Broken boost invariance in the Glasma via finite nuclei thickness

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Introduction

Various heavy-ion collision experiments:

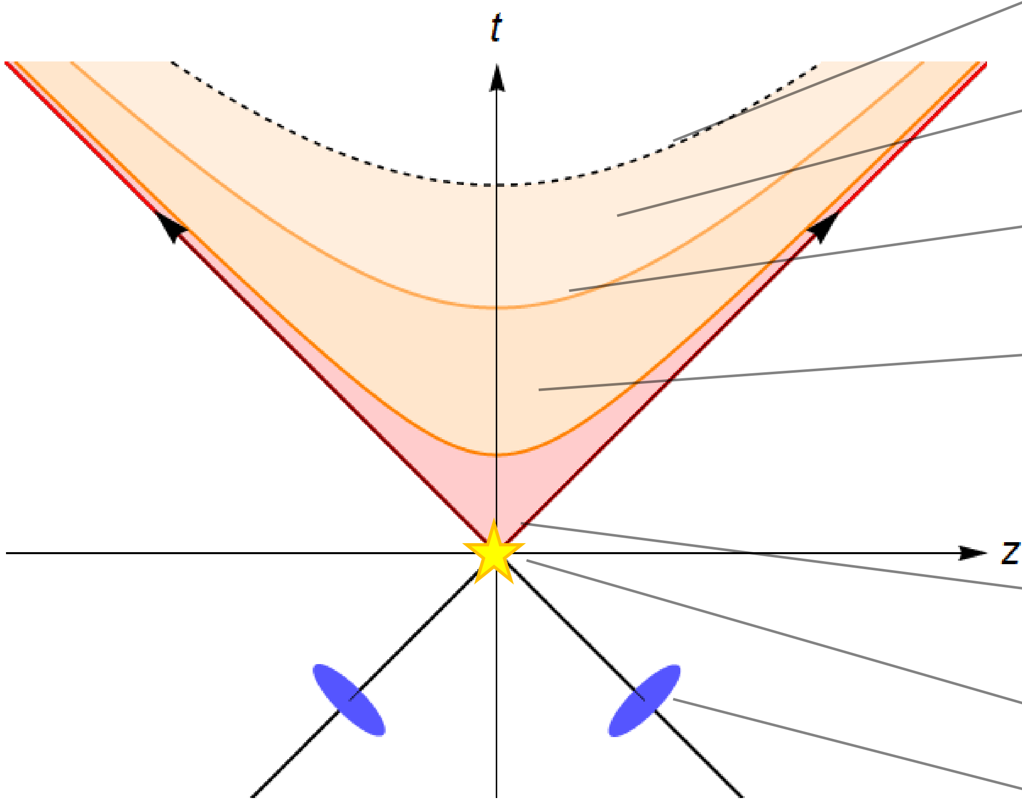
- LHC (ALICE) @ CERN: Pb+Pb with **~5.5 TeV** per nucleon pair. $(\gamma \approx 2700)$
- RHIC @ BNL: Au+Au with **~200 GeV** per nucleon pair. $(\gamma \approx 100)$
- RHIC beam energy scan: **~7.7 – 62.4 GeV** $(\gamma \approx 4 - 30)$

Early stages (few fm/c):

- Color glass condensate / Glasma: classical fields
- Finite longitudinal extent of the nuclei?

Goal: Simulate heavy-ion collisions in the color glass condensate (CGC) framework with finite nucleus thickness. Possible with colored particle-in-cell (CPIC) method.

Stages of a heavy-ion collision



Freeze-out ($\tau \approx$ 15fm/c)**:** interactions stop

Hadronic gas ($\tau \approx 10{-}15$fm/c)**:** hadrons (kinetic transport theory)

Hadronization ($\tau \approx$ 10fm/c)**:** confinement transition → hadron formation

QGP ($\tau \approx$ 1–10 fm/c)**:** quarks and gluons (relativistic viscous hydrodynamics)
(almost) isotropic and in thermal equilibrium

Glasma ($\tau \approx$ 0–1 fm/c)**:** quasi-classical fields (classical field equations)

Collision event

Initial state: Lorentz-contracted pancakes (color glass condensate)

scope of this project

$$1\ \mathrm{fm}/c \approx 3.3 \cdot 10^{-24}\ s \approx 3.3\ ys$$

Color glass condensate

Nuclei at ultrarelativistic speeds can be described by **classical effective theory** in the color glass condensate (CGC) framework.
[Gelis, Iancu, Jalilian-Marian, Venugopalan, Ann.Rev.Nucl.Part.Sci.60:463-489,2010]

Large gluon occupation numbers → coherent, classical gluon field

Split degrees of freedom into ...

- Hard partons = classical color charges
- Soft gluons = classical gauge field

generates..

- Static field configuration due to time dilation.
- Collision of two such classical fields creates the **Glasma**.
  [Gelis, Int.J.Mod.Phys. A28 (2013) 1330001]

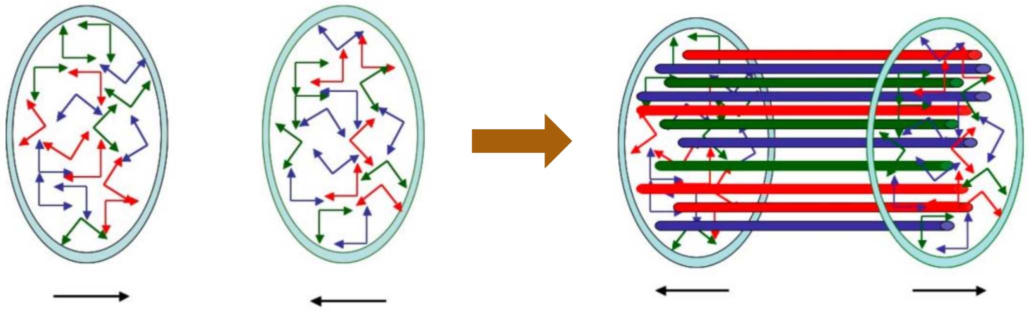
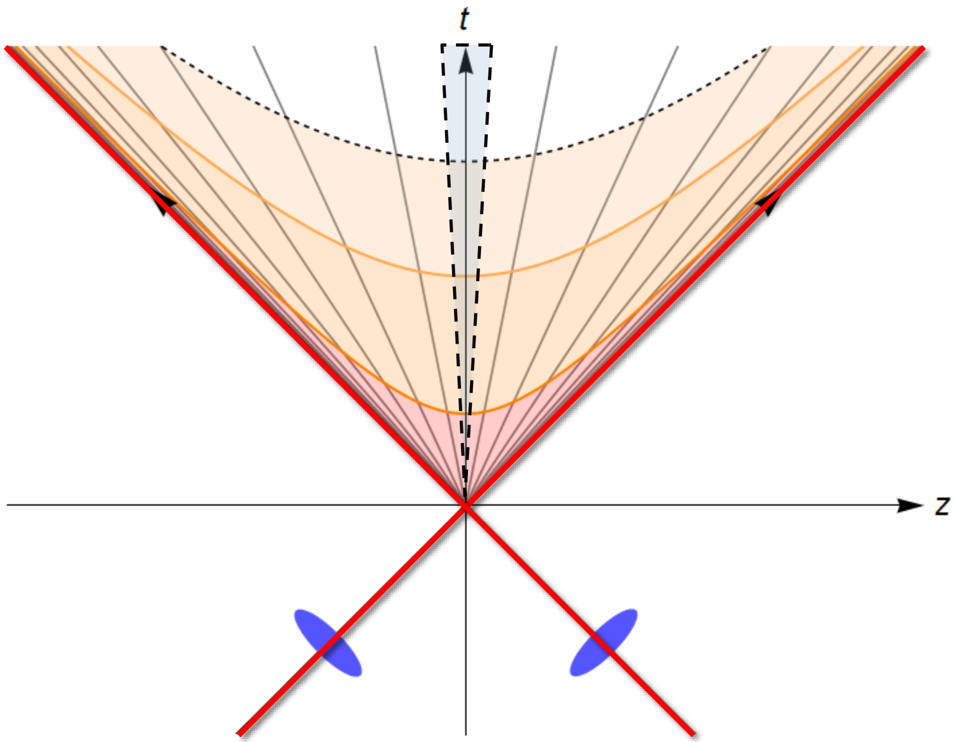


Figure from L. McLerran: Proceedings of ISMD08, p.3-18 (2008)

Boost-invariant collision

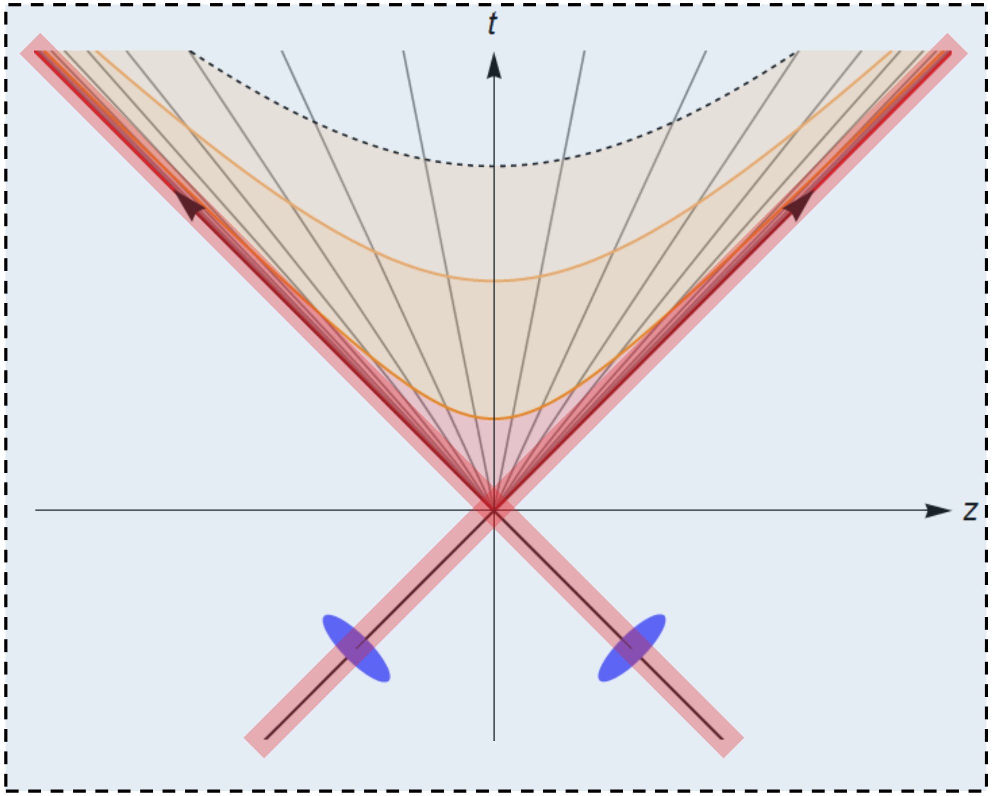


Description in τ, η (comoving) frame

- Color currents of the nuclei restricted to the light cone and infinitely thin
- Point-like collision region
- Analytical solutions exist for everything except the forward light cone. Glasma initial conditions on the boundary.
- Fields in the forward light cone are independent of rapidity η. Reduction from 3+1 to 2+1.
- Need to solve **2D+1 source-free Yang-Mills** equations in the forward light cone with Glasma initial conditions on the boundary of the light cone:

