Study of sensitivity to anomalous VVH couplings at the ILC

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Introduction

The SM has been successful to describe nature. However, there are phenomena we can’t explain only with the SM. (Dark matter, baryon asymmetry in our universe, … )

The Higgs is the key particle connecting to New Physics. The Higgs is a tool to identify the theory of nature.

The Lorentz structure of the VVH (V=Z/W/γ) couplings plays an important role to probe the new physics beyond the SM. (Higgs CP, the electroweak symmetry breaking mechanism)

In the framework of effective field theory, an effective Lagrangian containing ZZH couplings can be written:

\[ \mathcal{L}_{ZZH} = M_Z^2 \left( \frac{1}{\nu} + \frac{a}{\Lambda} \right) Z_\mu Z^\mu H + \frac{b}{2\Lambda} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} H + \frac{\tilde{b}}{2\Lambda} \tilde{Z}_{\mu\nu} \tilde{Z}^{\mu\nu} H \]

Expected deviations from the SM are small. ( ~ a few % or less )

Precise measurement is necessary, and the ILC is suitable for this purpose.

Tim Barklow et al.
https://agenda.linearcollider.org/event/7371/contributions/37884/

James E. Brau et al., arXiv:1210.0202 [hep-ex]

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Verification of the Lorentz structures

- “$a_z$”: a normalization parameter affecting the overall cross section. (rescales the SM-coupling)
- “$b_z$”: a different CP-even tensor structure affecting momentum and changes angular distribution.
- “$\tilde{b}_z$”: a CP-violating parameter affecting angular/spin correlations.

$$e^+e^- \rightarrow ZH \rightarrow l^+l^-H$$

- $\cos \theta_z$: a production angle of the Z.
- $\cos \theta_f^*$: a helicity angle of a Z’s daughter.
- $\Delta \Phi$: an angle between two production plane.

\[
\mathcal{L}_{ZZH} = M_Z^2 \left( \frac{1}{v} + \frac{a_z}{\Lambda} \right) Z_\mu Z^\mu H + b_Z \frac{\hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu}}{2\Lambda} H + \tilde{b}_Z \frac{\tilde{Z}_{\mu\nu} \tilde{Z}^{\mu\nu}}{2\Lambda} H
\]
Verification of the Lorentz structures

\[ ZH \rightarrow l^+l^-H , \quad \sqrt{s} = 250\text{GeV} \]

- “a\text{z}” : affect (rescales the SM-coupling)
- “b\text{z}” : affect (changes angular)
- “\tilde{b}\text{z}” : affect (change)

\[ \frac{1}{\sigma} \frac{d\sigma}{d\cos \theta_Z} \times 10^{-3} \]

Production angle in the Lab. frame

\[ \frac{1}{\sigma} \frac{d\sigma}{d\Delta \Phi_{(Z)\overline{ff}}} \times 10^{5} \]

In the H rest-frame

\[ \mathcal{L}_{ZZH} = M_Z^2 \left( \frac{1}{v} + \frac{a_Z}{\Lambda} \right) Z_\mu Z^\mu H + \frac{b_Z}{2\Lambda} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} H + \frac{\tilde{b}_Z}{2\Lambda} \hat{\tilde{Z}}_{\mu\nu} \hat{\tilde{Z}}^{\mu\nu} H \]

In the Laboratory frame
Verification of the Lorentz structures

\[ ZH \rightarrow t^+tH, \quad \sqrt{s} = 250\text{GeV} \]

\[ \sqrt{s} = 500\text{GeV} \]

- “\( a_z \)”: affect (rescaled)
- “\( b_z \)”: affect channel
- “\( \bar{a}_z \)”: angular

<table>
<thead>
<tr>
<th>( \cos\theta_z )</th>
<th>( 1/\sigma \frac{d\sigma}{d\cos\theta_Z} )</th>
<th>Impact of ( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cos\theta_z )</td>
<td>( \times 10^{-3} )</td>
<td>( 1/\sigma \frac{d\sigma}{d\cos\theta_Z} )</td>
</tr>
<tr>
<td>( \cos\theta_z )</td>
<td>( \times 10^{-3} )</td>
<td>( 1/\sigma \frac{d\sigma}{d\cos\theta_Z} )</td>
</tr>
<tr>
<td>( \Delta\Phi_{\text{plane}}^{(Z)ff} )</td>
<td>( 1/\sigma \frac{d\sigma}{d\Delta\Phi_{(Z)ff}} )</td>
<td>Impact of ( b )</td>
</tr>
<tr>
<td>( \Delta\Phi_{\text{plane}}^{(Z)ff} )</td>
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<td>( 1/\sigma \frac{d\sigma}{d\Delta\Phi_{(Z)ff}} )</td>
</tr>
</tbody>
</table>

