ON THE COHERENT INELASTIC BINARY AND MULTIPARTICLE PROCESSES IN ULTRARELATIVISTIC HADRON–NUCLEUS, PHOTON–NUCLEUS AND NUCLEUS–NUCLEUS COLLISIONS

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Abstract

The coherent inelastic processes of the type $a \to b$, which may take place in the interaction of hadrons and γ quanta with nuclei at very high energies (the nucleus remains the same), are theoretically investigated. For taking into account the influence of matter inside the nucleus, the optical model based on the concept of refraction index is applied . Analytical formulas for the effective cross section $\sigma_{\rm coh}(a \to b)$ are obtained, taking into account that at ultrarelativistic energies the main contribution into $\sigma_{\rm coh}(a \to b)$ is provided by very small transferred momenta in the vicinity of the minimum longitudinal momentum transferred to the nucleus. It is shown that the cross section $\sigma_{\rm coh}(a \to b)$ may be expressed through the "forward" amplitudes of inelastic scattering $f_{a+N\to b+N}(0)$ and elastic scattering $f_{a+N\to a+N}(0)$, $f_{b+N\to b+N}(0)$ on a separate nucleon, and it depends on the ratios L_a/R and L_b/R, where L_a, L_b are the respective mean free paths in the nucleus matter for the particles a, b and R is the nuclear radius. In doing so, several characteristic cases with different relations of the magnitudes L_a, L_b, R are considered in detail.

The formalism described above is generalized also for the case of coherent inelastic multiparticle processes on a nucleus of the type $a \to \{ b_1, b_2, b_3 .... b_i \}$ and for the case of coherent processes in collisions of two ultrarelativistic nuclei .

1 Momentum transfer at ultrarelativistic energies and coherent reactions on nuclei

In the present work we will investigate theoretically the processes of inelastic coherent scattering at collisions of particles with nuclei at very high energies. It is essential that at ultrarelativistic energies the minimum longitudinal momentum transferred to a nucleus tends to zero, and in connection with this the role of coherent processes increases.

Let $f_{a+N\to b+N}(\mathbf{q})$ be the amplitude of an inelastic process $a+N \to b+N$ on a separate nucleon in the rest frame of the nucleus (laboratory frame), $\mathbf{k}_a$ and $\mathbf{k}_b$ be the momenta of particles a and b, respectively, $\mathbf{q} = \mathbf{k}_b - \mathbf{k}_a$ be the momentum transferred to the nucleon. Let us find the effective cross section of the coherent inelastic process $a \to b$ on a nucleus at high energies.

In the framework of the impulse approximation [1] we can, taking into account the interference phase shifts at the inelastic scattering of particle a on the system of nucleons, present the expression for the effective cross section of the coherent inelastic scattering on a nucleus in the following form:

$$\sigma_{\rm coh}(a \to b) = \int |f_{a+N\to b+N}(\mathbf{q})|^2 \left| \sum_{i=1}^{A} \langle \exp(-i\mathbf{q}\mathbf{r}_i) \rangle \right|^2 d\Omega_b, \qquad (1)$$

where

$$f_{a+N\to b+N}(\mathbf{q}) = \frac{Z f_{a+p\to b+p}(\mathbf{q}) + (A-Z) f_{a+n\to b+n}(\mathbf{q})}{A}, \qquad (2)$$

Z is the number of protons in the target nucleus, $(A-Z)$ is the number of neutrons in the target nucleus, $\langle .... \rangle$ denotes averaging over the quantum states of the target nucleus, $d\Omega_b$ is the element of the solid angle of flight of particle b in the laboratory frame. The magnitude

$$P(\mathbf{q}) = \left| \sum_{i=1}^{A} \langle \exp(-i\mathbf{q}\mathbf{r}_i) \rangle \right|^2 \qquad (3)$$

has the meaning of the probability of the event that at the collision with particle a all the nucleons remain in the nucleus and the quantum state of the nucleus does not change. Let us introduce the nucleon density $n(\mathbf{r})$ normalized by the total number of nucleons in the nucleus:

$$\int_V n(\mathbf{r}) d^3\mathbf{r} = A, \qquad (4)$$

where the integration is performed over the volume of the nucleus. Then

$$P(\mathbf{q}) = |\int n(\mathbf{r}) \exp(-i\mathbf{q}\mathbf{r}) d^3\mathbf{r}|^2 =$$
$$= |\int_V n(\boldsymbol{\rho}, z) \exp(-i\mathbf{q}_\perp \boldsymbol{\rho}) \exp(-iq_{||} z) d^2\boldsymbol{\rho} dz|^2. \quad (5)$$

Here the axis z is parallel to the initial momentum $\mathbf{k}_a$, $\mathbf{q}_\perp$ is the transverse component of the transferred momentum , $q_{||}$ is the longitudinal component of the transferred momentum.

