Effective kinetic description of the early-time dynamics in heavy-ion collisions

Naoto Tanji
Institut für Theoretische Physik
Heidelberg University

Early times in heavy-ion collisions

Strongly interacting overoccupied gluonic plasma is produced by a collision. How does the system evolve toward thermalization or hydrodynamization?

QCD-based theoretical descriptions

- Classical-statistical: $f_g \gg 1$
  - Epelbaum, Gelis (2013)
  - Berges et al. (2014)

- Kinetic theory: $1/\alpha_s \gg f_g$
  - Kurkela, Zhu (2015)
  - Keegan et al. (2015)

Strong color fields: $A \sim Q_s/g$

High occupancy: $f_g \sim 1/g^2$
Early time dynamics of quarks

- Less understood compared with gauge fields
- Important implications
  - chemical equilibrium
  - electromagnetic probes
  - chiral magnetic effect

Investigate the early-time dynamics of quarks and overoccupied gluons by using kinetic theory
Kinetic equations

\[
\left( \frac{\partial}{\partial \tau} - \frac{p_z}{\tau} \frac{\partial}{\partial p_z} \right) f_g(\tau, p) = C_{\text{gluon}}[f_g, f_q].
\]

\[
\left( \frac{\partial}{\partial \tau} - \frac{p_z}{\tau} \frac{\partial}{\partial p_z} \right) f_q(\tau, p) = C_{\text{quark}}[f_q, f_g].
\]

Collision processes for $C'_{\text{gluon}}$

2 $\leftrightarrow$ 2

2 $\leftrightarrow$ 3 ($n \leftrightarrow n + 1$)

etc.

the same order in g

due to IR/collinear enhancement

Arnold, Moore, Yaffe (2003)
Kinetic equations

\[
\begin{align*}
\left( \frac{\partial}{\partial \tau} - \frac{p_z}{\tau} \frac{\partial}{\partial p_z} \right) f_g(\tau, p) &= C_{\text{gluon}}[f_g, f_q]. \\
\left( \frac{\partial}{\partial \tau} - \frac{p_z}{\tau} \frac{\partial}{\partial p_z} \right) f_q(\tau, p) &= C_{\text{quark}}[f_q, f_g].
\end{align*}
\]

Collision processes for \( C_{\text{gluon}} \)

\[ 2 \leftrightarrow 2 \]

\( \begin{array}{c}
\text{etc.} \\
\end{array} \]

For \( \tau \lesssim Q_s^{-1} \alpha_s^{-3/2} \), the 2-2 processes play a dominant role.

Baier, Mueller, Schiff, Son (2001)
Kinetic equations

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\left( \frac{\partial}{\partial \tau} - \frac{p_z}{\tau} \frac{\partial}{\partial p_z} \right) f_g(\tau, p) = C_{\text{gluon}}[f_g, f_q].
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Collision processes for \( C_{\text{gluon}} \)

\[ 2 \leftrightarrow 2 \]

\[ \begin{array}{ccc}
\text{exchange momentum} & \sim m_D & \ll Q_s
\end{array} \]

\[ \text{small-angle approximation} \]

\[ C \sim \nabla_p^2 f(\tau, p) \]

source terms

Mueller (2000)
Blaizot, Wu, Yan (2014)
Kinetic equations

\[
\left( \frac{\partial}{\partial \tau} - \frac{p_z}{\tau} \frac{\partial}{\partial p_z} \right) f_g(\tau, p) = -\nabla_p \cdot J_g + S_g
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Collision processes for $C_{\text{gluon}}$