\( e^+e^\rightarrow\mu\mu H @ 250\text{GeV} \)
\( \text{Pol}(e^+=, e^-=R) \)
Production angle in the Lab. frame

\( e^+e^\rightarrow\mu\mu H @ 500\text{GeV} \)
\( \text{Pol}(e^+=, e^-=R) \)
Production angle in the Lab. frame

\( e^+e^\rightarrow\mu\mu H @ 250\text{GeV} \)
\( \text{Pol}(e^+=, e^-=R) \)
Production angle in the Lab. frame

\( e^+e^\rightarrow\mu\mu H @ 500\text{GeV} \)
\( \text{Pol}(e^+=, e^-=R) \)
Production angle in the Lab. frame
Analysis Strategy on ZZH

**Available processes**

**Higgsstrahlung @250, 500GeV**

σ_{zh} is close to maximum @250GeV
main obs. ΔΦ, cosθz, cosθ_{f*}

**ZZ-fusion @250, 500GeV**

@500GeV σ_{eeh} is similar to lepton channels of the Higgsstrahlung.
main obs. ΔΦ, cosθh, P_{h}

→ All analysis is done based on the full simulation of the ILD
Analysis Strategy : $\chi^2$ definition

Our approach to evaluate the sensitivity to the anomalous couplings based on a combined chi2.

- Kinematical/Shape information

```
$\chi^2_s = \sum_{i=1}^{n} \left[ \frac{N_{SM}}{\sigma} \frac{d\sigma}{dX}(x_i) \cdot f_i - \frac{N_{SM}}{\sigma} \frac{d\sigma}{dX}(x_i; a_Z, b_Z, \tilde{b}_Z) \cdot f_i \right]^2 \delta N_{SM}(x_i)
```

“Generator level” distribution
Calculated $d\sigma/dX$ with explicit parameters.

Detector response function
→ Transfer to “Detector level” distribution

Poisson error on each bin
(SM Bkgs are taken into account)

- Normalization information

```
\chi^2_c = \left[ \frac{N_{SM} \cdot \epsilon - N_{BSM} \cdot \epsilon}{\delta\sigma \cdot N_{SM} \cdot \epsilon} \right]^2
```

Expected #events
with different models

Erros on $\sigma$

$\delta\sigma(ZH) = 2.0 \%$ and $3.0 \%$
for 250 and 500 GeV

$\delta\sigma(ZZf) = 28.0 \%$ and $5.0 \%$
for 250 and 500 GeV

full simulation, T. Barklow et al.,
Analysis Strategy: Migration effect

Distributions are subject to migration effects due to:
- finite detector resolution
- jet clustering,
- missing particles
- ...

→ detector response \( f \)

\[
N_{\text{Rec}}(x_j^{\text{Rec}}) = \sum_{i \text{ detector response}} f(x_j^{\text{Rec}}, x_i^{\text{Gen}}) \cdot N_{\text{Gen}}(x_i^{\text{Gen}})
\]

\[
N_{\text{Rec}}(x_j^{\text{Rec}}) = \sum_i f_{ji} \cdot N_i^{\text{Gen}} = \sum_i f_{ji} \cdot \eta_i \cdot N_i^{\text{Gen}}
\]

\[
\eta_i \equiv \frac{N_i^{\text{Accept}}}{N_i^{\text{Gene}}} \quad \text{(Event Acceptance)}
\]

\[
f_{ji} \equiv \frac{N_{ji}^{\text{Accept}}}{N_i^{\text{Accept}}} \quad \text{(Migration Matrix)}
\]

For a binned in N distribution, an NxN migration matrix is necessary to transfer the “generator” level to the “detector” level.
Sensitivity to ZZH couplings \( @ 250 \) GeV

\[ \Delta \chi^2 \text{ distribution in the two-parameter space. : } a_z \text{ vs } b_z \]

\textit{Higgsstrahlung} and \textit{ZZ-fusion} are combined.

\[ \sqrt{s}=250 \text{GeV and } \int \! dt = 250 \text{fb}^{-1} \text{ are assumed.} \]

\begin{itemize}
  \item \textbf{Shape information}
  \item \textbf{Normalization information}
  \item \textbf{Both information}
\end{itemize}

There is no shape info. along the parameter “\( a_z \)”,
the sensitivity is coming from normalization info.

Correlation between the parameters “\( a_z \)” and “\( b_z \)” is strong
and coming from normalization change.

The shape can improve partially.
Sensitivity to ZZH couplings @ 250 GeV

$\Delta \chi^2$ distribution in the two-parameter space: $b_z$ vs $\tilde{b}_z$

Higgsstrahlung and ZZ-fusion are combined.

