# Effective kinetic description of the early-time dynamics in heavy-ion collisions

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NT, R. Venugopalan, Phys.Rev. D95 (2017) 094009 [arXiv:1703.01372]

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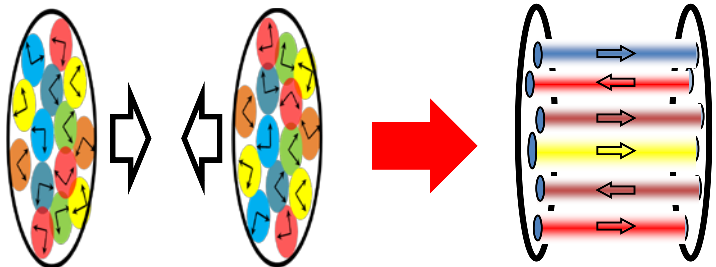
# Early times in heavy-ion collisions

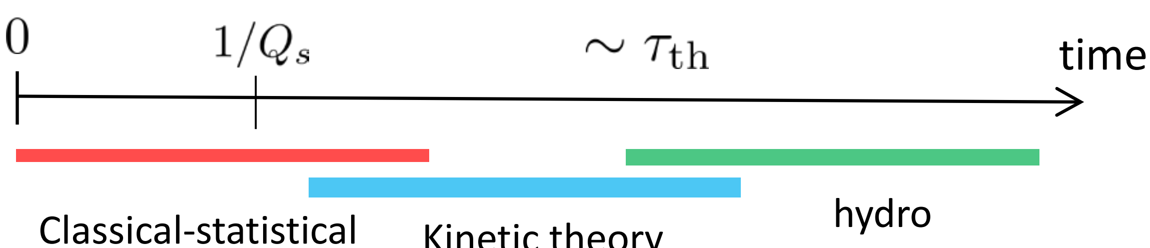



strong color fields $A \sim Q_s/g$

high occupancy $f_g \sim 1/g^2$

Strongly interacting overoccupied gluonic plasma is produced by a collision.
How does the system evolve toward thermalization or hydrodynamization?

QCD-based theoretical descriptions




$f_g \gg 1$

$1/\alpha_s \gg f_g$

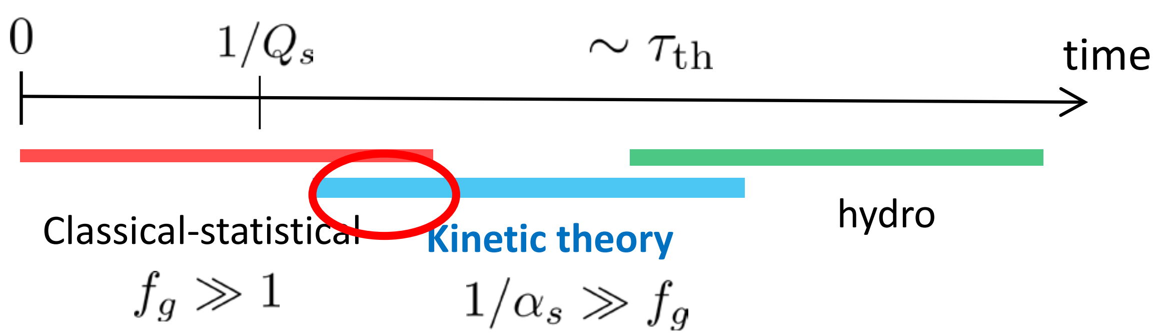
Epelbaum, Gelis (2013)
Berges et al. (2014)

Kurkela, Zhu (2015)
Keegan et al. (2015)

# Early time dynamics of quarks

- Less understood compared with gauge fields
- Important implications
    - chemical equilibrium
    - electromagnetic probes
    - chiral magnetic effect

Investigate the early-time dynamics of **quarks** and **overoccupied gluons** by using **kinetic theory**



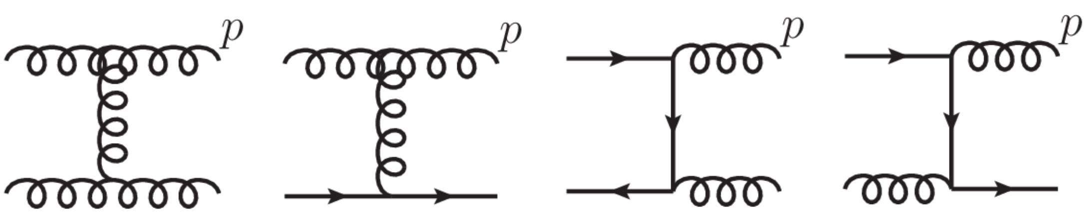
# Kinetic equations

$$\left(\frac{\partial}{\partial\tau}-\frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right)f_g(\tau,\boldsymbol{p})=C_{\text{gluon}}[f_g,f_q]$$

$$\left(\frac{\partial}{\partial\tau}-\frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right)f_q(\tau,\boldsymbol{p})=C_{\text{quark}}[f_q,f_g]$$

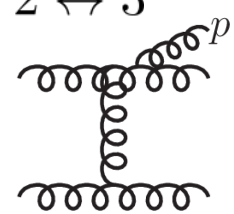
Collision processes for $C_{\text{gluon}}$

$2\leftrightarrow 2$




etc.

