# Chiral Magnetic Effect in the Dirac-Heisenberg-Wigner formalism

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07. 07. 2017.
EPS-HEP 2017, Venice

# Chiral Magnetic Effect

What is the Chiral Magnetic Effect?

- Given a background EM magnetic field, and the QCD gauge fields.
- An initially vanishing chiral imbalance could obtain non-zero value due to the interaction with the gauge fields with non-zero $Q_w$ winding number.

$$Q_w = \frac{g^2}{32\pi^2} \int \mathrm{d}^4 x F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a \in \mathbb{Z} \tag{1}$$

Axial charge:

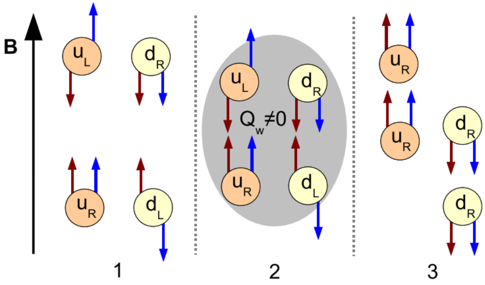
$$(N_L - N_R)_{t=\infty} = 2 N_f Q_w \tag{2}$$

Axial current (on the background field):

$$j^5_\mu = \langle \bar{\psi} \gamma_\mu \gamma_5 \psi \rangle_A \tag{3}$$

D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008).

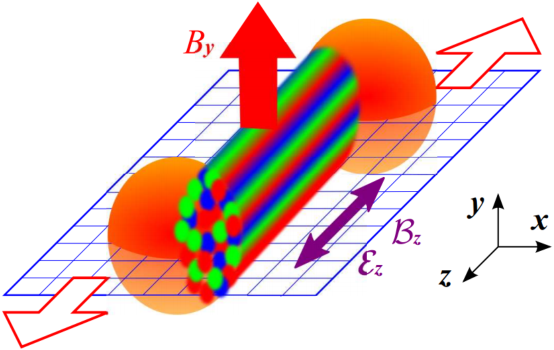
# Chiral Magnetic Effect




1. Chirally neutral mixture in very strong B field: particles constrained to Lowest Landau Level.
2. Gauge interaction with non-zero $Q_w$ fields change chirality.
3. Chirality separation leads to charge separation, that leads to current.

# Chiral Magnetic Effect

Possible realisation in heavy-ion collisions:




- Background: very strong B field due to highly charged nuclei passing near each other.
- Gauge: QCD gluons

# Chiral Magnetic Effect

- Transition between different topologies can happen via tunneling.
- The simplest configuration is a flux-tube, where the gauge fields are $E || B$.
- This can be described by the Schwinger effect $\rightarrow$ connection to pair production.
- Already investigated for constant fields

  Kenji Fukushima, Dmitri E. Kharzeev, and Harmen J. Warringa Phys. Rev. Lett. 104, 212001 2010.

- Main idea: color diagonalisation leads to QED description with $E_z, B_z$ from chromoelectric/magnetic fields and with $B_y$ from EM.

# Chiral Magnetic Effect

Main charachteristics of the CME (electric) current $j_\mu$:

- $E_z = B_z = B_y = 0$, nothing happens, everything is zero
- $E_z = 0, B_z \neq 0, B_y \neq 0$, B fields alone does not create any current
- $E_z \neq 0, B_z = 0, B_y = 0$, E field alone only drives current in its direction
- $E_z \neq 0, B_z \neq 0, B_y = 0$, still nothing...
- $E_z \neq 0, B_z = 0, B_y \neq 0$, still nothing...
- Only in the case, when none of the three is zero, is there a CME current!