$\sqrt{s}=250\text{GeV}$ and $\int L dt=250\text{fb}^{-1}$ are assumed.

With only normalization info, the sensitivity to “$\tilde{b}_z$” is limited.

The Shape info. is very useful to squeeze the sensitive parameter space.
Sensitivity to ZZH couplings \( 250 \text{ GeV} \) vs \( 500 \text{ GeV} \)

Simultaneous fitting is performed
in three-parameter space.

\[
\sqrt{s}=250\text{GeV and } \int L dt=250\text{fb}^{-1}
\]

The shape distributions quickly change at 500GeV
the correlation between “\(a_z\)” and “\(b_z\)” can be disentangled.
Sensitivity to ZZH couplings \(250 \text{ GeV} + 500 \text{ GeV}\)

A realistic ILC full operation is assumed\(^{1}\)

H20 scenario:
Total luminosities of 2000 fb\(^{-1}\) and 4000 fb\(^{-1}\) are planned to be accumulated at \(\sqrt{s} = 250\) and 500 GeV, respectively.

New physics scale \(\Lambda\) is assumed to be 1 TeV.

**A table showing sensitivity to ZZH at 250 + 500 GeV.**

<table>
<thead>
<tr>
<th>(\sigma) bounds</th>
<th>(a_Z)</th>
<th>(b_Z)</th>
<th>(\tilde{b}_Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(ZH) with shape</td>
<td>total</td>
<td>-</td>
<td>±0.0080</td>
</tr>
<tr>
<td>(ZH) with shape+(\sigma) total</td>
<td>±0.0307</td>
<td>±0.0074</td>
<td>±0.0070</td>
</tr>
<tr>
<td>(ZH+ZZ)-fusion with shape</td>
<td>total</td>
<td>-</td>
<td>±0.0079</td>
</tr>
<tr>
<td>(ZH+ZZ)-fusion with shape+(\sigma) total</td>
<td>±0.0218</td>
<td>±0.0058</td>
<td>±0.0067</td>
</tr>
</tbody>
</table>

For the parameter “a” (SM-like couplings) precision is \(a_z\) few \%. For new tensor structures precision of less than 1\% or better is possible to achieve.

Precision on \(\tilde{b}_z\) is decided by angular info.

---

Sensitivity to ZZH and $\gamma$ZH couplings

Once ZZH couplings are assumed, $\gamma$ZH couplings should be considered because of the electroweak mixing.

$$B_\mu = \cos \theta_W A_\mu - \sin \theta_W Z_\mu,$$

$$W_\mu^3 = \cos \theta_W Z_\mu + \sin \theta_W A_\mu.$$

$B$ couples to both $e_L$ and $e_R$ in the same way. $W^3$ couples to $e_L$ only.

By employing beam polarization (Left- and Right-state) it is possible to disentangle the ZZH and the $\gamma$ZH couplings.
Sensitivity to ZZH and $\gamma$ZH couplings

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Modification of the Lagrangian

$$\mathcal{L}_{ZZH} = M_Z^2 \left( \frac{1}{v} + \frac{a_Z}{\Lambda} \right) Z_\mu Z^\mu H + \frac{b_Z}{2\Lambda} \hat{Z}_{\mu \nu} \hat{Z}^{\mu \nu} H + \frac{\tilde{b}_Z}{2\Lambda} \tilde{Z}_{\mu \nu} \tilde{Z}^{\mu \nu} H,$$

$$\zeta_{ZZ} = \frac{v}{\Lambda} b_Z,$$

$$\tilde{\zeta}_{ZZ} = \frac{v}{\Lambda} \tilde{b}_Z.$$ 

$$\mathcal{L}_{VVH} = M_Z^2 \left( \frac{1}{v} + \frac{a_Z}{\Lambda} \right) Z_\mu Z^\mu H + \frac{1}{2v} \left( \zeta_{ZZ} \hat{Z}_{\mu \nu} \hat{Z}^{\mu \nu} + \zeta_{AZ} \hat{A}_{\mu \nu} \hat{Z}^{\mu \nu} \right) H + \frac{1}{2v} \left( \tilde{\zeta}_{ZZ} \tilde{Z}_{\mu \nu} \tilde{Z}^{\mu \nu} + \tilde{\zeta}_{AZ} \tilde{A}_{\mu \nu} \tilde{Z}^{\mu \nu} \right) H.$$
Sensitivity to ZZH and $\gamma$ZH couplings and general EFT coefficients

Through the sensitivity of ZZH and $\gamma$ZH couplings, sensitivity to general EFT coefficients describing the effective Lagrangian is given.

\[
\begin{align*}
  a_Z &= 1 - C_T - \frac{1}{2} C_H - C_H' \\
  \zeta_{ZZ} &= 8(c_0^2 C_{WW} + 2s_0^2 C_{WB} + \frac{s_0^4}{c_0^2} C_{BB}) \\
  \zeta_{AZ} &= 8s_0^2(C_{WW} - 2C_{WB} + C_{BB}) \\
  + &\text{ CP violating terms}
\end{align*}
\]

Under the assumption of the ILC full operation,

New physics scale $\Lambda$ is assumed to be 1 TeV.