$2\leftrightarrow 3$ $(n\leftrightarrow n+1)$




etc.

the same order in g
due to IR/collinear enhancement
Arnold, Moore, Yaffe (2003)

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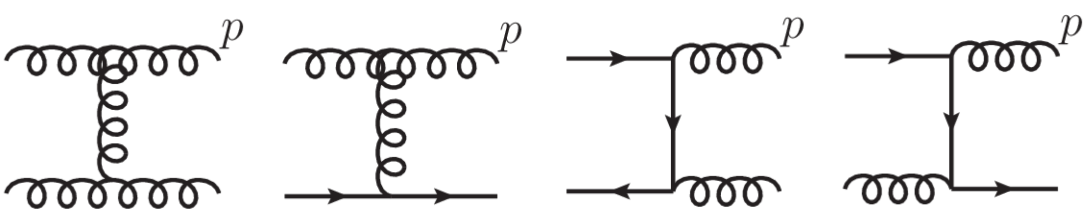


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For $\tau \lesssim Q_s^{-1}\alpha_s^{-3/2}$ , the 2-2 processes play a dominant role.
Baier, Mueller, Schiff, Son (2001)

# Kinetic equations

$$\left(\frac{\partial}{\partial\tau} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f_g(\tau, \boldsymbol{p}) = C_{\rm gluon}[f_g, f_q]$$

$$\left(\frac{\partial}{\partial\tau} - \frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right) f_q(\tau, \boldsymbol{p}) = C_{\rm quark}[f_q, f_g]$$

Collision processes for $C_{\rm gluon}$

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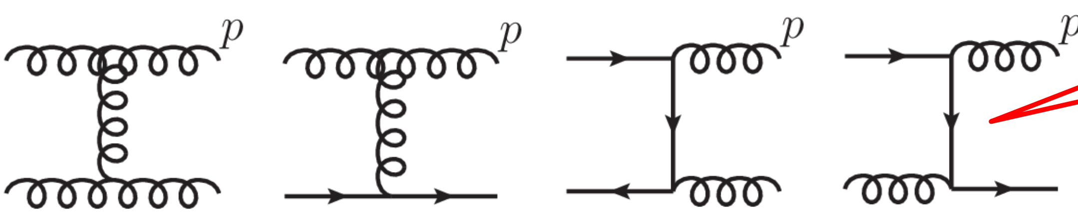
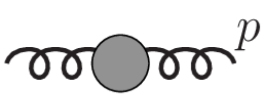
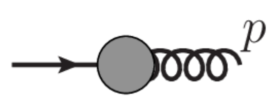



exchange momentum $\sim m_D \ll Q_s$

small-angle approximation




$C \sim \nabla_p^2 f(\tau, \boldsymbol{p})$




source terms

Mueller (2000)
Blaizot, Wu, Yan (2014)

# Kinetic equations

$$\left(\frac{\partial}{\partial\tau}-\frac{p_z}{\tau}\frac{\partial}{\partial p_z}\right)f_g(\tau,\boldsymbol{p})=-\nabla_p\cdot\boldsymbol{J}_g+S_g$$

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Fokker-Plank equations

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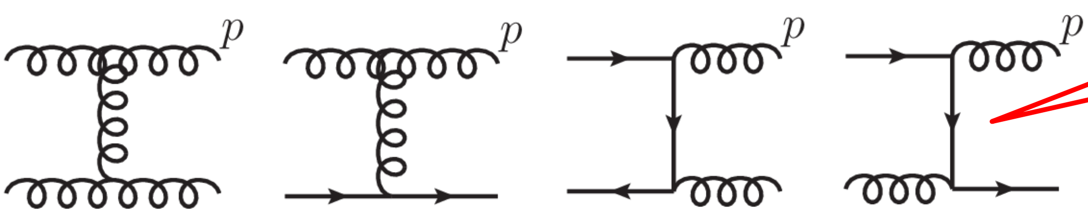


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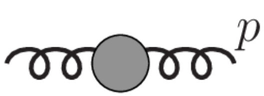


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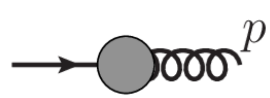