It is easy to see that the momenta transferred to a nucleon, which are less than the inverse radius of the nucleus in modulus or of the order of it, give the main contribution to the effective cross section of the coherent inelastic process $a \to b$ on the nucleus, having the form

$$\sigma_{\rm coh}(a \to b) = \int |f_{a+N\to b+N}(\mathbf{q})|^2 P(\mathbf{q}) d\Omega_b. \quad (6)$$

In accordance with this, we can consider in Eq. (6) only the values

$$|\mathbf{q}| <\sim \frac{1}{R}, \quad (7)$$

where R is the radius of the nucleus. It is obvious that at ultrarelativistic energies we have

$$E_a \gg m_a, \quad E_b \gg m_b, \quad E_a \gg \frac{1}{R}, \quad E_b \gg \frac{1}{R}, \quad (8)$$

where m_a and m_b are the masses of particles a and b, respectively. Taking into account Eqs. (7) and (8), the recoil energy of the nucleon

$$E_{\rm rec} \approx \frac{|\mathbf{q}|^2}{m_N} <\sim (m_N R^2)^{-1}$$

and the much smaller recoil energy of the nucleus can be neglected. In so doing, the transverse transferred momentum is equal in modulus to $|\mathbf{q}_\perp| = k \sin\theta$, where $k = E_a \approx E_b$, θ is the angle of flight of particle b, and the longitudinal transferred momentum is determined by the expression

$$q_{||} = \sqrt{k^2 - m_b^2} \cos\theta - \sqrt{k^2 - m_a^2} \approx \frac{m_a^2 - m_b^2}{2k} - k(1 - \cos\theta). \quad (9)$$

The inequalities (7) and (8) lead to small flight angles for the particle b:

$$\theta <\sim \frac{1}{kR} \ll 1.$$

Therefore, it is possible to assume in Eqs. (5) and (6) that:

$$|\mathbf{q}_\perp| = k\theta, \qquad q_{||} = q_{\min} = \frac{m_a^2 - m_b^2}{2k}. \quad (10)$$

Here $q_{\rm min}$ is the *minimum* transferred momentum corresponding to the "forward" direction. At small angles θ

$$d\Omega_b = \sin\theta d\theta d\phi \approx \frac{d^2\mathbf{q}_\perp}{k^2}. \tag{11}$$

In most cases the characteristic momentum transferred to the nucleus at the inelastic coherent scattering (see Eq. (7)) is small as compared with the characteristic momentum transferred to the nucleon in the process $a+N\to b+N$. In connection with this, the amplitude $f_{a+N\to b+N}(\mathbf{q})$ in Eq.(6) can be replaced by its value $f_{a+N\to b+N}(0)$ corresponding to the flight of particle b in the "forward" direction. Thus,

$$\sigma_{\rm coh}(a\to b) = |f_{a+N\to b+N}(0)|^2\times$$

$$\times\int\left(|\int n(\boldsymbol{\rho},z)\,\exp(-i\mathbf{q}_\perp\boldsymbol{\rho})\,\exp(-iq_{\rm min}z)d^2\boldsymbol{\rho}dz|^2\right)\frac{d^2\mathbf{q}_\perp}{k^2}, \tag{12}$$

where $q_{\rm min}$ is determined by Eq.(10). Integrating expression (12) over the transverse transferred momenta and over the volume of the nucleus, and taking into account that

$$\int\exp[-i\mathbf{q}_\perp(\boldsymbol{\rho}-\boldsymbol{\rho}')]d^2\mathbf{q}_\perp = (2\pi)^2\delta^2(\boldsymbol{\rho}-\boldsymbol{\rho}'),$$

we find:

$$\sigma_{\rm coh}(a\to b) = \frac{4\pi^2}{k^2}|f_{a+N\to b+N}(0)|^2\int\left(|\int_{-\infty}^{\infty}n(\boldsymbol{\rho},z)\exp(-iq_{\rm min}\,z)\,dz|^2\right)d^2\boldsymbol{\rho}. \tag{13}$$