<table>
<thead>
<tr>
<th>$1\sigma$ bounds</th>
<th>$a_Z$</th>
<th>$\zeta_{ZZ}$</th>
<th>$\zeta_{AZ}$</th>
<th>$\tilde{\zeta}_{ZZ}$</th>
<th>$\tilde{\zeta}_{AZ}$</th>
<th>$C_H$</th>
<th>$C_{WW}$</th>
<th>$\tilde{C}_{WW}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z\bar{H} + ZZ$-fusion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>with shape+$\sigma$</td>
<td>$\pm 0.022$</td>
<td>$\pm 0.0057$</td>
<td>$\pm 0.0028$</td>
<td>$\pm 0.00825$</td>
<td>$\pm 0.00049$</td>
<td></td>
<td></td>
<td>$\pm 0.0112$</td>
</tr>
</tbody>
</table>

The anomalous ZZH and $\gamma$ZH couplings can be measured to 1% or better.
Summary

The Higgs boson is the key particle to new physics. The new physics could be imprinted in the Lorentz structure of the VVH coupling.

Based on full simulation, the sensitivity to the anomalous ZZH couplings is evaluated, where backgrounds and detector response are taken into account.

Shape information is important to verify the new tensor structures and higher $\sqrt{s}$ is useful to disentangle the correlation of the parameters.

Beam polarization has power to disentangle the ZZH and $\gamma$ZH couplings and makes it possible to evaluate sensitivity to the $\gamma$ZH couplings.

The different Lorentz structures originating from the ZZH and $\gamma$ZH couplings can be measured to 1% or better with the ILC full operation.
A preprint of our ZZH study has been prepared …

**Study of sensitivity to anomalous $ZZH$ couplings at the International Linear Collider**

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(Dated: July 3, 2017)

In this report, we present prospective sensitivity to the anomalous couplings between the Higgs boson and the $Z$ boson at the future International Linear Collider (ILC) experiment. The analysis is performed by employing a framework of the Effective Field Theory (EFT) where general Lorentz tensor structures of the $ZZH$ couplings including both CP-even and -odd contributions of the Higgs boson is assumed with dimension-5 operators and a new physics scale $\Lambda$. The evaluation of the sensitivity is carried out based on full detector simulation in which background contributions are taken into account. Kinematical distributions of leading channels of main Higgs production processes $e^+e^- \rightarrow ZH \rightarrow f\bar{f}H$ and $e^+e^- \rightarrow ZZ \rightarrow e^+e^-H$ and information of cross sections are used for finding out deviations from the SM. Results are given with assumption of benchmark integrated luminosities and certain realistic running scenario of the ILC experiment for both center-of-mass energies $\sqrt{s}=250$ and $500$ GeV with two different beam polarization states. Sensitivity to the anomalous $\gamma ZH$ couplings are also given based on the framework of EFT by utilizing two beam polarizations. A discussion on sensitivity to general parameters describing the new Lorentz tensor structures related to the Higgs and the vector bosons is given at the end.
A complete set of an effective Lagrangian with dim-6 operators

Tim Barklow et al.
“Model-Independent Determination of the Triple Higgs Coupling at e+e- Colliders”
https://agenda.linearcollider.org/event/7371/contributions/37884/

\[
\Delta L = \frac{c_H}{2v^2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \overset{\leftrightarrow}{D} \mu \Phi)(\Phi^\dagger \overset{\leftrightarrow}{D}_\mu \Phi) - \frac{c_6}{v^2} (\Phi^\dagger \Phi)^3 \]

\[
+ \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger \Phi W^\mu W^\mu \nu + \frac{4gg' c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W^\mu_{\mu \nu} B^{\mu \nu} \]

\[
+ \frac{g^2 c_{BB}}{m_W^2} \Phi^\dagger \Phi B^\mu B^{\mu \nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W^\mu_{\mu \nu} W^{b \nu} W^{c \rho} \]

\[
+i \frac{c_{HL}}{v^2} (\Phi^\dagger \overset{\leftrightarrow}{D} \mu \Phi)(\overline{L} \gamma_\mu L) + +4i \frac{c_{HL}'}{v^2} (\Phi^\dagger t^a \overset{\leftrightarrow}{D} \mu \Phi)(\overline{L} \gamma_\mu t^a L) \]

\[
+i \frac{c_{HE}}{v^2} (\Phi^\dagger \overset{\leftrightarrow}{D} \mu \Phi)(\overline{\epsilon} \gamma_\mu \epsilon) .
\]