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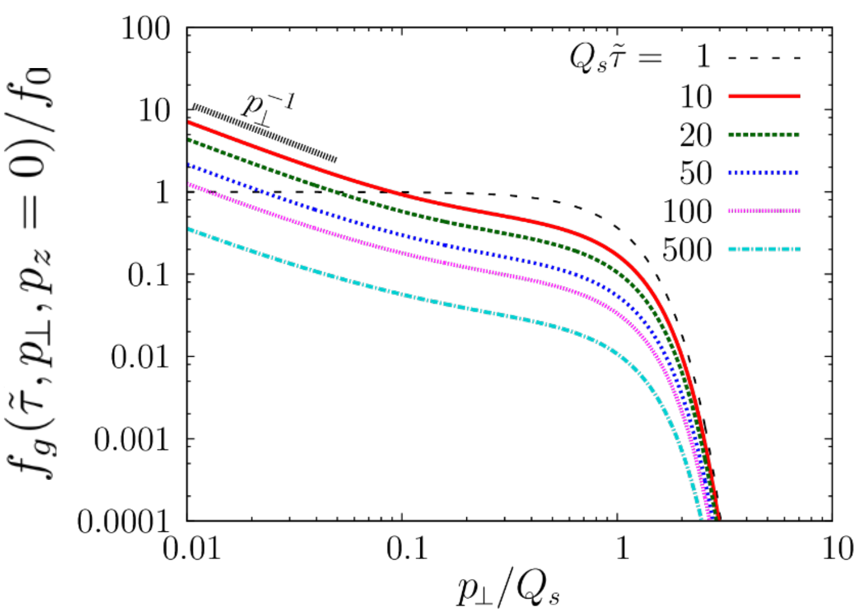
# Gluon distribution

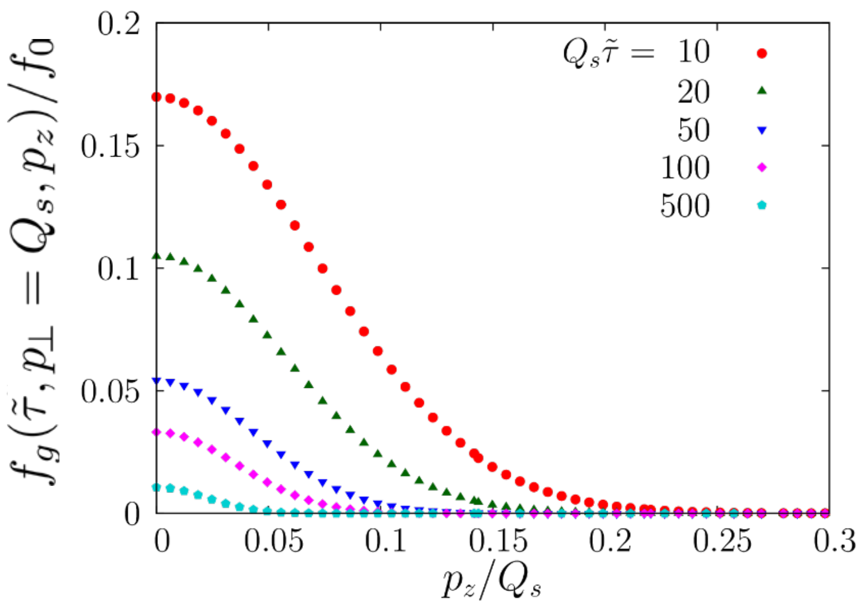
initial distribution $f_g(\tau_0, p_\perp, p_z) = f_0 \exp\left[-(p_\perp^2 + (\xi_0 p_z)^2)/Q_s^2\right]$

$f_0 = \dfrac{n_0}{g^2}$ with $g = 10^{-3}$ and $n_0 = 0.1$ (overoccupied kinetic regime) $\xi_0 = 2$

transverse momentum distribution at $p_z = 0$




longitudinal momentum distribution at $p_\perp = Q_s$



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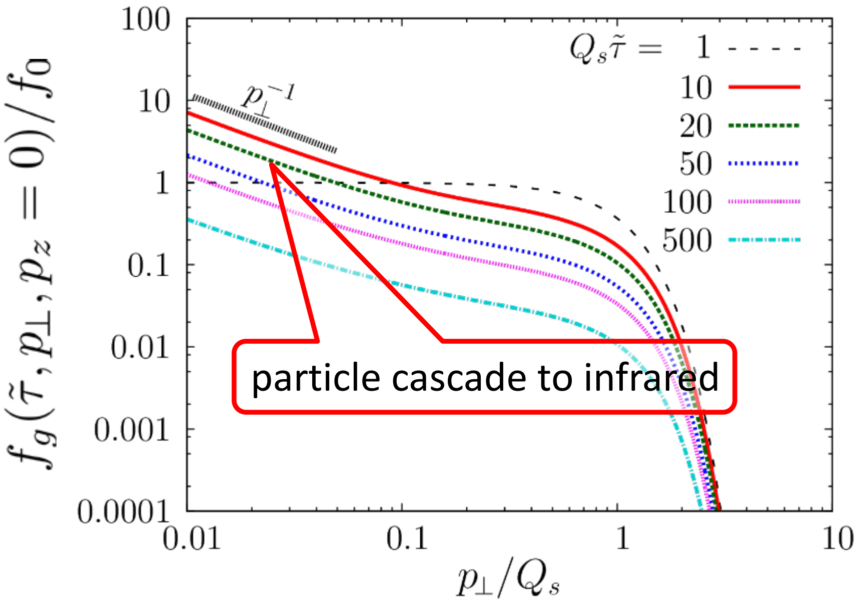


