^{i(\theta+1)H}\,\tau^-_{-\theta}$
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  - For the R-sector:

Positive angle θ		*Negative angle* θ	
$\vert\theta\rangle_{B\otimes R}$	$: \; e^{i(\theta-1/2)H}\,\sigma^+_\theta$	$\vert\theta\rangle_{B\otimes R}$	$: \; e^{i(\theta+1/2)H}\,\sigma^-_{-\theta}$

- The σ, τ, ω are twisted bosonic conformal fields.

Anastasopoulos Bianchi Richter

Vertex Operators

- Each physical VO has to obey:

$$[Q_{BRST}, V] = 0$$

where the BRST charge is given by:

$$Q_{BRST} = \oint \frac{dz}{2\pi i}\Big\{ e^{\varphi}\eta \frac{1}{\sqrt{2\alpha'}}\Big(i\partial X^{\mu}\,\psi_{\mu} + \sum_{I=1}^{3} \partial Z^{I}\, e^{-iH_I} + \sum_{I=1}^{3} \partial \overline{Z}^{I}\, e^{iH_I} \Big)$$
$$+ c\Big(\frac{1}{\alpha'} i\partial X^{\mu} i\partial X_{\mu} - \frac{1}{2}\psi^{\mu}\partial\psi_{\mu} + \sum_{I=1}^{3}\Big(\frac{1}{\alpha'}\partial Z^{I}\partial \overline{Z}_{I} - \frac{1}{2} e^{-iH_I}\partial e^{iH_I}\Big)\Big)\Big\} + \ldots$$

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- The physical condition typically gives:
  - a simple pole → the equations of motion.
  - a double pole → the energy-momentum equation.

R sectors and $[Q_{BRST}, V] = 0$

- The "zero" states (massless) are:

$$\Phi(k) \;:\; \psi_{-\frac{1}{2}-a^3_{ab}}|a^1_{ab}, a^2_{ab}, -a^3_{ab}\rangle_{NS} \qquad \Psi(k) \;:\; |a^1_{ab}, a^2_{ab}, -a^3_{ab}\rangle_R$$

- The "first" states with mass $\alpha' M^2 = a_1$ (or $\alpha' M^2 = a_2$ by the exchange $1 \leftrightarrow 2$) :

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$$V^{(-1/2)}_\psi = C_\psi g_\psi [\Lambda_{ab}]^\beta_\alpha\, v^\alpha_\psi\, e^{-\phi/2} S_\alpha \left(\sigma_{a^1_{ab}}\, e^{i\left(a^1_{ab}-\frac{1}{2}\right)H_1}\right)\left(\sigma_{a^2_{ab}}\, e^{i\left(a^2_{ab}-\frac{1}{2}\right)H_2}\right)\left(\sigma_{1+a^3_{ab}}\, e^{i\left(a^3_{ab}+\frac{1}{2}\right)H_3}\right) e^{ikX}$$

$$-\frac{1}{2} \quad \text{mod } 2 \quad : \quad \textit{chiral}$$
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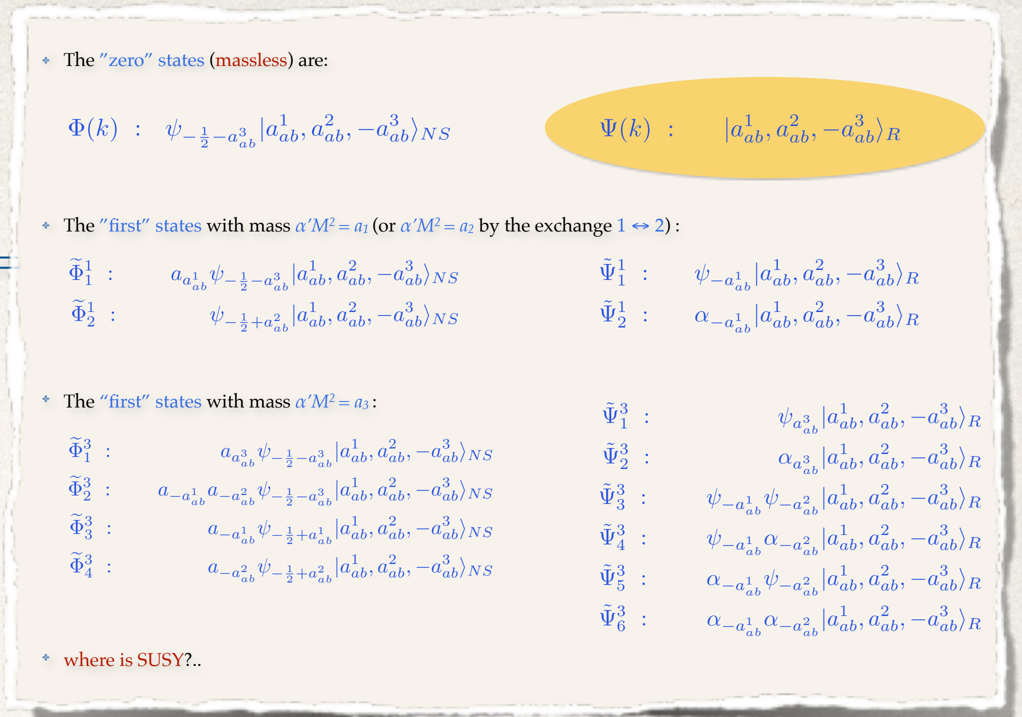

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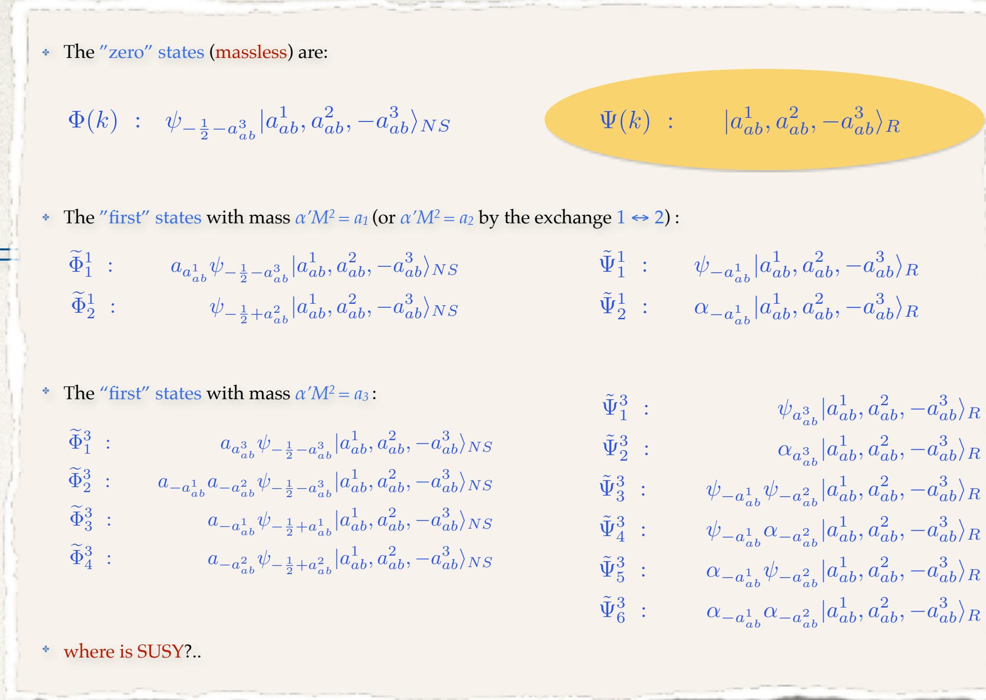
- Therefore, this is a physical massless fermion.

R sectors and $[Q_{BRST}, V] = 0$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- That could be any chiral fermion in the SM massless spectrum.

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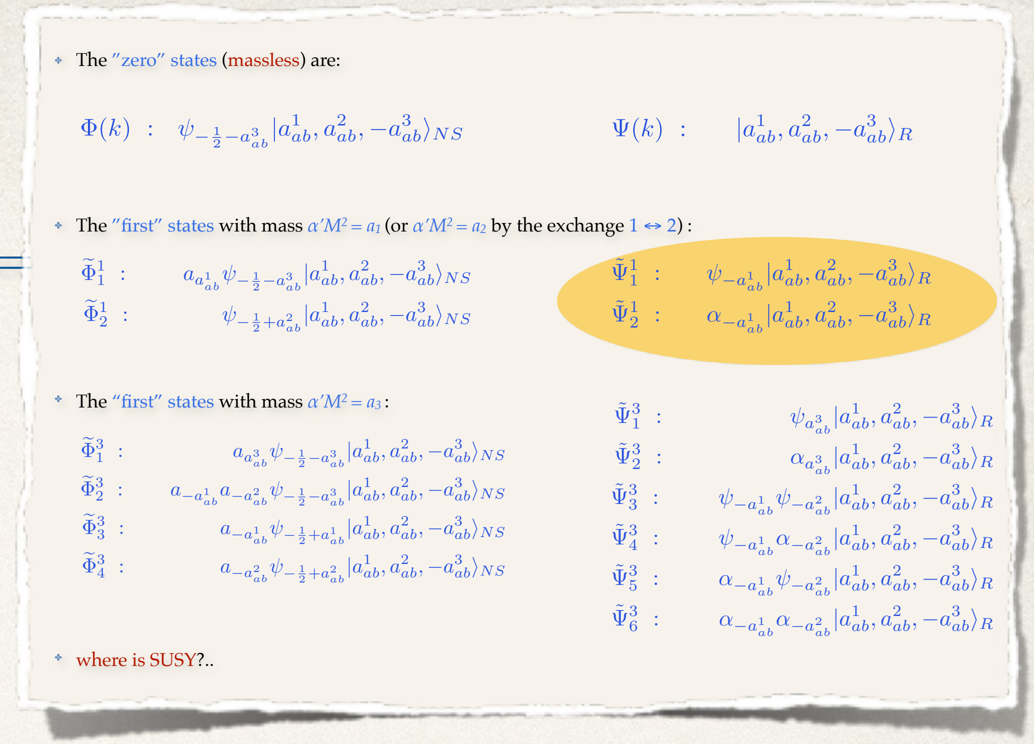

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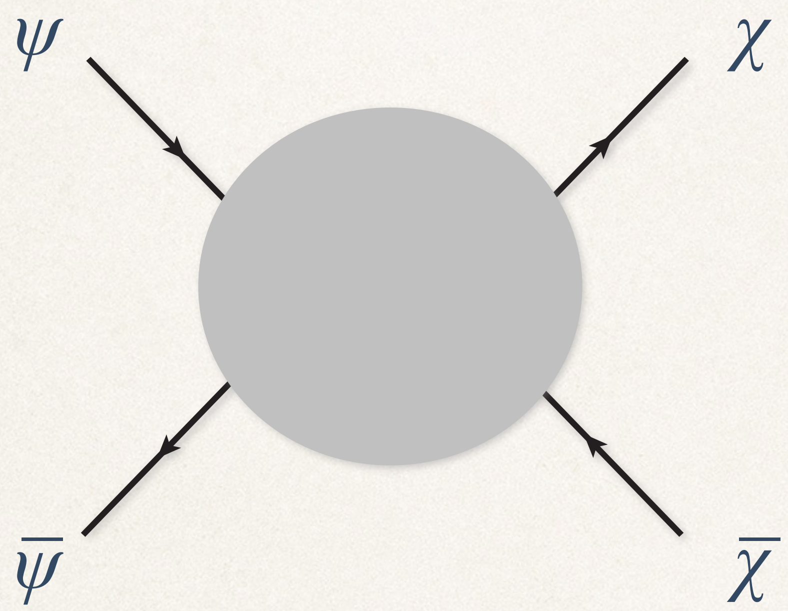

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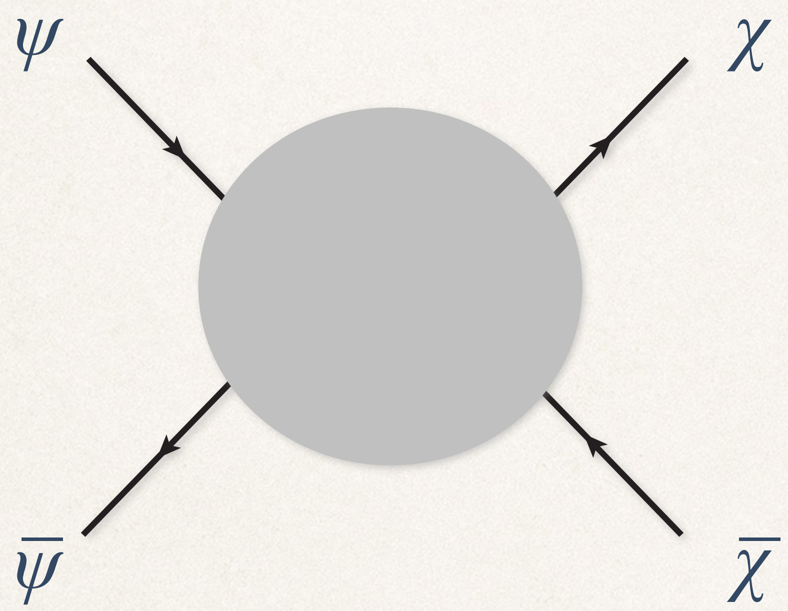

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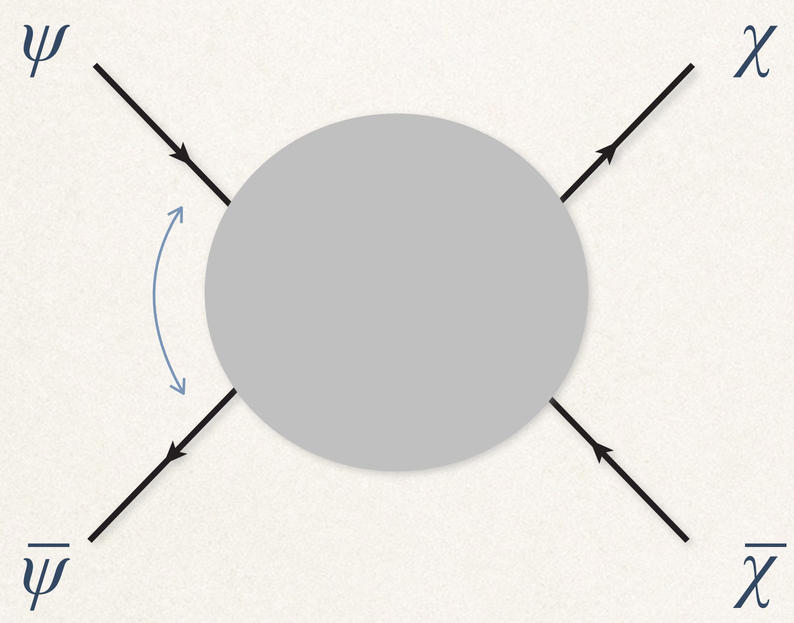

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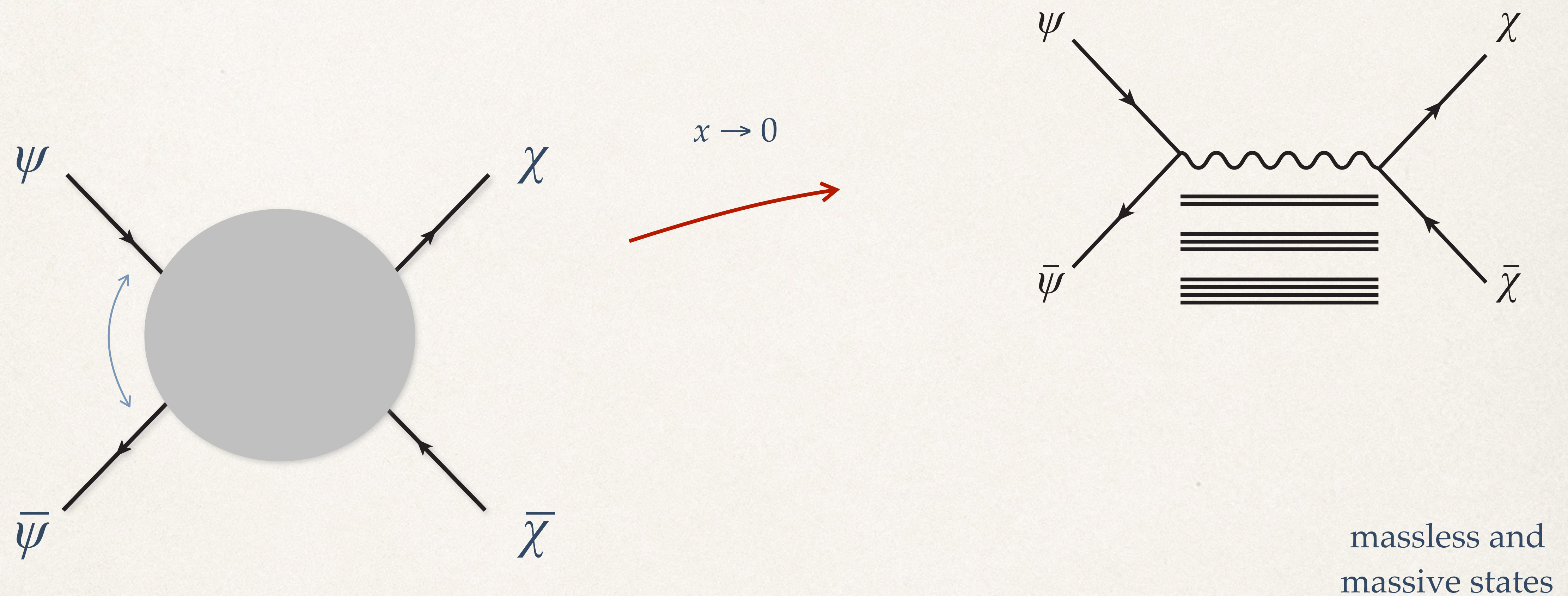

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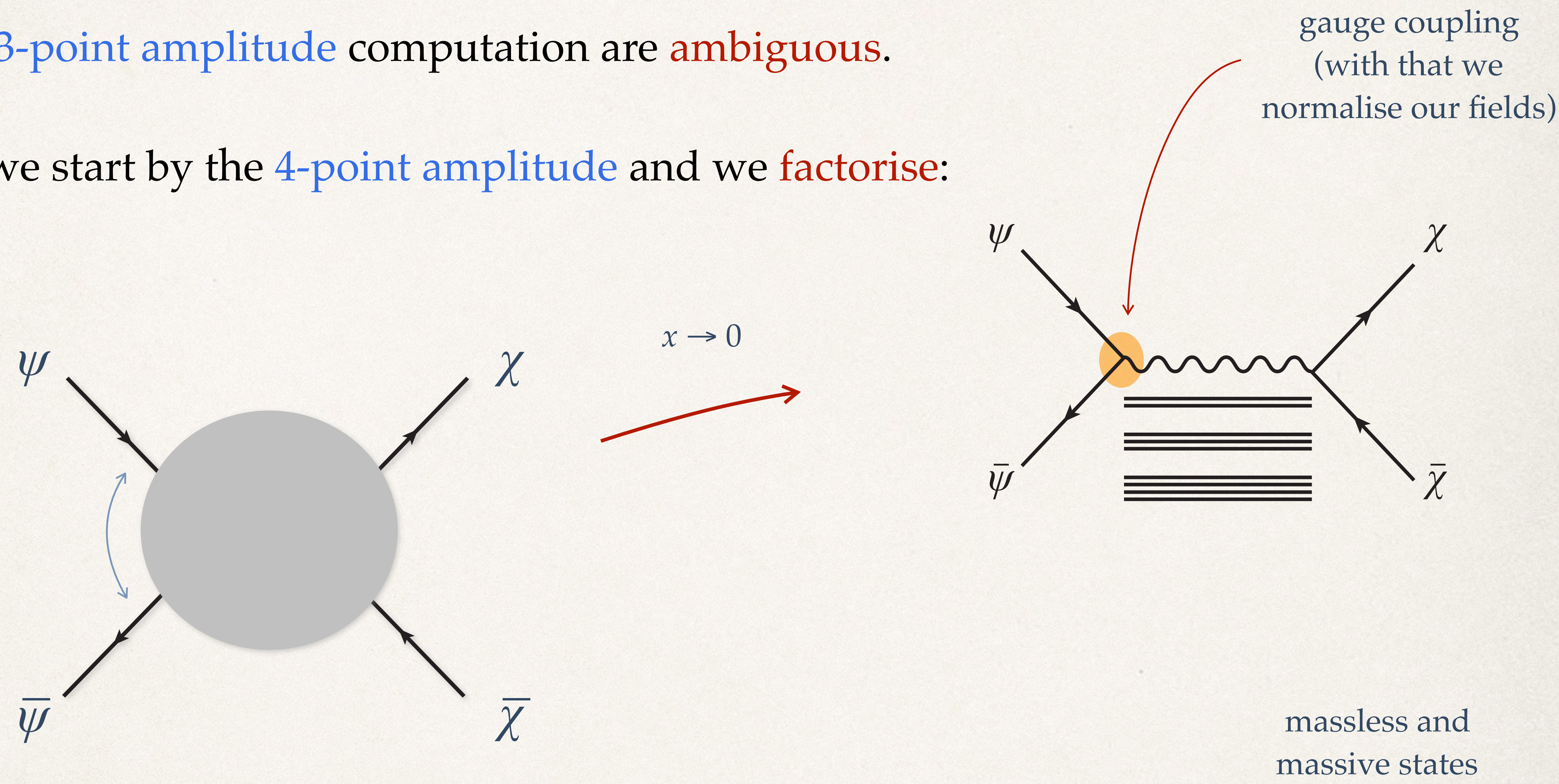