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\text{small-angle approximation} \\
C \sim \nabla_p^2 f(\tau, p)
\end{array}\]

exchange momentum $\sim m_D \ll Q_s$

source terms

Mueller (2000)
Blaizot, Wu, Yan (2014)
Gluon distribution

initial distribution \( f_g(\tau_0, p_\perp, p_z) = f_0 \exp \left[ -\left( p_\perp^2 + (\xi_0 p_z)^2 \right)/Q_s^2 \right] \)

\[ f_0 = \frac{n_0}{g^2} \] with \( g = 10^{-3} \) and \( n_0 = 0.1 \) (overoccupied kinetic regime) \( \xi_0 = 2 \)

transverse momentum distribution at \( p_z = 0 \)

longitudinal momentum distribution at \( p_\perp = Q_s \)
Gluon distribution

initial distribution \[ f_g(\tau_0, p_\perp, p_z) = f_0 \exp \left[ -\frac{(p_\perp^2 + (\xi_0 p_z)^2)}{Q_s^2} \right] \]

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\[ \xi_0 = 2 \]

transverse momentum distribution at \( p_z = 0 \)

longitudinal momentum distribution at \( p_\perp = Q_s \)

particle cascade to infrared
Gluon distribution

Initial distribution:

\[ f_g(\tau_0, p_\perp, p_z) = f_0 \exp \left[ -\left(\frac{p_\perp^2 + (\xi_0 p_z)^2}{Q_s^2}\right) \right] \]

\[ f_0 = \frac{n_0}{g^2} \]

with \( g = 10^{-3} \) and \( n_0 = 0.1 \) (overoccupied kinetic regime)

\( \xi_0 = 2 \)

Transverse momentum distribution at \( p_z = 0 \):

Longitudinal momentum distribution at \( p_\perp = Q_s \):

Particle cascade to infrared
Self-similar scaling behavior

Scaling law $f_g(\tau, p_\perp, p_z) = (Q_s\tau)^{-2/3} f_S \left( p_\perp, (Q_s\tau)^{1/3} p_z \right)$

- Universal attractor observed in classical statistical simulations by Berges et al. (2014)
- Consistent with the bottom-up thermalization scenario by Baier, Mueller, Schiff, Son (2001)
Quark distribution

massless $N_f = 3$

initial distribution $f_q(\tau_0, p_\perp, p_z) = F_0 \exp \left[ - (p_\perp^2 + (\xi_0 p_z)^2) / Q_s^2 \right]$

$F_0 = 0.5$ (assuming early-time quark production $\tau \lesssim Q_s^{-1}$)
Self-similar scaling behavior for quarks

- Quarks undergo the same small-angle scattering as gluons and it is Bose-enhanced.
- The 2-3 processes would not alter the scaling behavior.
- The scaling behavior appears independently of the initial quark occupancy \( F_0 \).
- Implication for photon production:
  - Early-time photon production is parametrically as important as thermal one.

\[ f_q(\tau, p_\perp, p_z) = (Q_s\tau)^{-2/3} F_S \left( p_\perp, (Q_s\tau)^{1/3} p_z \right) \]

Berges, Reygers, Tanji, Venugopalan (2017)
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Implication for photon production

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Berges, Reygers, Tanji, Venugopalan (2017)
Crank up the coupling $g = 0.5$

- Only with the 2-2 scatterings, the equilibration takes a parametrically long time.
  \[ \sim Q_s^{-1} \exp(\alpha_s^{-1/2}) \]

- The number ratio approaches its equilibrium value more quickly than the pressure ratio, because equipartition is not hindered by the expansion.
Summary and Outlook

- Far-from-equilibrium dynamics of quarks and overoccupied gluons is investigated by using the kinetic equations with the small-angle approximation.

- In weak coupling, gluons show the self-similar scaling behavior, that is consistent with the bottom-up thermalization scenario.

- Quarks obey the same scaling behavior as gluons.

• Later stages of the bottom-up thermalization scenario $Q_s^{-1} \alpha_s^{-3/2} \lesssim \tau \lesssim Q_s^{-1} \alpha_s^{-13/5}$, the roles of the 2-3 processes, chemical equilibrium.

• Classical-statistical simulations including quarks.