After electro weak symmetry breaking, Lorentz tensor structures between the Higgs and gauge bosons,

\[
\Delta L_h = -\eta_h \lambda_0 v_0 h^3 + \frac{\theta_h}{v_0} h \partial_\mu h \partial^\mu h + \eta_Z \frac{m_Z^2}{v_0} Z_\mu Z^\mu h + \frac{1}{2} \eta_{2Z} \frac{m_Z^2}{v_0^2} Z_\mu Z^\mu h^2 \]

\[
+ \eta_W \frac{2m_W^2}{v_0} W^+_\mu W^{-\mu} h + \eta_{2W} \frac{m_W^2}{v_0^2} W^+_\mu W^{-\mu} h^2 \]

\[
+ \frac{1}{2} \left( \zeta_Z \frac{h}{v_0} + \frac{1}{2} \zeta_{2Z} \frac{h^2}{v_0^2} \right) \hat{Z}_{\mu \nu} \hat{Z}^{\mu \nu} + \left( \zeta_W \frac{h}{v_0} + \frac{1}{2} \zeta_{2W} \frac{h^2}{v_0^2} \right) \hat{W}_{\mu \nu} \hat{W}^{-\mu \nu} \]

\[
+ \frac{1}{2} \left( \zeta_A \frac{h}{v_0} + \frac{1}{2} \zeta_{2A} \frac{h^2}{v_0^2} \right) \hat{A}_{\mu \nu} \hat{A}^{\mu \nu} + \left( \zeta_{AZ} \frac{h}{v_0} + \zeta_{2AZ} \frac{h^2}{v_0^2} \right) \hat{A}_{\mu \nu} \hat{Z}^{\mu \nu} .
\]
**Sensitivities to the a, b and bt with only the Higgsstrahlung**

**Nominal energies and luminosities**

$\sqrt{s}=250\text{GeV and } \int \mathcal{L} dt = 250\text{fb}^{-1}$

<table>
<thead>
<tr>
<th>$ZH$</th>
<th>$a_Z$</th>
<th>$b_Z$</th>
<th>$\tilde{b}_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_L^{-}e_R^{+}$</td>
<td>$e_L^{-}e_R^{+}$</td>
<td>$e_L^{-}e_R^{+}$</td>
<td>$e_L^{-}e_R^{+}$</td>
</tr>
<tr>
<td>with shape</td>
<td>-</td>
<td>$\pm 0.110$</td>
<td>$\pm 0.051$</td>
</tr>
<tr>
<td>$Z H$</td>
<td></td>
<td>$\pm 0.129$</td>
<td>$\pm 0.061$</td>
</tr>
<tr>
<td>$Z H$</td>
<td>$\pm 0.309$</td>
<td>$\pm 0.109$</td>
<td>$\pm 0.051$</td>
</tr>
<tr>
<td>with shape+σ</td>
<td>$\pm 0.356$</td>
<td>$\pm 0.125$</td>
<td>$\pm 0.061$</td>
</tr>
</tbody>
</table>

$\sqrt{s}=500\text{GeV and } \int \mathcal{L} dt = 500\text{fb}^{-1}$

<table>
<thead>
<tr>
<th>$ZH$</th>
<th>$a_Z$</th>
<th>$b_Z$</th>
<th>$\tilde{b}_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_L^{-}e_R^{+}$</td>
<td>$e_L^{-}e_R^{+}$</td>
<td>$e_L^{-}e_R^{+}$</td>
<td>$e_L^{-}e_R^{+}$</td>
</tr>
<tr>
<td>with shape</td>
<td>-</td>
<td>$\pm 0.0199$</td>
<td>$\pm 0.0183$</td>
</tr>
<tr>
<td>$Z H$</td>
<td></td>
<td>$\pm 0.0215$</td>
<td>$\pm 0.0198$</td>
</tr>
<tr>
<td>$Z H$</td>
<td>$\pm 0.116$</td>
<td>$\pm 0.0201$</td>
<td>$\pm 0.0183$</td>
</tr>
<tr>
<td>with shape+σ</td>
<td>$\pm 0.130$</td>
<td>$\pm 0.0217$</td>
<td>$\pm 0.0198$</td>
</tr>
</tbody>
</table>

**correlation matrix (w/ shape+σ P_{LR})**

\[
\rho = \begin{pmatrix}
1 & -0.9917 & 0.0064 \\
-0.9917 & 1 & -0.0051 \\
0.0064 & -0.0051 & 1
\end{pmatrix}
\]

\[
\rho = \begin{pmatrix}
1 & -0.848 & 0.0136 \\
-0.848 & 1 & -0.0124 \\
0.0136 & -0.0124 & 1
\end{pmatrix}
\]
Angular Asymmetry derived from the new structures