# Chiral Magnetic Effect

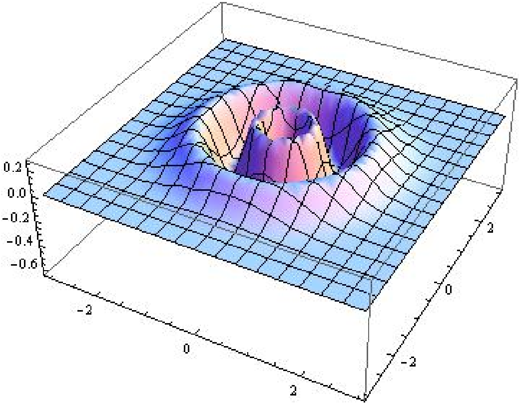
- Q: How can we investigate the time dependence of this process?

- A: Generalizing the Schwinger description as usual:
  Wigner functions in the real time formalism.

# Wigner function

Tool of description: the Wigner function:

- Quantum analogue of the classical phase space distribution.




Wigner function of an n=3 Fock state.

# Wigner function

How it is defined?

- Take the equal time density matrix in terms of 'center of mass' coordinates:

$$\hat{\rho}(\vec{x},\vec{s},t) = e^{-ig\int_{-1/2}^{1/2}\vec{\mathcal{A}}(\vec{x}+\lambda\vec{s},t)\vec{s}\mathrm{d}\lambda}\left[\Psi(\vec{x}+\frac{\vec{s}}{2},t),\bar{\Psi}(\vec{x}-\frac{\vec{s}}{2},t)\right] \tag{4}$$

- Take the expectation value.
- Fourier transform it w.r.t the coordinate difference:

$$W(\vec{x},\vec{p},t) = -\frac{1}{2}\int e^{-i\vec{p}\vec{s}}\langle\Omega|\hat{\rho}(\vec{x},\vec{s},t)|\Omega\rangle \mathrm{d}^3 s \tag{5}$$

# Wigner function

The evolution equation:

$$\color{red}{D_t} W = -\frac{1}{2}\color{red}{\vec{D}_{\vec{x}}}[\gamma^0\vec{\gamma}, W] - im[\gamma^0, W] - i\color{red}{\vec{P}}\{\gamma^0\vec{\gamma}, W\} \tag{6}$$

The equation has the following non-local differential operators:

$$\color{red}{D_t} = \partial_t + g\vec{\mathcal{E}}(\vec{x}, t)\vec{\nabla}_{\vec{p}} - \frac{g\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{E}}(\vec{x}, t)\vec{\nabla}_{\vec{p}} + \ldots \tag{7}$$

$$\color{red}{\vec{D}_{\vec{x}}} = \vec{\nabla}_{\vec{x}} + g\vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} - \frac{g\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} + \ldots \tag{8}$$

$$\color{red}{\vec{P}} = \vec{p} + \frac{g\hbar}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})\vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} + \ldots \tag{9}$$

For spin-1/2, the 4x4 gamma matrix basis is used:

$$W(x, p, t) = \frac{1}{4}\left[\mathbb{1}\mathbb{s} + i\gamma_5\mathbb{p} + \gamma^\mu\mathbb{v}_\mu + \gamma^\mu\gamma_5\mathbb{a}_\mu + \sigma^{\mu\nu}\mathbb{t}_{\mu\nu}\right] \tag{10}$$

# Wigner function

The equation has the following non-local differential operators
for homogeneous E and B:

$$D_t = \partial_t + g\vec{\mathcal{E}}(t)\vec{\nabla}_{\vec{p}} \tag{11}$$

$$\vec{D}_{\vec{x}} = g\vec{\mathcal{B}}(t) \times \vec{\nabla}_{\vec{p}} \tag{12}$$

$$\vec{P} = \vec{p} \tag{13}$$

# Equations of motion for the spin-1/2 Wigner function

## We arrive at a system for 16 unknown real functions:

$$D_t \mathbb{s} - 2\vec{P} \cdot \vec{\mathbb{t}}_1 = 0 \quad (14)$$

$$D_t \mathbb{p} + 2\vec{P} \cdot \vec{\mathbb{t}}_2 = 2m\mathbb{a}_0 \quad (15)$$