$$D_\mu F^{\mu\nu}(\tau, x_T) = 0$$

Broken boost invariance



Description in t, z (laboratory) frame

- Extended color currents
- Extended collision region
- Fields depend on rapidity
- Need to solve **full 3D+1 Yang-Mills** equations **with currents**!

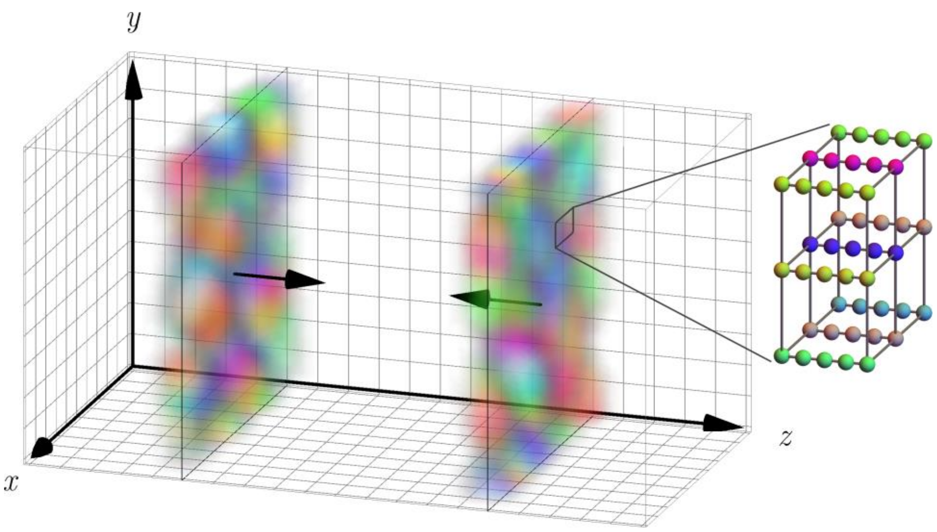
$$D_\mu F^{\mu\nu}(t, z, x_T) = J^\nu$$
$$D_\mu J^\mu(t, z, x_T) = 0$$

Colored particle-in-cell (CPIC) provides a framework to numerically solve the field and current equations on a lattice.

Colored particle-in-cell (CPIC) method

Generalization of the particle-in-cell (PIC) method for Abelian plasmas to non-Abelian gauge theories.

[A. Dumitru, Y. Nara, M. Strickland: Phys.Rev.D75:025016 (2007)]



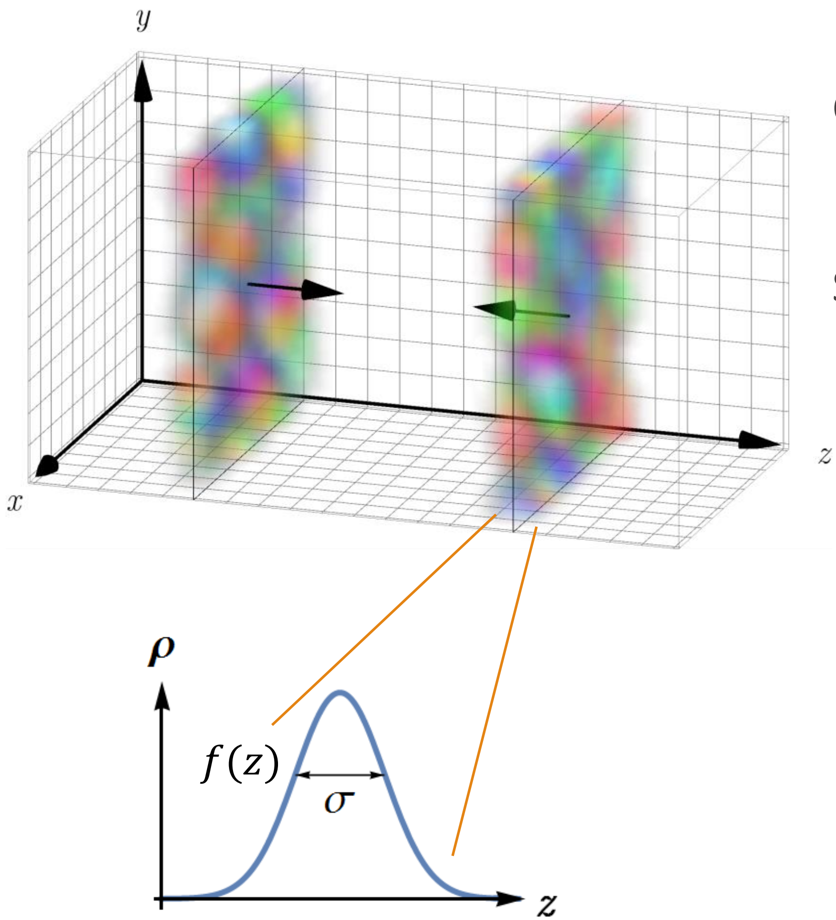
Heavy-ion collision in the lab frame with CPIC:

- Fields: discretized on a lattice
- **Color charge densities: replace with a large number of colored particles.**
- Current J_μ on the grid generated by particle movement → field equations

$$D_\mu F^{\mu\nu}(t,z,x_T) = J^\nu$$
$$D_\mu J^\mu(t,z,x_T) = 0$$

Non-Abelian gauge theory → parallel transport of charges

Initial conditions



longitudinal profile function $f(z)$ (Gaussian)

Color charges in the nuclei are randomly distributed

Models needed to fix the exact distribution

Simple model: **2D McLerran-Venugopalan (MV)**

[McLerran, Venugopalan: PRDD49 (1994) 3352-3355]

- Infinite transverse extent (periodic b.c.)
- **New:** embed 2D density into 3D space

$$\langle \hat{\rho}^a(\boldsymbol{x}_T)\hat{\rho}^b(\boldsymbol{x}'_T)\rangle = g^2\mu^2\delta^{(2)}(\boldsymbol{x}_T - \boldsymbol{x}'_T)\delta^{ab}$$

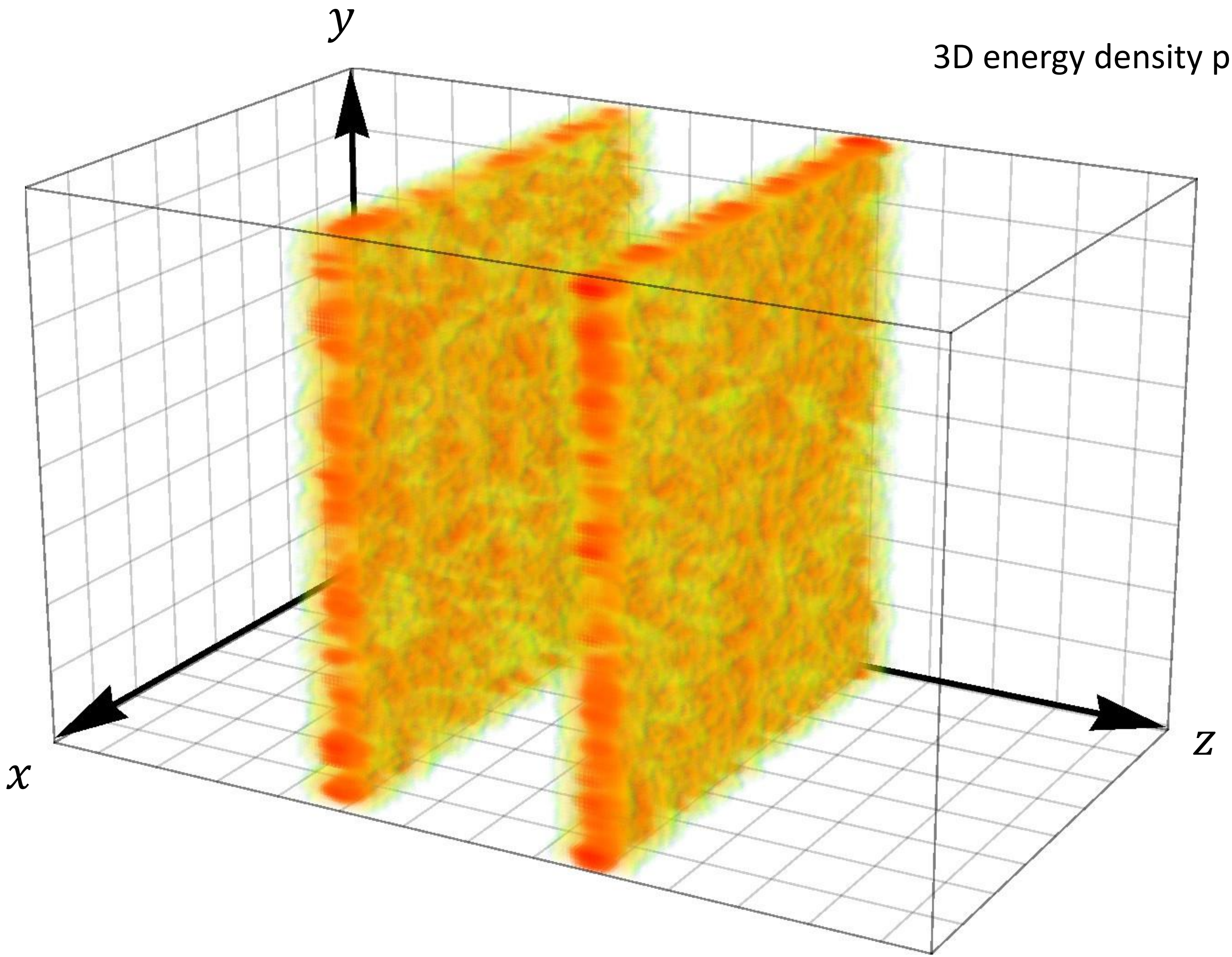
$$\rho\,(t, z, \boldsymbol{x}_T) = f(z-t)\hat{\rho}(\boldsymbol{x}_T)$$