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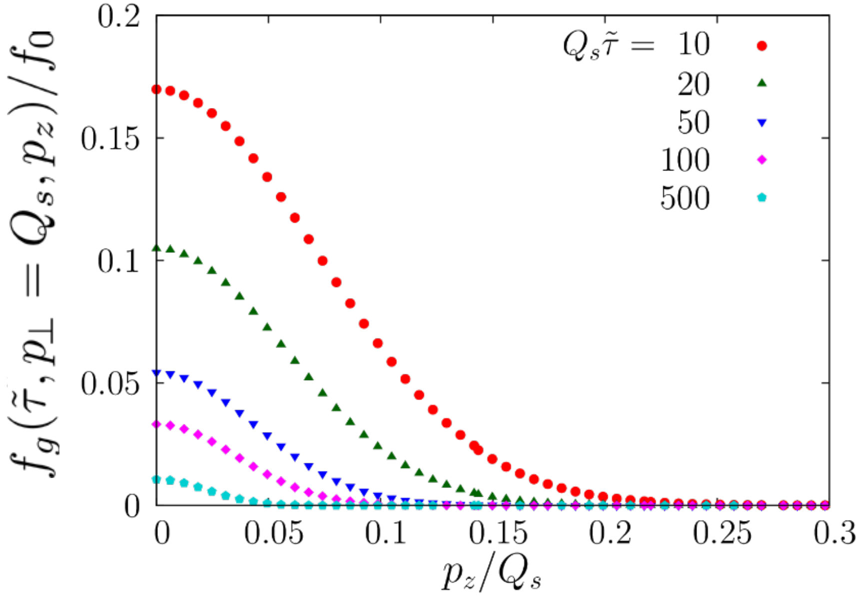

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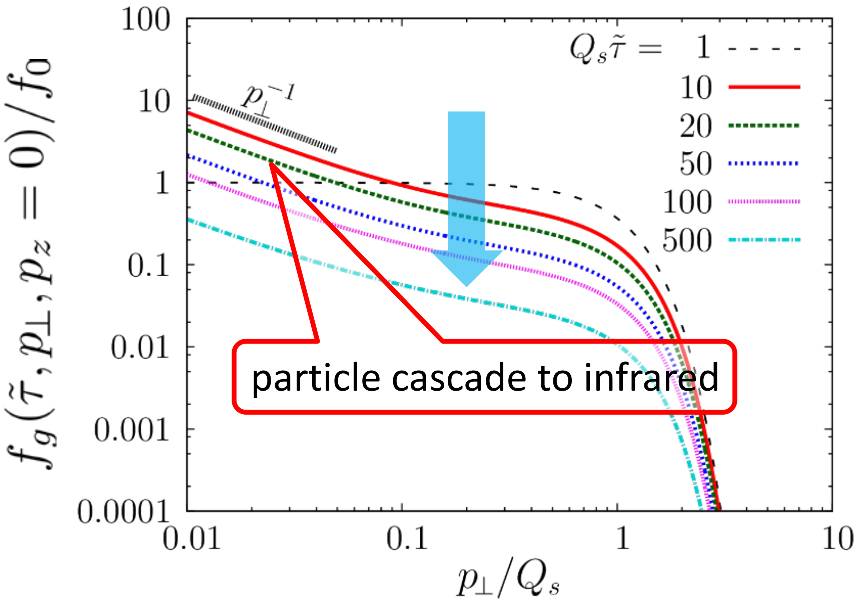