In the case of a spherical nucleus with the radius R and the constant density of nucleons

$$n_0 = \frac{3}{4\pi R^3}A \tag{14}$$

Eq. (13) gives:

$$\sigma_{\rm coh}(a\to b) = \frac{32\pi^3}{k^2q_{\rm min}^2}n_0^2\,|f_{a+N\to b+N}(0)|^2\int_0^R\sin^2(q_{\rm min}\sqrt{R^2-\rho^2})\,\rho\,d\rho. \tag{15}$$

According to Eq. (10), in the ultrarelativistic limit the minimum transferred momentum decreases with the energy as k^{-1}, so that at sufficiently high energies $q_{\rm min}R\ll 1$. Then

$$\sigma_{\rm coh}(a\to b) = \frac{8\pi^3}{k^2}n_0^2\,|f_{a+N\to b+N}(0)|^2R^4 = \frac{9\pi}{2k^2R^2}A^2\,|f_{a+N\to b+N}(0)|^2. \tag{16}$$

In so doing, the magnitude $\Delta\Omega_b = 9\pi/2k^2R^2$ has the meaning of the "effective" solid angle of flight of the final particle b in the vicinity of the "forward" direction.

It should be noted that our consideration relates not only to binary reactions but also to multiparticle coherent processes $a \to b_1 + b_2 + ...b_i$ on nuclei at very high energies. In the general case vector $\mathbf{k}_b$ has the meaning of the total momentum of the system $b = \{b_1, b_2...b_i\}$ with the *effective mass* m_b. In so doing, the magnitude $|f_{a+N\to b+N}(0)|^2$ determines the cross section of the production of the system b, moving as a whole in the "forward" direction, at the collision of particle a with the separate nucleon.

2 Effect of the nucleus matter on coherent processes

In the relations obtained above the multiple scattering of the initial and final particles on nucleons of the nucleus was neglected. This is possible when the mean free paths of particles a and b inside the nucleus are much greater than the nuclear radius R. Actually, the role of the nucleus matter may be essential,— especially in the case of medium and heavy nuclei. For the analysis of the effects of the nucleus matter we will apply the optical model of the nucleus at high energy based on the concept of refraction index [1,2]. Our analysis is close to the Glauber eikonal approach for taking into account the multiple scattering on nucleons in the limit of heavy nuclei [3,4] (see also [5]).

Further we will consider the influence of the nucleus matter for *binary* reactions. According to the known formula for the refraction index, being close to unity, the renormalized momenta of ultrarelativistic particles a and b inside the nucleus can be presented in the form:

$$\widetilde{\mathbf{k}_a} = \mathbf{k}_a + \frac{\mathbf{k}_a}{|\mathbf{k}_a|}\chi_a(\mathbf{r}), \quad \widetilde{\mathbf{k}_b} = \mathbf{k}_b + \frac{\mathbf{k}_b}{|\mathbf{k}_b|}\chi_b(\mathbf{r}),$$

where

$$\chi_a(\mathbf{r}) = \frac{2\pi n(\mathbf{r})}{k} f_{a+N\to a+N}(0), \quad \chi_b(\mathbf{r}) = \frac{2\pi n(\mathbf{r})}{k} f_{b+N\to b+N}(0). \qquad (17)$$

Here, as before, $n(\mathbf{r})$ is the density of nucleons inside the nucleus, $k = E_a$ is the initial energy in the rest frame of the nucleus (laboratory frame); $f_{a+N\to a+N}(0)$ and $f_{b+N\to b+N}(0)$ are the amplitudes of elastic scattering of particles a and b on a nucleon at zero angle in the laboratory frame, connected with the scattering amplitudes on a proton and on a neutron by the relations of the type of Eq. (2); χ_a and χ_b are complex magnitudes characterizing the phase shifts and the absorption of particles a and b at their passage through the nucleus matter. The relations (17) hold at $|\chi_a| \ll k$, $|\chi_b| \ll k$. Using the optical theorem [5], we can rewrite the relations (17) in the form:

$$\chi_a(\mathbf{r}) = i\,\frac{n(\mathbf{r})}{2}(1 - i\,\alpha_a)\sigma_{aN}, \qquad \chi_b(\mathbf{r}) = i\,\frac{n(\mathbf{r})}{2}(1 - i\,\alpha_b)\sigma_{bN}, \tag{18}$$

where

$$\sigma_{aN} = \frac{Z\sigma_{ap} + (A - Z)\,\sigma_{an}}{A}, \qquad \sigma_{bN} = \frac{Z\sigma_{bp} + (A - Z)\,\sigma_{bn}}{A}$$

are the total cross sections of interaction of particles a and b with nucleons, averaged over the protons and neutrons of the nucleus (compare with Eq. (2)), α_a and α_b are the ratios of the real parts of the amplitudes $f_{a+N\to a+N}(0)$ and $f_{b+N\to b+N}(0)$, respectively, to their imaginary parts.

Let us emphasize once again that the results of Section 1 are valid when all effects connected with rescattering of particles in the nucleus matter are practically absent. In this situation the probabilities of absorption of particles a and b and the additional phase shifts at their passage through the nucleus are close to zero. In case of spherical nucleus with the constant density of nucleons $n(\mathbf{r}) \equiv n_0$ (see Eq. (14)), this leads to the restrictions: $|\chi_a|R \ll 1$, $|\chi_b|R \ll 1$ or $L_a \gg R$, $L_b \gg R$, where

$$L_a = \frac{1}{n_0\sigma_{aN}}, \quad L_b = \frac{1}{n_0\sigma_{bN}} \tag{19}$$

are the mean free paths inside the nucleus.

Analysis shows that under the condition

$$|f_{a+N\to b+N}(0)| \ll |f_{a+N\to a+N}(0) - f_{b+N\to b+N}(0)| \tag{20}$$

it is possible to neglect the reverse transition $b \to a$ at the passage of the final particle b through the nucleus. Then taking into account the influence of the nucleus matter on the coherent inelastic processes implies the introduction of the additional complex phase shift into Eq. (13): the exponential factor $\exp(-iq_{\min}z)$ is replaced by $Q = \exp[-iq_{\min}z + i\delta(\boldsymbol{\rho},\, z)]$, where, in general,

$$\delta(\boldsymbol{\rho},\, z) = \int_{-\infty}^{z} \chi_a(\boldsymbol{\rho},\, z')\,dz' + \int_{z}^{\infty} \chi_b(\boldsymbol{\rho},\, z')\,dz' =$$

$$= \int_{-\infty}^{z} [\chi_a(\boldsymbol{\rho},\, z') \; - \chi_b(\boldsymbol{\rho},\, z')]\,dz' + \int_{-\infty}^{\infty} \chi_b(\boldsymbol{\rho},\, z')\,dz'; \tag{21}$$

in so doing, the quantities $\chi_a(\boldsymbol{\rho}, z')$ and $\chi_b(\boldsymbol{\rho}, z')$ are described by Eqs. (18) with the density of nucleons $n(\mathbf{r}) \equiv n(\boldsymbol{\rho}, z')$. It follows from Eq.(21) that at $n(\rho, z) = n_0$ in the interval $0 \le |z| \le \sqrt{R^2 - \rho^2}$ and $n(\rho, z) = 0$ outside this interval (the spherical nucleus with the uniform distribution of nucleons; $\rho = |\boldsymbol{\rho}|$) we have

$$\delta(\rho, z) = (\chi_a - \chi_b)\, z + (\chi_a + \chi_b)\sqrt{R^2 - \rho^2}\,, \qquad 0 \le |z| \le \sqrt{R^2 - \rho^2}. \quad (22)$$

Here χ_a and χ_b are determined by Eqs. (18) at $n(\mathbf{r}) = n_0$.