- "Trivial" longitudinal structure
- Regulation of IR and UV modes required

MV parameter $\mu \approx 0.5$ GeV (Au)

Numerical results

3D energy density plot



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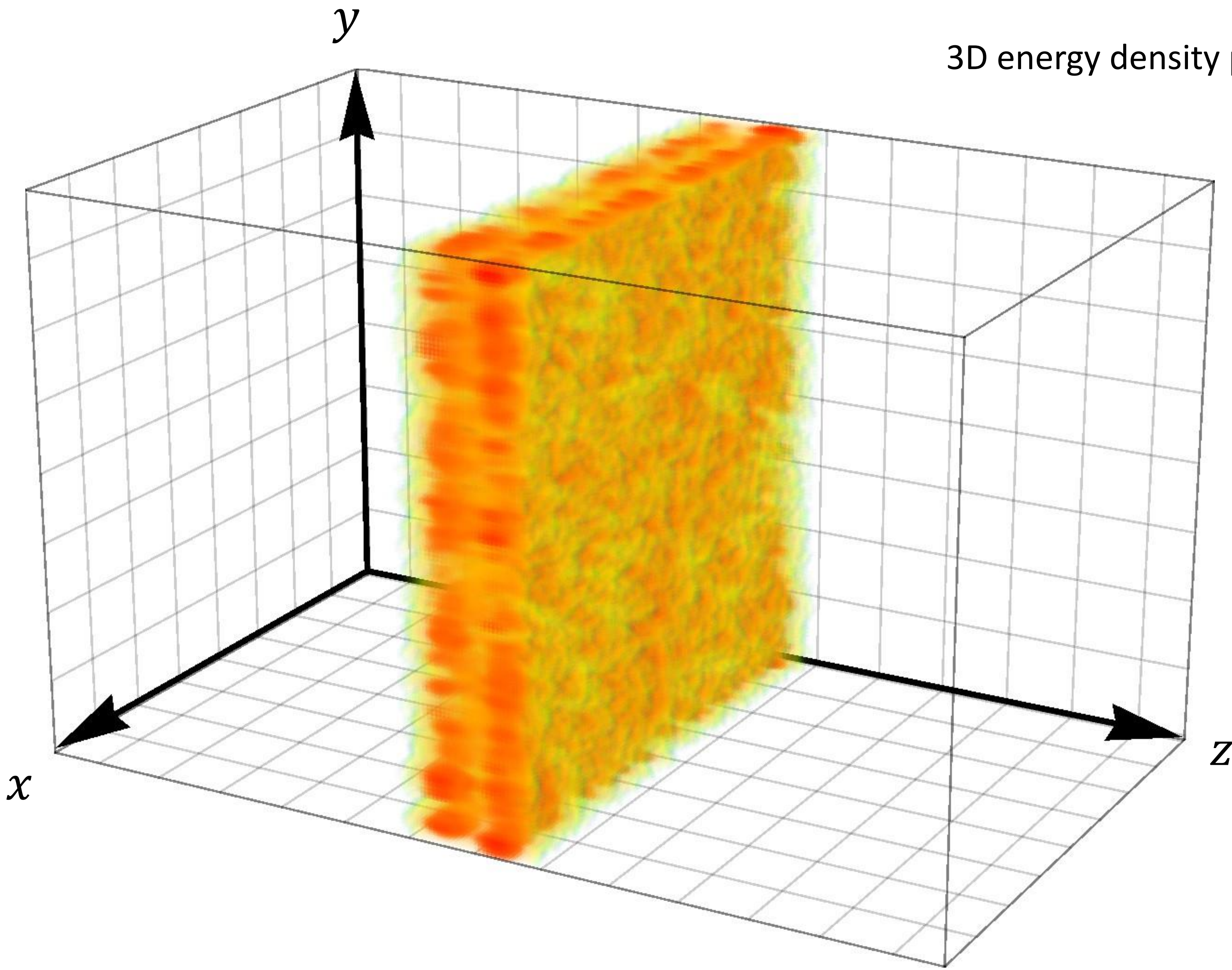

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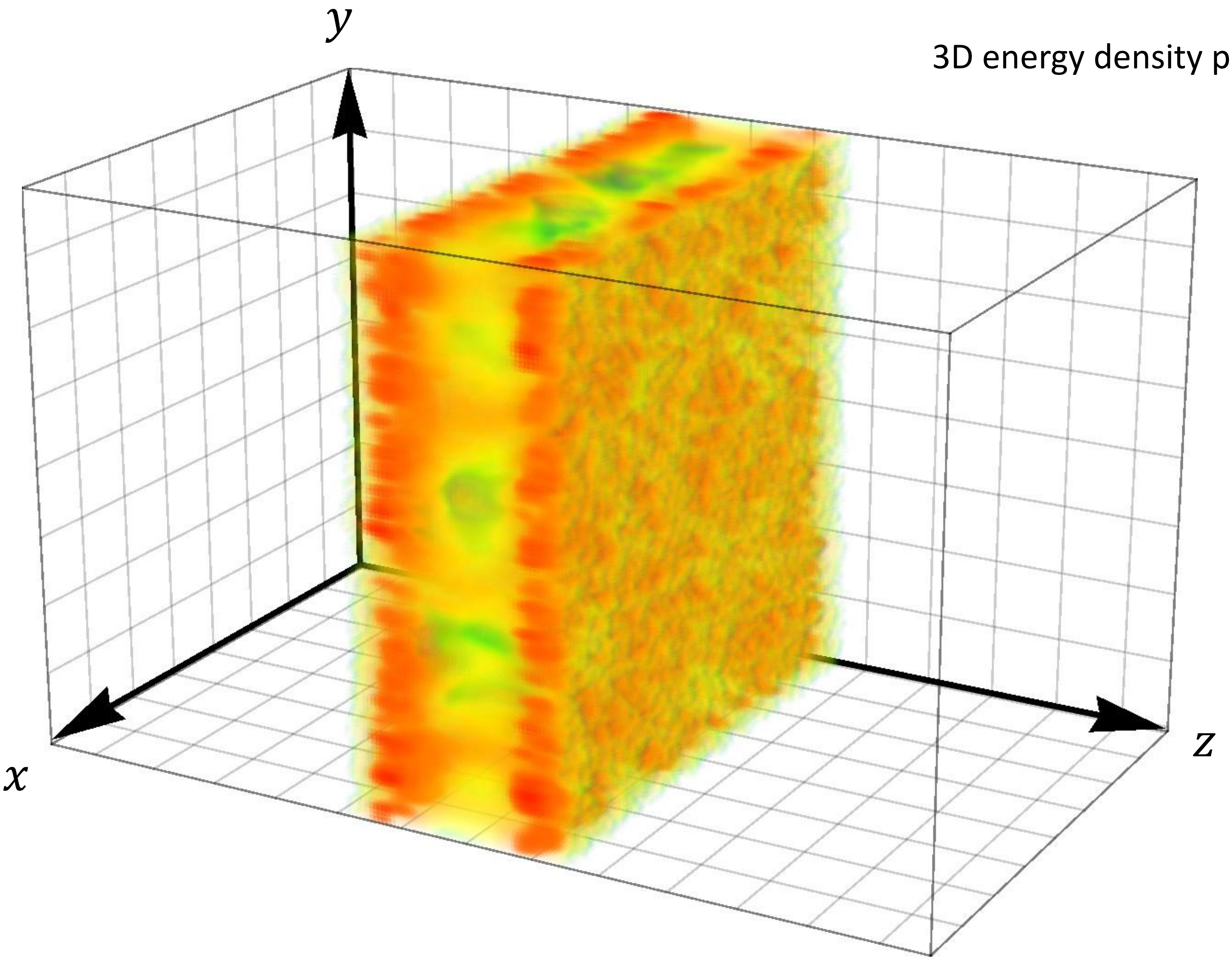