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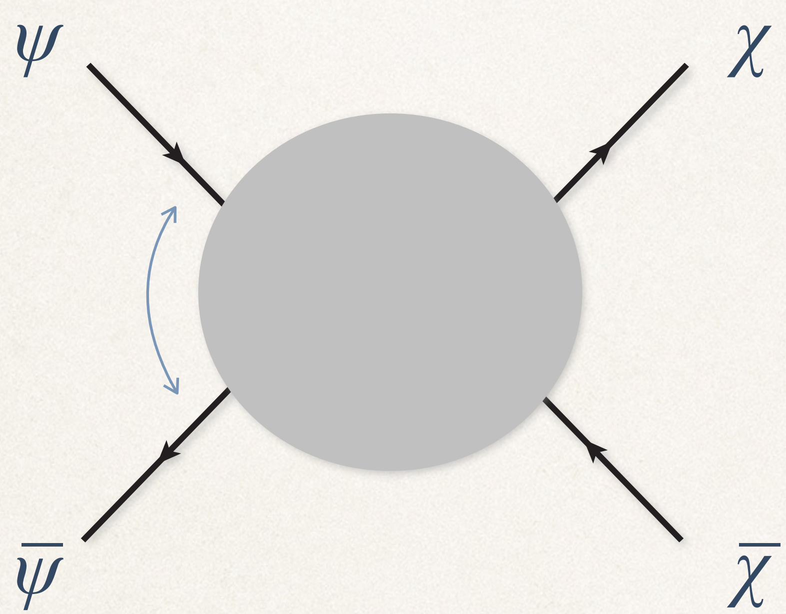

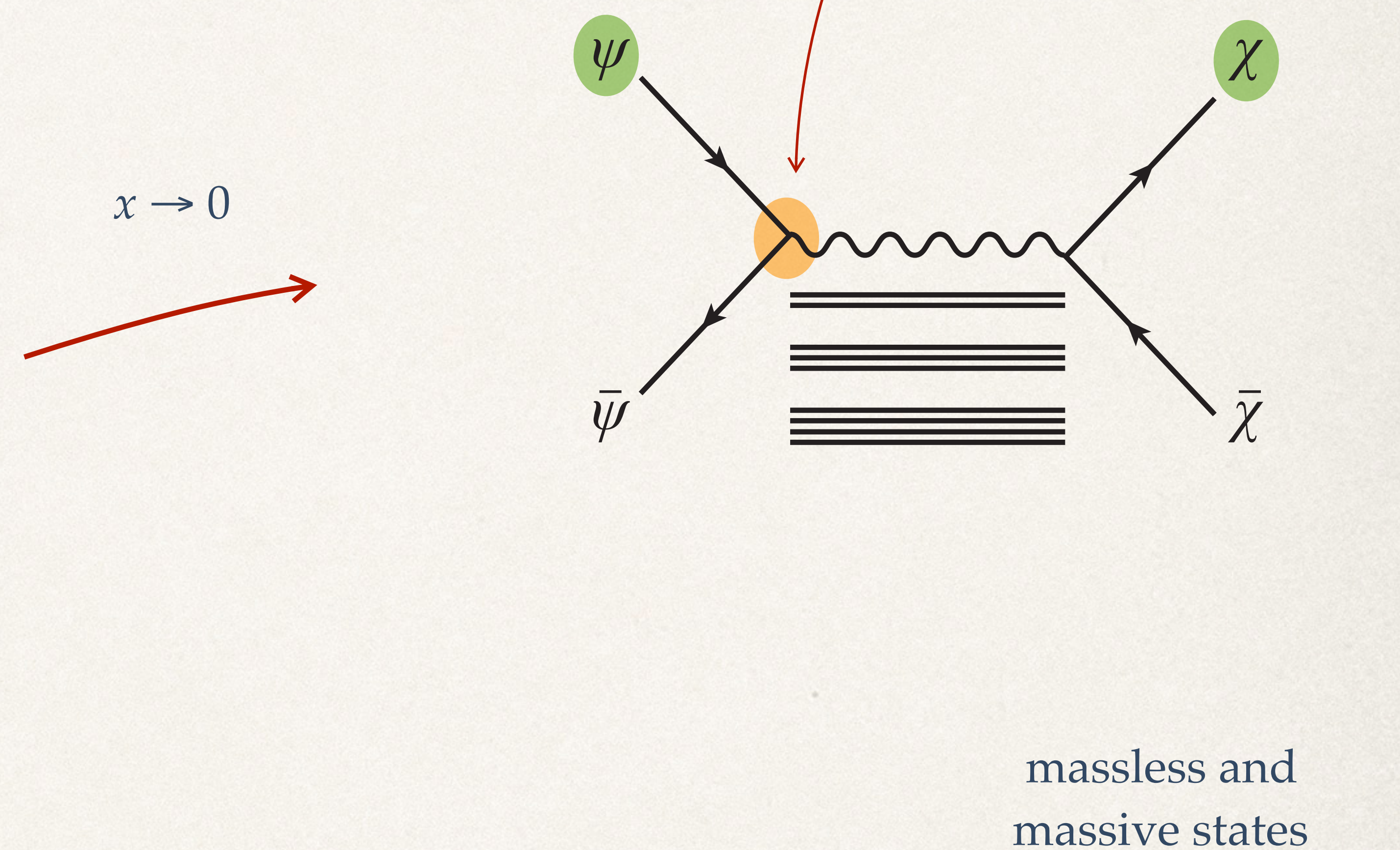

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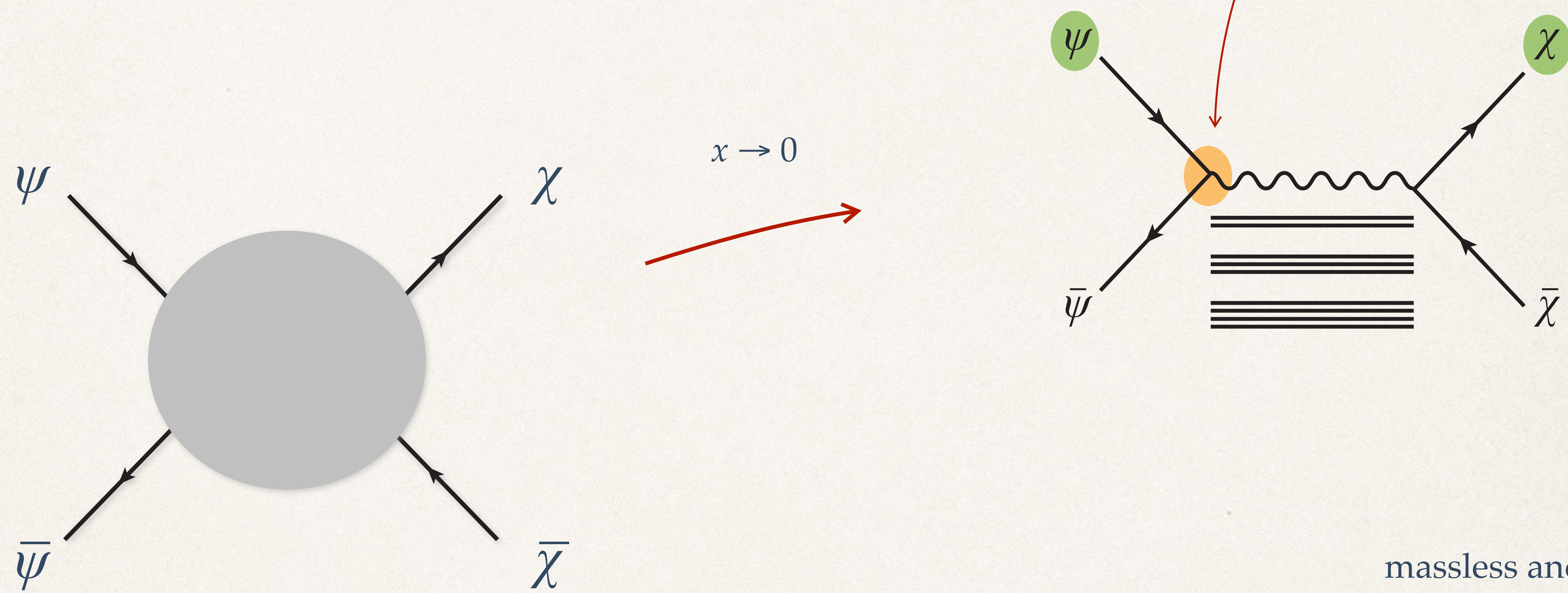

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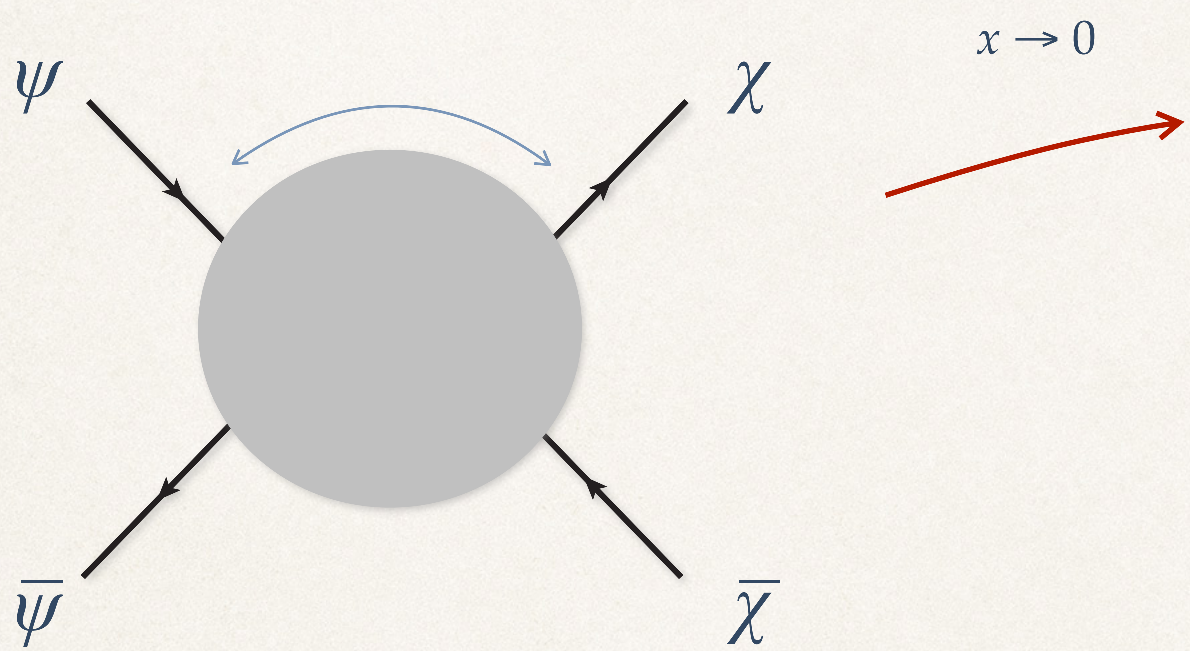

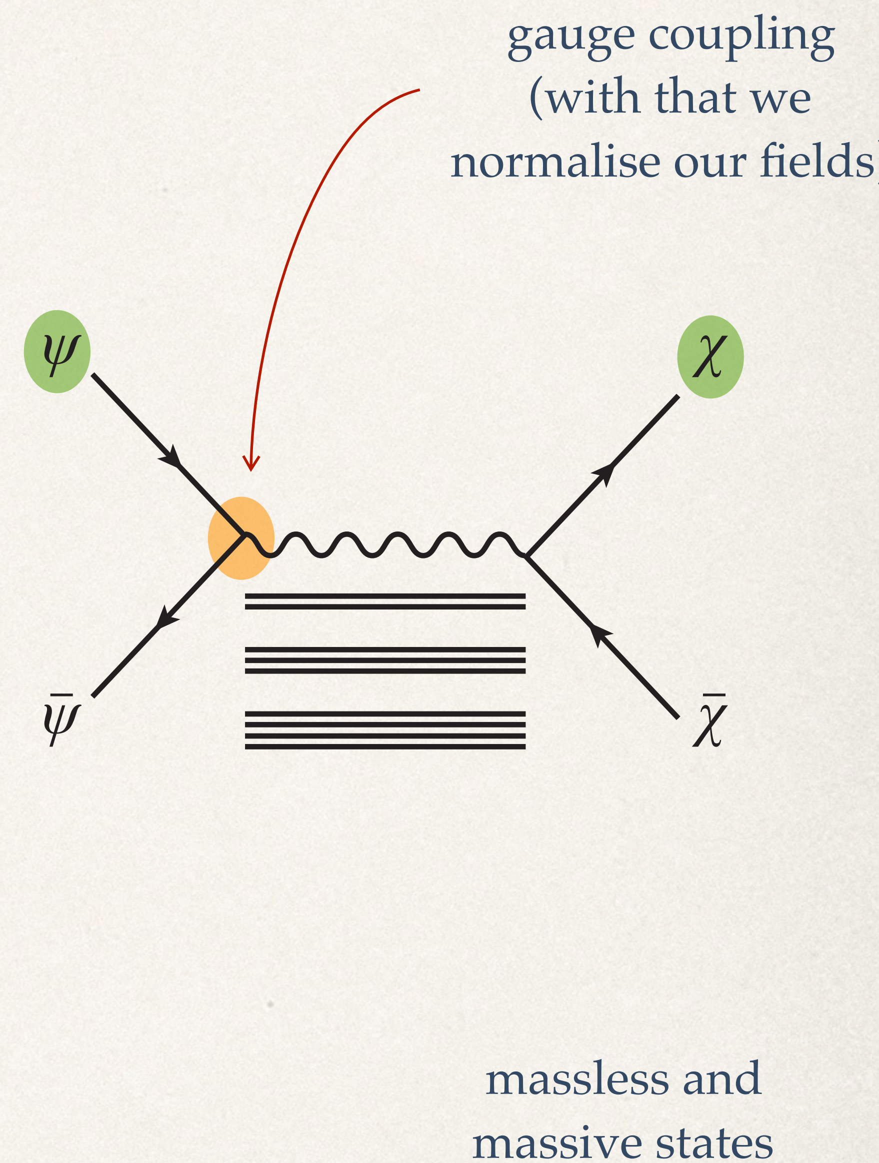

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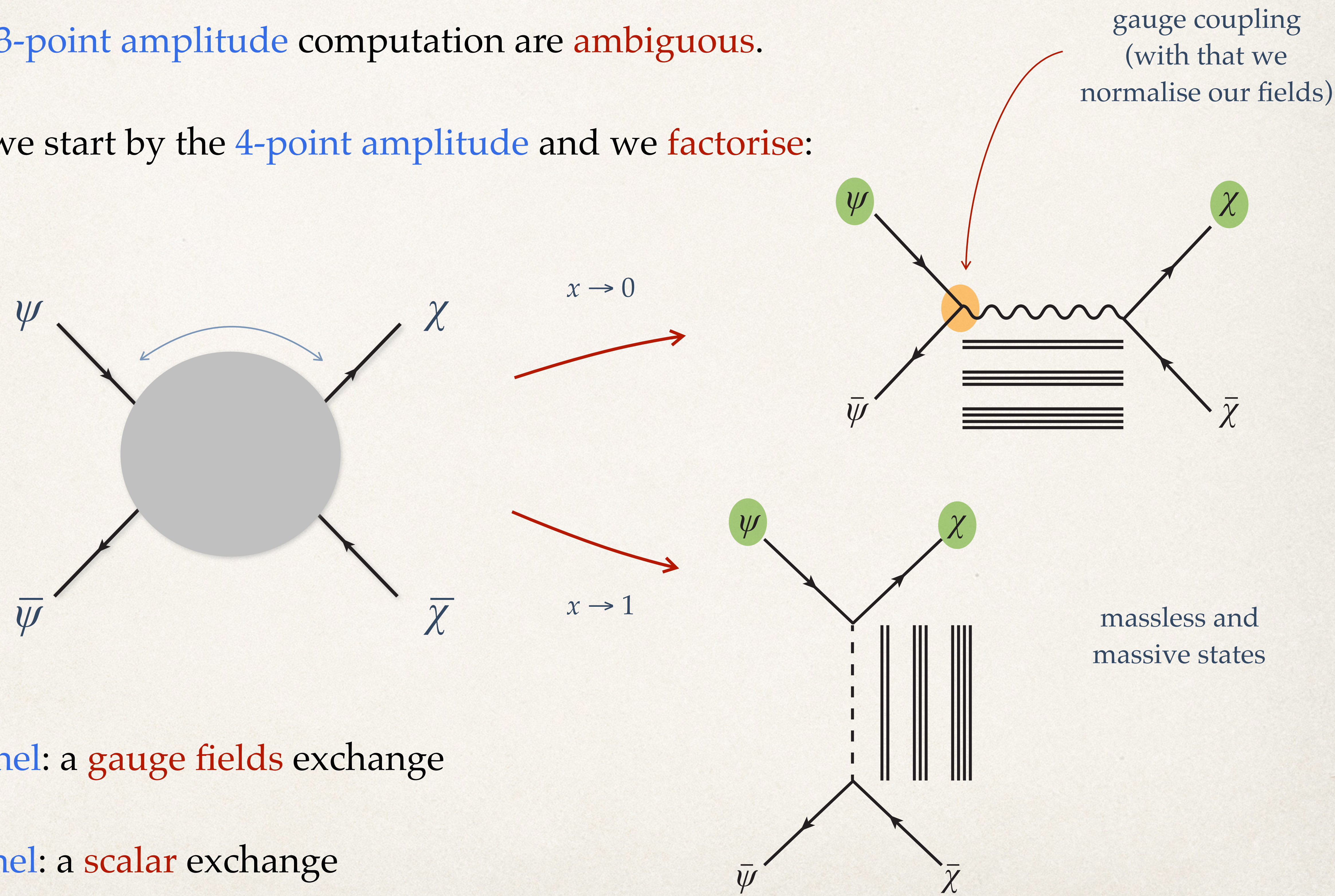