$$D_t \mathbb{v}_0 + \vec{D}_{\vec{x}} \cdot \vec{\mathbb{v}} = 0 \quad (16)$$

$$D_t \mathbb{a}_0 + \vec{D}_{\vec{x}} \cdot \vec{\mathbb{a}} = 2m\mathbb{p} \quad (17)$$

$$D_t \vec{\mathbb{v}} + \vec{D}_{\vec{x}} \mathbb{v}_0 + 2\vec{P} \times \vec{\mathbb{a}} = -2m\vec{\mathbb{t}}_1 \quad (18)$$

$$D_t \vec{\mathbb{a}} + \vec{D}_{\vec{x}} \mathbb{a}_0 + 2\vec{P} \times \vec{\mathbb{v}} = 0 \quad (19)$$

$$D_t \vec{\mathbb{t}}_1 + \vec{D}_{\vec{x}} \times \vec{\mathbb{t}}_2 + 2\vec{P} \mathbb{s} = 2m\vec{\mathbb{v}} \quad (20)$$

$$D_t \vec{\mathbb{t}}_2 - \vec{D}_{\vec{x}} \times \vec{\mathbb{t}}_1 - 2\vec{P} \mathbb{p} = 0 \quad (21)$$

# Equations of motion for the $m = 0$ spin-1/2 Wigner function

Simplification: $m = 0$.

Only the vector current / charge and the axial current / charge remains in the equations:

**A system for 8 unknown real functions remains:**

$$D_t \mathbb{v}_0 + \vec{D}_{\vec{x}} \cdot \vec{\mathbb{v}} = 0 \tag{22}$$

$$D_t \mathbb{a}_0 + \vec{D}_{\vec{x}} \cdot \vec{\mathbb{a}} = 0 \tag{23}$$

$$D_t \vec{\mathbb{v}} + \vec{D}_{\vec{x}} \mathbb{v}_0 + 2\vec{P} \times \vec{\mathbb{a}} = 0 \tag{24}$$

$$D_t \vec{\mathbb{a}} + \vec{D}_{\vec{x}} \mathbb{a}_0 + 2\vec{P} \times \vec{\mathbb{v}} = 0 \tag{25}$$

# Numerical Solution

Ingredients of the 3+1 D numerical solver:

- Pseudospectral collocation
- Rational Chebyshev polynomial basis
- 4th order Runge-Kutta
- GPU Acceleration (30x speed up)

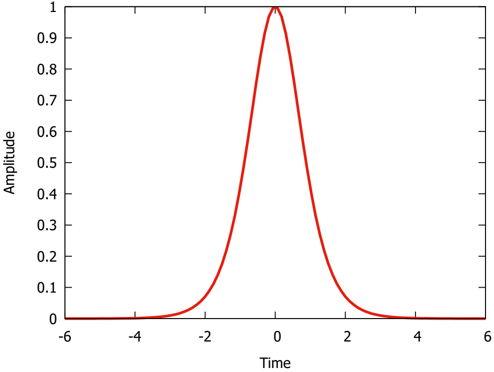
# Sauter field

Start with a toy field that we know very well:

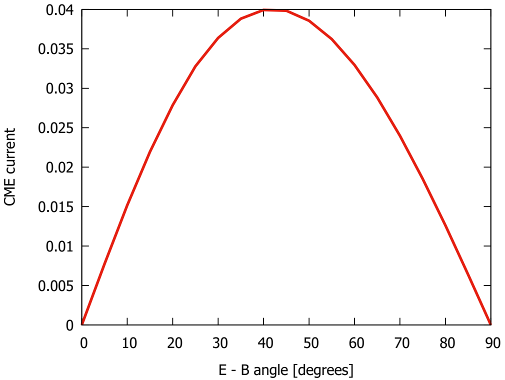
$$S(t) = \cosh^{-2}(t/\tau) \qquad (26)$$