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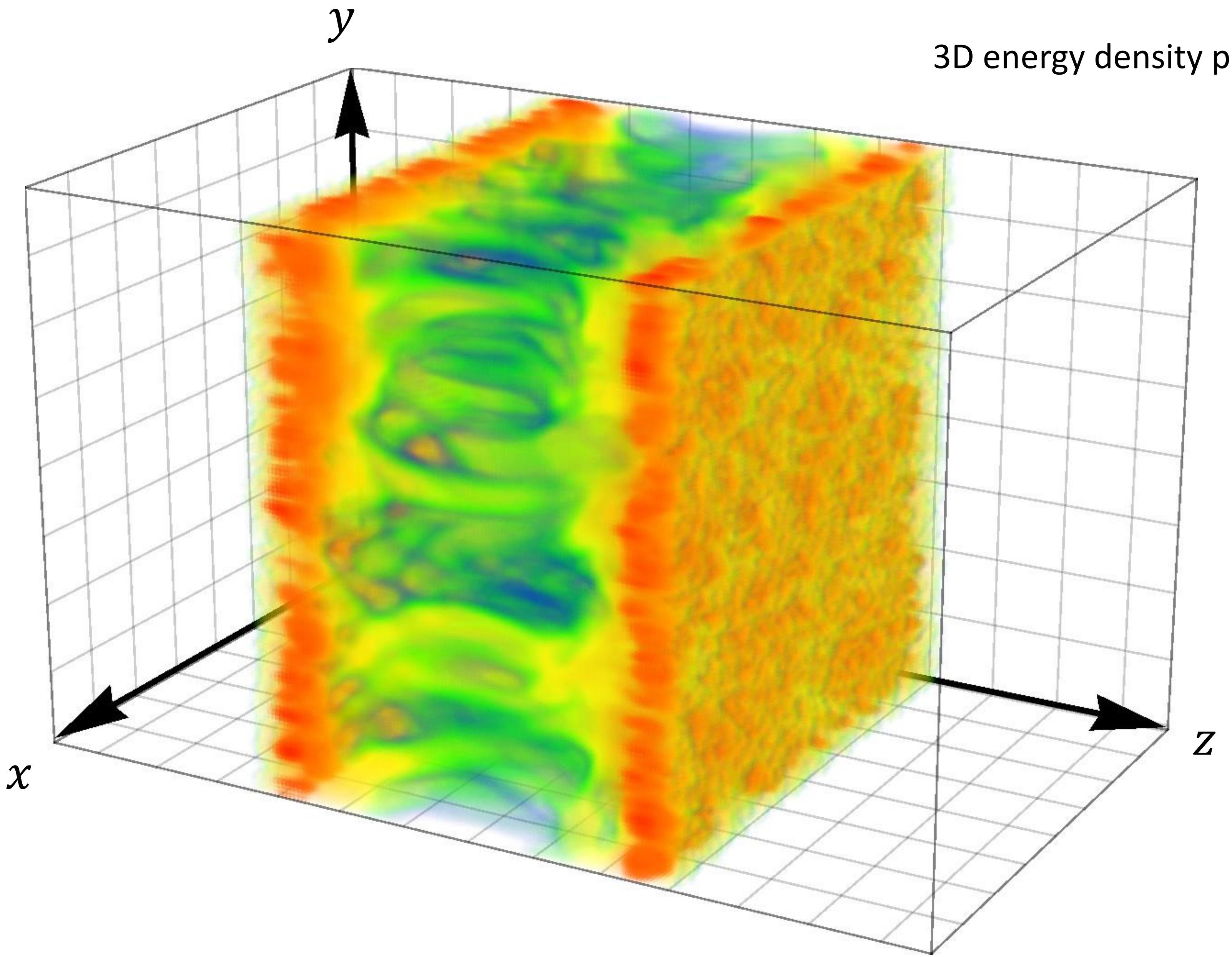
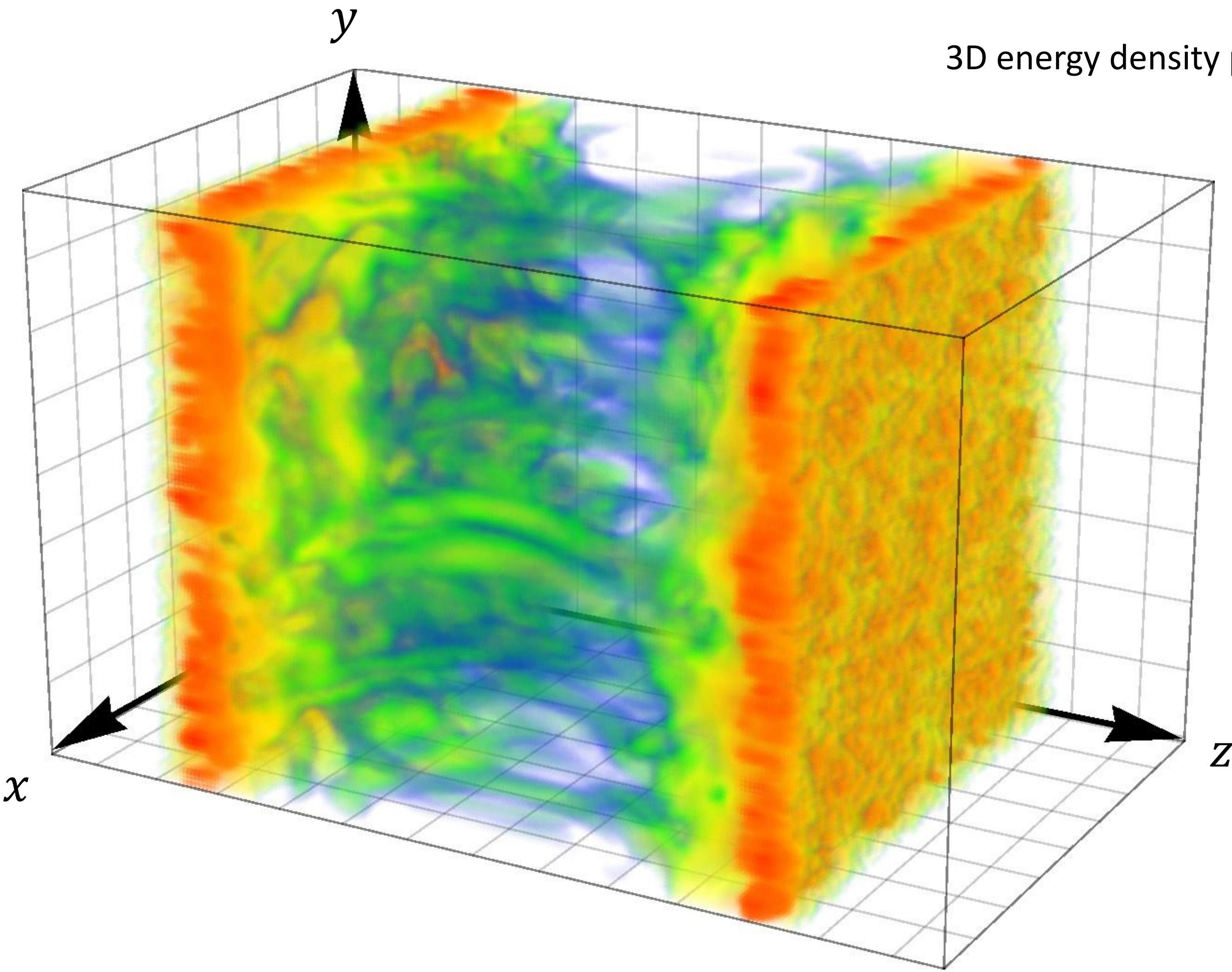
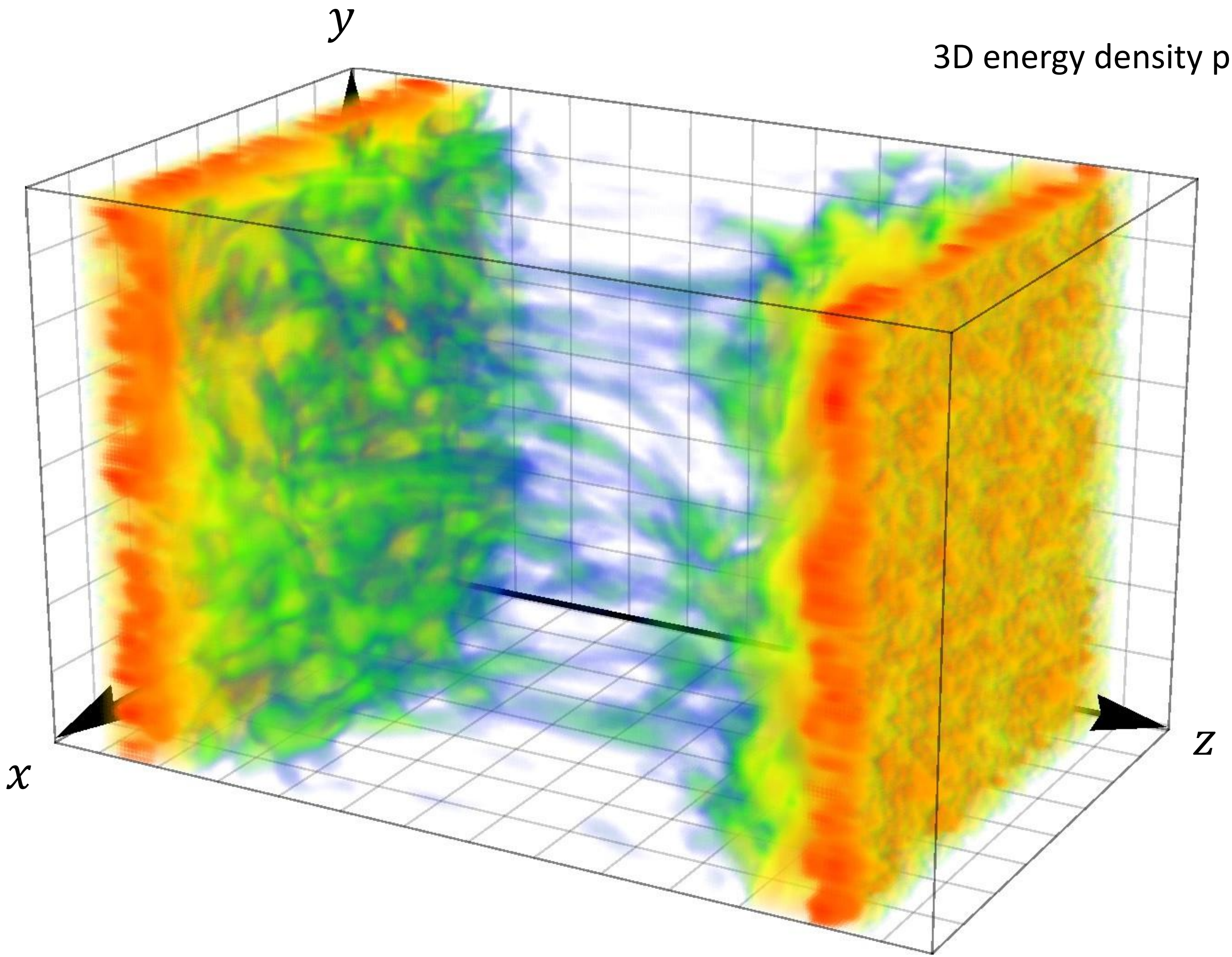