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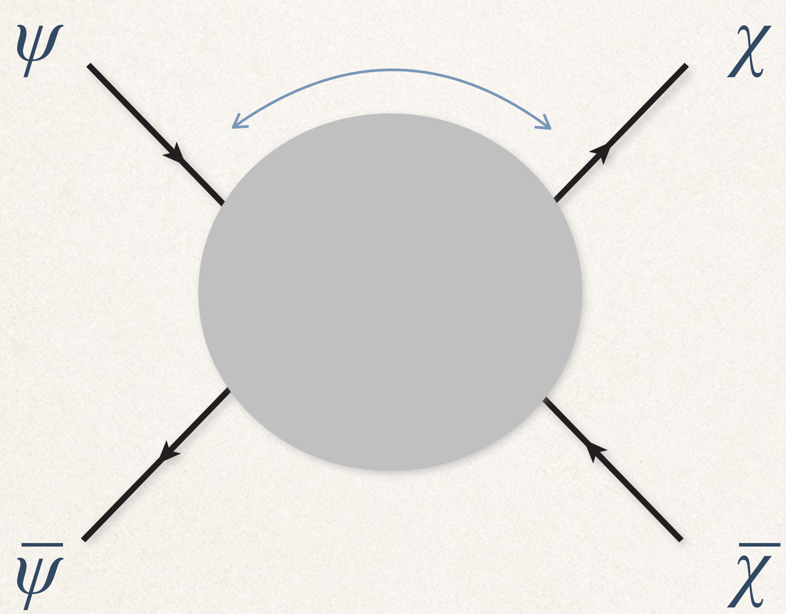

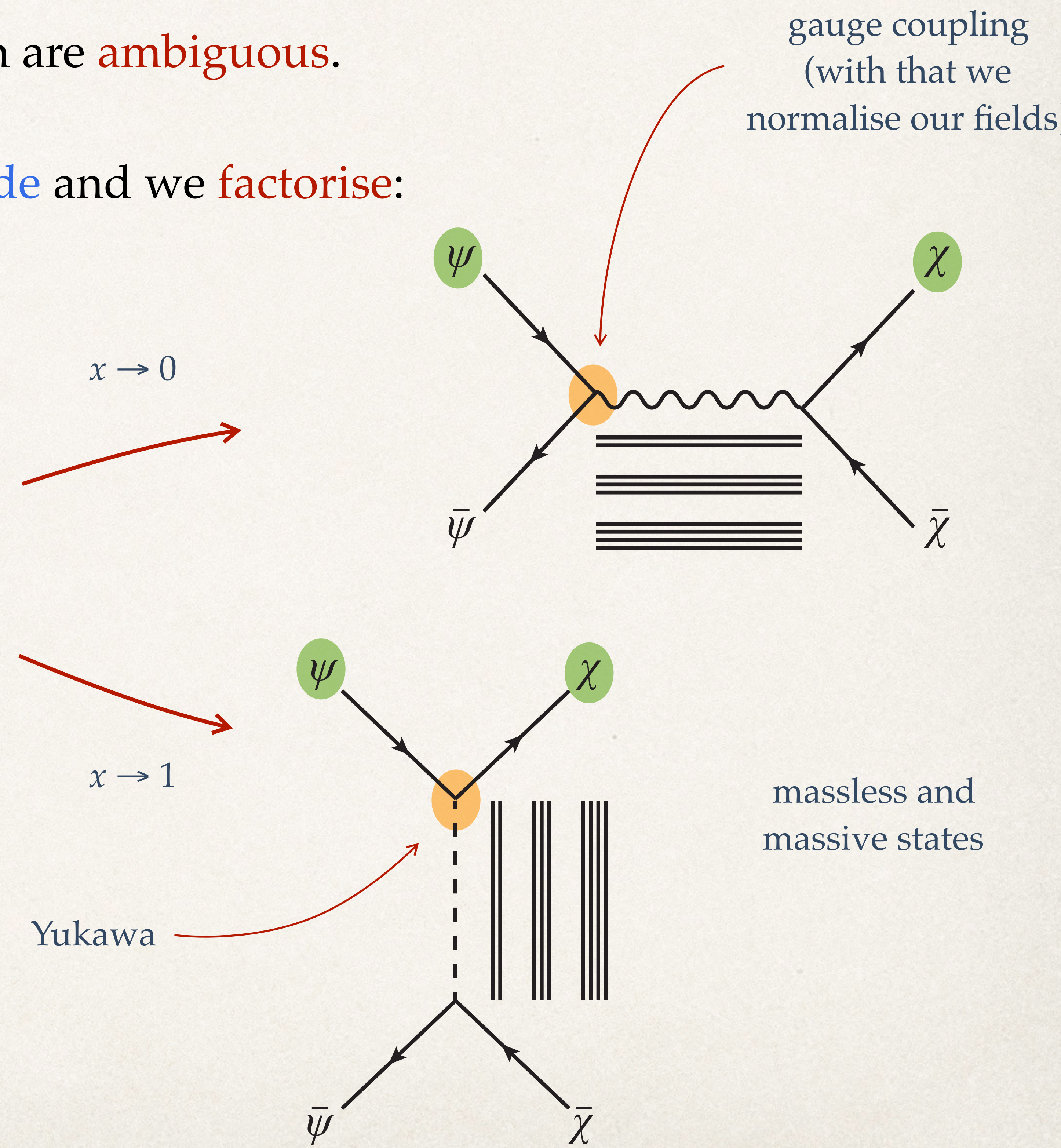

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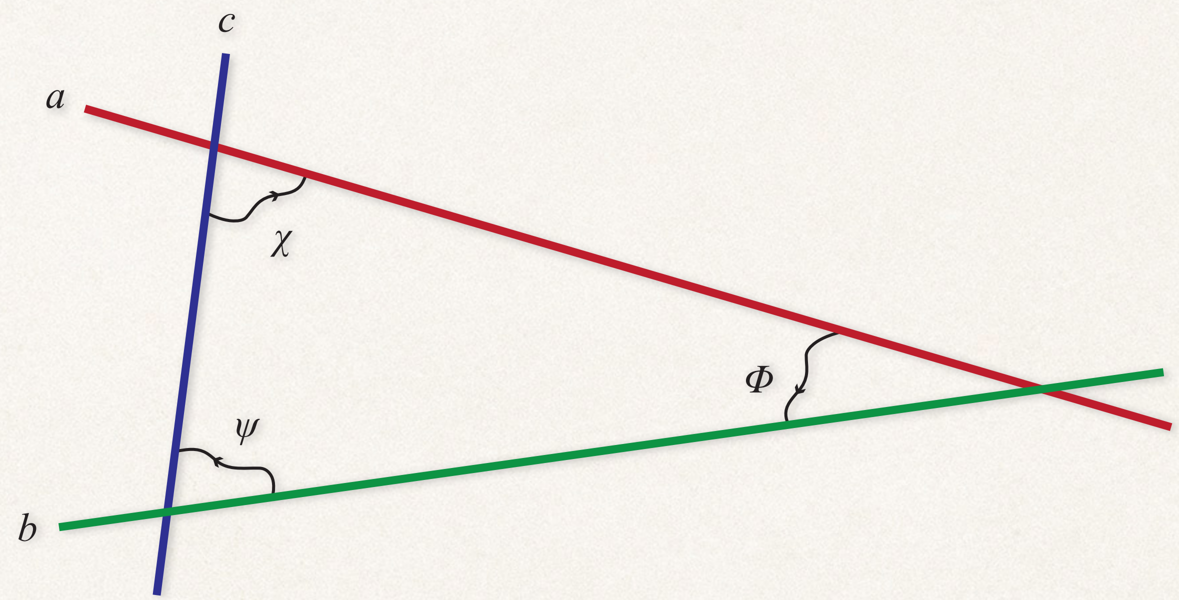

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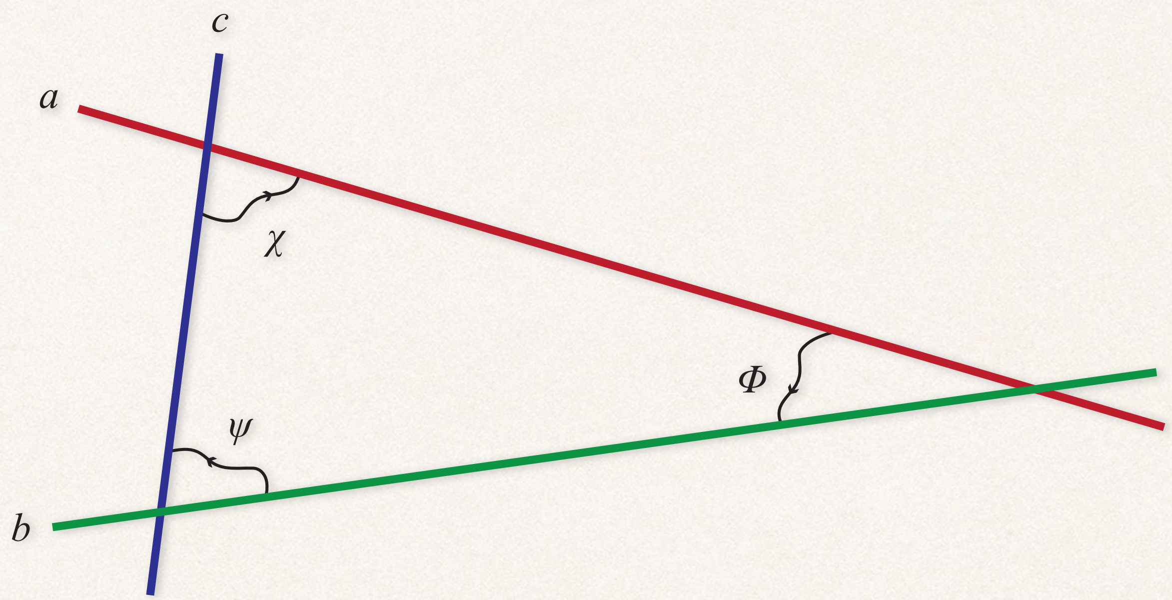

- For the sake of concreteness we choose a supersymmetric setup with:

$$\begin{array}{lll} \theta^1_{ab} > 0\,, & \theta^2_{ab} > 0\,, & \theta^3_{ab} < 0 \\ \theta^1_{bc} > 0\,, & \theta^2_{bc} > 0\,, & \theta^3_{bc} < 0 \\ \theta^1_{ca} < 0\,, & \theta^2_{ca} < 0\,, & \theta^3_{ca} < 0 \end{array} \quad \Rightarrow \quad \begin{array}{l} \theta^1_{ab} + \theta^2_{ab} + \theta^3_{ab} = 0 \\ \theta^1_{bc} + \theta^2_{bc} + \theta^3_{bc} = 0 \\ \theta^1_{ca} + \theta^2_{ca} + \theta^3_{ca} = -2 \end{array}$$

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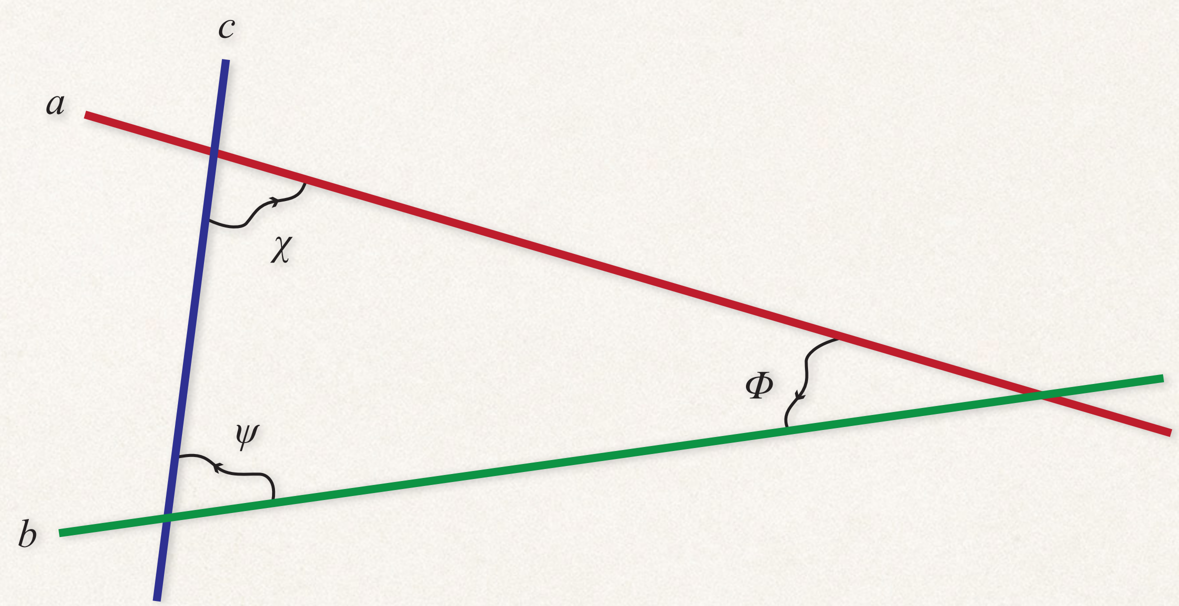

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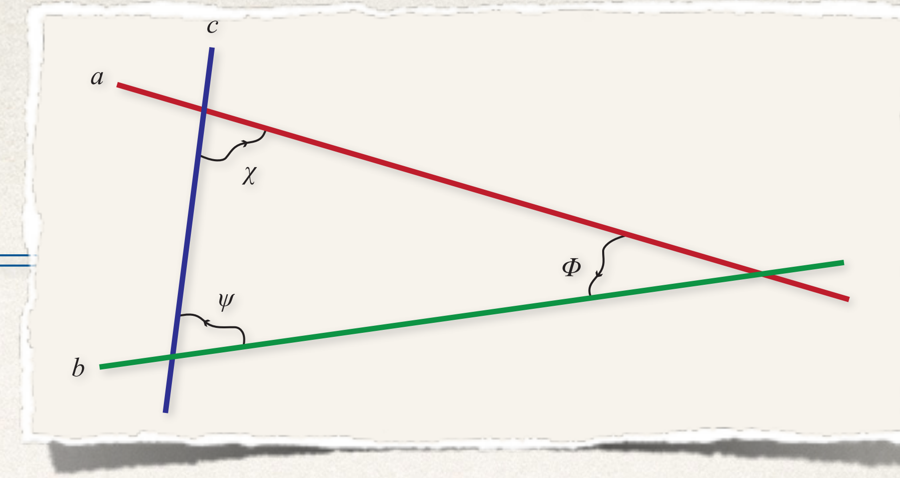
- At the intersections live chiral fermions ψ, $\bar{\psi}$, χ ,$\bar{\chi}$, ϕ, $\bar{\phi}$ and their superparteners Ψ, X, Φ.

Fields at angles



- VO's for the fields at the *ab*, *bc*, *ca* intersections.

$$V^{(-1)}_{\phi_0=\phi_0^{ab}} = C_{\phi_0} e^{-\phi_{10}} \phi_0 e^{-\varphi} \sigma_{a^1_{a,b}} \sigma_{a^2_{a,b}} \sigma_{1+a^3_{a,b}} e^{i[a^1_{a,b}\varphi_1 + a^2_{a,b}\varphi_2 + (a^3_{a,b}+1)\varphi_3]} e^{ikX}$$

$$V^{(-\frac{1}{2})}_{\psi_0=\chi_0^{bc}} = C_{\psi_0} e^{-\phi_{10}} \psi_0^\alpha S_\alpha e^{-\frac{\varphi}{2}} \sigma_{a^1_{b,c}} \sigma_{a^2_{b,c}} \sigma_{1+a^3_{b,c}} e^{i[(a^1_{b,c}-\frac{1}{2})\varphi_1 + (a^2_{b,c}-\frac{1}{2})\varphi_2 + (a^3_{b,c}+\frac{1}{2})\varphi_3]} e^{ikX}$$

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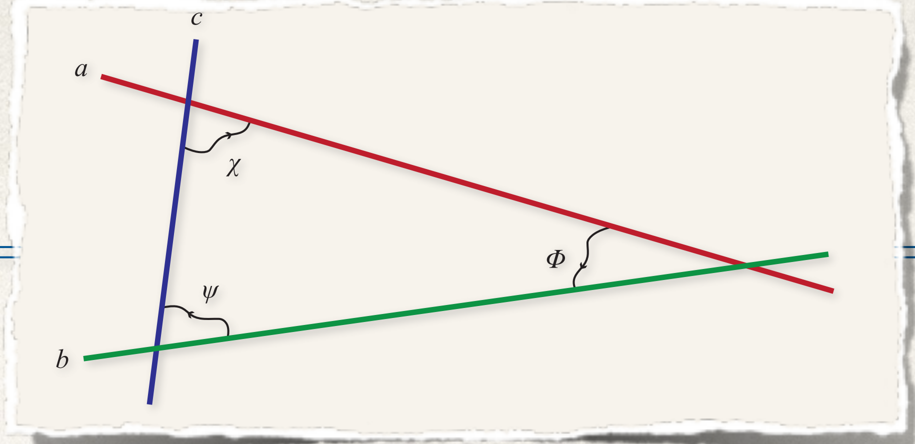
$$V^{(-\frac{1}{2})}_{\chi_0=\chi_0^{ca}} = C_{\chi_0} e^{-\phi_{10}} \chi_0^\alpha S_\alpha e^{-\frac{\varphi}{2}} \sigma_{1+a^1_{c,a}} \sigma_{1+a^2_{c,a}} \sigma_{1+a^3_{c,a}} e^{i[(a^1_{c,a}+\frac{1}{2})\varphi_1 + (a^2_{c,a}+\frac{1}{2})\varphi_2 + (a^3_{c,a}+\frac{1}{2})\varphi_3]} e^{ikX}$$

$$V^{(-\frac{1}{2})}_{\chi_1=\chi_1^{ca}} = C_{\chi_1} e^{-\phi_{10}} \chi_1^\alpha S_\alpha e^{-\frac{\varphi}{2}} \sigma_{1+a^1_{c,a}} \sigma_{1+a^2_{c,a}} \tau_{1+a^3_{c,a}} e^{i[(a^1_{c,a}+\frac{1}{2})\varphi_1 + (a^2_{c,a}+\frac{1}{2})\varphi_2 + (a^3_{c,a}+\frac{1}{2})\varphi_3]} e^{ikX}$$
$$+ C_{\tilde{\chi}_1} e^{-\phi_{10}} \tilde{\chi}^\dagger_{1\dot{\alpha}} C^{\dot{\alpha}} e^{-\frac{\varphi}{2}} \sigma_{1+a^1_{c,a}} \sigma_{1+a^2_{c,a}} \sigma_{1+a^3_{c,a}} e^{i[(a^1_{c,a}+\frac{1}{2})\varphi_1 + (a^2_{c,a}+\frac{1}{2})\varphi_2 + (a^3_{c,a}-\frac{1}{2})\varphi_3]} e^{ikX}$$

Fields at angles



- VO's for the fields at the *ab*, *bc*, *ca* intersections.