The Lorentz structure

\[ \mathcal{L}_{ZH} = M_Z^2 \left( \frac{1}{v} + \frac{a_Z}{\Lambda} \right) Z_\mu Z^\mu H \]

\[ + \frac{b_Z}{2\Lambda} \tilde{Z}_{\mu\nu} \tilde{Z}^{\mu\nu} H + \frac{\tilde{b}_Z}{2\Lambda} \tilde{Z}_{\mu\nu} \tilde{Z}^{\mu\nu} H \]

\[ \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \propto B_{p1} \cdot B_{p2} - E_{p1} \cdot E_{p2} \]

\[ \hat{F}_{\mu\nu} \tilde{F}^{\mu\nu} \propto E_{p1} \cdot B_{p2} \]

\textit{in the Higgs rest frame}

- “a_Z”: a normalization parameter affecting the overall cross section. (rescales the SM-coupling)
- “b_Z”: a different CP-even tensor structure affecting momentum and changes angular distribution.
- “\tilde{b}_Z”: a CP-violating parameter affecting angular/spin correlations.

The new tensor structures can be resulted in a simple field strength of the EM field.

\[ \Delta \Phi \] which is defined by two different planes can be the useful observable.

\[ \Delta \Phi \] tends to be parallel / perpendicular.

4 processes for different \( \sqrt{s} \) are available.

\[ ZH \rightarrow ee/\mu\mu H \text{ and } qqH(H\rightarrow bb) \]

\[ ZZf \rightarrow eeH(H\rightarrow bb) \]
Angular Asymmetry : 250GeV

The Lorentz structure

\[ \mathcal{L}_{ZZH} = M_Z^2 \left( \frac{1}{v} + \frac{a_Z}{\Lambda} \right) Z_\mu Z^\mu H + \frac{b_Z}{2\Lambda} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} H + \frac{\hat{b}_Z}{2\Lambda} \tilde{Z}_{\mu\nu} \tilde{Z}^{\mu\nu} H \]

In the Laboratory frame

![Diagram showing angular asymmetry and laboratory frame](image)
Angular Asymmetry : 500GeV

The Lorentz structure

\[ \mathcal{L}_{ZZH} = \frac{M_Z^2}{2} \left( \frac{1}{v} + \frac{a_Z}{\Lambda} \right) Z_\mu Z_\mu H + \frac{b_Z}{2\Lambda} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} H + \frac{\tilde{b}_Z}{2\Lambda} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} H \]

In the Laboratory frame

![Diagram showing the angular asymmetry and the Lorentz structure](image)
Kinematical distribution with the ZH : 2-dimensional @ 500GeV

\[ \text{ee} \rightarrow \text{Zh} \rightarrow \ell \ell h \]

\[ \Delta \Phi \text{ vs } \cos \theta_z \]

\[ \Phi [0 \sim 2\pi] \]
$L_{ZZH} = M_Z^2 \left( \frac{1}{v} + \frac{a_Z}{\Lambda} \right) Z_\mu Z^\mu H + \frac{b_Z}{2\Lambda} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} H + \bar{\theta} Z^{\mu\nu} \tilde{Z}^{\mu\nu} H$
Kinematical distribution with the ZZ-fusion: 500\text{GeV}

\[ \mathcal{L}_{ZH} = M_Z^2 \left( \frac{1}{v} + \frac{a_Z}{\Lambda} \right) Z_{\mu} Z^\mu H \\
+ \frac{b_Z}{2\Lambda} \hat{Z}_{\mu\nu} \hat{Z}^{\mu\nu} H + \frac{\hat{b}_Z}{2\Lambda} \hat{Z}_{\mu\nu} \tilde{Z}^{\mu\nu} H \]

In the Higgs rest frame
Analysis Strategy: $\chi^2$ test

Our approach to evaluate the sensitivity to the anomalous couplings is a binned analysis.

- Kinematical/Shape information (for an 1-dimensional distribution)

$$
\chi^2_s = \sum_{i=1}^{n} \left[ \frac{N_{SM}}{\sigma} \frac{d\sigma}{dX}(x_i) \cdot f_i - \frac{N_{BSM}}{\sigma} \frac{d\sigma}{dX}(x_i; a_Z, b_Z, \tilde{b}_Z) \cdot f_i \right]^2
$$

$i$ is Nth-bin

Computed differential cross section with SM parameters and BSM parameters → “Generator level” distribution

Normalized to #event expected with the SM for extracting impact of the shape.