y
3D energy density plot
x
z

3D energy density plot



Observables

Main observable: energy-momentum tensor $T^{\mu\upsilon}(x)$

- Compute $T^{\mu\upsilon}(x)$ from electric and magnetic fields $E_i^a(x), B_i^a(x)$
- Average over configurations and integrate over transverse plane

$$\langle T^{\mu\nu} \rangle = \begin{pmatrix} \langle \varepsilon \rangle & 0 & 0 & \langle S_L \rangle \\ 0 & \langle p_T \rangle & 0 & 0 \\ 0 & 0 & \langle p_T \rangle & 0 \\ \langle S_L \rangle & 0 & 0 & \langle p_L \rangle \end{pmatrix}$$

$$\langle \varepsilon \rangle = \frac{1}{2} \left\langle E_T^2 + B_T^2 + E_L^2 + B_L^2 \right\rangle$$

$$\langle p_T \rangle = \frac{1}{2} \left\langle E_L^2 + B_L^2 \right\rangle$$

$$\langle p_L \rangle = \frac{1}{2} \left\langle E_T^2 + B_T^2 - E_L^2 - B_L^2 \right\rangle$$

$$\langle S_L \rangle = \left\langle \left(\vec{E}^a \times \vec{B}^a \right)_L \right\rangle$$

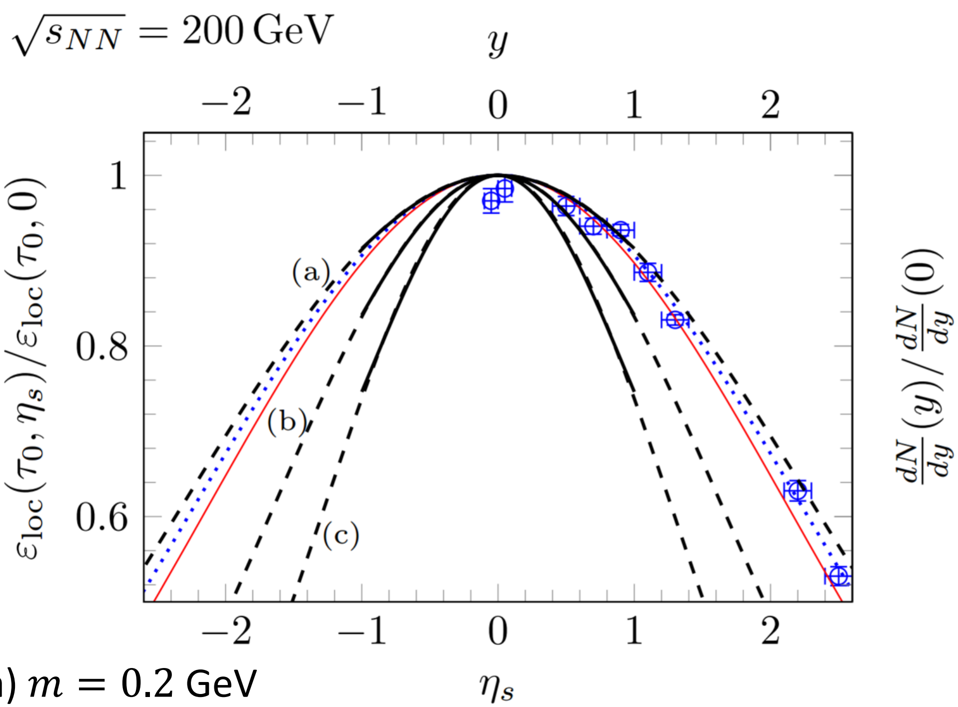
- Diagonalize, obtain local rest-frame energy density

$$\langle \varepsilon_{\mathrm{loc}} \rangle = \frac{1}{2} \left(\langle \varepsilon \rangle - \langle p_L \rangle + \sqrt{\left(\langle \varepsilon \rangle + \langle p_L \rangle \right)^2 - 4 \langle S_L \rangle^2} \right)$$

Rapidity profiles

Plot (space-time) rapidity profile of local rest-frame energy density

Compare to measured **rapidity profile of particle multiplicity (RHIC)** and **Landau model** prediction



(a) $m = 0.2$ GeV
(b) $m = 0.4$ GeV
(c) $m = 0.8$ GeV

- Simulation data in interval $\eta_s \in (-1,1)$ at $\tau = 1$ fm/c
- Fit to Gaussian profile (dashed)
- Dependency on thickness (or rather $\sqrt{s}$)
- Strong dependency on IR regulator, but $m = 0.2$ GeV gives realistic shape
- However: no hydrodynamic expansion included
- Limited rapidity interval

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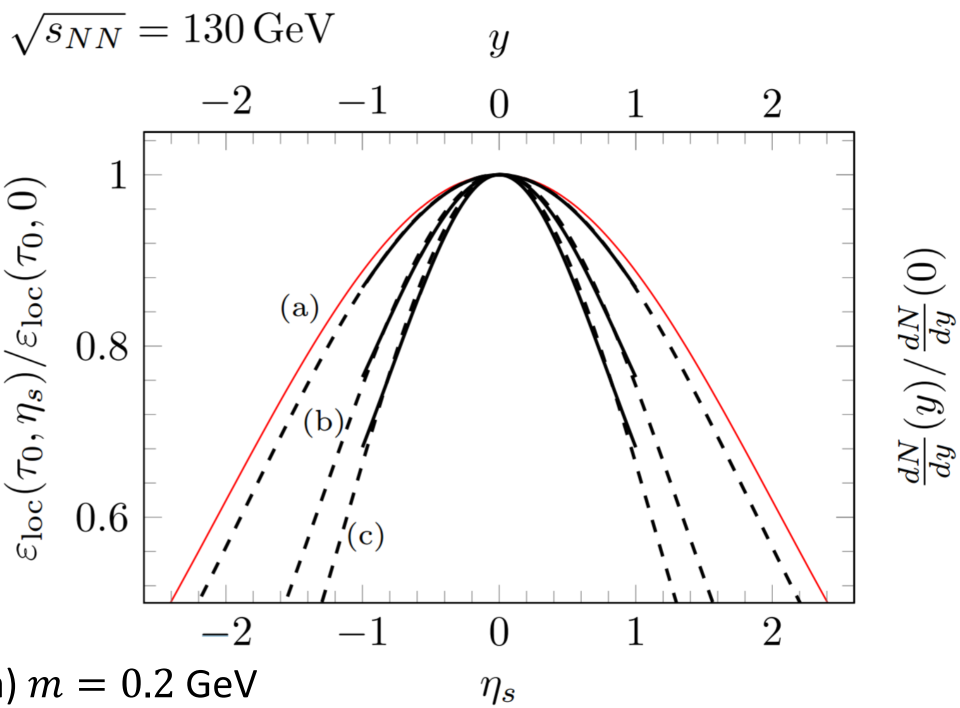

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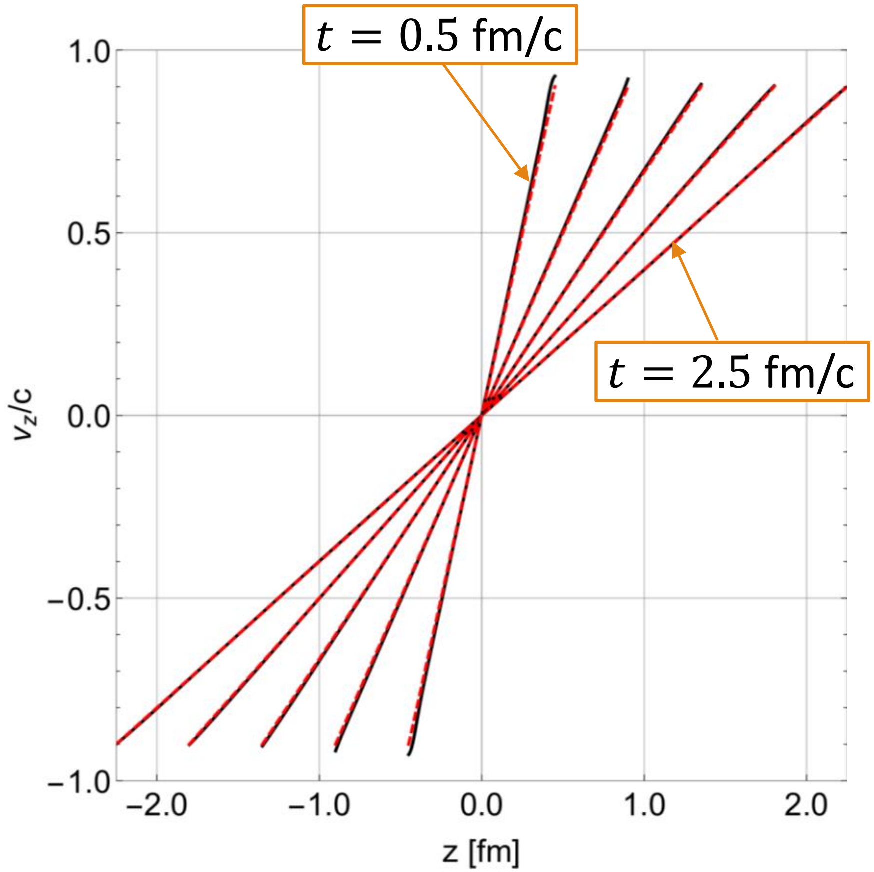
Longitudinal velocity v_z

How fast is the Glasma flowing in the longitudinal direction?

