

The turnaround radius as a probe of dark energy and modified gravity

Valerio Faraoni¹

¹Bishop's University, Sherbrooke, Canada

work with A. Prain, M. Lapierre-Léonard, V. Vitagliano *JCAP* 10, 013 (2015); *CQG* 33, 145008 (2016); *Phys. Dark Univ.* 11, 11 (2016)

Outline

- 1 Turnaround radius with Hawking mass in GR
- 2 Turnaround radius in scalar-tensor gravity
- 3 Conclusions

Turnaround radius with Hawking mass

Part of a larger program aiming at applying quasilocal mass in cosmology, already used to test Newtonian N -Body simulations of large scale structures and lensing by Λ (VF, Prain & Lapierre-Léonard PRD 2015, 2016)

Consider **present accelerated era** of the universe and the largest bound objects in the sky. The turnaround radius was suggested as a possible way to test dark energy (Roupas *et al.* 2014, PRD 89, 083002; Pavlidou & Tomaras 2014, JCAP 09, 020; Pavlidou, Tetradis, Tomaras 2014, JCAP 05, 017) but the concept of TR is older (Souriau 1981; Stuchlik 1983; Stuchlik *et al.* 1989-2005; Mizony & Lachiéze-Rey 2005; Blau & Rollier 2008, ...)

Consider an **accelerated FLRW universe with one spherical inhomogeneity**; massive test particles with zero radial initial velocity cannot collapse if they are outside a critical radius R_c (**turnaround radius**), but only expand.

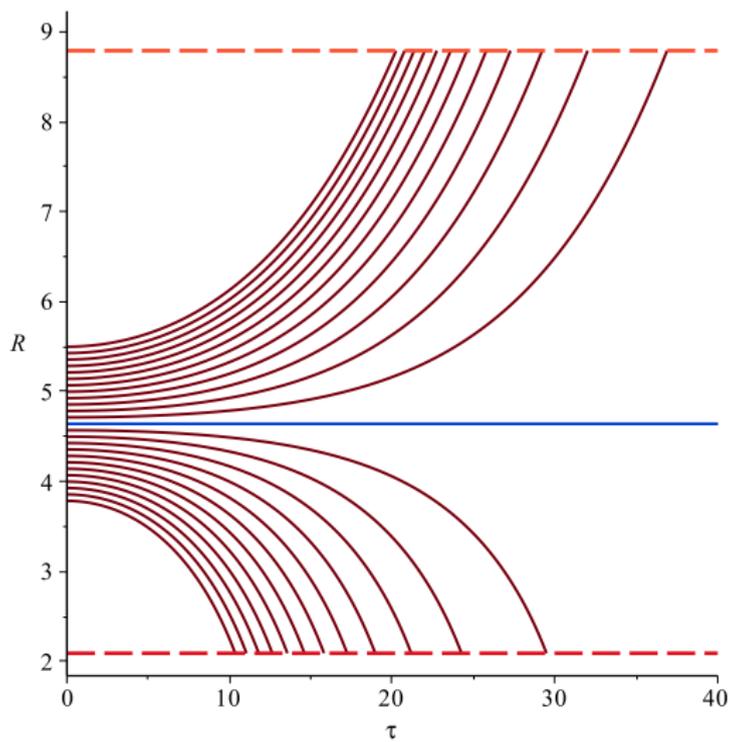
For $R < R_c$, dust shells reach zero radial acceleration and collapse under self-gravity. If you cross outside R_c in geodesic motion, you will never fall back.