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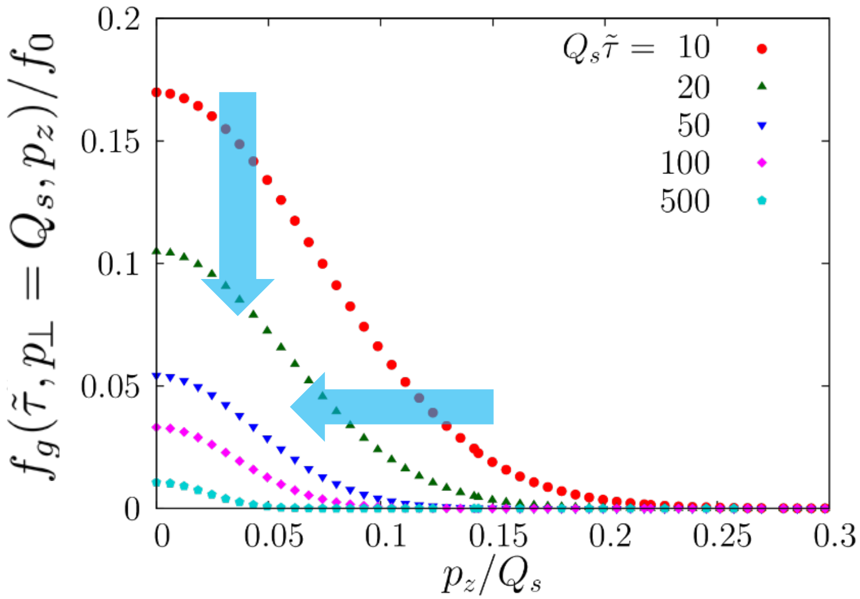
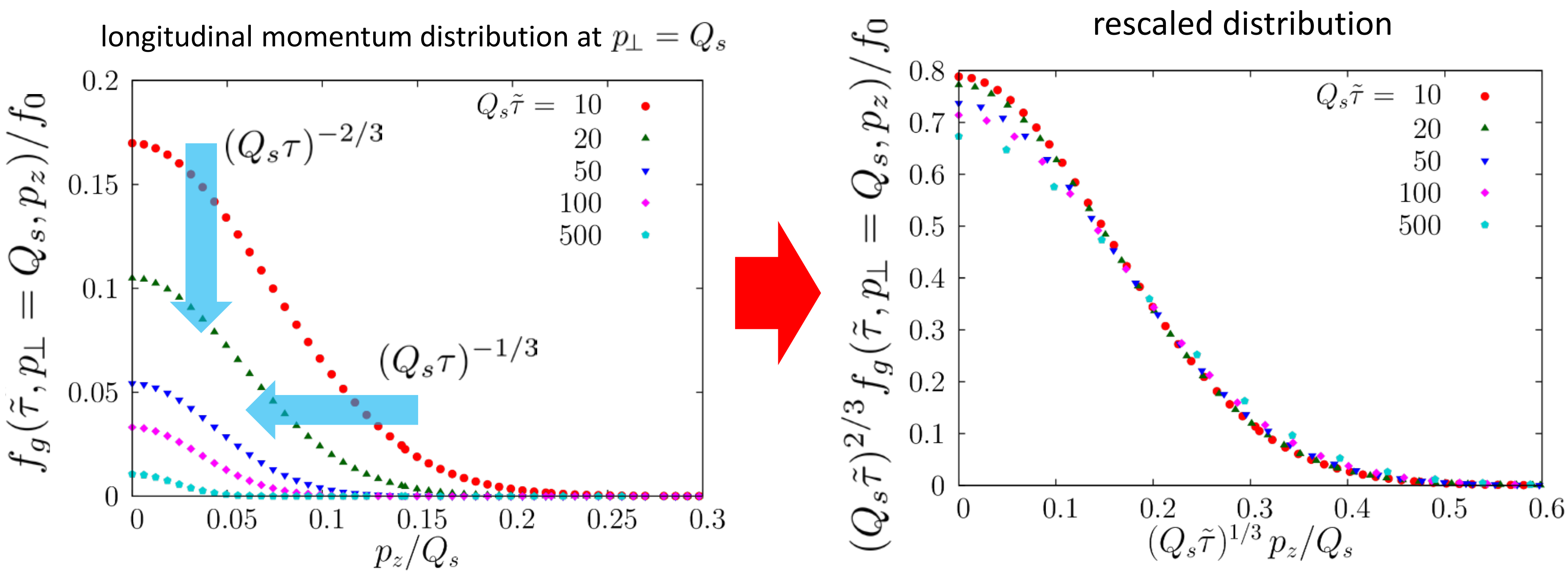


# Self-similar scaling behavior




Scaling law $f_g(\tau, p_\perp, p_z) = (Q_s\tau)^{-2/3} f_S\left(p_\perp, (Q_s\tau)^{1/3} p_z\right)$