Is the use of the τ, η frame justified?



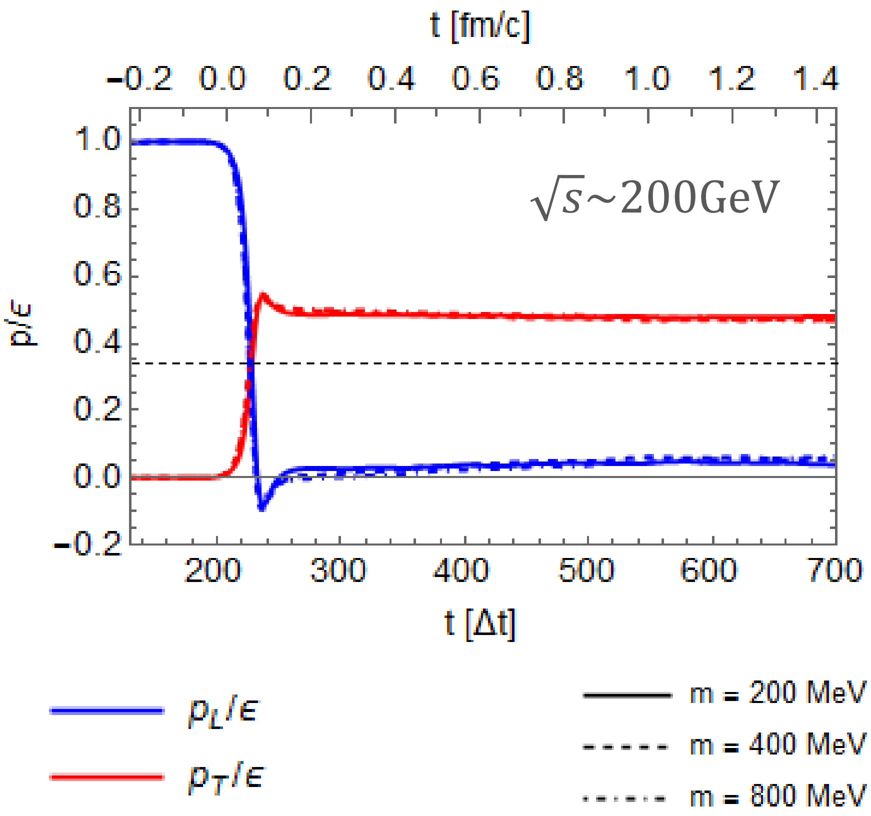
- Compute v_z from energy-momentum tensor **(black)**
- Compare to free-streaming case $v_z = z/t$ **(red, dashed)**
- Glasma fields depend on η, but still free-streaming!
- Local rest-frame $\cong \tau, \eta$ frame

Pressure anisotropy

Look at **pressure to energy density ratio** at mid-rapidity to investigate pressure anisotropy of the Glasma fields

$$\frac{\langle p_L \rangle}{\langle \varepsilon \rangle}, \frac{\langle p_T \rangle}{\langle \varepsilon \rangle} \text{ at } \eta = 0$$

- Strong anisotropy, no isotropization
- Similar to boost-invariant case
- No dependence on IR regulator m
- Initially negative longitudinal pressure is “hidden”



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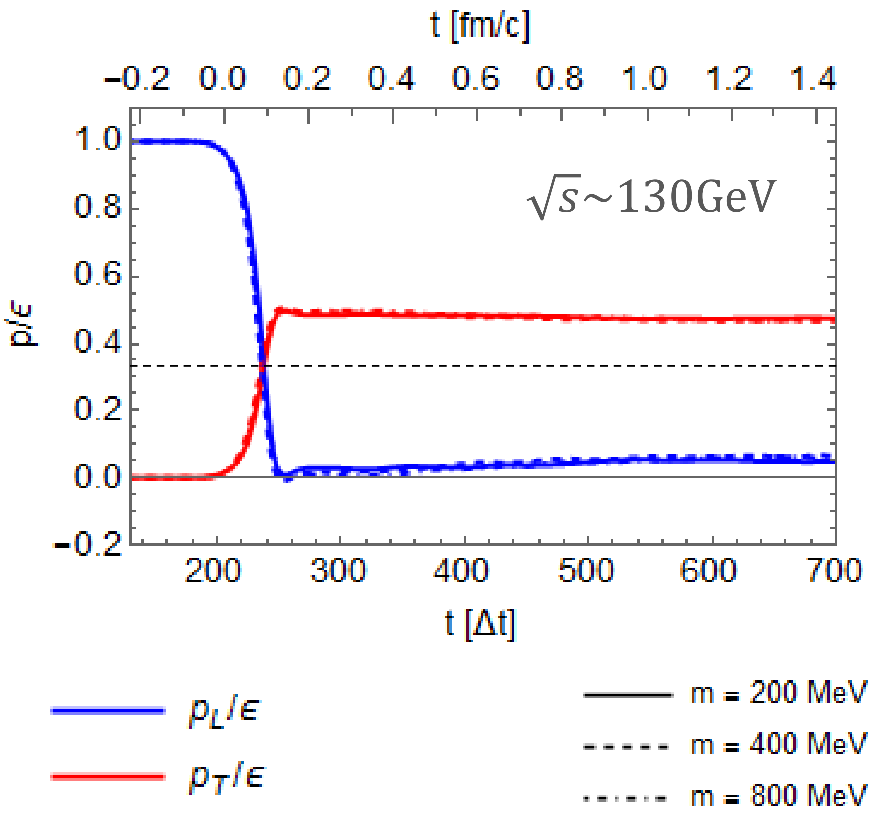
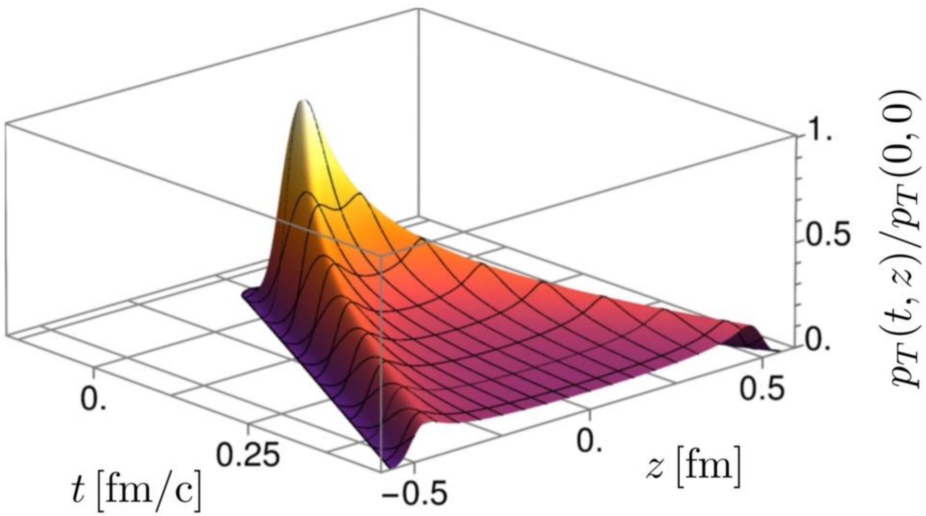


Transverse pressure distribution

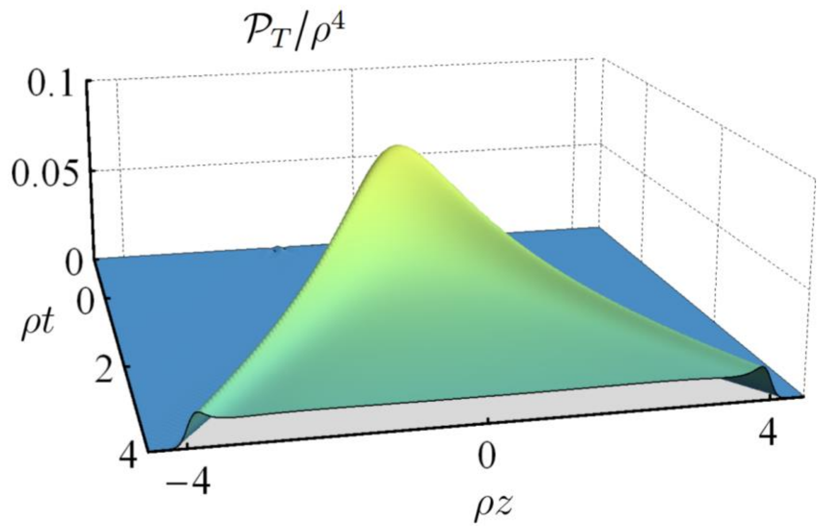
Transverse pressure $p_T(x)$ generated by longitudinal fields

$$\langle p_T \rangle = \frac{1}{2} \langle E_L^2 + B_L^2 \rangle$$

Boost-invariant case: initial conditions at $\tau = 0$ for longitudinal **E** and **B** fields, i.e. constant p_T along the boundary of the forward light cone



3+1 Yang-Mills



Holographic model

[Casalderrey-Solana et al., PRL (2013) 181601]

Conclusions and outlook

- Genuine 3D picture of heavy-ion collisions in the CGC framework
- Finite thickness breaks boost invariance → Gaussian rapidity profiles
- Strong pressure anisotropy, no isotropization
- Free-streaming flow