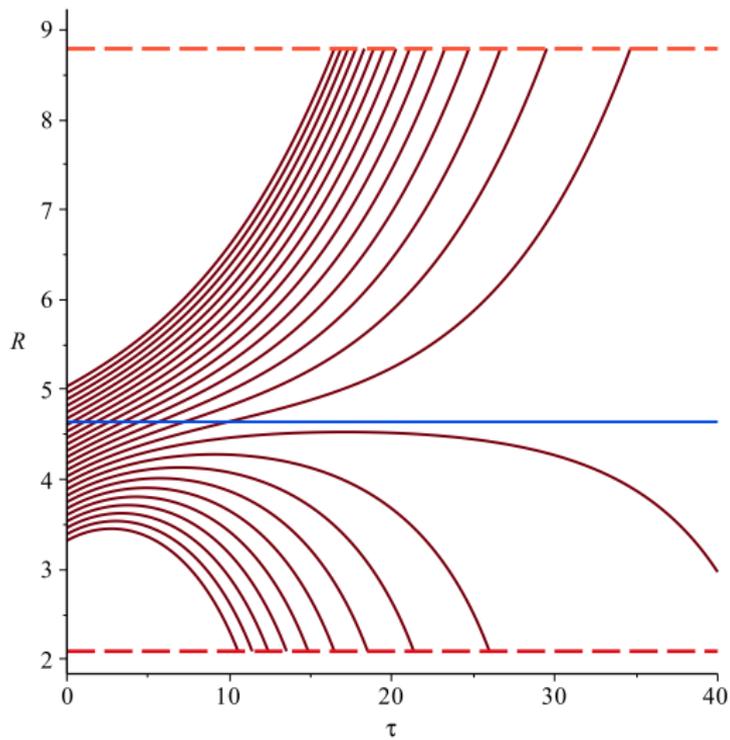
TR studied in Schwarzschild-de Sitter, Lemaître-Tolman-Bondi, and McVittie spacetimes.

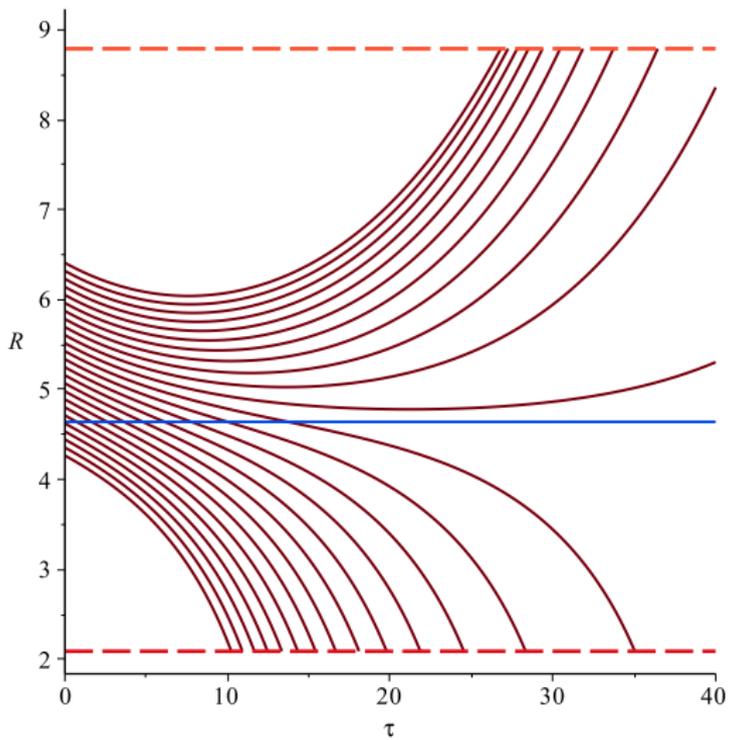
SdS (heuristic):

$$ds^2 = - \left(1 - \frac{2M}{R} - H^2 R^2 \right) dt^2 + \frac{dR^2}{1 - \frac{2M}{R} - H^2 R^2} + R^2 d\Omega_{(2)}^2$$

$$H = \sqrt{\Lambda/3}, \quad R_c = \left(\frac{3GM}{\Lambda} \right)^{1/3}$$







Radial timelike geodesics obey $\ddot{R}(\tau) = (R^3 - R_c^3) H^2 / R^2$
 LTB models (dust) Pavlidou, Tetradis & Tomaras 2014 have

$$ds^2 = -dt^2 + \frac{R'(t, r)}{1 + f(r)} dr^2 + R^2(t, r) d\Omega_{(2)}^2$$

with $' \equiv d/dr$, $f(r)$ related to initial density profile. Radial timelike geodesics obey

$$\ddot{R} = -\frac{GM(r)}{R^2} + \frac{\Lambda R}{3}$$

and the turnaround radius is $R_c = \left(\frac{3GM(r_c)}{\Lambda} \right)^{1/3}$ where
 $\mathcal{M}(r) = \int_0^R dR R^2 \rho$ Lemaître mass.