$$V^{(-1)}_{\phi_0=\phi_0^{ab}} = C_{\phi_0} e^{-\phi_{10}} \phi_0 e^{-\varphi} \sigma_{a^1_{a,b}} \sigma_{a^2_{a,b}} \sigma_{1+a^3_{a,b}} e^{i[a^1_{a,b}\varphi_1 + a^2_{a,b}\varphi_2 + (a^3_{a,b}+1)\varphi_3]} e^{ikX}$$

$$V^{(-\frac{1}{2})}_{\psi_0=\chi_0^{bc}} = C_{\psi_0} e^{-\phi_{10}} \psi_0^\alpha S_\alpha e^{-\frac{\varphi}{2}} \sigma_{a^1_{b,c}} \sigma_{a^2_{b,c}} \sigma_{1+a^3_{b,c}} e^{i[(a^1_{b,c}-\frac{1}{2})\varphi_1 + (a^2_{b,c}-\frac{1}{2})\varphi_2 + (a^3_{b,c}+\frac{1}{2})\varphi_3]} e^{ikX}$$

$$V^{(-\frac{1}{2})}_{\psi_1=\chi_1^{bc}} = C_{\psi_1} e^{-\phi_{10}} \psi_1^\alpha S_\alpha e^{-\frac{\varphi}{2}} \tau_{a^1_{b,c}} \sigma_{a^2_{b,c}} \sigma_{1+a^3_{b,c}} e^{i[(a^1_{b,c}-\frac{1}{2})\varphi_1 + (a^2_{b,c}-\frac{1}{2})\varphi_2 + (a^3_{b,c}+\frac{1}{2})\varphi_3]} e^{ikX}$$
$$+ C_{\tilde{\psi}_1} e^{-\phi_{10}} \tilde{\psi}^\dagger_{1\dot{\alpha}} C^{\dot{\alpha}} e^{-\frac{\varphi}{2}} \sigma_{a^1_{b,c}} \sigma_{a^2_{b,c}} \sigma_{1+a^3_{b,c}} e^{i[(a^1_{b,c}+\frac{1}{2})\varphi_1 + (a^2_{b,c}-\frac{1}{2})\varphi_2 + (a^3_{b,c}+\frac{1}{2})\varphi_3]} e^{ikX}$$

$$V^{(-\frac{1}{2})}_{\chi_0=\chi_0^{ca}} = C_{\chi_0} e^{-\phi_{10}} \chi_0^\alpha S_\alpha e^{-\frac{\varphi}{2}} \sigma_{1+a^1_{c,a}} \sigma_{1+a^2_{c,a}} \sigma_{1+a^3_{c,a}} e^{i[(a^1_{c,a}+\frac{1}{2})\varphi_1 + (a^2_{c,a}+\frac{1}{2})\varphi_2 + (a^3_{c,a}+\frac{1}{2})\varphi_3]} e^{ikX}$$

$$V^{(-\frac{1}{2})}_{\chi_1=\chi_1^{ca}} = C_{\chi_1} e^{-\phi_{10}} \chi_1^\alpha S_\alpha e^{-\frac{\varphi}{2}} \sigma_{1+a^1_{c,a}} \sigma_{1+a^2_{c,a}} \tau_{1+a^3_{c,a}} e^{i[(a^1_{c,a}+\frac{1}{2})\varphi_1 + (a^2_{c,a}+\frac{1}{2})\varphi_2 + (a^3_{c,a}+\frac{1}{2})\varphi_3]} e^{ikX}$$
$$+ C_{\tilde{\chi}_1} e^{-\phi_{10}} \tilde{\chi}^\dagger_{1\dot{\alpha}} C^{\dot{\alpha}} e^{-\frac{\varphi}{2}} \sigma_{1+a^1_{c,a}} \sigma_{1+a^2_{c,a}} \sigma_{1+a^3_{c,a}} e^{i[(a^1_{c,a}+\frac{1}{2})\varphi_1 + (a^2_{c,a}+\frac{1}{2})\varphi_2 + (a^3_{c,a}-\frac{1}{2})\varphi_3]} e^{ikX}$$

- The masses of the fields are:

$$m^2_{\phi_0} = 0 \quad , \quad m^2_{\psi_0} = 0 \quad , \quad m^2_{\chi_0} = 0$$
$$, \quad m^2_{\psi_1} = a^1_{bc}/\alpha' \quad , \quad m^2_{\chi_1} = (1 - |a^3_{ca}|)/\alpha' \quad .$$

The $A(\bar{\psi}_0\psi_0\bar{\psi}_0\psi_0)$ amplitude

- By the $A(\bar{\psi}_0\psi_0\bar{\psi}_0\psi_0)$ we fix the normalisation of the ψ fields.

$$\mathcal{A}(\bar{\psi}_0,\psi_0,\bar{\psi}_0,\psi_0) = |C_{\psi_0}|^4\sqrt{C_{D_b^2}C_{D_c^2}}\,g_{op}^2\psi_0(2)\cdot\psi_0(4)\bar{\psi}_0(1)\cdot\bar{\psi}_0(3)\int_0^1 dx\,x^{\alpha' s-1}(1-x)^{\alpha' t-1}$$

$$\times\prod_{I=1}^{3}\frac{4\pi^2\alpha' K_I^{c,b}}{L_{b,I}L_{c,I}F_{\alpha_I}(x)}\sum_{n_I,m_I}\exp\left[-\frac{\pi t_I(x)}{\sin\pi|a_{bc}^I|}\left(\frac{4\pi^2\alpha'}{L_{c,I}^2}m_I^2+\frac{\sin^2\pi|a_{bc}^I|}{4\pi^2\alpha'}n_I^2L_{b,I}^2\right)\right]$$

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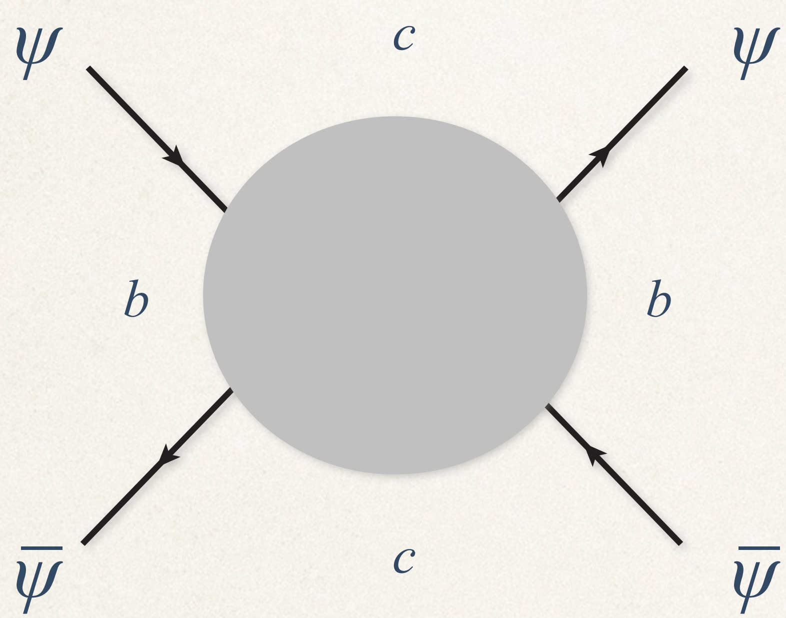
$$\times\prod_{I=1}^{3}\frac{4\pi^2\alpha' K_I^{c,b}}{L_{b,I}L_{c,I}F_{\alpha_I}(x)}\sum_{n_I,m_I}\exp\left[-\frac{\pi t_I(x)}{\sin\pi|a_{bc}^I|}\left(\frac{4\pi^2\alpha'}{L_{c,I}^2}m_I^2+\frac{\sin^2\pi|a_{bc}^I|}{4\pi^2\alpha'}n_I^2L_{b,I}^2\right)\right]$$



The $A(\bar{\psi}_0\psi_0\bar{\psi}_0\psi_0)$ amplitude

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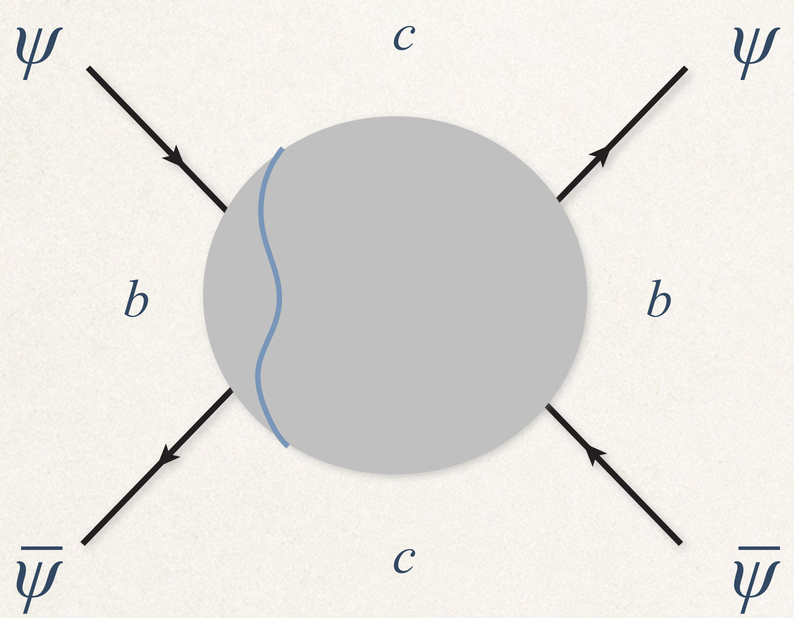
$$\times \prod_{I=1}^{3} \frac{4\pi^2\alpha' K_I^{c,b}}{L_{b,I} L_{c,I} F_{\alpha_I}(x)} \sum_{n_I,m_I} \exp\left[-\frac{\pi t_I(x)}{\sin\pi|a_{bc}^I|}\left(\frac{4\pi^2\alpha'}{L_{c,I}^2} m_I^2 + \frac{\sin^2\pi|a_{bc}^I|}{4\pi^2\alpha'} n_I^2 L_{b,I}^2\right)\right]$$



The $A(\bar{\psi}_0\psi_0\bar{\psi}_0\psi_0)$ amplitude

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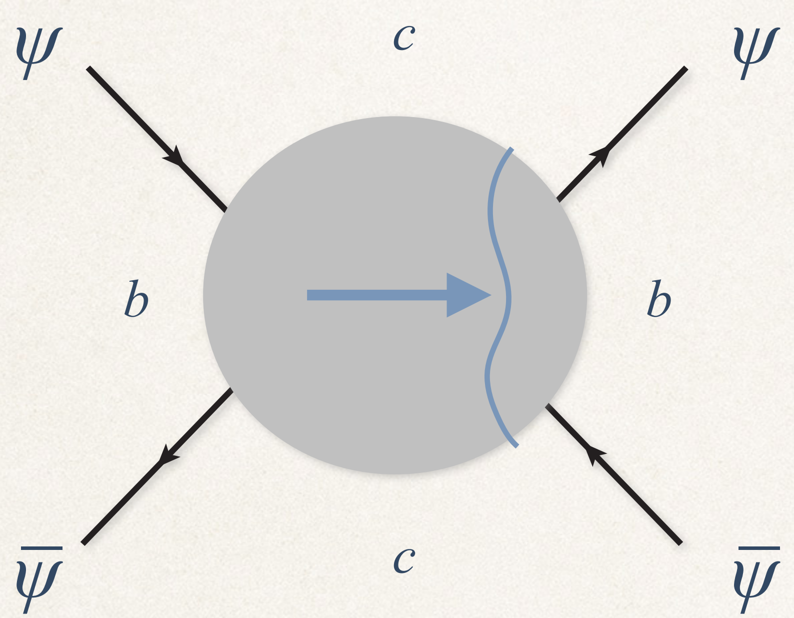
$$\times\prod_{I=1}^{3}\frac{4\pi^2\alpha' K_I^{c,b}}{L_{b,I}L_{c,I}F_{\alpha_I}(x)}\sum_{n_I,m_I}\exp\left[-\frac{\pi t_I(x)}{\sin\pi|a_{bc}^I|}\left(\frac{4\pi^2\alpha'}{L_{c,I}^2}m_I^2+\frac{\sin^2\pi|a_{bc}^I|}{4\pi^2\alpha'}n_I^2L_{b,I}^2\right)\right]$$



The $A(\bar{\psi}_0\psi_0\bar{\psi}_0\psi_0)$ amplitude

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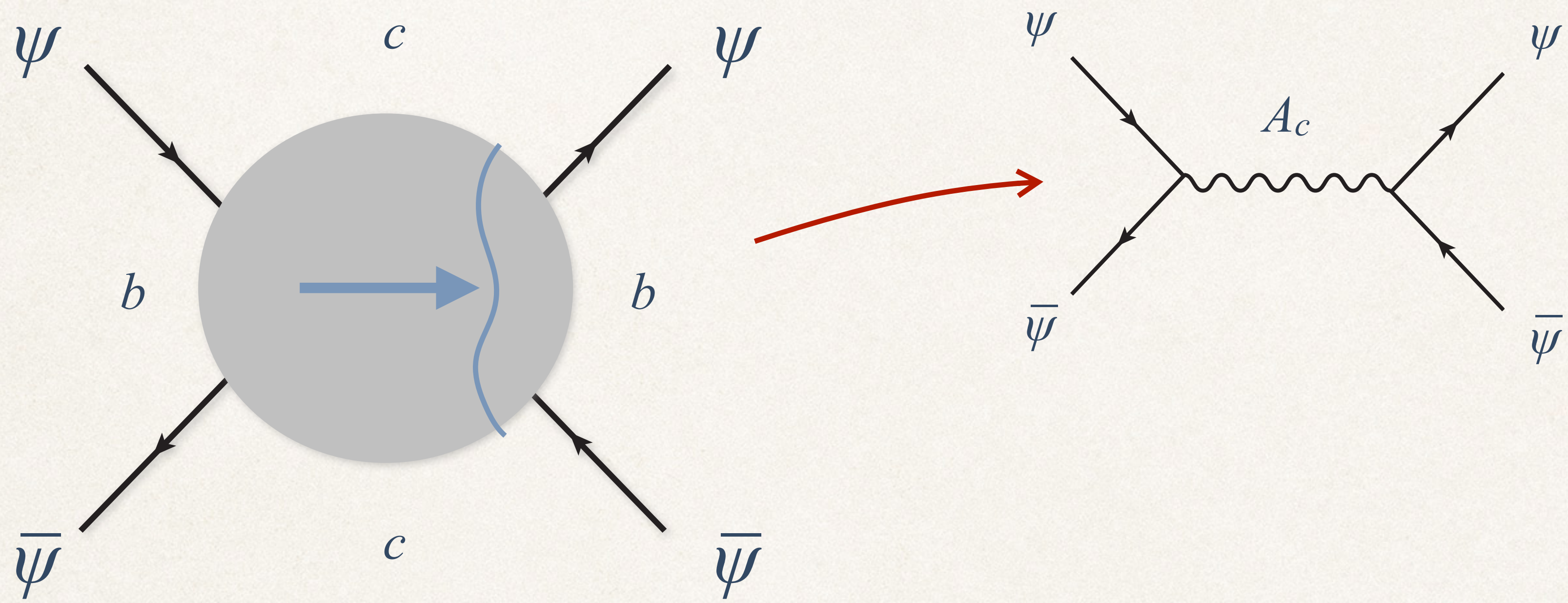
$$\mathcal{A}(\bar{\psi}_0,\psi_0,\bar{\psi}_0,\psi_0) = |C_{\psi_0}|^4\sqrt{C_{D_b^2}C_{D_c^2}}\,g_{op}^2\,\psi_0(2)\cdot\psi_0(4)\bar{\psi}_0(1)\cdot\bar{\psi}_0(3)\int_0^1 dx\, x^{\alpha' s-1}(1-x)^{\alpha' t-1}$$

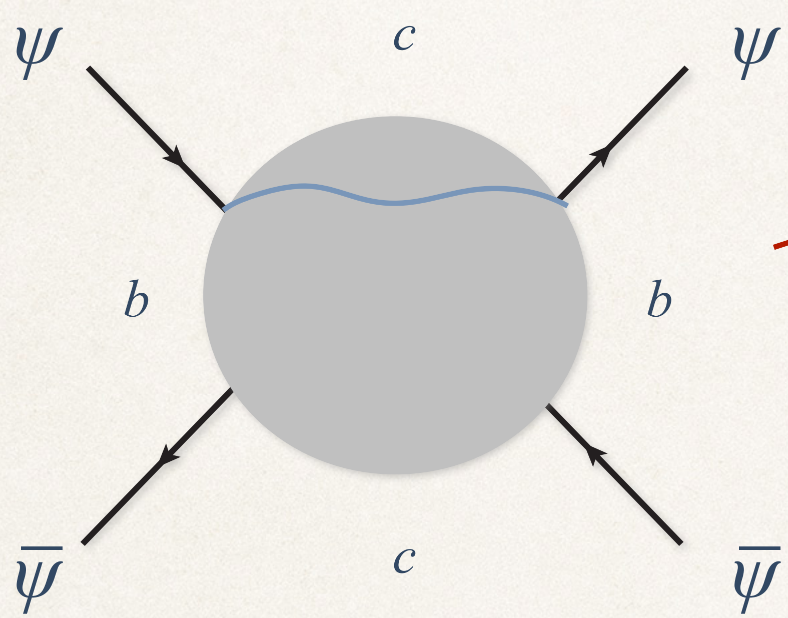
$$\times\prod_{I=1}^{3}\frac{4\pi^2\alpha' K_I^{c,b}}{L_{b,I}L_{c,I}F_{\alpha_I}(x)}\sum_{n_I,m_I}\exp\left[-\frac{\pi t_I(x)}{\sin\pi|a_{bc}^I|}\left(\frac{4\pi^2\alpha'}{L_{c,I}^2}m_I^2+\frac{\sin^2\pi|a_{bc}^I|}{4\pi^2\alpha'}n_I^2L_{b,I}^2\right)\right]$$

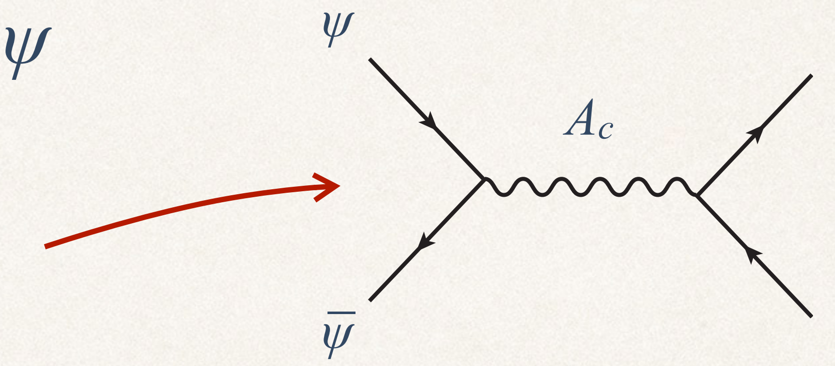


The $A(\bar{\psi}_0 \psi_0 \bar{\psi}_0 \psi_0)$ amplitude

- By the $A(\bar{\psi}_0 \psi_0 \bar{\psi}_0 \psi_0)$ we fix the normalisation of the ψ fields.

$$\mathcal{A}(\bar{\psi}_0, \psi_0, \bar{\psi}_0, \psi_0) = |C_{\psi_0}|^4 \sqrt{C_{D_b^2} C_{D_c^2}} g_{op}^2 \psi_0(2) \cdot \psi_0(4) \bar{\psi}_0(1) \cdot \bar{\psi}_0(3) \int_0^1 dx\, x^{\alpha' s - 1} (1-x)^{\alpha' t - 1}$$
$$\times \prod_{I=1}^{3} \frac{4\pi^2 \alpha' K_I^{c,b}}{L_{b,I} L_{c,I} F_{\alpha_I}(x)} \sum_{n_I, m_I} \exp\left[-\frac{\pi t_I(x)}{\sin \pi |a_{bc}^I|} \left(\frac{4\pi^2 \alpha'}{L_{c,I}^2} m_I^2 + \frac{\sin^2 \pi |a_{bc}^I|}{4\pi^2 \alpha'} n_I^2 L_{b,I}^2 \right) \right]$$