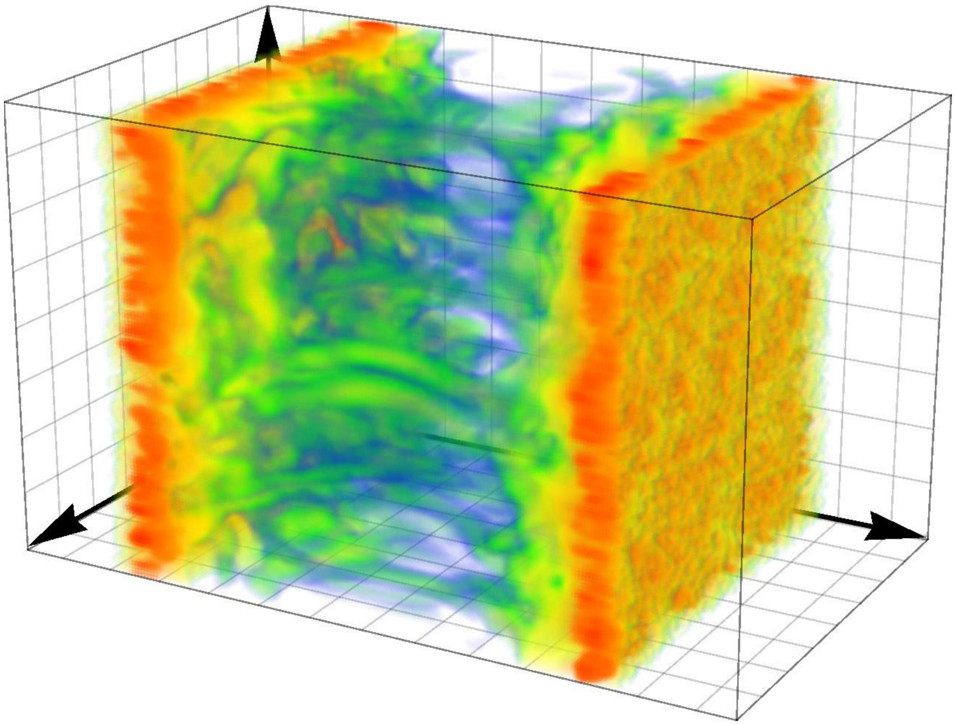
Future:

- Initial conditions with random (complicated) longitudinal structure
- Numerical improvements to study higher energies (LHC)
- JIMWLK evolved initial conditions

arXiv:1605.07184
arXiv:1703.00017

open source!
github.com/openpixi/openpixi (Java)
gitlab.com/monolithu/pyglasma3d (Python, Cython)

Thank you for your attention!



arXiv:1605.07184
arXiv:1703.00017

open source!
github.com/openpixi/openpixi (Java)
gitlab.com/monolithu/pyglasma3d (Python, Cython)

Backup slides

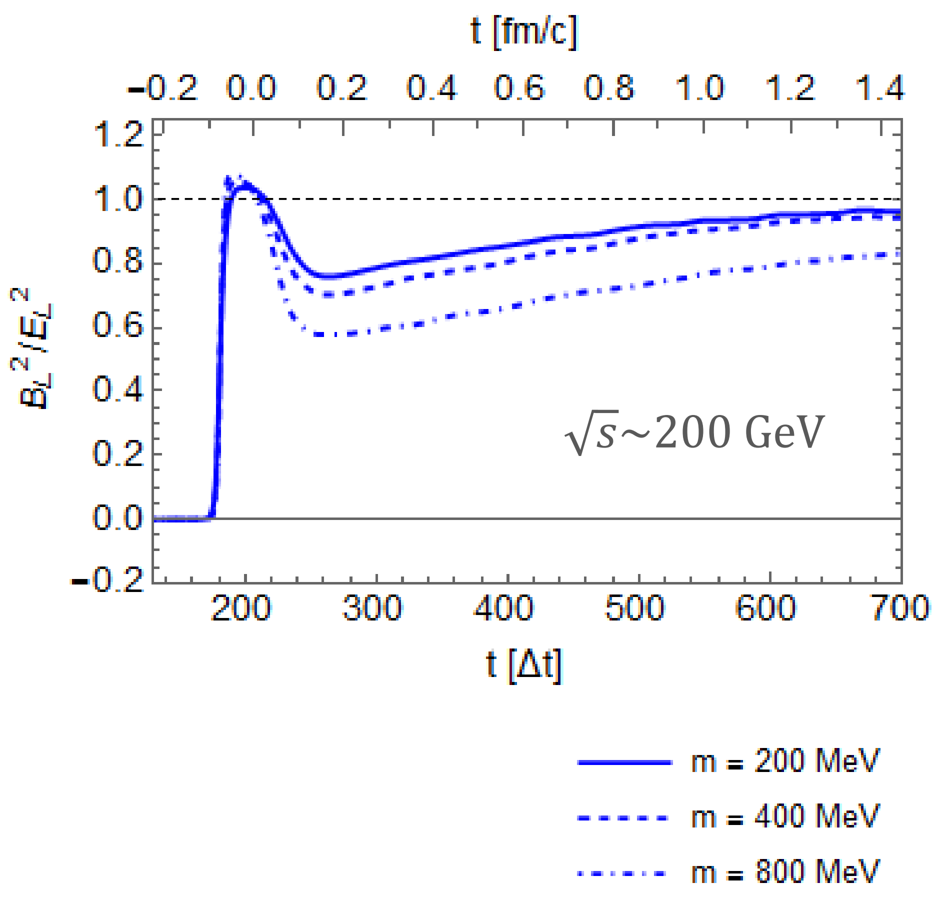
Chromo-magnetic and electric fields

Study creation and evolution of longitudinal fields

- Boost-invariant case: $\langle E_L^2 \rangle \sim \langle B_L^2 \rangle$
- Here: magnetic and electric fields are not created equally
- Strong dependence on IR regulator m and thickness

Large m, thick nuclei: low ratio
Small m, thin nuclei: ratio $\sim$ 1

Artefact of initial conditions?



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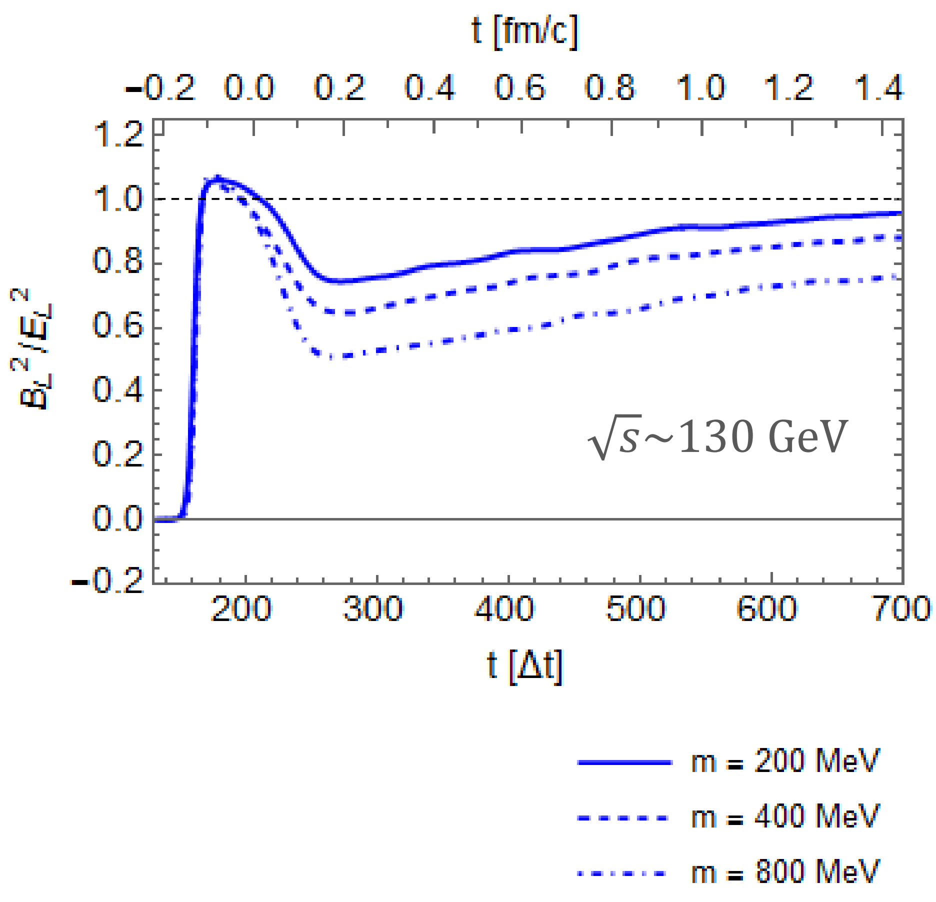


Longitudinal structure (1)

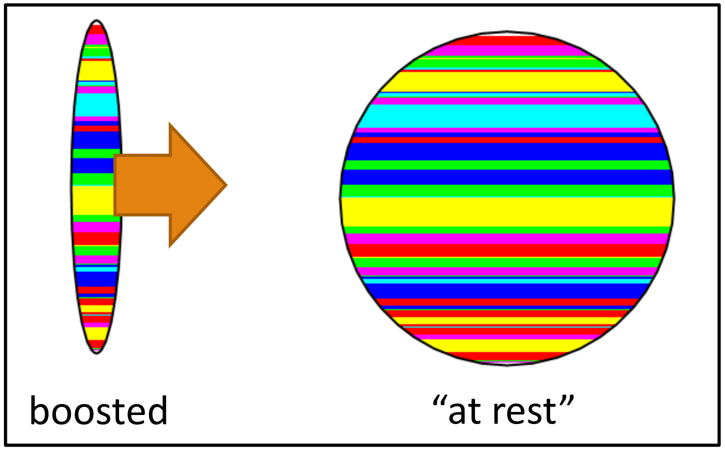
Initial conditions are still missing **random longitudinal structure:**

- Color charges (or fields) of the nuclei are correlated in the longitudinal direction (figure on the left)
- Realistic nuclei should consist of independent "sheets" (figure on the right) [Fukushima, PRD 77 (2008) 074005]

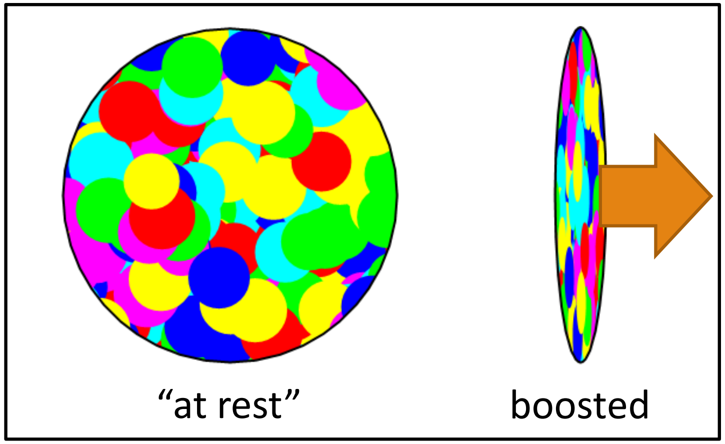
Consequences

- Higher energy density in the Glasma
- MV model: Higher Q_s for fixed μ [Lappi, Eur. Phys. J. C55 (2008) 285]
- Effect on ratio of longitudinal electric to magnetic field?