More realistic: post-FLRW space (1st order)

$$ds^2 = a^2(\eta) \left[- (1 + 2\phi) d\eta^2 + (1 - 2\phi) \left(dr^2 + r^2 d\Omega_{(2)}^2 \right) \right]$$

Pavlidou, Tetradis & Tomaras find timelike radial geodesics obey

$$\ddot{R} = \frac{\ddot{a}}{a} - \frac{GM(r)}{R^2}$$

where it is suggested (but not written down)

$$\mathcal{M}(r) = \int_0^R dR R^2 \rho_{\text{total}}$$

$$\rightarrow R_c = \left(\frac{3\mathcal{M}}{4(3w + 1)\pi\rho_{\text{DE}}} \right)^{1/3}$$

Questions (not answered, nor posed):

- gauge-invariance?
- What is the “mass in a sphere of radius R ”? Should it include ρ_{DE} ? If not, why? Should it include only $\rho_{\text{perturbation}}$? Why?

Use **Hawking-Hayward quasilocal energy** (reduces to **Misner-Sharp-Hernandez mass** in spherical symmetry) and a new splitting of it.

Assumptions:

- GR is valid
- 1st order in metric perturbations; spherical symmetry
 $\phi = \phi(r)$
- spatially flat FLRW background accelerated by dark energy with ρ_{DE} , $P_{\text{DE}} = w\rho_{\text{DE}}$

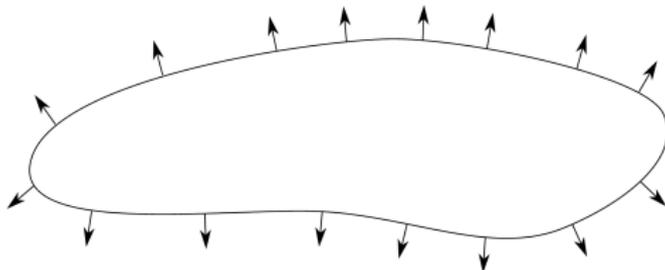
Physical mass is the **Hawking quasilocal energy**¹

Idea: total mass in a region bounded by a surface S is measured by behaviour of null geodesics at S

S = closed spacelike orientable 2-surface

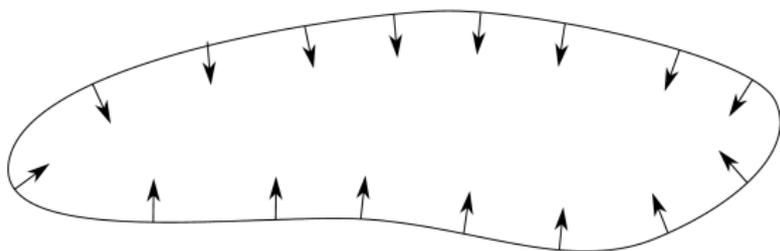
\mathcal{R} = induced Ricci scalar on S

$\theta_{(\pm)}$ = expansions of outgoing/ingoing null geodesic congruences from S



¹S.W. Hawking 1968, *J. Math. Phys.* 9, 568; S.A. Hayward 1994, *Phys. Rev. D* 49, 831

General perturbations of FLRW



$\sigma_{ab}^{(\pm)}$ = shear tensors of null congruences
 ω^a = projection on S of the commutator of null normal vectors to S (anholonomicity)

μ = volume 2-form on S

A = area of S

$$M_{\text{HH}} \equiv \frac{1}{8\pi} \sqrt{\frac{A}{16\pi}} \int_S \mu \left(\mathcal{R} + \theta_{(+)}\theta_{(-)} - \frac{1}{2} \sigma_{ab}^{(+)} \sigma_{(-)}^{ab} - 2\omega_a \omega^a \right)$$

Compute for

$$d\tilde{s}^2 = a^2(\eta) \underbrace{\left[-(1 + 2\phi_N) d\eta^2 + (1 - 2\phi_N) (dr^2 + r^2 d\Omega_{(2)}^2) \right]}_{\text{post-Newtonian}}$$

and attempt to decompose as

$$M_{\text{HH}} = (\text{local}) + (\text{cosmological})$$

to first order

Conformal factor of $g_{ab} \rightarrow \Omega^2 g_{ab}$ is $\Omega = a(\eta)$

Final result (with two methods) is

$$\tilde{M}_H = \underbrace{\Omega M_H - \frac{R\Omega_{,\eta}}{4\pi} \frac{\Omega_{,\eta}}{\Omega} \int_S \mu \phi_N}_{\text{local}} + \underbrace{\frac{R^3}{2} \frac{\Omega_{,\eta}^2}{\Omega}}_{\text{cosmological}}$$