As a result, the expression for the effective cross section of the coherent reaction $a \to b$ on the nucleus takes the following form:

$$\sigma_{\rm coh}(a \to b) = \frac{8\pi^3}{k^2} n_0^2 \, |f_{a+N\to b+N}(0)|^2 \times$$

$$\times \int_0^R \left\{ \left| \int_0^{2\sqrt{R^2-\rho^2}} \exp[-i(q_{\rm min} + \Delta\chi)\, z]\, dz \right|^2 \cdot |\exp(2i\chi_b \sqrt{R^2 - \rho^2})|^2 \right\} \rho\, d\rho. \quad (23)$$

Let us note that the quantity $\mathrm{Re}\,(\Delta\chi) = \mathrm{Re}\,(\chi_b - \chi_a)$ determines the additional longitudinal transferred momentum connected with the presence of the matter. After the integration over z we obtain

$$\sigma_{\rm coh}(a \to b) = \frac{8\pi^3}{k^2} n_0^2 \, \frac{|f_{a+N\to b+N}(0)|^2}{|q_{\rm min} + \Delta\chi|^2} \times$$

$$\times \int_0^R \left| \exp[-2i\,(q_{\rm min} - \chi_a)\sqrt{R^2 - \rho^2}] - \exp[2i\chi_b\sqrt{R^2 - \rho^2}] \right|^2 \rho\, d\rho. \quad (24)$$

Taking into account Eqs. (10), (17) and (18), it follows from Eq. (24) that

$$\sigma_{\rm coh}(a \to b) = \frac{32\pi^3 n_0^2 \, |f_{a+N\to b+N}(0)|^2}{|m_a^2 - m_b^2 + 4\pi n_0 (f_{b+N\to b+N}(0) - f_{a+N\to a+N}(0))|^2} \times$$

$$\times \int_0^R \Big\{ \exp(-2n_0\sigma_{aN} r) + \exp(-2n_0\sigma_{bN} r) -$$

$$-2\exp[-n_0(\sigma_{aN} + \sigma_{bN}) r] \cos\left[\left(\frac{m_a^2 - m_b^2}{2k} + n_0\sigma_{aN}\alpha_a - n_0\sigma_{bN}\alpha_b \right) r \right] \Big\} r\, dr. \quad (25)$$

At very high energies the minimum longitudinal momentum transferred to the nucleus tends to zero and the terms in Eq.(25), depending on the masses of particles a and b, disappear.

3 Dependence of the cross sections of inelastic coherent processes on the nuclear radius

In the case of medium and heavy nuclei the radius of the nucleus $R \approx 1.1 \times 10^{-13}\, A^{1/3}$ cm; then, according to Eq. (14), the density of nucleons, incorporated in Eq. (24), amounts to $n_0 \approx 0.28 \times 10^{39}$ cm^{-3}.

It follows from Eqs. (25) and (19) that when the mean free paths are different, and both of them are small as compared to the nuclear radius ($L_a \neq L_b$, $L_a \ll R$, $L_b \ll R$), the coherent processes are conditioned only by the peripheral collisions of the initial particle a with the nucleons located in the surface layer of the nucleus. In the considered case, neglecting the particle masses (see the remark after Eq.(25)), the expression for the effective cross section $\sigma_{\rm coh}(a \to b)$ at $f_{b+N\to b+N}(0) \neq f_{a+N\to a+N}(0)$ takes the following form:

$$\sigma_{\rm coh}(a \to b) = \pi \frac{|f_{a+N\to b+N}(0)|^2}{|f_{b+N\to b+N}(0) - f_{a+N\to a+N}(0)|^2} \times$$

$$\times \left[\frac{L_a^2}{2} + \frac{L_b^2}{2} + 4\, L_a^2 L_b^2 {\rm Re} \left(\frac{1}{L_a + L_b + i(L_a\alpha_b - L_b\alpha_a)} \right)^2 \right]. \qquad (26)$$

Let us consider now the situation when the total cross section of the interaction of the initial particle a with nucleons is small, so that

$$\sigma_{aN} \ll \sigma_{bN}, \quad L_a \gg R, \quad L_b <\sim R;$$

in doing so, according to the condition (20), the relation $|f_{a+N\to b+N}(0)| \ll |f_{b+N\to b+N}(0)|$ should hold. In particular, we can deal with the coherent production of vector mesons ρ^0, ω, ϕ at the interaction of very high energy photons with nuclei.