Current implementation



Longitudinal randomness



Longitudinal structure (2)

Wilson line expectation value $\langle \mathrm{tr}(V) \rangle$ of a single nucleus is sensitive to longitudinal structure.

Embedded 2D MV-model:

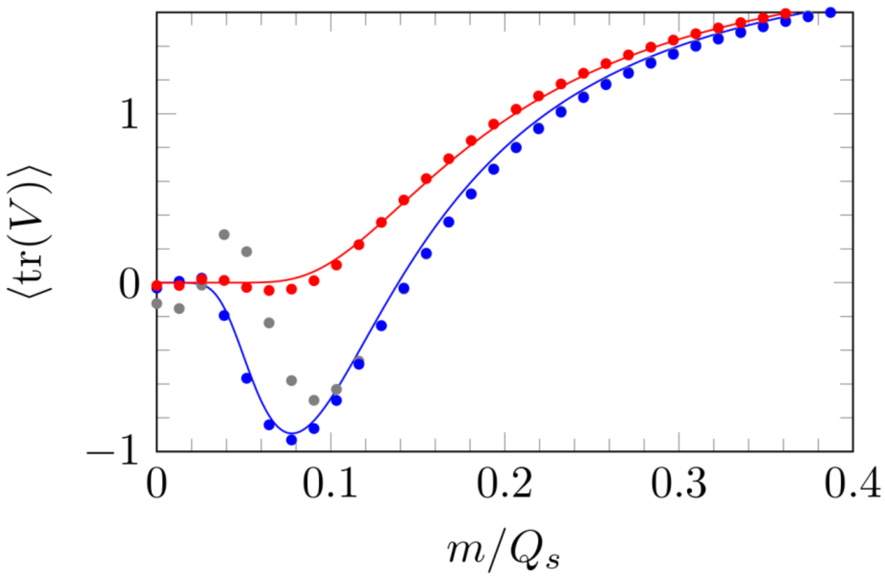
$$\langle \hat{\rho}^a(\boldsymbol{x}_T)\hat{\rho}^b(\boldsymbol{x}'_T)\rangle = g^2\mu^2\delta^{(2)}(\boldsymbol{x}_T - \boldsymbol{x}'_T)\delta^{ab}$$

$$\rho\,(t, z, \boldsymbol{x}_T) = f(z-t)\hat{\rho}(\boldsymbol{x}_T)$$

3D MV-model:
(with random longitudinal structure)

$$\langle \rho^a(t,\boldsymbol{x})\rho^b(t,\boldsymbol{x}')\rangle = g^2\mu^2 f(z)\delta^{(3)}(\boldsymbol{x}-\boldsymbol{x}')\delta^{ab}$$

$f(z)$ … longitudinal profile function



Lines: analytical result
Dots: numerical result

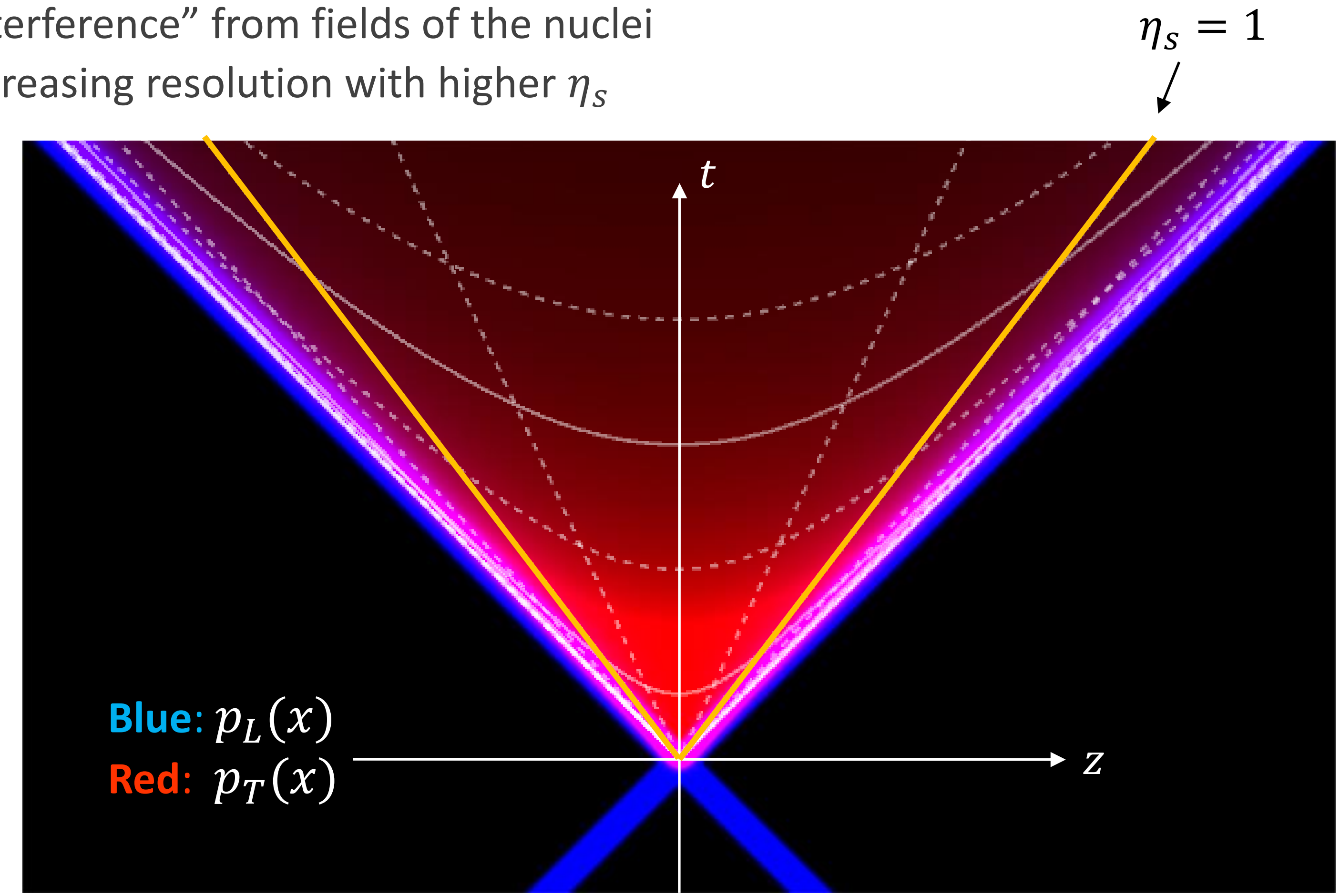
Blue: 2D MV-model
Red: 3D MV-model
Gray: intermediate

Introducing independent "sheets" in longitudinal direction
[Fukushima, PRD 77 (2008) 074005]

Extending the rapidity interval (1)

Limited rapidity interval due to ..

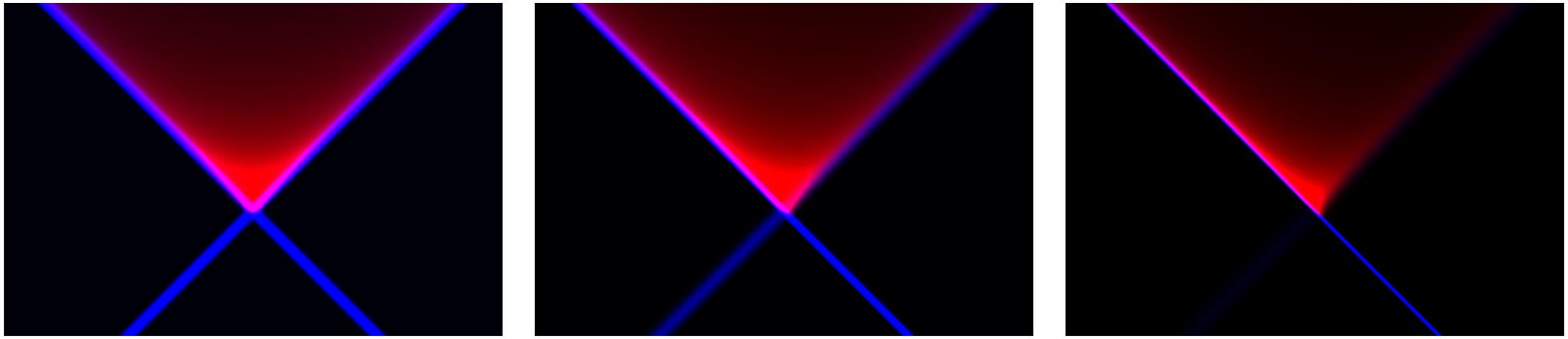
- longitudinal simulation box length / simulation time
- "interference" from fields of the nuclei
- decreasing resolution with higher η_s



Extending the rapidity interval (2)

Idea:

- Simulate collisions in boosted frame (asymmetric collision), parametrize with η'
- Record data only in "safe" interval $\eta_s \in (-1,1)$
- Boost back to center of mass frame



(a) $\eta' = 0$ (b) $\eta' = 0.5$ (c) $\eta' = 1$

Extending the rapidity interval (3)

Result:

- Rapidity profiles can be extended to higher ranges of η_s
- Non-trivial test of the boost invariance of the code