(Prain, Vitagliano, VF & Lapierre-Léonard 2016, *Class. Quantum Grav.* 33, 145008)

Now adapt to spherical symmetry \rightarrow

$$M_H = ma + \frac{H^2 R^3}{2} (1 - \phi) \simeq ma + \frac{H^2 R^3}{2}$$

Criterion for a system on the verge of breaking down is now

local part $ma = \frac{H^2 R^3}{2}$ cosmological part \rightarrow

$$R_c(t) = \left(\frac{2ma}{H^2} \right)^{1/3}$$

Now $H^2 = 8\pi G\rho_{DE}/3 \rightarrow R_c(t) = \left(\frac{3ma}{4\pi\rho_{DE}} \right)^{1/3}$ and, if

$w = \text{const.}, R_c = \left(\frac{3ma}{4\pi\rho_0} \right)^{1/3} a^{(3w+4)/3}$

Compare with Pavlidou, Tetradis & Tomaras

$$\frac{R_c}{R_c^{(PTT)}} = \left(\frac{|3w + 1|}{2} \right)^{1/3} \approx 1 \quad \text{if } w \approx -1$$

but now

- no ambiguities in “mass inside a sphere of radius R_c ”
- rigorous derivation of turnaround radius R_c
- important if you want to constrain w

Can express $R_c = R_c(z)$ and invert to obtain

$$\int dz \frac{w(z) + 1}{z + 1} = \ln \left[\left(\frac{3ma}{4\pi\rho} \right)^{1/3} \frac{1}{R(z)} \right]$$

If $w = \text{const.}$ reduces to

$$w = -1 + \frac{\ln \left[\left(\frac{3ma}{4\pi\rho_0} \right)^{1/3} \frac{1}{R_c(z)} \right]}{\ln(z + 1)}$$

constrain w if ma and R_c are known

Turnaround radius in ST gravity

$$ds^2 = a^2(\eta) \left[- (1 + 2\psi) d\eta^2 + (1 - 2\phi) \left(dr^2 + r^2 d\Omega_{(2)}^2 \right) \right],$$

$\phi = \phi(r)$, $\psi = \psi(r)$. Massive test particles follow timelike geodesics

$$\frac{du^a}{d\tau} + \Gamma_{bc}^a u^b u^c = 0,$$

$u_c u^c = -1$ and the geodesic eq. give

$$\frac{du^0}{d\tau} + \frac{a_\eta}{a} (u^0)^2 + 2\psi' u^0 u^1 + \frac{a_\eta}{a} (1 - 2\phi - 2\psi) (u^1)^2 = 0$$

$$\frac{du^1}{d\tau} + \psi' (u^0)^2 + \frac{2a_\eta}{a} u^0 u^1 - \phi' (u^1)^2 = 0$$

Areal radius is $R(t, r) = ar\sqrt{1 - 2\phi} \simeq ar(1 - \phi)$, further manipulation yields

$$\frac{d^2 R}{dt^2} = \left[\ddot{a}r + \frac{\dot{a}u^1}{au^0} + \frac{1}{au^0} \frac{d}{d\tau} \left(\frac{u^1}{u^0} \right) \right] (1 - \phi)$$

Criterion locating the (unique) turnaround radius is

$$\boxed{\frac{d^2 R}{dt^2} = 0}$$

which becomes

$$\ddot{a}r - \frac{\psi'}{a} = 0$$

In terms of the *areal* turnaround radius,

$$R_c = a(t)r_c [1 - \phi(r_c)]$$

or, using the gravitational slip $\xi \equiv (\phi - \psi) / \phi$,

$$\ddot{a} R_c (1 + \phi_c) - \phi'_c (1 - \xi_c) + \phi_c \xi'_c = 0$$

Conclusions

- Turnaround radius is an opportunity to test gravity and/or the Λ CDM model.
- Split M_H for spherical perturbations of FLRW \rightarrow rigorous derivation of R_C , small correction, much needed clarification of “mass”.
- In modified gravity, no accepted M_H , use criterion $\ddot{R} = 0 \rightarrow$ eq. for R_C in ST gravity.
- Claim by astronomers that the upper bound on R_C in GR is exceeded by far in galaxy group NGC 5353/4 (Lee *et al.* 2015, *Astrophys. J.* 815, 43; Lee 2016, *ApJ* 832, 123). Taken back in Lee & Yepes 2016, *ApJ* 832, 185. Wait and see!

THANK YOU