The $A(\bar{\psi}_0\psi_0\bar{\psi}_0\psi_0)$ amplitude

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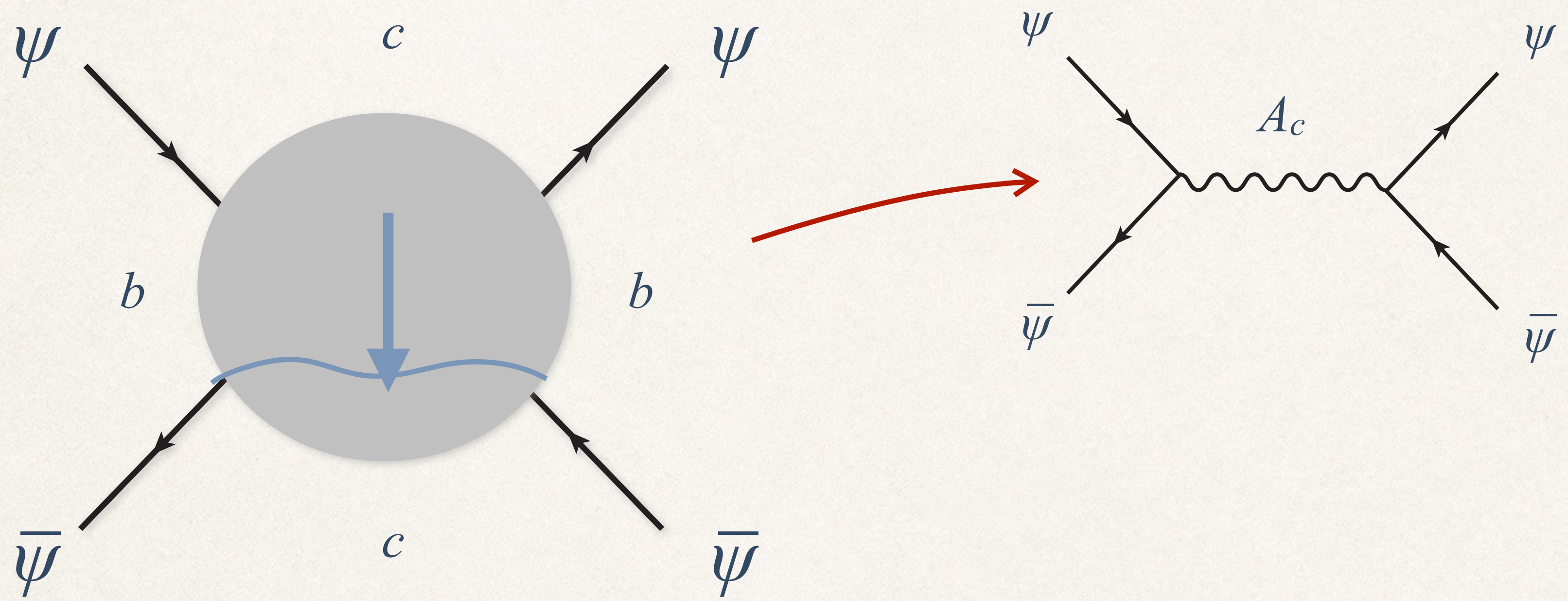
$$\mathcal{A}(\bar{\psi}_0,\psi_0,\bar{\psi}_0,\psi_0) = |C_{\psi_0}|^4\sqrt{C_{D_b^2}C_{D_c^2}}\,g_{op}^2\,\psi_0(2)\cdot\psi_0(4)\bar{\psi}_0(1)\cdot\bar{\psi}_0(3)\int_0^1 dx\, x^{\alpha' s-1}(1-x)^{\alpha' t-1}$$
$$\times\prod_{I=1}^{3}\frac{4\pi^2\alpha' K_I^{c,b}}{L_{b,I}L_{c,I}F_{\alpha_I}(x)}\sum_{n_I,m_I}\exp\left[-\frac{\pi t_I(x)}{\sin\pi|a_{bc}^I|}\left(\frac{4\pi^2\alpha'}{L_{c,I}^2}m_I^2+\frac{\sin^2\pi|a_{bc}^I|}{4\pi^2\alpha'}n_I^2L_{b,I}^2\right)\right]$$



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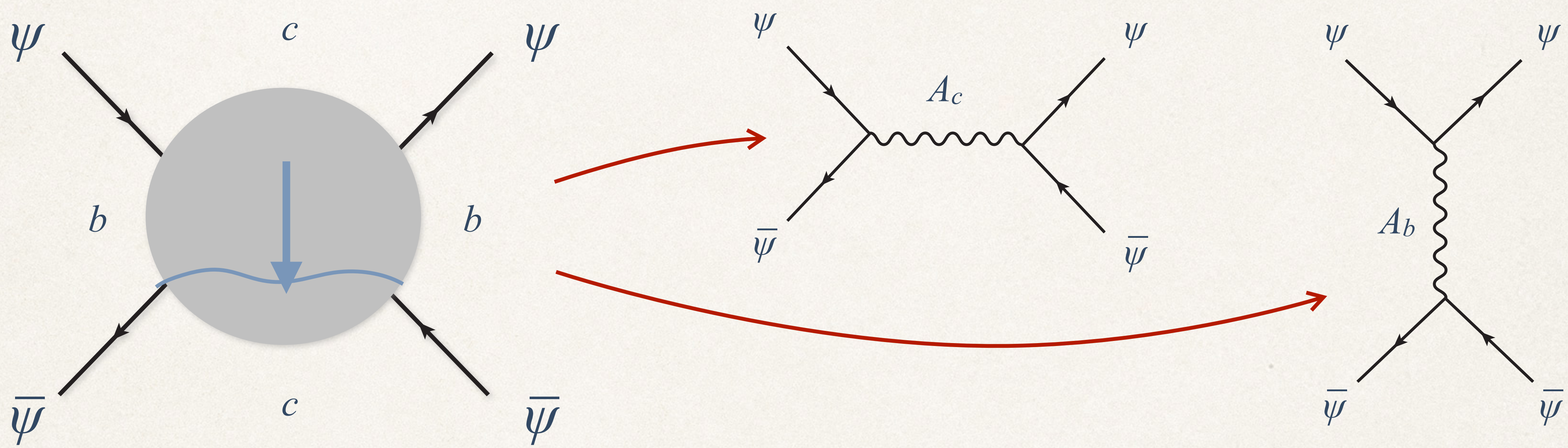
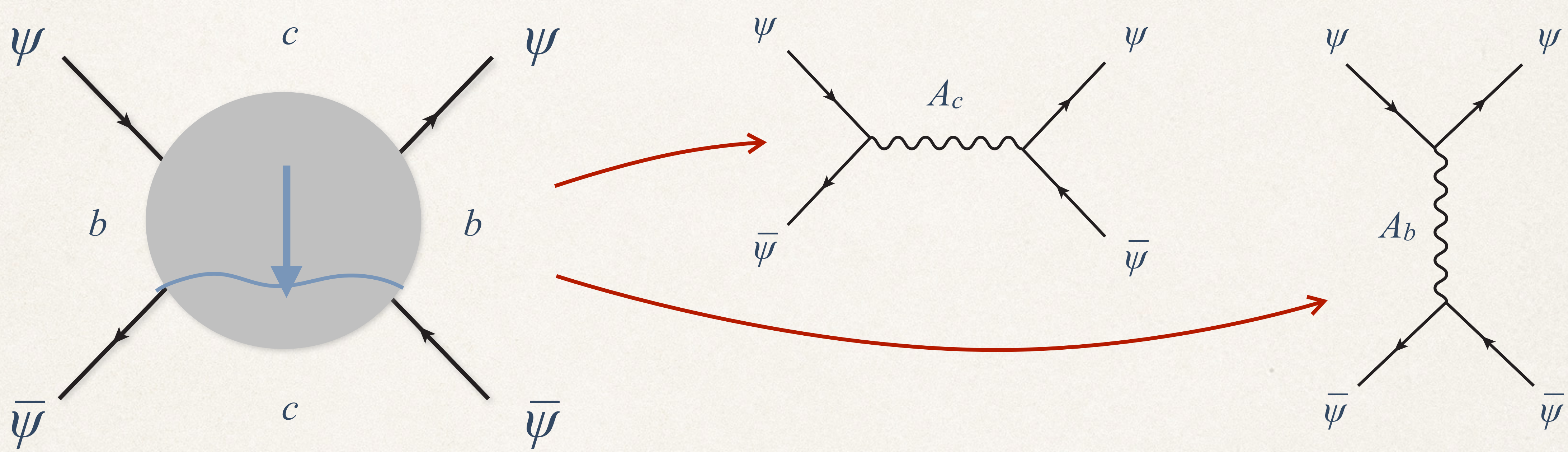
$$\mathcal{A}(\bar{\psi}_0,\psi_0,\bar{\psi}_0,\psi_0) = |C_{\psi_0}|^4 \sqrt{C_{D_b^2} C_{D_c^2}}\, g_{op}^2\, \psi_0(2)\cdot\psi_0(4)\bar{\psi}_0(1)\cdot\bar{\psi}_0(3) \int_0^1 dx\, x^{\alpha' s-1}(1-x)^{\alpha' t-1}$$
$$\times \prod_{I=1}^{3} \frac{4\pi^2\alpha' K_I^{c,b}}{L_{b,I}L_{c,I}F_{\alpha_I}(x)} \sum_{n_I,m_I} \exp\left[-\frac{\pi t_I(x)}{\sin\pi|a_{bc}^I|}\left(\frac{4\pi^2\alpha'}{L_{c,I}^2}m_I^2 + \frac{\sin^2\pi|a_{bc}^I|}{4\pi^2\alpha'}n_I^2 L_{b,I}^2\right)\right]$$



The $A(\bar{\psi}_0\psi_0\bar{\psi}_0\psi_0)$ amplitude

- By the $A(\bar{\psi}_0\psi_0\bar{\psi}_0\psi_0)$ we fix the normalisation of the ψ fields.

$$\mathcal{A}(\bar{\psi}_0,\psi_0,\bar{\psi}_0,\psi_0) = |C_{\psi_0}|^4\sqrt{C_{D_b^2}C_{D_c^2}}\,g_{op}^2\,\psi_0(2)\cdot\psi_0(4)\bar{\psi}_0(1)\cdot\bar{\psi}_0(3)\int_0^1 dx\, x^{\alpha' s-1}(1-x)^{\alpha' t-1}$$
$$\times\prod_{I=1}^3 \frac{4\pi^2\alpha' K_I^{c,b}}{L_{b,I}L_{c,I}F_{\alpha_I}(x)}\sum_{n_I,m_I}\exp\left[-\frac{\pi t_I(x)}{\sin\pi|a_{bc}^I|}\left(\frac{4\pi^2\alpha'}{L_{c,I}^2}m_I^2+\frac{\sin^2\pi|a_{bc}^I|}{4\pi^2\alpha'}n_I^2L_{b,I}^2\right)\right]$$



- The ratio of $g_{YM,c}\,/\,g_{YM,b}$ depends only on the length of the branes and therefore

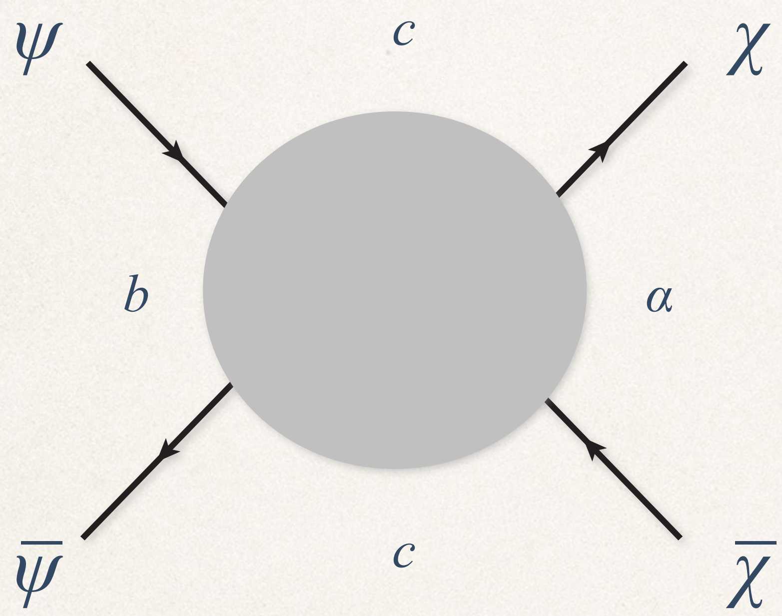
$$g_{\mathrm{YM},a} = g_{\mathrm{op}}\prod_I\sqrt{\frac{\sqrt{\alpha'}}{L_{a,I}}}\qquad C_{A_a} = \sqrt{2\alpha'}\prod_I\sqrt{\frac{\sqrt{\alpha'}}{L_{a,I}}}\qquad K_I^{c,b} = \frac{\sqrt{L_{b,I}L_{c,I}}\,L_{b,I}}{4\pi^2\alpha'}$$

$$C_{\chi_0^{bc}} = e^{i\gamma_0^{bc}}(\alpha')^{1/4}\sqrt{2\alpha'}\prod_I\left[\frac{\alpha'}{L_{b,I}L_{c,I}}\right]^{1/4}\qquad C_{\phi_0^{bc}} = e^{i\gamma_0^{bc}}\sqrt{2\alpha'}\prod_I\left[\frac{\alpha'}{L_{b,I}L_{c,I}}\right]^{1/4}$$

The $A(\bar\psi_0\psi_0\bar\chi_0\chi_0)$ amplitude

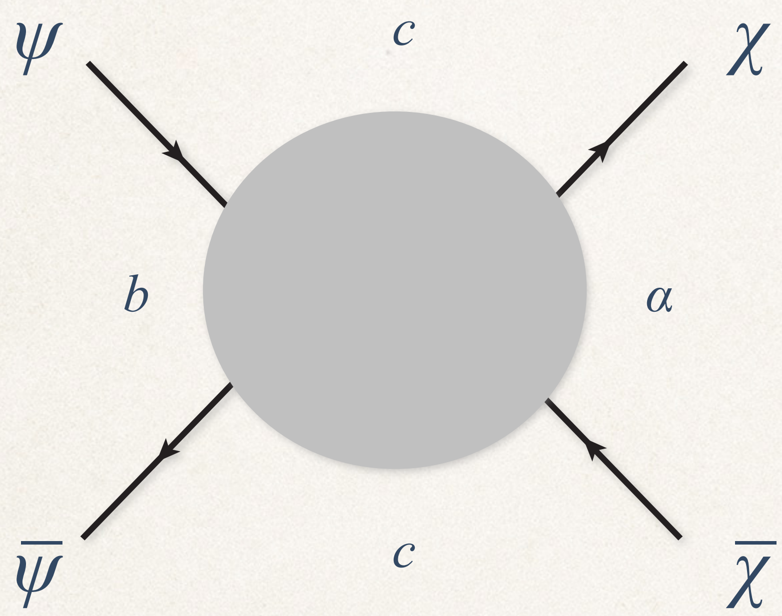
$$\mathcal{A}(\bar\psi_0^{cb},\psi_0^{bc},\chi_0^{ca},\bar\chi_0^{ac}) = g_{\rm op}^2\alpha'\psi_0(2)\cdot\chi_0(3)\bar\psi_0(1)\cdot\bar\chi_0(4)\int_0^1 dx\, x^{\alpha' s-1}(1-x)^{\alpha' t-1}\times$$

$$\times\prod_{I=1}^{3}\frac{4\pi^2 K_I^{c,ab}\alpha'}{L_{a,I}^{1/4}L_{b,I}^{5/4}L_{c,I}^{1/2}}\frac{\sqrt{\alpha'}}{L_{c,I}G_1^{(I)}(x)}\sum_{n_I,m_I} e^{-S_{\rm Ham}^{(I)}(m_I,n_I)}$$



The $A(\bar{\psi}_0\psi_0\bar{\chi}_0\chi_0)$ amplitude

$$\mathcal{A}(\bar{\psi}_0^{cb},\psi_0^{bc},\chi_0^{ca},\bar{\chi}_0^{ac}) = g_{\rm op}^2\alpha'\psi_0(2)\cdot\chi_0(3)\bar{\psi}_0(1)\cdot\bar{\chi}_0(4)\int_0^1 dx\, x^{\alpha' s-1}(1-x)^{\alpha' t-1}\times$$
$$\times\prod_{I=1}^{3}\frac{4\pi^2K_I^{c,ab}\alpha'}{L_{a,I}^{1/4}L_{b,I}^{5/4}L_{c,I}^{1/2}}\frac{\sqrt{\alpha'}}{L_{c,I}G_1^{(I)}(x)}\sum_{n_I,m_I}e^{-S_{\rm Ham}^{(I)}(m_I,n_I)}$$

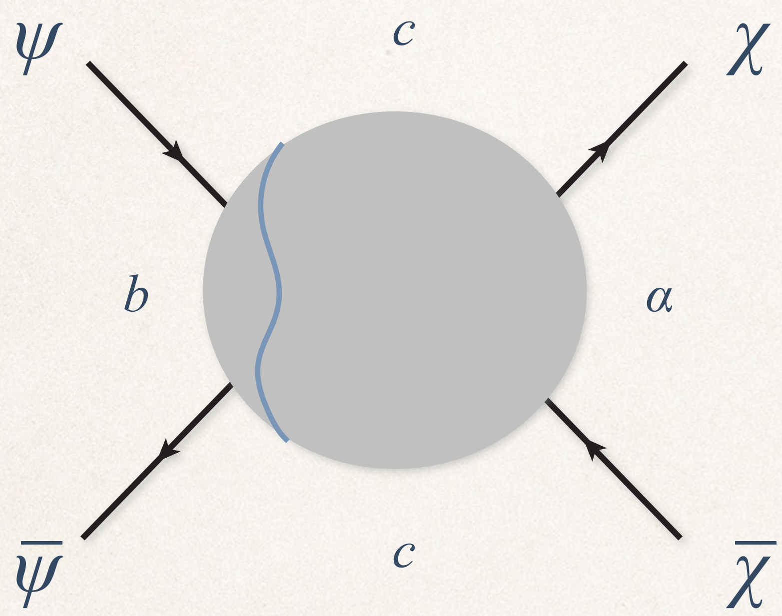


- The *s*-channel normalise the fields.

The $A(\bar{\psi}_0\psi_0\bar{\chi}_0\chi_0)$ amplitude

$$\mathcal{A}(\bar{\psi}_0^{cb}, \psi_0^{bc}, \chi_0^{ca}, \bar{\chi}_0^{ac}) = g_{\text{op}}^2 \alpha' \psi_0(2)\cdot\chi_0(3)\bar{\psi}_0(1)\cdot\bar{\chi}_0(4) \int_0^1 dx\, x^{\alpha' s - 1}(1-x)^{\alpha' t - 1} \times$$

$$\times \prod_{I=1}^{3} \frac{4\pi^2 K_I^{c,ab}\alpha'}{L_{a,I}^{1/4} L_{b,I}^{5/4} L_{c,I}^{1/2}} \frac{\sqrt{\alpha'}}{L_{c,I} G_1^{(I)}(x)} \sum_{n_I, m_I} e^{-S_{\text{Ham}}^{(I)}(m_I, n_I)}$$

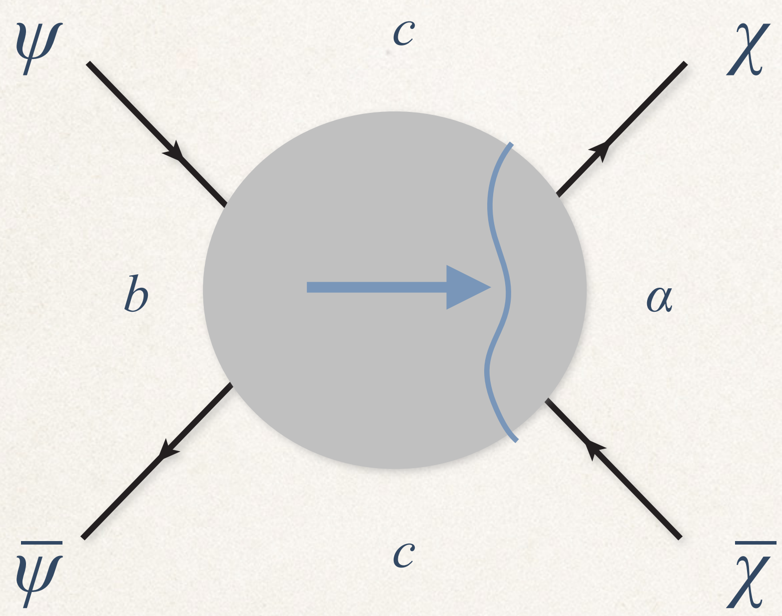


- The *s*-channel normalise the fields.