Detector acceptance function evaluated with full detector simulation
(event acceptance and detector migration is considered)
→ Transfer to “Detector level” distribution

Poisson error on each bin (full simulation, SM Bkgs are taken into account)

- Cross section information

$$
\chi^2_c = \left[ \frac{N_{SM} \cdot \epsilon - N_{BSM} \cdot \epsilon}{\delta \sigma \cdot N_{SM} \cdot \epsilon} \right]^2
$$

Deviation of cross section of corresponding processes is also important information.

$\delta\sigma(ZH) = 2\%$ and $3\%$ for $250$ and $500\text{GeV}$

$\delta\sigma(ZZf) = 28\%$ and $5\%$ for $250$ and $500\text{GeV}$

**A reconstructed distribution receives a migration effect due to**

Detector finite resolution
Jet clustering, missing particles etc...

\[
N_{\text{Rec}}(x_j^{\text{Rec}}) = \sum_i f(x_j^{\text{Rec}}, x_i^{\text{Gen}}) \cdot N_{\text{Gen}}(x_i^{\text{Gen}})
\]

\[
N_{\text{Rec}}(x_j^{\text{Rec}}) = \sum_i f_{ji} \cdot N_i^{\text{Gen}} = \sum_i \bar{f}_{ji} \cdot \eta_i \cdot N_i^{\text{Gen}}
\]

\[
\eta_i \equiv \frac{N_i^{\text{Accept}}}{N_i^{\text{Gene}}} \quad \text{(Event Acceptance)}
\]

\[
\bar{f}_{ji} \equiv \frac{N_{ji}^{\text{Accept}}}{N_{ji}^{\text{Accept}}} \quad \text{(Migration Matrix)}
\]

\[
N_{\text{Rec}}(x_4^{\text{Rec}}) = \sum_i \bar{f}_{4i} \cdot \eta_i \cdot N_i^{\text{Gene}}
\]

\[
= \eta_1 \bar{f}_{41} \cdot N_1^{\text{Gene}} + \eta_2 \bar{f}_{42} \cdot N_2^{\text{Gene}} + \eta_3 \bar{f}_{43} \cdot N_3^{\text{Gene}}
\]

\[+ \eta_4 \bar{f}_{44} \cdot N_4^{\text{Gene}} + \eta_5 \bar{f}_{45} \cdot N_5^{\text{Gene}}
\]

In order to transfer the generator distribution (binned in \( N \)) to the detector level distribution, a \( N \times N \) matrix is needed.
**Examples: Reconstructed angular distribution & Migration matrix**

ZH → \(\mu\mu H\) @ 250GeV

ZH → qqH(H→bb) @ 250GeV

Reconstructed distribution of \(\Delta \Phi\) vs \(\cos \theta_z\) binned in 10x10

Migration matrices on several bins

Matrix element [0~1]

Lepton channel is very clean signature.
Hadron channel has relatively large migration.
2d detector acceptance

\[ N^{Reco}(x_{j\beta}) = \sum_i \sum_{\alpha} f(x_{j\beta}^{Reco}, x_{i\alpha}^{Gene}) \cdot N^{Gene}(x_{i\alpha}^{Gene}) = \sum_i \sum_{\alpha} f_{j\beta i\alpha} \cdot N_{i\alpha} \]

\[ N^{Reco}(x^{Reco}) = \sum_i \sum_{\alpha} \tilde{f}_{j\beta i\alpha} \cdot \eta_{i\alpha} \cdot N_{i\alpha} \]

\[ \eta_{i\alpha} = \frac{N_{Accept}}{N_{i\alpha}} \quad \text{(Event Acceptance)} \]

\[ \tilde{f}_{j\beta i\alpha} = \frac{N_{Accept}}{N_{i\alpha}} \quad \text{(Migration Matrix)} \]

**FIG. 7.** Distributions showing the migration matrix on several bins when the two-dimensional distribution \(X(\cos \theta_Z, \Delta \Phi_{ff})\) binned in 10×10 is used. Compared with the channel \(q\bar{q}b\bar{b}(H)\) showing in the next section, the migration is almost nothing because of the clear signature of the signal process. (\(\tilde{f}_{j\beta i\alpha}\); the index \(j\beta\) denotes certain reconstructed bin in the two-dimensional distribution, and the index \(i\alpha\) corresponds to the other axes defining MC truth directions which are shown in the distributions. The range of the \(Z\) axis is [0, 1] and the normalization of each distribution is \(\sum_{j, \beta} \tilde{f}_{j\beta i\alpha} = 1\).)

**FIG. 8.** Distributions showing the migration matrix on several bins in which the two-dimensional distribution \(X(\cos \theta_Z, \Delta \Phi_{ff})\) binned in 10×10 is used. Compared with the channel \(\mu\mu H\), clear the bin-by-bin migration can be seen, which is originating from the multiple jet environment.
Sensitivity to ZZH couplings

Contours showing sensitivities with three parameter space. 250fb\(^{-1}\) and 500fb\(^{-1}\) are assumed as the integrated luminosity for 250 and 500GeV.