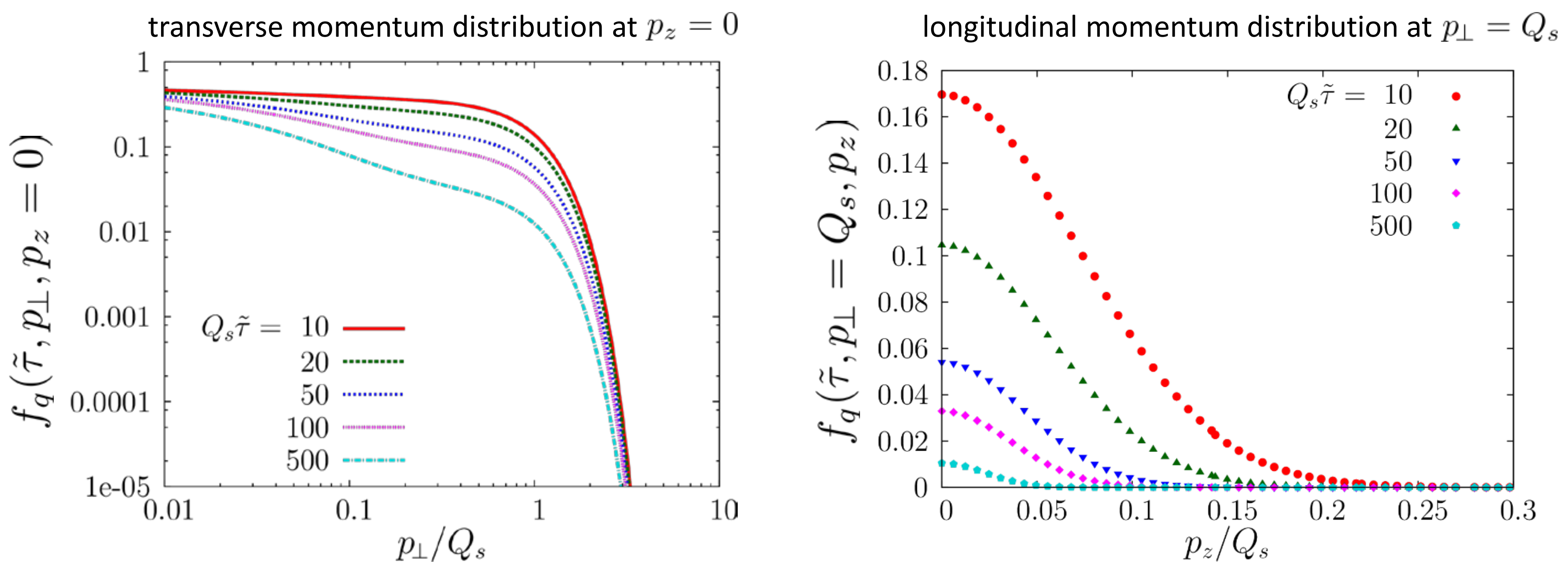
- Universal attractor observed in classical statistical simulations by Berges et al. (2014)
- Consistent with the bottom-up thermalization scenario by Baier, Mueller, Schiff, Son (2001)

# Quark distribution

massless $N_f = 3$

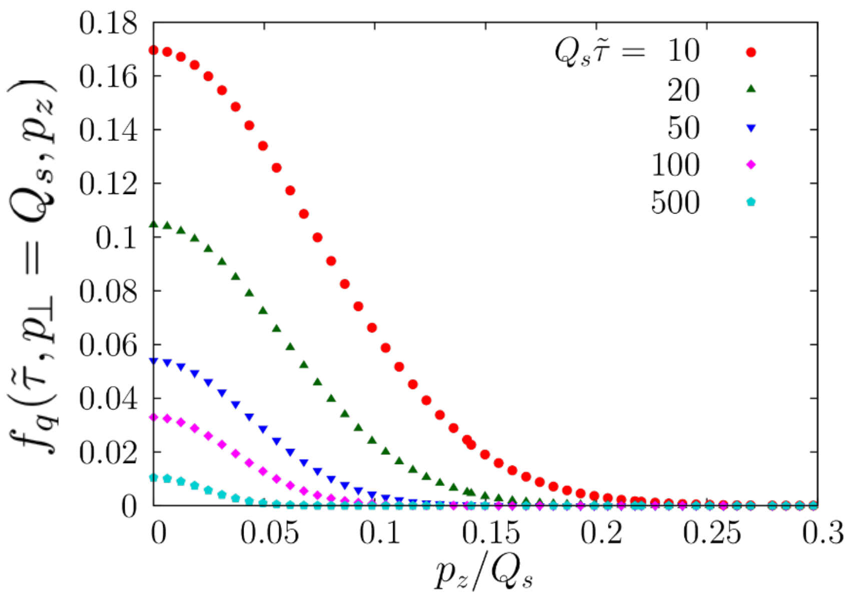
initial distribution $f_q(\tau_0, p_\perp, p_z) = F_0 \exp\left[-(p_\perp^2 + (\xi_0 p_z)^2)/Q_s^2\right]$

$F_0 = 0.5$ (assuming early-time quark production $\tau \lesssim Q_s^{-1}$)