The $A(\bar{\psi}_0\psi_0\bar{\chi}_0\chi_0)$ amplitude

$$\mathcal{A}(\bar{\psi}_0^{cb},\psi_0^{bc},\chi_0^{ca},\bar{\chi}_0^{ac}) = g_{\rm op}^2\alpha'\psi_0(2)\cdot\chi_0(3)\bar{\psi}_0(1)\cdot\bar{\chi}_0(4)\int_0^1 dx\, x^{\alpha' s-1}(1-x)^{\alpha' t-1}\times$$

$$\times\prod_{I=1}^{3}\frac{4\pi^2 K_I^{c,ab}\alpha'}{L_{a,I}^{1/4}L_{b,I}^{5/4}L_{c,I}^{1/2}}\frac{\sqrt{\alpha'}}{L_{c,I}G_1^{(I)}(x)}\sum_{n_I,m_I}e^{-S_{\rm Ham}^{(I)}(m_I,n_I)}$$

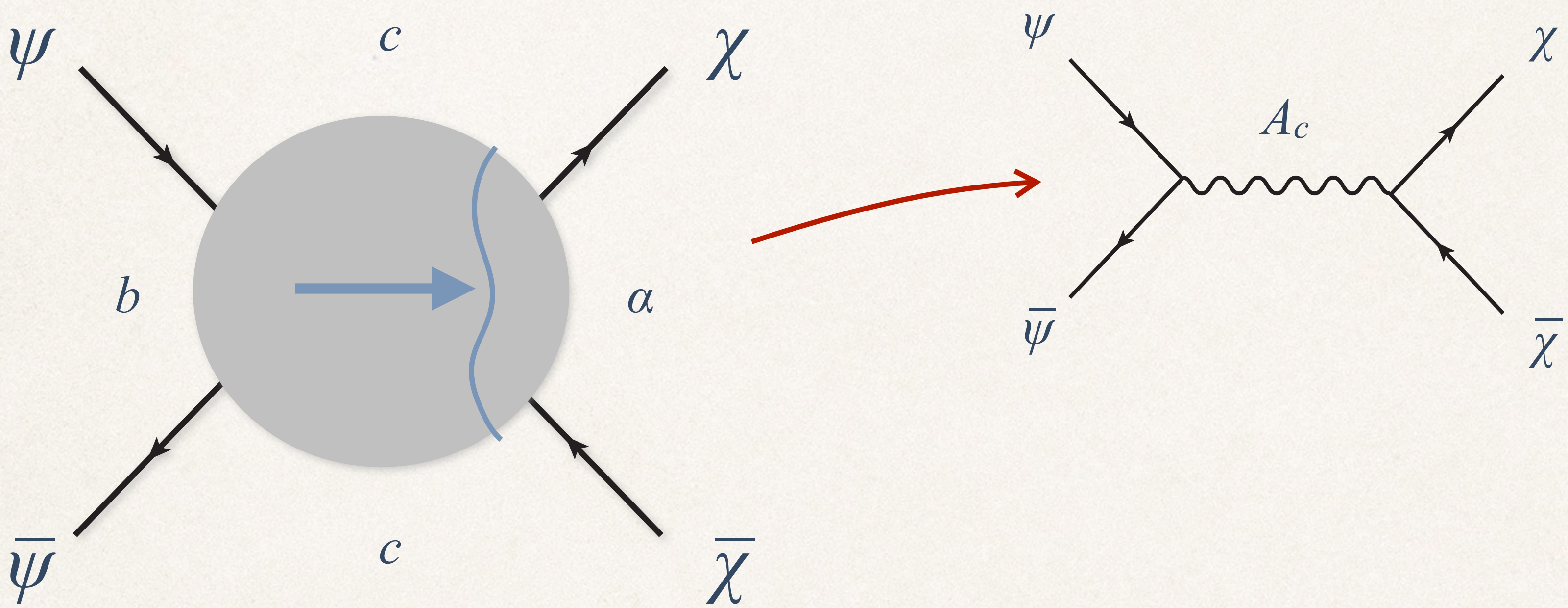


- The *s*-channel normalise the fields.

The $A(\bar{\psi}_0\psi_0\bar{\chi}_0\chi_0)$ amplitude

$$\mathcal{A}(\bar{\psi}_0^{cb},\psi_0^{bc},\chi_0^{ca},\bar{\chi}_0^{ac}) = g_{\rm op}^2\alpha'\psi_0(2)\cdot\chi_0(3)\bar{\psi}_0(1)\cdot\bar{\chi}_0(4)\int_0^1 dx\, x^{\alpha' s-1}(1-x)^{\alpha' t-1}\times$$

$$\times\prod_{I=1}^{3}\frac{4\pi^2 K_I^{c,ab}\alpha'}{L_{a,I}^{1/4}L_{b,I}^{5/4}L_{c,I}^{1/2}}\frac{\sqrt{\alpha'}}{L_{c,I}G_1^{(I)}(x)}\sum_{n_I,m_I} e^{-S^{(I)}_{\rm Ham}(m_I,n_I)}$$

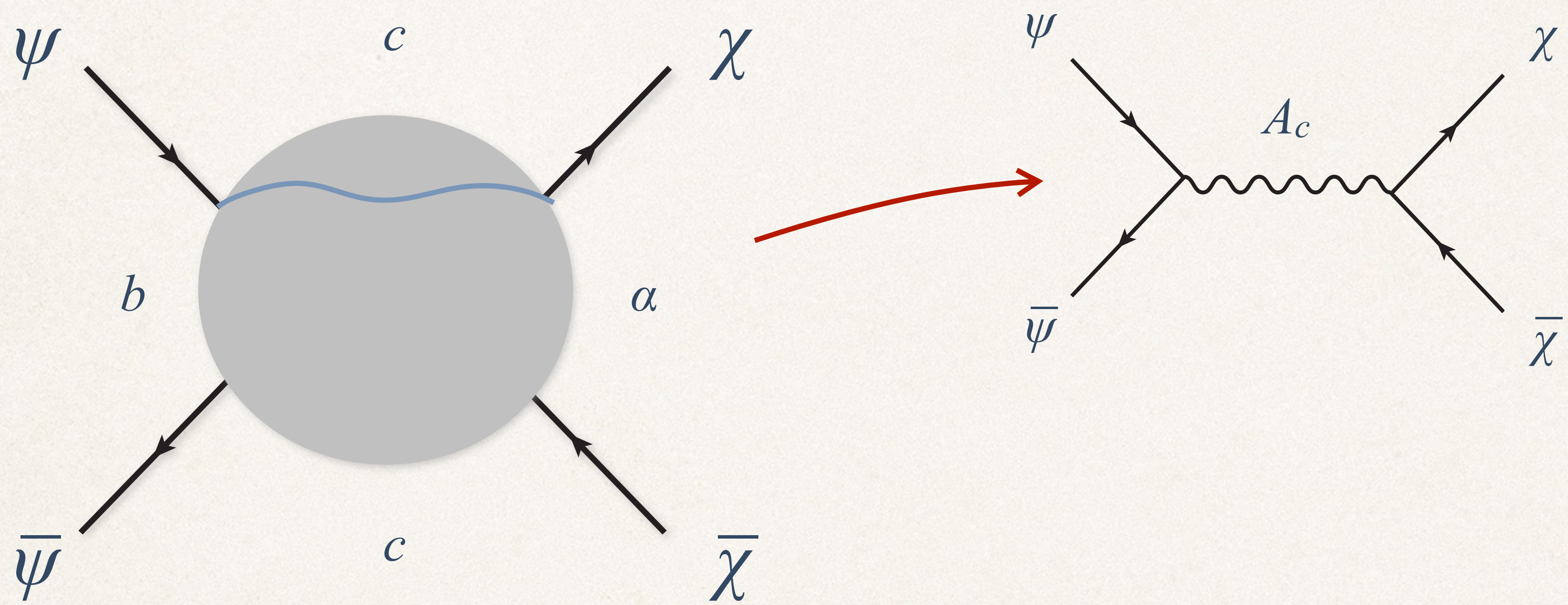


- The *s*-channel normalise the fields.

The $A(\bar{\psi}_0\psi_0\bar{\chi}_0\chi_0)$ amplitude

$$\mathcal{A}(\bar{\psi}_0^{cb}, \psi_0^{bc}, \chi_0^{ca}, \bar{\chi}_0^{ac}) = g_{\rm op}^2 \alpha' \psi_0(2)\cdot\chi_0(3)\bar{\psi}_0(1)\cdot\bar{\chi}_0(4) \int_0^1 dx\, x^{\alpha' s-1}(1-x)^{\alpha' t-1} \times$$

$$\times \prod_{I=1}^{3} \frac{4\pi^2 K_I^{c,ab}\alpha'}{L_{a,I}^{1/4} L_{b,I}^{5/4} L_{c,I}^{1/2}} \frac{\sqrt{\alpha'}}{L_{c,I} G_1^{(I)}(x)} \sum_{n_I,m_I} e^{-S_{\rm Ham}^{(I)}(m_I,n_I)}$$



- The *s*-channel normalise the fields.
- The *t*-channel provides the Yukawas.

The $A(\bar{\psi}_0\psi_0\bar{\chi}_0\chi_0)$ amplitude

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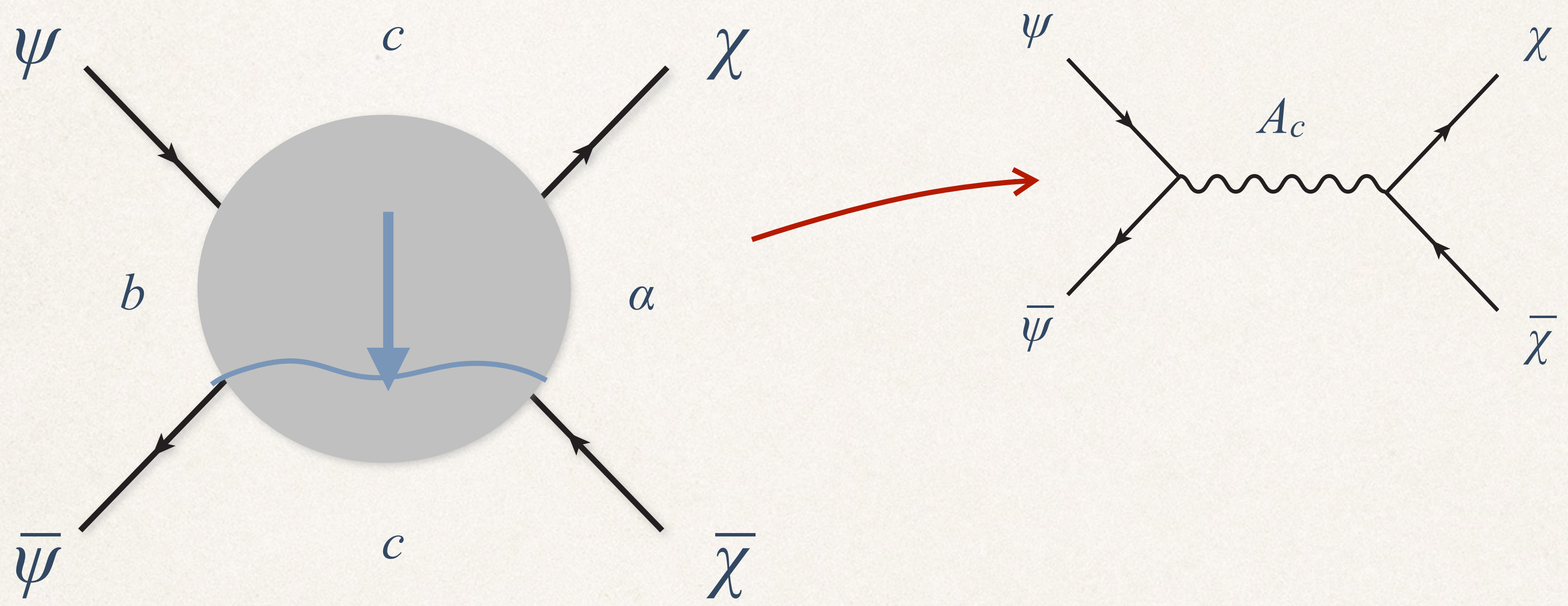

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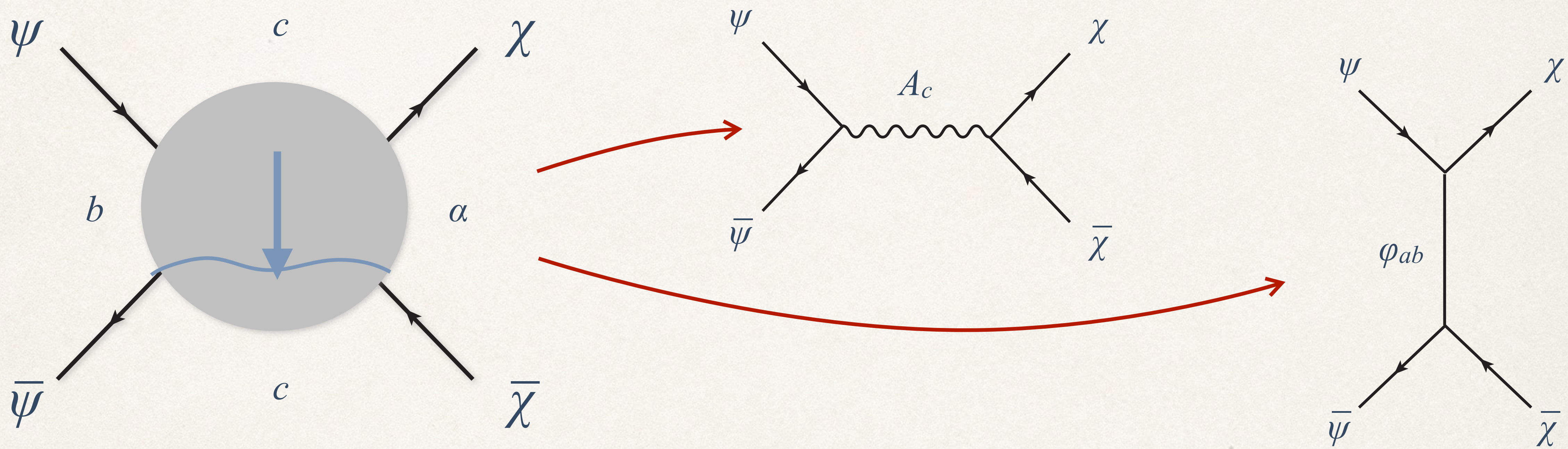

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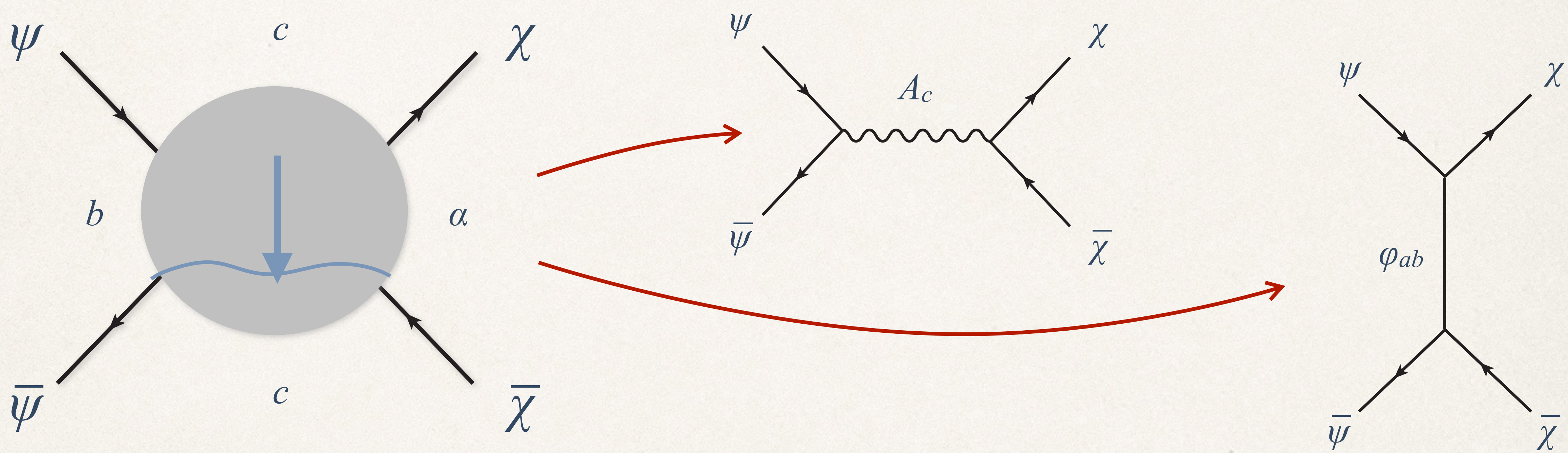

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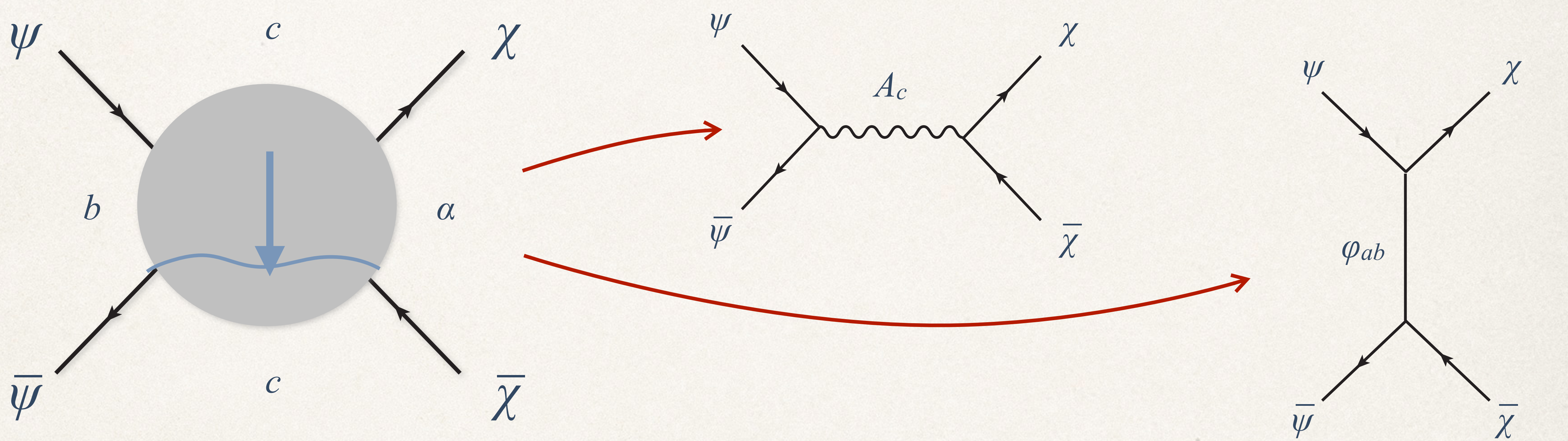

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$$\mathcal{A}(\bar\psi_0,\psi_0,\chi_0,\bar\chi_0)\xrightarrow{t\to n\,a_{ab}^1/\alpha'}|Y_{n00}|^2\psi_0(2)\cdot\chi_0(3)\frac{1}{t-n\,a_{ab}^1/\alpha'}\bar\chi_0(4)\cdot\bar\psi_0(1)$$

Yukawas between SM and l.s.s

- The Yukawas are (by direct computations and using some SUSY Ward ID's)

$$|Y_{000}| = g_{\rm op}(2\pi)^{-3/4}[\Gamma_{1-a^1_{ab},1-a^1_{bc},-a^1_{ca}}\Gamma_{1-a^2_{ab},1-a^2_{bc},-a^2_{ca}}\Gamma_{-a^3_{ab},-a^3_{bc},-a^3_{ca}}]^{1/4}\prod_{I=1}^{3}\exp\left[-\frac{A^{(I)}_{\phi\psi\chi}}{2\pi\alpha'}\right]$$

usual SM Yukawa's (three SM particles)