In the considered case Eq. (25) (without the terms, depending on the masses m_a and m_b; see the remark after Eq. (25)) gives:

$$\sigma_{\rm coh}(a \to b) = \pi R^2 \left| \frac{f_{a+N\to b+N}(0)}{f_{b+N\to b+N}(0)} \right|^2 \times$$

$$\times \left\{ 1 + \frac{1}{x^2} \left[\frac{1}{2}(1 - e^{-2x}) - 4 \frac{1-\alpha^2}{(1+\alpha^2)^2} (1 - e^{-x} \cos \alpha x) - \frac{8\alpha}{(1+\alpha^2)^2} e^{-x} \sin \alpha x \right] + \right.$$

$$\left. + \frac{1}{x} \left[\frac{4}{1+\alpha^2} e^{-x} \cos \alpha x - \frac{4\alpha}{1+\alpha^2} e^{-x} \sin \alpha x - e^{-2x} \right] \right\}, \qquad (27)$$

where $\alpha \equiv \alpha_b$, $x = n_0 \sigma_{bN} R = R/L_b$. At $x \gg 1$ (large cross sections σ_{bN}, heavy nuclei) we obtain the simple expression

$$\sigma_{\rm coh}(a \to b) = \pi R^2 \left| \frac{f_{a+N\to b+N}(0)}{f_{b+N\to b+N}(0)} \right|^2 . \quad (28)$$

Let us emphasize that, according to Eq. (28), the effective cross section of the coherent process $a \to b$ on a nucleus at very high energies has the *same* dependence on the number of nucleons (proportional to $A^{2/3}$) as the cross section of scattering of the final particle b on the "black" nucleus, despite the smallness of the cross section of interaction of the initial particle a (for example, γ quantum) with a separate nucleon (in connection with this, see [6,7]).

For the coherent process $\gamma \to \rho^0$ on the lead nucleus ($R = 1.1 \cdot 10^{-13} A^{1/3}$ cm ≈ 6.5 Fm, $L_\rho \sim 1.5$ Fm, $|f_{\gamma+N\to\rho+N}(0)/f_{\rho+N\to\rho+N}(0)|^2 \sim 10^{-3}$), the formula (28) is applicable at the energies of γ-quanta above several tens of GeV in the nucleus rest frame ($k \gg m_\rho^2 L_\rho \sim 4.5$ GeV). In doing so, $\sigma_{\rm coh}(\gamma + Pb \to \rho^0 + Pb) \sim 1.3$ mbn.

When, on the contrary,

$$\sigma_{aN} \gg \sigma_{bN}, \quad L_b \gg R, \quad L_a \sim R, \quad |f_{a+N\to b+N}(0)| \ll |f_{a+N\to a+N}(0)|,$$

then the effective cross section of the coherent production of the particle b is described by the same formulae (27), (28), in which one should take $x = R/L_a$, $\alpha \equiv \alpha_a$ and replace the amplitude $f_{b+N\to b+N}(0)$ by $f_{a+N\to a+N}(0)$.

It should be emphasized that at $L_a \gg R$, $L_b \ll R$ the coherent process $a \to b$ is conditioned by the interaction of particle a with nucleons located near the surface of the nucleus in the back hemisphere. On the contrary, at $L_a \ll R$, $L_b \gg R$ this coherent process is conditioned by the interaction of particle a with nucleons located in the vicinity of the nuclear surface in the front hemisphere.

Taking into account the equality

$$f_{b+N\to b+N}(0) = i \frac{k}{4\pi} \sigma_{bN}(1 - i\,\alpha_b), \quad (29)$$

following from the optical theorem [5], it is easy to verify that the expansion of expression (27) into the power series over the parameter x leads at $x \ll 1$ to the relation (16), as one would expect at the conditions $L_a \gg R, L_b \gg R$. In this limit, $\sigma_{\rm coh}(a \to b)$ is proportional to R^4 (or to $A^{4/3}$).[1)]

[1)] Let us recall: we have assumed in the present work that the characteristic momenta

Let us note that the ratio of the values of the cross sections calculated according to the formulae (28) and (16), respectively, is the following, taking into account Eqs. (19) and (29):

$$\eta_b = \frac{k^2}{8\pi^2 |f_{b+N\to b+N}(0)|^2 \, n_0^2 \, R^2} = 2 \left(\frac{L_b}{R}\right)^2 \frac{1}{1+|\alpha_b|^2}. \tag{30}$$

It is clear that the factor η_b has the magnitude of the order of the squared ratio of the "transparency" volume for particle b in the vicinity of the back hemisphere of the nuclear surface to the total volume of the nucleus. At $L_a \ll R$, $L_b \gg R$ the ratio of the corresponding cross sections $\eta_a \sim (L_a/R)^2$ has the analogous meaning with reference to particle a in the vicinity of the front hemisphere of the nuclear surface.