<table>
<thead>
<tr>
<th>Couplings</th>
<th>(a_Z)</th>
<th>(b_Z)</th>
<th>(\bar{b}_Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z H)</td>
<td>(e^{-}e^{+})</td>
<td>(e^{-}e^{+})</td>
<td>(e^{-}e^{+})</td>
</tr>
<tr>
<td>with shape</td>
<td>-</td>
<td>(\pm 0.110)</td>
<td>(\pm 0.051)</td>
</tr>
<tr>
<td>(Z H)</td>
<td>(e^{-}e^{+})</td>
<td>(e^{-}e^{+})</td>
<td>(e^{-}e^{+})</td>
</tr>
<tr>
<td>with shape+(\sigma)</td>
<td>(e^{-}e^{+})</td>
<td>(\pm 0.309)</td>
<td>(\pm 0.109)</td>
</tr>
<tr>
<td>(Z H+ZZ)-fusion</td>
<td>(e^{-}e^{+})</td>
<td>(e^{-}e^{+})</td>
<td>(e^{-}e^{+})</td>
</tr>
<tr>
<td>with shape</td>
<td>(e^{-}e^{+})</td>
<td>(\pm 0.110)</td>
<td>(\pm 0.051)</td>
</tr>
<tr>
<td>(Z H+ZZ)-fusion</td>
<td>(e^{-}e^{+})</td>
<td>(e^{-}e^{+})</td>
<td>(e^{-}e^{+})</td>
</tr>
<tr>
<td>with shape+(\sigma)</td>
<td>(e^{-}e^{+})</td>
<td>(\pm 0.238)</td>
<td>(\pm 0.084)</td>
</tr>
</tbody>
</table>

bt can be evaluated through only shape information @ 250 and 500GeV

Correlation a and b is strong because \(\sigma\) info. is much stronger than that of the shape

@ 500GeV the shape quickly changes the correlation can be disentangled.
Power of each process for the anomalous couplings

ZH : leptonic(e/µ) / hadronic (q)

ZZ : H → bb

![Graph showing sensitivity to anomalous ZZH couplings](image)

**FIG. 25.** A plot shows the sensitivity to the anomalous ZZH couplings. Fitting is performed with simultaneous fitting in three free parameter space, and each contour showing impact of each channel are projected into the $a_Z$-$b_Z$ parameter space. The integrated luminosity is assumed to be 250 $fb^{-1}$ with left-handed polarization $e_L e_R^+$. 

![Graph showing sensitivity to anomalous ZZH couplings](image)

**FIG. 26.** A plot shows the sensitivity to the anomalous ZZH couplings. Fitting is performed with simultaneous fitting in three free parameter space, and each contour showing impact of each channel are projected into the $a_Z$-$b_Z$ parameter space. The integrated luminosity is assumed to be 500 $fb^{-1}$ with left-handed polarization $e_L e_R^+$. 

TABLE XI. 

<table>
<thead>
<tr>
<th>Expected number of remaining signal and background events after applying each cut on the combined $Z$ and $W$ spectrum at $500$ GeV with the beam polarization $e^+ e^-$.</th>
<th>$e_L e_R^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+ e^- ZH$</td>
<td>$e^+ e^- ZZ$</td>
</tr>
<tr>
<td>$e^+ e^- ZH$</td>
<td>$e^+ e^- ZZ$</td>
</tr>
</tbody>
</table>

$\bar{b}$ and signal significance

$\bar{b}$
anomalous WWH couplings

The Lorentz tensor structures described by the anomalous parameters $b$ and $b_t$ affect angular and momentum distributions of the $W$ boson and helicity angle of a daughter particle.

**In the Higgs rest frame**

- The direction of motion of the $f$ in the $W$ rest frame.
- The momentum of the $W$.
- The angle between the $W$ in the Higgs rest frame and the $f$ in the $W$ rest frame.
- The direction of motion of the $W$ in the Higgs rest frame.
- The difference of Higgs decay plane.
anomalous WWH couplings

The Lorentz tensor structures described by the anomalous parameters $b$ and $b_t$ affect angular and momentum distributions of the W boson and helicity angle of a daughter particle.

The most effective observable to catch $b_t$ is $\Delta \Phi$.

$ZH \rightarrow vvH (H \rightarrow WW \rightarrow qqqq)$
The final state is multi-jet state.

An original idea to keep sensitivity of $\Delta \Phi$ is to use Jets deriving from Cs in the W bosons.

two c-jets: sensitivity become half
No id: sensitivity become quarter.