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$$|Y_{110}| = |Y_{000}|\left|\frac{2A^{(1)}_{\phi\psi\chi}}{\pi\alpha'}-1\right|\frac{1}{\sqrt{a^1_{ab}a^1_{bc}}}\Gamma_{1-a^1_{ab},1-a^1_{bc},-a^1_{ca}}$$

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Yukawa's between three l.s.s.

$$|Y_{111}| = \frac{|Y_{000}|}{\sqrt{a^1_{ab}a^1_{bc}(1+a^3_{ca})}}\Gamma_{1-a^1_{ab},1-a^1_{bc},-a^1_{ca}}\Gamma^{1/2}_{-a^3_{ab},-a^3_{bc},-a^3_{ca}}\left|\frac{2A^{(1)}_{\phi\psi\chi}}{\pi\alpha'}-1\right|\sqrt{\frac{2A^{(3)}_{\phi\psi\chi}}{\pi\alpha'}}$$

$$|Y_{211}| = \frac{|Y_{000}|}{\sqrt{2}a^1_{ab}\sqrt{a^1_{bc}(1+a^3_{ca})}}\Gamma^{3/2}_{1-a^1_{ab},1-a^1_{bc},-a^1_{ca}}\Gamma^{1/2}_{-a^3_{ab},-a^3_{bc},-a^3_{ca}}\left|\frac{2A^{(1)}_{\phi\psi\chi}}{\pi\alpha'}-3\right|\sqrt{\frac{2A^{(1)}_{\phi\psi\chi}}{\pi\alpha'}\frac{2A^{(3)}_{\phi\psi\chi}}{\pi\alpha'}}$$

etc etc…

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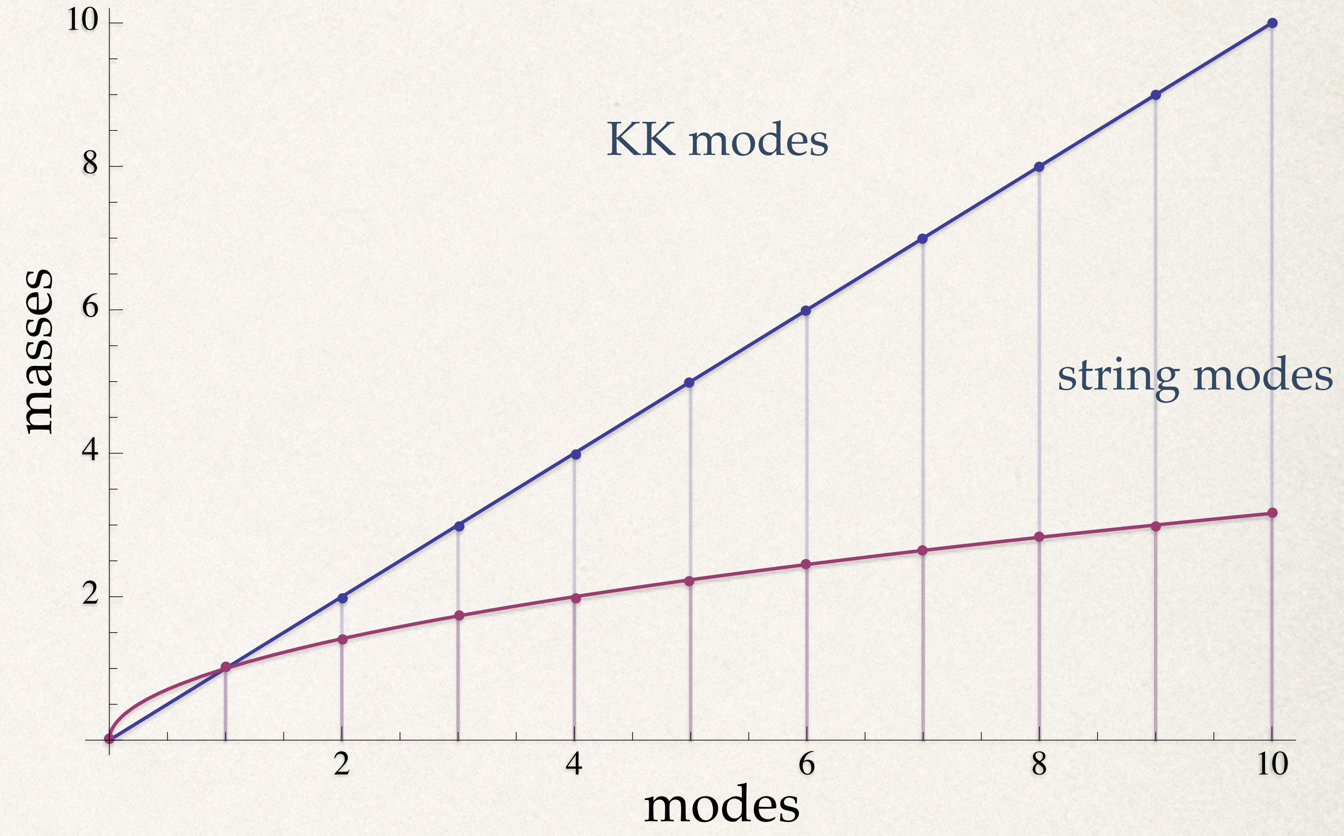

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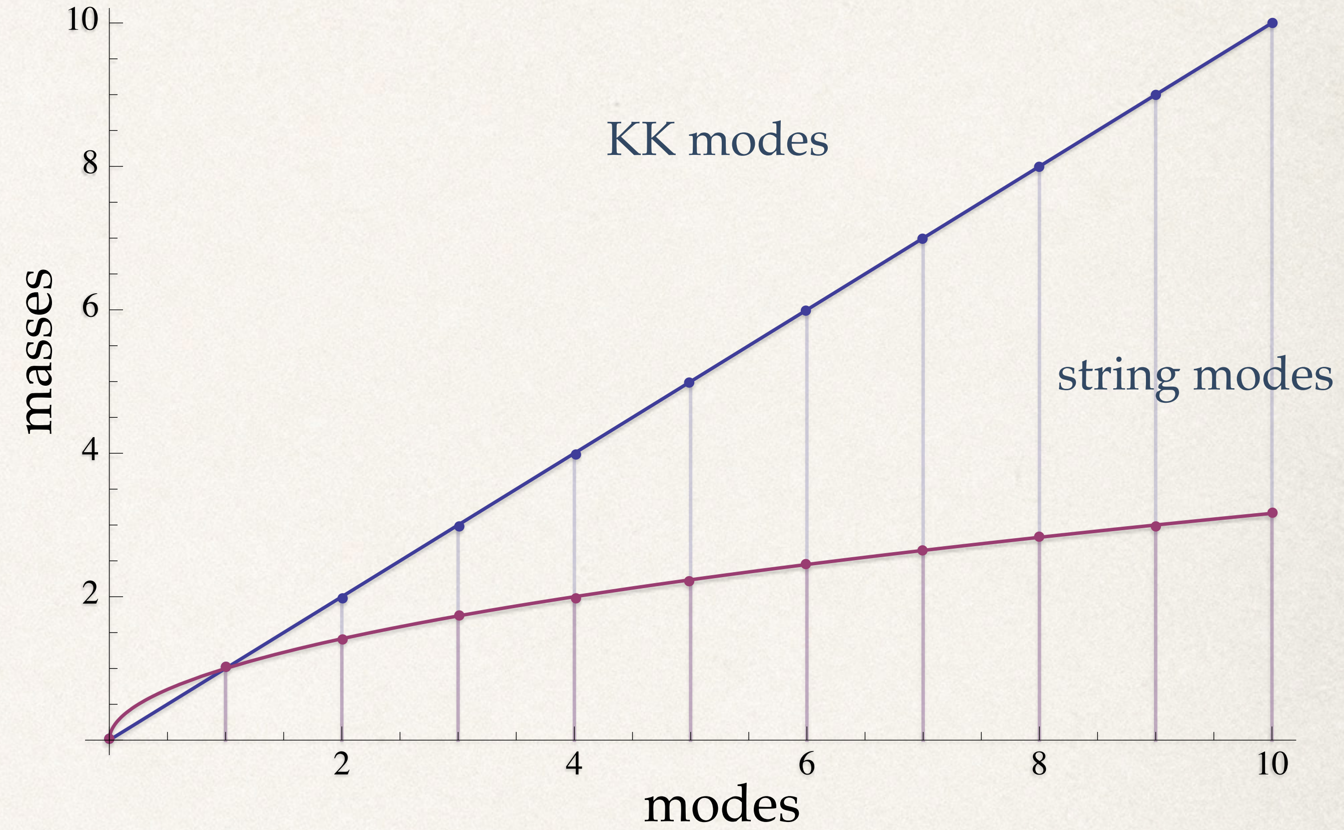

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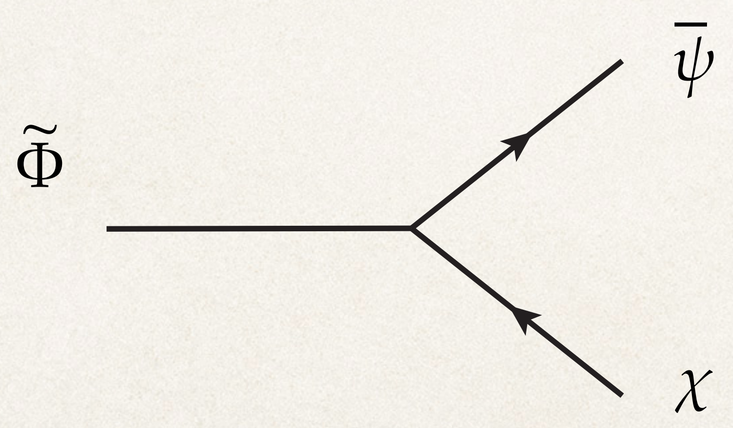

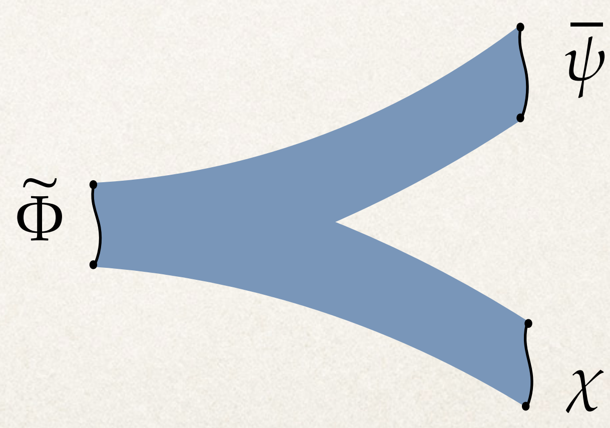

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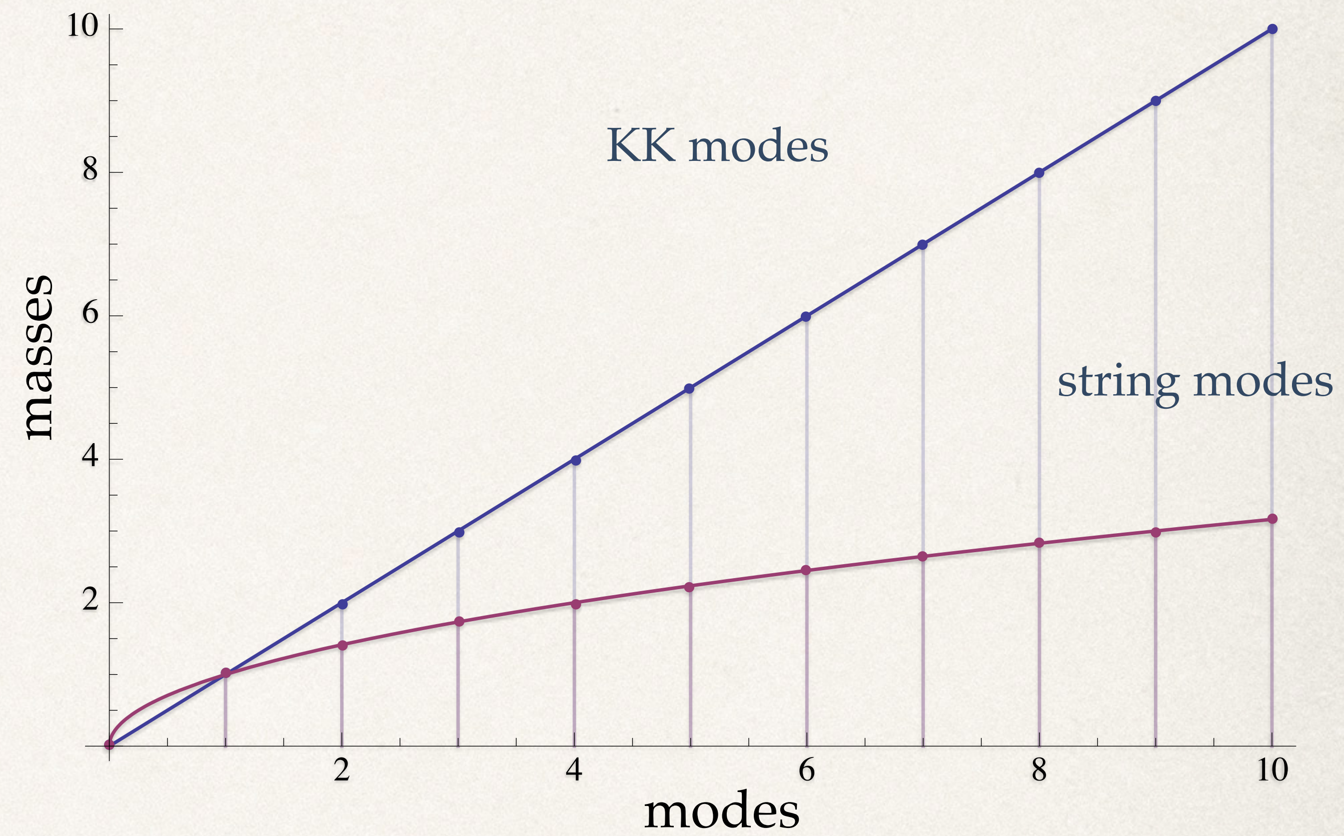

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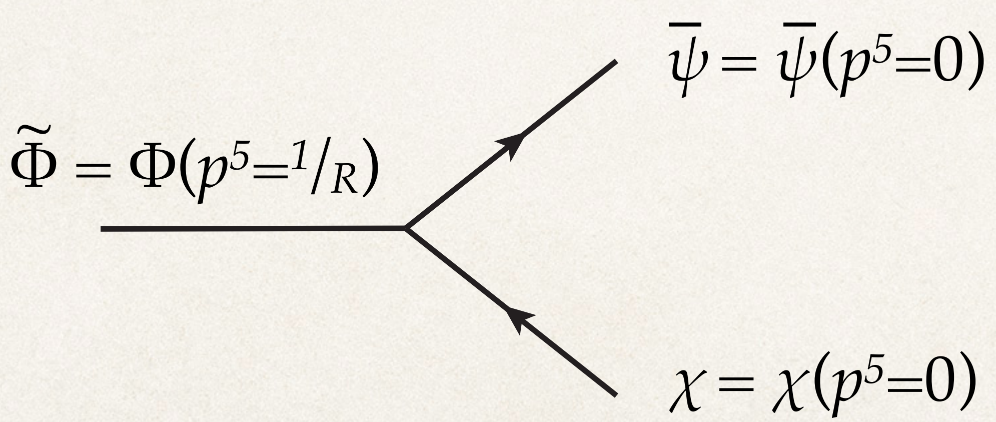

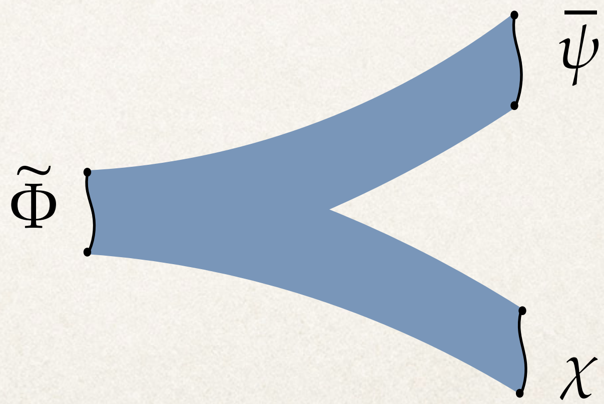

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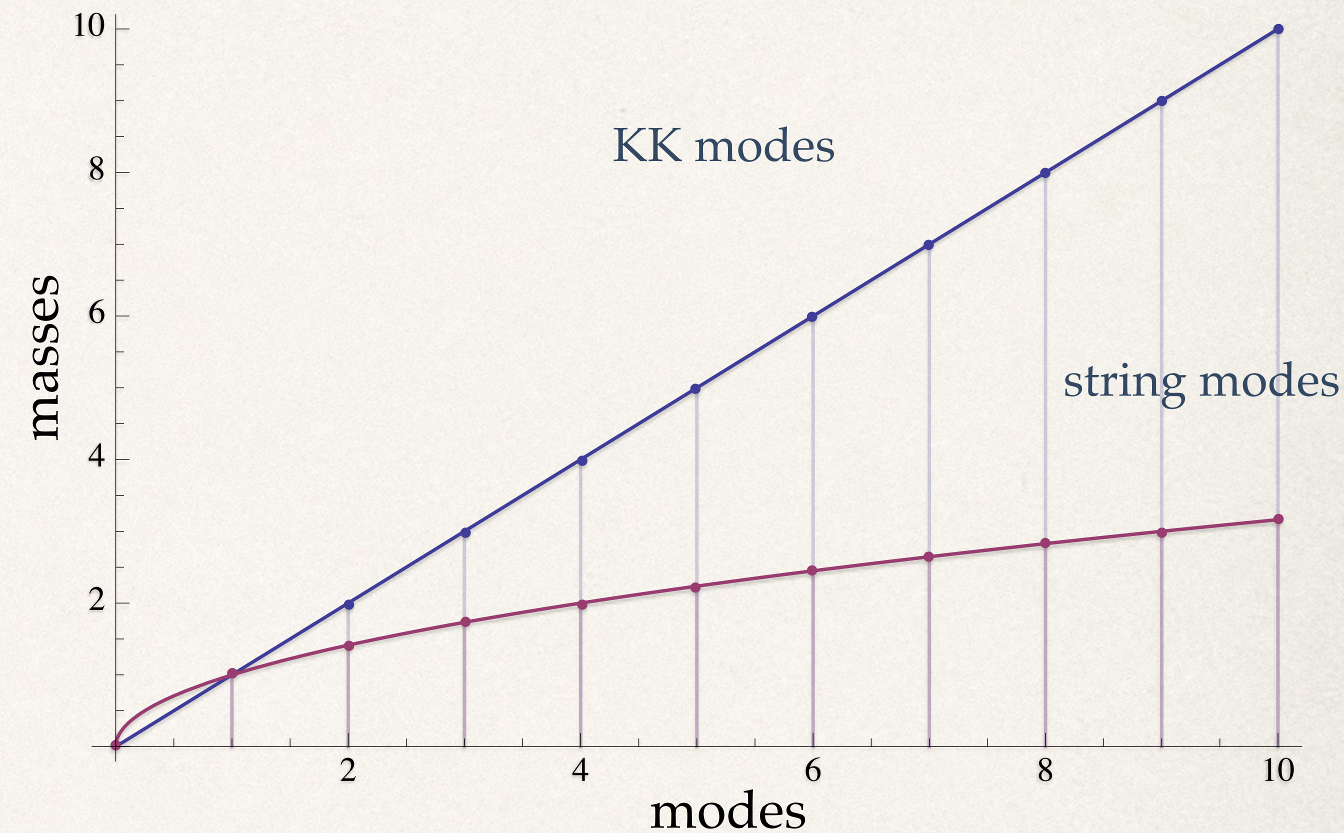