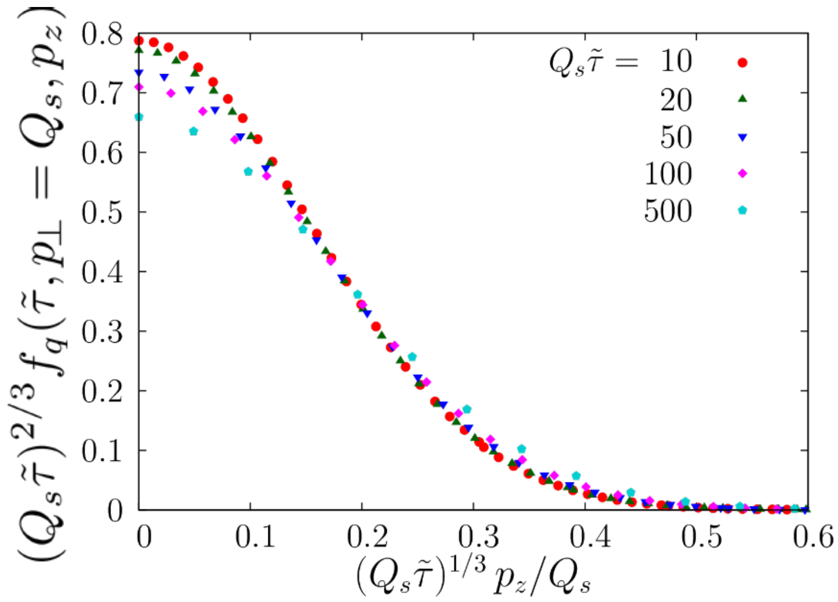


# Self-similar scaling behavior for quarks

longitudinal momentum distribution at $p_\perp = Q_s$




rescaled distribution




the same scaling law as gluons

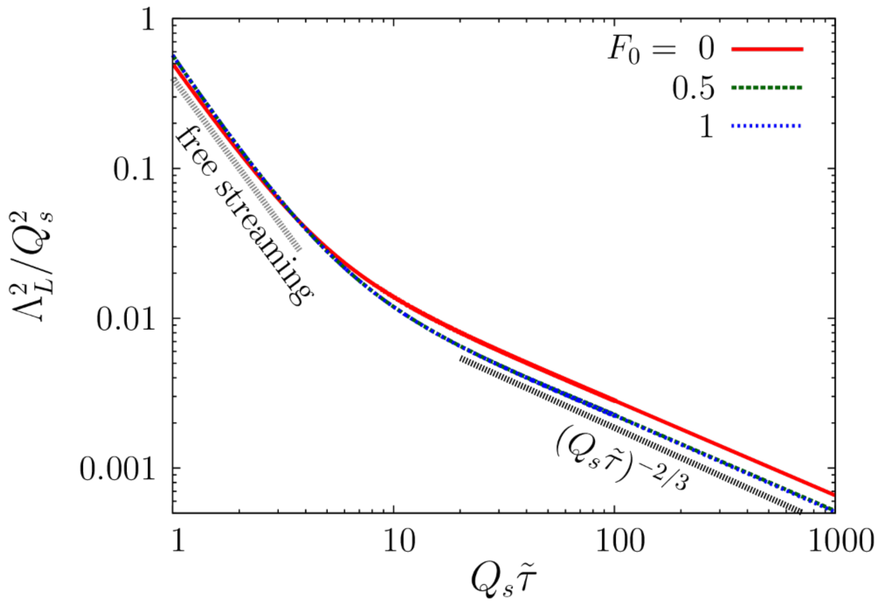
$$f_q(\tau, p_\perp, p_z) = (Q_s\tau)^{-2/3} F_S\left(p_\perp, (Q_s\tau)^{1/3} p_z\right)$$

- Quarks undergo the same small-angle scattering as gluons and it is Bose-enhanced.
- The 2-3 processes would not alter the scaling behavior.
- The scaling behavior appears independently of the initial quark occupancy $F_0$.
- Implication for photon production
  - Early-time photon production is parametrically as important as thermal one.

Berges, Reygers, Tanji, Venugopalan (2017)

# Self-similar scaling behavior for quarks

evolution of the typical longitudinal scale

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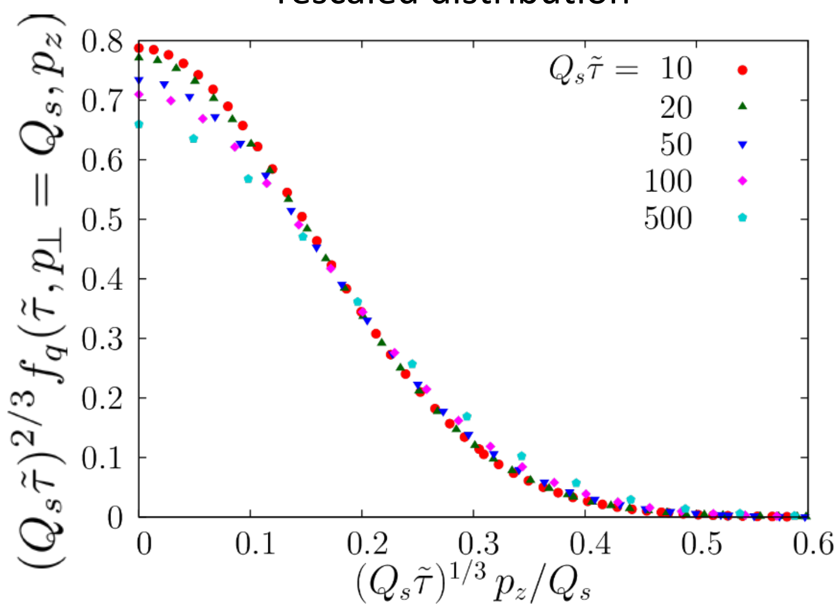