In the given report we have performed the concrete calculations for the case of a spherical nucleus with the sharp boundary and the constant nucleon density. However, our general relations contain the nucleon density depending on coordinates (see Eqs. (13), (17), (19)) and make it possible, in principle, to take into account the role of the nuclear surface. It is evident that when the thickness of the boundary layer is very small as compared with the radius of the nucleus core, then expression (16) at $L_a \gg R$, $L_b \gg R$ does not change practically. But, in the case of very small free paths, the "transparency" parameters η_b or η_a and, hence, the cross section of the coherent inelastic process can depend essentially on the concrete structure of the surface of the nucleus.

4 The case of collisions of two ultrarelativistic nuclei

At collisions of two ultrarelativistic nuclei, the coherent inelastic processes of the type $N \to b = N + c$ ($c = \pi, 2\pi, ...$) on the nucleus 2, induced by the nucleons of nucleus 1, and the analogous ones on the nucleus 1, induced by the nucleons of nucleus 2, may take place . In doing so, the nucleus being incident in fact (nucleus 1 in the first case and nucleus 2 in the second case) is disintegrated and, thus, the contributions of constituent

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$\widetilde{q_N}$, transferred to a nucleon in the reaction $a+N \to b+N$ on a separate nucleon, are much greater than the characteristic momenta of the order of $1/R$ transferred to a nucleus (see Eq. (12) and further). When, on the contrary, $\widetilde{q_N} \ll 1/R$, the effective cross section of the coherent reaction is proportional just to the nucleon number squared A^2. In particular, the cross section of photoproduction of electron–positron pairs [8] or positronium [9] in the Coulomb field of a nucleus at ultrarelativistic energies is proportional to the proton number squared Z^2, since in the framework of our consideration in this case the amplitude of the corresponding process on a neutron, incorporated in Eq. (2), is equal to zero.

nucleons into the effective cross section of the coherent process are summed up incoherently. This holds due to the following : if the coherent process takes place, e.g., on the nucleus 2, then in the rest frame of this nucleus the energy transfer to the nucleons at very high energies equals zero in fact, whereas the longitudinal transferred momentum amounts to :

$$|\, q_{||}\,| = |\, q_{\min}\,| << \frac{1}{R}\,, \tag{31}$$

where R is the nuclear radius . Then, according to the Lorentz transformation, in the rest frame of the incident nucleus 1 we have :

$$|\, \tilde{q}_{||}\,| = \gamma |\, q_{\min}\,| = \frac{|\, m_N^2 - m_{N+c}^2\,|}{m_N}\,. \tag{32}$$

Here γ is the Lorentz factor . In case of sufficiently large differences of the masses m_N and m_{N+c}, the magnitude $|\, \tilde{q}_{||}\,| >> \frac{1}{R}$. Thus, we obtain :

$$\sigma_{\rm coh}^{(A_1, A_2)} = A_1\, \sigma_{\rm coh}^{(2)}(N \to N + c) + A_2\, \sigma_{\rm coh}^{(1)}(N \to N + c)\,, \tag{33}$$

where A_1 and A_2 are respective numbers of nucleons in the nuclei 1 and 2, and $\sigma_{\rm coh}(N \to N + c)$ is determined in accordance with the general formula for the cross section of the coherent inelastic process at the particle collision with a nucleus. Both the nuclei may remain unchanged, if the mass difference $|\, m_N - m_{N+c}\,|$ is very small - for example, in the case of production of electron-positron pairs at the collision of two ultrarelativistic nuclei ($c = e^+e^-$) .

5 Summary

In the present report (see also, e.g., our works [10 -13]) the coherent processes at the interaction of ultrarelativistic particles with atomic nuclei are investigated. The role of these processes essentially increases at very high energies due to the fact that the minimal momentum, transferred to a nucleon, tends to zero with increasing energy. Using the concept of refraction index, the analysis of influence of the nucleus matter on coherent reactions is performed. Analytical expressions for the effective cross sections of coherent inelastic processes are obtained, depending on the nuclear radius and the mean free paths of the initial and final particles in the nucleus matter .

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