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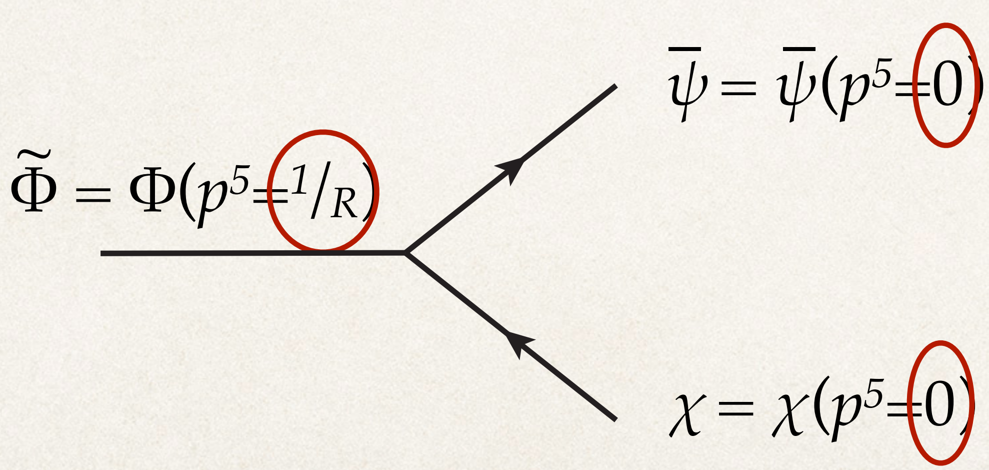

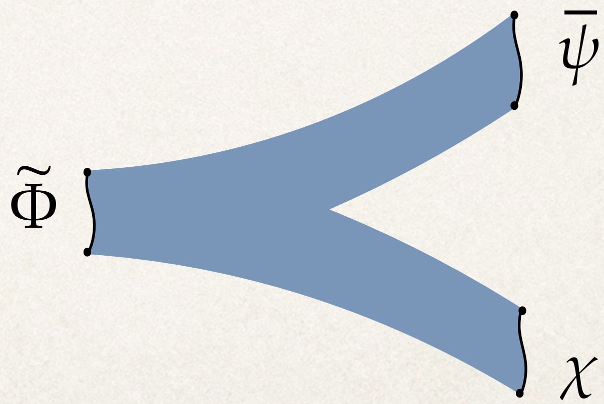

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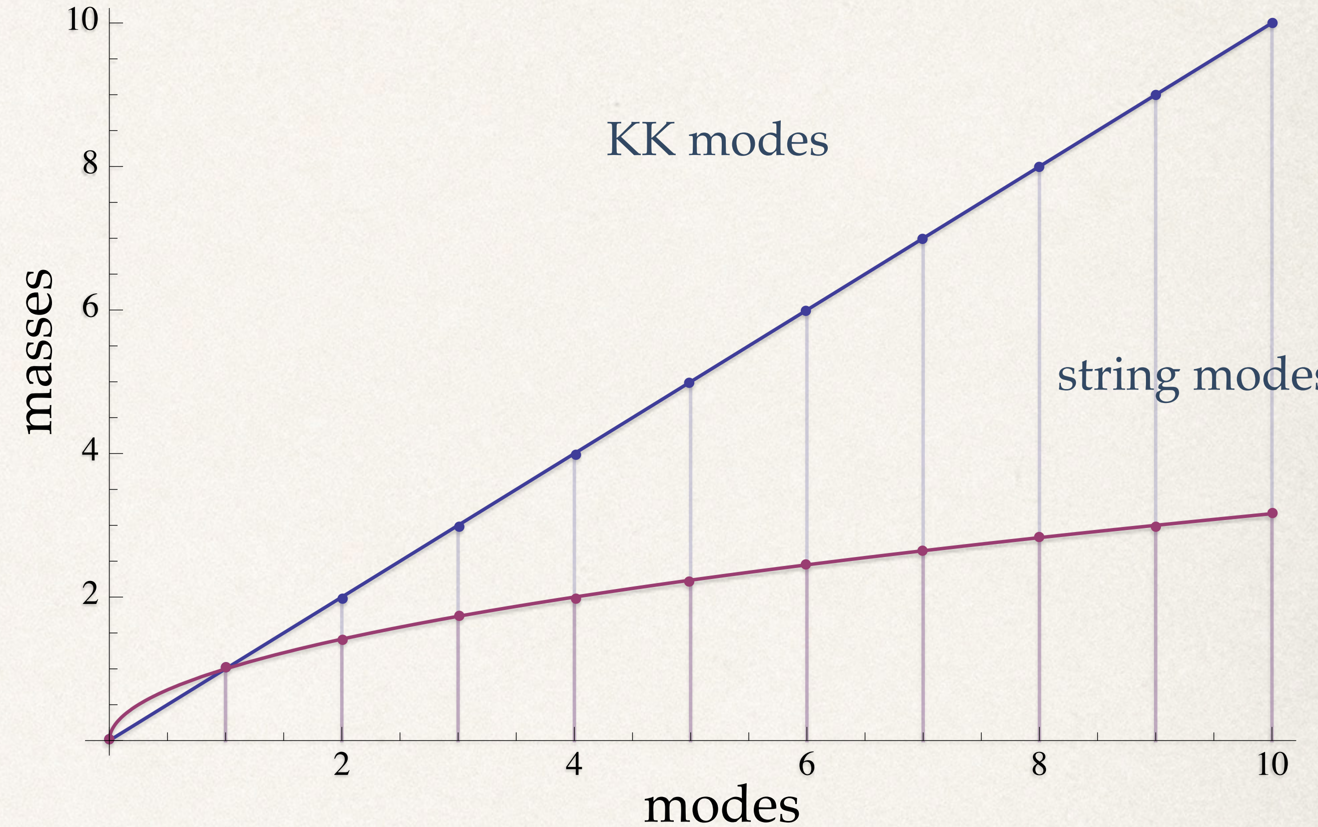

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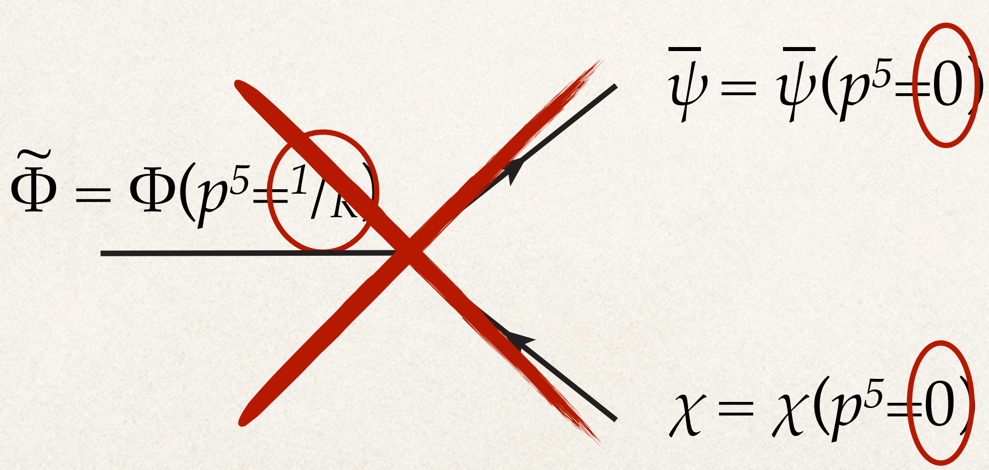

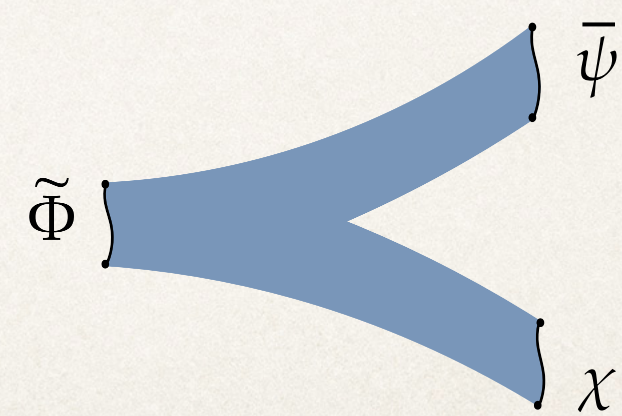

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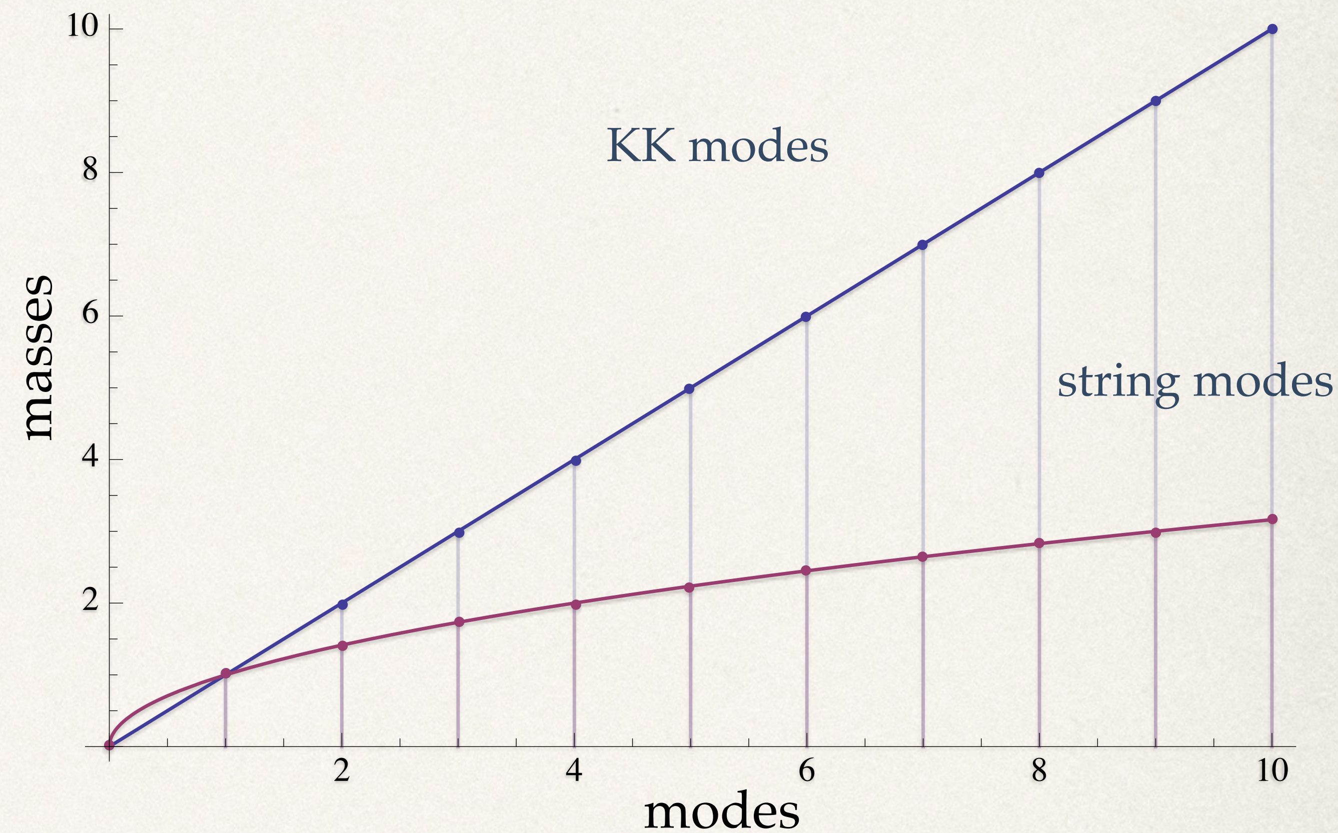

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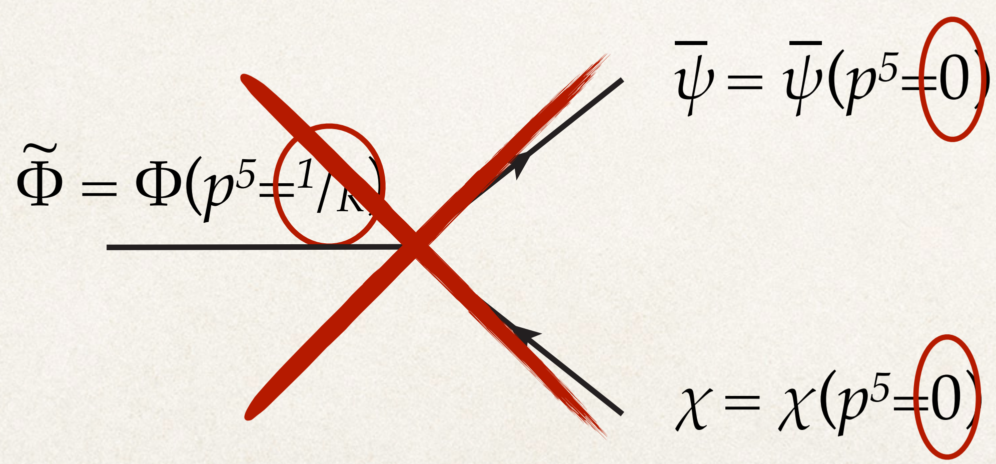

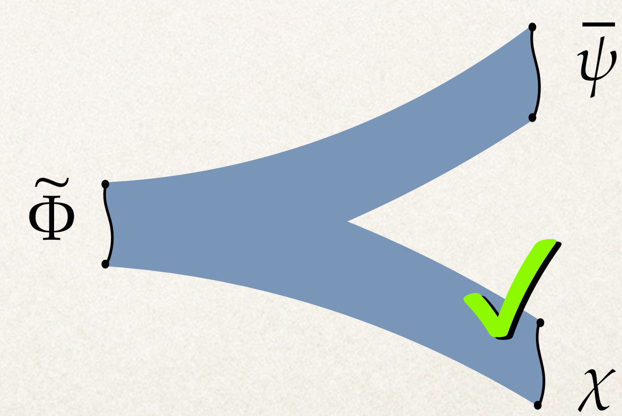

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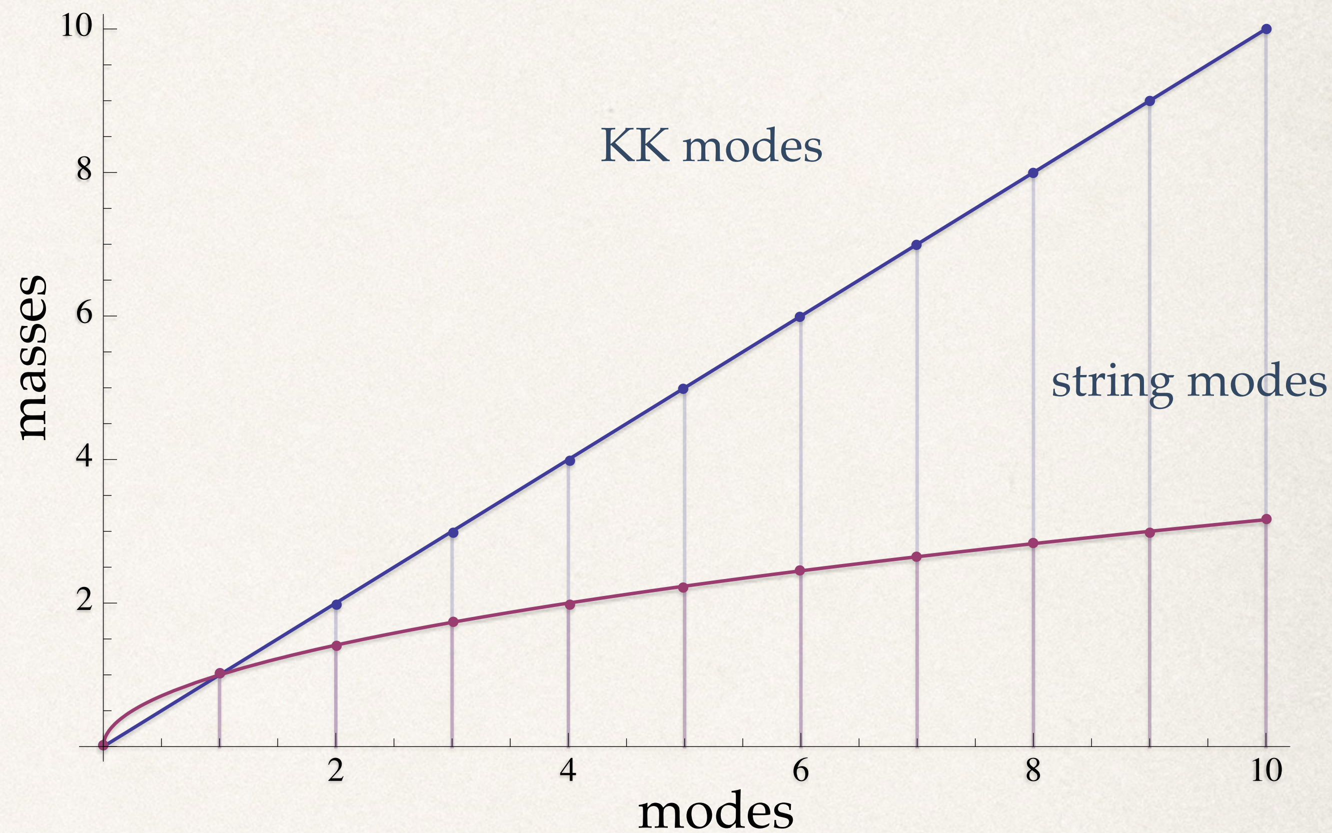

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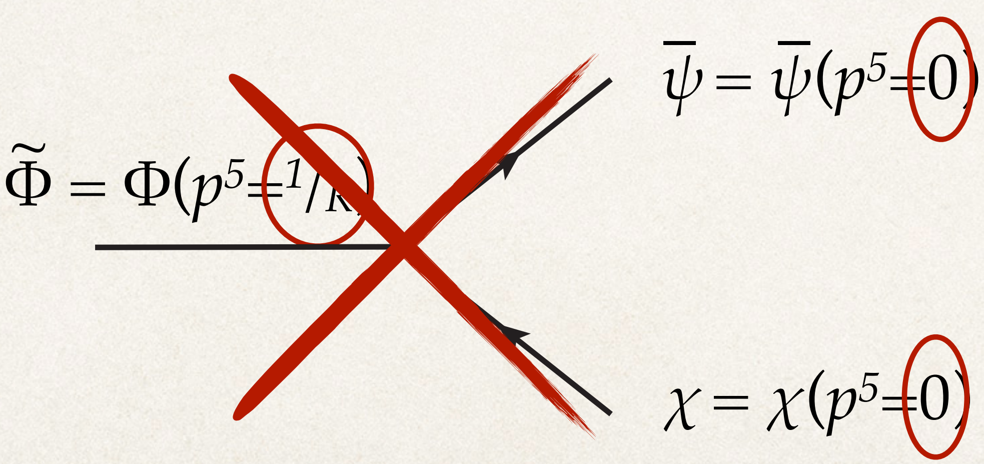

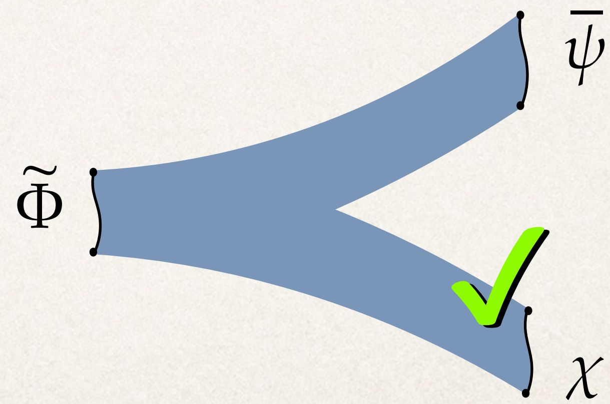

- Therefore, the two scenarios are easily distinguishable.

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- Decays of such states might be visible at LHC.