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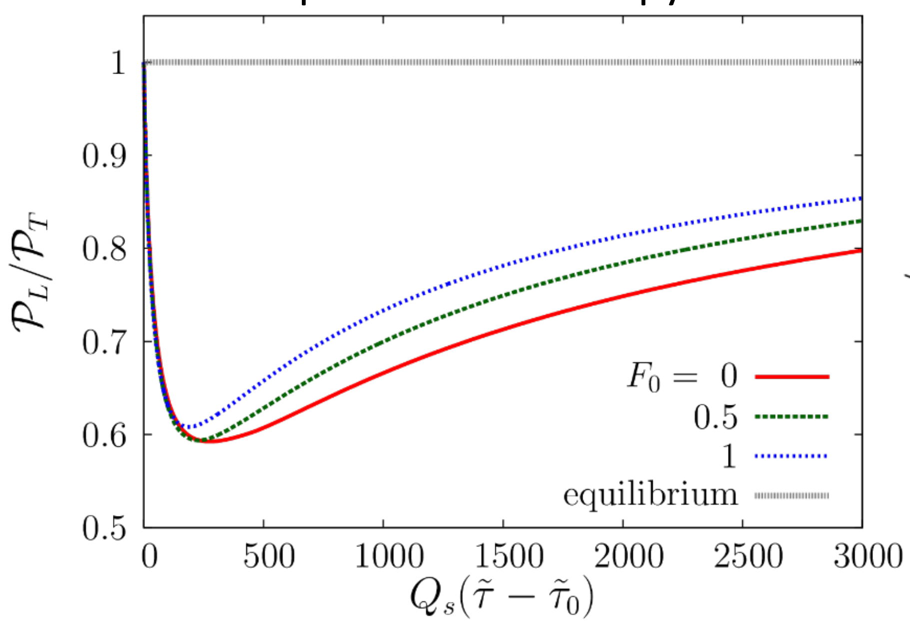
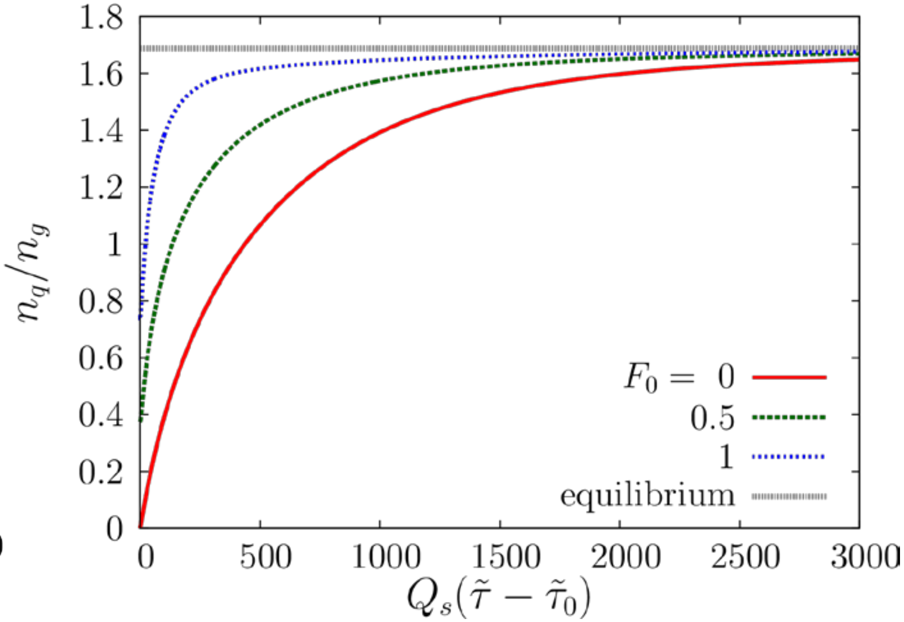
# Toward equilibrium

Crank up the coupling $g = 0.5$

pressure anisotropy




quark-gluon number ratio




- Only with the 2-2 scatterings, the equilibration takes a parametrically long time.

  $\sim Q_s^{-1} \exp(\alpha_s^{-1/2})$

- The number ratio approaches its equilibrium value more quickly than the pressure ratio, because equipartition is not hindered by the expansion.

# Summary and Outlook

- ➢ Far-from-equilibrium dynamics of quarks and overoccupied gluons is investigated by using the kinetic equations with the small-angle approximation.
- ➢ In weak coupling, gluons show the self-similar scaling behavior, that is consistent with the bottom-up thermalization scenario.
- ➢ Quarks obey the same scaling behavior as gluons.

- Later stages of the bottom-up thermalization scenario $Q_s^{-1}\alpha_s^{-3/2} \lesssim \tau \lesssim Q_s^{-1}\alpha_s^{-13/5}$, the roles of the 2-3 processes, chemical equilibrium.
- Classical-statistical simulations including quarks.