Let the fields be:

$$E_z = A \cdot S(t),$$
$$B_z = A \cdot \cos(\alpha) S(t),$$
$$B_y = A \cdot \sin(\alpha) S(t)$$



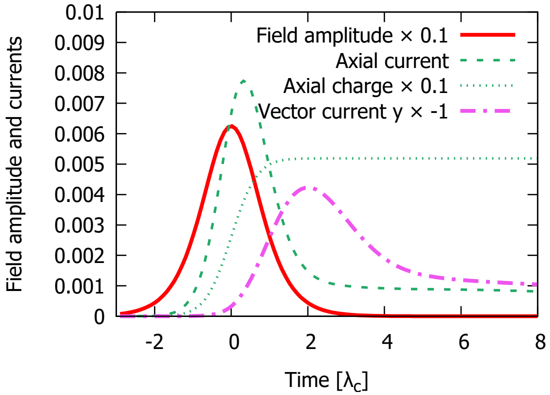
# Sauter field




Only in the case, when none of the three is zero, is there a CME current!

# Results

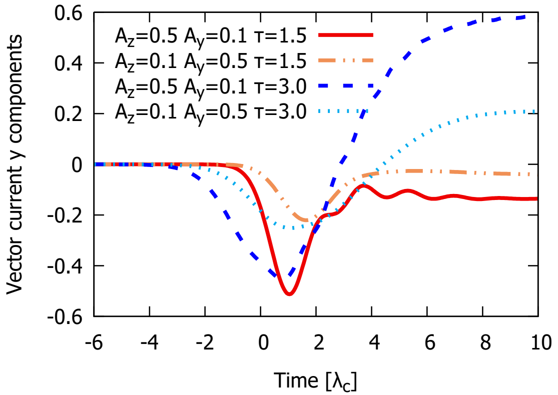
Chiral Magnetic Current formation during the interaction:




Field comes first, then axial current, axial charge separation and finally the electric current!

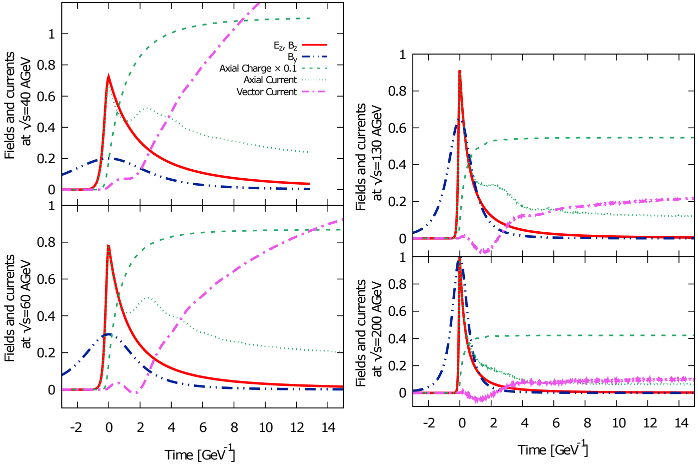
# Results

Interplay of Amplitude and time extent:




Longer time scales lead to larger effect together with a sign change.

CEM effect at different collision energies:




Sign change at 40-60 AGeV, disappearance above 200 AGeV

# Summary

- Heavy-ion collisions at RHIC energies indicated the appearance of a specific phenomena, the Chiral Magnetic Effect, which is generated by a strong gluon field modifying the chirality of the plasma.
- The CME effect can be successfully modelled by the Dirac-Heisenberg-Wigner description of time dependent strong fields.
- The detailed calculations indicate the expected disappearance of CME at energies above 200 AGeV.
- Between 40-60 AGeV we have found an interesting sign change in the CME current.

Supporters: OTKA Grants No. NK-106119, K16-120660, and Wigner GPU Laboratory